The MOND-cosmology connection

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Overview

- Introduction to MOND
- Appearances of the MOND constant, a_0, in the data
- The “coincidence”
- Some “practical” implications
- Possible origins

MOND – basic tenets

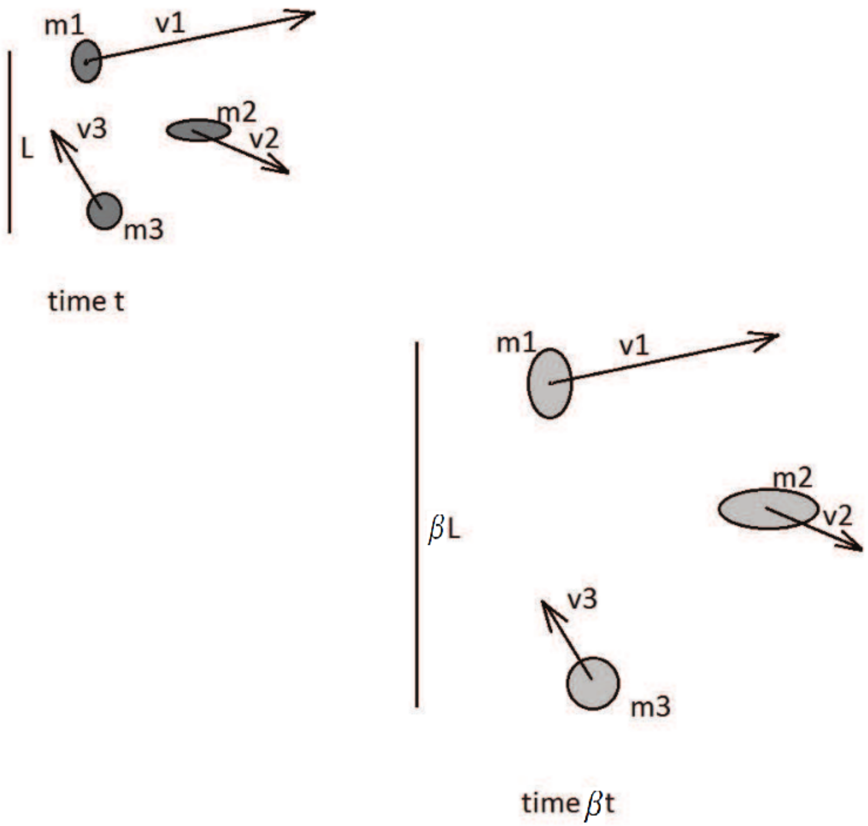
- A theory of dynamics (gravity/inertia) involving a new constant a_0 (beside G, ...)
- Standard limit ($a_0 \to 0$): The Newtonian limit
- Scale invariance: $(t, \mathbf{r}) \to \lambda(t, \mathbf{r})$

  in the deep-MOND limit : $a_0 \to \infty, \quad \mathcal{A}_0 \equiv G a_0 \ fixed \qquad (G \to 0)$:
- Small print: no other new constants, dimensionful or dimensionless
- a_0 is analog to c in relativity or $\hbar$ in QM

The effective definition of MOND at present

Not modified gravity at large distances

Scale invariance



X $\quad m\mathbf{a} = \mathbf{F}, \qquad F = mMG/r^2$

V $\quad ma^2/a_0 = F, \qquad F = mMG/r^2,$

or $\quad ma = F, \qquad F \propto m(MGa_0)^{1/2}/r$

Nonrelativistic theories

Poisson: $\vec{\nabla} \cdot [\vec{\nabla}\phi_N] = 4\pi G\rho$

AQUAL: $\vec{\nabla} \cdot [\mu(\frac{|\vec{\nabla}\phi|}{a_0})\vec{\nabla}\phi] = 4\pi G\rho = \vec{\nabla} \cdot [\vec{\nabla}\phi_N]$

QUMOND: $\vec{\nabla} \cdot [\vec{\nabla}\phi] = \vec{\nabla} \cdot [\nu(\frac{|\vec{\nabla}\phi_N|}{a_0})\vec{\nabla}\phi_N]$

Derivable from actions

Limits of relativistic theories

$$\text{Schematic :} \quad g\mu(g/a_0) = g_N, \qquad or \qquad g = g_N\nu(g_N/a_0)$$

$$g = g_N \qquad \Rightarrow g^2 = g_N a_0$$

"Modified inertia"?

Relativistic MOND theories

- Tensor-Vector-Scalar Gravity (TeVeS–Bekenstein 2004, ideas from Sanders 1997) Gravity is described by $g_{\alpha\beta}$, $\mathcal{U}_\alpha$, ϕ: $\tilde{g}_{\alpha\beta} = e^{-2\phi}(g_{\alpha\beta} + \mathcal{U}_\alpha \mathcal{U}_\beta) - e^{2\phi}\mathcal{U}_\alpha \mathcal{U}_\beta$

- MOND adaptations of Aether theories (Zlosnik, Ferreira, & Starkman 2007, Hossenfelder 2017)

$$\mathcal{L}(A, g) = \frac{a_0^2}{16\pi G}\mathcal{F}(\mathcal{K}) + \lambda(A^\mu A_\mu + 1);$$

$$\mathcal{K} = a_0^{-2} A^\gamma{}_{;\alpha} A^\sigma{}_{;\beta}(c_1 g^{\alpha\beta} g_{\gamma\sigma} + c_2 \delta^\alpha_\gamma \delta^\beta_\sigma + c_3 \delta^\alpha_\sigma \delta^\beta_\gamma + c_4 A^\alpha A^\beta g_{\gamma\sigma}).$$

- Galileon k-mouflage MOND adaptation (Babichev, Deffayet, & Esposito-Farese 2011)

  Also a tensor-vector-scalar theory. Said to improve on TeVeS in various regards (e.g., small enough departures from GR in high-acceleration environments)

- Nonlocal metric MOND theories (Soussa & Woodard 2003; Deffayet, Esposito-Farese, & Woodard 2011, 2014) Pure metric, but highly nonlocal in that they involve $F(\Box)$.

- BIMOND (Bimetric MOND) (Milgrom 2009-2013)

$$I = -\frac{1}{16\pi G}\int [R + \hat{R} + \ell_M^{-2}\mathcal{M}(\ell_M^2 C^2)]dv + I_M + \hat{I}_M$$

- MOND from a specialized formulation of $f(R)$ theories (Bernal, Capozziello, Hidalgo, & Mendoza 2011, Barrientos & Mendoza 2016)

- Massive bi-gravity plus a polarizable medium (Blanchet & Heisenberg 2015)

- $f(Q)$ versions of MOND (Milgrom 2019, D'Ambrosio et al. 2020)

Some of the MOND laws

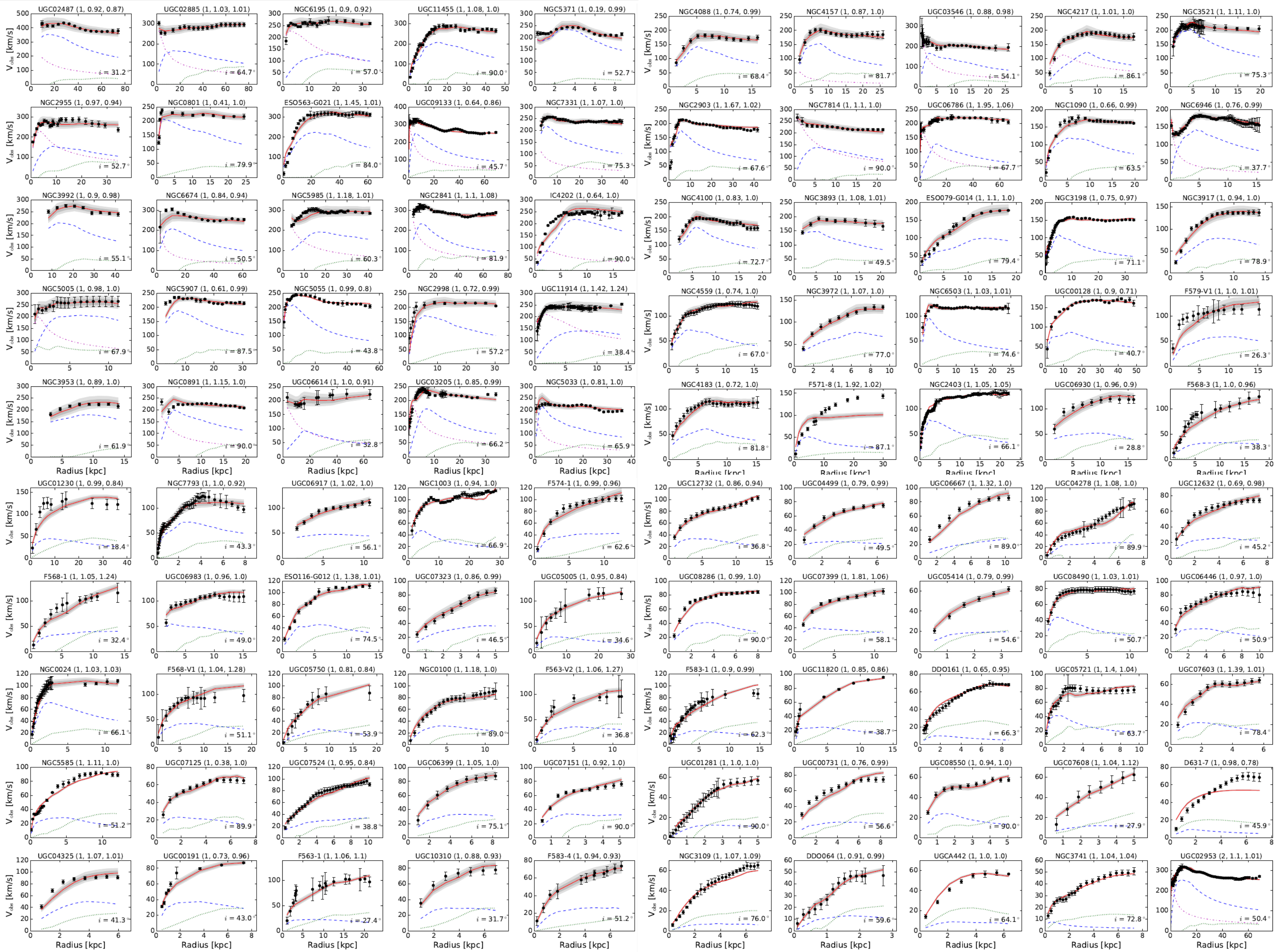
- Asymptotic circular speed (and bending angle): $V(r) \to V_\infty$ (H)
- The velocity mass relation (BTFR): $V_\infty^4 = M\mathcal{A}_0$ (H-B)
- DML virial relation: $\sigma^4 \sim M\mathcal{A}_0$ (H-B)
- Discrepancy appears always at $V^2/R = a_0$ (H-B)
- Isothermal spheres have surface densities $\bar{\Sigma} \lesssim \Sigma_M \equiv a_0/2\pi G$; (B)
- Universal dyn.-bar. central surface densities relation (H-B).

$$\Sigma^0_{Dyn} = \Sigma^0_B Q(\Sigma^0_B/\Sigma_M)$$

$$Q(y \ll 1) \approx 2y^{-1/2}, \quad Q(y \gg 1) \approx 1; \qquad \Sigma^0_P(y \gg 1) \approx \Sigma_M$$

- Full rotation curves from baryon distribution alone (H-B)

Rotation curves



Rotation curves

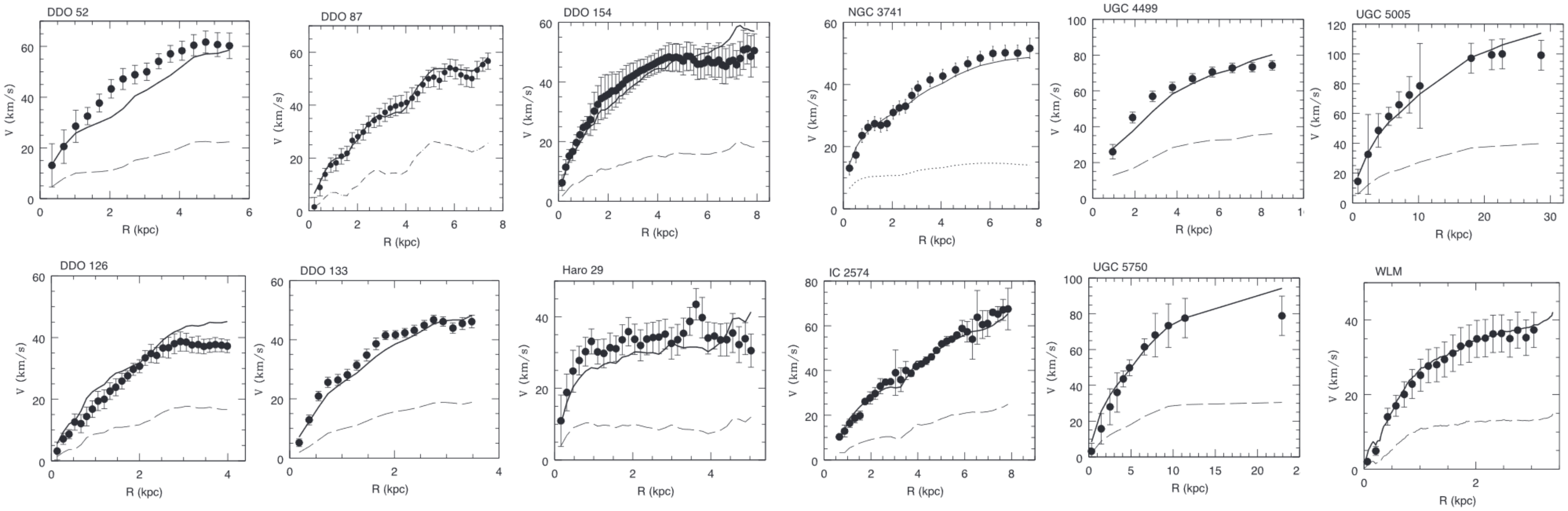
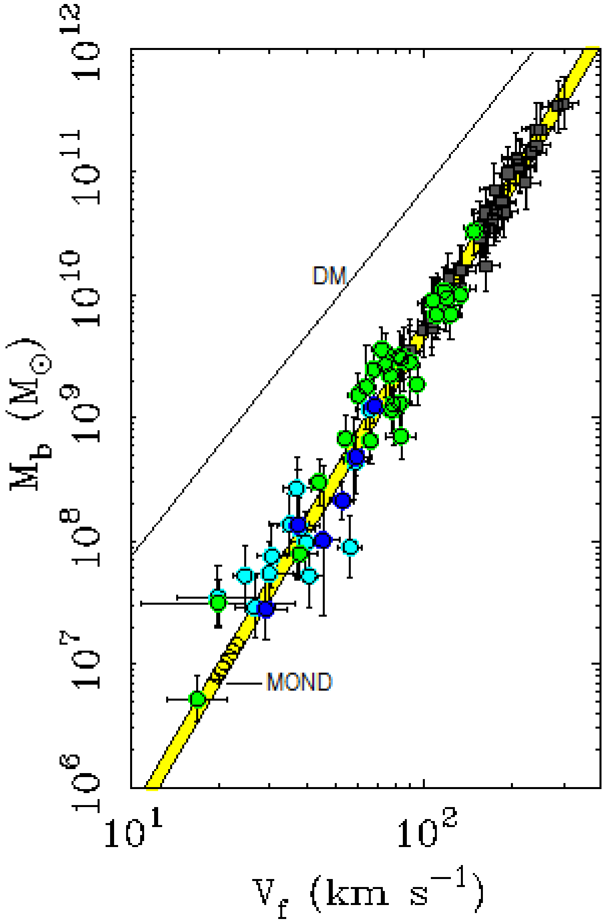


Figure 1: From Sanders 2019

Mass-asymptotic-speed relation



Mass-discrepancy-acceleration relation

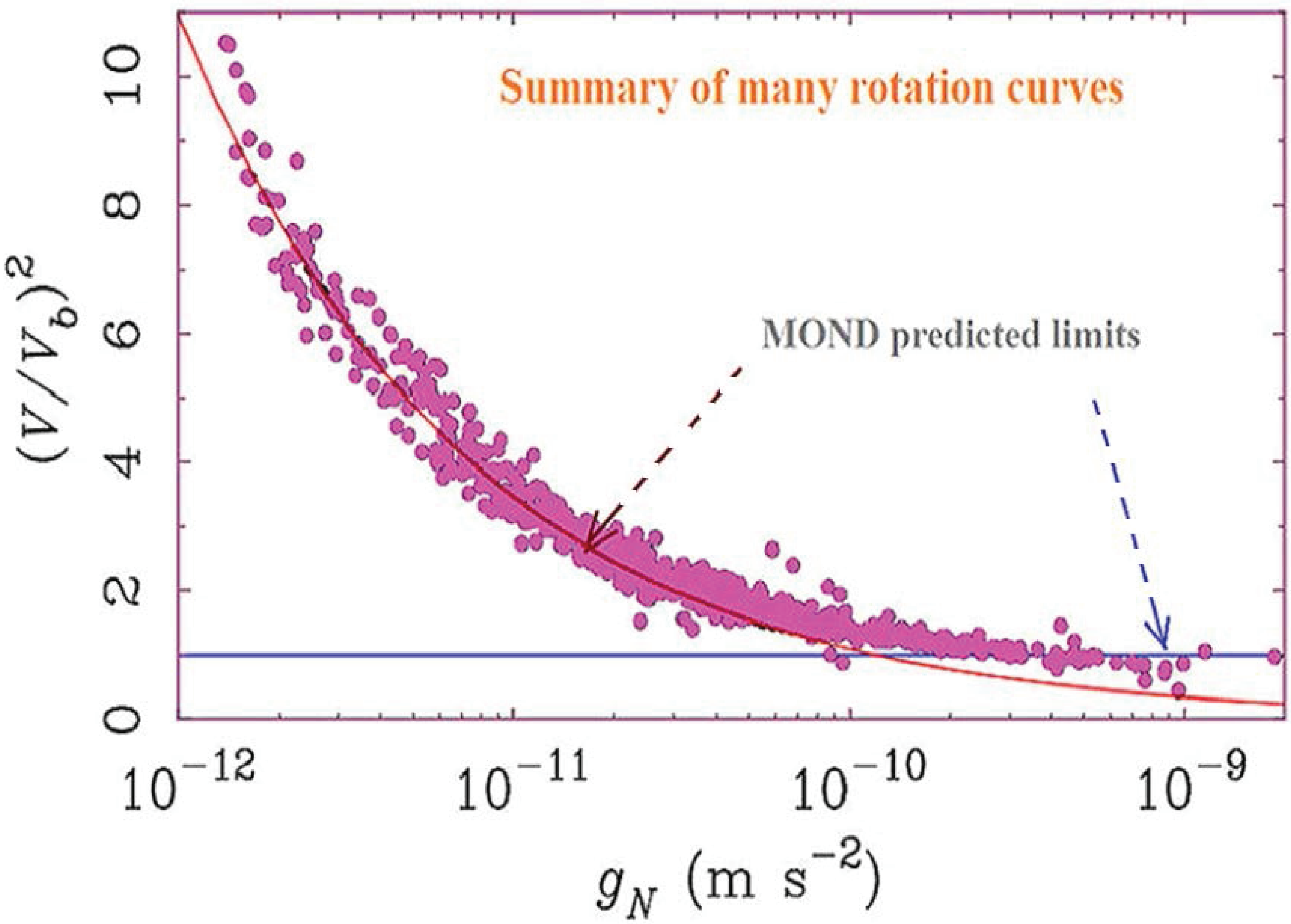
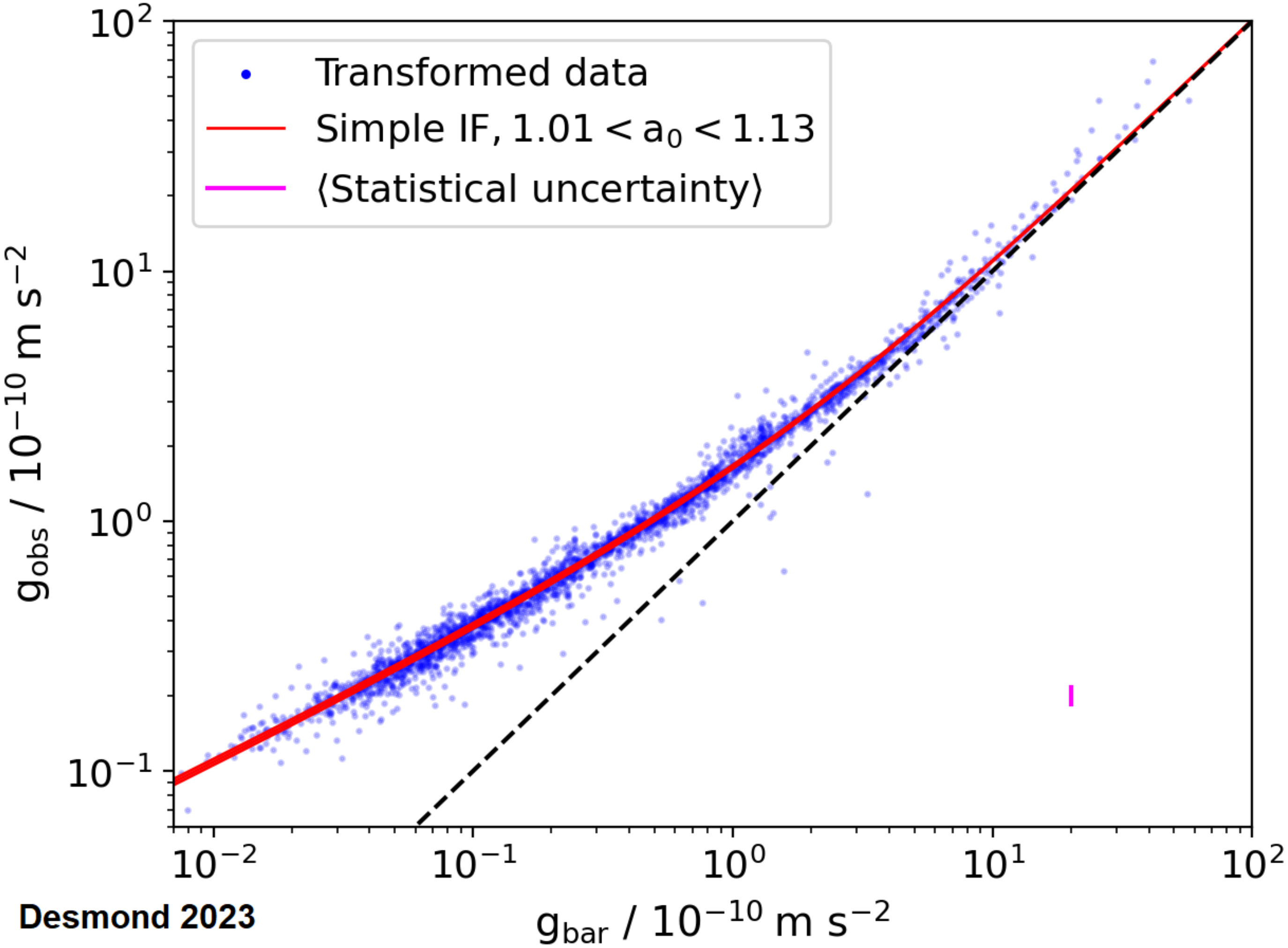


Figure 2: McGaugh

Rotation curves: observed-vs-Newtonian-acceleration relation



Desmond 2023

Lensing: acceleration relation

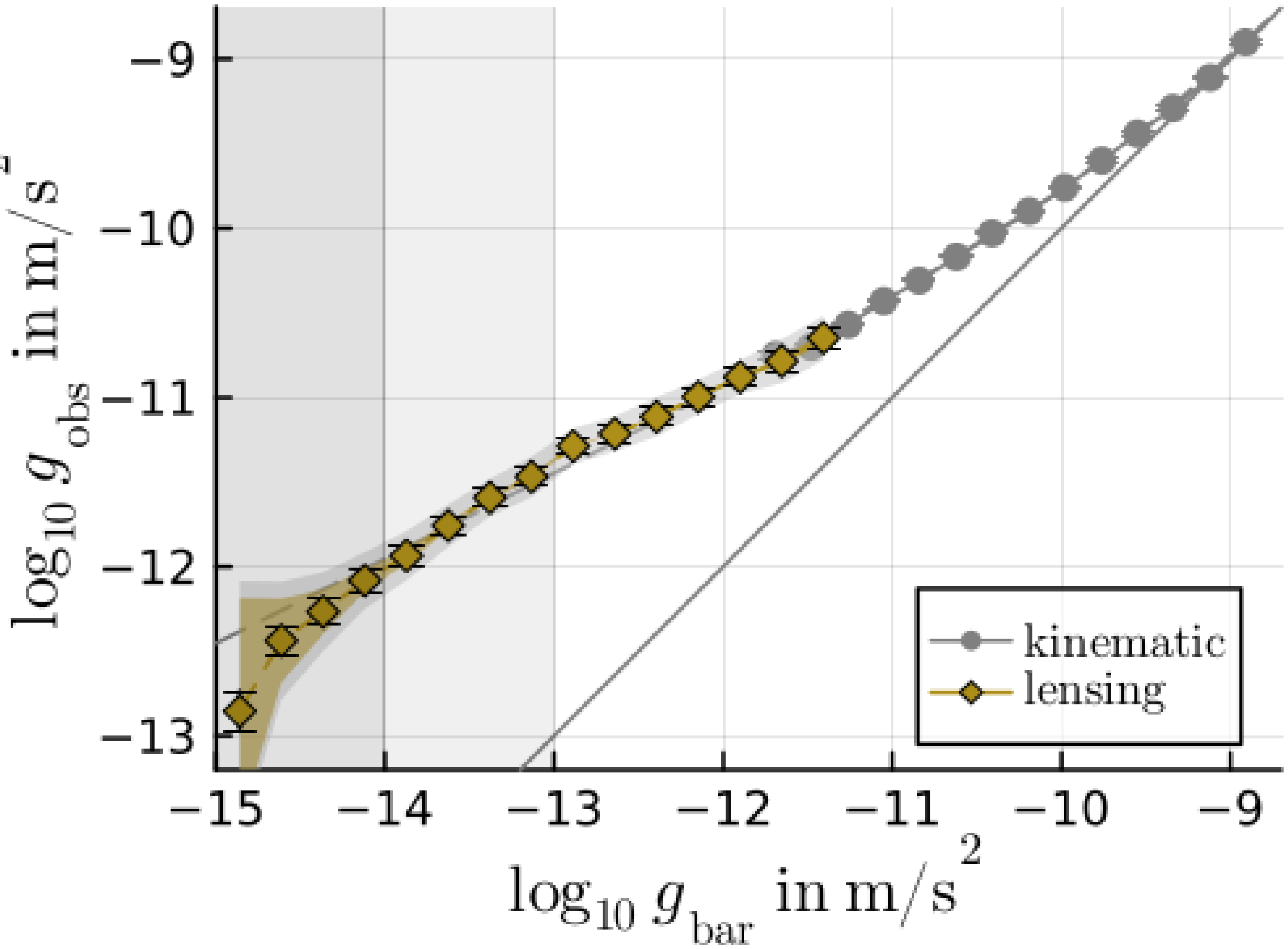


Figure 3: From Mistele et al. 2023. Lensing data from Brouwer et al. 2021

Central-surface-densities relation

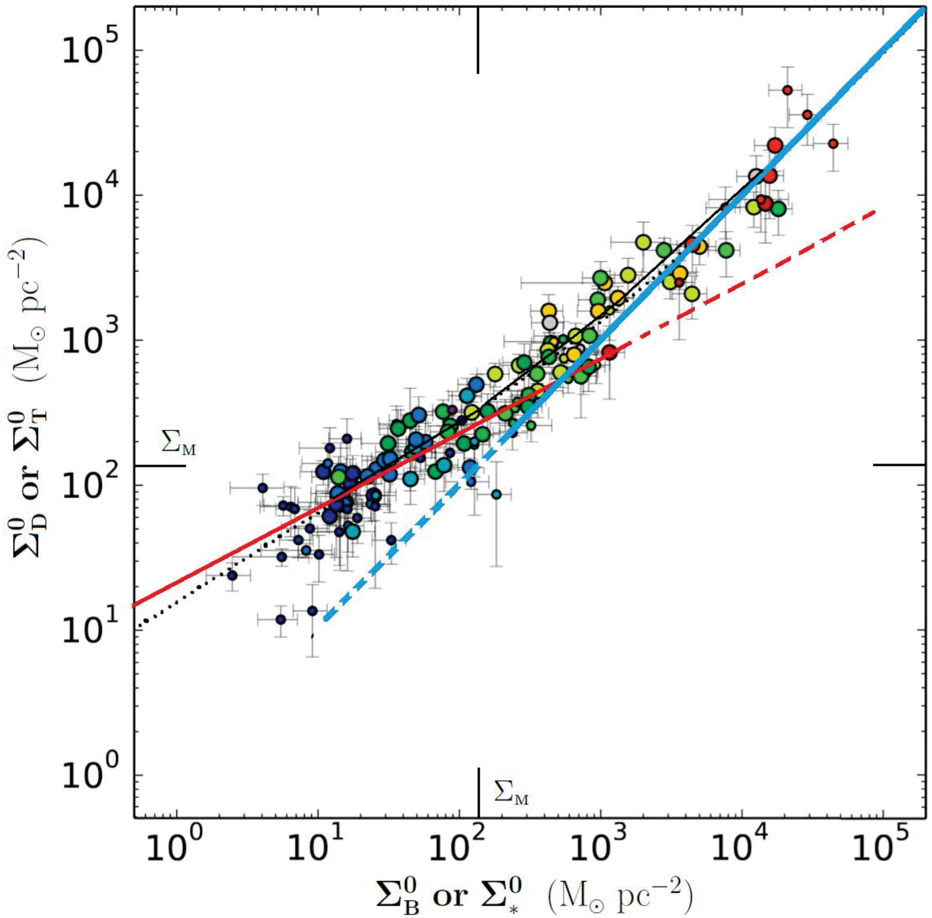


Figure 4: From Lelli et al. 2016

Phantom halo central surface density

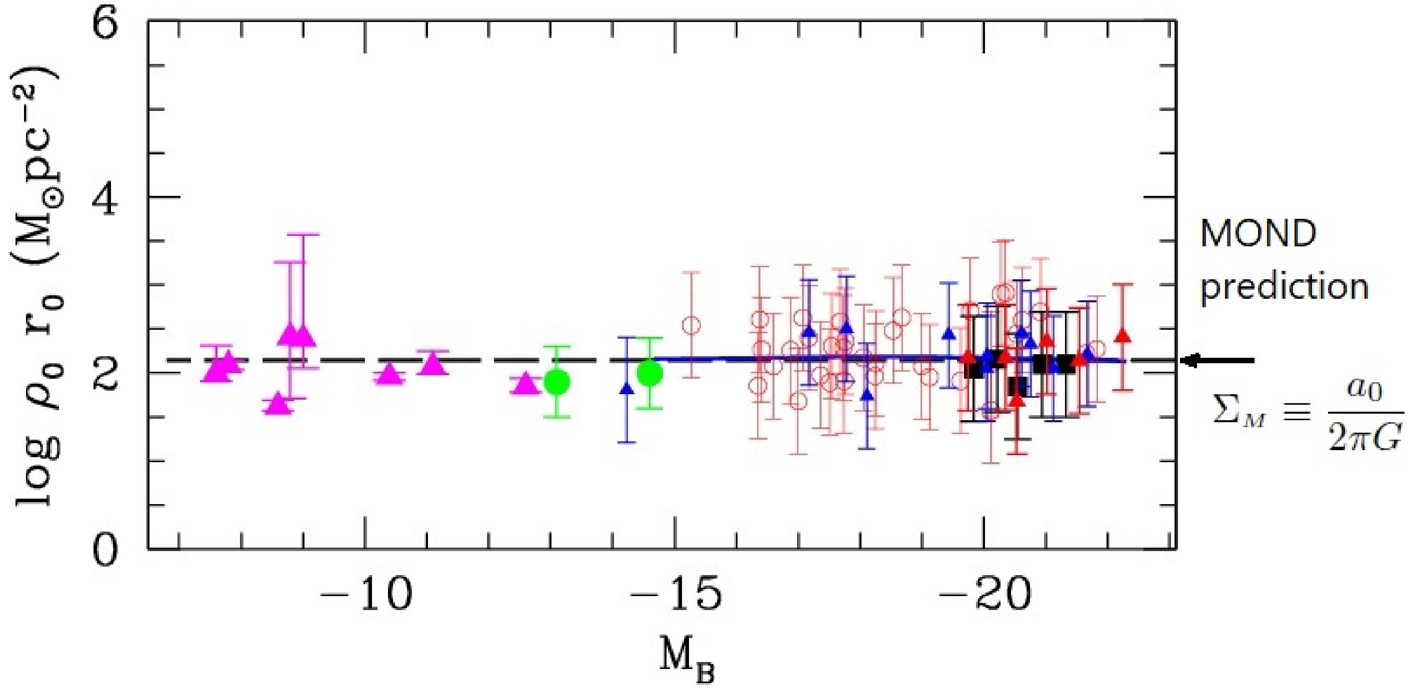
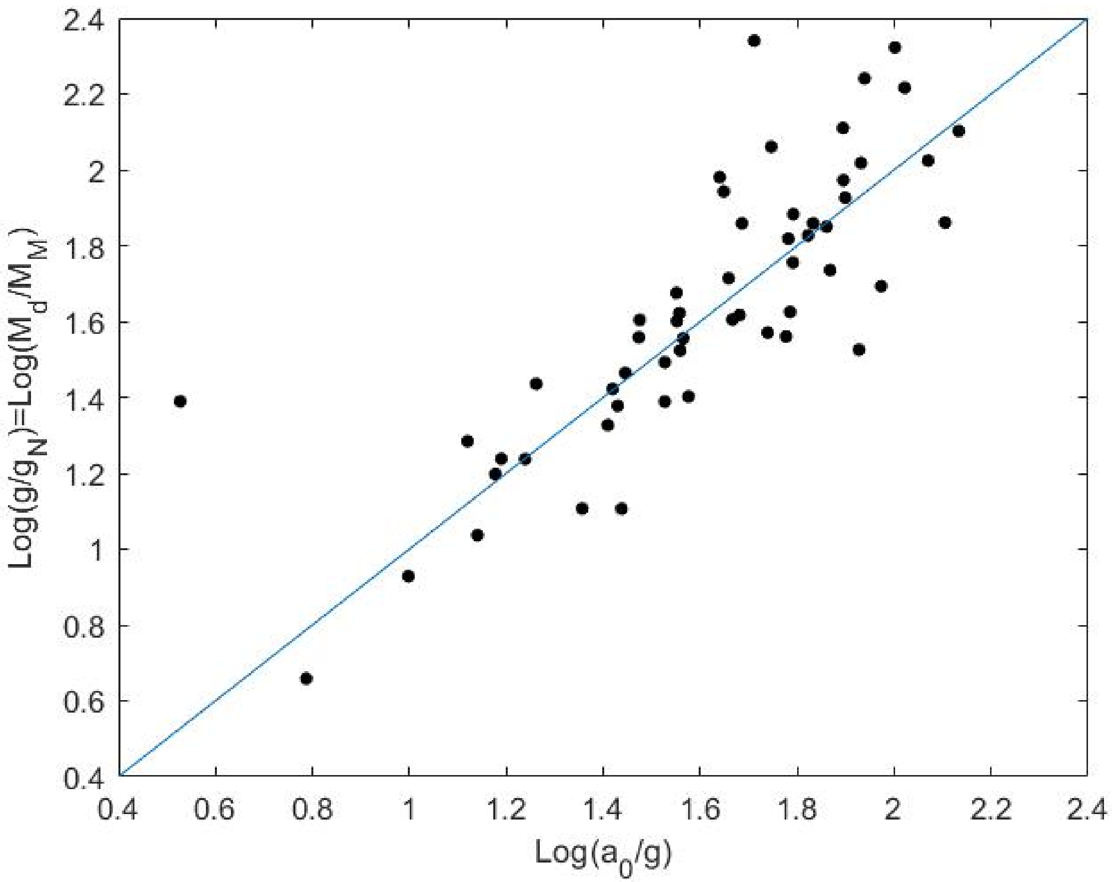


Figure 5: From Donato et al. 2008

Galaxy groups



Covergence

Table 8.1 *Jean Perrin's (1916) tabulation of experimental determinations of Avogadro's constant*

Phenomena observed		$N/10^{22}$
Viscosity of gases (van der Waal's equation)		62
Brownian movement	Distribution of grains	68.3
	Displacements	68.8
	Rotations	65
	Diffusion	69
Irregular molecular distribution	Critical opalescence	75
	The blue of the sky	60 (?)
Black-body spectrum		64
Charged spheres (in a gas)		68
Radioactivity	Charges produced	62.5
	Helium engendered	64
	Radium lost	71
	Energy radiated	60

Our wonder is aroused at the very remarkable agreement found between values [of N] derived from the consideration of such widely different phenomena. Seeing that not only is the same magnitude obtained by each method when the conditions under which it is applied are varied as much as possible, but that the numbers thus established also agree among themselves, without discrepancy, for all the methods employed, the real existence of the molecule is given a probability bordering on certainty (Perrin, 1916, p. 206–207).

MOND and cosmology

New Relativistic Theory for Modified Newtonian Dynamics

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We propose a relativistic gravitational theory leading to modified Newtonian dynamics, a paradigm that explains the observed universal galactic acceleration scale and related phenomenology. We discuss phenomenological requirements leading to its construction and demonstrate its agreement with the observed cosmic microwave background and matter power spectra on linear cosmological scales. We show that its action expanded to second order is free of ghost instabilities and discuss its possible embedding in a more fundamental theory.

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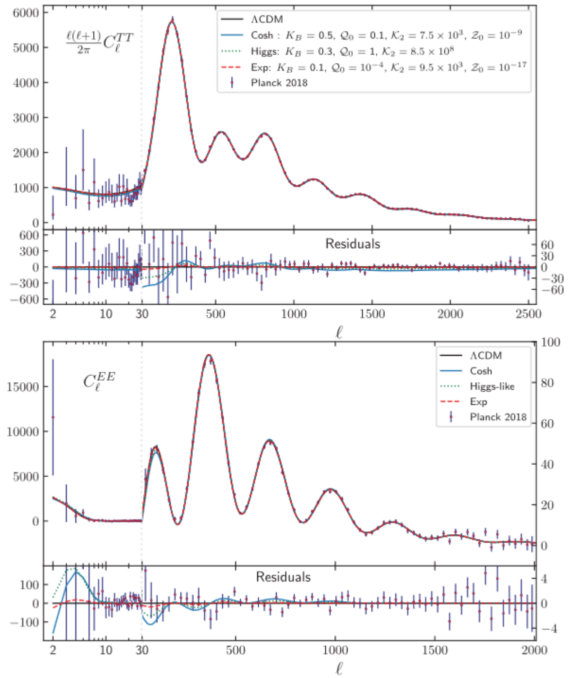


FIG. 1. The CMB temperature (T) C_ℓ^{TT} and E-mode polarization C_ℓ^{EE} angular power spectra for ΛCDM and this theory for a collection of functions and parameter values. The ΛCDM parameters are angular acoustic scale $100\theta_s = 1.04171$, DM density $\Omega_c h^2 = 0.1202$, baryon density $\Omega_b h^2 = 0.02235$, reionization optical depth $\tau = 0.049$, helium fraction $Y_{\rm He} = 0.242$, primordial scalar amplitude $10^9 A_s = 2.078$, and spectral index $n_s = 0.963$, while the MOND curves deviate from these within $\sim\{0.07, 0.33, 3.98, 14.29, 1.57, 0.58, 2.60\}\%$. MOND models have $\lambda_s = \infty$, and their other parameters are shown in the C_ℓ^{TT} panel, with $\mathcal{Q}_0$ and $\mathcal{Z}_0$ in Mpc^{-1}. The "Higgs-like" function parameters are incompatible with a MOND limit.

a_0-cosmology connection

$$\bar{a}_0 \equiv 2\pi a_0 \approx 7.5 \times 10^{-8}\ \mathrm{cm\ s^{-2}}$$

$$\bar{a}_0 \approx cH_0 \equiv c^2/\ell_H \qquad\qquad \bar{a}_0 \approx c(\Lambda/3)^{1/2} \equiv c^2/\ell_\Lambda$$

$$\ell_M \equiv c^2/a_0 \approx \ell_U = \ell_\Lambda,\ \ell_H,\ \ldots$$

Shall we take it seriously?

Weber & Kohlrausch: $\mathrm{v}_e = (\epsilon_0 \mu_0)^{-1/2} \approx c$

$\Rightarrow$ MOND as an effective theory

$$a \Rightarrow \ell_a \equiv c^2/a$$

(Rindler horizon, typical wavelength of Unruh radiation)

$$a \lessgtr a_0 \qquad \Leftrightarrow \qquad \ell_a \gtrless \ell_U$$

Does a_0 vary on cosmological time scale?
High-z rotation curves

‘Practical’ implications

- No deep-MOND ($g \ll a_0$), strong gravity ($\phi \sim c^2$) systems (e.g., no deep-MOND black holes)

  $(g/a_0) \cdot (c^2/\phi) \approx \ell_M/r$

- No ‘cosmological’ deep-MOND, strong lensing

  $r_E/r_M = (4D_{ls}D_l/D_s\ell_M)^{1/2} \approx (4D_l/\ell_M)^{1/2} \quad [r_M \equiv (MG/a_0)^{1/2}]$

a_0 as a Universal constant

Effective (Lagrangian) theories in which ℓ_M or a_0 is the only dimensioned new constant.

It then enters both as CC and in local dynamics.

Einstein-Hilbert Lagrangian density:

$$R \quad \rightarrow \quad R + \ell^{-2}\mathcal{F}(\ell^2 Q)$$

Nonrelativistic limit: $Q \rightarrow (\vec{\nabla}\phi)^2/c^4$

$c^4 \times$Lagrangian $\rightarrow (\vec{\nabla}\phi)^2 + a_0^2\mathcal{F}[(\vec{\nabla}\phi)^2/a_0^2]$
(up to derivatives)

Happens in BIMOND, MOND adaptations of Einstein-Aether, $f(Q)$ versions of MOND, dipolar medium ...

MOND may emerges on a de Sitter background

Does not attempt to explain Λ. Takes the background geometry as given.

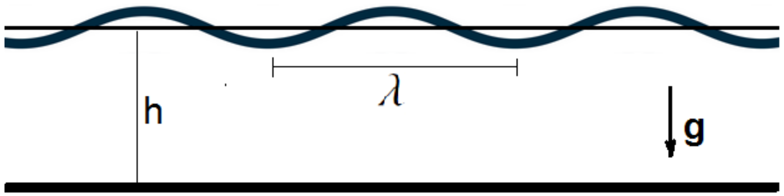
'Boxed' systems sometimes know about the size of the box even if they seem 'local':

$$g = V_e^2/2R_\oplus$$

Quantum particle in a box of size L: 'critical momentum' $P_c = \hbar/L$.

Accelerated charge in a conducting cavity: radiation zone $R > c^2/a$.

Gravity waves



$$\omega^2 = gk \cdot \tanh(kh) \qquad (k \equiv 2\pi/\lambda, \quad \delta h \ll h)$$

Constants: $g, \quad h \Rightarrow M\bar{G} \equiv c_\ell^2 h = gh^2$

$$(c_\ell^2 \equiv gh \ll c_{bulk}^2)$$

Degrees of freedom:

$$\omega, \;\; k \Rightarrow r_\omega \equiv c_\ell/\omega, \;\; a \equiv c_\ell^2 k$$

$$\frac{M\bar{G}}{r_\omega^2} = a \cdot \tanh(a/g), \qquad \mu(x) = \tanh(x)$$

$$g \leftrightarrow a_0, \quad c_\ell \leftrightarrow c, \quad h \leftrightarrow \ell_M, \quad M\bar{G} \leftrightarrow MG,$$
$$M\bar{G}g \leftrightarrow M\mathcal{A}_0$$

For $a \ll g$ ($\lambda \gg h$): $\frac{M\mathcal{A}_0}{r_\omega^2} = a^2$, Scale invariant

'Newtonian limit': $M\bar{G}/r_\omega^2 = a$, Not SI

MOND as a vacuum effect

Gibbons-Hawking:

$$a_\Lambda \equiv (\Lambda/3)^{1/2} = \kappa T_\Lambda, \qquad \kappa \equiv 2\pi c k_B/\hbar$$

Unruh: $a = \kappa T_U \qquad \Rightarrow \qquad F = m\kappa T$?? $\quad$ (**a** $\parallel \vec{\nabla} T$)

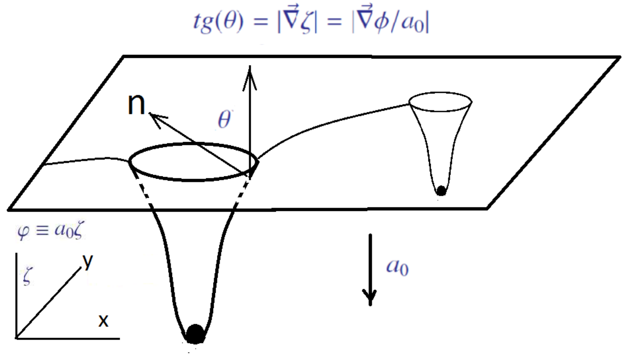
Combined: $T(a, \Lambda) = \kappa (a^2 + a_\Lambda^2)^{1/2}$

$$a_5 = (a^2 + a_\Lambda^2)^{1/2}$$

$$F = m\kappa(T - T_\Lambda) = ma\hat{\mu}(a/a_\Lambda)$$

$$\hat{\mu}(x) = [1 + (2x)^{-2}]^{1/2} - (2x)^{-1},$$

MOND from Membrane



Effective gravity depends on the ‘energy function’ of the brane

E.g., brane area

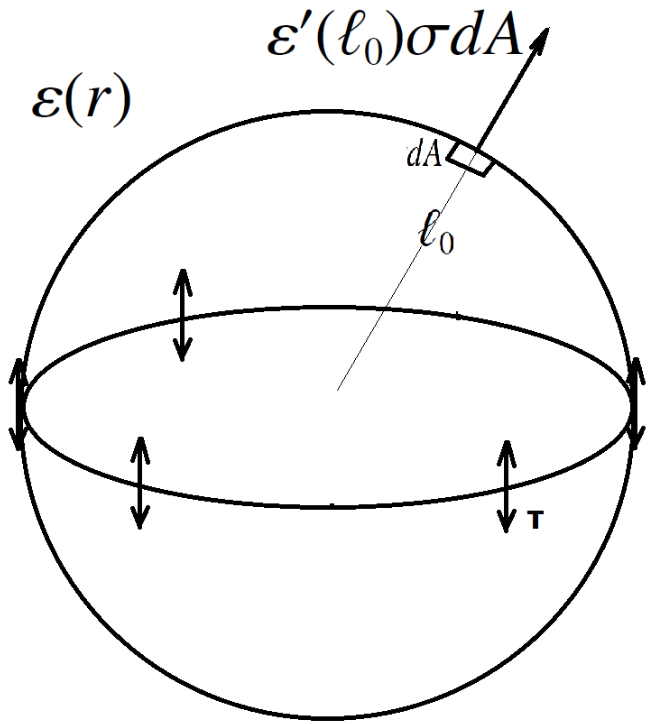
$$A = \int d^2x[1+(\vec{\nabla}\zeta)^2]^{1/2} = \int d^2x[1+(\vec{\nabla}\phi/a_0)^2]^{1/2}$$

For ‘masses’ $E = m\zeta a_0 = m\phi$

MOND from Membrane

NR version: 3-D almost spherical membrane in a 4-D Euclidean space subject to a radial potential field $\varepsilon(r)$ that couples to ‘masses’, both that of the membrane and to those of bodies on it. Bodies could be made of the same ‘stuff’ as the membrane only concentrated by other forces (EM, strong, etc.)

Membrane equilibrium

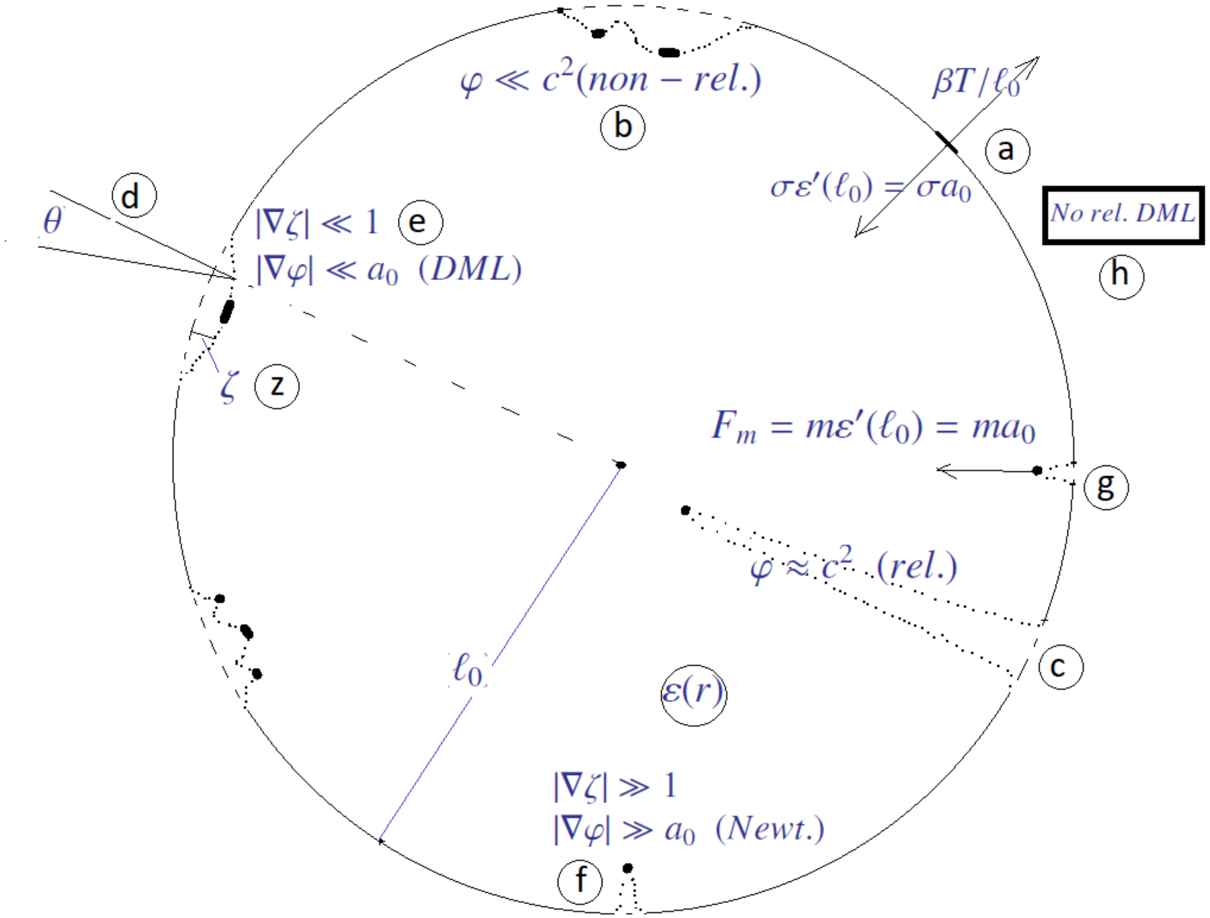


$$2\pi\ell_0 T = \pi\ell_0^2\varepsilon'(\ell_0)\sigma$$

$$\boxed{a_0 \equiv \varepsilon'(\ell_0) = 2c^2/\ell_0} \quad (c^2 \equiv T/\sigma)$$

$T = c^2\sigma \approx$ 'vacuum energy', but does not enter $\Lambda \approx c^2/\ell_0$

MOND from Membrane



$\varphi \equiv \varepsilon \approx \varepsilon(\ell_0) + \varphi_0 + \varepsilon'(\ell_0)\zeta = a_0\zeta$ $\qquad$ $tg(\theta) = |\vec{\nabla}\zeta| = |\vec{\nabla}\phi/a_0|$

$\varphi/c^2 = a_0\zeta/c^2 = \zeta/\ell_0$ $\qquad$ $a_0 \equiv |\varepsilon'(\ell_0)|$ $\qquad$ $T/\sigma = \alpha c^2$