Singularity at the demise of a black hole

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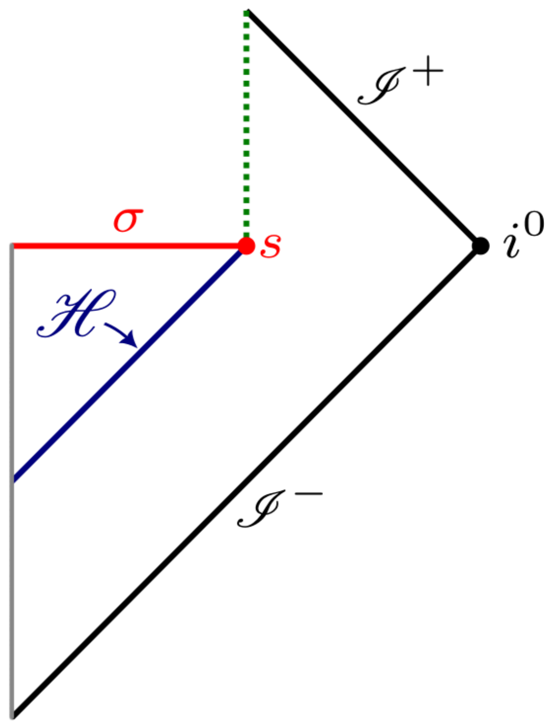
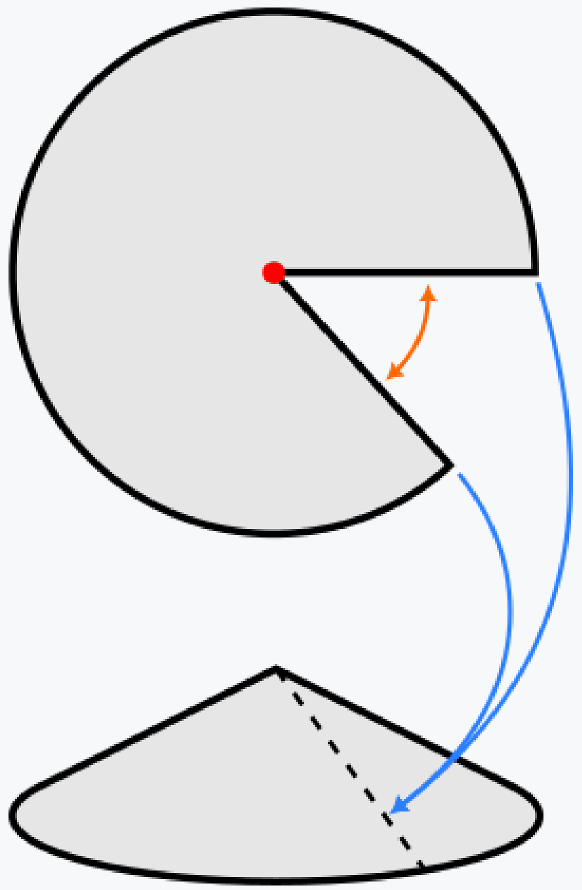
End state of a Black Hole



- What is the end state of an evaporating black hole?
  - Remnants, naked singularities, white holes, etc.[†]
- One possibility illustrated on left (horizon disappears)
- Can imagine regularizing singularity[†‡] σ, but what about s?
- In this talk, I claim that:
  - i. If σ is regularized in the $1+1$ case, then s is a quasiregular singularity.
  - ii. $\exists$ theories that can describe quasiregular singularities.

[†] S Hossenfelder, L Smolin, Phys.Rev.D 81 (2010) 064009; P Martin-Dussaud, C Rovelli, Class. Quantum Grav. 36, 245002 (2019)
[‡] A Simpson, M Visser, JCAP 02 (2019) 042 [arxiv:1812.07114]

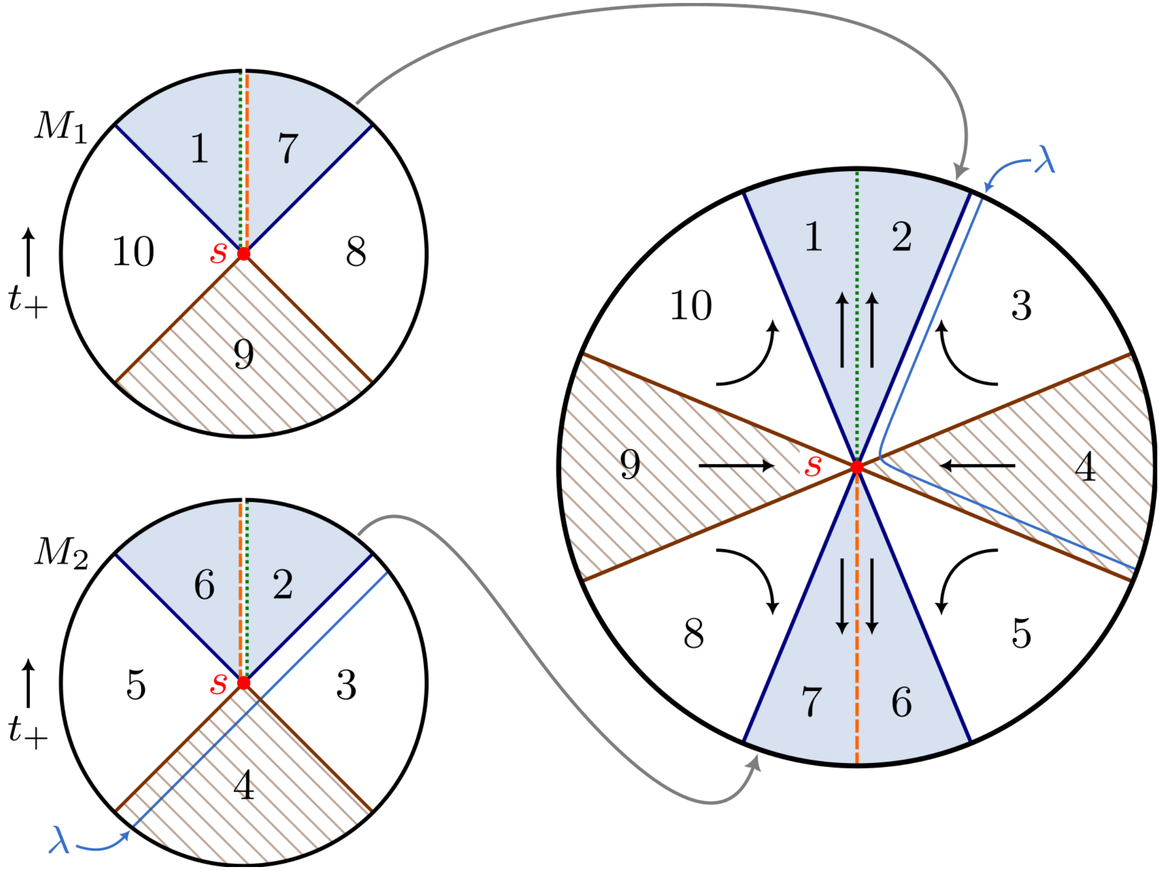
Quasiregular singularities



- Here, singularities defined as (boundary) points on which inextendible geodesics terminate
- Curvature singularities defined by diverging curvature in parallel frame along geodesic
- Quasiregular singularity[†] has well-behaved curvature (can even be zero) in its neigborhood
    - Can easily construct with cut-and-paste procedures
    - Conical singularity is an example

[†]G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

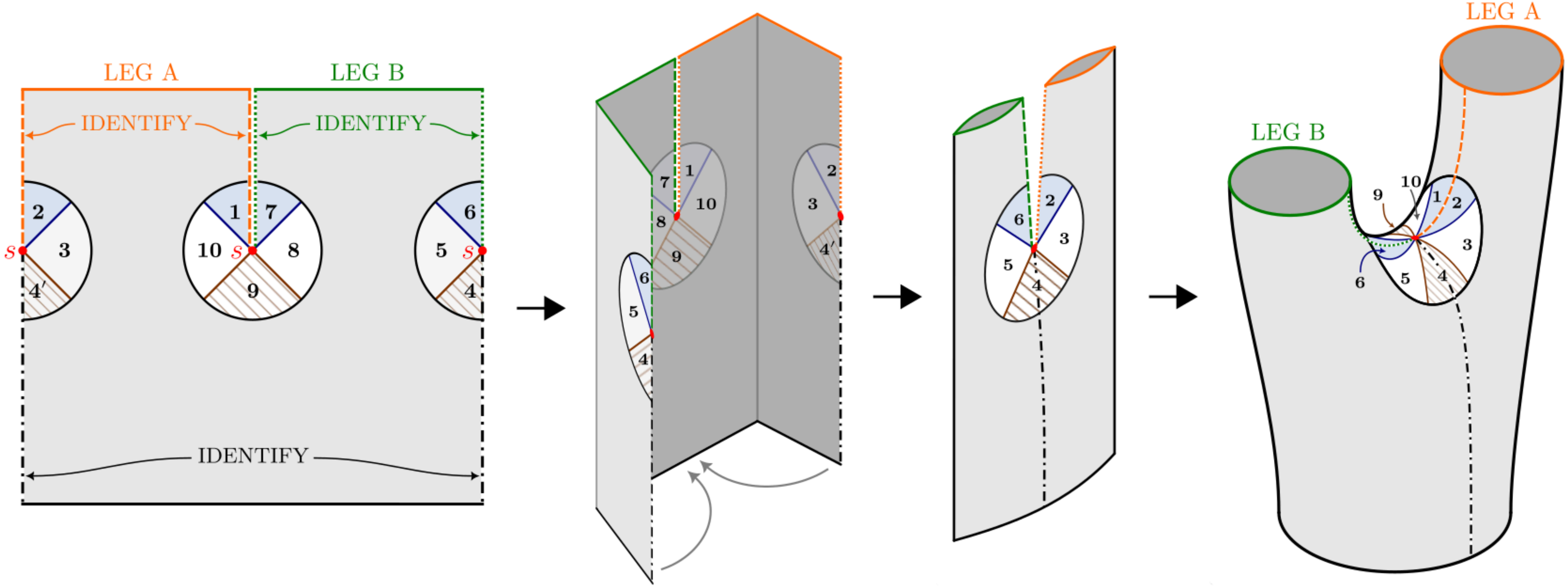
Saddlelike causally discontinuous singularity (SCDS)



- Can construct by cut-and-paste procedure in $1+1$ flat spacetime
- Two regions of $1+1$ flat spacetime illustrated on left; nonconformal cartoon of result on right
- In $1+1$ flat spacetime, each point has one future light cone and one past light cone
- Point s (SCDS) characterized by *two* future and *two* past light cones

cf. Fig 4(e) of G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).
There are further generalizations with more light cones---see G F R Ellis and B G Schmidt, Gen. Rel. Grav. 8, 915 (1977).

$1+1$ Trousers[†*] spacetime



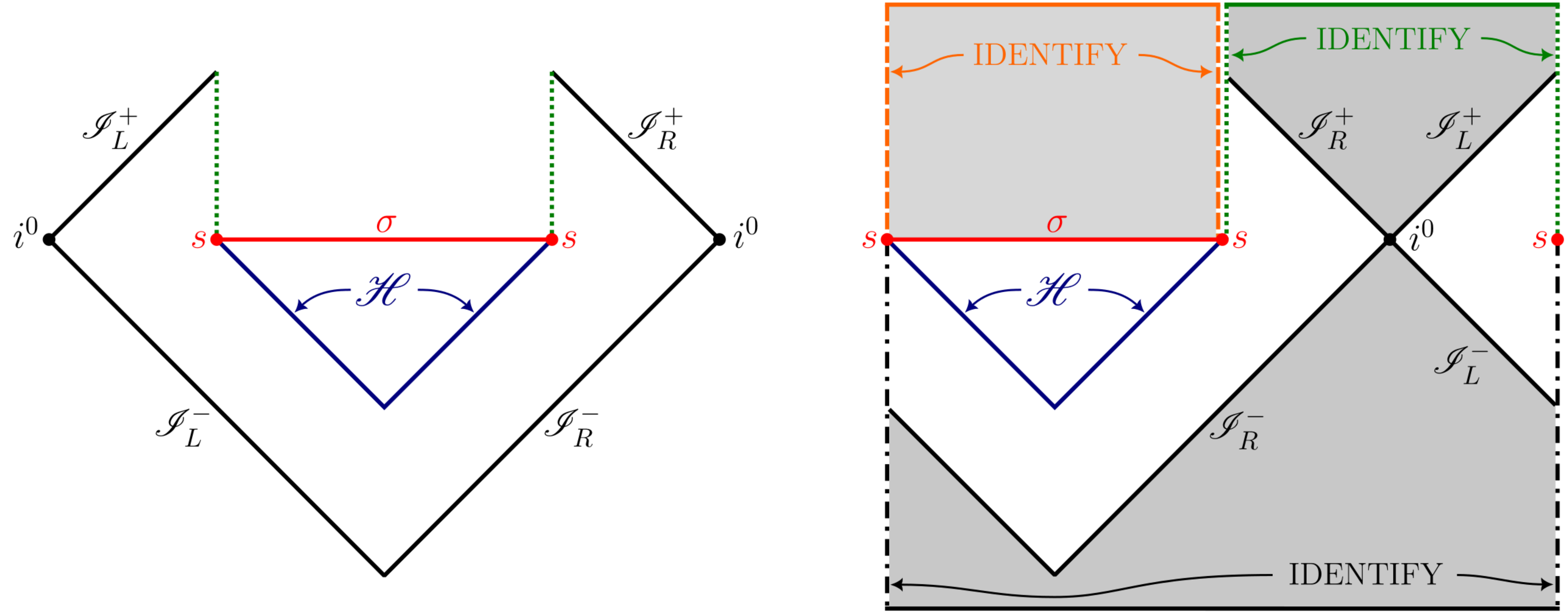
s is a SCDS, characterized by 2 future and 2 past light cones.

[†] A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105

[*] F. Dowker, S. Surya, PRD 58, 124019 (1998) [arXiv:gr-qc/9711070]; Buck et al., Class.Quant.Grav. 34 (2017) 5, 055002 [arXiv:1609.03573]

$1+1$ Black hole and trousers[†] spacetime

Schwarzschild in $1+1$ is conformal to trousers: s is quasiregular[*] singularity!

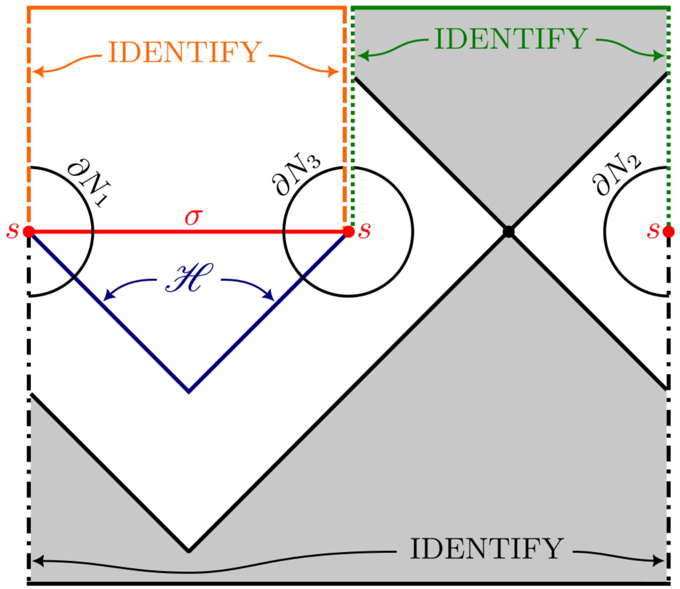


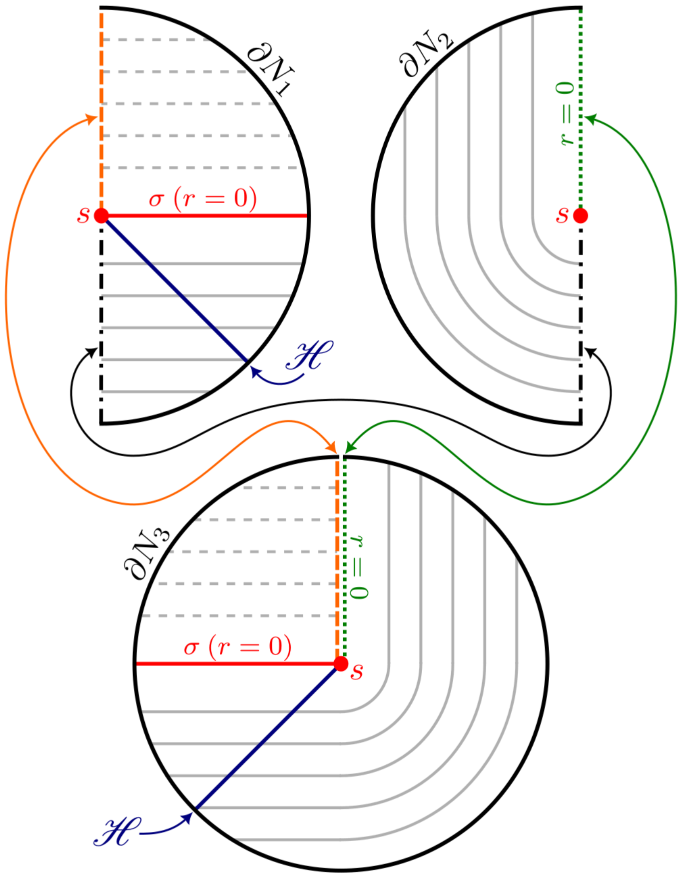
Planar slice of $d+1$ BH through origin can be regarded similarly.

[†] A Anderson, B S DeWitt, Found.Phys. 16 (1986) 91-105

[*] Provided that σ is regularized and spacetime analytically extended. If future topology of $\mathscr{H}$ differs, this is still true but s may not be SCDS.

Generalization to $d + 1$ spherically symmetric case





- $1 + 1$ plane for $d + 1$ evaporating BH on left, neighborhood ∂N of $\boldsymbol{s}$ on right.
- One way to understand $d + 1$: treat areal radius r as a scalar function (contours of r in gray)

Emergent Lorentz signature theory

- There is one theory that can describe a regularization of a SCDS.
- Postulate Euclidean-signature g_{ab} with shift-symmetric scalar-tensor action:[†]

$$S = \int_M d^4x\sqrt{|g|}L, \quad \varphi_a := \nabla_a\varphi, \quad \varphi_{ab} := \nabla_a\nabla_b\varphi, \quad X := \varphi^a\varphi_a$$

$$L = c_1R^2 + c_2R_{ab}R^{ab} + c_3R_{abcd}R^{abcd} + c_4XR + c_5R^{ab}\varphi_a\varphi_b \\ + c_6\mathrm{X}^2 + c_7(\Box\varphi)^2 + c_8\varphi_{ab}\varphi^{ab} + c_9R + c_{10}X + c_{11}$$

- At long distance scales, matter coupled to $\mathbf{g}_{ab}$:

$$\mathbf{g}_{ab} = g_{ab} - \varphi_a\varphi_b/X_C$$

- Can show S reduces to Lorentzian scalar-tensor theory in long-distance limit.[†‡]
- Can avoid Ostrogradsky instability for S bounded below, theory is renormalizable[*]

[†] S. Mukohyama, Phys. Rev. D 87, 085030 (2013)
[‡] S Mukohyama, J Uzan, Phys. Rev D. 87:065020 (2013)
[*] K Muneyuki, N Ohta, Phys. Lett. B 725 (2013) 495-499

Regularized SCDS in quadratic ELST

Consider a saddle-like scalar field profile and flat metric:

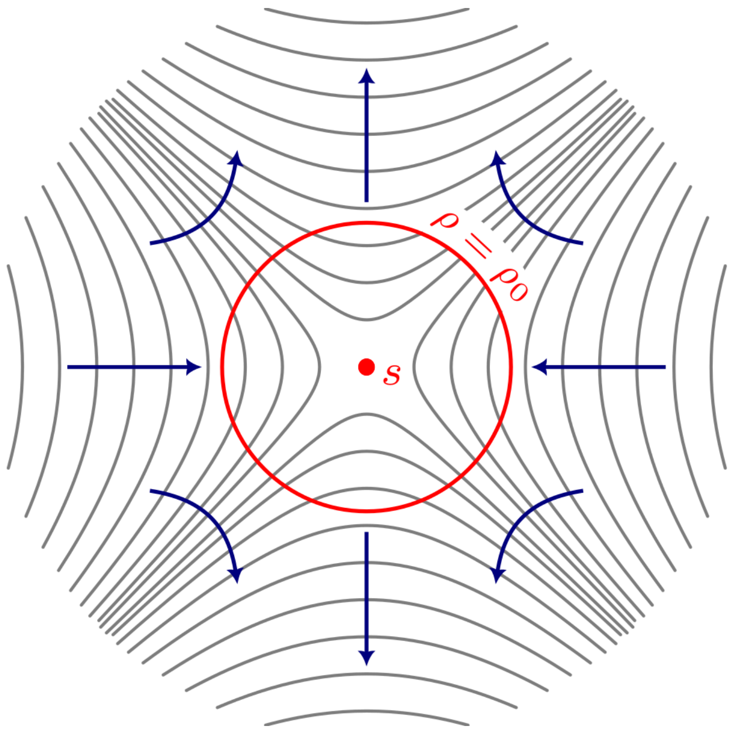
$$\varphi = (u^2 - v^2)/(2L_0)$$
$$ds^2 = du^2 + dv^2 + dy^2 + dz^2$$

These form a soln. for the parameter choices
$c_4 = c_6 = c_{10} = 0$ and $c_{11} = 8(c_5 - c_8)$.

Can get approximate soln. using Riemann normal coords:†

$$g_{\mu\nu} = \delta_{\mu\nu} - \frac{1}{3}\left[R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta - \frac{1}{6}\left[\nabla_\gamma R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma$$
$$-\left[\frac{2}{45} R_{\mu\alpha\lambda\beta} R^\lambda{}_{\gamma\delta\nu} + \frac{1}{20}\nabla_\gamma \nabla_\delta R_{\mu\alpha\nu\beta}\right]_0 x^\alpha x^\beta x^\gamma x^\delta + O(x^{\cdot 5})$$

where s is the origin.*



†U Muller, C Schubert, and A M E van de Ven, Gen. Rel. Grav. 31, 1759 (1999);
A Z Petrov, *Einstein Spaces*, Pergamon (1969); E Kreysig, *Intro. to Diff. Geom. and Riem. Geom.*, U Toronto Press (1968)

*For this to model an evaporating black hole, one should assume $r = 0$ contour is far from s.

Issues/questions to think about

- Result compatible with a "baby universe" resolution[†] to BH information paradox (but meaning of "time" evolution in quantum theory needs to be clarified)
- Singularity regularized only in fundamental metric and scalar field; effective metric still singular
- Analysis is very preliminary; a more comprehensive analysis is needed to determine whether the solutions are realized at the end of BH evaporation
- How might other theories handle quasiregular singularities?

[†] S Hossenfelder, L Smolin, Phys.Rev.D81:064009,2010 [arXiv:0901.3156]

Finite areal radius at the origin

Consider line element for $R^2 \times S^2$ manifold:

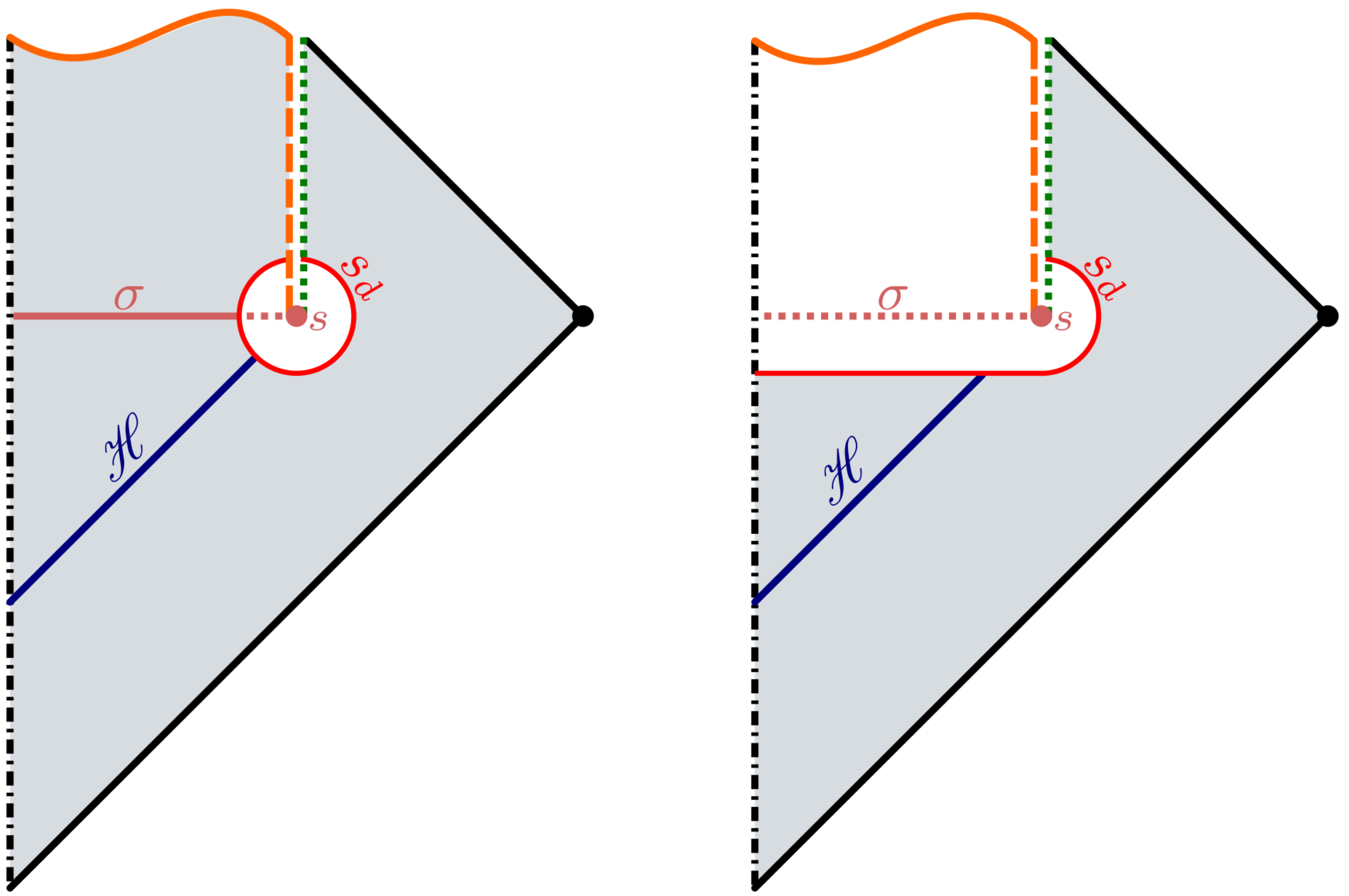
$$ds^2 = du^2 + dv^2 + r(u,v)^2(d\theta^2 + \sin^2\phi d\phi^2)$$

For the $v = 0$ surface, the extrinsic curvature is:

$$K^a{}_b = \mathrm{diag}\left(0, \frac{\partial_v r(u,v)|_{v=0}}{r(u,0)}, \frac{\partial_v r(u,v)|_{v=0}}{r(u,0)}\right)$$

Assuming symmetry about $v = 0$, can have finite areal radius r at $v = 0$ and smooth (C^1 at least) geometry provided that $\partial_v r(u,v)|_{v=0} = 0$.

Regularization possibilities



Cf. Fig. 4 of S Hossenfelder, L Smolin, Phys.Rev.D81:064009,2010 [arXiv:0901.3156], but they did not consider microscopic description for s