Kinematic and thermodynamic properties of dynamical regular black holes

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YITP, Kyoto, JP

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References

PHYSICAL REVIEW D
covering particles, fields, gravitation, and cosmology

Phys. Rev. D **108**, 044002 (2023) arXiv:2304.05421 [gr-qc]

Regular black holes and the first law of black hole mechanics

Sebastian Murk[1,*] and Ioannis Soranidis[2,†]

[1]*Okinawa Institute of Science and Technology, 1919-1 Tancha, Onna-son, Okinawa 904-0495, Japan*
[2]*School of Mathematical and Physical Sciences, Macquarie University, Sydney, New South Wales 2109, Australia*

2304.05421

Phys. Rev. D **108**, 124007 (2023) arXiv:2309.06002 [gr-qc]

Kinematic and energy properties of dynamical regular black holes

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2309.06002

Spherically symmetric dynamical RBHs in semiclassical gravity

Spherical symmetry:

$$ds^2 = -f(v,r)dv^2 + 2dvdr + r^2 d\Omega^2$$

Metric function and Misner–Sharp mass:

$$f(v,r) := \partial_\mu r \partial^\mu r = 1 - \frac{C(v,r)}{r}$$

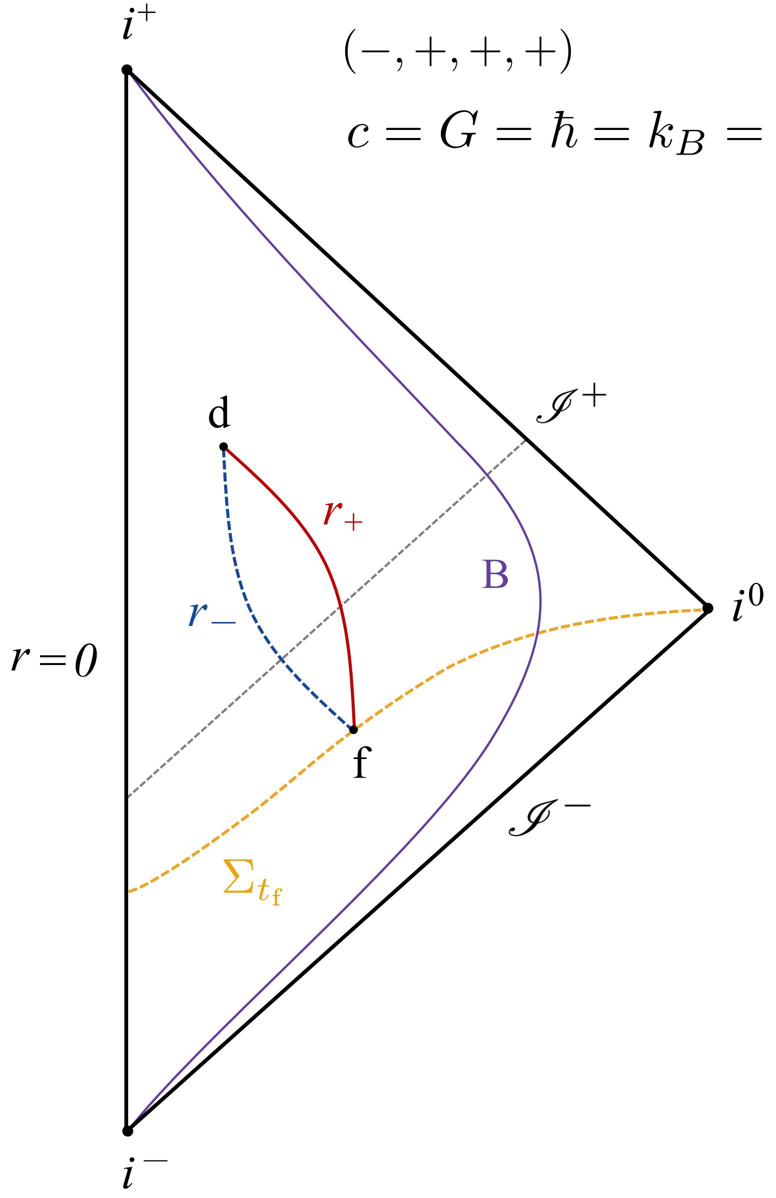
$$C(v,r) = r_+(v) + \sum_{i=1}^{\infty} w_i(v) \big(r - r_+(v)\big)^i$$

Dynamical RBHs: $a, b \in \mathbb{N}_{\text{odd}} = \{1, 3, 5, \dots\}$

$$f(v,r) = \underbrace{g(v,r)}_{>0} \big(r - r_-(v)\big)^a \big(r - r_+(v)\big)^b$$

$(-,+,+,+)$

$c = G = \hbar = k_B = 1$



Dynamical solutions in spherical symmetry

Only two metric families can describe the geometry near an apparent horizon formed in finite time of a distant observer:

Review article:

Mann, SM, Terno
Int. J. Mod. Phys. D **31**, 2230015 (2022)

Table 2. Properties of the four types of Vaidya metrics. The Einstein equations have real solutions at finite time $t > t_S$ only if the NEC is violated.

$\mathrm{sgn}(T_{tt})$	$\mathrm{sgn}(T_t{}^r)$	Time-evolution of Vaidya mass function	Black/ White hole	NEC violation
$-$	$-$	$C'(v) < 0$	B	✓
$-$	$+$	$C'(u) > 0$	W	✓
$+$	$-$	$C'(u) < 0$	W	✗
$+$	$+$	$C'(v) > 0$	B	✗

Evaporating black holes

Accreting white holes

Surface gravity and the first law of BH mechanics

First law of BH mechanics: (for $\delta J = \delta Q = 0$)

$$\delta M = \frac{\kappa}{8\pi}\delta A$$

Bardeen, Carter, Hawking
Commun. Math. Phys. **31**, 161 (1973)

Horizon area: $A = 4\pi r_+^2$

$$\Rightarrow \frac{\delta M}{\delta r_+} = \frac{\kappa}{8\pi}\frac{\delta A}{\delta r_+} \Rightarrow \kappa = \frac{1}{2r_+}$$

Definition of κ is unambigous only in stationary spacetimes: $\kappa = 2\pi T_H$

SM, Terno, Phys. Rev. D **103**, 064082 (2021)
Mann, SM, Terno, Phys. Rev. D **105**, 124032 (2022)

Dynamical generlizations: [1] peeling surface gravity → ill-defined for transient object!

[2] Kodama surface gravity: $\kappa_K K_\nu := \frac{1}{2}K^\mu\left(\nabla_\mu K_\nu - \nabla_\nu K_\mu\right), \quad K^\mu = (1,0,0,0)$

Nielsen, Yoon, Class. Quant. Grav. **25**, 085010 (2008)
Cropp, Liberati, Visser, Class. Quant. Grav. **30**, 125001 (2013)

Kodama, Prog. Theor. Phys. **63**, 1217 (1980)
Abreu, Visser, Phys. Rev. D **82**, 044027 (2010)
Kurpicz, Pinamonti, Verch, Lett. Math. Phys. **111**, 110 (2021)

Generalized dynamical first law

$$C(v,r) = r_+(v) + \sum_{i=1}^{\infty} w_i(v)\big(r - r_+(v)\big)^i$$

Generalized dynamical first law: $$\delta\left(\frac{r_+}{2}\right) = \frac{1-w_1}{16\pi r_+}\delta A + \frac{w_1}{8\pi r_+^2}\delta V$$

$$\delta M = \frac{\kappa}{8\pi}\delta A - p\delta V$$

$$p = -\frac{w_1}{8\pi r_+^2}$$

➡ **Consistency condition:** $$w_1\big|_{r=r_+} = 0$$

Note:
Applies generically to dynamical black holes!

Now: Focus on dynamical RBH models $f(v,r) = g(v,r)\big(r - r_-(v)\big)^a\big(r - r_+(v)\big)^b$ with $b = 1$.

$$\kappa_K\big|_{r=r_+} = \frac{1}{2}\partial_r f(v,r)\big|_{r=r_+} = \lim_{r\to r_+}\frac{(r-r_+)^{-1+b}bg(v,r)(r-r_-)^a}{2}$$

Nonzero Kodama surface gravity only possible for nondegenerate outer horizon.

Generalized dynamical first law

$$C(v,r) = r_+(v) + \sum_{i=1}^{\infty} w_i(v)\big(r - r_+(v)\big)^i$$

Generalized dynamical first law: $$\delta\left(\frac{r_+}{2}\right) = \frac{1-w_1}{16\pi r_+}\delta A + \frac{w_1}{8\pi r_+^2}\delta V$$

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Applies generically to dynamical black holes!

Now: Focus on dynamical RBH models $f(v,r) = g(v,r)\big(r - r_-(v)\big)^a\big(r - r_+(v)\big)^b$ with $b = 1$.

➡ $$w_1\big|_{r=r_+} = 1 - g(v,r_+)r_+(r_+ - r_-)^a$$

$$\Leftrightarrow \quad \boxed{g\left(v,r_+\right)r_+\left(r_+ - r_-\right)^a = 1}$$

Nondegenerate dynamical RBH models $(a=1, b=1)$

Popular examples: Bardeen, Dymnikova, Hayward.

Bardeen in *Proceedings of the International Conference GR5* (Tbilisi University Press, Tbilisi, 1968)

Dymnikova, Gen. Relativ. Gravit. **24**, 235 (1992)
Hayward, Phys. Rev. Lett. **96**, 031103 (2006)

Dynamical Hayward RBH: $f(v,r) = 1 - \dfrac{r_g(v) r^2}{r^3 + r_g(v) l(v)^2}$

Using the roots of $f=0$: $r_0 = -l + \dfrac{l^2}{2r_g} + \mathcal{O}(l^3) < 0\,,\quad r_- = l + \dfrac{l^2}{2r_g} + \mathcal{O}(l^3)\,,\quad r_+ = r_g - \dfrac{l^2}{r_g} + \mathcal{O}(l^4)$

$$\Rightarrow \quad f(v,r) = \frac{r - r_0}{r^3 + r_g l^2}(r - r_-)(r - r_+)$$

Comparison with $f(v,r) = g(v,r)\big(r - r_-(v)\big)\big(r - r_+(v)\big) \Rightarrow g(v,r) = \dfrac{r - r_0}{r^3 + r_g l^2} > 0$

Expansion of MS mass about the outher horizon $r = r_+$: $\boxed{w_1\big|_{r=r_+} = \dfrac{3l^2}{r_g^2} + \mathcal{O}(l^4) \geqslant 0}$

Not covered in talk:
Degenerate models $(a \geqslant 3)$;
Charged Hayward-Frolov BH

Analogous expressions are obtained for other nondegenerate models.

SM, Soranidis
Phys. Rev. D **108**, 044002 (2023)

Summary #1 [Phys. Rev. D **108**, 044002 (2023)]

1. First law receives corrections that can be interpreted as an additional work term of an extended first law:

$$\delta\left(\frac{r_+}{2}\right) = \frac{1 - w_1}{16\pi r_+}\delta A + \boxed{\frac{w_1}{8\pi r_+^2}\delta V}$$

2. Linear coefficient of Misner–Sharp suffices to determine the relevant thermodynamic properties.

3. Need for corrections is linked to introduction of minimal length scale (consequence of spacetime regularization).

Phys. Rev. D **108**, 044002 (2023)   arXiv:2304.05421 [gr-qc]

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Null energy condition for dynamical RBHs

$$T_{\mu\nu}\ell^\mu\ell^\nu \overset{?}{\geqslant} 0$$

$$\ell^\mu = (1, f/2, 0, 0)$$

$$T_{\mu\nu}\ell^\mu\ell^\nu = -\frac{\partial_v f}{8\pi r}$$

$$ds^2 = -f(v,r)dv^2 + 2dvdr + r^2 d\Omega^2$$

$$f(v,r) = g(v,r)\big(r - r_-(v)\big)^a \big(r - r_+(v)\big)^b$$

Evolution of the NEC violation for the model proposed in

Carballo-Rubio *et al.*
J. High Energy Phys. **09**, 118 (2022)

At outer apparent horizon:

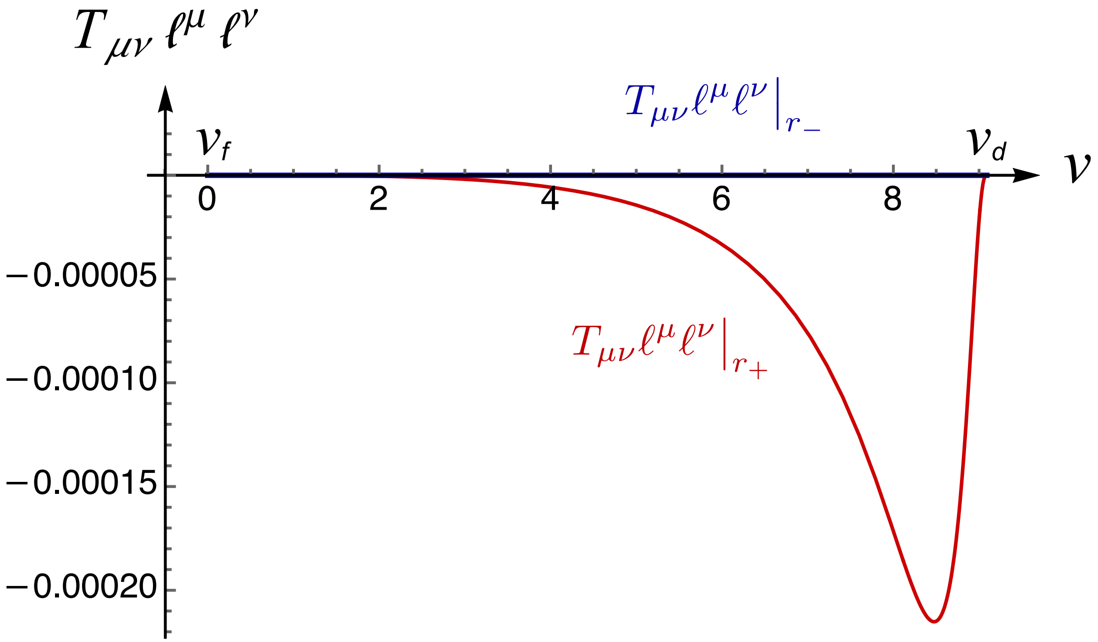
$$T_{\mu\nu}\ell^\mu\ell^\nu\Big|_{r_+} = \frac{-bg(v,r_+)(-r_+')(r_+ - r_-)^a}{8\pi r_+}(r - r_+)^{b-1} + \mathcal{O}(r - r_+)^b$$

$$< 0$$

At inner apparent horizon:

$$T_{\mu\nu}\ell^\mu\ell^\nu\Big|_{r_-} = (-1)^{b+1}\frac{ag(v,r_-)(-r_-')}{8\pi r_-}(r_+ - r_-)^b \cdot (r - r_-)^{a-1} + \mathcal{O}(r - r_-)^a.$$

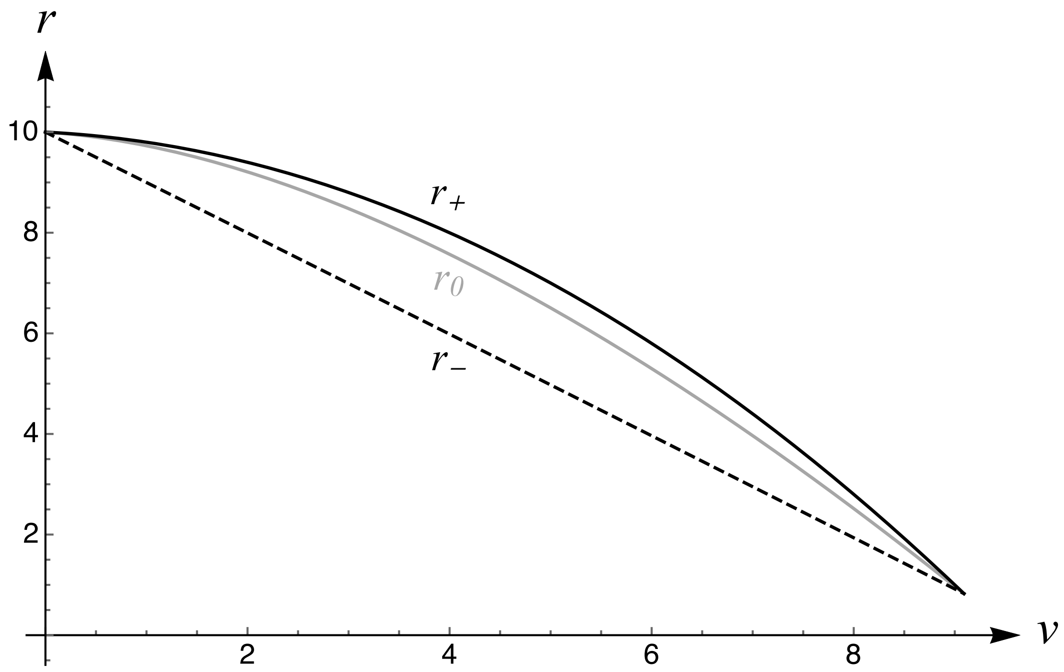
$$\geqslant 0$$



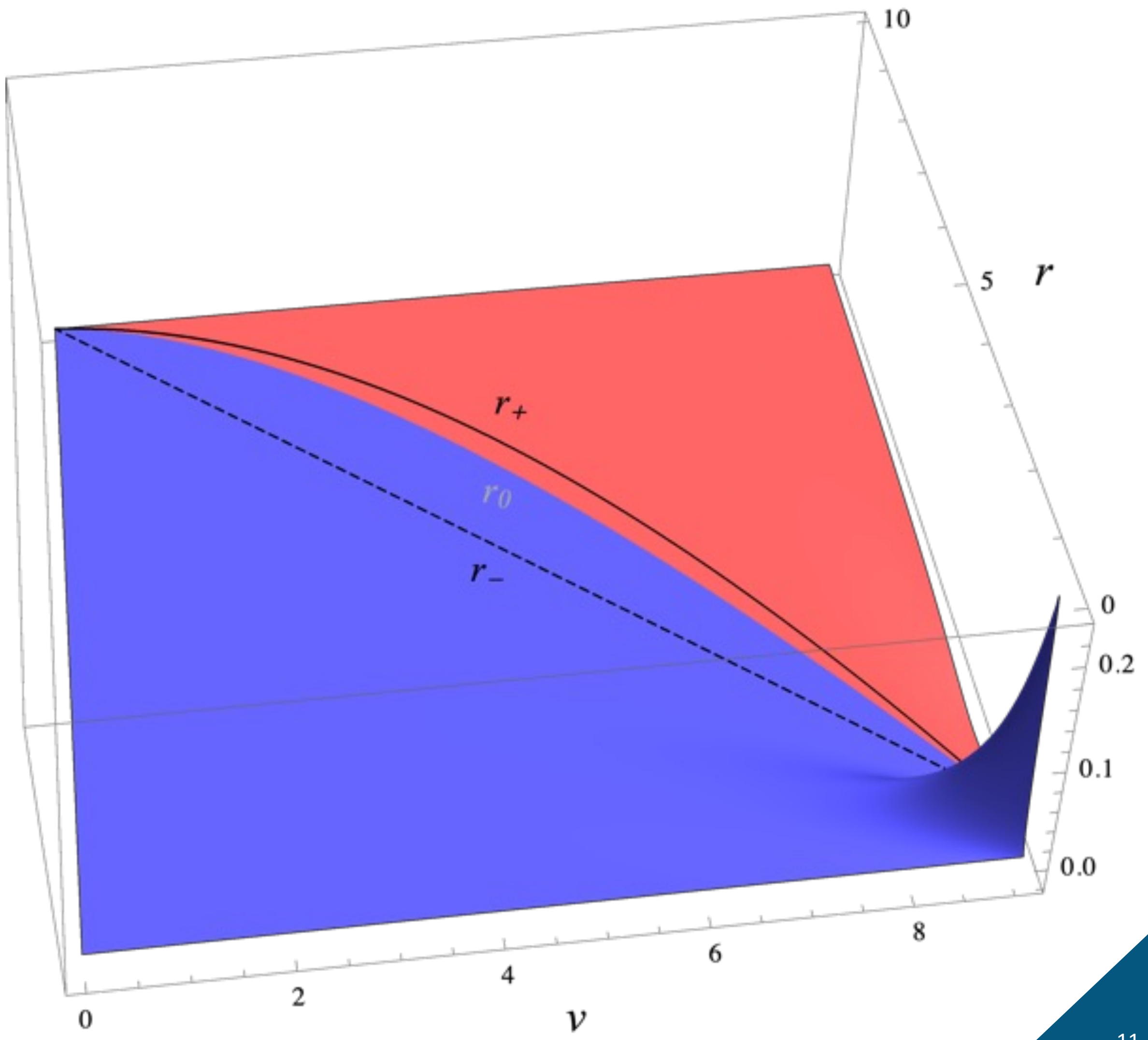
Null energy condition for dynamical RBHs

	$0 \leqslant r \leqslant r_-$	$r_- < r \leqslant r_0$	$r_0 < r \leqslant r_+$
$T_{\mu\nu}\ell^\mu\ell^\nu \overset{?}{\geqslant} 0$	✓	✓	✗

TABLE I. Overview of NEC-non-violating (✓) and NEC-violating (✗) regions of an evaporating RBH with a nondegenarate outer horizon [$b = 1$]. If the outer horizon is degenerate [$b > 1$], the NEC-violating region is given by $r_0 < r < r_+$, i.e. it no longer includes the outer horizon $r = r_+$ itself.



$$f(v,r) = g(v,r)\big(r - r_-(v)\big)^a \big(r - r_+(v)\big)^b$$



Trajectories of timelike observers/massive particles

Lagrangian for radially moving timelike observers: $\mathcal{L} = \frac{1}{2} f \dot{v}^2 - \dot{v}\dot{r}$

Euler–Lagrange equations:

$$\ddot{v} = -\frac{1}{2}(\partial_r f)\dot{v}^2$$
$$\ddot{r} = \frac{1}{2}(\partial_v f)\dot{v}^2 - \frac{1}{2}(\partial_r f)$$

$$u^\mu u_\mu = -1 \quad \Rightarrow \quad -f\dot{v}^2 + 2\dot{v}\dot{r} = -1$$

$$\Rightarrow \quad \dot{v} = \frac{\dot{r} \pm \sqrt{\dot{r}^2 + f}}{f}$$

Real-valued solutions inside of trapped region if $\dot{r} \leqslant -\sqrt{-f}$

Untrapped region:

$$0 \leqslant r < r_- \quad \& \quad r > r_+$$

"+" (for both ingoing/outgoing trajectories)

Trapped region:

$$r_- < r < r_+$$

"+" (ingoing trajectories)

"−" (outgoing trajectories)

Trajectories of timelike observers/massive particles

$$r < r_-$$

Alice begins her journey from the untrapped region on an outgoing geodesic.

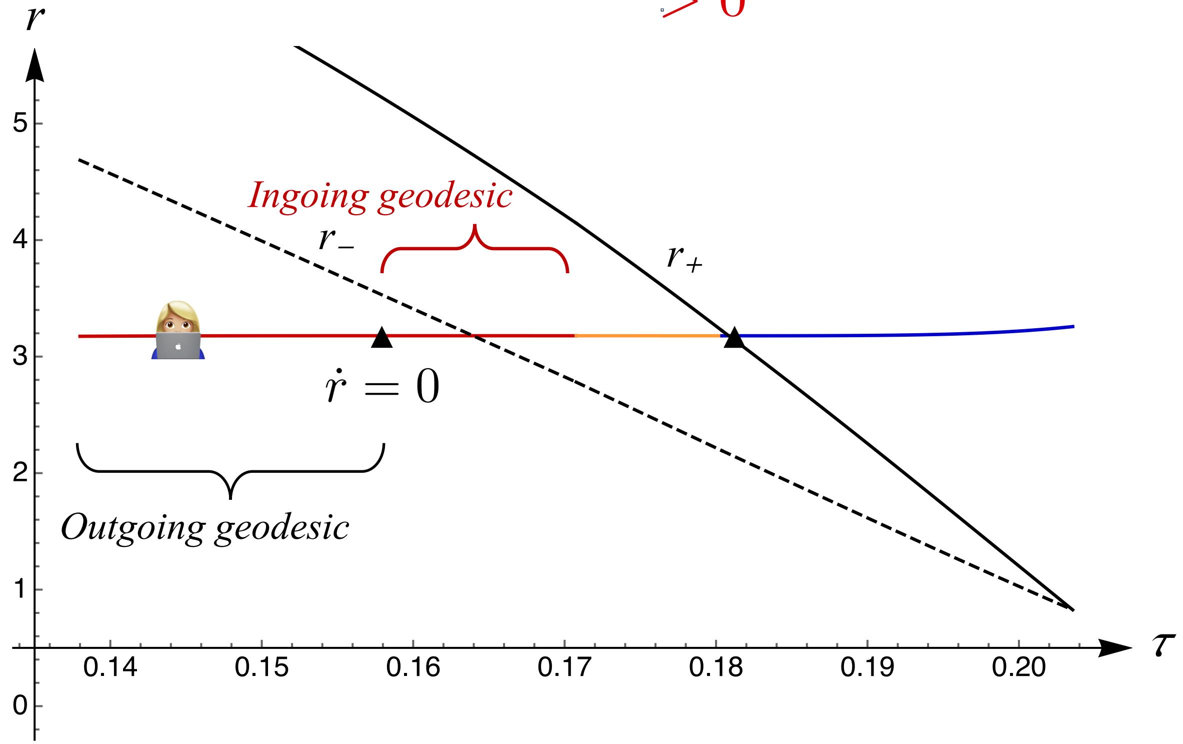
$$\dot{v} = \frac{\dot{r} + \sqrt{\dot{r}^2 + f}}{f}$$

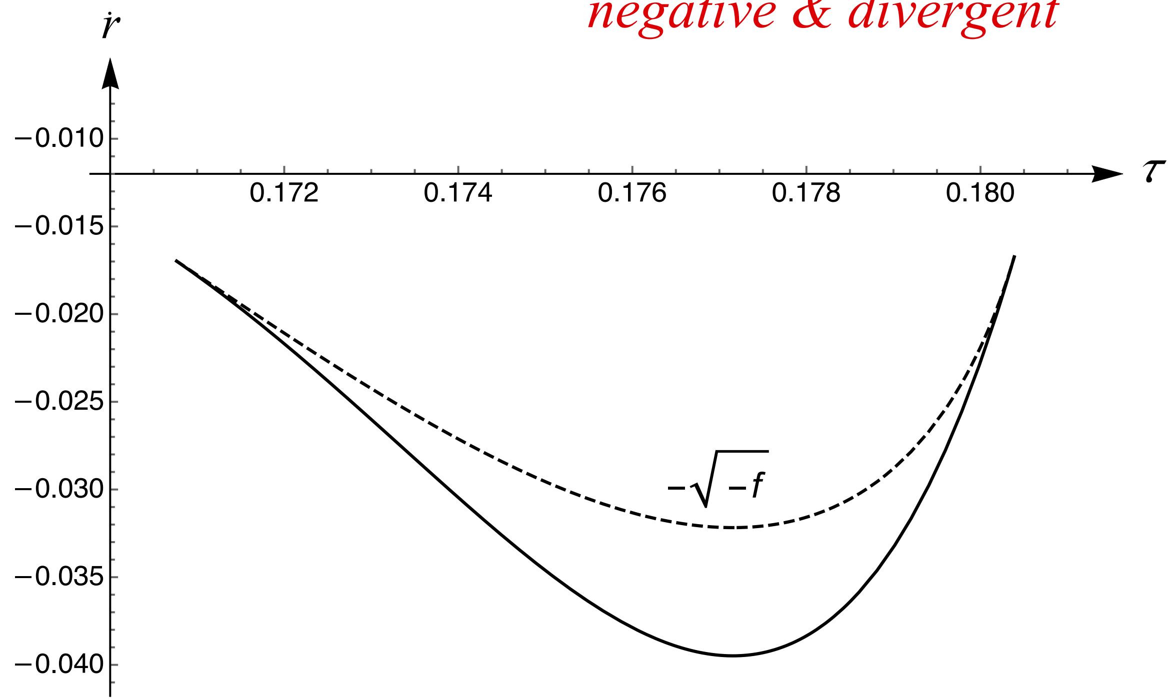
Radial acceleration on approach to the inner horizon r_- :

$$\ddot{r} = \frac{(-1)^{a+b} 2a\dot{r}^2}{g(v, r_-)(r_+ - r_-)^b} \frac{r'_-(v)}{|r - r_-|^{a+1}} + \mathcal{O}\left(\frac{1}{(r - r_-)^a}\right) \Rightarrow \ddot{r} < 0$$

even ($a+b$); $r'_-(v) < 0$; $g(v, r_-) > 0$

negative & divergent





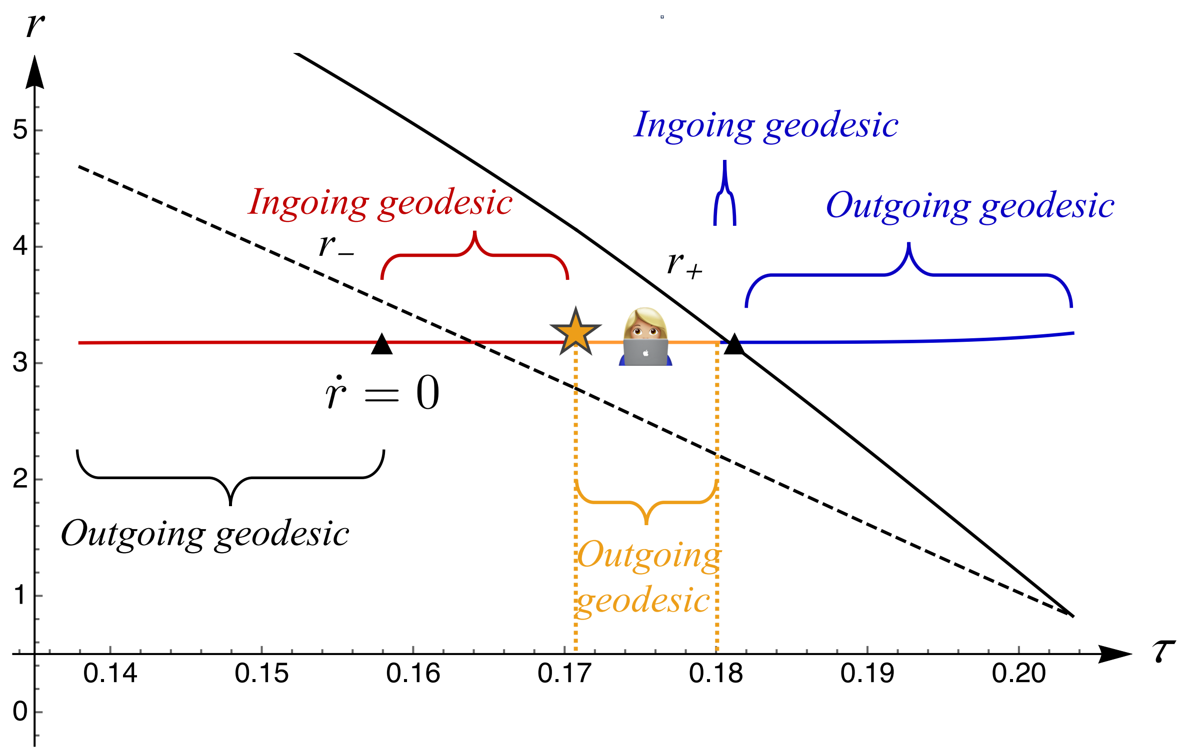
Trajectories of timelike observers/massive particles

Inside of the trapped region: $\dot{r} < 0$

$r_- < r < r_+$ $\quad \ddot{r} < 0$

At ★: Transition from ingoing to outgoing geodesic

$$\dot{v} = \frac{\dot{r} + \sqrt{\dot{r}^2 + f}}{f} \quad \Rightarrow \quad \dot{v} = \frac{\dot{r} - \sqrt{\dot{r}^2 + f}}{f}$$

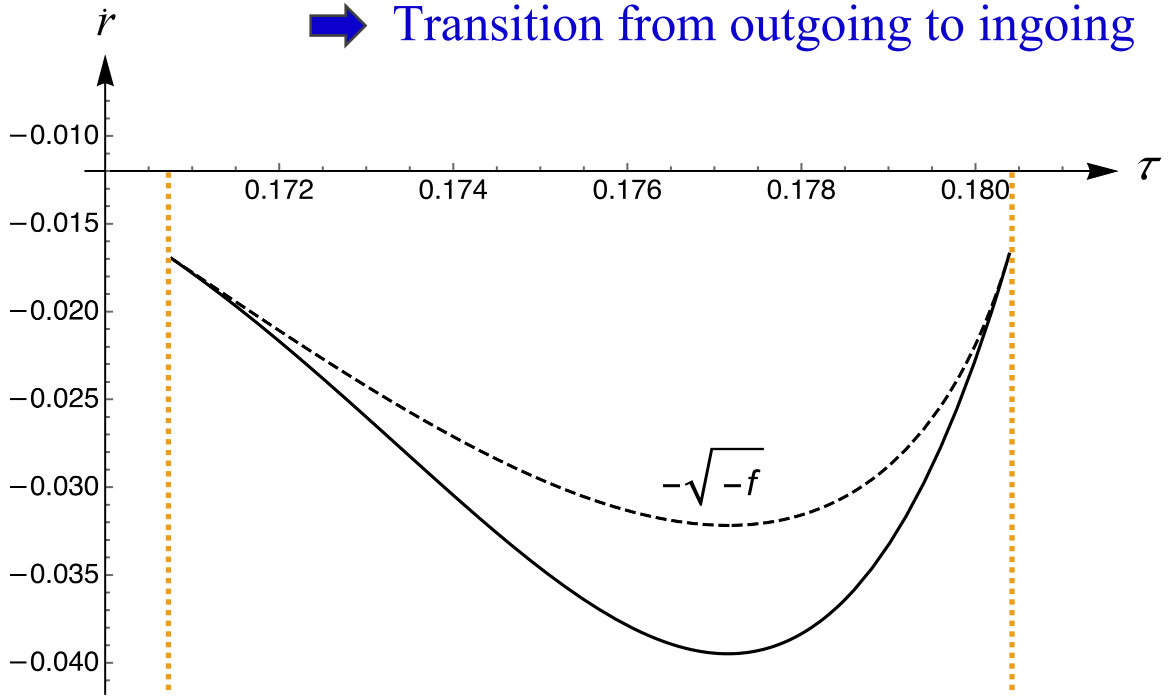


Radial acceleration on approach to the outer horizon r_+ :

$$\ddot{r} = \frac{2b\dot{r}^2}{g(v, r_+)(r_+ - r_-)^a} \frac{-r'_+(v)}{|r - r_+|^{b+1}} + \mathcal{O}\left(\frac{1}{|r - r_+|^b}\right)$$

$$\Rightarrow \quad \ddot{r} > 0$$

➡ Transition from outgoing to ingoing



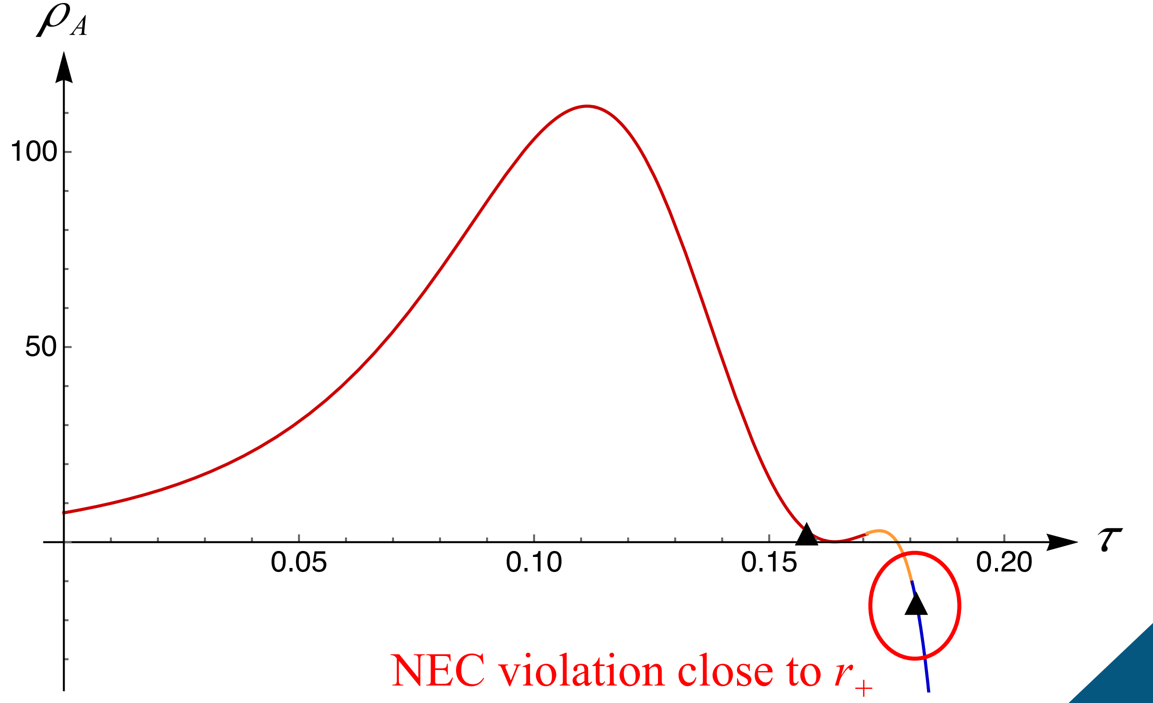
Energy density at the horizon crossing

Horizon crossings always occur on an ingoing geodesic. Close to both horizons $f \simeq 0$ and expansion of the ingoing trajectory leads to

$$\dot{v}\big|_{r_\pm} = -\frac{1}{2\dot{r}} + \frac{f}{8\dot{r}^3} + \mathcal{O}(f^2)$$

$$\rho_A\big|_{r_\pm} = \left[\frac{1}{8\pi r^2}\left(1 - r\partial_r f - \frac{r}{4\dot{r}^2}\partial_v f\right)\right]\Bigg|_{r_\pm} + \mathcal{O}(f)$$

➡ Energy density ρ_A observed by Alice upon crossing the horizons is finite (i.e. no firewalls).



Energy density ρ_A measured by Alice throughout her trajectory for the model proposed in

Carballo-Rubio *et al.*
J. High Energy Phys. **09**, 118 (2022)

Energy density at the horizon crossing

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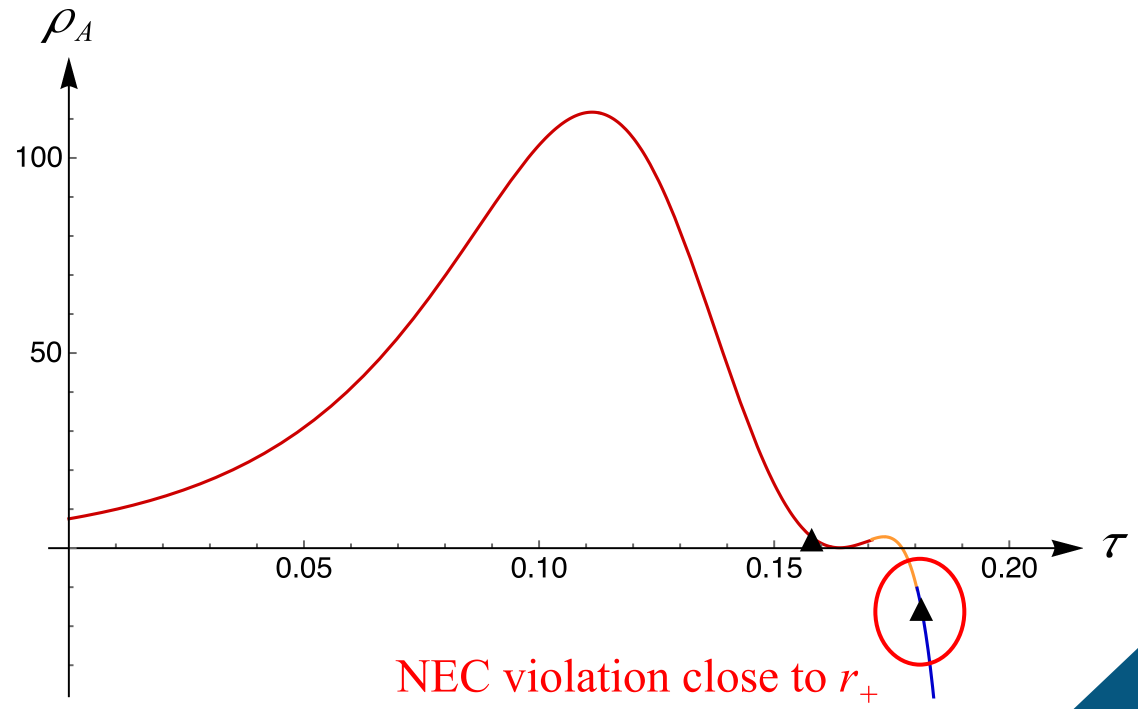
	$0 \leqslant r \leqslant r_-$	$r_- < r \leqslant r_0$	$r_0 < r \leqslant r_+$
$T_{\mu\nu}\ell^\mu\ell^\nu \overset{?}{\geqslant} 0$	✓	✓	✗

TABLE I. Overview of NEC-non-violating (✓) and NEC-violating (✗) regions of an evaporating RBH with a nondegenarate outer horizon [$b = 1$]. If the outer horizon is degenerate [$b > 1$], the NEC-violating region is given by $r_0 < r < r_+$, i.e. it no longer includes the outer horizon $r = r_+$ itself.



Trajectories of timelike observers/massive particles

What do we learn from this?

There is a unique way for massive observers and particles to escape the trapped region on a geodesic trajectory, whereby crossing the horizons is only possible on an ingoing geodesic.

This result has two important (one may argue pleasant) physical implications:

1. **Absence of firewalls**
   Energy densities measured by geodesic observers do not diverge.

2. **Natural resolution of the information loss problem**
   Particles and any information content associated with their existence on the manifold can escape the supposedly trapped spacetime region.

Summary #2 [Phys. Rev. D **108**, 124007 (2023)]

1. NEC is violated near r_+ and satisfied near r_- . Trapped spacetime region is separated.

	$0 \leqslant r \leqslant r_-$	$r_- < r \leqslant r_0$	$r_0 < r \leqslant r_+$
$T_{\mu\nu}\ell^\mu\ell^\nu \overset{?}{\geqslant} 0$	✓	✓	✗

2. Quantum effects are more pronounced near r_+ and towards the final stages of the evaporation process.

3. Timelike trajectories of massive observers/particles can exit the supposedly trapped region on an ingoing geodesic. No firewalls & no information loss.

Phys. Rev. D **108**, 124007 (2023)   arXiv:2309.06002 [gr-qc]

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BACKUP SLIDES

Semiclassical considerations

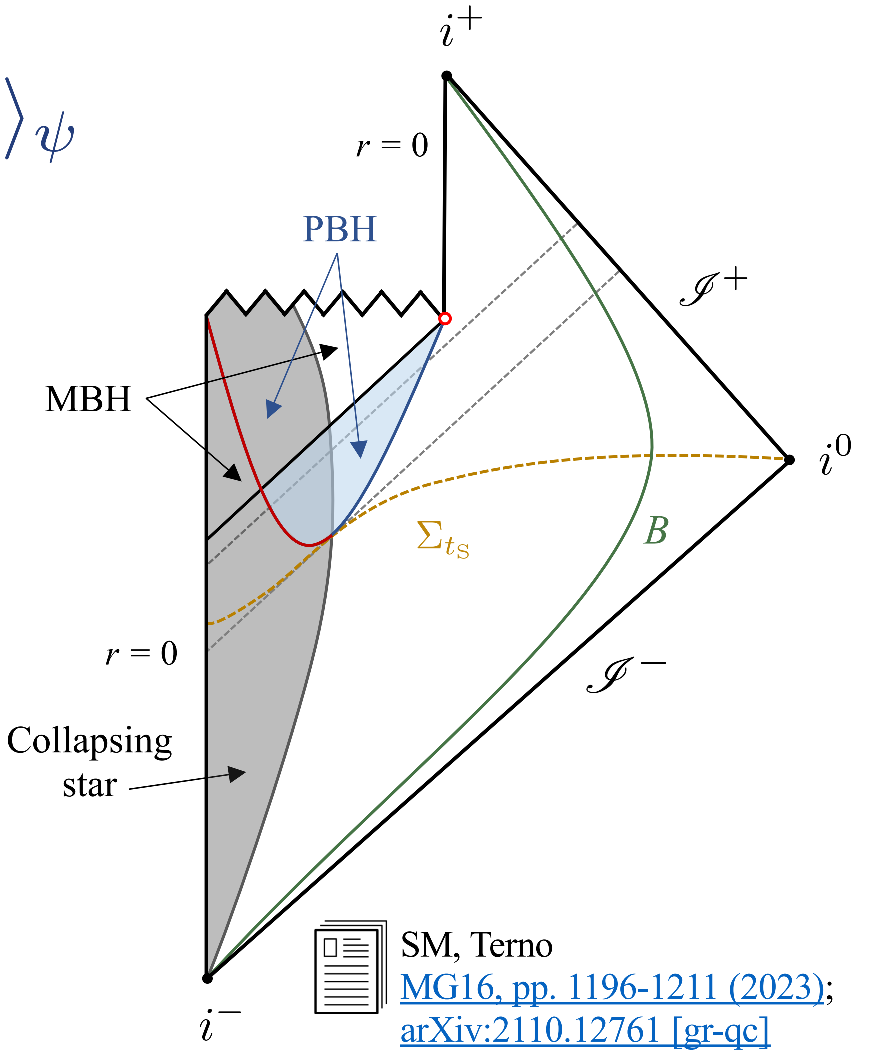
Semiclassical gravity: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \langle \hat{T}_{\mu\nu} \rangle_\psi$

1. Metric is modified by quantum effects.
   The resulting curvature satisfies semiclassical Einstein equations.

2. Renormalised EMT describes total matter content, i.e. both the original collapsing matter and the produced quantum excitations. Dynamics of collapsing matter is still described classically using metric.

3. Classical spacetime structure is still meaningful and described by metric; classical notions (e.g. horizons, trajectories) can be used.

<ins>No assumptions about:</ins>
global/asymptotic structure of spacetime; quantum state ψ; status of energy conditions; presence or absence of singularity; presence or absence of Hawking radiation.



SM, Terno
MG16, pp. 1196-1211 (2023); arXiv:2110.12761 [gr-qc]

Mann, SM, Terno
Int. J. Mod. Phys. D **31**, 2230015 (2022)

Review article with more details

International Journal of Modern Physics D | Vol. 31, No. 09, 2230015 (2022) | Review Paper

Black holes and their horizons in semiclassical and modified theories of gravity

Robert B. Mann, Sebastian Murk and Daniel R. Terno

https://doi.org/10.1142/S0218271822300154

International Journal of Modern Physics D
GRAVITATION ▪ ASTROPHYSICS ▪ COSMOLOGY

arXiv:2112.06515 [gr-qc]

Int. J. Mod. Phys. D **31**, 2230015 (2022)

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Abstract

For distant observers, black holes are trapped spacetime domains bounded by apparent horizons. We review properties of the near-horizon geometry emphasizing the consequences of two common implicit assumptions of semiclassical physics. The first is a consequence of the cosmic censorship conjecture, namely, that curvature scalars are finite at apparent horizons. The second is that horizons form in finite asymptotic time (i.e. according to distant observers), a property implicitly assumed in conventional descriptions of black hole formation and evaporation. Taking these as the only requirements within the semiclassical framework, we find that in spherical symmetry only two classes of dynamic solutions are admissible, both describing evaporating black holes and expanding white holes. We review their properties and present the implications. The null energy condition is violated in the vicinity of the outer horizon and satisfied in the vicinity of the inner apparent/anti-trapping horizon. Apparent and anti-trapping horizons are timelike surfaces of intermediately singular behavior, which manifests itself in negative energy density firewalls. These and other properties are also present in axially symmetric solutions. Different generalizations of surface gravity to dynamic spacetimes are discordant and do not match the semiclassical results. We conclude by discussing signatures of these models and implications for the identification of observed ultra-compact objects.

Dynamical solutions in spherical symmetry

Only two metric families can describe the geometry near an apparent horizon formed in finite time of a distant observer:

Review article:

Mann, SM, Terno
Int. J. Mod. Phys. D **31**, 2230015 (2022)

Table 2. Properties of the four types of Vaidya metrics. The Einstein equations have real solutions at finite time $t > t_S$ only if the NEC is violated.

	$\mathrm{sgn}(T_{tt})$	$\mathrm{sgn}(T_t{}^r)$	Time-evolution of Vaidya mass function	Black/ White hole	NEC violation	
Evaporating black holes	$-$	$-$	$C'(v) < 0$	B	✓	
	$-$	$+$	$C'(u) > 0$	W	✓	Accreting white holes
	$+$	$-$	$C'(u) < 0$	W	✗	
	$+$	$+$	$C'(v) > 0$	B	✗	

Each of the two metric families has two classes of solutions: $\lim_{r \to r_g} T^{\text{eff}}_{\mu\nu} \sim \pm \Upsilon(t)^2 f(t,r)^k, \quad k \in \{0, 1\}$

$k = 1$ at formation/disappearance $v = v_{\text{f}|\text{d}}$, $k = 0 \; \forall \; v \in (v_{\text{f}}, v_{\text{d}})$

Semiclassical considerations

Semiclassical gravity: $$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi \langle \hat{T}_{\mu\nu} \rangle_\psi$$

$(-,+,+,+)$

$c = G = \hbar = k_B = 1$

Spherical symmetry: $$ds^2 = -e^{2h(v,r)} f(v,r) dv^2 + 2e^{h(v,r)} dv dr + r^2 d\Omega^2$$

Integrating factor in coordinate transformations, e.g.

$$dt = e^{-h} \left(e^{h_+} dv - f^{-1} dr \right)$$

$$f(v,r) := \partial_\mu r \partial^\mu r = 1 - \frac{C(v,r)}{r}$$

Misner-Sharp mass

Misner, Sharp
Phys. Rev. **136**, B571 (1964)

Only assumption: Regular apparent horizon forms in finite time of a distant observer.

(Scenario III of gravitational collapse)

Unique!

SM, Terno
Phys. Rev. D **103**, 064082 (2021)

Mann, SM, Terno
Int. J. Mod. Phys. D **31**, 2230015 (2022)

Semiclassical gravity: spherically symmetric setup

$$ds^2 = -e^{2h(t,r)} f(t,r) dt^2 + f(t,r)^{-1} dr^2 + r^2 d\Omega$$

where $f(v,r) := \partial_\mu r \partial^\mu r = 1 - \frac{C(v,r)}{r}$

Misner-Sharp mass

Misner, Sharp
[Phys. Rev. **136**, B571 (1964)]

Einstein equations

$$\partial_r C = 8\pi r^2 \tau_t / f$$
$$\partial_t C = 8\pi r^2 e^h \tau_t{}^r$$
$$\partial_r h = 4\pi r \left(\tau_t + \tau^r\right) / f^2,$$

Curvature scalars:

$$\mathrm{T} := \left(\tau^r - \tau_t\right) / f$$
$$\mathfrak{T} := \left(\left(\tau^r\right)^2 + \left(\tau_t\right)^2 - 2\left(\tau_t{}^r\right)^2\right) / f^2$$

Effective EMT components:

$$\tau_t := e^{-2h} T_{tt} \ , \quad \tau_t{}^r := e^{-h} T_t{}^r \ , \quad \tau^r := T^{rr}$$

Solutions are characterised by scaling behaviour of EMT close to horizon: $\lim_{r \to r_g} \tau \sim \pm \Upsilon(t)^2 f(t,r)^k$

Terno, [Phys. Rev. D **101**, 124053 (2020)]
SM, Terno, [Phys. Rev. D **103**, 064082 (2021)]

Only two values of k are consistent: $k \in \{0, 1\}$

Both classes violate the NEC.

Dynamical solutions in spherical symmetry

SEBASTIAN MURK

PHYS. REV. D **105,** 044051 (2022)

	$k = 0$ solutions		$k = 1$ solution	
Metric functions	$C = r_g - c_{12}\sqrt{x} + \sum_{j\geq 1}^{\infty} c_j x^j$	(k0.1)	$C = r_g + x - c_{32}x^{3/2} + \sum_{j\geq 2}^{\infty} c_j x^j$	(k1.1)
$x := r - r_g$	$h = -\frac{1}{2}\ln\frac{x}{\xi} + \sum_{j\geq\frac{1}{2}}^{\infty} h_j x^j$	(k0.2)	$h = -\frac{3}{2}\ln\frac{x}{\xi} + \sum_{j\geq\frac{1}{2}}^{\infty} h_j x^j$	(k1.2)
Leading coefficient	$c_{12} = 4\sqrt{\pi} r_g^{3/2}\Upsilon$	(k0.3)	$c_{32} = 4r_g^{3/2}\sqrt{-\pi e_2/3}$	(k1.3)
Horizon dynamics	$r_g' = \pm c_{12}\sqrt{\xi}/r_g$	(k0.4)	$r_g' = \pm c_{32}\xi^{3/2}/r_g$	(k1.4)

Describes black holes immediately after their formation (and for the rest of their lifetime).

Describes the formation of black holes.

Both violate the NEC near the horizon!

The formation of black holes follows a unique scenario that involves both classes of solutions!

The transition between them is continuous.

Details: SM, Terno, Phys. Rev. D **103**, 064082 (2021)

Nonsingular trapped spacetime regions

Radial null geodesic congruences:

$$\theta_- = -\frac{2}{r}\,, \quad \theta_+ = \frac{f(v,r)}{r}$$

Hayward
Phys. Rev. D **49**, 6467 (1994)

⇨ Existence of trapped region: $\theta_-\theta_+ \overset{?}{\lessgtr} 0$

Presence of trapped region is signified by $\theta_-\theta_+ > 0$, which implies $f < 0$ inside, $f > 0$ outside of the trapped region.

$$f(v,r) = g(v,r)\big(r - r_-(v)\big)^a\big(r - r_+(v)\big)^b$$

➡ $g > 0$ and b odd.

"Disappearance point":

$r_-(v_d) \equiv r_+(v_d)$

$$\theta_-\theta_+\big|_{v_\mathrm{d}} = -\frac{2}{r^2} g(v_\mathrm{d}, r)\big(r - r_+(v_\mathrm{d})\big)^{a+b} \leqslant 0 \quad \forall\, r$$

➡ Sum $a + b$ must be even.

➡ a odd.

SM, Soranidis
Phys. Rev. D **108**, 044002 (2023)

Generalized dynamical first law

Using $f(v,r) := \partial_\mu r \partial^\mu r = 1 - \dfrac{C(v,r)}{r}$

$$C(v,r) = r_+(v) + \sum_{i=1}^{\infty} w_i(v)\left(r - r_+(v)\right)^i$$

$$\Rightarrow \quad \kappa_K\Big|_{r=r_+} = \frac{1}{2}\partial_r f(v,r) = \frac{1}{2}\left[C(v,r) - r\partial_r C(v,r)\right]\Big|_{r=r_+} = \boxed{\frac{1-w_1}{2r_+}}$$

1st law: $$\frac{\delta M}{\delta r_+} = \frac{\kappa}{8\pi}\frac{\delta A}{\delta r_+} \Rightarrow \delta M\Big|_{r=r_+} = \frac{1-w_1}{16\pi r_+}\delta A$$

MS mass: $$\delta M\Big|_{r=r_+} = \frac{\delta C}{2}\Big|_{r=r_+} = \frac{1}{2}\left(1-w_1\right)\delta r_+$$

$$\delta\left(\frac{r_+}{2}\right) = \frac{1-w_1}{16\pi r_+}\delta A + \frac{w_1}{2}\delta r_+$$

$$\delta M = \frac{\kappa}{8\pi}\delta A - p\delta V$$

$$p = -\frac{w_1}{8\pi r_+^2}$$

Using $A = 4\pi r_+^2$, $V = \frac{4}{3}\pi r_+^3$

$\left(\Rightarrow \delta A = 8\pi r_+ \delta r_+ = \frac{2}{r_+}\delta V\right)$

Generalized dynamical first law:

$$\Rightarrow \quad \boxed{\delta\left(\frac{r_+}{2}\right) = \frac{1-w_1}{16\pi r_+}\delta A + \frac{w_1}{8\pi r_+^2}\delta V}$$

Dynamical generalization of inner-extremal RBH model $(a = 3, b = 1)$

Recall: $\kappa_K\big|_{r_+} = \dfrac{1 - w_1(v)}{2r_+(v)}$

$$w_1\big|_{r=r_+} = 1 - g(v, r_+)r_+(r_+ - r_-)^a$$

At formation/disappearance: $k = 1$

$r_-(v_{\mathrm{f|d}}) \equiv r_+(v_{\mathrm{f|d}})$ ➡ $w_1(v_{\mathrm{f|d}}) = 1$

Value of $w_1(v)$ indicates transition from

$k = 1 \to k = 0$ at formation

$[w_1(v_{\mathrm{f}}) = 1 \to w_1(v > v_{\mathrm{f}}) < 1]$

$k = 0 \to k = 1$ at disappearance

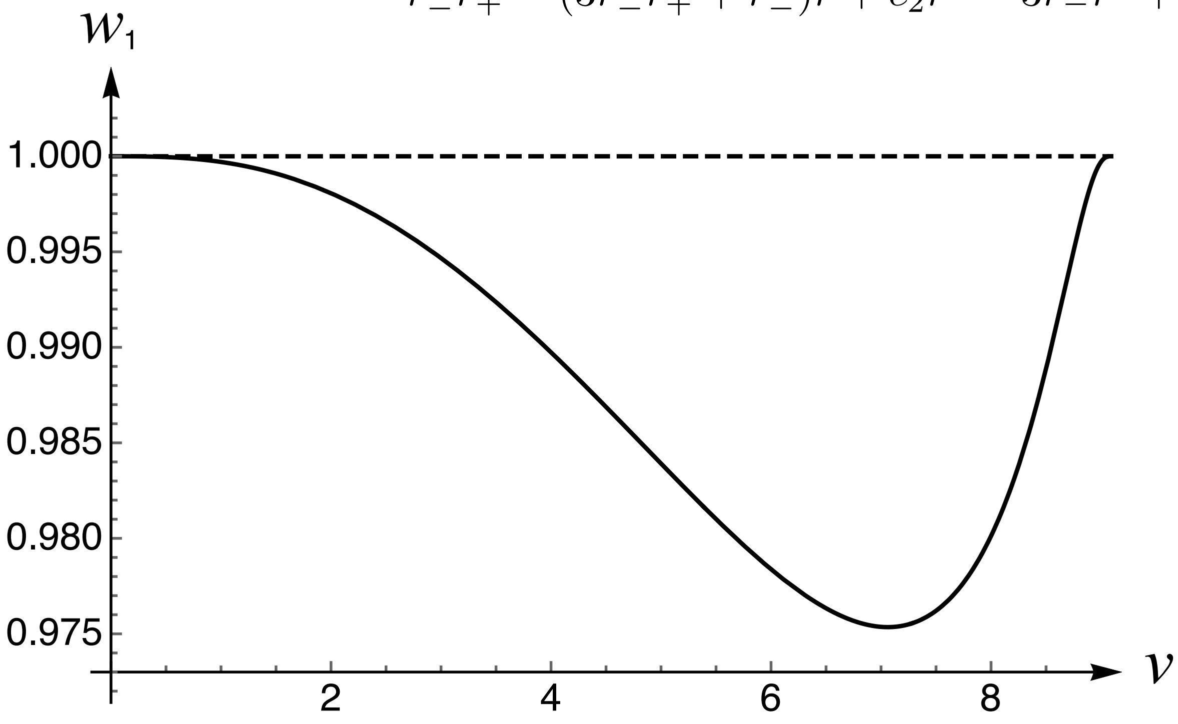
$[w_1(v < v_{\mathrm{d}}) < 1 \to w_1(v_{\mathrm{d}}) = 1]$

Evolution of the linear coefficient in the MS mass for the model proposed in

Carballo-Rubio *et al.*
J. High Energy Phys. **09**, 118 (2022)

$$f(v, r) = g(v, r)(r - r_-)^3(r - r_+)$$

$$g(v, r) = \frac{1}{r_-^3 r_+ - (3r_-^2 r_+ + r_-^3)r + c_2 r^2 - 3r_- r^3 + r^4}$$



For evaporating RBHs r_- and r_+ are timelike

Define hypersurface Σ by restricting the coordinates via $\Phi(\Sigma_{r_\xi}) =: r - r_\xi \equiv 0$

Inner and outer horizon correspond to the constraint $\Phi(\Sigma_{r_\pm}) = r - r_\pm \equiv 0$

which leads to a normal vector defined by $n_\mu =: \eta \partial_\mu \Phi(\Sigma_{r_\pm}) = \eta \left(-r'_\pm, 1, 0, 0\right)$

normalisation factor

➡ Inner product at r_- / r_+ : $n_\mu n^\mu \Big|_{r_\pm} = -2\eta^2 r'_\pm$

For evaporating RBHs $[r'_\pm < 0]$, this inner product is spacelike: $n_\mu n^\mu > 0$

➡ Causal character of the horizons is timelike.

Generalized dynamical charged Hayward–Frolov RBH

SM, Soranidis
Phys. Rev. D **108**, 044002 (2023)

Generalized dynamical metric function: $f(v,r) = 1 - \frac{\left(r_g(v)r - q(v)^2\right) r^2}{r^4 + \left(r_g(v)r + q(v)^2\right) l(v)^2}$

Kodama surface gravity: $\kappa_{K_{\mathrm{HF}}} = \frac{1 - w_1(v,l)}{2r_+(v,l)}$

Horizons:

$$r_-(v,l) = r_-(v) + \beta_-(v)l^2 + \mathcal{O}\left(l^3\right)$$
$$r_+(v,l) = r_+(v) + \beta_+(v)l^2 + \mathcal{O}\left(l^4\right)$$

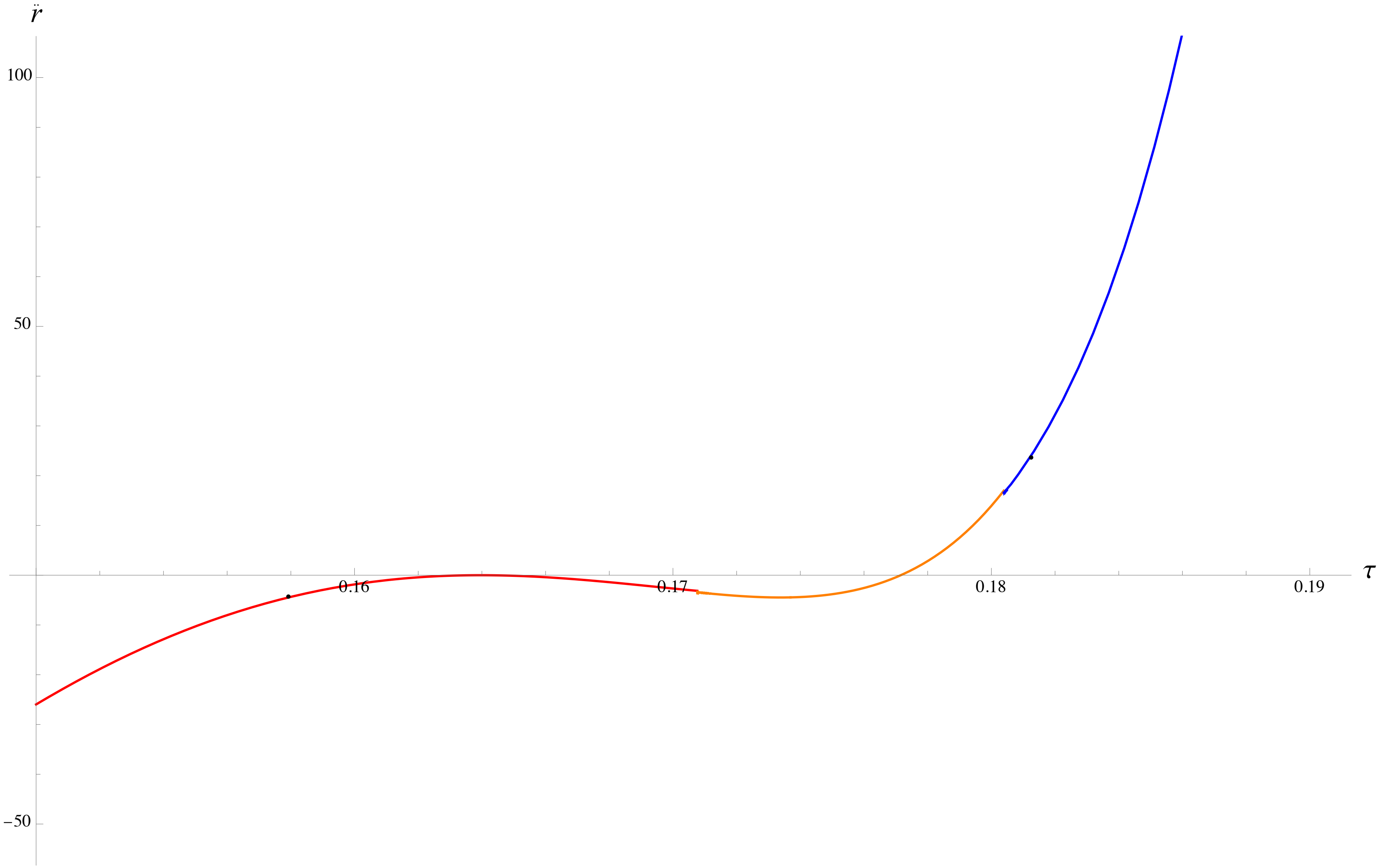
$$r_-(v) = m(v) - \sqrt{m(v)^2 - q(v)^2}$$
$$r_+(v) = m(v) + \sqrt{m(v)^2 + q(v)^2}$$

Consider MS expansion: $w_1(v,l) = \frac{q(v)^2}{r_+(v)^2} + \beta(v)l^2 + \mathcal{O}\left(l^4\right) \Rightarrow \kappa_{K_{\mathrm{HF}}} = \frac{r_+(v) - r_-(v)}{2r_+(v)^2} + \mathcal{O}\left(l^2\right)$

Differences:

1. Inner horizon $r_- \neq 0$ even if $l = 0$ due to the presence of a charged term that is independent of l.
2. Compatibility with the first law is no longer encoded by $w_1 = 0$. For $l \to 0$, the new compatibility condition can be stated as $$w_1(v,0) = \frac{q(v)^2}{r_+(v)^2}$$

Radial acceleration as function of Alice’s proper time



Kodama surface gravity

$$ds^2 = -f(v,r)dv^2 + 2dvdr + r^2 d\Omega^2 \qquad K^\mu = (1,0,0,0)$$

Kodama surface gravity: $$\kappa_K K_\nu := \frac{1}{2} K^\mu \left(\nabla_\mu K_\nu - \nabla_\nu K_\mu \right)$$

$$\nabla_\mu K^\mu = 0$$

$$\nabla_\mu J^\mu = 0\,, \quad J^\mu := G^{\mu\nu} K_\nu$$

Kodama surface gravity evaluated at outer horizon: $$\kappa_K \Big|_{r=r_+} = \frac{1}{2} \partial_r f(v,r) \Big|_{r=r_+}$$

$$= \lim_{r \to r_+} \frac{(r-r_+)^{-1+b} b g(v,r) (r-r_-)^a}{2}$$

Note: Nonzero Kodama surface gravity requires that outer horizon is nondegenerate, i.e. $b = 1$.

Surface gravity in stationary spacetimes

Hawking temperature: $$T_\mathrm{H} = \frac{\kappa}{2\pi}$$ (for observer at infinity)

Several equivalent definitions, related to either

Killing vector field

with norm $\sqrt{\xi^\mu \xi_\mu} = 0$

Inaffinity of null geodesics on the horizon: $$\xi^\mu{}_{;\nu}\xi^\nu := \kappa \xi^\mu$$

or

$x := r - r_\mathrm{g}$

$r \gtrsim r_\mathrm{g}$

Peeling off properties of null geodesics near the horizon: $$\frac{dr}{dt} = \pm 2\kappa_\mathrm{peel}(t) x + \mathcal{O}\left(x^2\right)$$

Surface gravity in dynamical spacetimes

In general dynamical spacetimes: no asymptotically timelike Killing vector.

Kodama, Prog. Theor. Phys. **63**, 1217 (1980)
Abreu, Visser, Phys. Rev. D **82**, 044027 (2010)
Kurpicz, Pinamonti, Verch, Lett. Math. Phys. **111**, 110 (2021)

Role of Hawking temperate captured either by **peeling** or **Kodama surface gravity**.

Barceló, Liberati, Sonego, Visser,
Phys. Rev. D **83**, 041501(R) (2011)

Indistinguishable for sufficiently slowly evolving horizons with properties close to their classical counterparts.

However: the similarity fails for dynamical spherically symmetric solutions!

Mann, SM, Terno,
Phys. Rev. D **105**, 124032 (2022)

Surface gravity in dynamical spacetimes: peeling surface gravity

Nielsen, Yoon, Class. Quantum Gravity **25**, 085010 (2008)
Cropp, Liberati, Visser, Class. Quantum Gravity **30**, 125001 (2013)

For example: k=0

$$C = r_g - c_{12}\sqrt{x} + \sum_{j\geqslant 1}^{\infty} c_j x^j$$

$$h = -\frac{1}{2}\ln\frac{x}{\xi} + \sum_{j\geqslant\frac{1}{2}}^{\infty} h_j x^j$$

Consider **peeling surface gravity:** $\kappa_{\text{peel}} = \dfrac{e^{h(t,r_g)}\left(1 - C'(t,r_g)\right)}{2r_g}$

With the metric functions C and h of the k=0 and k=1 solutions: $\kappa_{\text{peel}} \rightarrow \infty$

$$\frac{dr}{dt} = \pm r_g' + a_{12}(t)\sqrt{x} + \mathcal{O}(x)$$

Cf. stationary expression: $\dfrac{dr}{dt} = \pm 2\kappa_{\text{peel}}(t)x + \mathcal{O}\left(x^2\right)$

Nielsen, Visser,
Class. Quantum Gravity **23**, 4637 (2006)

Using Painlevé–Gullstrand coordinates $(\bar{t}, r)$: $\kappa_{\text{PG}_1} = \dfrac{1}{2r_g}\left(1 - \partial_r \bar{C}\right)\Big|_{r=r_g} \longrightarrow \kappa_{\text{PG}_1} = 0$

Mann, SM, Terno,
Phys. Rev. D **105**, 124032 (2022)

$\kappa_{\text{PG}_2} = \dfrac{1}{2r_g}\left(1 - \partial_r \bar{C} + \partial_{\bar{t}} \bar{C}\right)\Big|_{r=r} \longrightarrow$ 3 possibilities (0,∞,finite) depending on behaviour of $\bar{t}$

Surface gravity in dynamical spacetimes: Kodama surface gravity

Defined via $$\frac{1}{2}K^{\mu}\left(\nabla_{\mu}K_{\nu}-\nabla_{\nu}K_{\mu}\right):=\kappa_{\mathrm{K}}K_{\nu}$$ evaluated at horizon.

(v,r) coordinates

Kodama vector field: $$K^{\mu}=\left(e^{-h_{+}},0,0,0\right)$$

covariantly conserved: $$\nabla_{\mu}K^{\mu}=0,$$
$$\nabla_{\mu}J^{\mu}=0,\quad J^{\mu}:=G^{\mu\nu}K_{\nu}$$

Result: $$\kappa_{\mathrm{K}}=\frac{1}{2}\left(\frac{C_{+}(v,r)}{r^{2}}-\frac{\partial_{r}C_{+}(v,r)}{r}\right)\bigg|_{r=r_{+}}=\frac{(1-w_{1})}{2r_{+}}$$

- → 0 at formation of black hole.
- → Approaches static value $\kappa=1/(4M)$ only if metric is close to pure Vaidya metric. → **Contradicts semiclassical results.**

Mann, SM, Terno, Phys. Rev. D **105**, 124032 (2022)

Page evaporation law

Mass loss due to emission of Hawking radiation: $$\frac{dM}{dt} = -\sum_{j,\ell,m,p} \frac{1}{2\pi} \int_0^\infty \frac{\omega \Gamma_{j\omega\ell mp}}{e^{2\pi\omega/\kappa} - 1} d\omega$$

SM, Soranidis
Phys. Rev. D **108**, 044002 (2023)

Simplifying assumptions: $$m = \ell = 0$$

$$\Gamma \simeq \omega^2 r_g^2$$

Note:
Effects of Hawking radiation are described by ingoing Vaidya metric with decreasing mass $(C'(v) < 0)$.

$$\frac{dM}{dv} \simeq -\frac{a}{M^2} \Leftrightarrow \frac{dr_+}{dv} \simeq -\frac{\alpha}{r_+^2} \quad \Rightarrow \quad t_e \sim M_0^3$$

Explicit form of the coefficients and their expansion about $w_1 = 0$:

$$\alpha = 8a = -\frac{4}{\pi} \frac{1}{e^{\frac{4\pi}{1-w_1}} - 1},$$

$$\alpha = -\frac{4}{\pi} \frac{1}{e^{4\pi} - 1} + \mathcal{O}(w_1),$$

⇨ Standard Page evaporation law is modified if $w_1 = 0$ is not satisfied.

Information loss

"an isolated black hole will evaporate completely via the Hawking process within a finite time. If the correlations between the inside and outside of the black hole are not restored during the evaporation process, then by the time that the black hole has evaporated completely, an initial pure state will have evolved to a mixed state, i.e., information will have been lost. In a semiclassical analysis of the evaporation process, such information loss does occur and is ascribable to the propagation of the quantum correlations into the singularity within the black hole."

Wald,
Living Rev. Relativ. **4**, 6 (2001)

Review article

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International Journal of Modern Physics D | Vol. 31, No. 09, 2230015 (2022) | Review Paper

Black holes and their horizons in semiclassical and modified theories of gravity

Robert B. Mann, Sebastian Murk and Daniel R. Terno

https://doi.org/10.1142/S0218271822300154

arXiv:2112.06515 [gr-qc]

Int. J. Mod. Phys. D **31**, 2230015 (2022)

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Abstract

For distant observers, black holes are trapped spacetime domains bounded by apparent horizons. We review properties of the near-horizon geometry emphasizing the consequences of two common implicit assumptions of semiclassical physics. The first is a consequence of the cosmic censorship conjecture, namely, that curvature scalars are finite at apparent horizons. The second is that horizons form in finite asymptotic time (i.e. according to distant observers), a property implicitly assumed in conventional descriptions of black hole formation and evaporation. Taking these as the only requirements within the semiclassical framework, we find that in spherical symmetry only two classes of dynamic solutions are admissible, both describing evaporating black holes and expanding white holes. We review their properties and present the implications. The null energy condition is violated in the vicinity of the outer horizon and satisfied in the vicinity of the inner apparent/anti-trapping horizon. Apparent and anti-trapping horizons are timelike surfaces of intermediately singular behavior, which manifests itself in negative energy density firewalls. These and other properties are also present in axially symmetric solutions. Different generalizations of surface gravity to dynamic spacetimes are discordant and do not match the semiclassical results. We conclude by discussing signatures of these models and implications for the identification of observed ultra-compact objects.

Keywords: Semiclassical gravity ▪ modified gravity ▪ black holes ▪ apparent horizon ▪ evaporation ▪ white holes ▪ energy conditions ▪ thin shell collapse ▪ surface gravity ▪ information loss

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