Seeded vacuum decay with Gauss Bonnet

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Gravity and Cosmology 2024 @ YITP, Kyoto University, Kyoto, Japan

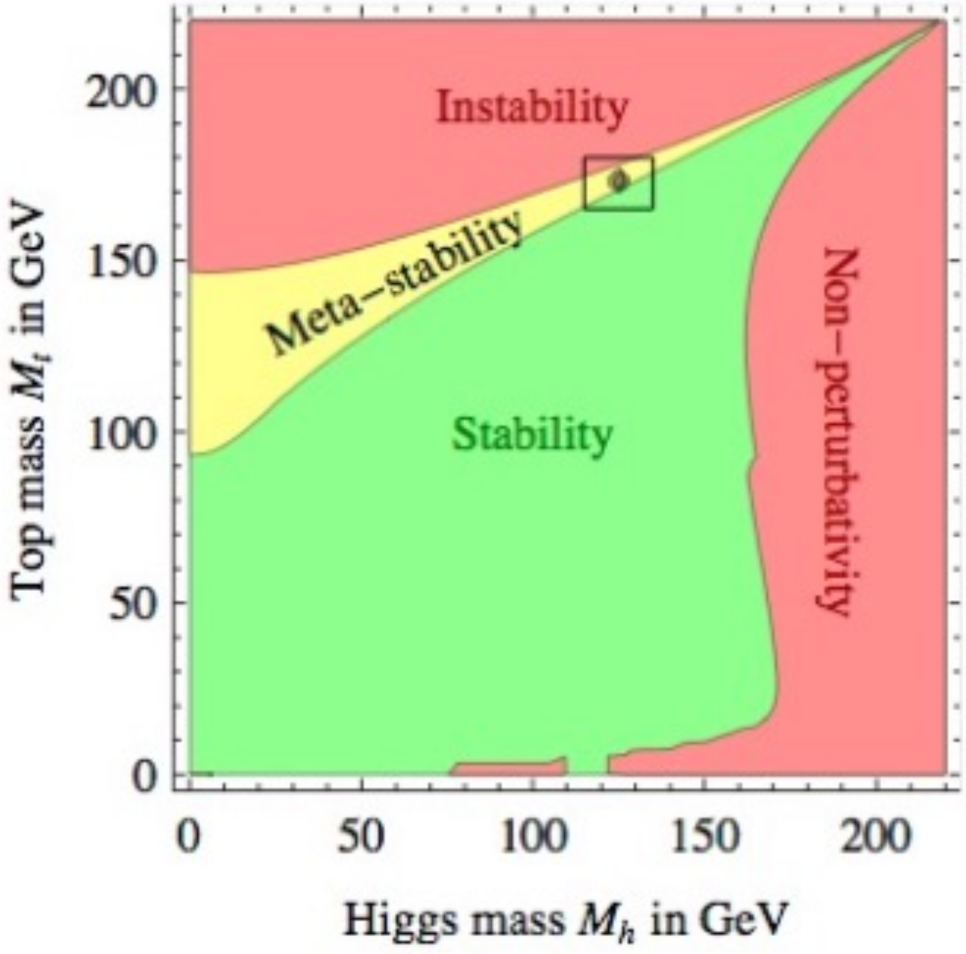
28/02/2024

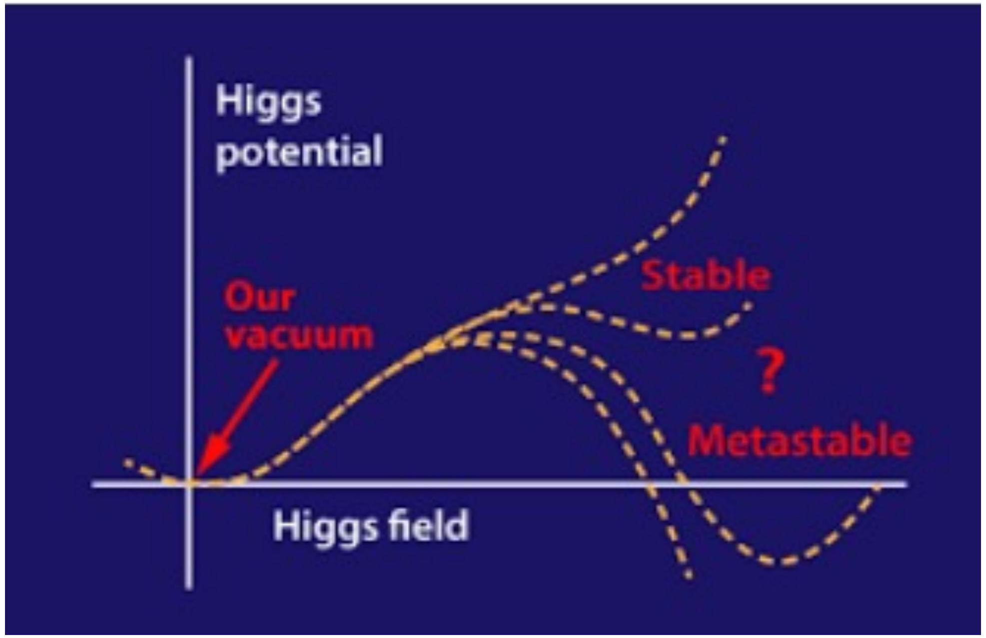
Based on: Seeded vacuum decay with Gauss-Bonnet, JHEP 11 (2023) 072

Testing Higher Derivative Gravity Through Tunnelling, Particles 7 (2024) 144-160

Higgs Instability

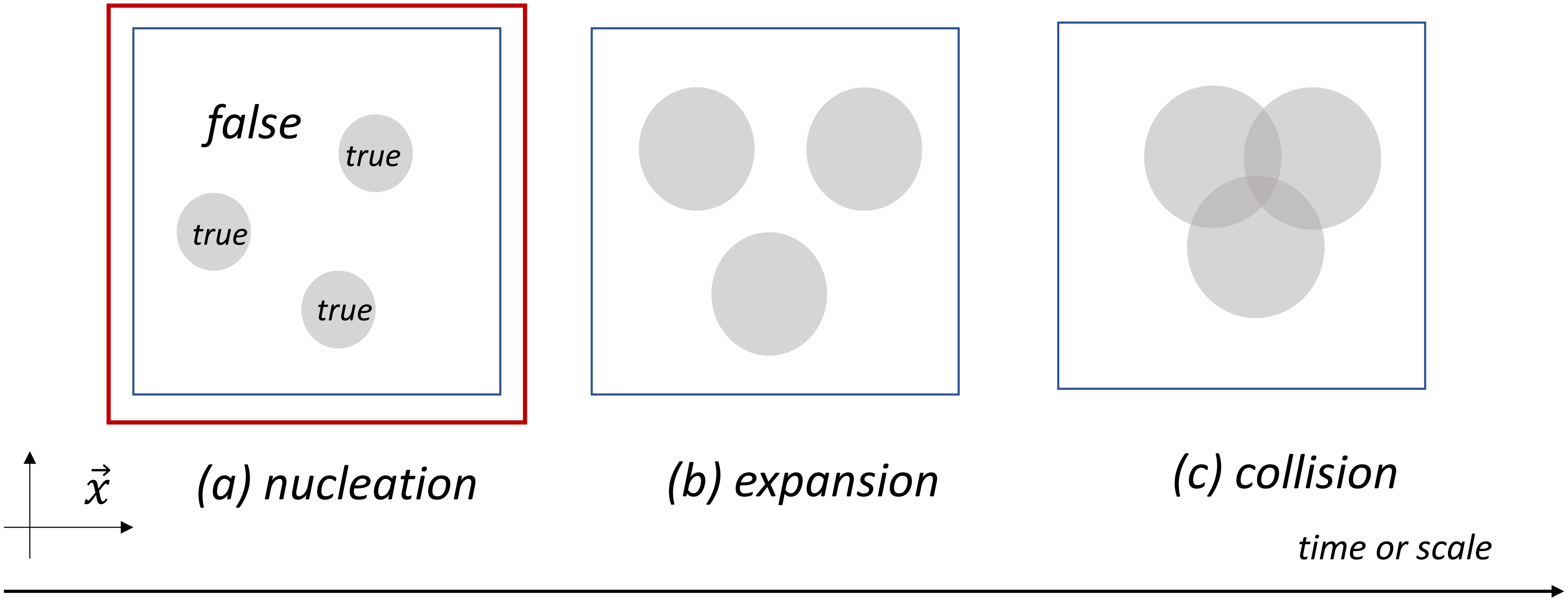
The Higgs field defines our vacuum – but the picture from the Standard Model is that our universe may not be entirely stable.





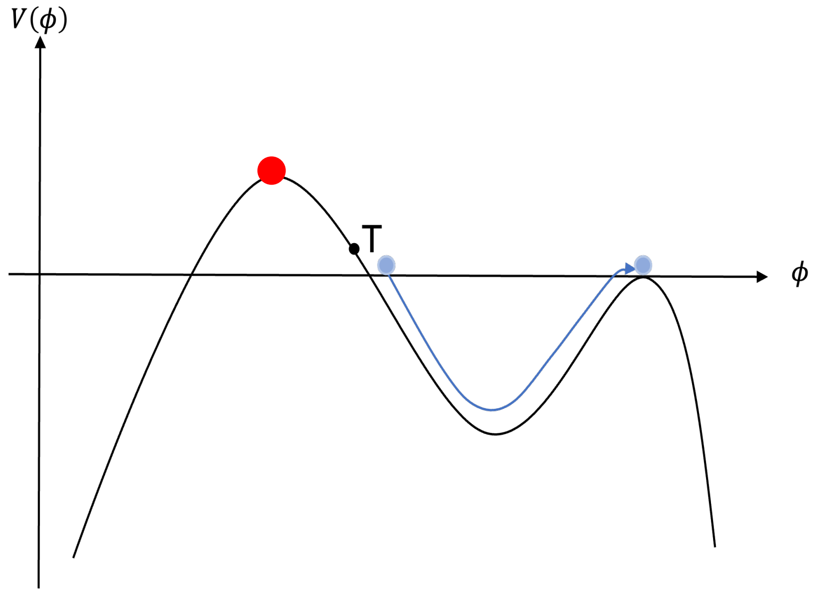
First order phase transition in quantum field theory

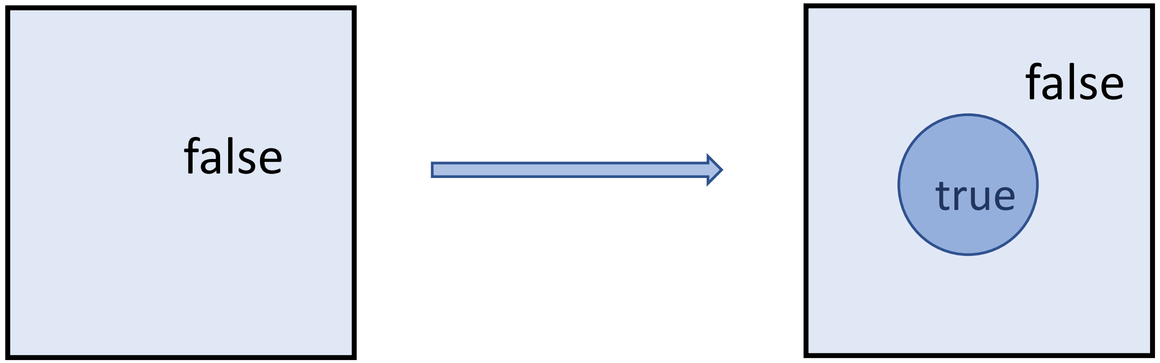
First-order phase transition: bubble nucleation, expansion, and collision



Instanton

In the 1970s, Coleman and collaborators describe vacuum decay via a mathematical tool of analytically continuing to imaginary (Euclidean) time, wick rotation, $\tau = it$.





They describe this in field theory by the Euclidean solution of a bubble of true vacuum bubble inside the false vacuum.

Goldilocks bubble

Gain energy from moving to true vacuum but cost energy to form bubble wall

Too small ⟶ Bubble has too much surface area ⟶ Re-collapse

Too large ⟶ Bubble wall is expensive to form ⟶ Can't afford

"Just Right" means the bubble will not re-collapse, but is still "cheap enough" to form.

$$\delta E = 2\pi^2 \underset{\substack{\big\uparrow \\ \text{Cost of wall}}}{R^3\sigma} - \frac{\pi^2}{2} \underset{\substack{\big\uparrow \\ \text{Gain from vacuum}}}{\epsilon R^4}$$

The critical bubble radius $R = 3\sigma/\epsilon$, and the amplitude for decay

$$\delta E = \frac{\pi^2 R^3}{2}(4\sigma + \epsilon R) \sim \frac{27\pi^2 \ \sigma^4}{2\epsilon^3}$$

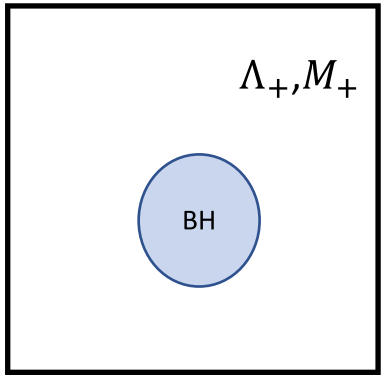
Coleman bounce and decay rate

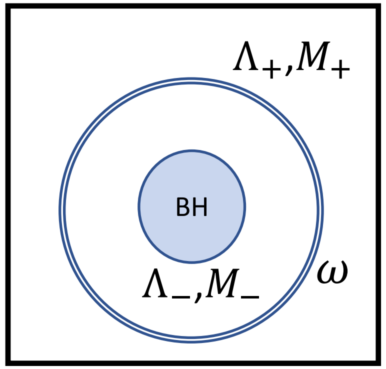
The impact of gravity was first worked out in [Coleman and de Luccia, 1980] by extending the partition function to include the Einstein-Hilbert action under thin wall approximation.

The decay rate is found to be

$$\Gamma \sim e^{-B} = \mathrm{e}^{-(I-I_0)}$$

The Coleman universe is empty and featureless - too idealized - throw in a little **impurity**.





Higher Curvature Gravity

The action for the EGB gravity with a cosmological constant is

$$S = -\frac{1}{16\pi G}\int_M d^D x\,\sqrt{g}\,(R - 2\Lambda + \alpha\, L_{GB}),$$

$$L_{GB} = R^2 - 4R_{ab}^2 + R_{abcd}^2$$

The metrics for the exterior and interior regions of the wall are given by

$$ds_\pm^2 = f_\pm\, d\tau_\pm^2 + f_\pm^{-1} dr^2 + r^2 d\Omega_{D-2}^2$$

The solutions on each side of the wall are

$$f_\pm = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{8\alpha\Lambda_\pm}{(D-1)(D-2)} + \frac{4\alpha\mu_\pm}{r^{D-1}}}\right), \qquad \mu_\pm = \frac{16\pi G M_\pm}{(D-2)A_{D-2}}$$

Equations of motion

Israel junction conditions determine the equation of motion of bubble wall [Davis, 2003]

$$\Delta K_{ab} - \Delta K h_{ab} + 2\alpha \left[3\Delta \mathcal{J}_{ab} - \Delta \mathcal{J} h_{ab} - 2\mathcal{P}_{acbd}\Delta K^{cd}\right] = 8\pi G \sigma h_{ab}$$

$$\mathcal{P}_{abcd} = \mathcal{R}_{abcd} + 2\mathcal{R}_{b[c}h_{d]a} - 2\mathcal{R}_{a[c}h_{d]b} + \mathcal{R}h_{a[c}h_{d]b}$$

The wall trajectory $R(\tau)$ satisfies :

$$(f_+\dot{\tau}_+ - f_-\dot{\tau}_-)\left[1 + \frac{2\tilde{\alpha}}{R^2} - \frac{4\tilde{\alpha}}{3}\frac{\dot{R}^2}{R^2}\right] - \frac{2\tilde{\alpha}}{3R^2}\left(f_+^2\dot{\tau}_+ - f_-^2\dot{\tau}_-\right) = -\frac{8\pi G\sigma R}{(D-2)},$$

Equations of motion for $R(\lambda)$:

$$\frac{\dot{R}^2}{R^2} = \frac{\bar{f}_E}{R^2} - \frac{(\Delta f_E)^2}{16R^4\bar{\sigma}^2} - \bar{\sigma}^2 + \frac{\tilde{\alpha}}{3R^4}\left\{3(\bar{f}_E - 1)^2 + \frac{(\Delta f_E)^2}{4} + 12R^2\bar{\sigma}^2(1 - \bar{f}_E) + 8R^4\bar{\sigma}^4\right\}$$

$f_E = 1 - \frac{\mu}{r^{D-3}} - \frac{2\Lambda r^2}{(D-1)(D-2)}$ $\quad \bar{\sigma} = \frac{2\pi G\sigma}{D-2}$ $\quad \bar{f}_E = (f_{E+} + f_{E-})/2, \ \Delta f = f_{E+} - f_{E-}$

Background action

The full action of the composite system is $I = I_{bulk} + I_{brane}$

$$I_{bulk} = \frac{1}{16\pi G}\int_{M_-} d^D x \sqrt{g_-}\,(R_- - 2\Lambda_- + \alpha\, L_{GB}^-) + \int_{M_+} d^D x \sqrt{g_+}\,(R_+ - 2\Lambda_+ + \alpha\, L_{GB}^+)$$

$$I_{brane} = \frac{1}{16\pi G}\int_W d^{D-1}x \sqrt{h}\left(\sigma + \frac{1}{8\pi G}(\Delta K - 2\alpha[2\,\mathrm{G_{ab}}\Delta K^{ab} - \Delta J])\right)$$

$$J_{ab} = \frac{1}{3}(2KK_{ac}K_b^c + K_{cd}K^{cd}K_{ab} - 2K_{ac}K^{cd}K_{db} - K^2K_{ab})$$

Integrating the bulk and wall contribution, together with the conical contribution, giving action of seed black hole as:

$$I_{seed} = I_C - S_{BH}(M_+, \Lambda_+)$$

Contribution either from a cutoff boundary ($\Lambda \leq 0$), or a cosmological horizon ($\Lambda > 0$).

Bubble action and decay rate

The bubble action:

$$I_{bubble} = I_{wall} + I_C - S_{BH}(M_-, \Lambda_-)$$

The integral over the wall:

$$I_{wall} = \frac{\mathcal{A}_{D-2}}{8\pi G} \int d\lambda R^{D-2} \Delta \left[-\frac{4(D-2)\tilde{\alpha}}{(D-4)} \frac{\ddot{R}}{R} K_1 + \left(\frac{\dot{R}^2}{f} K_0 - \dot{\tau} \ddot{R} \right) \left(1 + \frac{2(D-2)\tilde{\alpha}(1-f)}{(D-4)R^2} \right) \right]$$

Giving the tunnelling exponent:

$$I_B = I_{bubble} - I_{seed} = S_{BH}(M_+, \Lambda_+) - S_{BH}(M_-, \Lambda_-) + I_{wall}$$

↑ Static

$$I_{wall} = 0$$

Gauss-Bonnet in 4D

In 4D, the action of the instanton is:

$$I_B = I_{bubble} - I_{seed} = S_{BH}(M_+, \Lambda_+) - S_{BH}(M_-, \Lambda_-) + I_{wall}$$

$$S_{BH} = \frac{\pi r_+^2}{G}(1 + 4\alpha/r_+^2)$$

Hence if we tunnel with a black hole seed and have a remnant, there is no alteration to the action, **however,** tunnelling from a seed to no remnant increases the action,

$$I_{B,NR} = I_{bubble} - I_{seed} = \frac{\pi r_+^2}{G} + \frac{4\pi\alpha}{G} + \underset{\uparrow}{I_{wall}}$$

There is no α dependent contribution from the wall

For positive α, suppressing the topology-changing transitions.

Entropy shift

Tunnelling from a positive vacuum energy with a seed black hole to the Minkowski vacuum.

$$\Delta\mu = \mu_+ = 2\bar{\mu}, \text{ and } \Delta\Lambda = \Lambda_+ = 2\Lambda$$

The critical instanton seed mass

$$\mu_{crit} = \frac{4\bar{\sigma}_e \gamma_e^{D-2}}{(1-4\tilde{\alpha}\bar{\sigma}^2)} \frac{(D-2)^{D-2}}{(D-1)^{D-1}}$$

μ_0 is the critical mass at $\tilde{\alpha}$=0

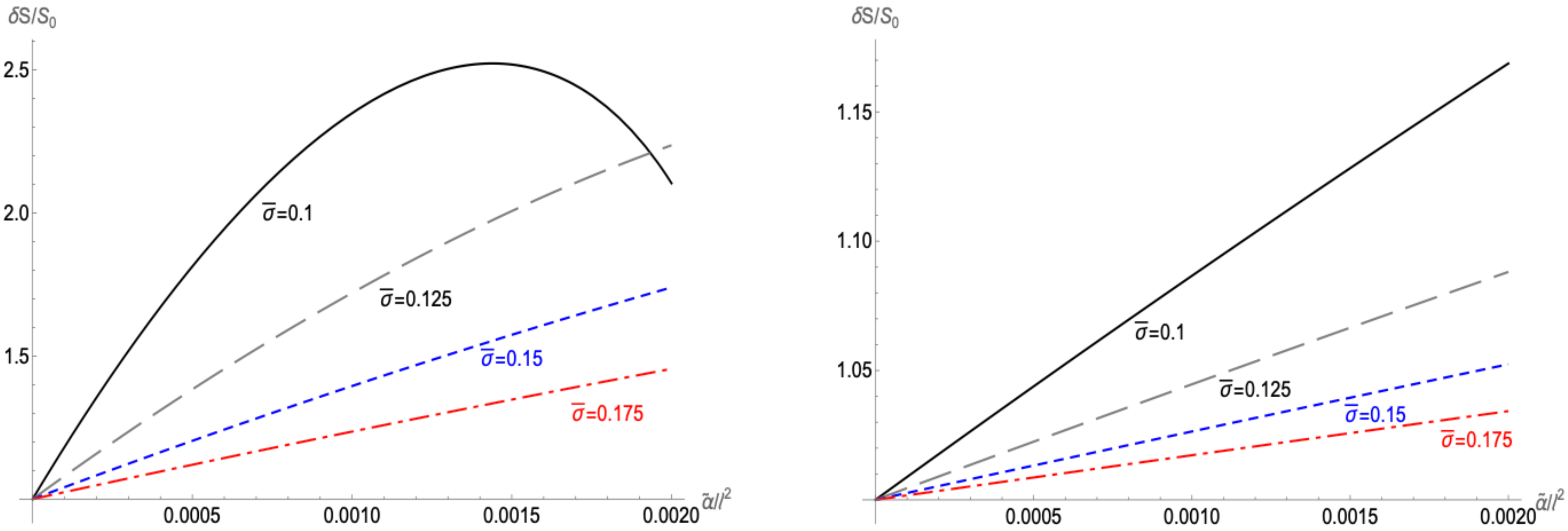
$$\frac{\delta\mu_{crit}}{\mu_0} = \frac{8}{3}\tilde{\alpha}\bar{\sigma}^2[1+(D-2)(1-2\bar{\sigma}\gamma_0)]$$

The entropy shift

$$\frac{\delta S}{S_0} = \frac{(D-2)\tilde{\alpha}}{r_0^3 f_E'(r_0)}\left(\frac{2r_0 f_E'(r_0)}{(D-4)} + \frac{8\mu_0\bar{\sigma}^2}{3r_0^{D-5}}[1+(D-2)(1-2\bar{\sigma}\gamma_0)] - 1\right)$$

The impact of GB correction

The critical static bubble is the instanton with the lowest action for a given wall tension σ.



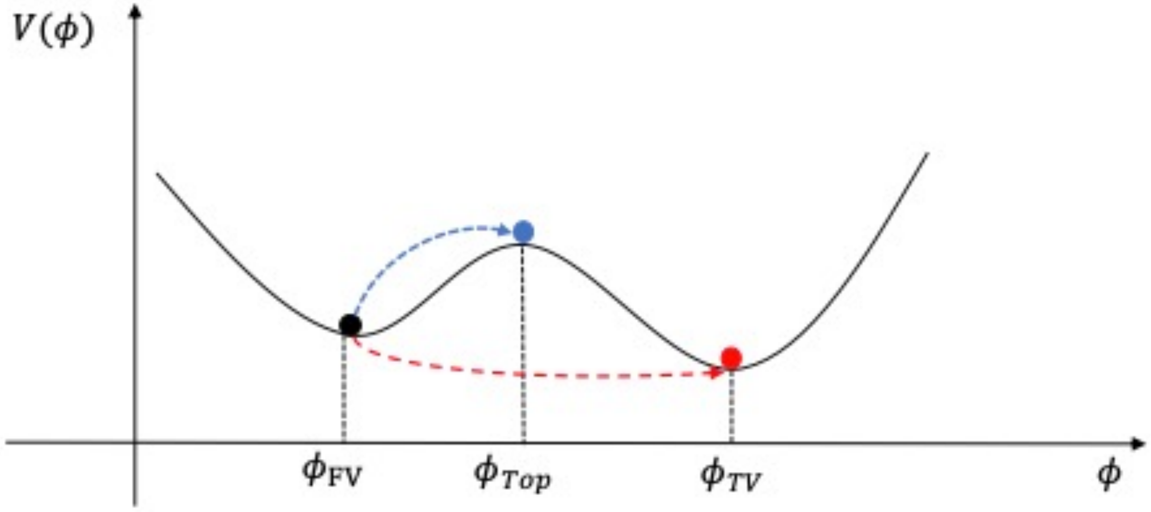
(a) Entropy variation in 5D

(b) Entropy variation in 6D

The variation in entropy of the seed black hole for both 5 and 6 dimensions.

Hawking Moss instanton

The Hawking Moss instanton is a transition from a false vacuum to a higher vacuum energy (from which we can roll to a lower energy).



The change in action is the difference in the entropies: $\Gamma_{Fv\to Top}{\sim}e^{-B}$

$$B_{FV\to Top} = I_T - I_F = [S_{CH} + S_{BH}]_F - [S_{CH} + S_{BH}]_T$$

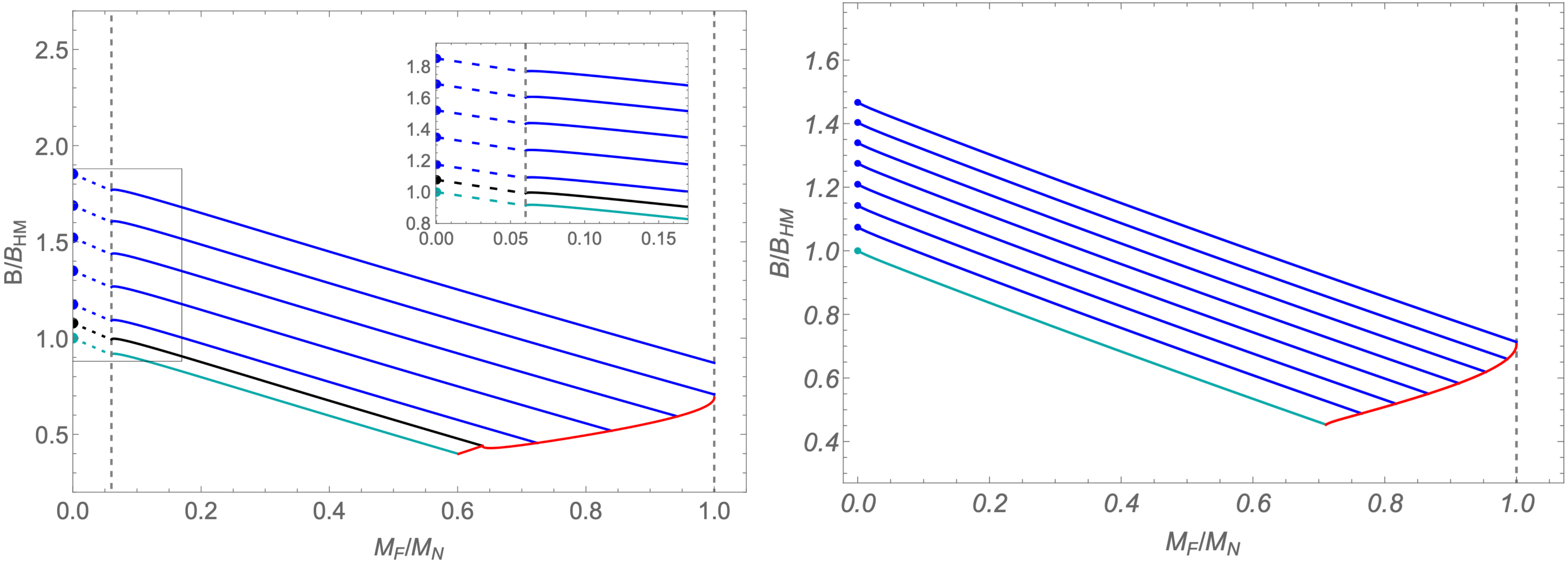
CAUTION

Cosmological Area Principle:

$$S_c|_T \leq S_c|_F$$

Hawking Moss instanton

Mass gap in 5D: $$M = \frac{(D-2)\mathcal{A}_{D-2}}{16\pi G}\mu = \frac{(D-2)\mathcal{A}_{D-2} r_h^{D-5}}{16\pi G}\left(\tilde{\alpha} + r_h^2 - \frac{r_h^4}{\ell^2}\right).$$



The ratios of tunnelling components B/B_{HM} change with the seed black hole mass in 5D and 6D spacetime.

Summary

- Vacuum decay is an example of quantum effects in action with gravity – we have good tools, but they are idealised.

- Tunnelling amplitudes are significantly enhanced in the presence of a black hole – a bubble forms around the black hole and can remove it altogether. Important if Higgs vacuum metastable.

- Gauss-Bonnet gravity suppresses topology changing transitions in 4D, and generally suppresses tunnelling – though only mildly.

- Gauss-Bonnet term lowered the black hole temperature and enhanced the variation of the entropy (for positive α).

Thanks for your attention & any questions?