



# Resonant orbits and chaos of rotating black holes beyond circularity

Che-Yu Chen

**iTHEMS (SPDR), RIKEN, JAPAN**

**CYC**, Hsu-Wen Chiang (NTU), Avani Patel (NCU)

arXiv:2306.08356, Phys. Rev. D



**RIKEN's**  
Programs for  
Junior Scientists

**iTHEMS**

RIKEN Interdisciplinary  
Theoretical and Mathematical  
Sciences Program

*YITP workshop, February 28 (2024)*



# Testing Kerr hypothesis





# What do we know about Kerr?

1. Axisymmetry and stationarity:  
Two Killing vectors

2. Equatorial reflection symmetry

3. Liouville Integrability

4. Circularity

- Testing individual symmetry = testing Kerr hypothesis
- Understand the feature when individual symmetry is broken



# Non-circularity?

1. Axisymmetry and stationarity:  
Two Killing vectors

2. Equatorial reflection symmetry

3. Liouville Integrability

4. Circularity

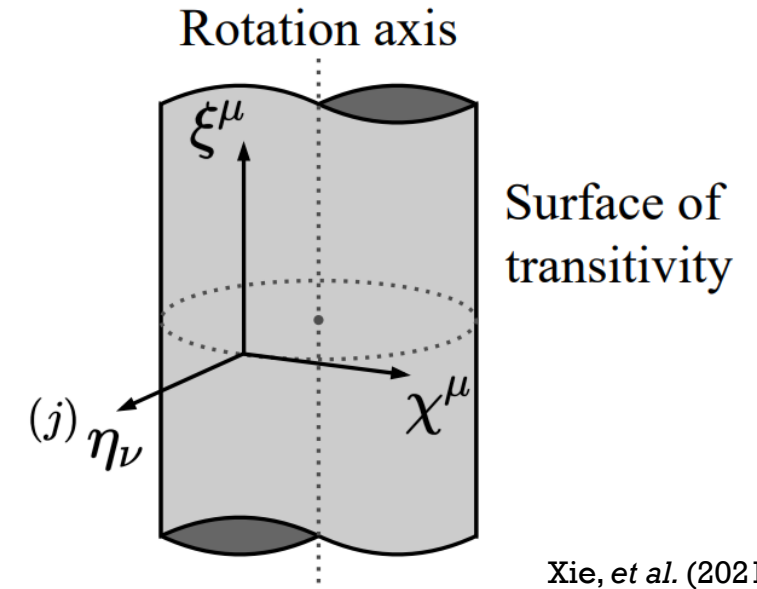
- Breaking 3, 4: chaotic orbital dynamics
- Bound orbits of timelike geodesics, e.g. EMRIs



# Circular spacetimes

- Two Killing vectors  $\xi^\mu$  (time translation) and  $\chi^\mu$  (rotation) span the “**surface of transitivity**”  $\Sigma_t$
- Circularity: There exists a family of 2-surfaces (meridional surface) that are everywhere orthogonal to  $\Sigma_t$
- In GR, circularity implies some constraints on matter fields that support the geometry (e.g. fluids with only circular motion)

Gourgoulhon, Bonazzola (1993), Ioka, Sasaki (2003)(2004), Gourgoulhon (2010) Birkl, *et al.* (2011)



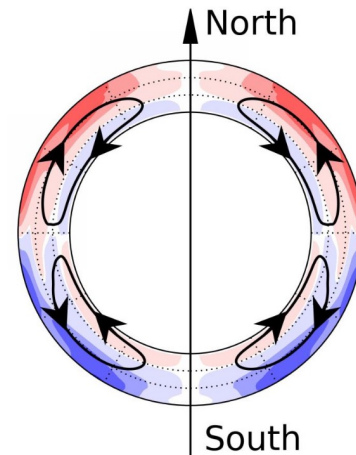
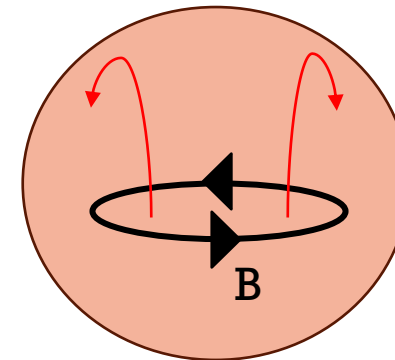
Xie, *et al.* (2021)

- Non-circularity happens in the contexts of:
  - Stellar objects with meridional circulation (e.g. NS with toroidal B-field)
  - BHs in some modified gravity
  - Quantum effects on BH, (e.g. asymptotic safety)
  - General Parameterized metric beyond KRZ

Van Aelst, *et al.* (2020), Ben Achour, *et al.* (2020), Anson, *et al.* (2021), Minamitsuji (2020)

Held, *et al.* (2019), Eichhorn, Held (2022), Fernandes (2023)

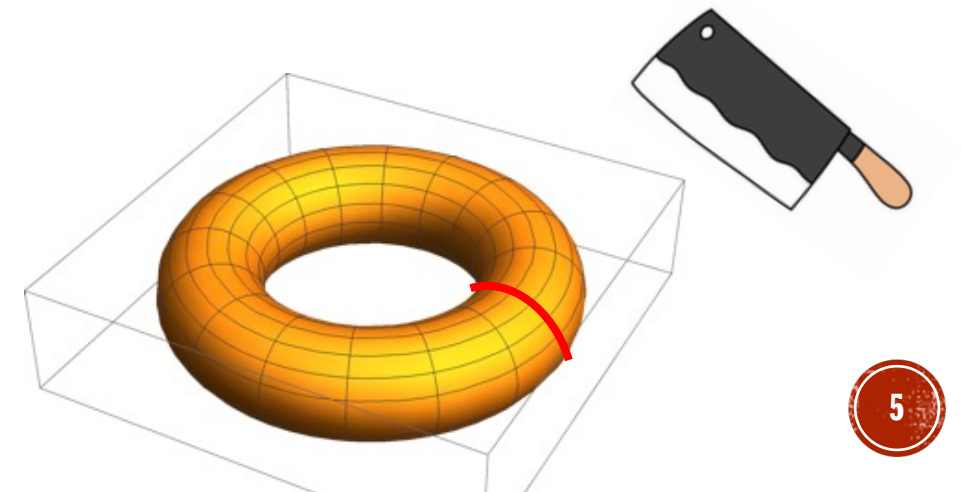
Delaporte, Eichhorn, Held (2022)



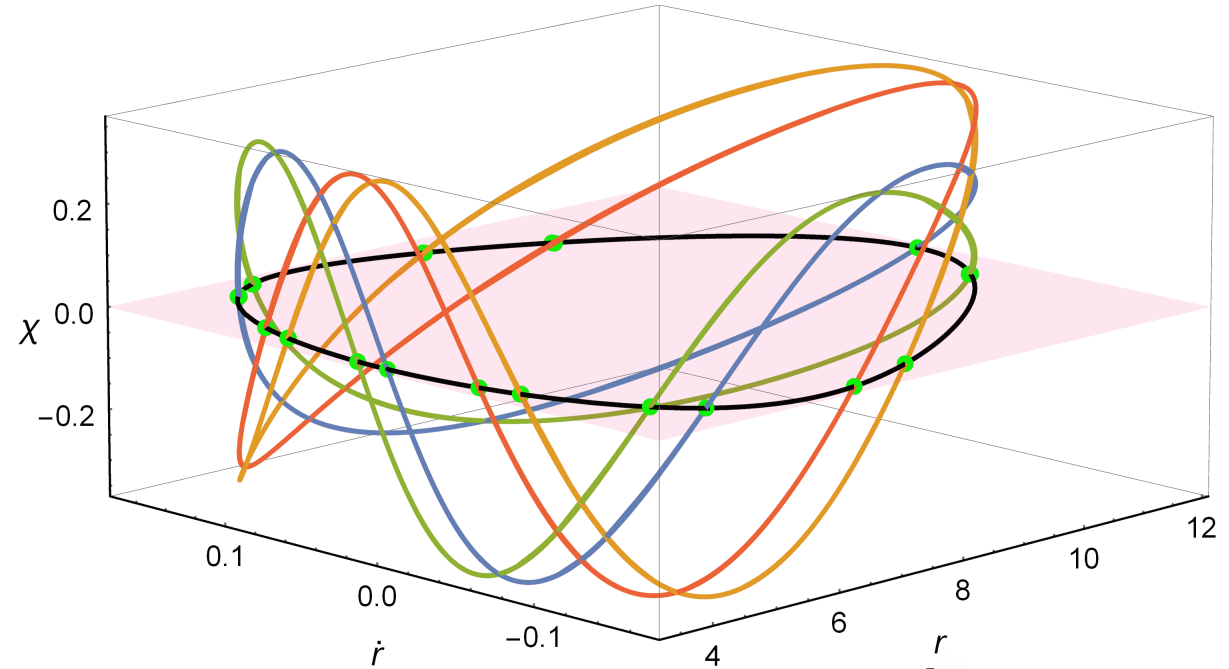
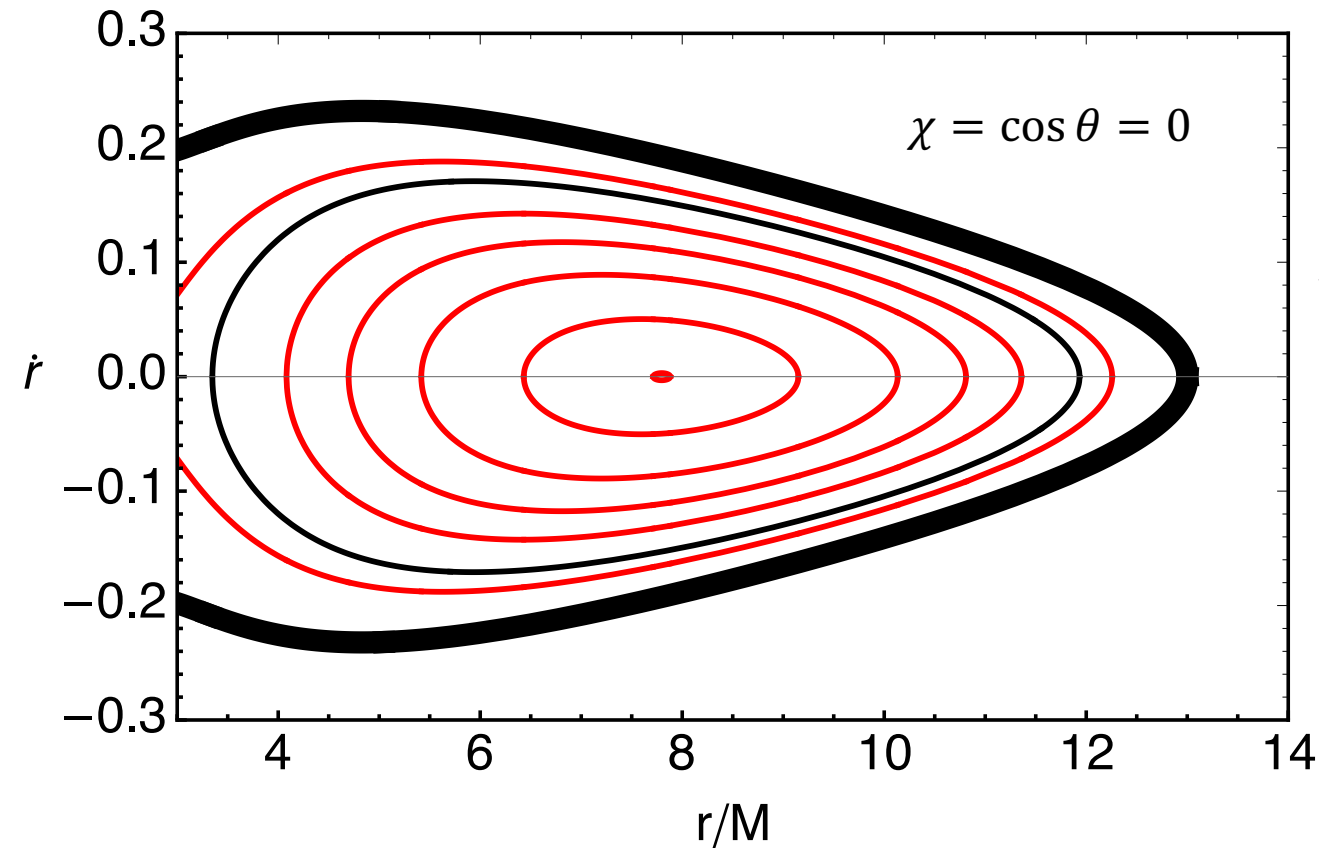


# Integrability and circularity

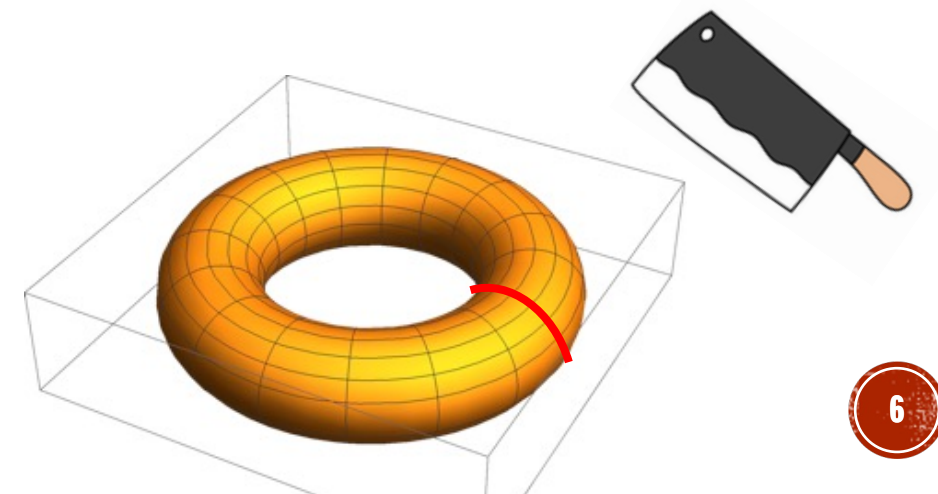
- Suppose a rank-2 Killing tensor exists:
  - Integrable orbital dynamics
  - Circular
  - 2D torus on phase space for each bound trajectory
- The integrability can be broken via
  - Breaking circularity
  - Keeping circularity
- Chaotic dynamics for bound orbits
  - Using Poincaré map to identify chaotic imprints



# Integrable orbital dynamics

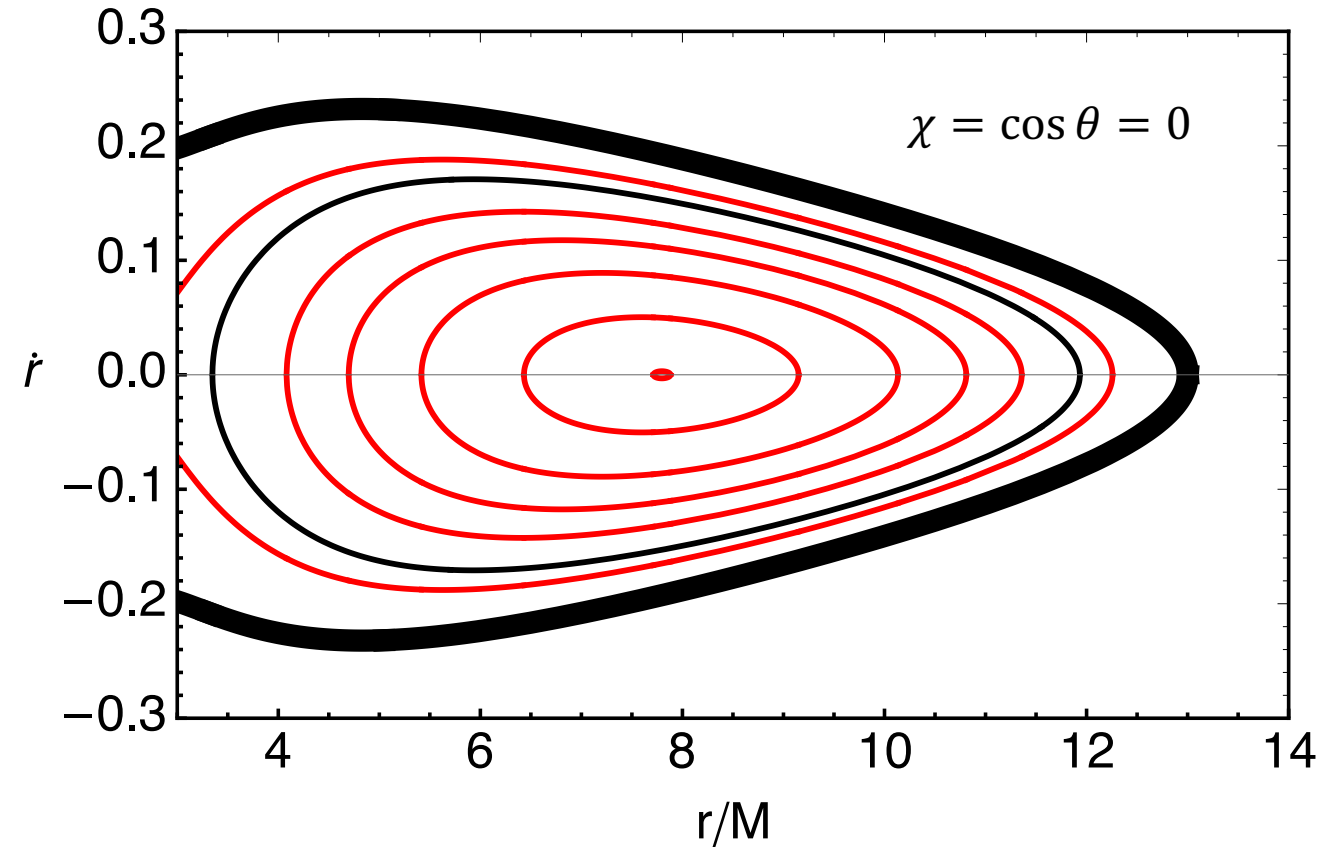


- **Red:**  $\nu_\theta \equiv \omega_r/\omega_\theta$  is irrational (quasi-periodic curves)
- **Black:**  $\nu_\theta \equiv \omega_r/\omega_\theta$  is rational (resonant tori)





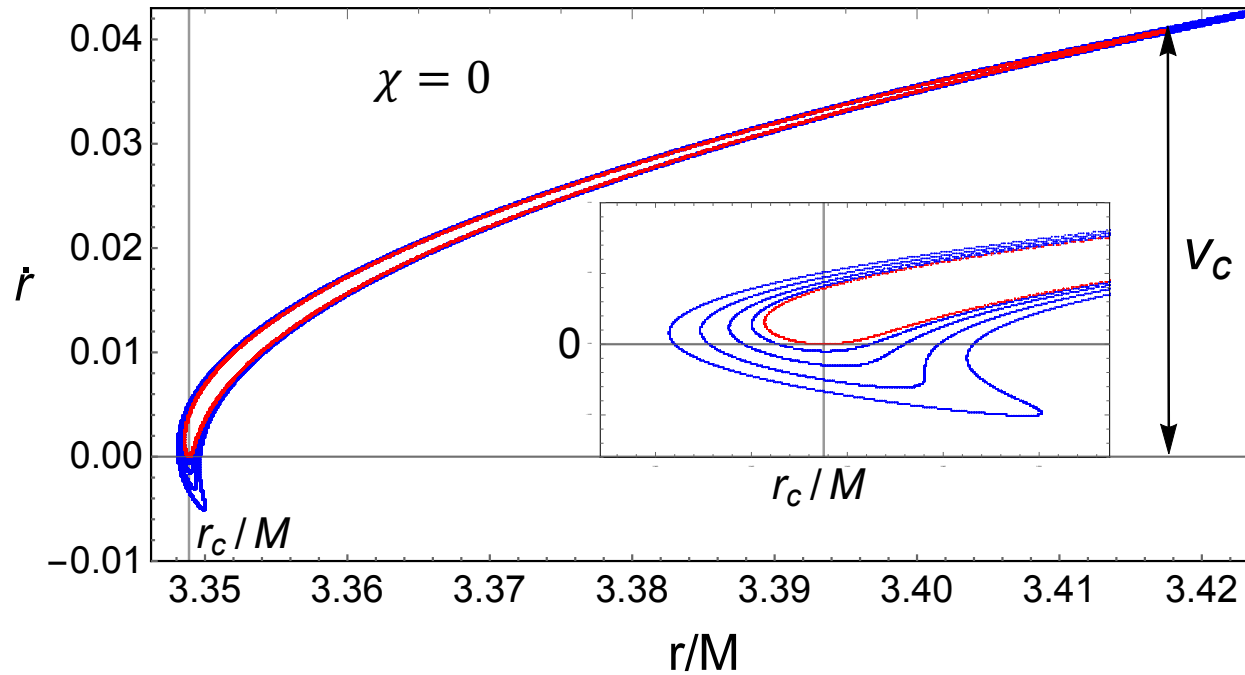
# Non-integrable deformations



$$H \rightarrow H + \delta H$$



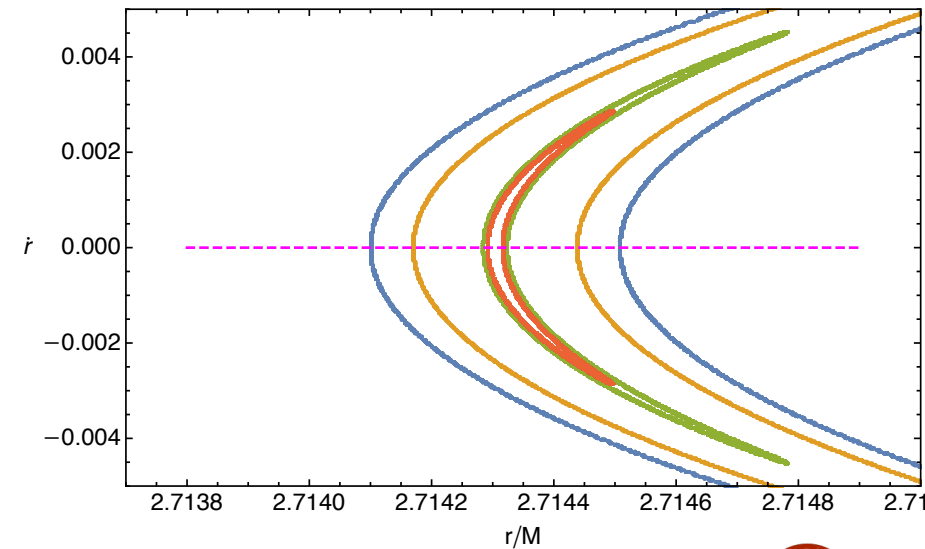
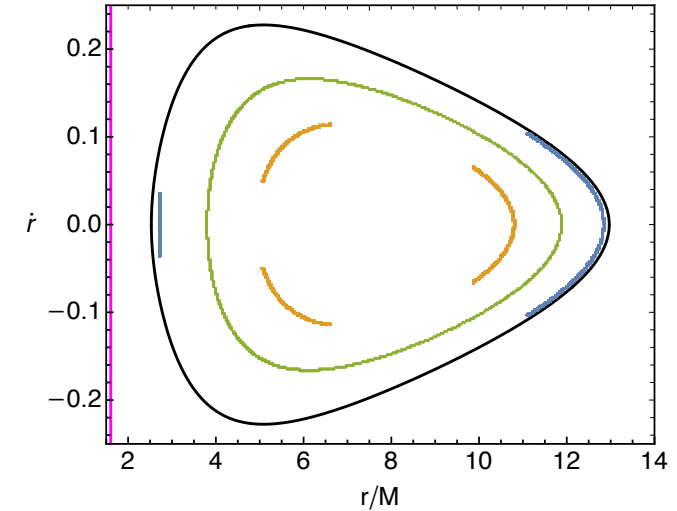
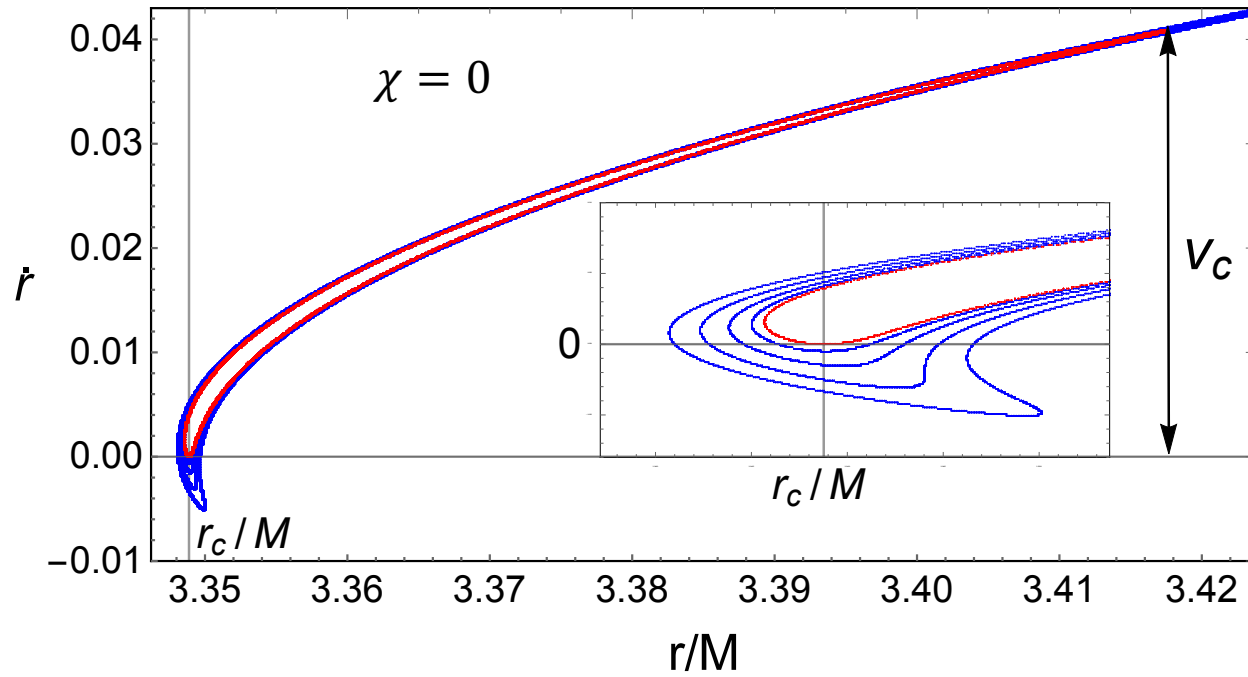
# Non-circular v.s circular deformations



- Resonant orbits disintegrate, form chains of islands (Poincaré-Birkhoff theorem)
- At  $\chi = 0$  surface, islands are asymmetric w.r.t.  $\dot{r} = 0$  axis

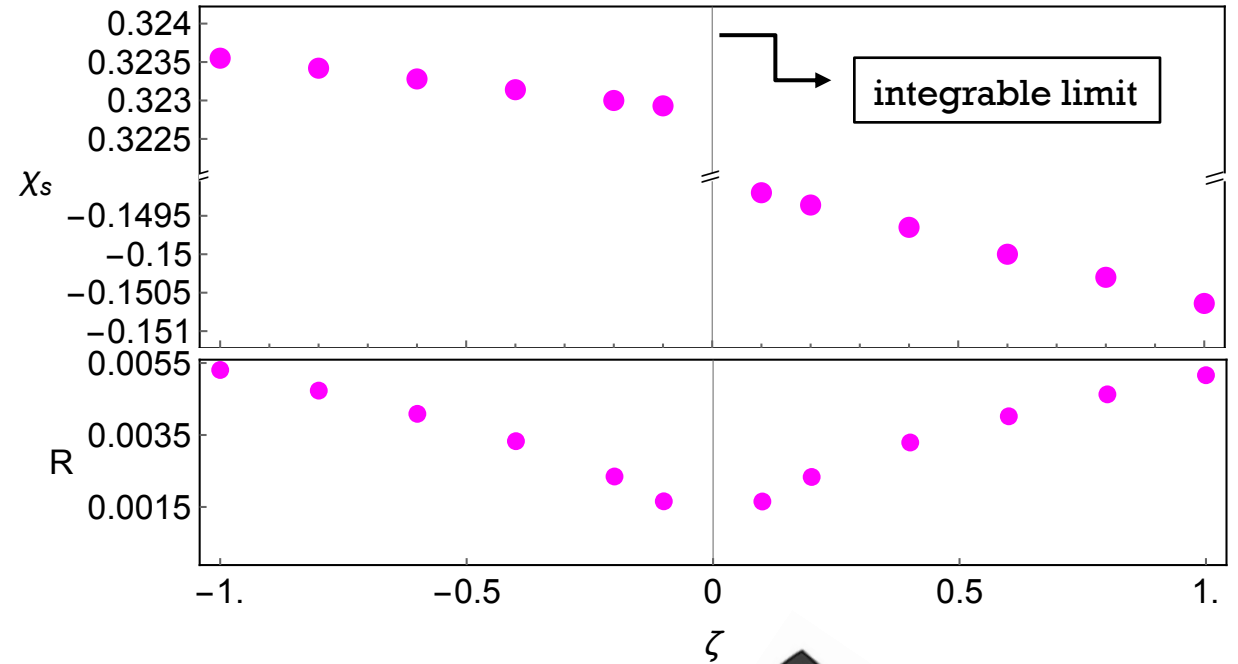
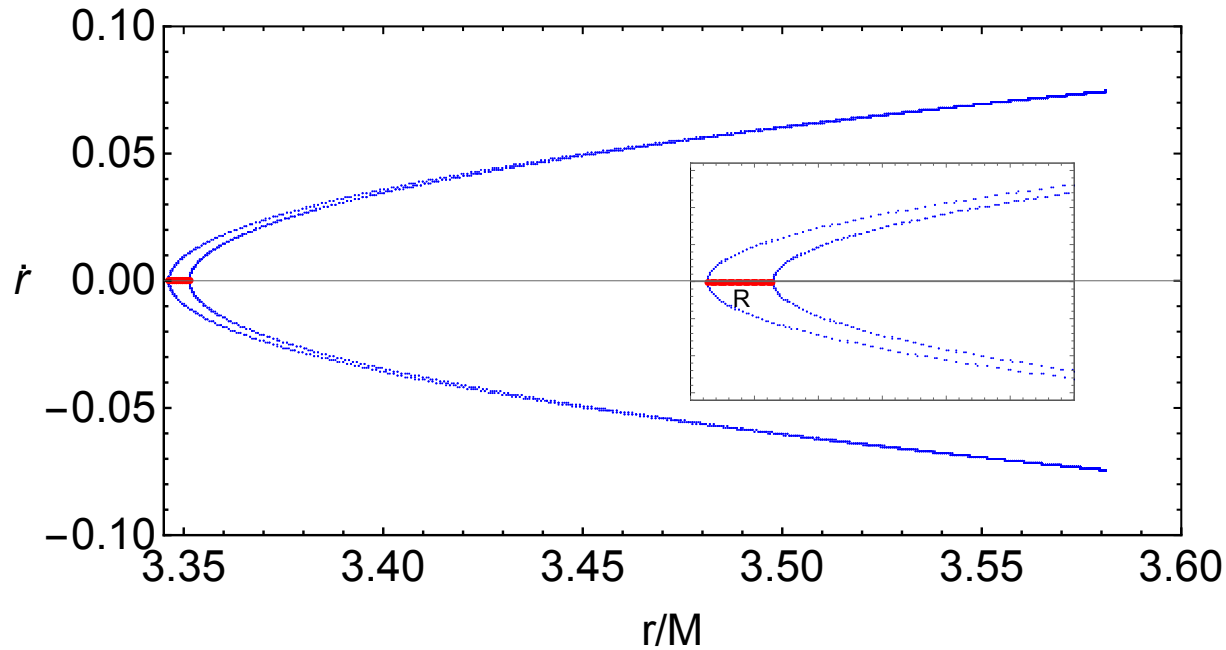


# Non-circular v.s circular deformations

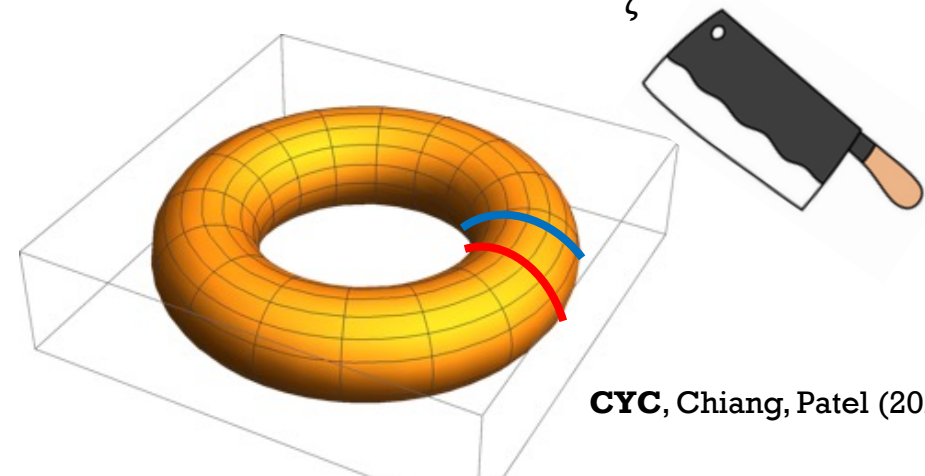


- Resonant orbits disintegrate, form chains of islands (Poincare-Birkhoff theorem)
- At  $\chi = 0$  surface, islands are asymmetric w.r.t.  $\dot{r} = 0$  axis

# Quantify the asymmetry: choose a different $\chi$

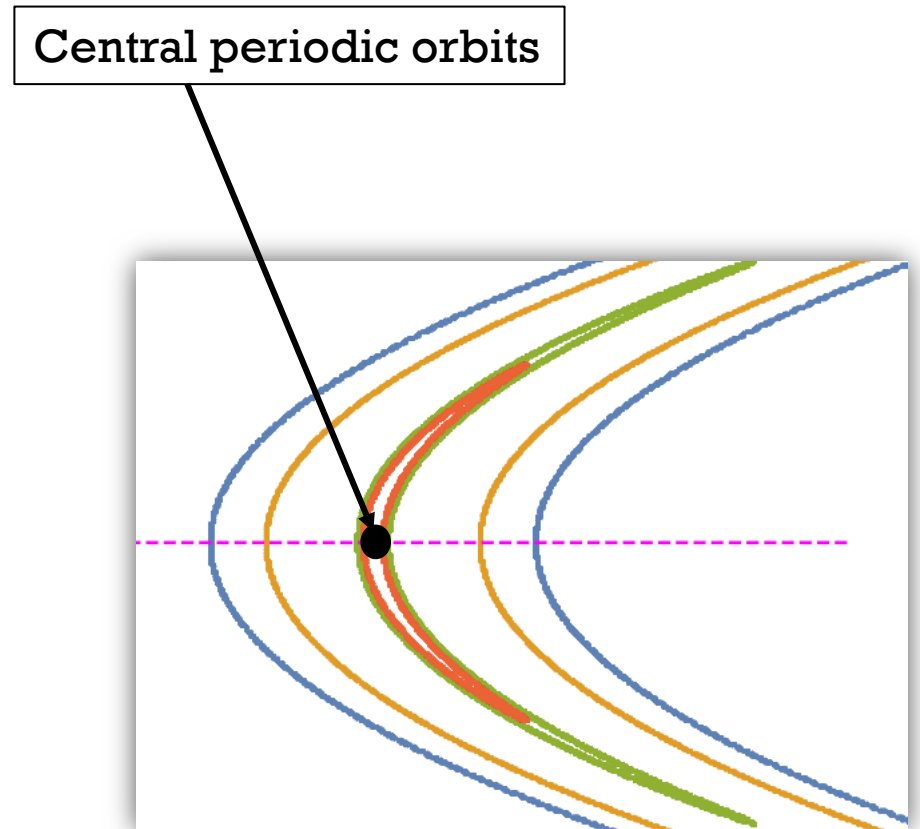


- Latitude of symmetry: Symmetric islands at the surface of section  $\chi = \chi_s \neq 0$
- Discontinuous jump of  $\chi_s$  at  $\zeta = 0$
- The width of island  $R \propto \sqrt{\zeta}$

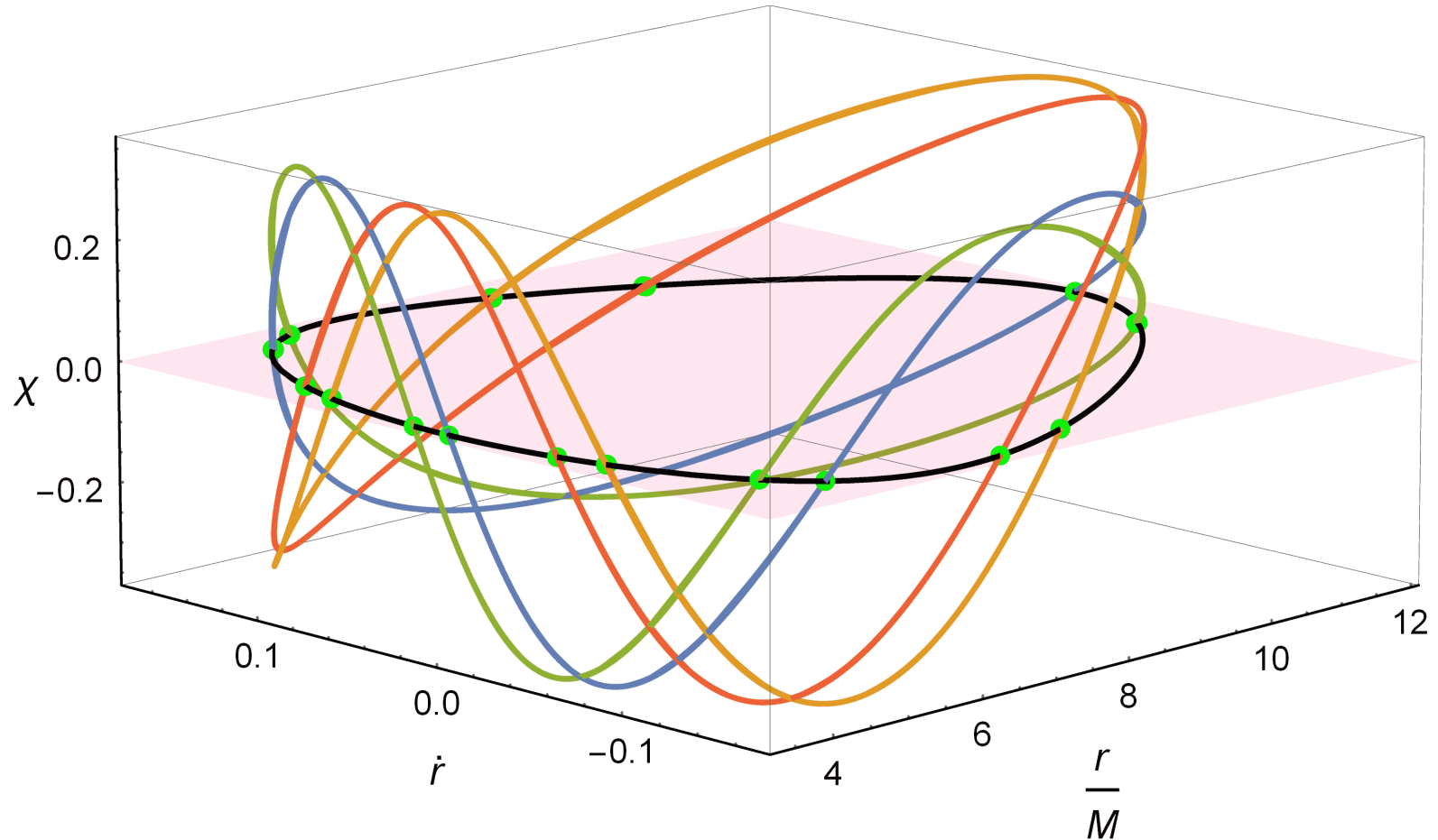




- The non-circular metric has equatorial reflection symmetry
- How could resonant islands become asymmetric?
- It is all about how the resonant islands form
- In particular, it is about how the central periodic orbits emerge



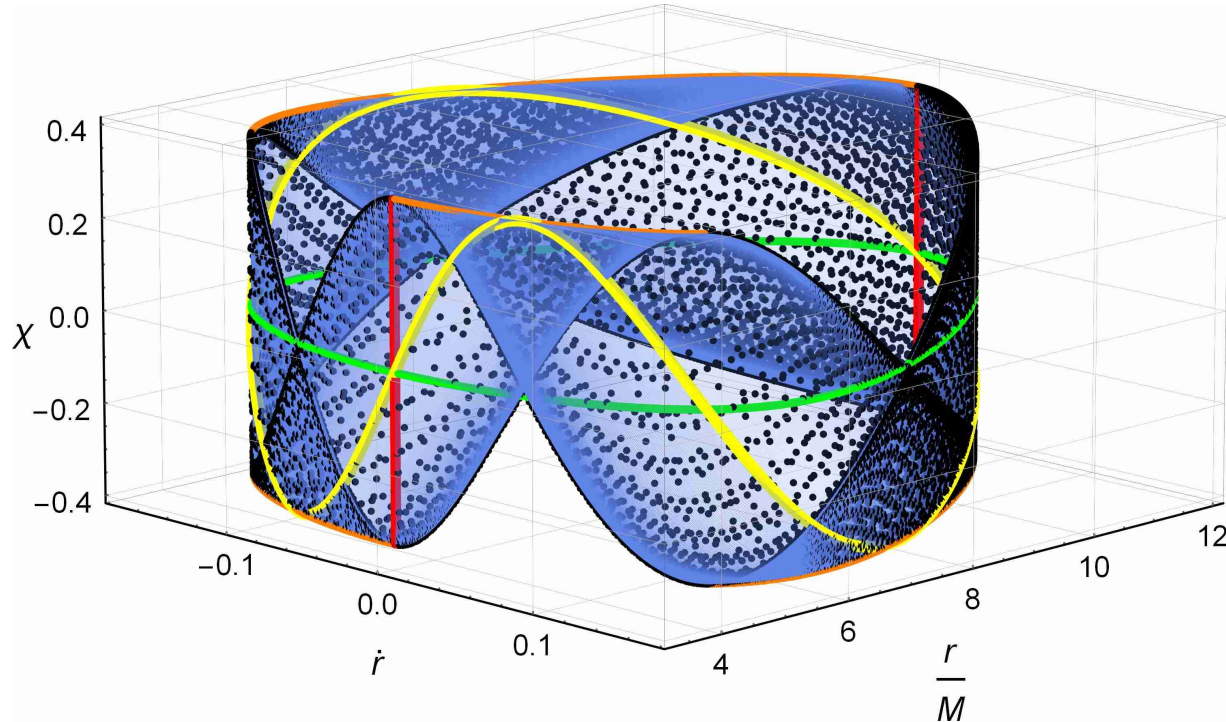
# Resonant orbits: who will be chosen?



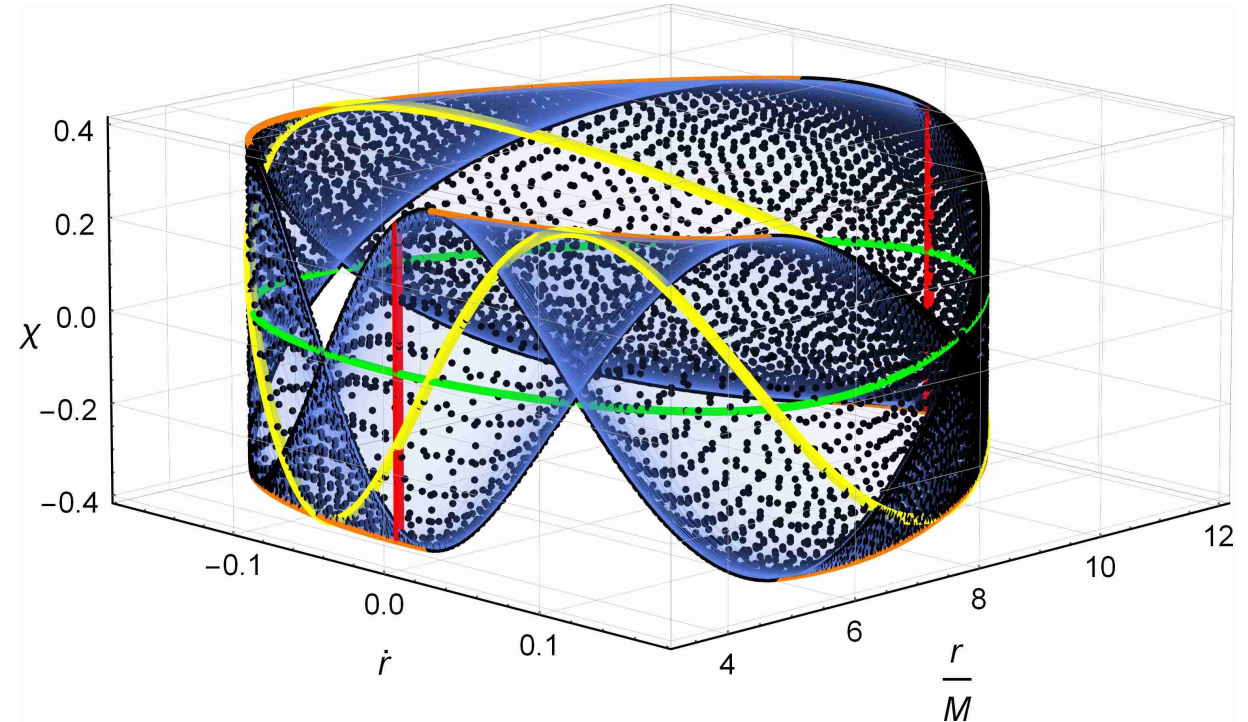


# Central periodic orbits

- Different kinds of deformations select different resonant orbits to form **central periodic orbits (yellow)**



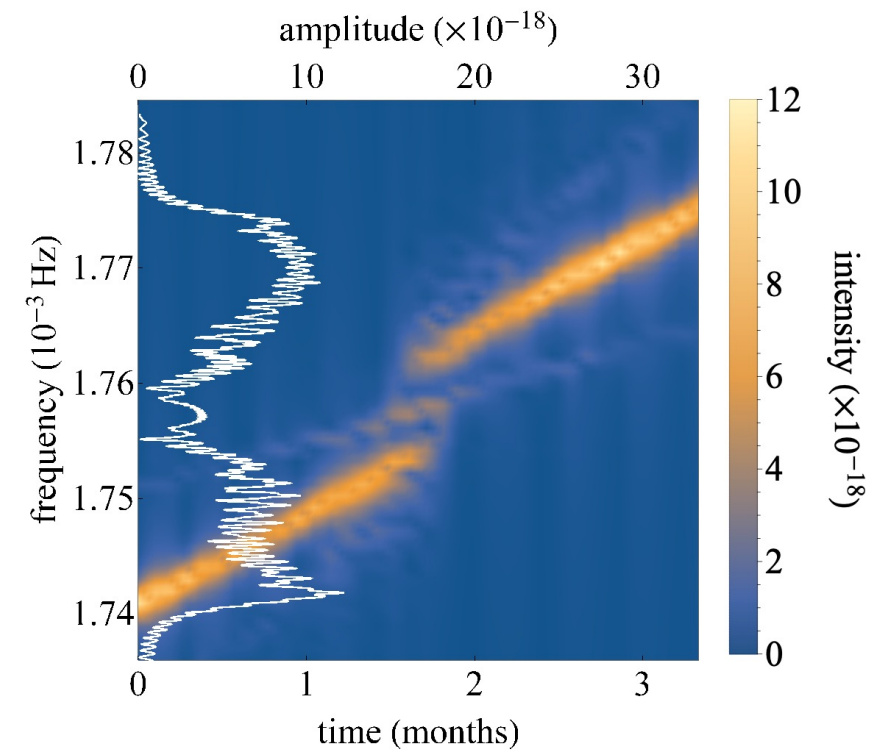
Circular deformations



Non-circular deformations

# Conclusions

- Non-integrable orbital dynamics and chaos
- Asymmetric islands from non-circularity
- Abrupt jump of the amount of asymmetry at the integrable limit
- **The way that central periodic point develops islands sensitively depends on the form of non-integrable deformations introduced**
- Future work:
  - Observational imprints on GWs from EMRIs
  - Can we test circularity using the asymmetry of islands?



# Intercontinental Seminars on Gravity and Cosmology



<https://forms.gle/pAW3iqdeXtmiHjcZA>

- Che-Yu Chen
- Torben Frost
- Ryo Namba
- Kostas Dialektopoulos