Gravity and Cosmology (week 5) @YITP 2/28, 2024

# Physically consistent gravitational waveform for capturing beyond General Relativity effects in the compact object merger phase

Based on arXiv:2309.14061

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# Outline

# 1. Background and Motivation



## - Search for Physics in Strong Gravitational Fields by Observation

- **Few tests of General Relativity (GR) have been made in strong gravity regime, where the nonlinearities of gravity come into play.**
  - In the weak fields, GR has been verified typically with an accuracy of $10^{-5}$ [C.Will, 2014] , but with an accuracy of $\leq 10\%$ in the strong gravity regime [B.Abbott+, 2021].
  - More precise tests of GR in the strong gravity regime are needed !
  - **GR would break down at some point in the strong gravity regime, but where the break would occur and what theory would arise there ?**

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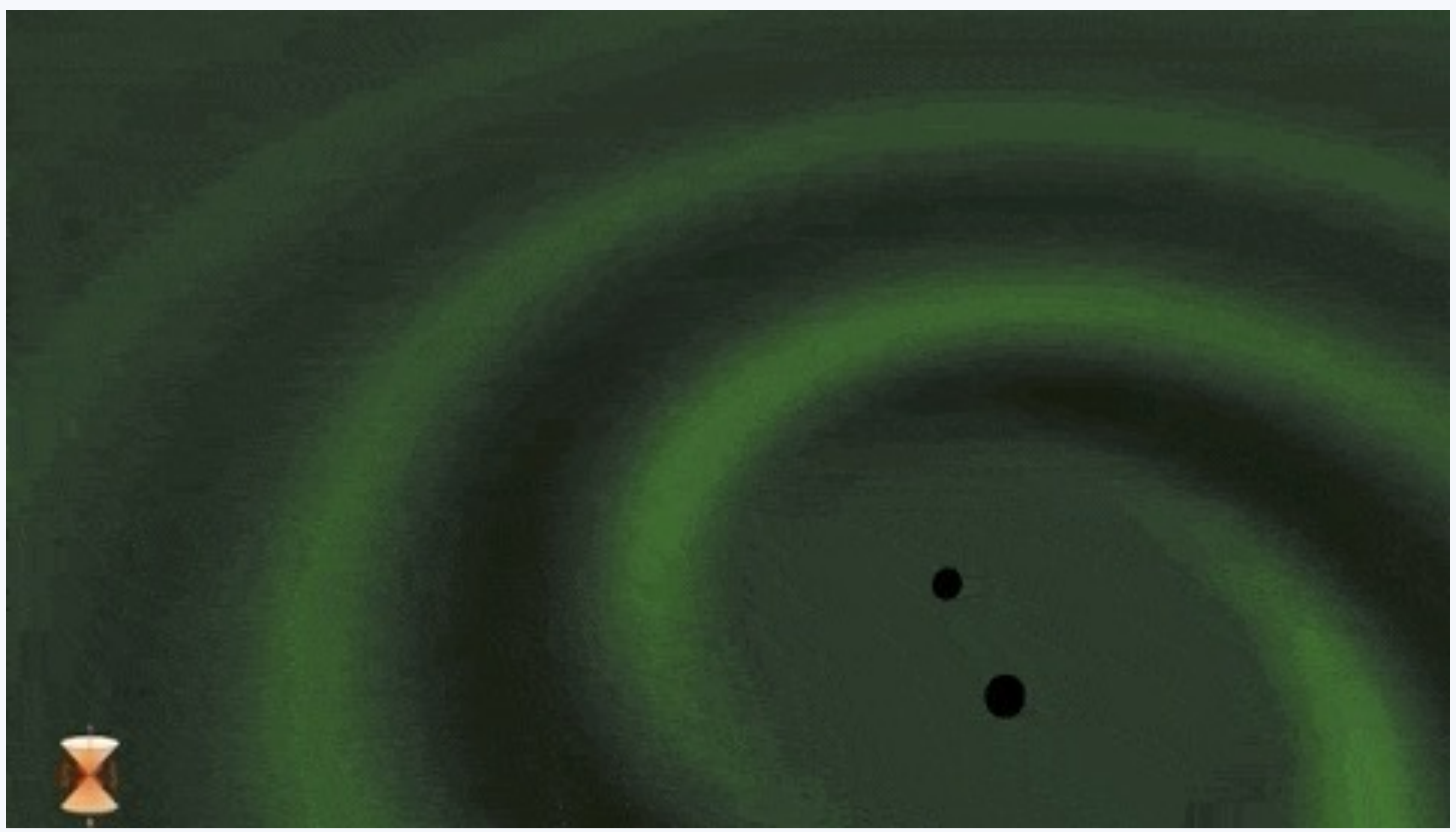
  - Further precise tests of GR in the nonlinear regime are needed !
  - GR would break down at some point in the strong gravity regime, but where the break would occur and what theory would arise there ?
- **The golden age of gravitational wave (GW) astronomy has begun !**
  - The 4$^{th}$ GW observation by LIGO/Virgo/KAGRA (LVK) collaboration has been operated and more precise tests of GR are expected.
  - In the next decades, more sensitive and multiband observations would be achieved by the 3$^{rd}$ generation detectors, LISA and so on.
  - **It is crucial to prepare a proper waveform depending on purpose.**

# 1. Background and Motivation
## - Binary Black Hole Coalescence

- Binary black hole (BBH) coalescences are the strongest gravitational phenomena that can be observed so far.

Dynamics of BBH coalescence



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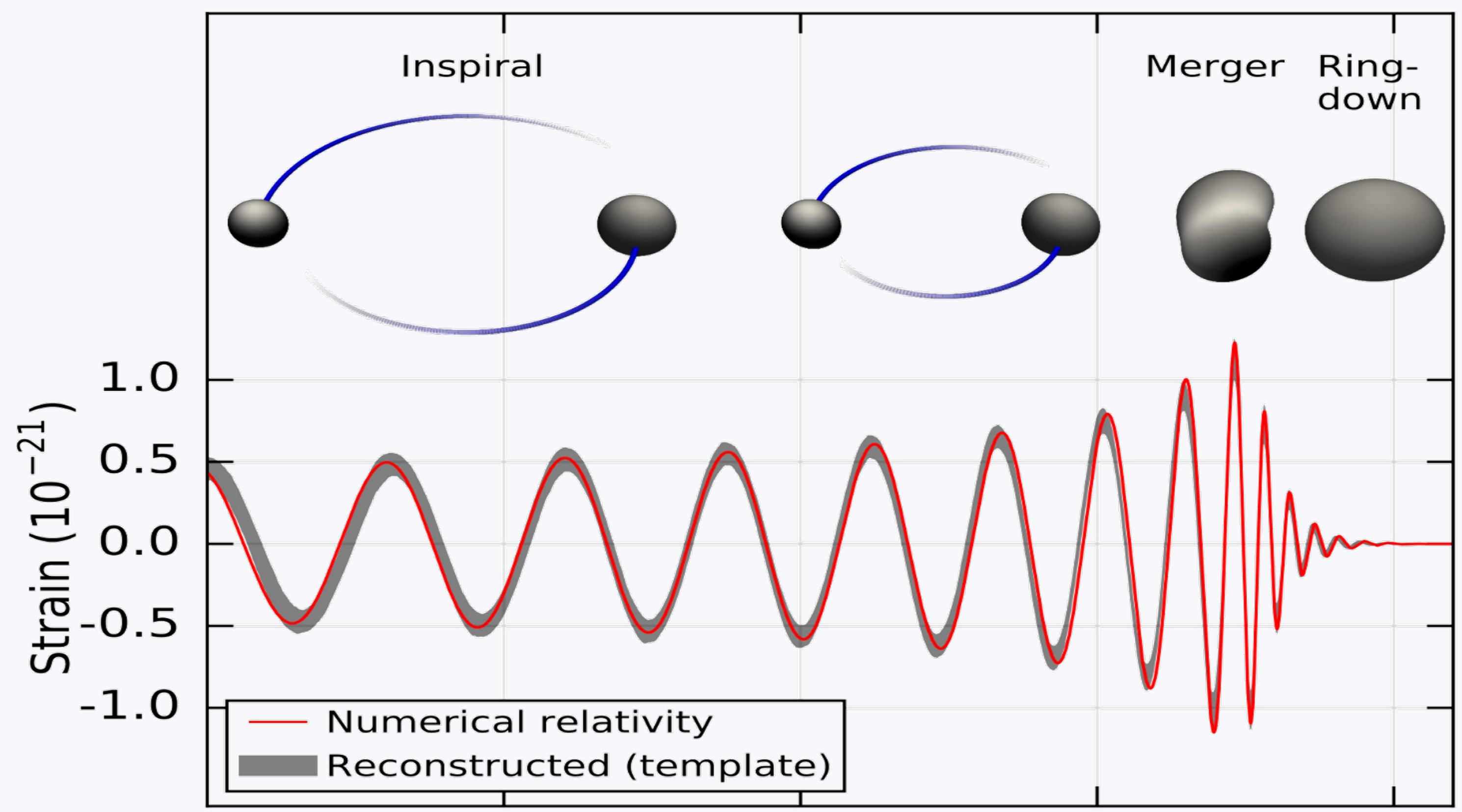
Corresponding GW signal

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# 1. Background and Motivation

## - Binary Black Hole Coalescence

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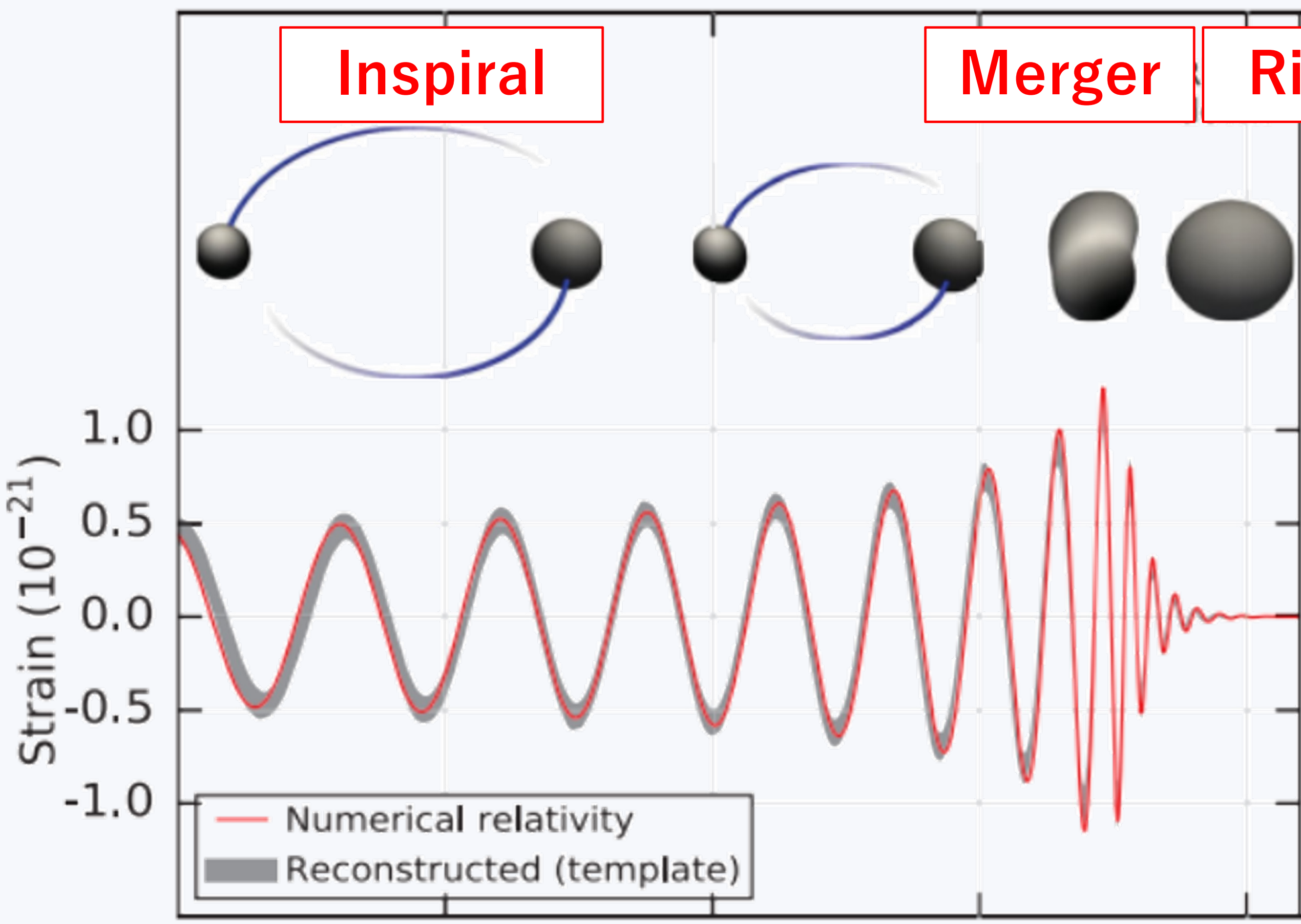


| | Inspiral | Merger | Ringdown |
|---|---|---|---|
| Dynamics | Quasi-circular orbit | Velocity $v \sim 0.5c$<br>Highly dynamical | Settling down to one stationary BH |
| Theoretical Approach | Post-Newtonian expansion | Numerical Relativity (NR) | BH perturbation theory |
| Signal | Periodic | Large amplitude<br>Rapid oscillation | Damped sinusoid |

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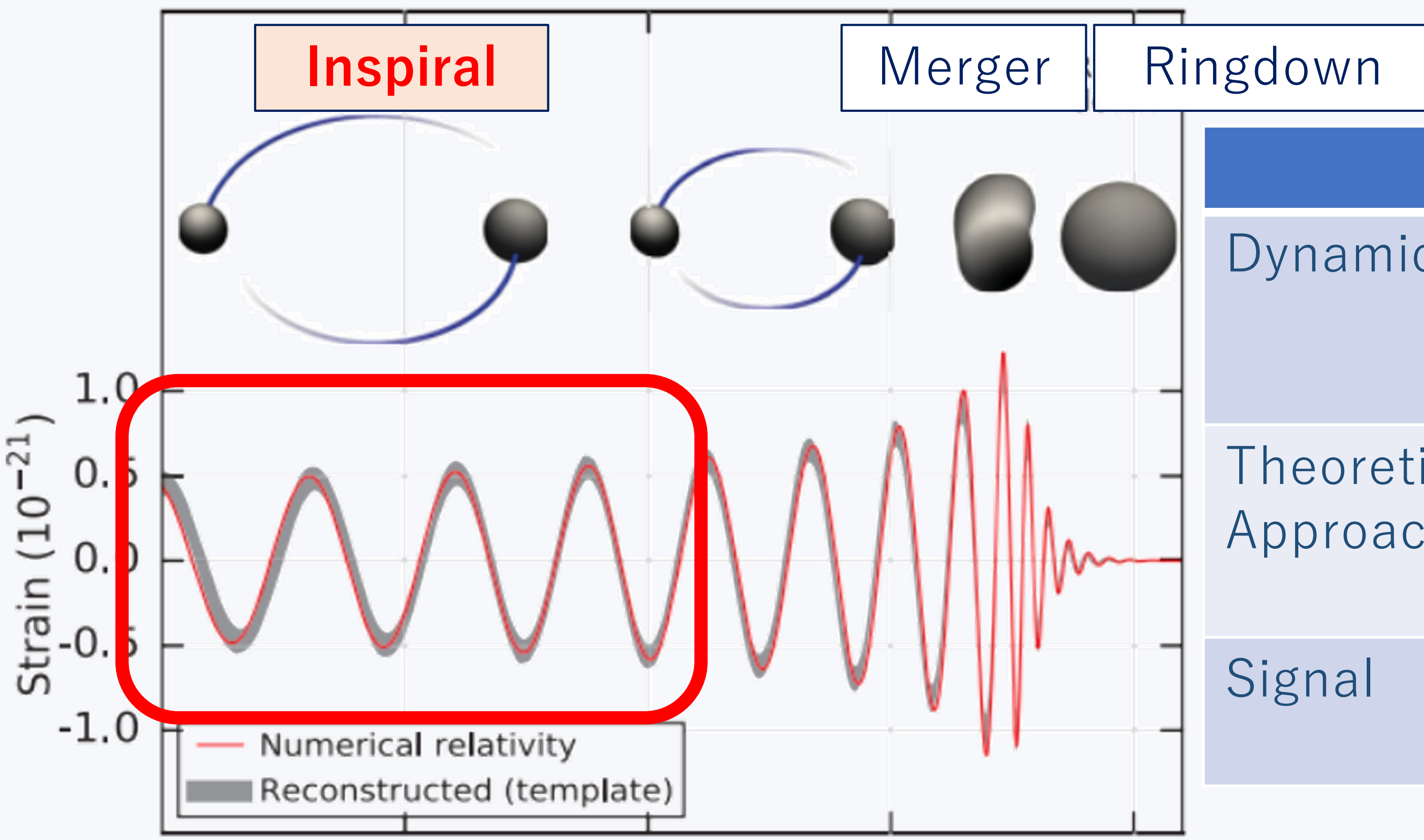


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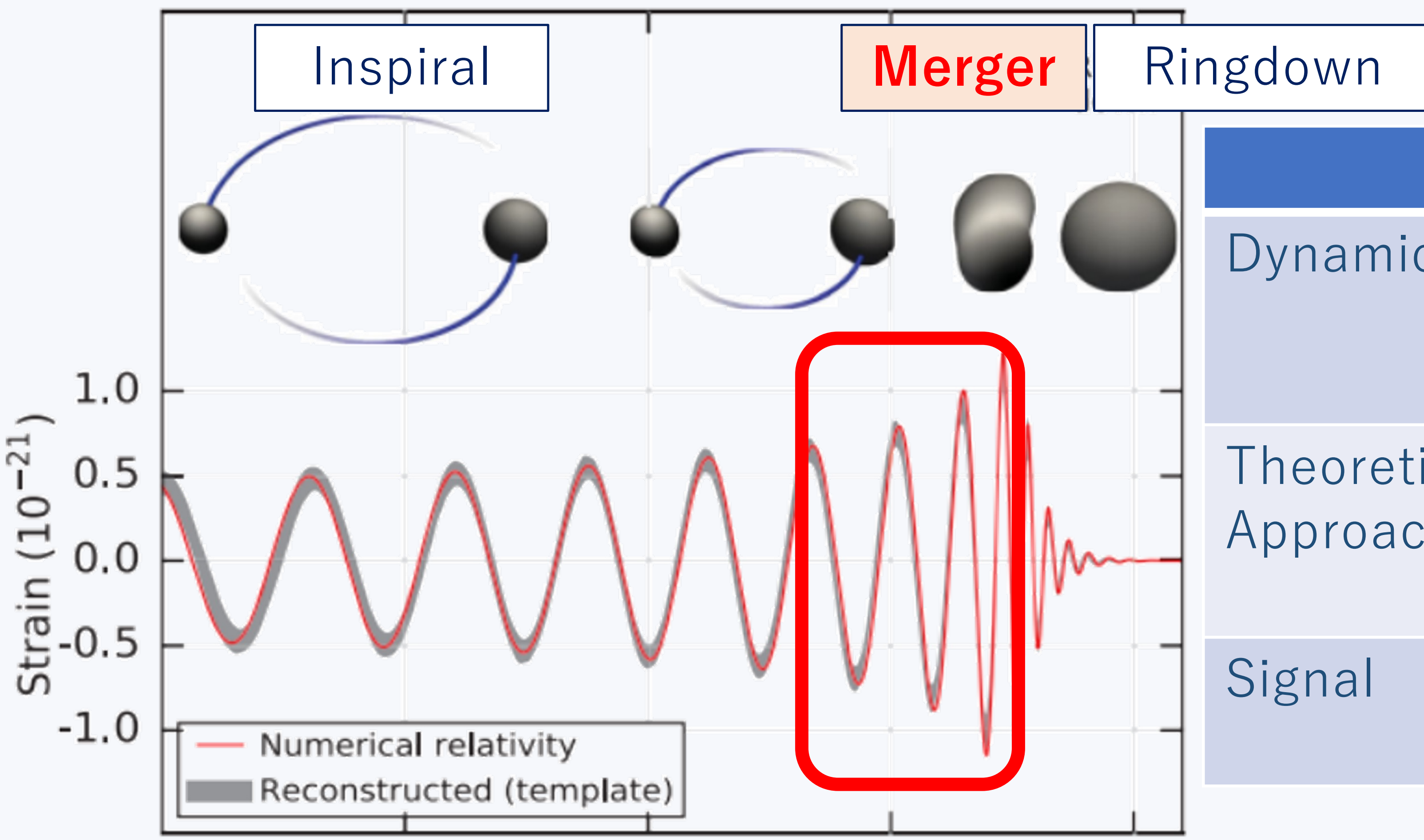


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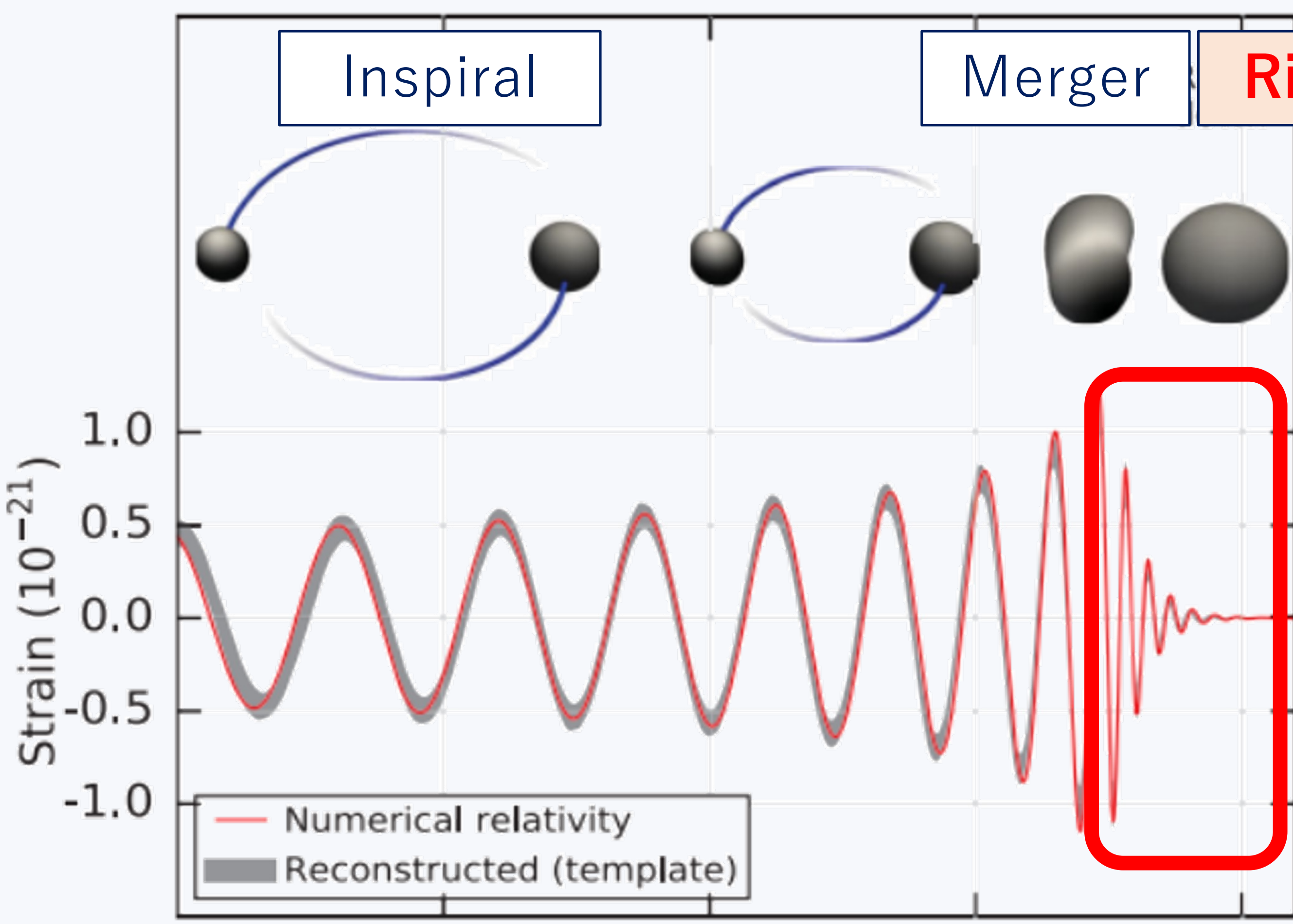


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- **However, it is difficult to constrain beyond-GR theories at the merger stage !**
  - This is mainly because of the difficulty of the numerical calculation of GWs based on specific models (in general, the well-posedness of the initial value problems in the extended models is not known) .
  - However, we should know the theoretical predictions in advance to give the constrains.
  - While recently the full simulation of binary BH coalescences based on the extended models have been performed (e.g., P.Figueras+ 2022, R.Cayuso+ 2023), there are still few predictions.

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  - ➢ However, we should know the theoretical predictions in advance to give the constrains.
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**At the first step, need to test GR at the merger stage more precisely !**

# 1. Background and Motivation



## - The Merger stage vs. the Inspiral and Ringdown stages in Data Analysis

- **The estimated results of the merger stage are difficult to be physically interpreted !**
  - ➢ The most crucial reason is that there is no theoretical approach other than NR to understand the merger stage.
  - ➢ Since the theoretical waveform for the merger stage is constructed by fitting to the NR waveform, the estimated results are for the phenomenological parameters whose physical meanings are not clear (fitting parameters to the NR waveform).

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⇒ **contrary to the cases of the inspiral and ringdown stages !**

- ✓ For the inspiral and ringdown, physically motivated analytical waveforms can be derived based on the perturbative approaches, even in several extended theories.
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**Want to extract "physical" information from the merger data !**

# 1. Background and Motivation

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Our aim: **Create a modified waveform that can capture deviations from GR at the merger stage !**

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Our aim: **Create a modified waveform that can capture deviations from GR at the merger stage !**

- **We focus on the merger stage, assuming that GR is valid in the inspiral and ringdown stages for brevity.**
  - ✓ Since there are already many frameworks for the inspiral and ringdown (e.g., Yunes&Pretorius 2009 and Gregorio+ 2019), more precise tests are expected as SNRs increase in the future.
  - ✓ Connection with the modifications in the inspiral and ringdown is left for future study.

- **The physical consistency in the whole coalescence process is considered in our waveform.** ( different point from G. Bonilla+ 2023, E. Maggio+ 2022 )
  - ✓ This enables us to utilize the IMR data in the analysis using our waveform.
  - ✓ Our waveform can constrain additional energy and angular momentum radiations in the nonlinear regime from GR (physical information).

# 2. Creation of Our Modified Waveform



## - Outline

- Strategies
- Modifications to GW Phase and Amplitude
- Physical Consistency

# 2. Creation of Our Modified Waveform



## - Strategies

Basic Idea

**Perturbative modifications to a GR waveform can capture possible deviations in the nonlinear regime that are difficult to model in a specific theory of gravity.**

# 2. Creation of Our Modified Waveform



## - Strategies

**Basic Idea**

Perturbative modifications to a GR waveform can capture possible deviations in the nonlinear regime that are difficult to model in a specific theory of gravity.

**Keys of the creation**

### Specify the principal component of the merger waveform

- The dominant component is the most sensitive combination to the deviation from GR ( though it is difficult to grasp the physical meanings of the fitting parameters ).

### Include the radiation reaction due to our modifications

- Any deviation from GR should change the properties of the remnant BH, i.e., the ringdown part should be modified.
- Take this into account and create a physically consistent waveform throughout the entire coalescence process based on the radiation formula in GR.

# 2. Creation of Our Modified Waveform
## - GR Waveform as a Basis



- **As a basis, we use a GR waveform, IMRPhenomD** (S.Khan+, 2016) **, which describes the dominant GW modes ( $(l,m) = (2, \pm 2)$ ) from the non-precessing BBH coalescences throughout the inspiral, merger, and ringdown (IMR) stages.**

  - IMRPhenomD is a frequency domain GR waveform.

$$\tilde{h}_{22}(f;\Xi) = A_{GR}(f;\Xi) e^{-i\phi_{GR}(f;\Xi)}$$

$\Xi$ : component masses and spins of BBH

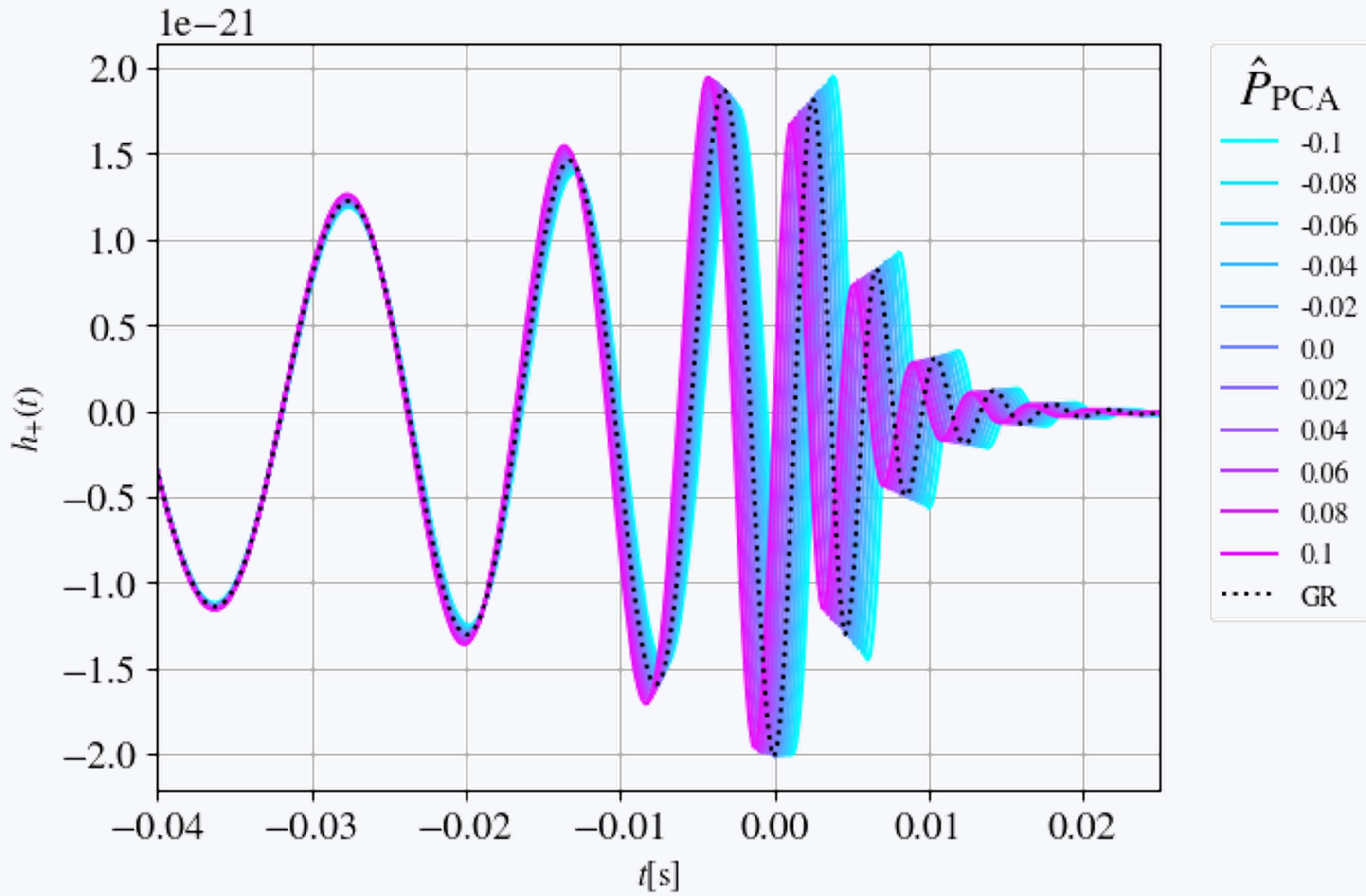
  - The amplitude, $A_{GR}(f)$, and phase, $\phi_{GR}(f)$, are modeled independently.

- **We create a modified waveform by adding beyond-GR parameters to $A_{GR}(f)$ and $\phi_{GR}(f)$.**

# 2. Creation of Our Modified Waveform
## - Phase Modification, $\hat{P}_{PCA}$



- **4 fitting parameters → 1 combination is important !**
  - Although IMRPhenomD has 4 fitting parameters in the nonlinear regime, principal component analysis finds that only the leading component is significant.
  - We adopt the leading component of the fractional errors of these parameters as a beyond-GR parameter, $\hat{P}_{PCA}$ .



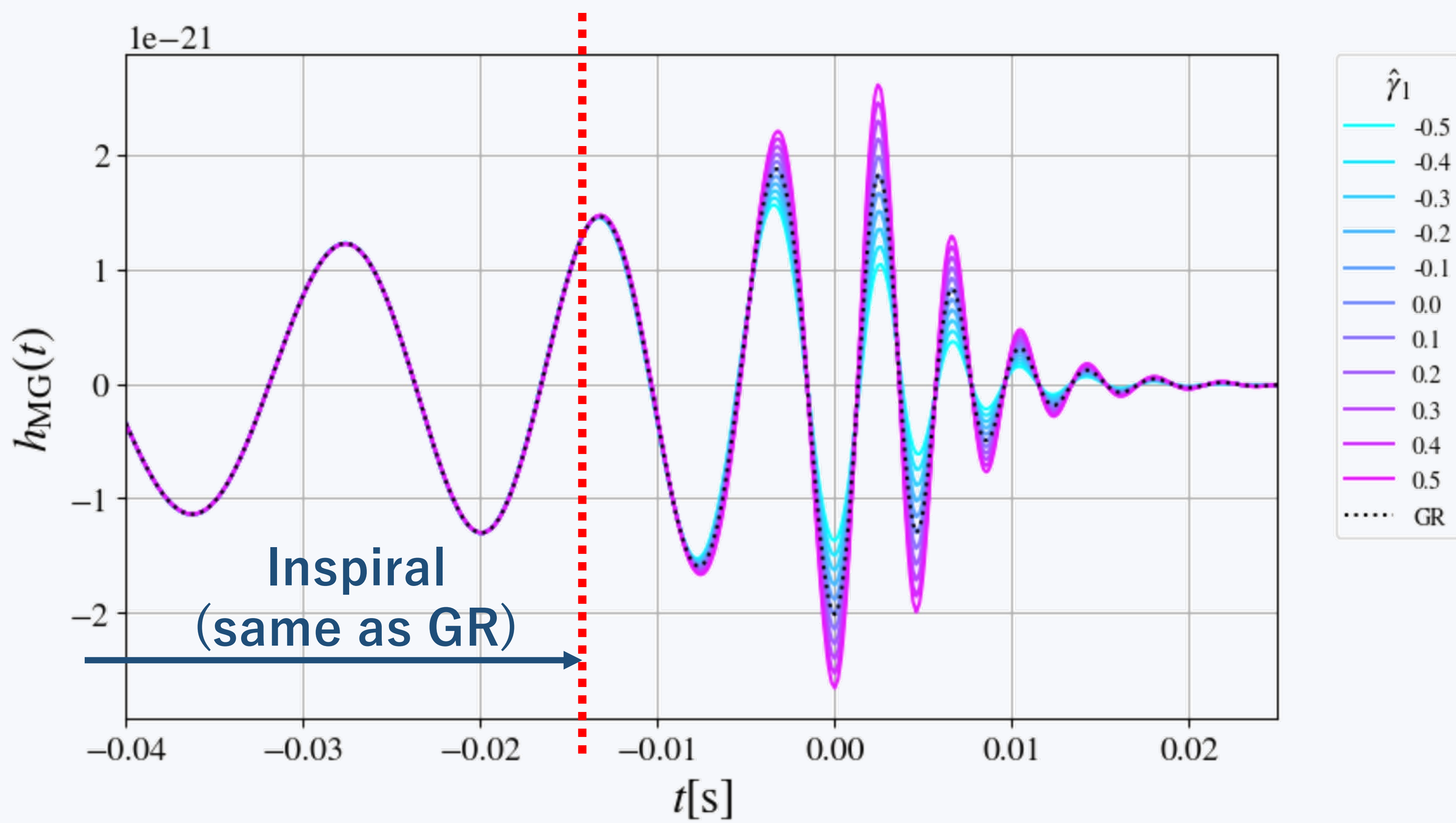

- **$\hat{P}_{PCA}$ shifts the time and phase in the nonlinear regime compared to GR waveform**

# 2. Creation of Our Modified Waveform



## - Amplitude Modification, $\hat{\gamma}_1$

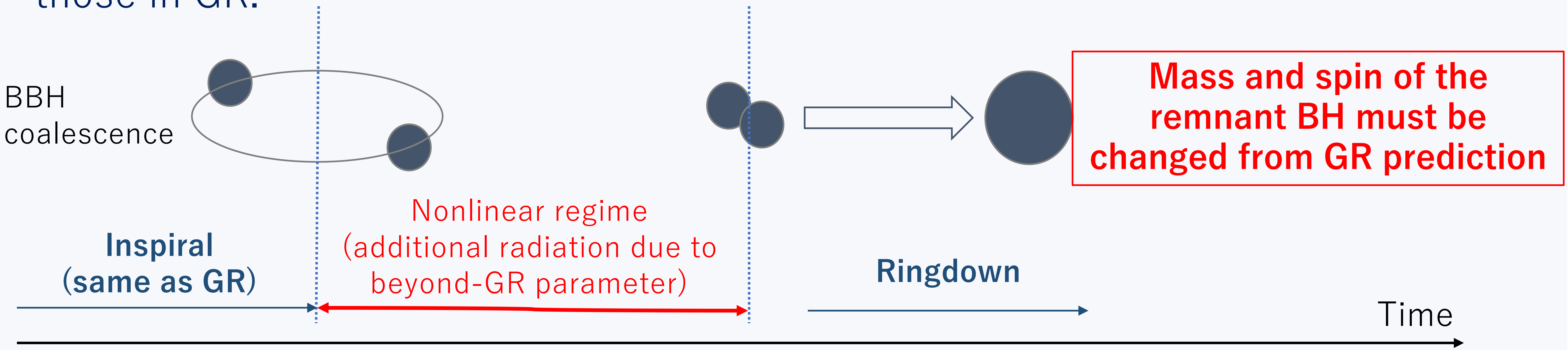
- One beyond-GR parameter, $\hat{\gamma}_1$ , that describes the amplification in the nonlinear regime, is introduced.
  - ➢ $\hat{\gamma}_1$ causes additional energy and angular momentum radiation !



# 2. Creation of Our Modified Waveform
## - Physical Consistency



- **Include the radiation reaction due to beyond-GR parameter to ensure physical consistency**, otherwise the resultant waveform describes a physically unnatural situation in which the properties of a remnant BH are the same as those in GR.




➢ Done by calculating the additional radiations due to $\hat{\gamma}_1$ and modify the remnant mass and spin.

# 2. Creation of Our Modified Waveform

## - Prescription to Ensure Physical Consistency



### 1. Calculate the radiations of the tentative waveform

- These are valid even for the nonlinear dynamics if the distance is large.

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{c^3 r^2}{16\pi G} \int \mathrm{d}\Omega \, |\dot{H}|^2,$$

$$\frac{\mathrm{d}J^z}{\mathrm{d}t} = -\frac{c^3 r^2}{16\pi G} \, \mathrm{Re}\left[ \int \mathrm{d}\Omega \, \frac{\partial H}{\partial \phi} \dot{H}^* \right],$$

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**2. Calculate deviations from GR of the final mass and spin**

- We can numerically calculate changes of mass and spin of the remnant BH associated with the beyond-GR parameter.
- The ringdown part, which is fully characterized by the mass and spin, should be modified.

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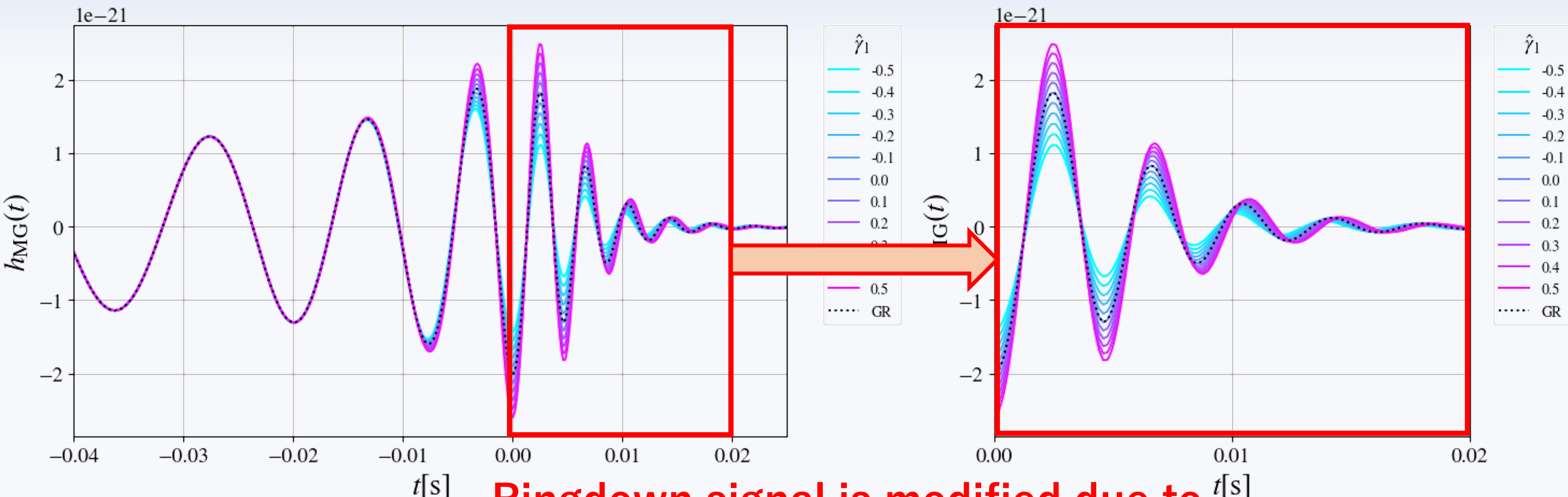
**3. Create a waveform that has the modified ringdown part**

- Inspiral (GR) + Modified Merger + Modified Ringdown (New!)
- The resultant waveform is physically consistent in the sense that the additional radiation reactions due to the modifications are included.

# 2. Creation of Our Modified Waveform
## - Physically Consistent Waveform




**Ringdown signal is modified due to the additional radiations !**

- As the amplitude becomes larger than GR, the ringdown frequency and damping time become smaller and shorter, respectively.

# 3. Evaluation of Our Parametrization



We consider two questions for the O4 and O5 runs by LVK collaboration.

1. **Is our waveform able to capture beyond-GR effects in the observational data ?**
   - ➢ We should check correlations between beyond-GR parameters and others ( The strong correlations mean difficulty to identify beyond-GR effects ).

   **Answer: Yes ! The Fisher analysis finds that the degeneracies are not so significant.**

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2. **Can our waveform reproduce a GW signal in a specific extended theory ?**

   - We should understand what kind theories our parametrization is compatible with.
   - We show that **our waveform can reproduce GW signals of Einstein dilaton Gauss-Bonnet gravity within the measurement errors**.

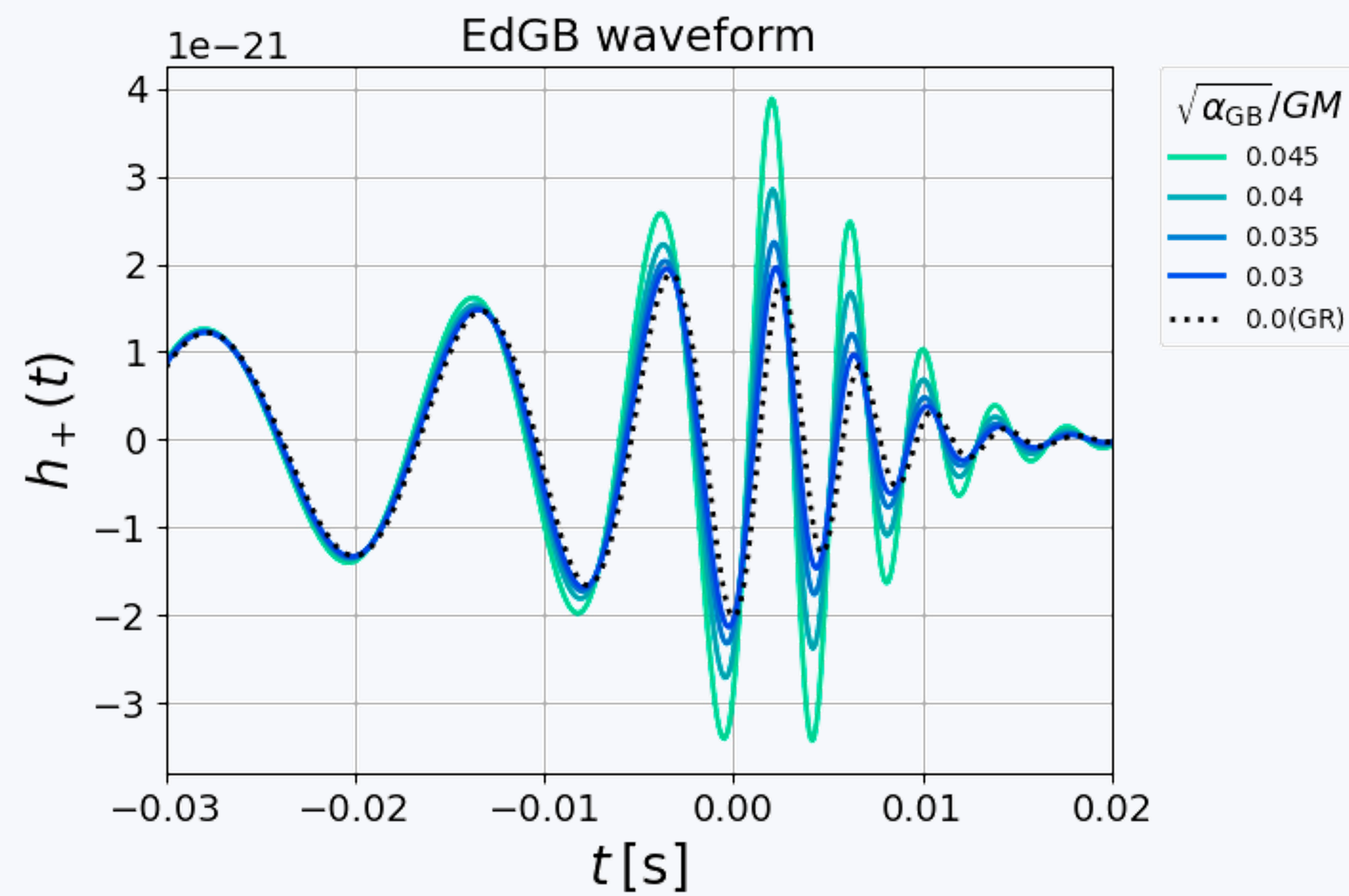
# 3. Evaluation of Our Parametrization
## - Reproduction of EdGB Gravity Waveform



- Einstein dilaton Gauss-Bonnet (EdGB) gravity is one of the extended theories whose IMR waveforms are available. (M.Okounkova 2020)

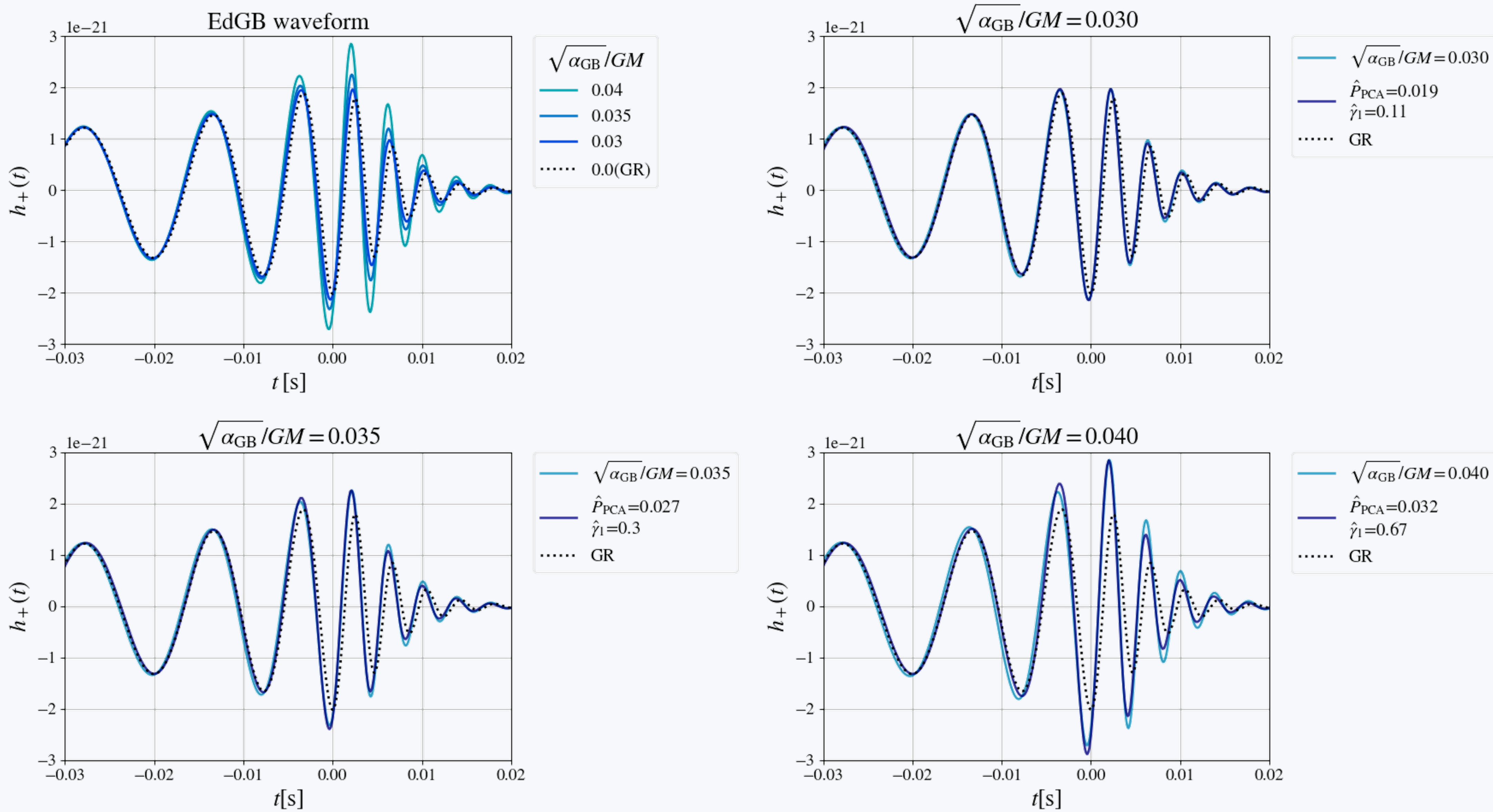
$$S \equiv \int \frac{m_{\rm pl}^2}{2} d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial\vartheta)^2 + 2\alpha_{\rm GB} f(\vartheta)\mathcal{R}_{\rm GB}\right],$$

- ➢ The dilatonic scalar field sources GWs though the coupling.
- ➢ The waveforms include only the leading contributions of the scalar field (not derived by the full simulation).



# 3. Evaluation of Our Parametrization
## - Reproduction of EdGB Gravity Waveform

- $\hat{P}_{PCA}$ and $\hat{\gamma}_1$ can capture the shifts of time and phase and the amplification in the nonlinear regime.



# 4. Remark and Prospect



## - Remark

- **Our modification is minimal in the sense that we introduce only one beyond-GR parameter to GW phase and amplitude, respectively.**
    - $\hat{P}_{PCA}$, which is the most sensitive combination of the fractional errors of the fitting parameters, measures the time and phase shifts at the merger stage.
    - $\hat{\gamma}_1$ measures the amplification at the merger stage.

- **Estimating $\hat{\gamma}_1$, we can infer additional energy and angular momentum radiations in the nonlinear regime.**
    - Our waveform would provide constraints on physical quantities in the nonlinear regime, while the results for the merger stage by LVK collaboration (testing GR papers) are difficult to be interpreted physically.

# 4. Summary and Prospect



## - Future Study (data analysis & waveform creations)

- **Analysis of the observational data**
  - We are now working on the analysis of GW150914.
  - First constraints on the additional energy and angular momentum radiations in the nonlinear regime would be provided.

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    - ✓ For brevity, we use IMRPhenomD waveform, but there are various types of more accurate GR waveforms ( including sub-dominant modes or precession effects )
  - Introduce other parameters such that the modified waveform is flexible enough to cover broader classes of beyond-GR theories.
    - ✓ In this study, we do not specify what causes additional radiations and focus only on the tensor field.
    - ✓ The recent developments of the numerical simulations based on extended models would guide us to introduce suitable additional parameters.

## Thank you for your attention !