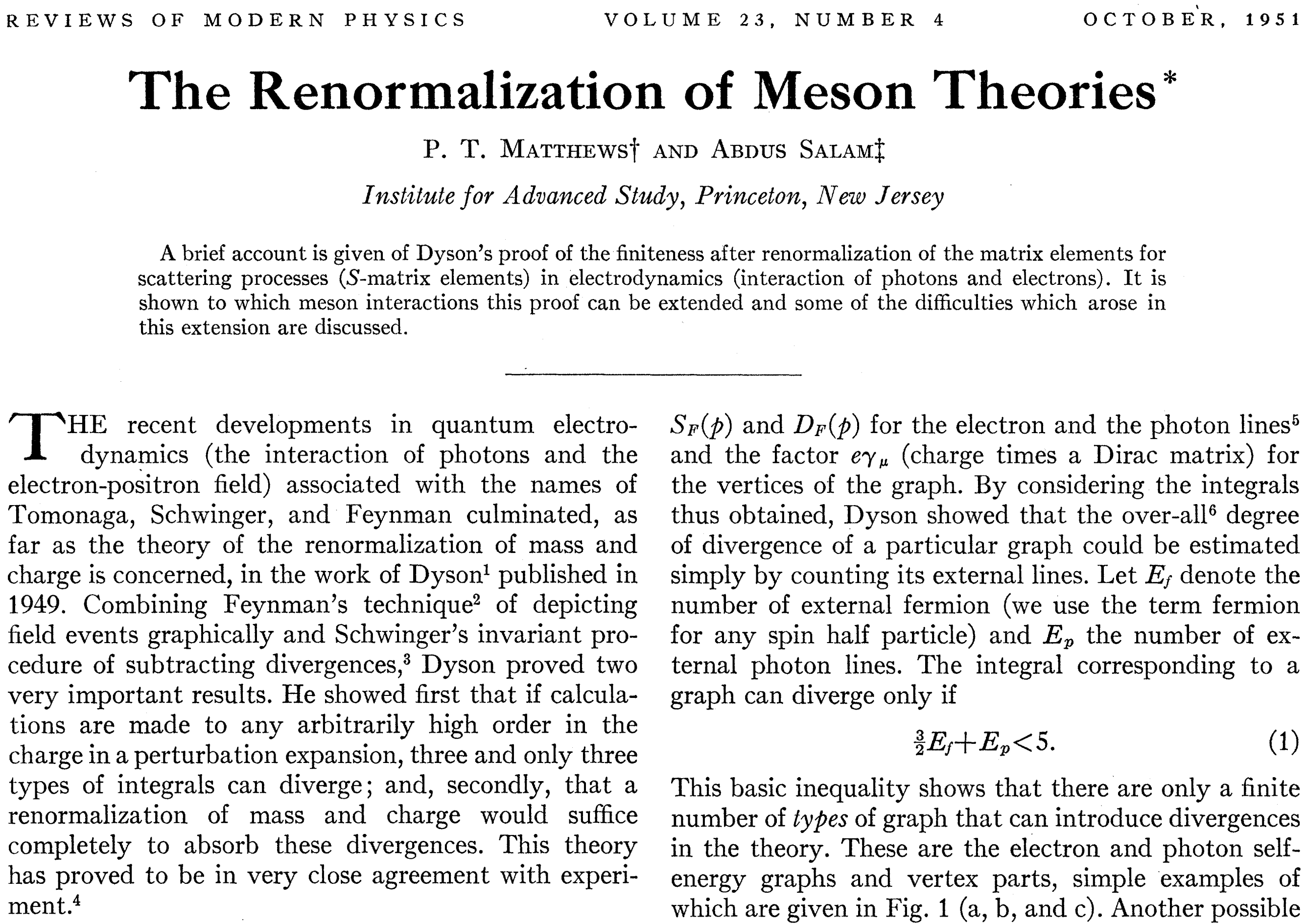
Vainshtein Screening, Galileons and Massive Gravity

Andrew J. Tolley
Imperial College London

GC2024, *Yukawa Institute*, Feb 29 2024



REVIEWS OF MODERN PHYSICS VOLUME 23, NUMBER 4 OCTOBER, 1951

The Renormalization of Meson Theories*

P. T. Matthews† and Abdus Salam‡
Institute for Advanced Study, Princeton, New Jersey

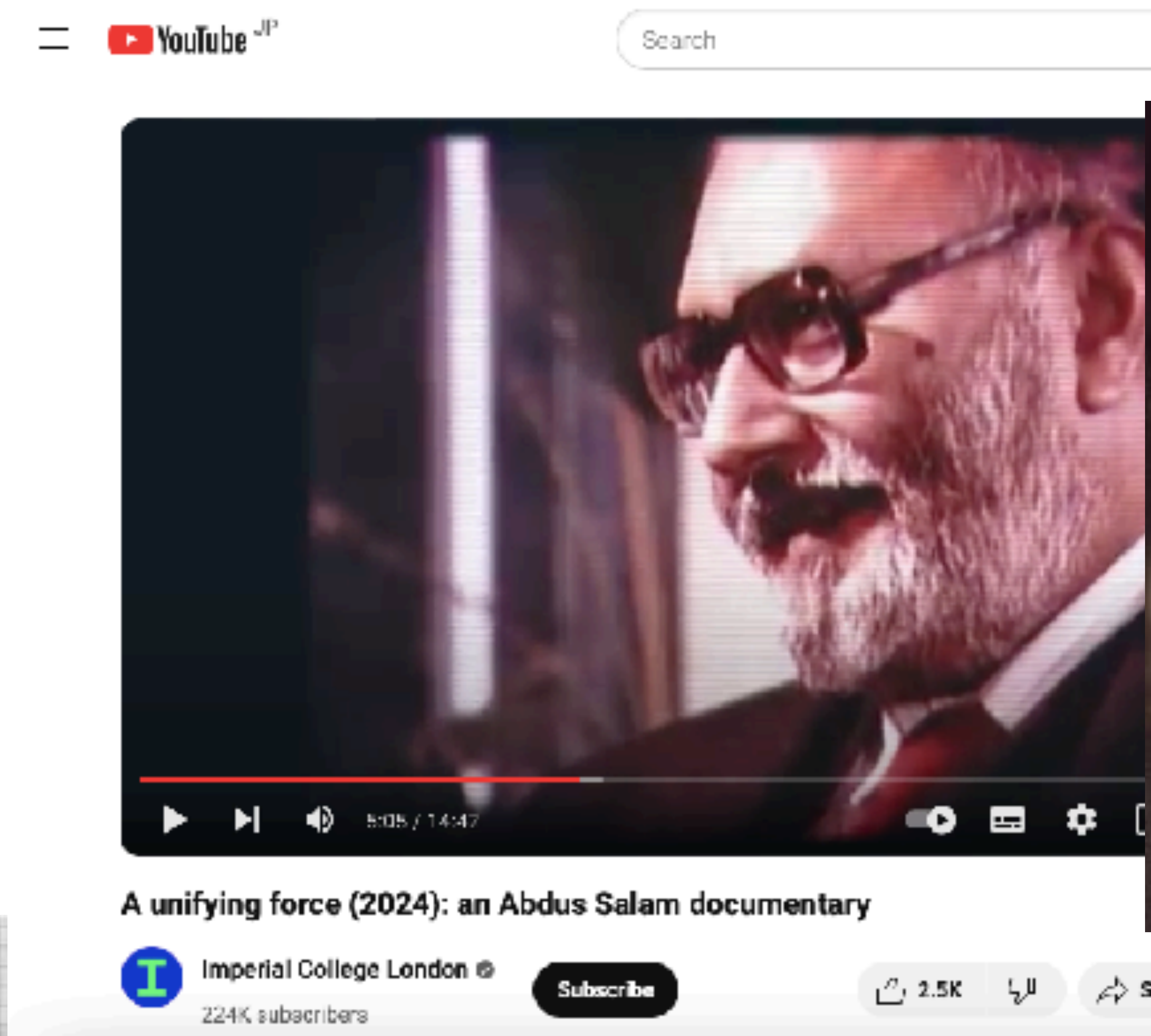
A brief account is given of Dyson's proof of the finiteness after renormalization of the matrix elements for scattering processes (*S*-matrix elements) in electrodynamics (interaction of photons and electrons). It is shown to which meson interactions this proof can be extended and some of the difficulties which arose in this extension are discussed.

THE recent developments in quantum electrodynamics (the interaction of photons and the electron-positron field) associated with the names of Tomonaga, Schwinger, and Feynman culminated, as far as the theory of the renormalization of mass and charge is concerned, in the work of Dyson[1] published in 1949. Combining Feynman's technique[2] of depicting field events graphically and Schwinger's invariant procedure of subtracting divergences,[3] Dyson proved two very important results. He showed first that if calculations are made to any arbitrarily high order in the charge in a perturbation expansion, three and only three types of integrals can diverge; and, secondly, that a renormalization of mass and charge would suffice completely to absorb these divergences. This theory has proved to be in very close agreement with experiment.[4]

$S_F(p)$ and $D_F(p)$ for the electron and the photon lines[5] and the factor $e\gamma_\mu$ (charge times a Dirac matrix) for the vertices of the graph. By considering the integrals thus obtained, Dyson showed that the over-all[6] degree of divergence of a particular graph could be estimated simply by counting its external lines. Let E_f denote the number of external fermion (we use the term fermion for any spin half particle) and E_p the number of external photon lines. The integral corresponding to a graph can diverge only if

$$\tfrac{3}{2}E_f+E_p<5. \qquad (1)$$

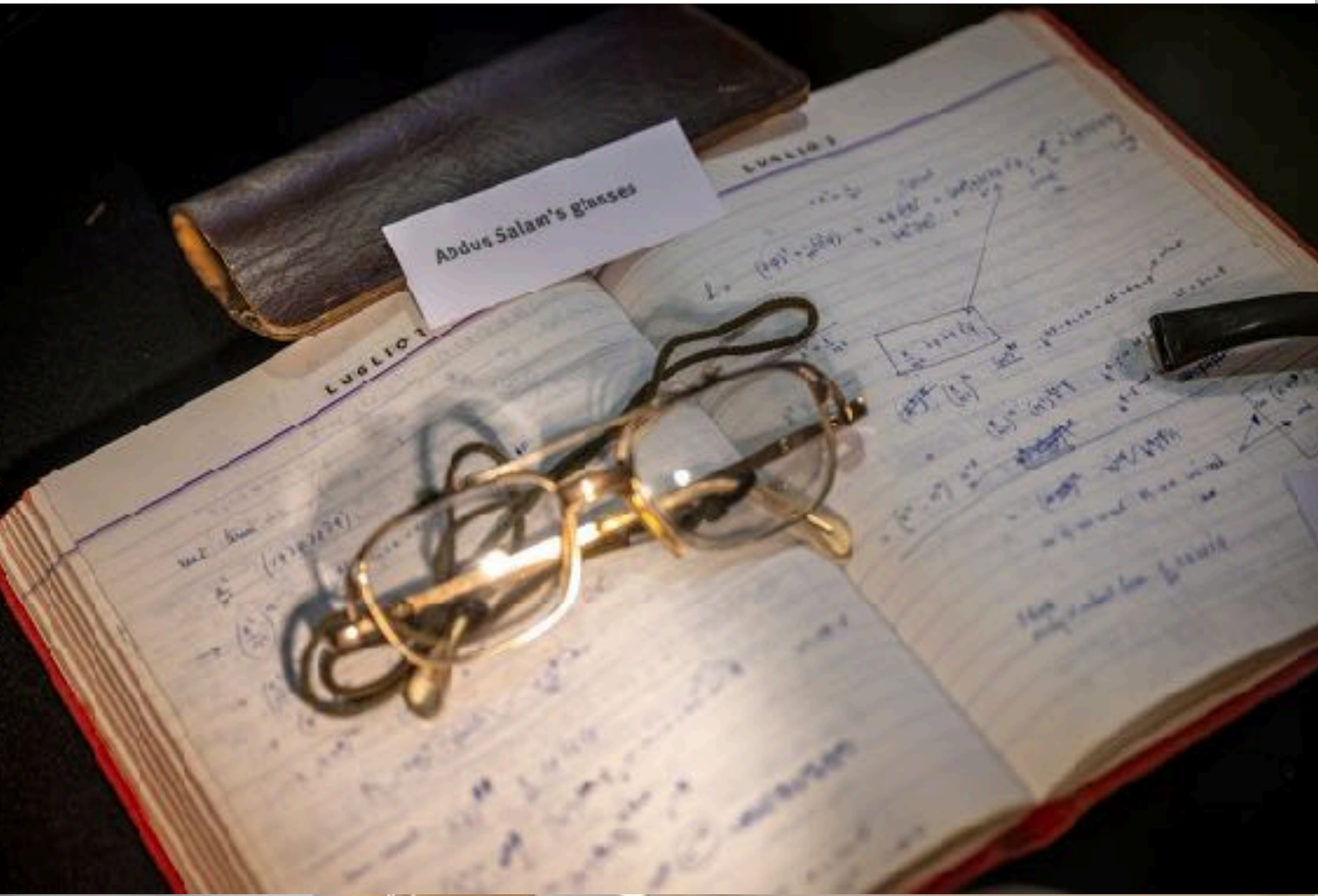
This basic inequality shows that there are only a finite number of *types* of graph that can introduce divergences in the theory. These are the electron and photon self-energy graphs and vertex parts, simple examples of which are given in Fig. 1 (a, b, and c). Another possible

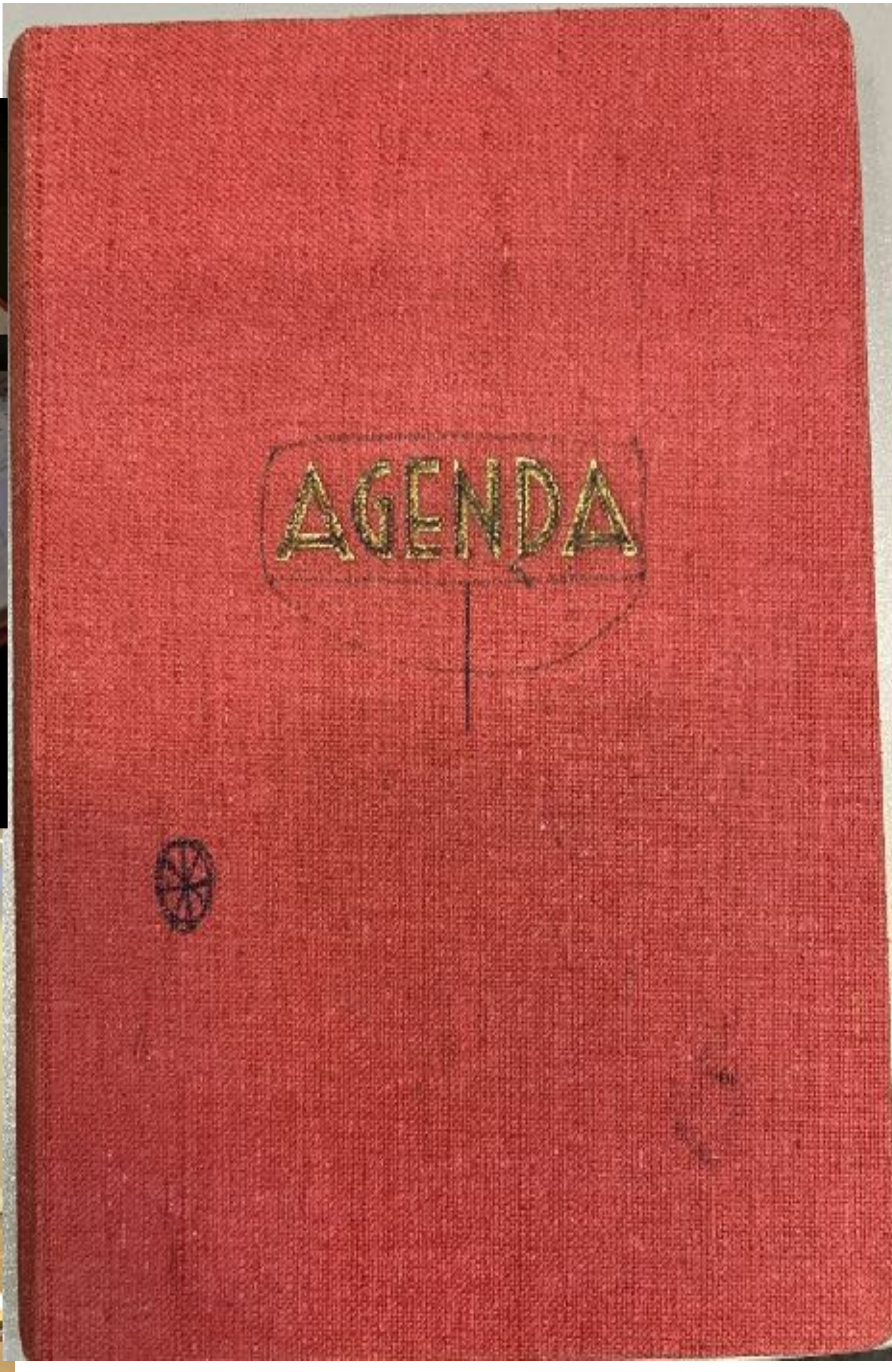


A unifying force (2024): an Abdus Salam documentary

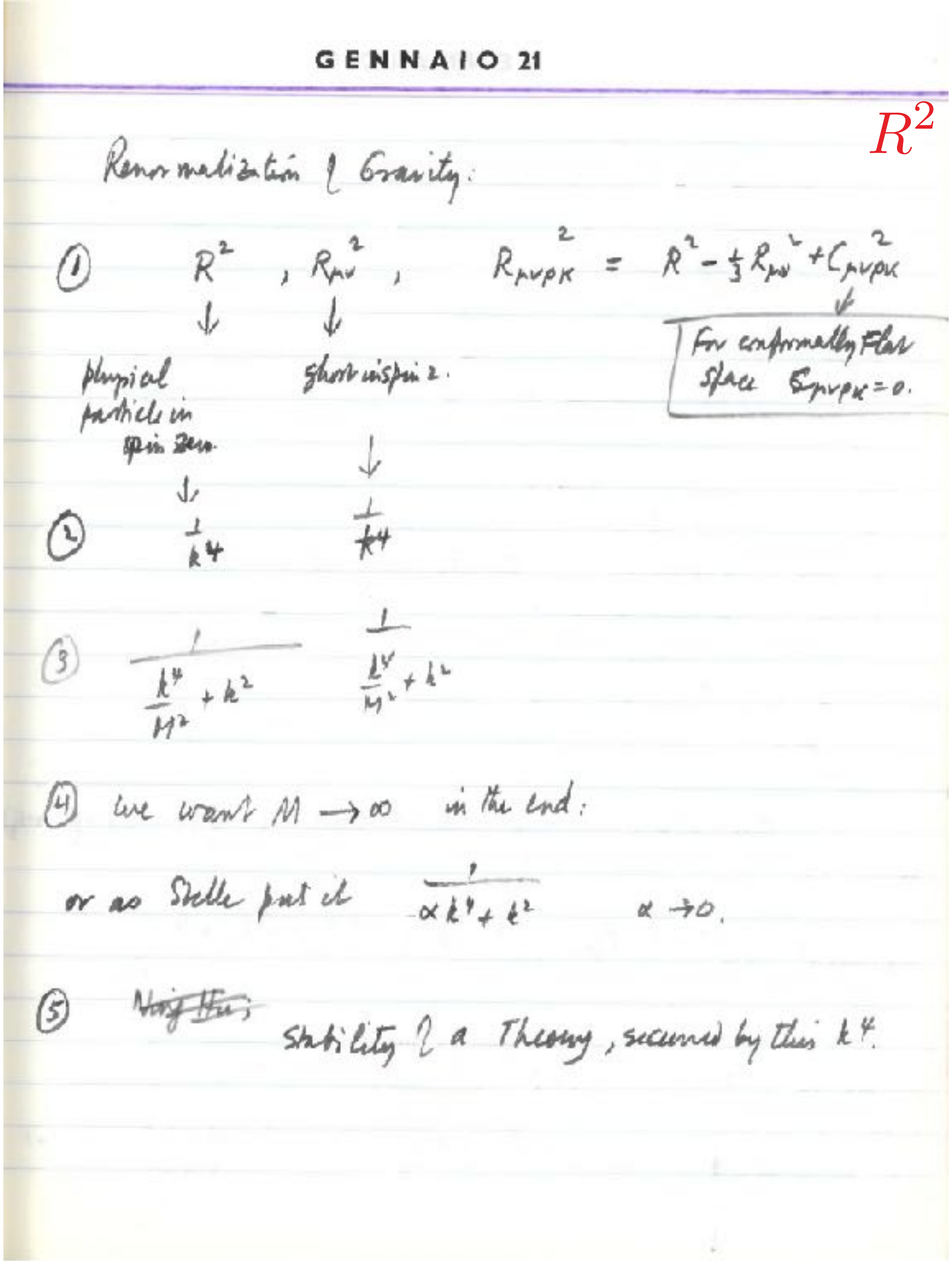
Imperial College London
224K subscribers

Abdus Salam’s 1977 notebook





R^2 gravity



GENNAIO 21

Renormalization of Gravity:

① R^2 , $R_{\mu\nu}^2$, $R_{\mu\nu\rho\kappa}^2 = R^2 - \frac{1}{3} R_{\mu\nu}^2 + C_{\mu\nu\rho\kappa}^2$

$R^2 \downarrow$ physical particle in spin zero.

$R_{\mu\nu}^2 \downarrow$ ghost in spin 2.

$C_{\mu\nu\rho\kappa}^2 \downarrow$ For conformally Flat space $C_{\mu\nu\rho\kappa} = 0$.

② $\frac{1}{k^4}$ $\frac{1}{k^4}$

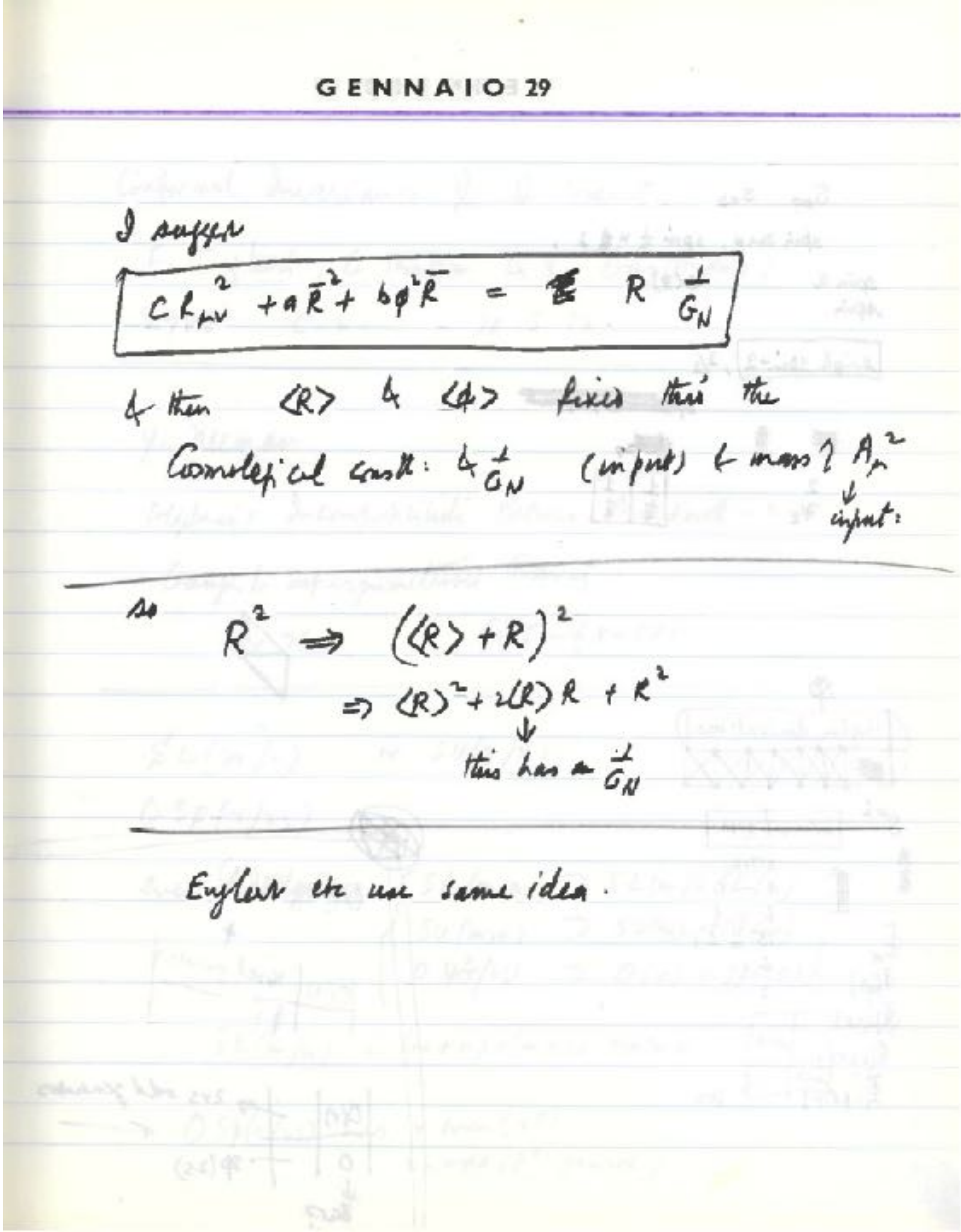
③ $\frac{1}{\frac{k^4}{M^2} + k^2}$ $\frac{1}{\frac{k^4}{M^2} + k^2}$

④ we want $M \to \infty$ in the end:

or as Stelle put it $\frac{1}{\alpha k^4 + k^2}$ $\alpha \to 0$.

⑤ ~~Nonf this~~ Stability of a Theory, secured by this k^4.

R^2 gravity



GENNAIO 29

I suggest

$$\boxed{c R_{\mu\nu}^2 + a\bar{R}^2 + b\phi^2\bar{R} = R\,\frac{1}{G_N}}$$

& then $\langle R\rangle$ & $\langle\phi\rangle$ fixes this the

Cosmological const: & $\frac{1}{G_N}$ (input) & mass? $\underset{\substack{\downarrow\\ \text{input:}}}{A_\mu^2}$

---

so $R^2 \Rightarrow (\langle R\rangle + R)^2$

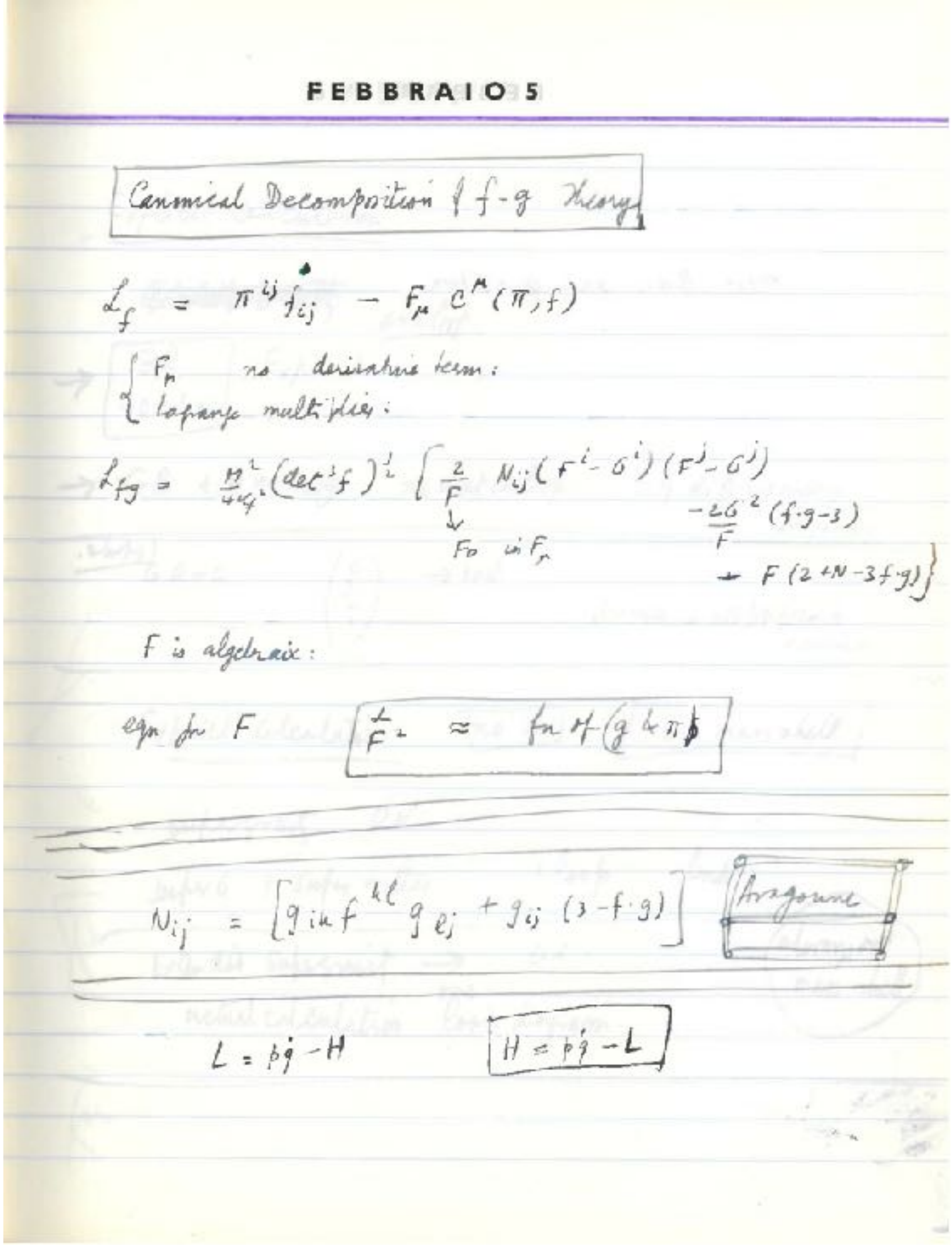
$$\Rightarrow \langle R\rangle^2 + \underset{\substack{\downarrow\\ \text{this has a } \frac{1}{G_N}}}{2\langle R\rangle R} + R^2$$

Englert etc use same idea.

Bigravity

more than one graviton

Bigravity



FEBBRAIO 5

Canonical Decomposition of f-g Theory

$$\mathcal{L}_f = \pi^{ij}\dot{f}_{ij} - F_\mu C^\mu(\pi, f)$$

$\Big\{$ F_μ no derivative term:
lagrange multiplier:

$$\mathcal{L}_{fg} = \frac{M^2}{4\kappa^2}(\det{}^3 f)^{\frac{1}{2}}\Big\{\underset{\substack{\downarrow \\ F_0 \text{ in } F_\mu}}{\frac{2}{F}} N_{ij}(F^i - G^i)(F^j - G^j) - \frac{2G^2}{F}(f\cdot g - 3) + F(2 + N - 3f\cdot g)\Big\}$$

F is algebraic:

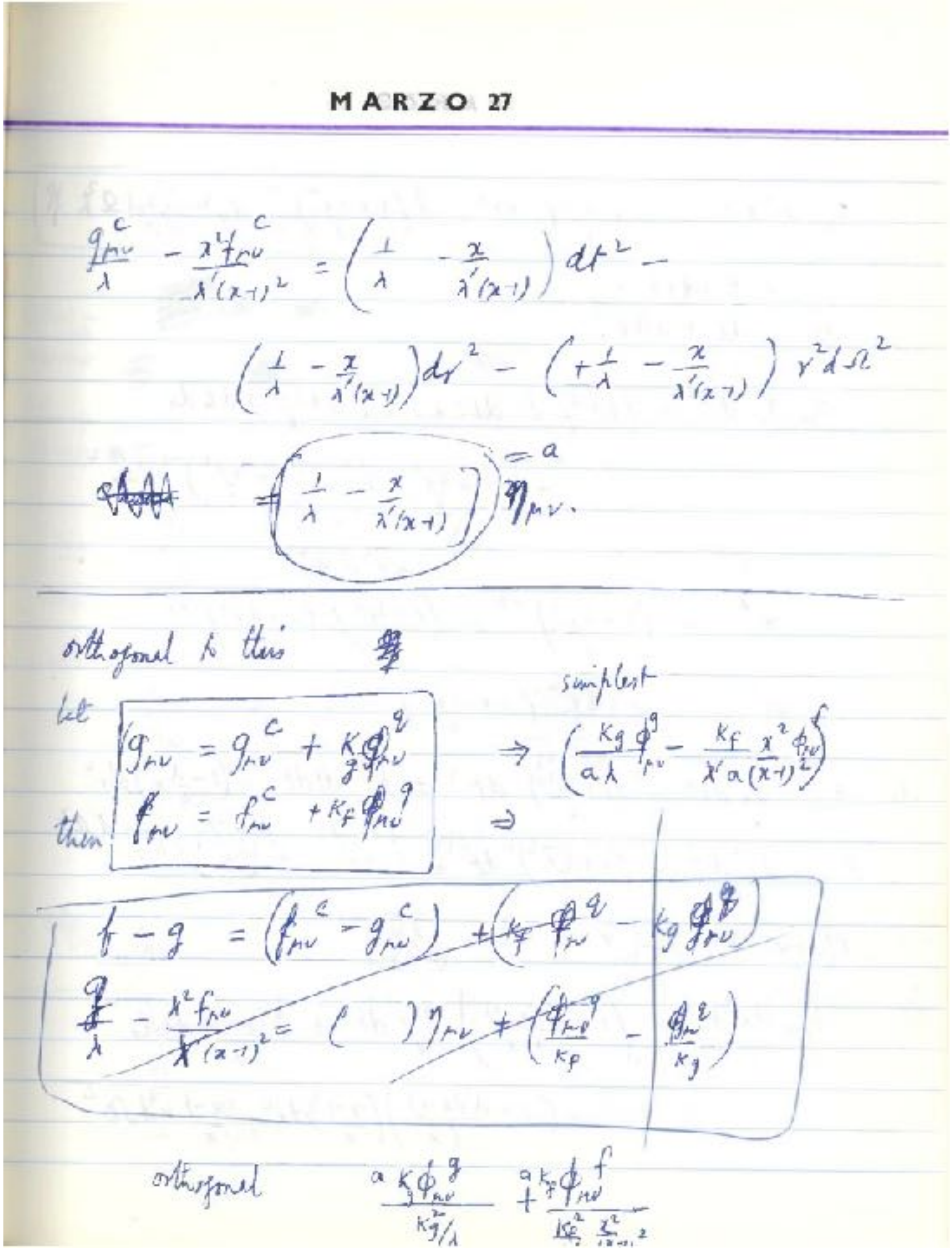
eqn for F $\boxed{\frac{1}{F^2} \approx \text{fn of } (g \,\&\, \pi)}$

$$N_{ij} = \left[g_{ik} f^{kl} g_{lj} + g_{ij}(3 - f\cdot g)\right]$$

Arragoune

$$L = p\dot{q} - H \qquad \boxed{H = p\dot{q} - L}$$

Bigravity



MARZO 27

$$\frac{g^c_{\mu\nu}}{\lambda} - \frac{x^2 f^c_{\mu\nu}}{\lambda'(x-1)^2} = \left(\frac{1}{\lambda} - \frac{x}{\lambda'(x-1)}\right) dt^2 -$$

$$\left(\frac{1}{\lambda} - \frac{x}{\lambda'(x-1)}\right) dr^2 - \left(+\frac{1}{\lambda} - \frac{x}{\lambda'(x-1)}\right) r^2 d\Omega^2$$

$$= \overset{= a}{\boxed{\left(\frac{1}{\lambda} - \frac{x}{\lambda'(x-1)}\right)}} \eta_{\mu\nu}.$$

---

orthogonal to this

let $\boxed{g_{\mu\nu} = g^c_{\mu\nu} + \kappa_g \phi^g_{\mu\nu}}$ $\Rightarrow$ simplest $\left(\frac{\kappa_g}{a\lambda}\phi^g_{\mu\nu} - \frac{\kappa_f}{\lambda' a}\frac{x^2}{(x-1)^2}\phi^f_{\mu\nu}\right)$

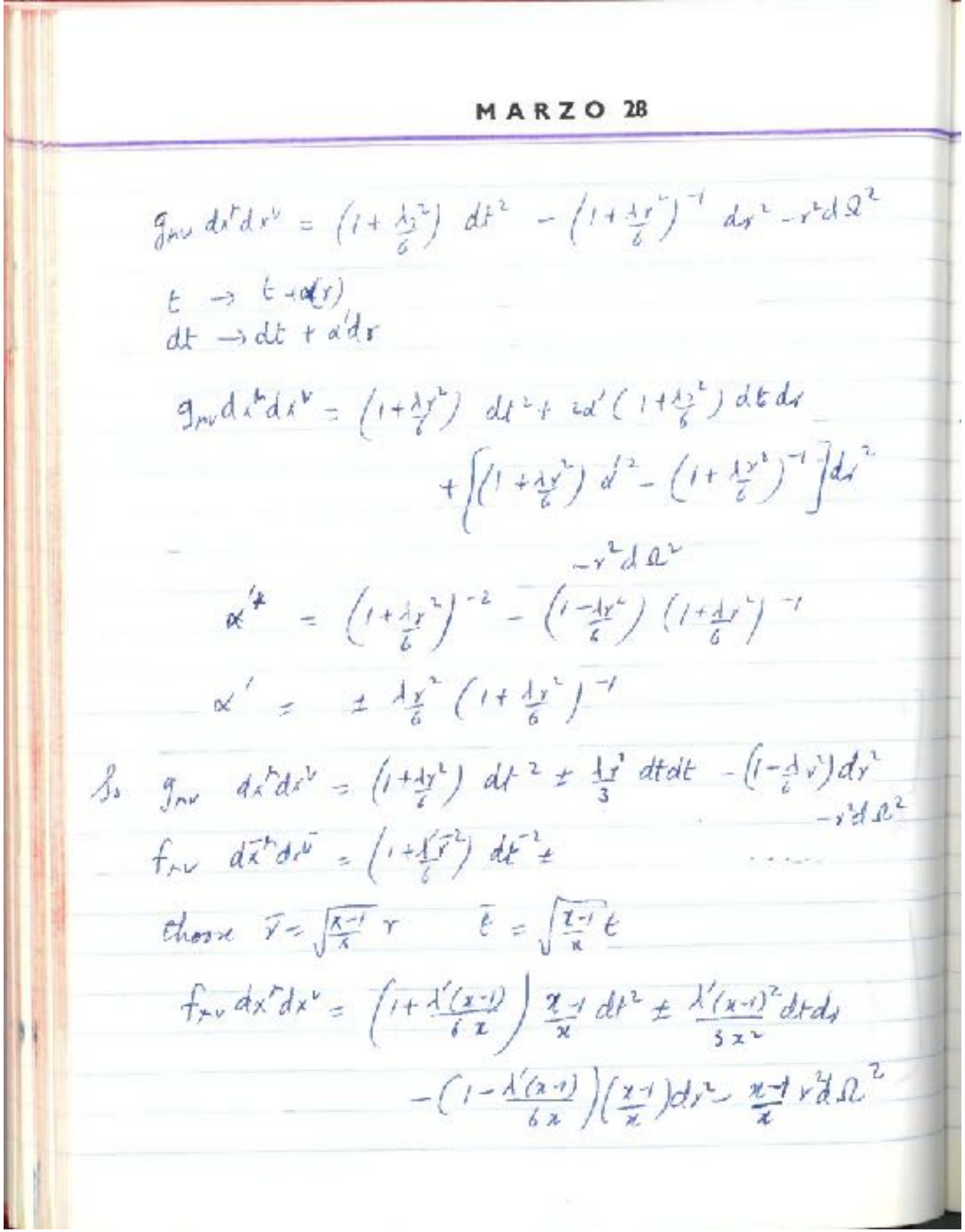
then $\boxed{f_{\mu\nu} = f^c_{\mu\nu} + \kappa_f \phi^g_{\mu\nu}}$ $\Rightarrow$

$$f - g = \left(f^c_{\mu\nu} - g^c_{\mu\nu}\right) + \left(\kappa_f \phi^g_{\mu\nu} - \kappa_g \phi^g_{\mu\nu}\right)$$

$$\frac{g}{\lambda} - \frac{x^2 f_{\mu\nu}}{\lambda'(x-1)^2} = (\quad)\eta_{\mu\nu} + \left(\frac{\phi^g_{\mu\nu}}{\kappa_f} - \frac{\phi^g_{\mu\nu}}{\kappa_g}\right)$$

orthogonal $\quad \frac{a\,\kappa_g \phi^g_{\mu\nu}}{\kappa_g^2/\lambda} + \frac{a\,\kappa_f \phi^f_{\mu\nu}}{\kappa_f^2 \frac{x^2}{(x-1)^2}}$

Bigravity



MARZO 28

$$g_{\mu\nu}dx^\mu dx^\nu = \left(1+\frac{\lambda r^2}{6}\right)dt^2 - \left(1+\frac{\lambda r^2}{6}\right)^{-1}dr^2 - r^2 d\Omega^2$$

$t \to t + \alpha(r)$

$dt \to dt + \alpha' dr$

$$g_{\mu\nu}dx^\mu dx^\nu = \left(1+\frac{\lambda r^2}{6}\right)dt^2 + 2\alpha'\left(1+\frac{\lambda r^2}{6}\right)dt\,dr + \left[\left(1+\frac{\lambda r^2}{6}\right)\alpha'^2 - \left(1+\frac{\lambda r^2}{6}\right)^{-1}\right]dr^2 - r^2 d\Omega^2$$

$$\alpha'^2 = \left(1+\frac{\lambda r^2}{6}\right)^{-2} - \left(1-\frac{\lambda r^2}{6}\right)\left(1+\frac{\lambda r^2}{6}\right)^{-1}$$

$$\alpha' = \pm\frac{\lambda r^2}{6}\left(1+\frac{\lambda r^2}{6}\right)^{-1}$$

So $g_{\mu\nu}dx^\mu dx^\nu = \left(1+\frac{\lambda r^2}{6}\right)dt^2 \pm \frac{\lambda r^2}{3}dt\,dt - \left(1-\frac{\lambda}{6}r^2\right)dr^2 - r^2 d\Omega^2$

$f_{\mu\nu}d\bar{x}^\mu d\bar{x}^\nu = \left(1+\frac{\lambda'\bar{r}^2}{6}\right)d\bar{t}^2 \pm \cdots\cdots$

choose $\bar{r} = \sqrt{\frac{x-1}{x}}\,r \qquad \bar{t} = \sqrt{\frac{x-1}{x}}\,t$

$$f_{\mu\nu}dx^\mu dx^\nu = \left(1+\frac{\lambda'(x-1)}{6x}\right)\frac{x-1}{x}dt^2 \pm \frac{\lambda'(x-1)^2}{3x^2}dt\,dr - \left(1-\frac{\lambda'(x-1)}{6x}\right)\left(\frac{x-1}{x}\right)dr^2 - \frac{x-1}{x}r^2 d\Omega^2$$

Equivalence Principle

Principle of uniqueness of free fall

APRILE 8

Dicke 1964

p 1051

~~rsponse of~~
response of

n } → $(.6-.5)\epsilon^{-1}$ ← Gold is 60% neutron
alum 50% neutron
parts in 10^{11} ie 1 in 10^{10}

p } same

e → 2 in 10^7

e^+e^- virtual → more in Gold than in Al: 1 in 10^4

nuclear BE → 1 in 10^7

Electrostatic energy → 3 in 10^9.

to Sun's gravity

1 : 10^{11} Sun's acc: of alumi: & gold

1 : 10^{12} " " Platinum & aluminium:

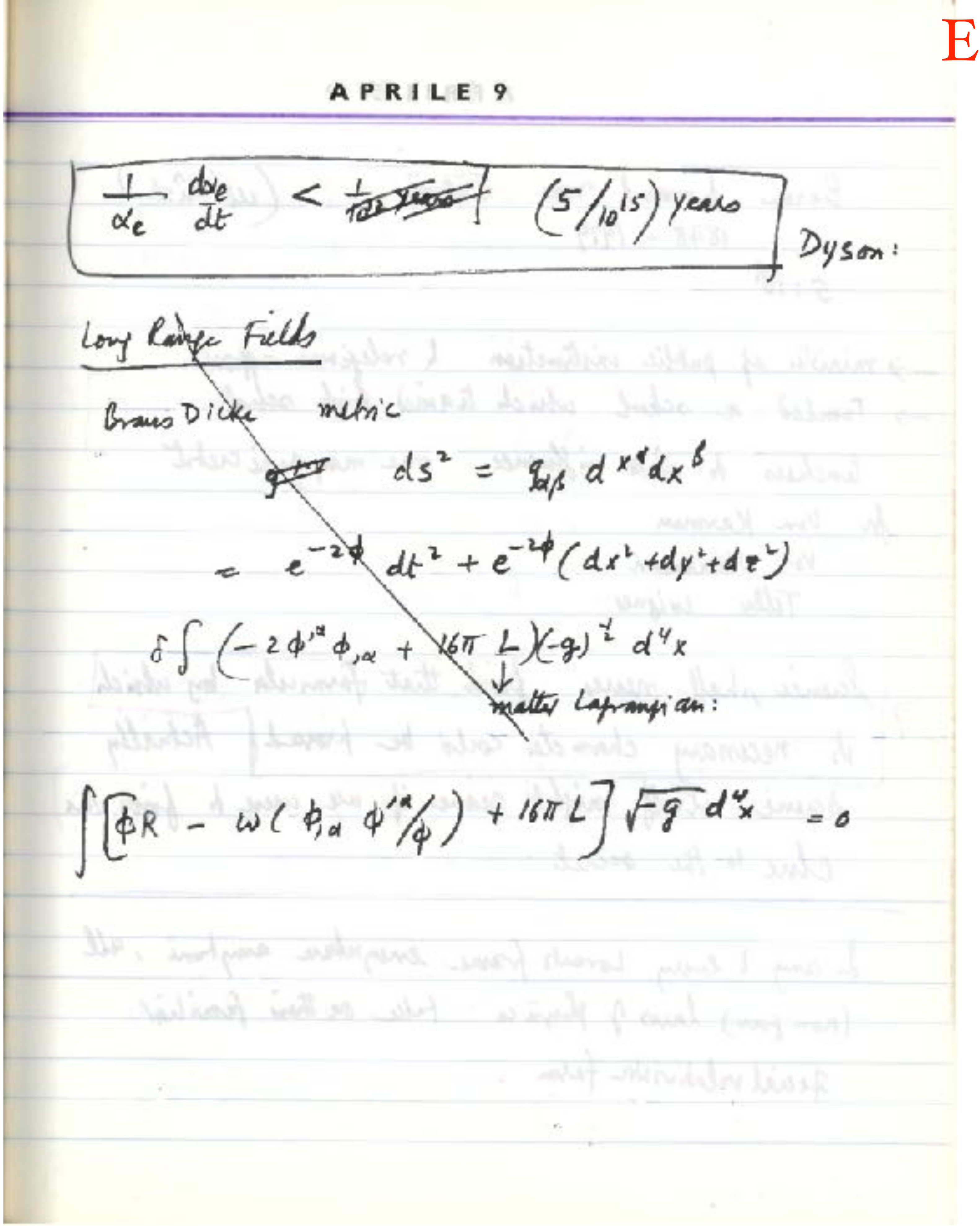
Eötvös (Etvash) 5 : 10^9 Earth imparts same acc to wood platinum, copper, asbestos, water,

Dicke Shapiro:

~~Eo~~ Sun acc: → to earth, → to moon } same:

Dicke (It is well known that carbon is required to make physicists)

Equivalence Principle



APRILE 9

$$\boxed{\frac{1}{\alpha_e}\frac{d\alpha_e}{dt} < \cancel{\frac{1}{10^{12}\text{ years}}}\ \left(5/10^{15}\right)\text{ years}}$$ Dyson:

Long Range Fields

Brans Dicke metric

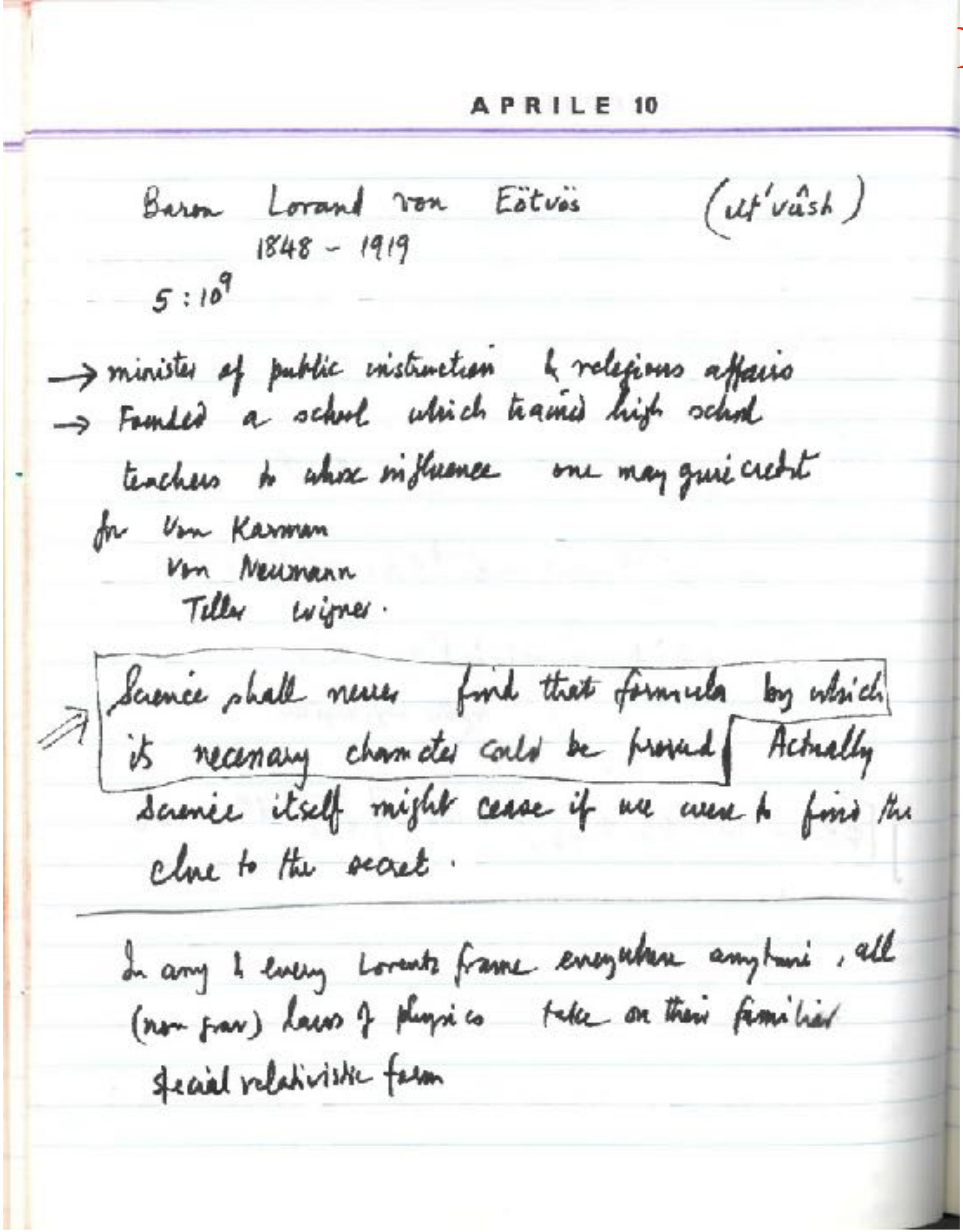
$$\cancel{g^{\mu\nu}}\quad ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$$

$$= e^{-2\phi} dt^2 + e^{-2\phi}(dx^2 + dy^2 + dz^2)$$

$$\delta \int \Big(-2\phi^{,\alpha}\phi_{,\alpha} + 16\pi \underset{\substack{\downarrow \\ \text{matter Lagrangian:}}}{L}\Big)(-g)^{\frac{1}{2}}\, d^4x$$

$$\int\left[\phi R - \omega\left(\phi_{,\alpha}\phi^{,\alpha}/\phi\right) + 16\pi L\right]\sqrt{-g}\, d^4x = 0$$

Equivalence Principle



APRILE 10

Baron Lorand von Eötvös (ut'vâsh)
1848 - 1919

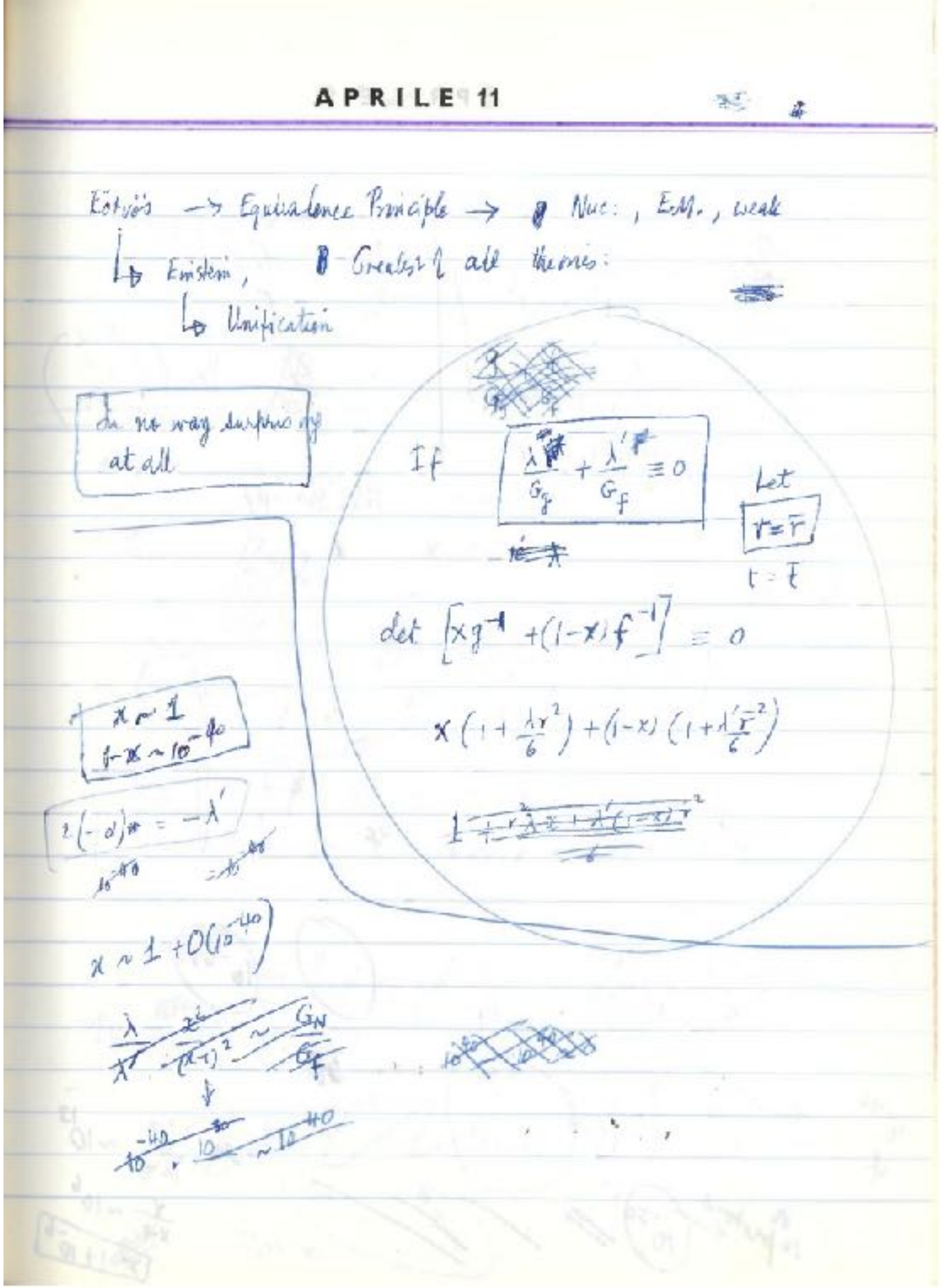
$5 : 10^9$

→ minister of public instruction & religious affairs
→ Founded a school which trained high school teachers to whose influence one may give credit for Von Karman
Von Neumann
Teller Wigner.

Science shall never find that formula by which its necessary character could be proved. Actually science itself might cease if we were to find the clue to the secret.

In any & every Lorentz frame everywhere anytime, all (non grav) laws of physics take on their familiar special relativistic form

Equivalence Principle + bigravity



APRILE 11

Eötvös → Equivalence Principle → Nuc., E.M., weak

↳ Einstein, Greatest of all theories:

↳ Unification

In no way surprising at all

If $\boxed{\frac{\lambda}{G_g} + \frac{\lambda'}{G_f} \equiv 0}$

Let $\boxed{r=\bar{r}}$

$t=\bar{t}$

$$\det\left[x g^{-1} + (1-x) f^{-1}\right] = 0$$

$$x\left(1+\frac{\lambda r^2}{6}\right) + (1-x)\left(1+\frac{\lambda' \bar{r}^2}{6}\right)$$

$\boxed{x \sim 1 \\ 1-x \sim 10^{-40}}$

$\boxed{\lambda(-\alpha) = -\lambda'}$

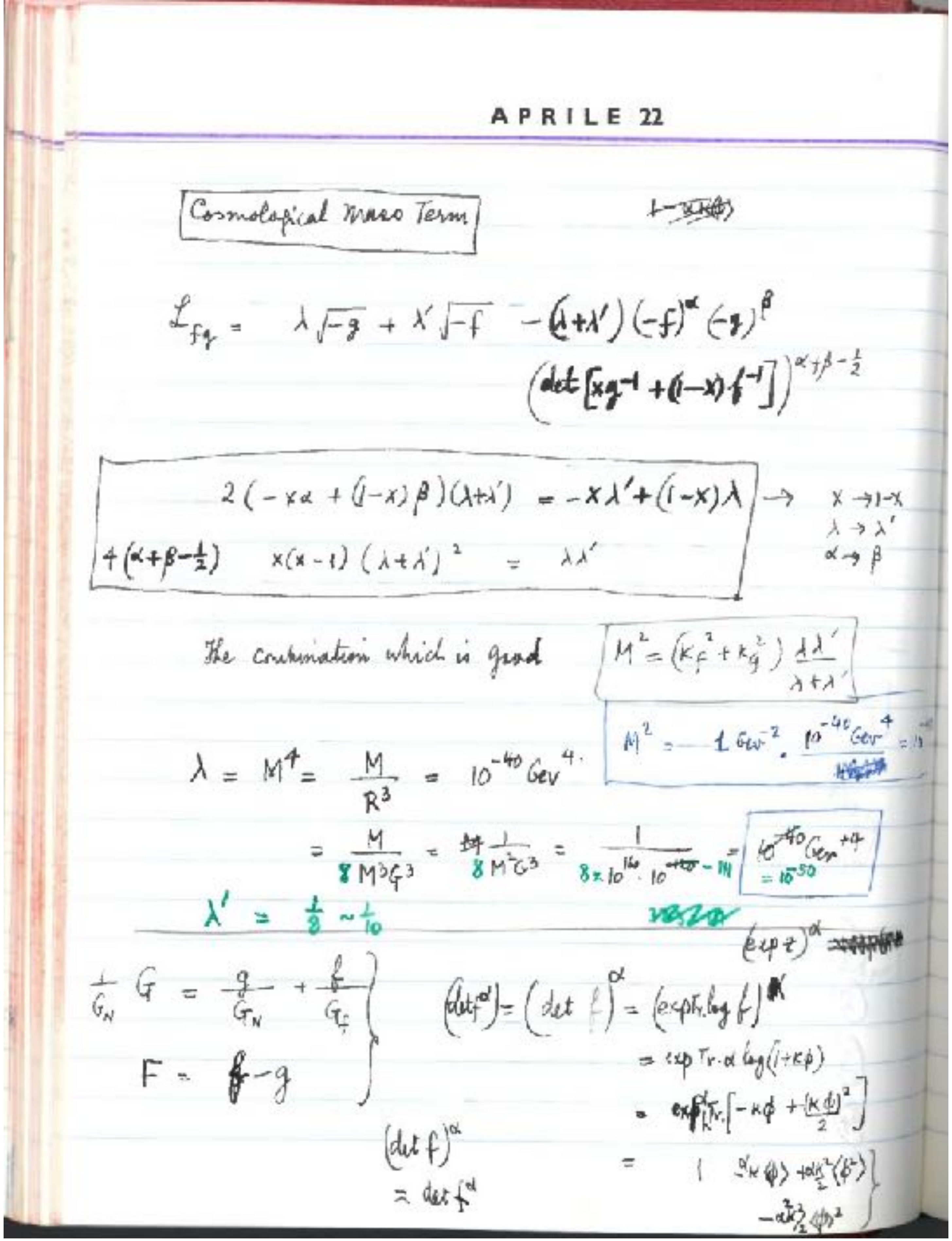
$x \sim 1 + O(10^{-40})$

~~$\frac{\lambda}{\lambda'} \frac{x^2}{(x-1)^2} \sim \frac{G_N}{G_f}$~~

↓

~~$10^{-40} \cdot 10^{80} \sim 10^{40}$~~

Cosmological Bigravity



APRILE 22

$\boxed{\text{Cosmological mass Term}}$

$$\mathcal{L}_{fg} = \lambda\sqrt{-g} + \lambda'\sqrt{-f} - (\lambda+\lambda')(-f)^{\alpha}(-g)^{\beta}$$
$$\left(\det\left[xg^{-1} + (1-x)f^{-1}\right]\right)^{\alpha+\beta-\frac{1}{2}}$$

$$2(-x\alpha + (1-x)\beta)(\lambda+\lambda') = -x\lambda' + (1-x)\lambda \rightarrow \quad x \to 1-x \quad \lambda \to \lambda' \quad \alpha \to \beta$$
$$4\left(\alpha+\beta-\tfrac{1}{2}\right) \quad x(x-1)(\lambda+\lambda')^2 = \lambda\lambda'$$

The combination which is good

$$\boxed{M^2 = (\kappa_f^2 + \kappa_g^2)\frac{\lambda\lambda'}{\lambda+\lambda'}}$$

$$\boxed{M^2 = 1\,\text{Gev}^{-2} \cdot \frac{10^{-40}\,\text{Gev}^4}{} = 10}$$

$$\lambda = M^4 = \frac{M}{R^3} = 10^{-40}\,\text{Gev}^4.$$
$$= \frac{M}{8M^3G^3} = \frac{1}{8M^2G^3} = \frac{1}{8\times10^{16}\cdot10^{-114}} = \boxed{10^{-40}\,\text{Gev}^{+4} = 10^{-50}}$$

$$\lambda' = \frac{1}{8} \sim \frac{1}{10}$$

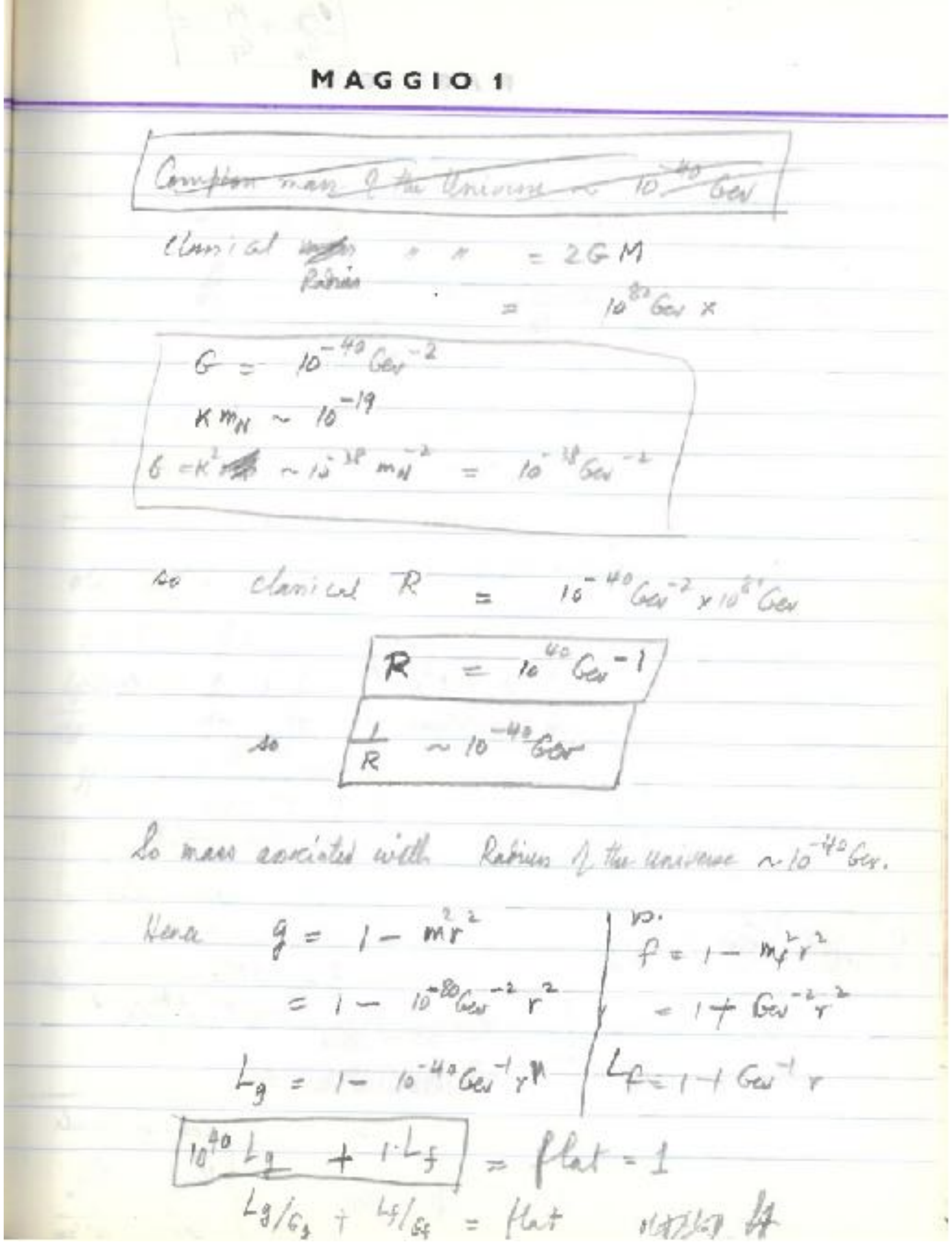
$$(\exp t)^{\alpha}$$

$$\left.\begin{aligned} \frac{1}{G_N}G &= \frac{g}{G_N} + \frac{f}{G_f} \\ F &= f - g \end{aligned}\right\}$$

$$(\det f^{\alpha}) = (\det f)^{\alpha} = (\exp \mathrm{Tr.}\log f)^{\alpha}$$
$$= \exp \mathrm{Tr.}\,\alpha\log(1+\kappa\phi)$$
$$= \exp \alpha\,\mathrm{Tr.}\left[-\kappa\phi + \frac{(\kappa\phi)^2}{2}\right]$$
$$= \left\{1 \quad \alpha\kappa\langle\phi\rangle + \frac{\alpha\kappa^2}{2}\langle\phi^2\rangle - \frac{\alpha^2\kappa^2}{2}\langle\phi\rangle^2\right\}$$

$$(\det f)^{\alpha} \simeq \det f^{\alpha}$$

Cosmological Bigravity



MAGGIO 1

~~Compton mass of the Universe ~ 10^{-40} GeV~~

Classical ~~length~~ Radius $= 2GM$

$= 10^{80}\,\mathrm{GeV}\, x$

$$\boxed{\begin{array}{l} G = 10^{-40}\,\mathrm{GeV}^{-2} \\ K m_N \sim 10^{-19} \\ G = K^2 \sim 10^{-38}\, m_N^{-2} = 10^{-38}\,\mathrm{GeV}^{-2} \end{array}}$$

so classical $R = 10^{-40}\,\mathrm{GeV}^{-2} \times 10^{80}\,\mathrm{GeV}$

$$\boxed{R = 10^{40}\,\mathrm{GeV}^{-1}}$$

so $\boxed{\frac{1}{R} \sim 10^{-40}\,\mathrm{GeV}}$

So mass associated with Radius of the universe $\sim 10^{-40}$ GeV.

Hence $g = 1 - m^2 r^2$

$= 1 - 10^{-80}\,\mathrm{GeV}^{-2} r^2$

$L_g = 1 - 10^{-40}\,\mathrm{GeV}^{-1} r$

vs. $f = 1 - m_f^2 r^2$

$= 1 + \mathrm{GeV}^{-2} r^2$

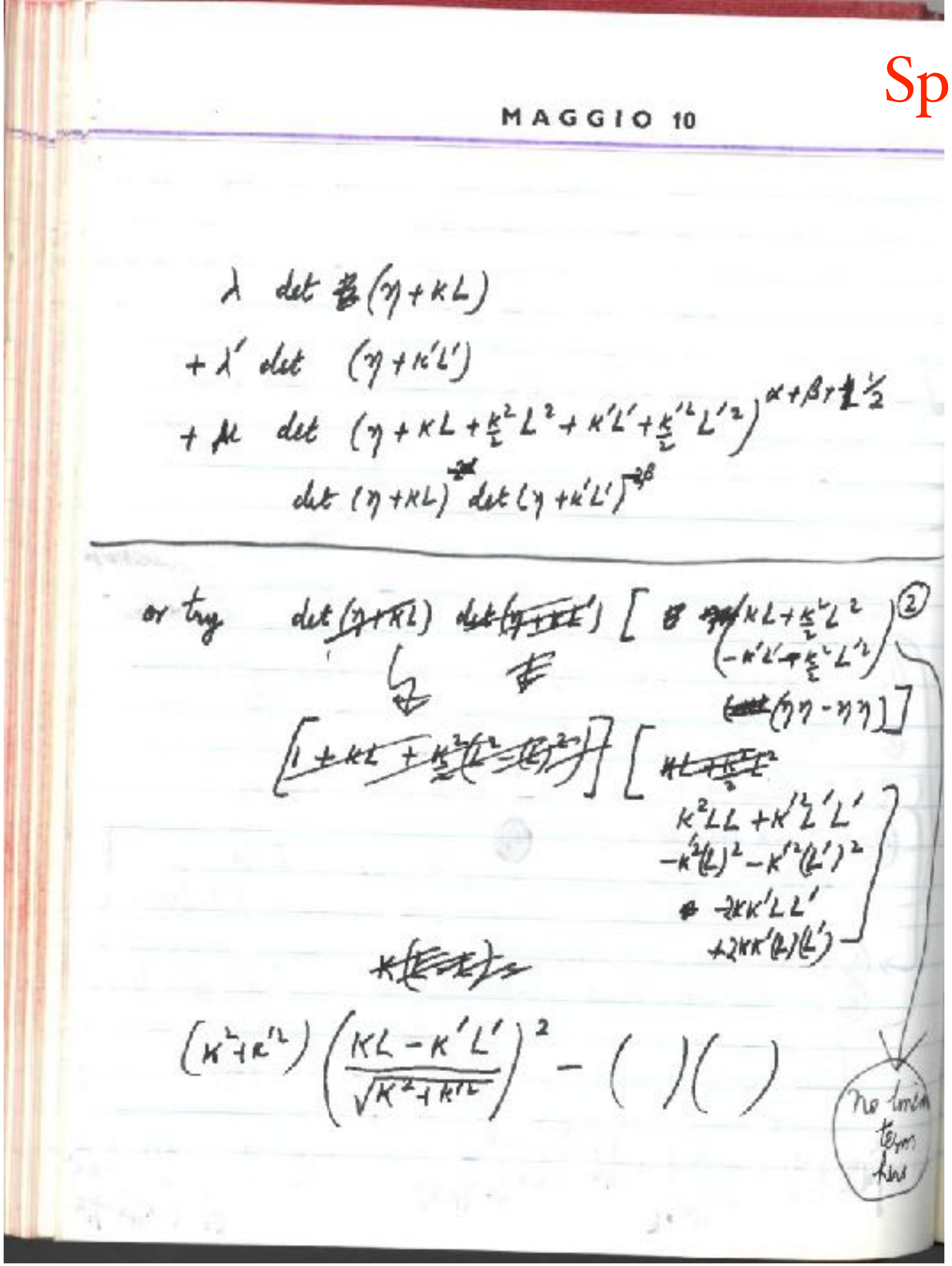
$L_f = 1 - 1\,\mathrm{GeV}^{-1} r$

$$\boxed{10^{40} L_g + 1 \cdot L_f} = \text{flat} = 1$$

$$L_g/G_g + L_f/G_f = \text{flat}$$

[illegible]

Spin-2 Mass terms



MAGGIO 10

$$\lambda \det (\eta + \kappa L)$$

$$+ \lambda' \det (\eta + \kappa' L')$$

$$+ \mu \det \left(\eta + \kappa L + \frac{\kappa^2}{2} L^2 + \kappa' L' + \frac{\kappa'^2}{2} L'^2\right)^{\alpha+\beta+\frac{1}{2}}$$

$$\det (\eta + \kappa L)^{-2\alpha} \det (\eta + \kappa' L')^{-2\beta}$$

---

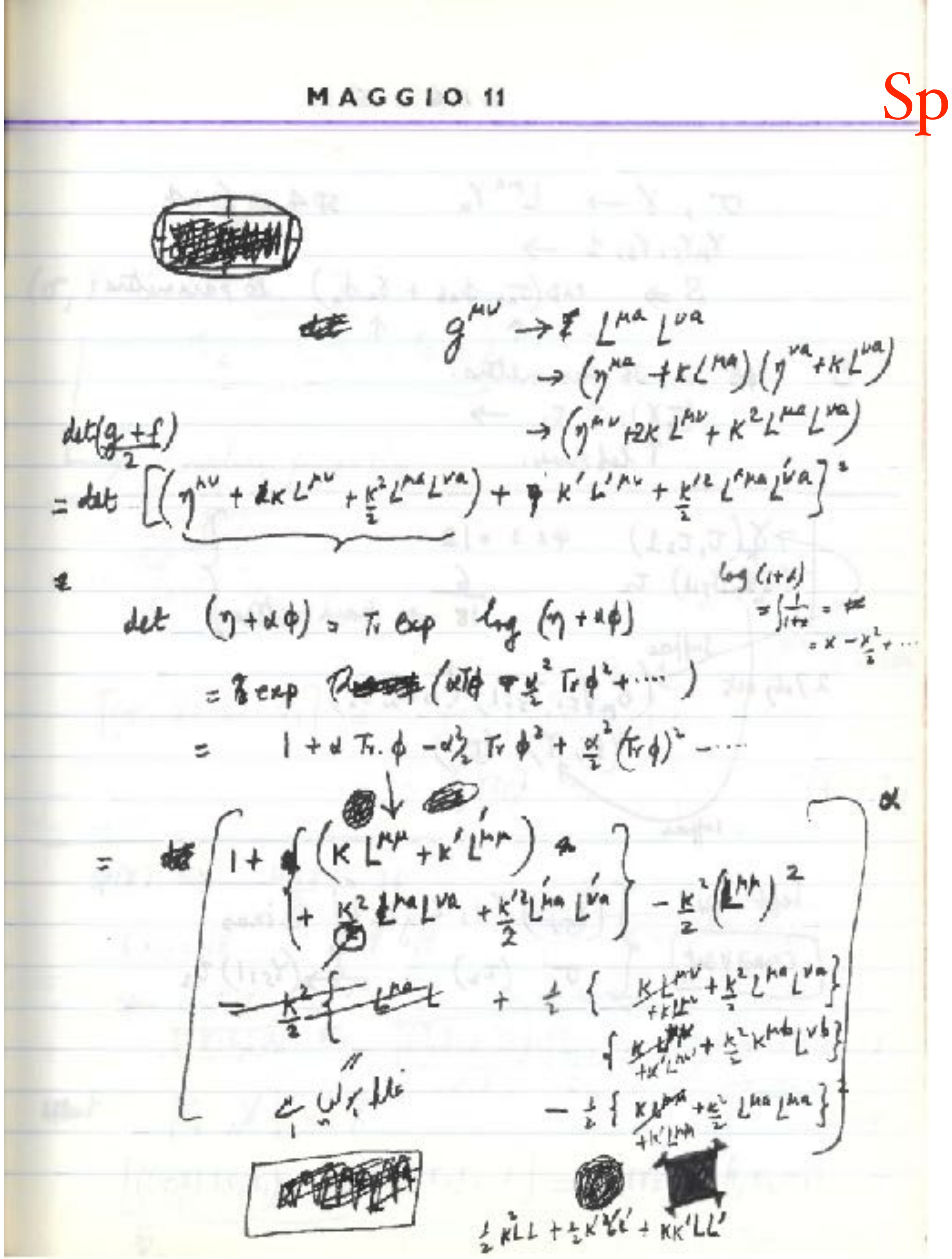
or try $\det(\eta+\kappa L)$ ~~$\det(\eta+\kappa' L')$~~ $\left[\left(\kappa L + \frac{\kappa^2}{2} L^2 - \kappa' L' + \frac{\kappa'^2}{2} L'^2\right)^{2} (\eta\eta - \eta\eta)\right]$ ②

~~$\left[1 + \kappa L + \frac{\kappa^2}{2}\left(L^2 - (L)^2\right)\right]$~~ $\left[\kappa^2 L L + \kappa'^2 L' L' - \kappa^2 (L)^2 - \kappa'^2 (L')^2 - 2\kappa\kappa' L L' + 2\kappa\kappa' (L)(L') \right]$

$$(\kappa^2+\kappa'^2)\left(\frac{\kappa L - \kappa' L'}{\sqrt{\kappa^2+\kappa'^2}}\right)^2 - (\ )(\ )$$

no linear terms here

Spin-2 Mass terms



MAGGIO 11

$$g^{\mu\nu} \to L^{\mu a} L^{\nu a}$$

$$\to (\eta^{\mu a} + \kappa L^{\mu a})(\eta^{\nu a} + \kappa L^{\nu a})$$

$$\to (\eta^{\mu\nu} + 2\kappa L^{\mu\nu} + \kappa^2 L^{\mu a} L^{\nu a})$$

$$\det\left(\frac{g+f}{2}\right)$$

$$= \det\left[\underbrace{\left(\eta^{\mu\nu} + \kappa L^{\mu\nu} + \frac{\kappa^2}{2} L^{\mu a} L^{\nu a}\right)}_{} + \kappa' L'^{\mu\nu} + \frac{\kappa'^2}{2} L'^{\mu a} L'^{\nu a}\right]^2$$

$$\det(\eta + \alpha\phi) = \mathrm{Tr}\exp\log(\eta + \alpha\phi)$$

$$\log(1+x) = \int \frac{1}{1+x} = x - \frac{x^2}{2} + \cdots$$

$$= \exp\left(\alpha\mathrm{Tr}\phi - \frac{\alpha^2}{2}\mathrm{Tr}\phi^2 + \cdots\right)$$

$$= 1 + \alpha\,\mathrm{Tr}.\phi - \frac{\alpha^2}{2}\mathrm{Tr}\,\phi^2 + \frac{\alpha^2}{2}(\mathrm{Tr}\,\phi)^2 - \cdots$$

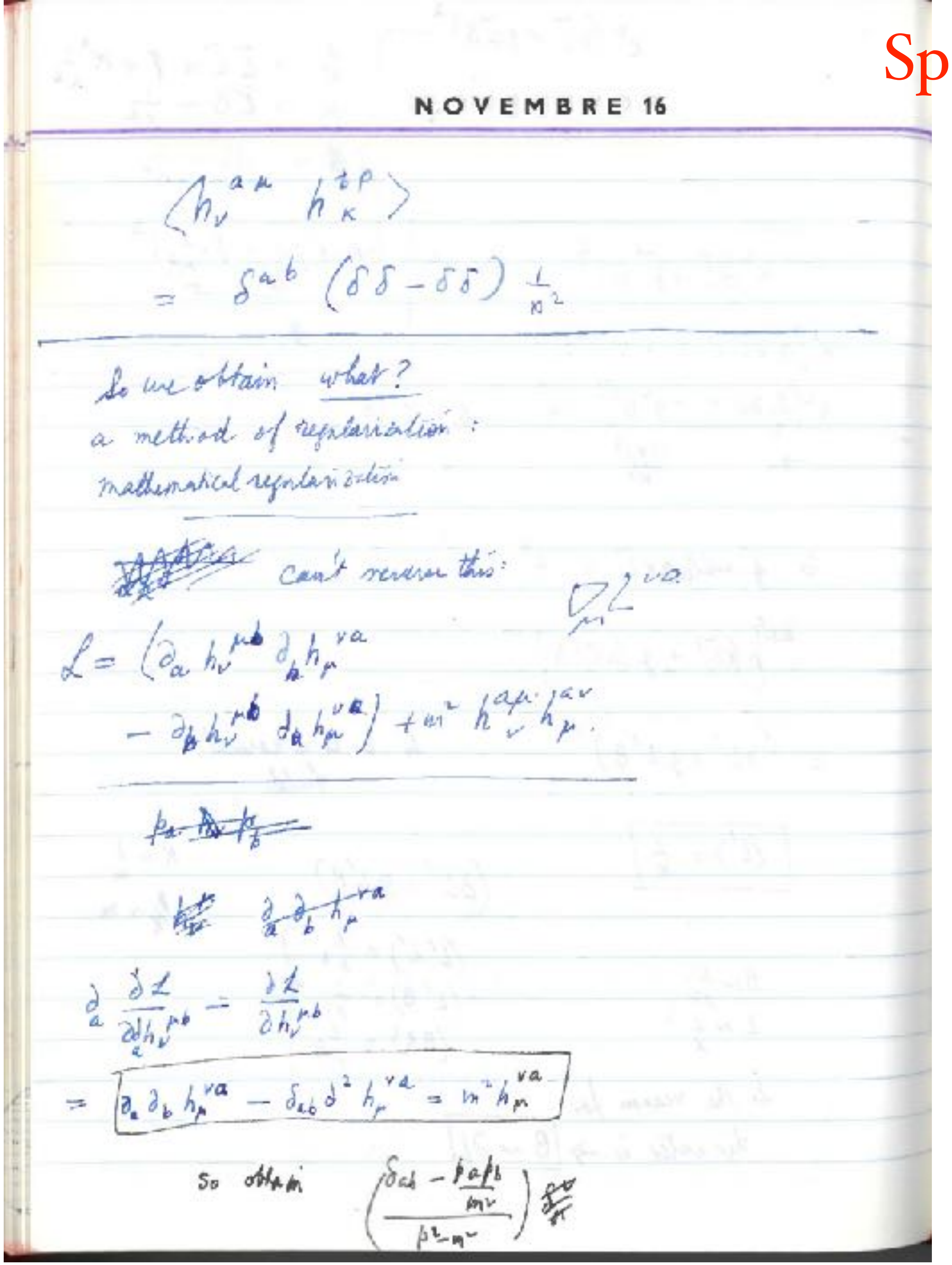
$$= \left[1 + \left\{\begin{array}{l}(\kappa L^{\mu\mu} + \kappa' L'^{\mu\mu}) \\ + \frac{\kappa^2}{2} L^{\mu a} L^{\nu a} + \frac{\kappa'^2}{2} L'^{\mu a} L'^{\nu a}\end{array}\right\} - \frac{\kappa^2}{2}(L^{\mu\mu})^2 \right.$$

$$~~- \frac{\kappa^2}{2}\{ L^{\mu a} L~~ + \frac{1}{2}\left\{\kappa L^{\mu\nu} + \kappa' L'^{\mu\nu} + \frac{\kappa^2}{2} L^{\mu a} L^{\nu a}\right\}\left\{\kappa' L'^{\mu\nu} + \frac{\kappa^2}{2} \kappa^{\mu b} L^{\nu b}\right\}$$

$$\left. - \frac{1}{2}\left\{\kappa L^{\mu\mu} + \kappa' L'^{\mu\mu} + \frac{\kappa^2}{2} L^{\mu a} L^{\mu a}\right\}^2 \right]^{\alpha}$$

$$\frac{1}{2}\kappa^2 L L + \frac{1}{2}\kappa'^2 L' L' + \kappa\kappa' L L'$$

Spin-2 propagator



NOVEMBRE 16

$$\langle h_\nu^{a\mu} \, h_\kappa^{b\rho} \rangle$$

$$= \delta^{ab} (\delta\delta - \delta\delta) \frac{1}{k^2}$$

So we obtain <u>what?</u>

a method of regularization :

mathematical regularization

~~[illegible]~~ can't reverse this:

$$\nabla_\mu L^{\nu a}$$

$$\mathcal{L} = \big(\partial_a h_\nu^{\mu b} \, \partial_b h_\mu^{\nu a} - \partial_b h_\nu^{\mu b} \, \partial_a h_\mu^{\nu a} \big) + m^2 h_\nu^{a\mu} h_\mu^{a\nu}.$$

~~$p_a \; p_b$~~

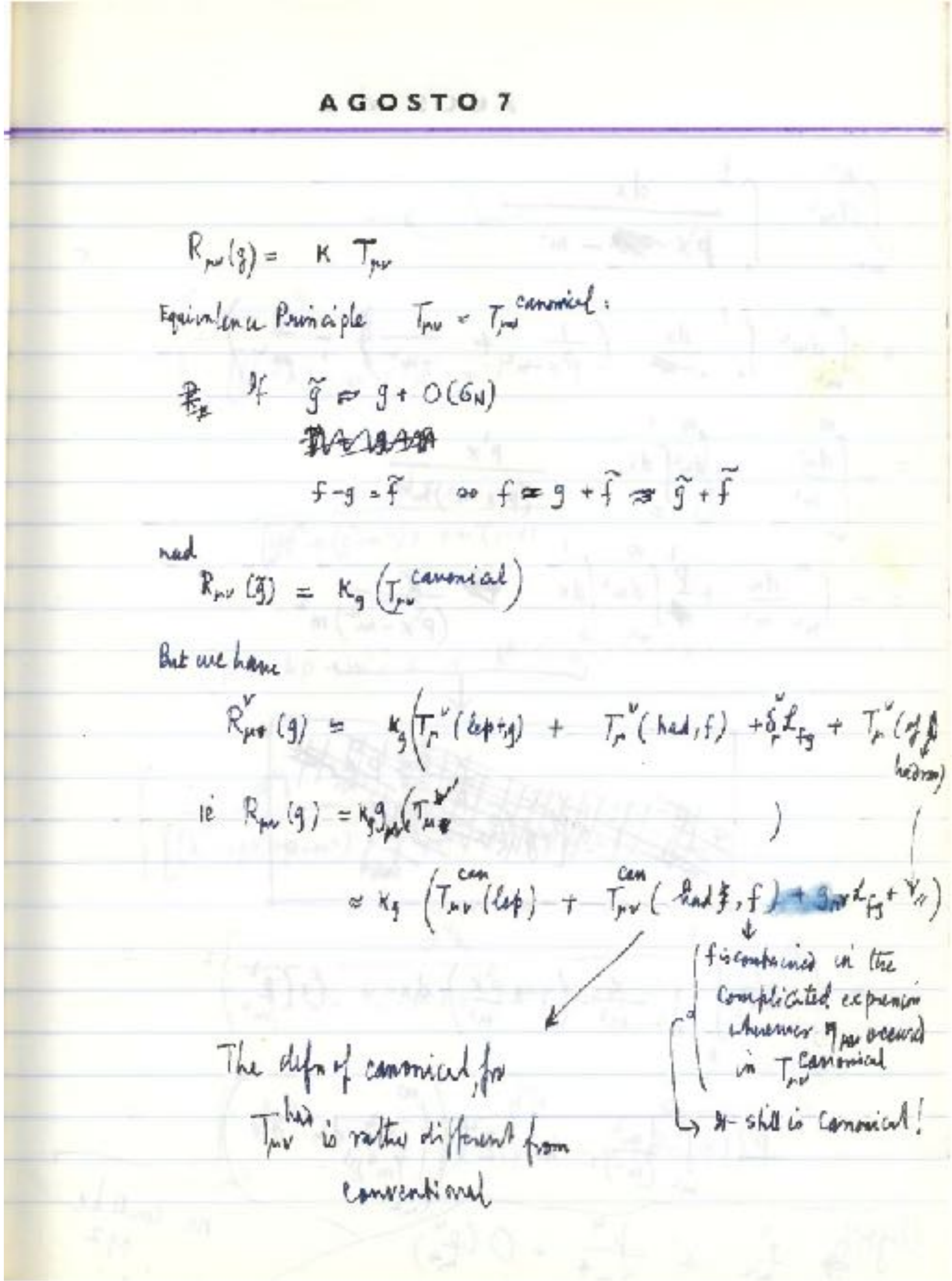
$$\partial_a \partial_b h_\mu^{\nu a}$$

$$\partial_a \frac{\partial \mathcal{L}}{\partial \partial_a h_\nu^{\mu b}} - \frac{\partial \mathcal{L}}{\partial h_\nu^{\mu b}}$$

$$= \boxed{\partial_a \partial_b h_\mu^{\nu a} - \delta_{ab} \partial^2 h_\mu^{\nu a} = m^2 h_\mu^{\nu a}}$$

So obtain $\left(\frac{\delta_{ab} - \frac{p_a p_b}{m^2}}{p^2 - m^2} \right)$

Spin-2 mixing



AGOSTO 7

$$R_{\mu\nu}(g) = \kappa\, T_{\mu\nu}$$

Equivalence Principle $T_{\mu\nu} = T_{\mu\nu}^{\text{canonical}}$:

If $\tilde{g} \approx g + O(G_N)$

$$f - g = \tilde{f} \quad \infty \quad f \approx g + \tilde{f} \approx \tilde{g} + \tilde{f}$$

need

$$R_{\mu\nu}(\tilde{g}) = \kappa_g \left(T_{\mu\nu}^{\text{canonical}}\right)$$

But we have

$$R^{\nu}_{\mu}(g) = \kappa_g\left(T_{\mu}^{\nu}(\text{lep}+g) + T_{\mu}^{\nu}(\text{had}, f) + \delta^{\nu}_{\mu}\mathcal{L}_{fg} + T_{\mu}^{\nu}(\text{of hadron})\right)$$

ie $R_{\mu\nu}(g) = \kappa_g g_{\mu\nu}\left(T_{\mu}^{\nu} \qquad\qquad \right)$

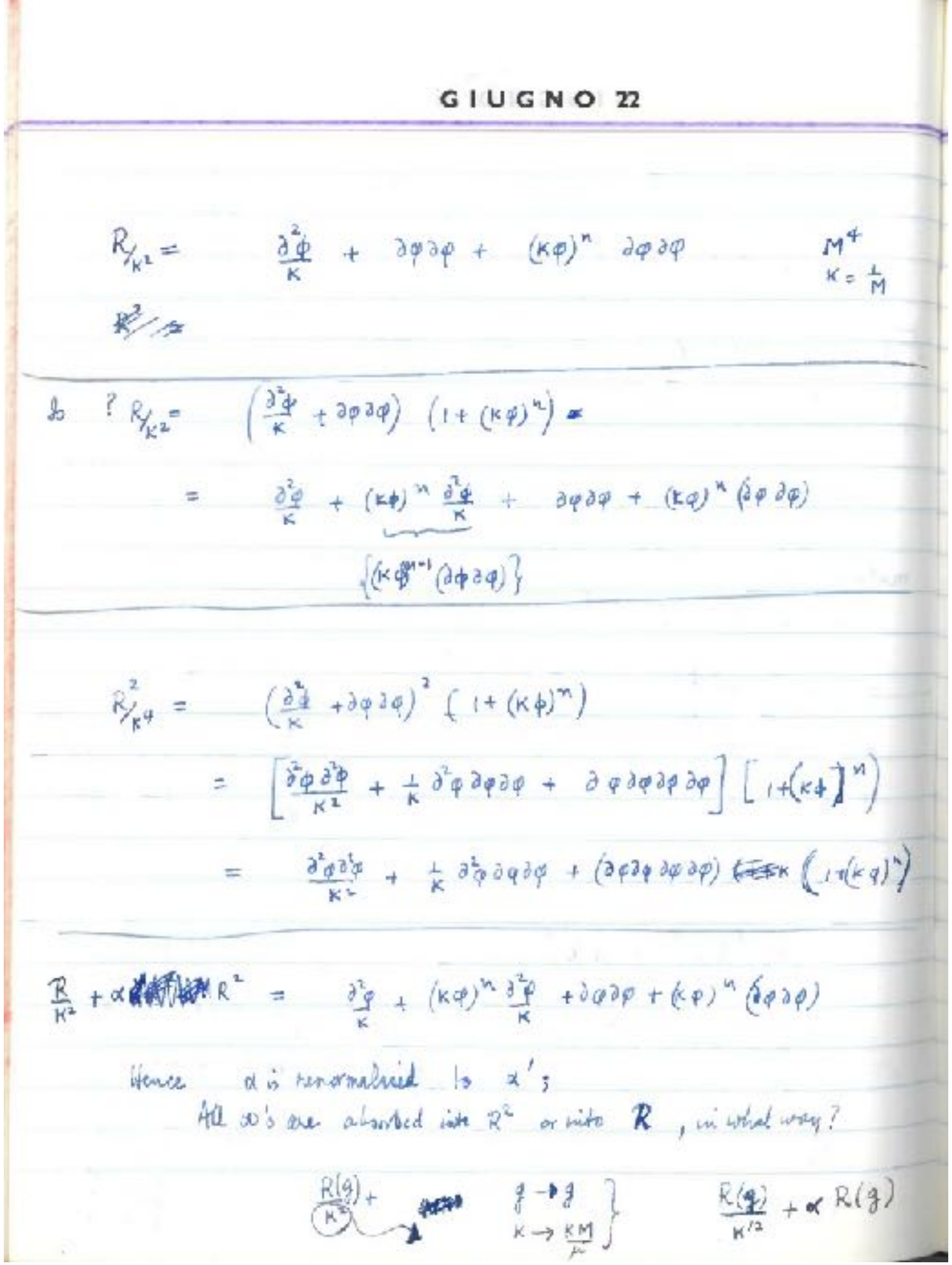
$$\approx \kappa_g\left(T_{\mu\nu}^{\text{can}}(\text{lep}) + T_{\mu\nu}^{\text{can}}(\text{had}\,\xi, f) + g_{\mu\nu}\mathcal{L}_{fg} + \gamma_{\prime\prime}\right)$$

f is contained in the complicated expression whenever $\eta_{\mu\nu}$ occurs in $T_{\mu\nu}^{\text{canonical}}$

→ It still is canonical!

The defn of canonical for $T_{\mu\nu}^{\text{had}}$ is rather different from conventional

R^2 gravity



GIUGNO 22

$$R/\kappa^2 = \frac{\partial^2\phi}{\kappa} + \partial\varphi\partial\varphi + (\kappa\varphi)^n\,\partial\varphi\partial\varphi \qquad M^4 \quad \kappa = \frac{1}{M}$$

So ? $R/\kappa^2 = \left(\frac{\partial^2\phi}{\kappa} + \partial\varphi\partial\varphi\right)\left(1+(\kappa\phi)^n\right)$

$$= \frac{\partial^2\varphi}{\kappa} + \underbrace{(\kappa\phi)^n\frac{\partial^2\phi}{\kappa}}_{\{(\kappa\phi)^{n-1}(\partial\phi\partial\varphi)\}} + \partial\varphi\partial\varphi + (\kappa\varphi)^n(\partial\varphi\partial\varphi)$$

$$R^2/\kappa^4 = \left(\frac{\partial^2\phi}{\kappa} + \partial\varphi\partial\varphi\right)^2\left(1+(\kappa\phi)^n\right)$$

$$= \left[\frac{\partial^2\phi\,\partial^2\phi}{\kappa^2} + \frac{1}{\kappa}\partial^2\varphi\,\partial\varphi\partial\varphi + \partial\varphi\partial\varphi\partial\varphi\partial\varphi\right]\left[1+(\kappa\phi)^n\right)$$

$$= \frac{\partial^2\varphi\,\partial^2\varphi}{\kappa^2} + \frac{1}{\kappa}\partial^2\varphi\,\partial\varphi\partial\varphi + (\partial\varphi\partial\varphi\,\partial\varphi\partial\varphi)\left(1+(\kappa\varphi)^n\right)$$

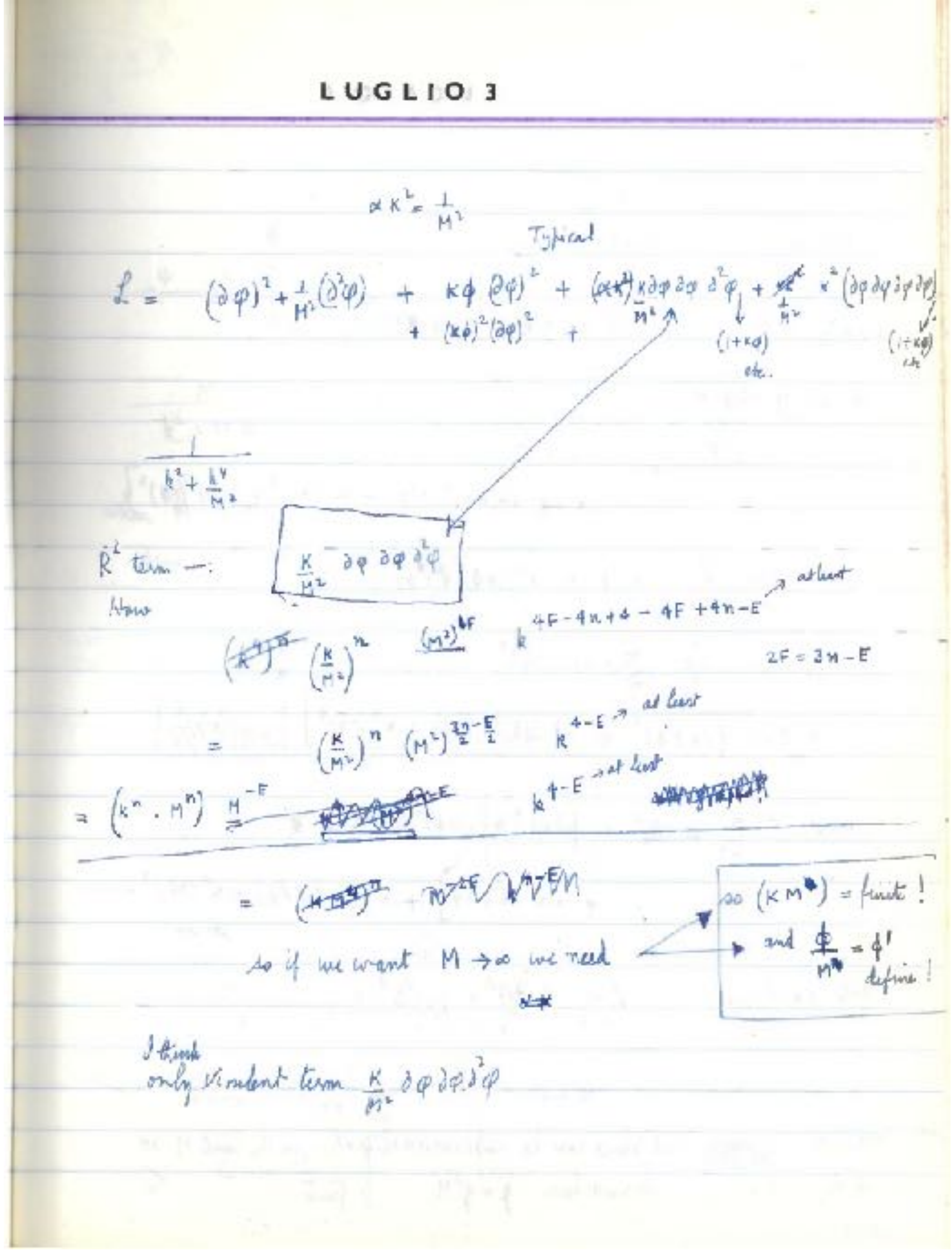
$$\frac{R}{\kappa^2} + \alpha R^2 = \frac{\partial^2\varphi}{\kappa} + (\kappa\varphi)^n\frac{\partial^2\varphi}{\kappa} + \partial\varphi\partial\varphi + (\kappa\varphi)^n(\partial\varphi\partial\varphi)$$

Hence α is renormalized to α';

All ∞'s are absorbed into R^2 or into R, in what way?

$$\frac{R(g)}{\kappa^2} + \quad \left.\begin{matrix} g \to g \\ \kappa \to \frac{\kappa M}{\mu}\end{matrix}\right\} \quad \frac{R(g)}{\kappa'^2} + \alpha R(g)$$

R^2 gravity



LUGLIO 3

$\alpha K^2 = \frac{1}{M^2}$

Typical

$$\mathcal{L} = (\partial\varphi)^2 + \frac{1}{M^2}(\partial^2\varphi) + K\phi(\partial\varphi)^2 + \frac{K}{M^2}\partial\varphi\partial\varphi\,\partial^2\varphi + \frac{1}{M^2}K^2(\partial\varphi\partial\varphi\partial\varphi\partial\varphi)$$

$$+ (K\phi)^2(\partial\varphi)^2 +$$

$(1+K\phi)$ etc.

$(1+K\phi)$ etc.

$$\frac{1}{k^2+\frac{k^4}{M^2}}$$

R^2 term —: $\boxed{\frac{K}{M^2}\partial\varphi\,\partial\varphi\,\partial^2\varphi}$

Now

$$\left(\frac{K}{M^2}\right)^n \quad \underline{(M^2)^{4F}} \quad k^{4F-4n+4-4F+4n-E} \rightarrow \text{at least}$$

$2F = 3n - E$

$$= \left(\frac{K}{M^2}\right)^n (M^2)^{\frac{3n}{2}-\frac{E}{2}} \quad k^{4-E} \rightarrow \text{at least}$$

$$= (K^n \cdot M^n)\ M^{-E} \quad k^{4-E} \rightarrow \text{at least}$$

= [illegible]

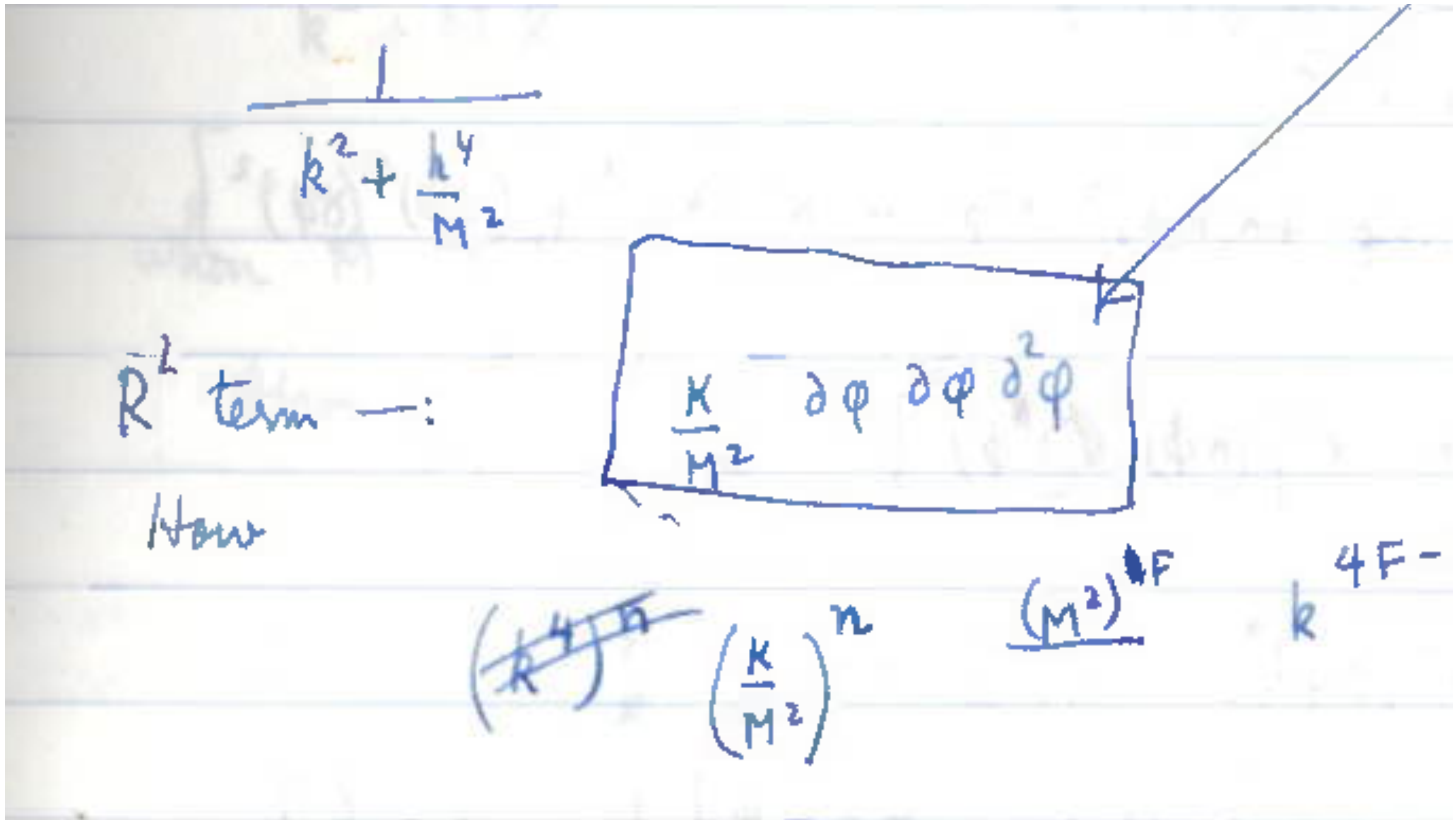
so if we want $M \to \infty$ we need

so (KM) = finite!

and $\frac{\phi}{M} = \phi'$ define!

I think only virulent term $\frac{K}{M^2}\partial\varphi\partial\varphi\partial^2\varphi$

Cubic Galileon!!!

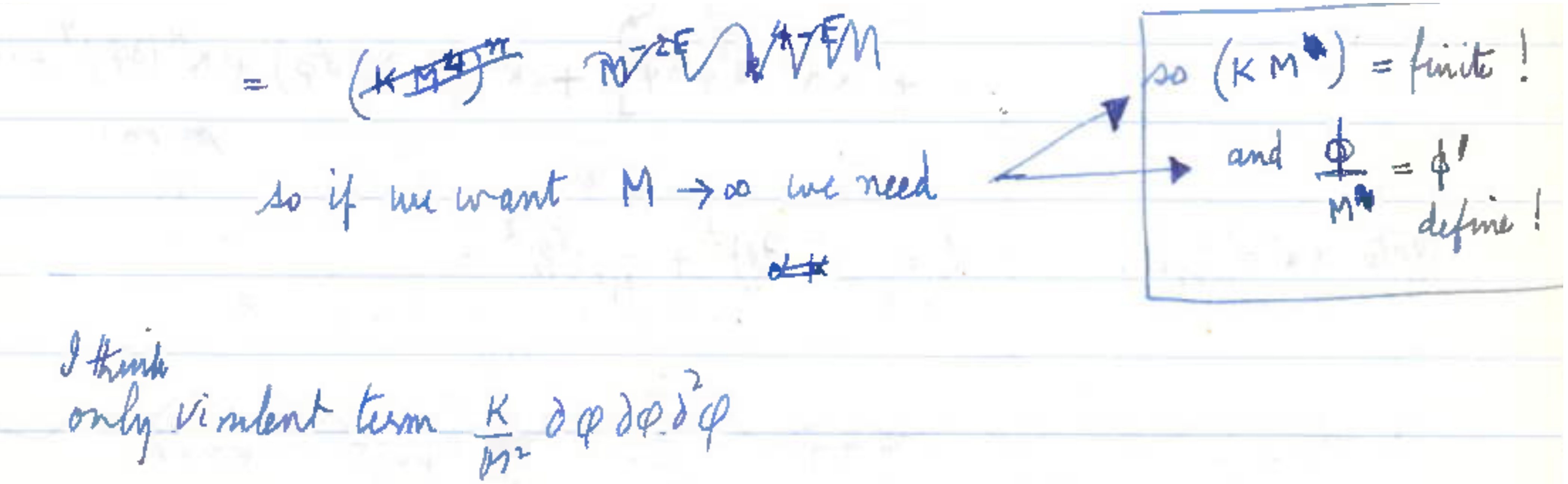


$$\frac{1}{k^2+\frac{k^4}{M^2}}$$

$\bar{R}^2$ term —: $\boxed{\frac{K}{M^2}\,\partial\varphi\,\partial\varphi\,\partial^2\varphi}$

How

$\cancel{(k^4)^n}$ $\left(\frac{K}{M^2}\right)^n$ $\underline{(M^2)^{F}}$ k^{4F-}



$= \cancel{(K M^4)^n}\ \cancel{M^{-2F}\ M^{4-F}}$

so if we want $M \to \infty$ we need

so $(K M^4)$ = finite !

and $\frac{\Phi}{M^4} = \phi'$ define !

I think only virulent term $\frac{K}{M^2}\,\partial\varphi\,\partial\varphi\,\partial^2\varphi$

R^2 gravity

de Rham, AJT et al 2011

$$\mathcal{L}_{\rm 3d,NMG} = \frac{M_3}{2}\int \mathrm{d}^3x\sqrt{-g}\left(-R + \frac{1}{m^2}\left(R_{\mu\nu}^2 - \frac{3}{8}R^2\right)\right)$$

$$\mathcal{L}_{\rm 3d,NMG} = \frac{M_3}{2}\int \mathrm{d}^3x\ \sqrt{-g}\left[-R - f^{\mu\nu}G_{\mu\nu} - \frac{1}{4}m^2(f^{\mu\nu}f_{\mu\nu} - f^2)\right]$$

- Restore the 2nd copy of (linear) diff invariance with Stü. fields

$$h_{\mu\nu} = \frac{\bar{h}_{\mu\nu}}{\sqrt{M_3}}, \qquad f_{\mu\nu} = \frac{\bar{f}_{\mu\nu}}{\sqrt{M_3}} + \nabla_\mu V_\nu + \nabla_\nu V_\mu$$

Decoupling limit

- Restore the 2nd copy of (linear) diff invariance with Stü. fields

$$\mathcal{L}^{(\mathrm{dec})}_{\mathrm{3d,NMG}} = -\frac{1}{4}F^2_{\mu\nu} - 2(\partial\pi)^2 - \frac{1}{2}(\partial\pi)^2\Box\pi$$

Salam 1977 was right!

- Splitting the Stü. field into scalar and vector parts,

$$h_{\mu\nu} = \frac{\bar{h}_{\mu\nu}}{\sqrt{M_3}}, \qquad f_{\mu\nu} = \frac{\bar{f}_{\mu\nu}}{\sqrt{M_3}} + \nabla_\mu V_\nu + \nabla_\nu V_\mu$$

$$V_\mu = \frac{A_\mu}{\sqrt{M_3 m}} + \frac{\nabla_\mu \pi}{\sqrt{M_3 m^2}}$$

Decoupling limit

$$\mathcal{L}^{(\mathrm{dec})}_{\mathrm{3d,NMG}} = -\frac{1}{4}F^2_{\mu\nu} - 2(\partial\pi)^2 - \frac{1}{2}(\partial\pi)^2\Box\pi$$

de Rham, AJT et al 2011

Salam 1977 was right!

$$\mathcal{L}^{(\mathrm{dec})}_{D} = -\frac{1}{4}F^2_{\mu\nu} - \frac{(D-1)}{(D-2)}(\partial\pi)^2 - \frac{(D-4)}{(D-2)}(\partial\pi)^2\Box\pi$$

Except in D=4

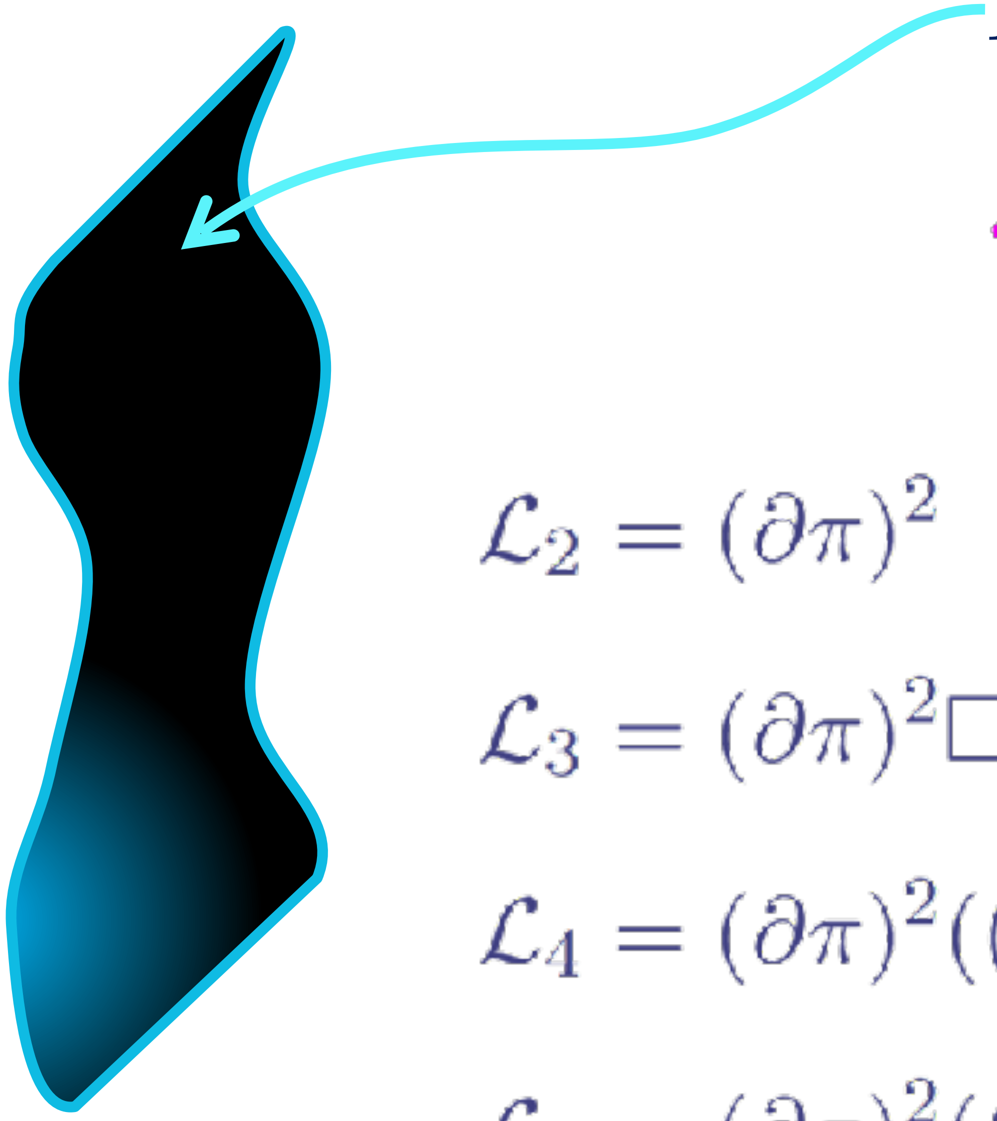
Hinterbichler, Saravani,2015

$\pi \to \pi + v_\mu x^\mu + c$

Nicolis, Rattazzi, Trincherini 2008

Galileons

de Rham, AJT, 1003.5917



Induced Metric

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu \pi \partial_\nu \pi$$

(disformal coupling)

$\mathcal{L}_2 = (\partial\pi)^2$ From λ

$\mathcal{L}_3 = (\partial\pi)^2 \Box\pi$ From $R^{(5)}$

$\mathcal{L}_4 = (\partial\pi)^2((\Box\pi)^2 + \cdots)$ From $R^{(4)}$

$\mathcal{L}_5 = (\partial\pi)^2((\Box\pi)^3 + \cdots)$ From $\mathcal{L}_{\rm GB}$

Geometric Unification

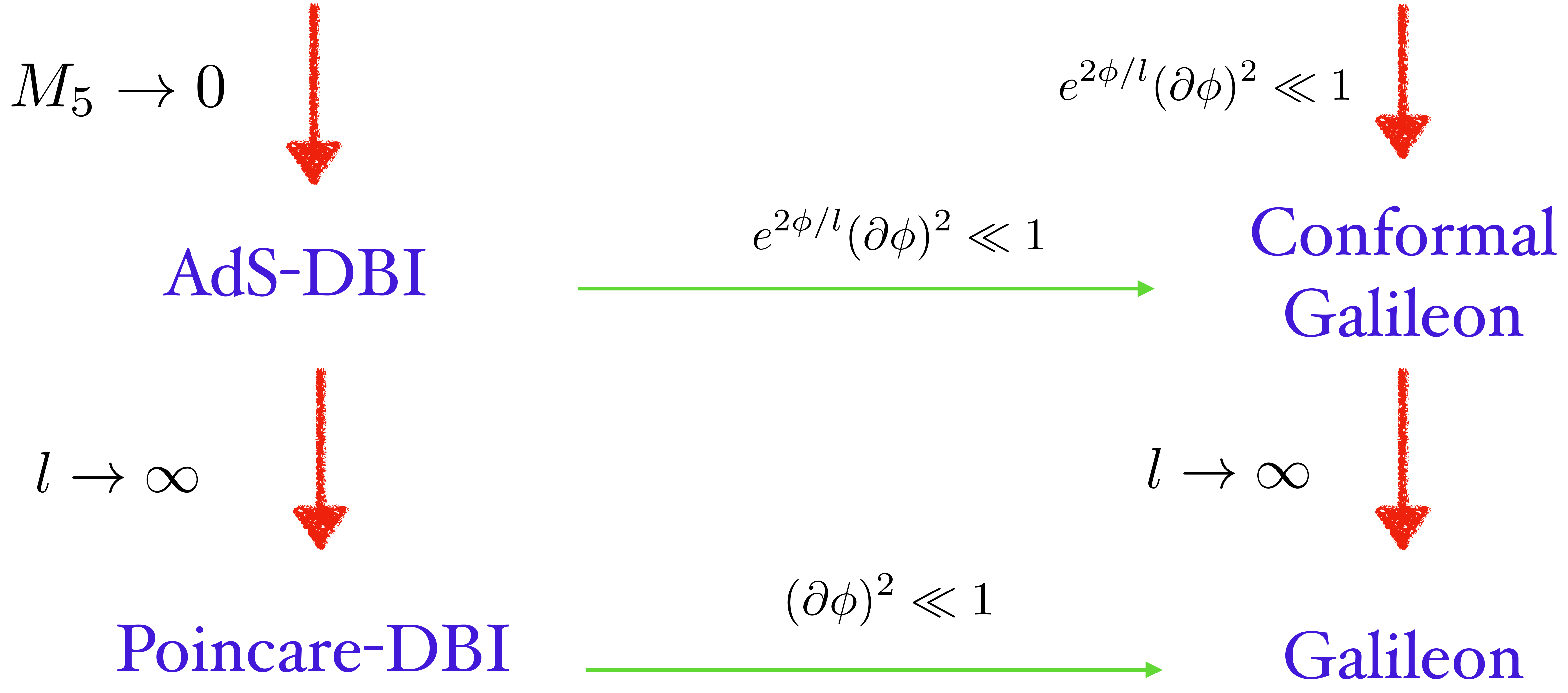
AJT, de Rham (2010)

AdS-DBI-Galileon

Gibbons-Hawking terms for bulk Ricci+Gauss-Bonnet

$$\mathcal{L} = \sqrt{-g}\left(-\lambda + M_4^2 R - M_5^3 K - \beta \frac{M_5^3}{m^2}\mathcal{K}_{GB}\right)$$

$$g_{\mu\nu} = e^{-2\phi/l}\gamma_{\mu\nu} + \partial_\mu\phi\partial_\nu\phi \qquad K_{\mu\nu} = -\frac{1}{\sqrt{1+e^{2\phi/l}(\partial\phi)^2}}\left(\partial_\mu\partial_\nu\phi + \frac{1}{l}\partial_\mu\phi\partial_\nu\phi + \frac{1}{l}g_{\mu\nu}\right)$$



$M_5 \to 0$

$e^{2\phi/l}(\partial\phi)^2 \ll 1$

AdS-DBI

$e^{2\phi/l}(\partial\phi)^2 \ll 1$

Conformal Galileon

$l \to \infty$

$l \to \infty$

Poincare-DBI

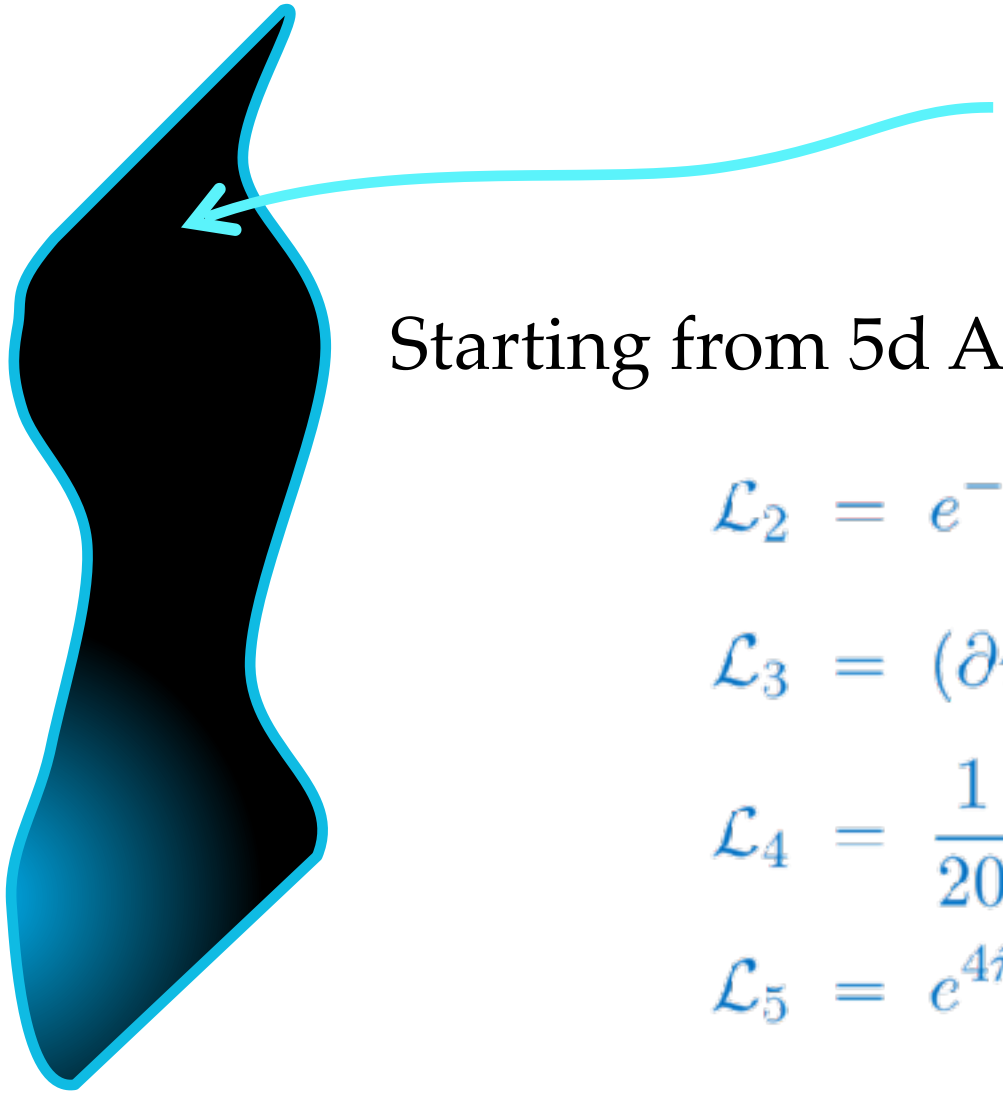
$(\partial\phi)^2 \ll 1$

Galileon

AJT, de Rham (2010)

Conformal Galileons

de Rham, AJT, 1003.5917



$$g_{\mu\nu} = e^{-2\pi/\ell}\eta_{\mu\nu} + \partial_\mu\pi\partial_\nu\pi$$

Starting from 5d AdS, we get the conformal Galileon

$$\begin{aligned}
\mathcal{L}_2 &= e^{-2\hat{\pi}}(\partial\hat{\pi})^2 \\
\mathcal{L}_3 &= (\partial\hat{\pi})^2\Box\hat{\pi} - \frac{1}{2}(\partial\hat{\pi})^4 \\
\mathcal{L}_4 &= \frac{1}{20}e^{2\hat{\pi}}(\partial\hat{\pi})^2\left(10([\hat{\Pi}]^2 - [\hat{\Pi}^2]) + 4((\partial\hat{\pi})^2\Box\hat{\pi} - [\partial \right. \\
\mathcal{L}_5 &= e^{4\hat{\pi}}(\partial\hat{\pi})^2\left([\hat{\Pi}]^3 + \cdots\right)
\end{aligned}$$

Decoupling limits of Massive Gravity Theories

- Decoupling limit of DGP: Galileon (cubic)
- Decoupling limit of Massive Gravity: Galileon (quintic)
- Decoupling limit of BiGravity: Galileon (quintic)
- Decoupling limit of New Massive Gravity: Galileon (cubic)
- Decoupling limit of Zwei-Dreibein Gravity: Galileon (quartic)

Emergence of Galileon Symmetry

Spin-2 Helmholtz
Or Helicity
Decomposition

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} + \partial_\mu \partial_\nu \phi \, + \ldots$$

Galileon symmetry

$$\delta\phi(x) = c + v_\mu x^\mu$$

$$\delta \partial_\mu \partial_\nu \phi = 0$$

Galileon Operators

$$\mathcal{L}_2 = \pi\epsilon^{abcd}\epsilon^{ABCD}\eta_{aA}\eta_{bB}\eta_{cC}\partial_d\partial_D\pi$$

$$\mathcal{L}_3 = \pi\epsilon^{abcd}\epsilon^{ABCD}\eta_{aA}\eta_{bB}\partial_c\partial_C\pi\partial_d\partial_D\pi$$

$$\mathcal{L}_4 = \pi\epsilon^{abcd}\epsilon^{ABCD}\eta_{aA}\partial_b\partial_B\pi\partial_c\partial_C\pi\partial_d\partial_D\pi$$

$$\mathcal{L}_5 = \pi\epsilon^{abcd}\epsilon^{ABCD}\partial_a\partial_A\pi\partial_b\partial_B\pi\partial_c\partial_C\pi\partial_d\partial_D\pi$$

Characteristic polynomials

$$\pi\,\det[\alpha\partial_a\partial_b\pi + \beta\eta_{ab}]$$

Galileon Helicity-2 Interactions

de Rham, Gabadadze 2009

$$\mathcal{L}_{2'} = \epsilon^{abcd}\epsilon^{ABCD} h_{aA}\eta_{bB}\eta_{cC}\partial_d\partial_D\pi$$

$$\mathcal{L}_{3'} = \epsilon^{abcd}\epsilon^{ABCD} h_{aA}\eta_{bB}\partial_c\partial_C\pi\partial_d\partial_D\pi$$

$$\mathcal{L}_{4'} = \epsilon^{abcd}\epsilon^{ABCD} h_{aA}\partial_b\partial_B\pi\partial_c\partial_C\pi\partial_d\partial_D\pi$$

Characteristic polynomials

$$\det[h_{ab} + \alpha\partial_a\partial_b\pi + \beta\eta_{ab}]$$

Helicity zero mode enters reference metric **squared**

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu \phi^A \partial_\nu \phi^B \qquad \phi^a = x^a + \frac{1}{mM_P}A^a + \frac{1}{\Lambda^3}\partial^a \pi$$

$$F_{\mu\nu} \approx \eta_{\mu\nu} + \frac{2}{\Lambda^3}\partial_\mu\partial_\nu\pi + \frac{1}{\Lambda^6}\partial_\mu\partial_\alpha\pi\partial^\alpha\partial_\nu\pi$$

To extract dominant helicity zero interactions we need
to take a square root

$$\left[\sqrt{g^{-1}F}\right]_{\mu\nu} \approx \eta_{\mu\nu} + \frac{1}{\Lambda^3}\partial_\mu\partial_\nu\pi$$

de Rham, Gabadadze, AJT 2010

Hard Λ_3 Massive Gravity

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2\, R[g] - m^2 \sum_{n=0}^{4} \beta_n\, \mathcal{U}_n \right) + \mathcal{L}_M$$

$$K = 1 - \sqrt{g^{-1} f}$$

$$\mathrm{Det}[1 + \lambda K] = \sum_{n=0}^{d} \lambda^n \mathcal{U}_n(K)$$

Characteristic Polynomials

Double epsilon structure!!!!!

Unique Lorentz invariant low energy EFT where the strong coupling scale is $\Lambda_3 = (m^2 M_P)^{1/3}$

5 propagating degrees of freedom!!!!

Currently being translated into Japanese by Bungeishunju Ltd.

Decoupling Limit of Bigravity

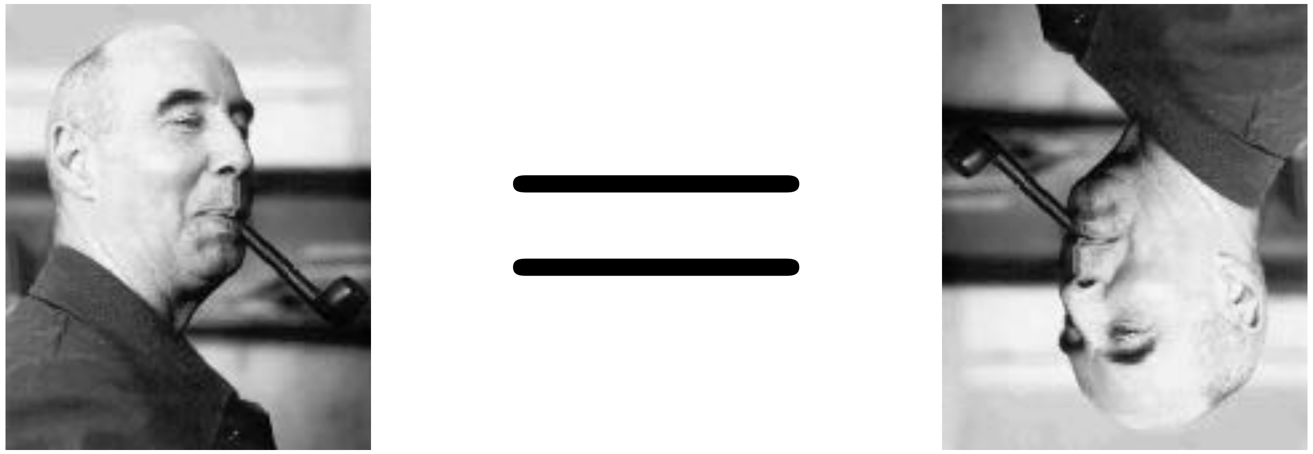
Fasiello, AJT 2013

In **Massive Gravity** - Mass term breaks a single copy of local Diffeomorphism Group down to a global Lorentz group

$$Diff(M) \to \text{Global Lorentz}$$

In **Bigravity** - Mass term breaks two copies of local Diffeomorphism Group down to a single copy of Diff group

$$Diff(M) \times Diff(M) \to Diff(M)_{\text{diagonal}}$$



Fasiello, AJT 2013

There are two ways to introduce Stuckelberg fields!

Dynamical metric I

$$g_{\mu\nu}(x)$$

Dynamical metric II

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu\phi^A\partial_\nu\phi^B$$

$$\tilde{x}^A = \phi^A(x) = x^A + \partial^A\pi(x)$$

OR

Dynamical metric I

$$\tilde{G}_{AB}(\tilde{x}) = g_{\mu\nu}(Z)\partial_A Z^\mu \partial_B Z^\nu$$

Dynamical metric II

$$f_{AB}(\tilde{x})$$

$$x^\mu = Z^\mu(\tilde{x}) = \tilde{x}^\mu + \partial^\mu\tilde{\pi}(\tilde{x})$$

Galileon Duality!!!!!

Explicitly Decoupling limit for Bigravity

de Rham, Gabadadze 2009
Fasiello, AJT 2013

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_P} h_{\mu\nu} \qquad f_{\mu\nu} = \eta_{\mu\nu} + \frac{1}{M_P} v_{\mu\nu}$$

massless helicity 2

massless helicity 0

$$S_{\text{helicity}-2/0} = \int \mathrm{d}^4x \left[-\frac{1}{4} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} - \frac{1}{4} v^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} v_{\alpha\beta} \right.$$
$$\left. + \ \frac{\Lambda_3^3}{2} h^{\mu\nu}(x) X^{\mu\nu} + \frac{M_{\rm p}\Lambda_3^3}{2M_{\rm f}} v_{\mu A}[x^a + \Lambda_3^{-3}\partial^a \pi](\eta^A_\nu + \Pi^A_\nu) Y^{\mu\nu} \right]$$

$$X^{\mu\nu} = -\frac{1}{2} \sum_{n=0}^{4} \frac{\hat{\beta}_n}{(3-n)!n!} \varepsilon^{\mu\cdots}\varepsilon^{\nu\cdots}(\eta + \Pi)^n \eta^{3-n}$$

$$\Pi_{ab} = \frac{\partial_a \partial_b \pi}{\Lambda_3^3}$$

$$Y^{\mu\nu} = -\frac{1}{2} \sum_{n=0}^{4} \frac{\hat{\beta}_n}{(4-n)!(n-1)!} \varepsilon^{\mu\cdots}\varepsilon^{\nu\cdots}(\eta + \Pi)^{(n-1)} \eta^{4-n}$$

Vainshtein effect

When curvature is large $R \gg m^2$ recover GR

When curvature is small $R \ll m^2$ fifth force propagates

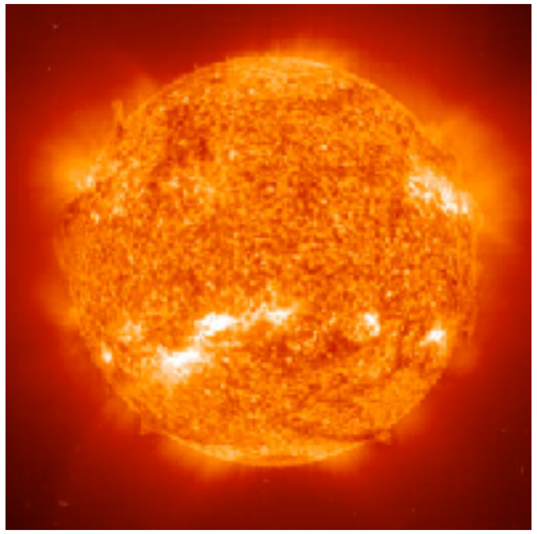
Determines characteristic Vainshtein radius $\frac{M}{M_P^2 r_V^3} \sim m^2$

Screened region $r \ll r_V$

$r_V = (r_s m^{-1})^{1/3}$

Weak coupling region $r \gg r_V$

For Sun



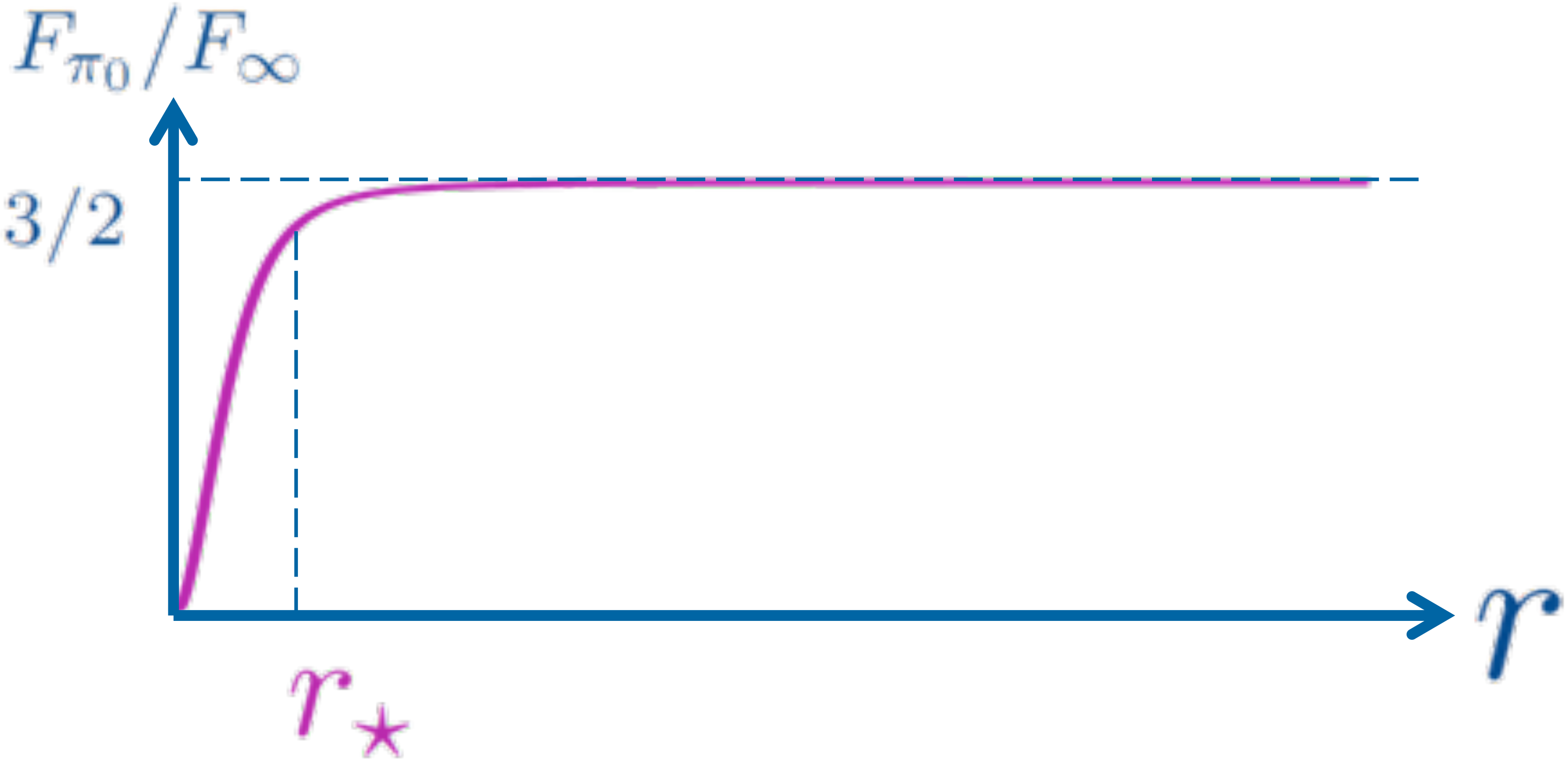
$m^{-1} \sim 4000 Mpc$

$r_s \sim 3km$

$r_V \sim 250pc$

Vainshtein in static and spherical symmetry case

$$\frac{F_{r \ll r_\star}}{F_{r \gg r_\star}} = \left(\frac{r}{r_\star}\right)^{3/2}$$



$$r_\star = \frac{1}{\Lambda}\left(\frac{M_\odot}{M_{\rm Pl}}\right)^{1/3} \quad \Lambda = (m^2 M_{\rm Pl})^{1/3}$$

slides courtesy of C.de Rham

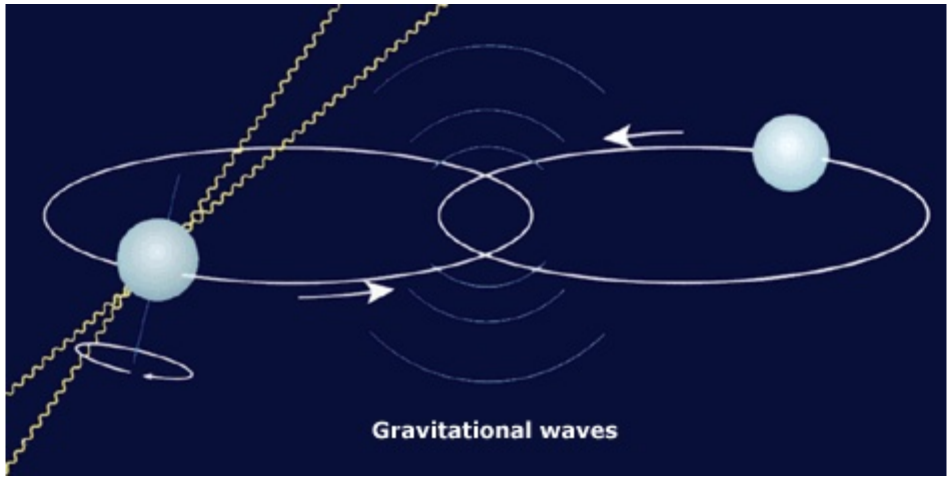
Vainshtein in static and spherical symmetric case

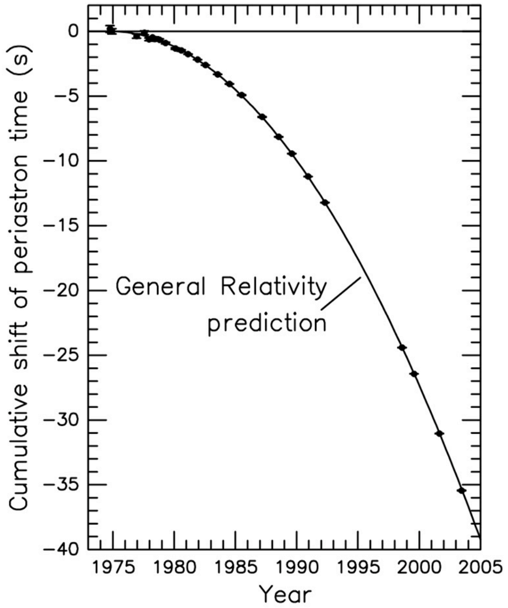
$$\frac{F_{r\ll r_\star}}{F_{r\gg r_\star}} = \left(\frac{r}{r_\star}\right)^{3/2} \sim 10^{-12}$$ For Sun-Earth System

$$\sim 10^{-13}$$ For Earth-Moon System

$$\sim 10^{-15}$$ For Hulse-Taylor Pulsar

expected





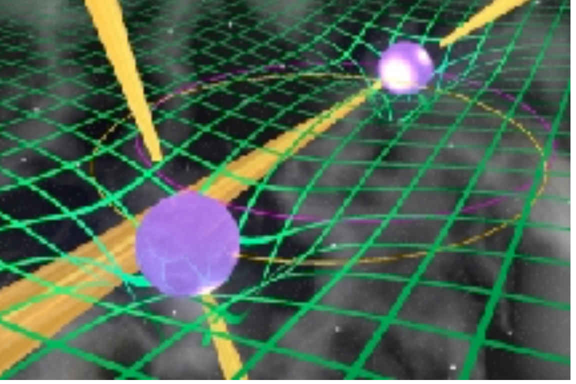
slides courtesy of C.de Rham

Vainshtein radius without spherical symmetry

An important question to address for theory and simulations is how well-do the screening mechanisms work *away* from the STATIC- SPHERICALLY symmetric situations in which they are usually described

e.g.:

- In time-dependent systems, screening may be different, computed exactly for binary pulsar systems
- When the spherical symmetry is broken



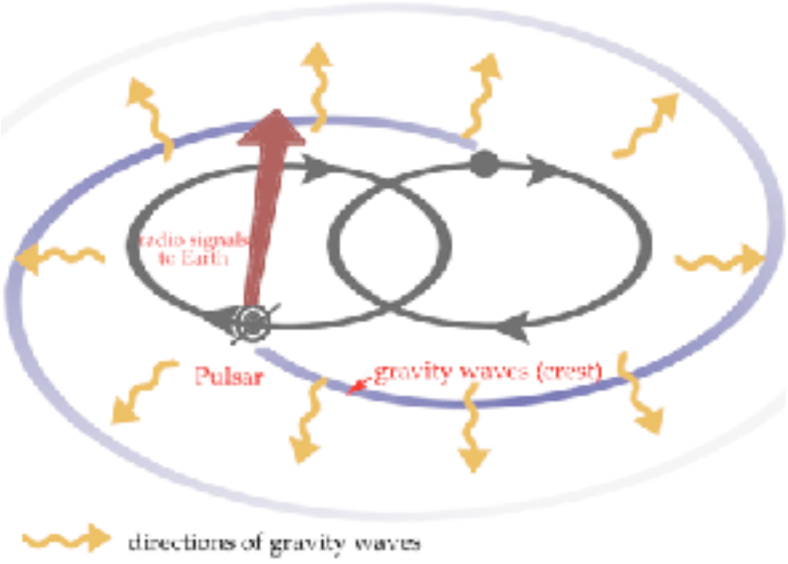
Binary Pulsars

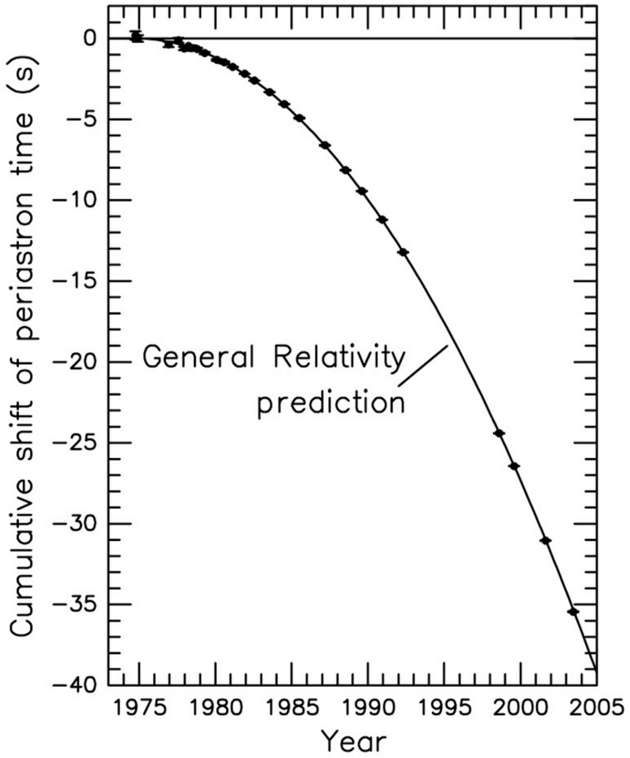
de Rham, AJT, Wesley 2012
de Rham, Matas, AJT 2013
Dar, de Rham, Deskins, Giblin, AJT 2018
Gerhardinger, Giblin, AJT, Trodden 2022
Gerhardinger, Giblin, AJT, Trodden 2024
de Rham, Giblin, AJT 2024

Extra polarizations of graviton = extra modes of gravitational wave

Binary pulsars lose energy faster than in GR so the orbit slows down more rapidly

Well approximated by decoupling limit!! (Unlike BH mergers etc) Helicity 2 graviton is always weakly coupled.





One-body approximation

$$S = \int \mathrm{d}^4x \left(-\frac{3}{4}(\partial\pi)^2 - \frac{1}{4\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{2M_{\mathrm{Pl}}}\pi T \right)$$

$$\pi(t,\vec{x}) = \pi_0(r) + \sqrt{2/3}\phi(t,\vec{x})$$

Radiation emitted by that scalar

Background due to centre of mass

$$\frac{E(r)}{r} + \frac{2}{3\Lambda^3}\left(\frac{E(r)}{r}\right)^2 = \frac{1}{12\pi r^3}\frac{M}{M_{\mathrm{Pl}}}$$

$$E(r) = \partial_r \pi_0(r)$$

Action for fluctuations

$$\mathcal{S}_\phi = \int \mathrm{d}^4x \left(-\frac{1}{2} Z^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{\phi \delta T}{\sqrt{6} M_{\mathrm{Pl}}} \right)$$

Vainshtein effect

$$Z^{tt}(r) = -\left[1 + \frac{2}{3\Lambda^3}\left(2\frac{E(r)}{r} + E'(r)\right)\right]$$

$$Z^{rr}(r) = 1 + \frac{4}{3\Lambda^3}\frac{E(r)}{r},$$

$$Z^{\Omega\Omega}(r) = 1 + \frac{2}{3\Lambda^3}\left(\frac{E(r)}{r} + E'(r)\right).$$

$$\phi_{lm\omega}(t, r, \theta, \phi) = u_{l\omega}(r) Y_{lm}(\theta, \phi) e^{-i\omega t}$$

Vainshtein region $Z \gg 1$ fifth force suppressed by $\frac{1}{Z}$

Power emitted

Goldberger+ Rothstein hep-th/0409156

$$\frac{2\mathrm{Im}(S_{\mathrm{eff}})}{T_{\mathrm{P}}} = \int_0^\infty \mathrm{d}\omega\, f(\omega) \qquad P = \int_0^\infty \mathrm{d}\omega\, \omega f(\omega)$$

Radiated power is $$P = \frac{\pi}{3M_{\mathrm{Pl}}^2} \sum_{n=0}^{\infty} \sum_{lm} n\Omega_{\mathrm{p}} |M_{lmn}|^2$$

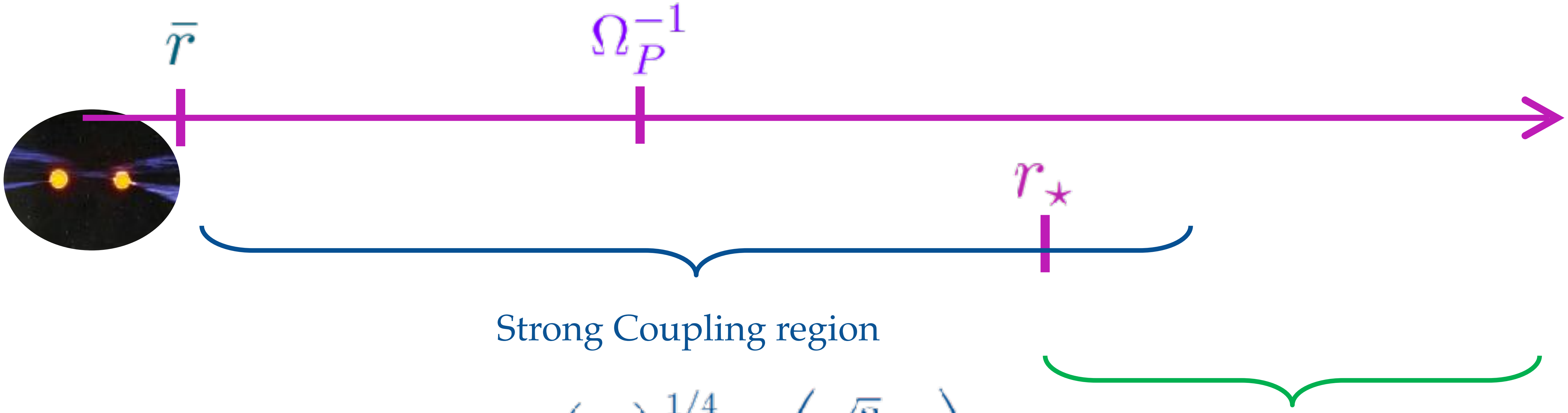
where $$\mathcal{M}_{lmn} = \frac{1}{T_P} \int_0^{T_p} dt \int d^3x u_{ln}(r) Y_{lm}(\theta, \phi) e^{-int/T_p} \delta T(x,t)$$

and modes satisfy

$$\partial_\mu (Z^{\mu\nu}(\pi_0) \partial_\nu \left[u_\ell(r) Y_{\ell m}(\Omega) e^{-i\omega t} \right]) = 0$$

WKB Matching

$$\partial_\mu\left(Z^{\mu\nu}(\pi_0)\partial_\nu\left[u_\ell(r)Y_{\ell m}(\Omega)e^{-i\omega t}\right]\right)=0$$



Strong Coupling region

$$u_\ell=\bar{u}\left(\frac{r}{r_\star}\right)^{1/4}J_\nu\left(\frac{\sqrt{3}}{2}\omega r\right)$$

$$\nu=\begin{cases}(2\ell+1)/4 & \text{for} \quad \ell>0\\ -1/4 & \text{for} \quad \ell=0\end{cases}$$

Free Field
in Minkowski

$$u_\ell=\frac{1}{\sqrt{\pi\omega r}}\cos\left(\omega r\right)$$

Scalar Gravitational Waves: Power Radiated

$$P = \frac{\pi}{3M_{\rm Pl}^2}\sum_{n=0}^{\infty}\sum_{lm}\frac{n}{T_P}|\mathcal{M}_l mn|^2 \qquad \mathcal{M}_{lmn} = \frac{1}{T_P}\int_0^{T_p} dt \int d^3x u_{ln}(r) Y_{lm}(\theta,\phi) e^{-int/T_p}\delta T(x,t)$$

Dominated by Quadrupole Radiation:

$$P_{\rm quadrupole} = 2^{7/2}\frac{5\lambda_1^2}{32}\frac{(\Omega_P\bar{r})^3}{(\Omega_P r_*)^{3/2}}\frac{M_Q^2}{M_{\rm pl}^2}\Omega_P^2$$

relative to GR result:

$$\frac{P^{\rm Galileon}_{\rm quadrupole}}{P^{\rm GR}_{\rm quadrupole}} = q(\Omega_P r_*)^{-3/2}(\Omega_P\bar{r})^{-1}$$

For realistic binary pulsars suppressed by 10^{-9}-10^{-7}

Static Suppression $\propto (\Omega_P r_*)^{-5/2}$

Vainshtein in static and spherical symmetry case

$$\frac{F_{r\ll r_\star}}{F_{r\gg r_\star}} = \left(\frac{r}{r_\star}\right)^{3/2} \sim 10^{-12}$$ **For Sun-Earth System**

$$\sim 10^{-13}$$ **For Earth-Moon System**

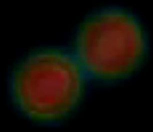
$$\sim \xcancel{10^{-15}}$$ **For Hulse-Taylor Pulsar**

expected

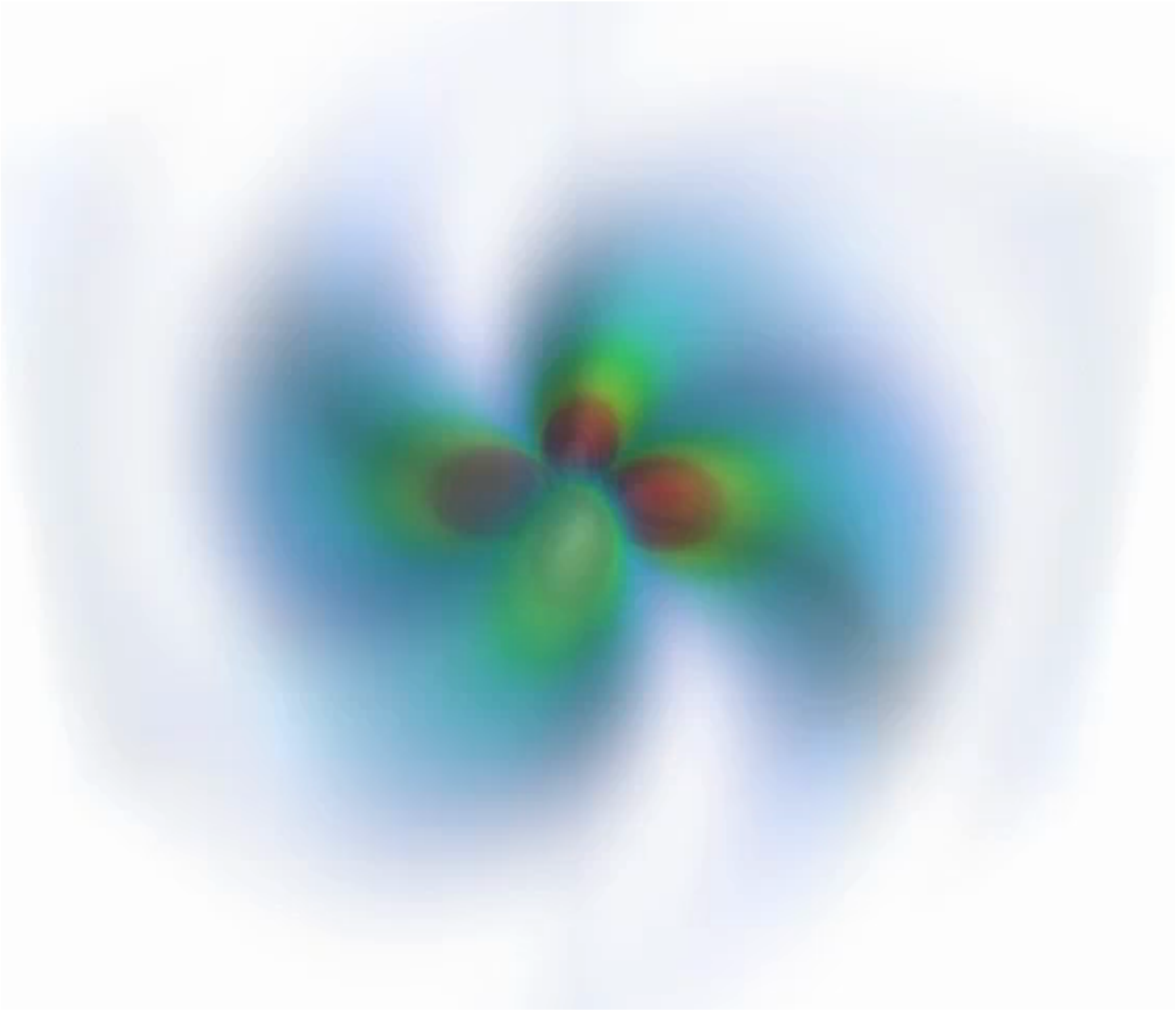
$$\text{Vainshtein Suppression in the Monopole} \sim \frac{1}{(\Omega_p r_\star)^{3/2}}$$

$$\text{Vainshtein Suppression in the Quadrupole} \sim \frac{1}{(\Omega_p r_\star)^{3/2}} \frac{1}{\Omega_p \bar{r}} \sim 10^{-8}$$

slides courtesy of C.de Rham

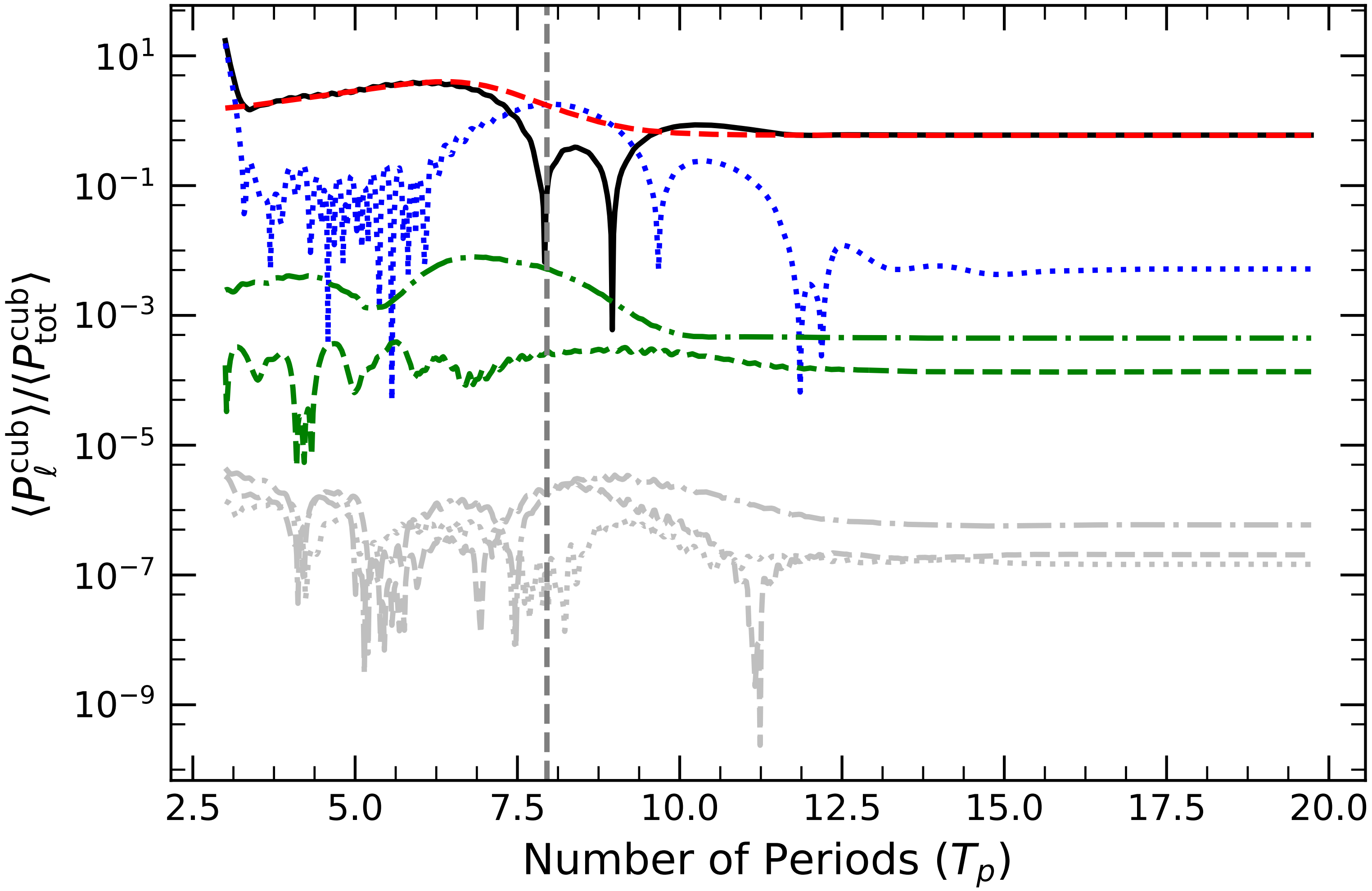


Movie



Power per multipole (numerics)

Dar, de Rham, Deskins, Giblin, AJT 2018
Gerhardinger, Giblin, AJT, Trodden 2022



Black: Total power
Dotted Blue: Monopole
Dotted Grey: Dipole
Dashed Red: Quadrupole

Consistent with analytic estimate:

$$\left.\frac{P_2^{\rm cub}}{P_2^{\rm KG}}\right|_{\rm numeric} \propto \Omega_{\rm p}^{-2.49}$$

$$\left.\frac{P_2^{\rm cub}}{P_2^{\rm KG}}\right|_{\rm analytic} \propto \Omega_{\rm p}^{-5/2}$$

Numerical simulation of Galileons and Massive Gravity

- Simulations problematic due to lack of manifest well-posedness
- Truncated EFTs,
- Tricomi or keldysh problem (Enrico's talk)
- Two successful approaches - High pass filter - Fixing equations (numerical UV completion)

Gerhardinger, Giblin, AJT, Trodden 2022

Numerical UV Completion I

$$\Box\pi + \frac{1}{3\Lambda^3}\left(H^{\mu\nu}H_{\mu\nu} - (H^{\nu}_{\nu})^2\right) = -\frac{T}{3M_{\mathrm{Pl}}}$$

$$\Box A_\mu - \frac{1}{\tau}\partial_t A_\mu - M^2 A_\mu = -M^2\partial_\mu\pi$$

$$\Box H_{\mu\nu} - \frac{1}{\tau}\partial_t H_{\mu\nu} - M^2 H_{\mu\nu} = -\frac{M^2}{2}\left(\partial_\mu A_\mu + \partial_\nu A_\mu\right)$$

Numerical UV Completion II

$$M \to \infty$$

$$\hat{\tau} = \frac{1}{\tau M}$$

$$\Box\pi + \frac{1}{3\Lambda^3}\left(H^{\mu\nu}H_{\mu\nu} - (H^{\nu}_{\nu})^2\right) = -\frac{T}{3M_{\mathrm{Pl}}}$$

$$\ddot{H}_{\mu\nu} = \frac{1}{\hat{\tau}^2}\left(\partial_\mu\partial_\nu\pi\right) - \frac{2}{\hat{\tau}}\dot{H}_{\mu\nu} - \frac{1}{\hat{\tau}^2}H_{\mu\nu}$$

Similar `Fixing Equations' (Luis Lehner and co), Israel-Stewart method (Discussed in Enrico's talk yesterday)

Extension to Quartic Galileon?

de Rham, Matas, AJT 2013

Background $$\left(\frac{E}{r}\right)+\frac{2}{3\Lambda_3^3}\left(\frac{E}{r}\right)^2+\frac{2}{\Lambda_4^6}\left(\frac{E}{r}\right)^3=\frac{1}{12\pi}\frac{M_{\rm tot}}{M_{\rm Pl}}\frac{1}{r^3}$$

Perturbations $$\begin{aligned}\pi(\vec{x},t)&=\pi(r)+\sqrt{2/3}\phi^{(1)}(\vec{x},t)+\cdots\\ T&=T_0+\delta T\,,\end{aligned}$$

Deep in quartic Vainshtein region:

$$\hat{\Box}\phi^{(1)}=\frac{128\times 3^{1/3}}{\pi^{2/3}}\left(\frac{\Lambda_4}{\Lambda_3}\right)^6\left(\frac{r_{\star,4}}{r}\right)^2\left[-\frac{1}{c_r^2}\partial_t^2\phi+\partial_r^2\phi+\frac{k_\Omega}{r_{\star,4}^2}\nabla_\Omega^2\phi\right]$$

No-centrifugal repulsion - high momentum modes are not sufficiently suppressed -

analytic approximation fails disastrously!!!!

+ numerics is naively not well-posed

Numerical Quartic Galileon

Gerhardinger, Giblin, AJT, Trodden 2024
de Rham, Giblin, AJT 2024

$$\Box\pi + \frac{1}{3\Lambda_3^3}\left((\Box\pi)^2 - (\partial_\mu\partial_\nu\pi)^2\right) + \frac{1}{9\Lambda_4^6}\left((\Box\pi)^3 - 3\Box\pi(\partial_\mu\partial_\nu\pi)^2 + 2(\partial_\mu\partial_\nu\pi)^3\right) = \frac{T}{3M_{\rm Pl}}$$

High Pass Filter + Smooth turn of sources

$$\mathcal{O}_2 + f_3(t_{\rm pr})\kappa_3\mathcal{O}_3 + f_4(t_{\rm pr})\kappa_4\mathcal{O}_4 = -f_1(t_{\rm pr})J_{\rm pr}$$

Numerical UV completion

$$\ddot{\pi} = \nabla^2\pi + \frac{1}{3\Lambda_3^3}\left((H^\nu_\nu)^2 - H_{\mu\nu}H^{\mu\nu}\right) + \frac{1}{9\Lambda_4^6}\;\left((H^\nu_\nu)^3 - 3H^\alpha_\alpha H_{\mu\nu}H^{\mu\nu} + 2(H_{\mu\nu})^3\right) - \frac{T}{3M_{\rm Pl}}$$

$$\ddot{A}_\mu = \nabla^2 A_\mu - \frac{1}{\tau}\partial_0 A_\mu - M^2(A_\mu - \partial_\mu\pi)\,,$$

$$\ddot{H}_{\mu\nu} = \nabla^2 H_{\mu\nu} - \frac{1}{\tau}\partial_0 H_{\mu\nu} - M^2(H_{\mu\nu} - \frac{1}{2}(\partial_\mu A_\nu + \partial_\nu A_\mu))\ .$$

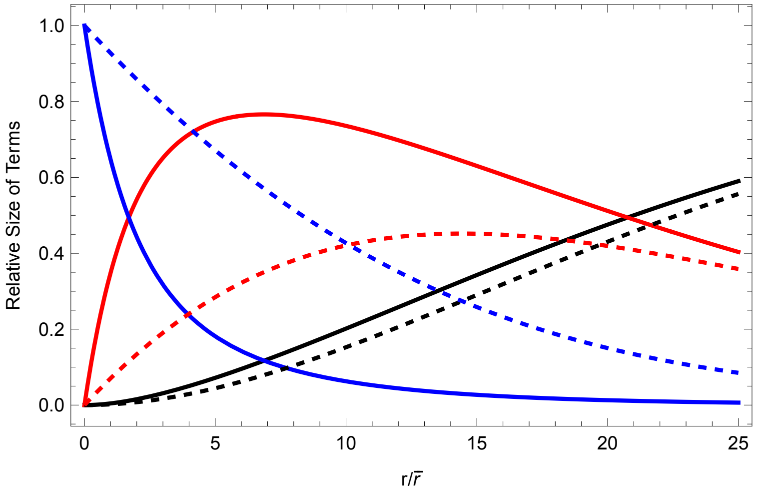


Figure 1. The relative contributions of the different terms on the left hand side of the spherically symmetric equation of motion (2.2). We show the relative contributions of the quartic (blue), cubic (red) and Klein-Gordon (black) contributions for our choice of $\xi = 0.6$ (solid) and what would happen in the case of a larger, $\xi = 0.95$ (dashed). In both cases we set the size of κ_3 such that the Vainstein radius is approximately $r/\bar{r} = 20$.

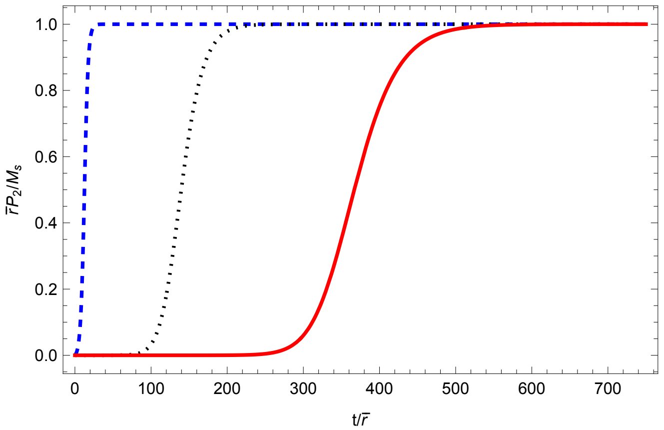


Figure 2. The turn-on functions, $f_1(t)$ (blue, dashed), $f_3(t)$ (black, dotted), and $f_4(t)$ (red solid). In the fiducial case, $\Omega_p \bar{r} = 0.2$ so the system orbits about every $t/\bar{r} = 30$.

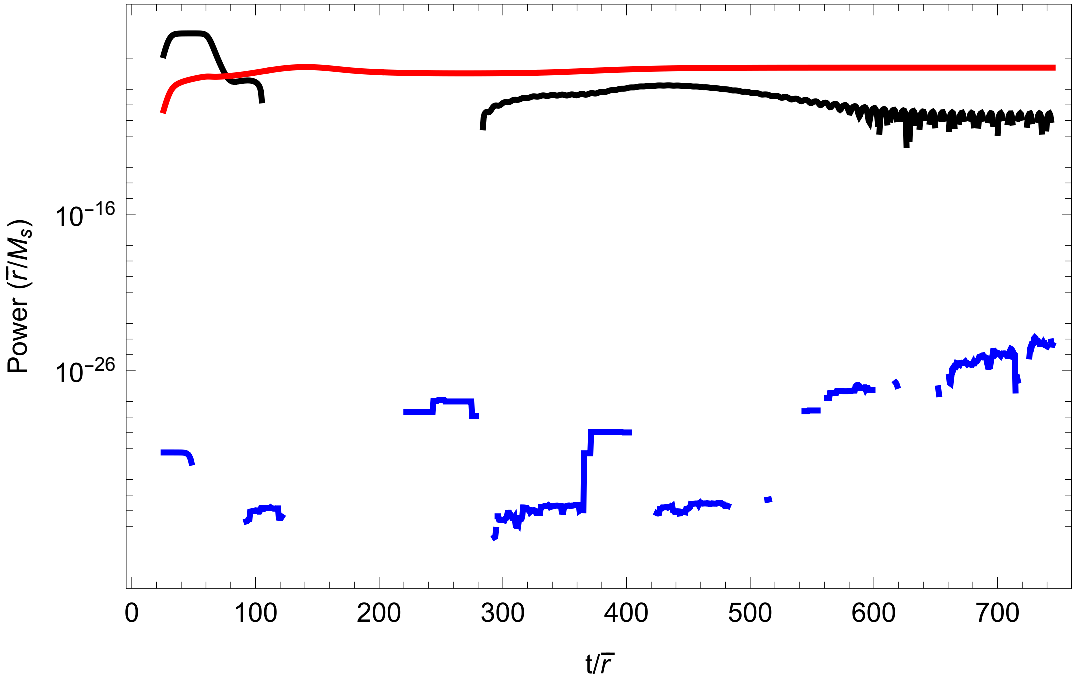


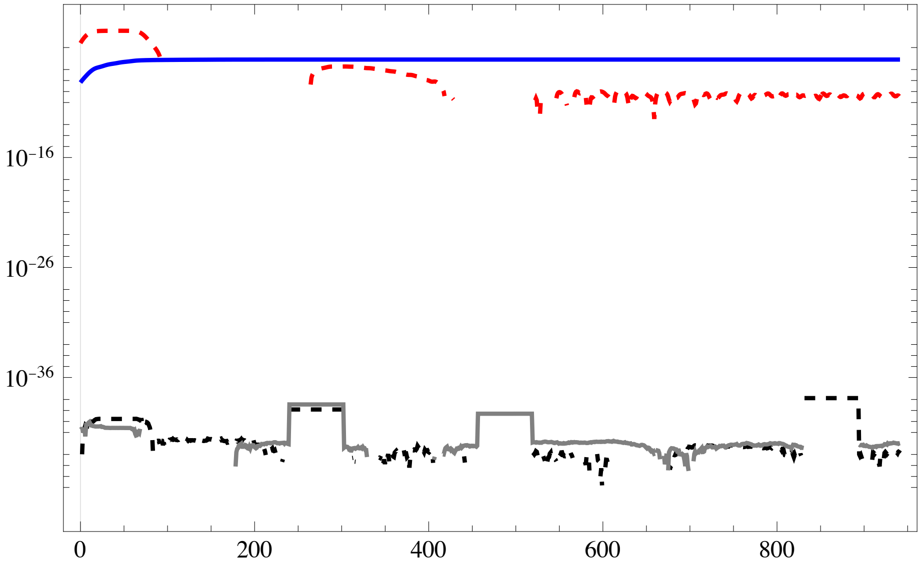
Figure 4. Power as a function of time for the fiducial, $\Omega_p \bar{r} = 0.2$ and $\xi = 0.6$ model. The curves show the period-averaged power in the monopole (black), dipole (blue) and quadrupole (red) as a function of time.

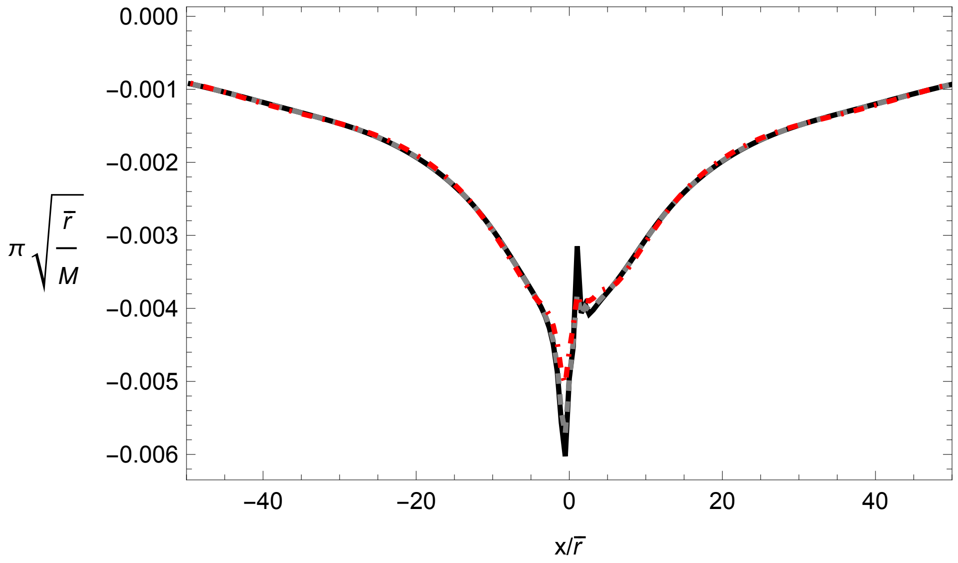
Quartic Galileon Conclusions

- For a binary source (circular orbit) dominant radiation continues to be quadrupole
- Odd multipoles vanish (machine noise)
- Parameterically distinct scaling when source likes within quartic Vainshtein region

$$\frac{\langle P_2\rangle}{\langle P_2^{\rm KG}\rangle} \propto (\Omega_p\bar{r})^{-2.07} \qquad \frac{\langle P_2^{\rm Cubic}\rangle}{\langle P_2^{\rm KG}\rangle} \propto (\Omega_p\bar{r})^{-2.5}$$

- Time averaged monopole matches perfectly analytic solution (outside of source)





Dynamical formulation of Massive Gravity

De Rham, Kozuszek, AJT, Wiseman 2023

Work with vierbein $g_{\mu\nu} = (f^{-1})^{\alpha\beta} E_{\alpha\mu} E_{\beta\nu}$

$$S = \frac{1}{16\pi G_N} \int \mathrm{d}^4x \sqrt{-g} \left(R[g] - m_1^2 (2[E] - 6) - \frac{m_2^2}{2} \left([E]^2 - [E^2] - 6 \right) \right) + S^{(\mathrm{matter})}[g, \psi_i]$$

$$\Downarrow$$

$$S = \int \mathrm{d}^4x |\det E| \left(-\frac{1}{2} A_{(1)}^{\alpha\beta\gamma\mu\nu\sigma} \partial_{[\alpha} E_{\beta]\gamma} \partial_{[\mu} E_{\nu]\sigma} - m^2 \mathcal{L}_{\mathrm{mass}} + \mathcal{L}_{\mathrm{matter}} \right)$$

$$\begin{aligned} A_{(1)}^{\alpha\beta\gamma\mu\nu\rho} &= \eta^{\gamma\rho} g^{\alpha[\mu} g^{\nu]\beta} - 2 (E^{-1})^{\rho[\alpha} g^{\beta][\mu} (E^{-1})^{\nu]\gamma} + 4 (E^{-1})^{\gamma[\alpha} g^{\beta][\mu} (E^{-1})^{\nu]\rho} \,, \\ A_{(2)}^{\alpha\beta\gamma\mu\nu\rho} &= [E] \left(\frac{1}{2} \eta^{\gamma\rho} g^{\alpha[\mu} g^{\nu]\beta} - (E^{-1})^{\rho[\alpha} g^{\beta][\mu} (E^{-1})^{\nu]\gamma} + 2 (E^{-1})^{\gamma[\alpha} g^{\beta][\mu} (E^{-1})^{\nu]\rho} \right) \\ &\quad - 2 \eta^{\gamma\rho} g^{\alpha[\mu} E^{\nu]\beta} + (E^{-1})^{\gamma\rho} g^{\nu[\alpha} g^{\beta]\mu} - 4 (E^{-1})^{\rho[\mu} g^{\nu][\alpha} g^{\beta]\gamma} \\ &\quad + 2 (E^{-1})^{\rho[\alpha} E^{\beta][\mu} (E^{-1})^{\nu]\gamma} - 4 (E^{-1})^{\gamma[\alpha} E^{\beta][\mu} (E^{-1})^{\nu]\rho} \,. \end{aligned}$$

Dynamical formulation of Massive Gravity

De Rham, Kozuszek, AJT, Wiseman 2023

Work with vierbein $g_{\mu\nu} = (f^{-1})^{\alpha\beta} E_{\alpha\mu} E_{\beta\nu}$

$$S = \int \mathrm{d}^4 x |\mathrm{det} E| \left(-\frac{1}{2} A_{(1)}^{\alpha\beta\gamma\mu\nu\sigma} \partial_{[\alpha} E_{\beta]\gamma} \partial_{[\mu} E_{\nu]\sigma} - m^2 \mathcal{L}_{\mathrm{mass}} + \mathcal{L}_{\mathrm{matter}} \right)$$

Conjugate variables $P_i = \partial_{[t} E_{i]t} \;, \quad P_{ij} = \partial_{[t} E_{i]j}$

BD Constraint!!!

$$\frac{\partial \mathcal{L}}{\partial \partial_t E_{tt}} = 0$$

Is algebraic!!!

$$|E|^2 A_{(1)}^{\alpha\beta\gamma\mu\nu\sigma} \partial_{[\alpha} E_{\beta]\gamma} \partial_{[\mu} E_{\nu]\sigma} = C_2' E_{tt}^2 + C_1' E_{tt} + C_0'$$

$$|E|^3 A_{(2)}^{\alpha\beta\gamma\mu\nu\sigma} \partial_{[\alpha} E_{\beta]\gamma} \partial_{[\mu} E_{\nu]\sigma} = C_3'' E_{tt}^3 + C_2'' E_{tt}^2 + C_1'' E_{tt} + C_0''$$

Dynamical formulation of Massive Gravity

De Rham, Kozuszek, AJT, Wiseman 2023

Decomposing - trace + trace free

$$E_{ij} = \tilde{E}_{ij} + \tilde{E}\delta_{ij} \ , \quad P_{ij} = \tilde{P}_{ij} + \tilde{P}\delta_{ij} \quad , \qquad \delta^{ij}\tilde{E}_{ij} = \delta^{ij}\tilde{P}_{ij} = 0$$

Equations reduce to

$$\partial_t \tilde{P}_{ij} = \mathcal{S}_{ij} \ , \quad \partial_t \tilde{E}_{ij} = \mathcal{U}_{ij} \ , \quad \partial_t E_i = \mathcal{V}_i \ , \quad \partial_t \tilde{E} = \mathcal{W}$$

$$\mathcal{S}_{ij} = \mathcal{J}_{ij}{}^{klmn}\partial_k\partial_l\tilde{E}_{mn} + \mathcal{J}_{ij}{}^{klm}\partial_k\partial_l\tilde{E}_m + \mathcal{J}_{ij}{}^{kl}\partial_k\partial_l\tilde{E} + \dots$$

$$\mathcal{U}_{ij} \ , \mathcal{V}_i , \mathcal{W} \sim \text{first order spatial derivatives}$$

Dynamical formulation of Massive Gravity

De Rham, Kozuszek, AJT, Wiseman 2023

Well-posed Fixing approach

$$\partial_t \tilde{P}_{ij} = \mathcal{S}_{ij} + \ell^2 \delta^{mn} \partial_m \partial_n \tilde{P}_{ij} \, , \quad \partial_t \tilde{E}_{ij} = \mathcal{U}_{ij} + \ell^2 \delta^{mn} \partial_m \partial_n \tilde{E}_{ij} \, ,$$
$$\partial_t E_i = \mathcal{V}_i + \ell^2 \delta^{mn} \partial_m \partial_n E_i \, , \quad \partial_t \tilde{E} = \mathcal{W} + \ell^2 \delta^{mn} \partial_m \partial_n \tilde{E} \, ,$$

Add diffusion terms in phase space

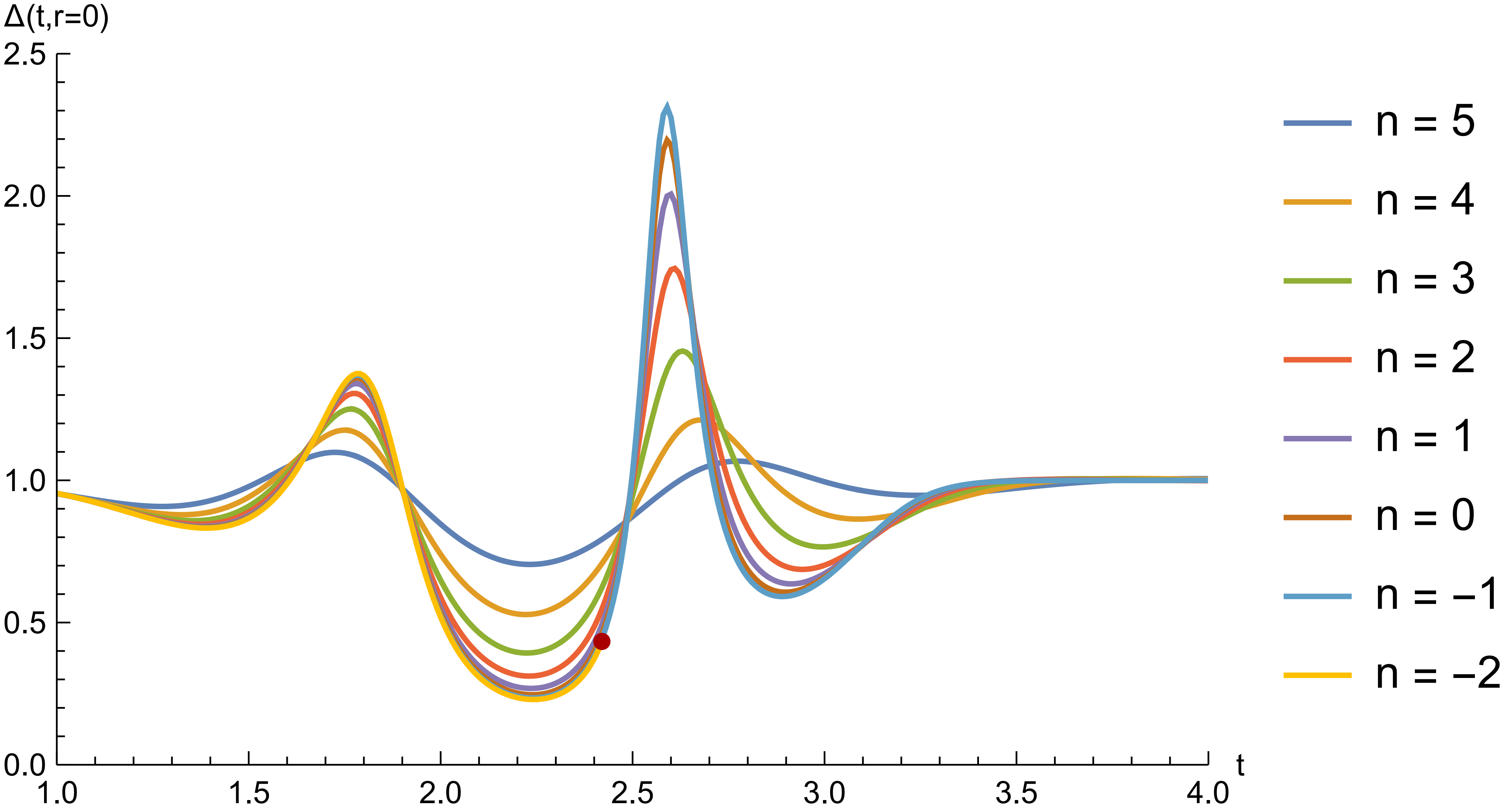
$$\partial_t \begin{pmatrix} \delta \tilde{P}_{ij} \\ \delta \tilde{E}_{ij} \\ \delta E_i \\ \delta \tilde{E} \end{pmatrix} = \begin{pmatrix} \ell^2 \delta_i^m \delta_j^n \delta^{kl} & \mathcal{J}_{ij}{}^{klmn} & \mathcal{J}_{ij}{}^{klm} & \mathcal{J}_{ij}{}^{kl} \\ 0 & \ell^2 \delta_i^m \delta_j^n \delta^{kl} & 0 & 0 \\ 0 & 0 & \ell^2 \delta_i^m \delta^{kl} & 0 \\ 0 & 0 & 0 & \ell^2 \delta^{kl} \end{pmatrix} \partial_k \partial_l \begin{pmatrix} \delta \tilde{P}_{mn} \\ \delta \tilde{E}_{mn} \\ \delta E_m \\ \delta \tilde{E} \end{pmatrix} + \dots ,$$

UV dispersion

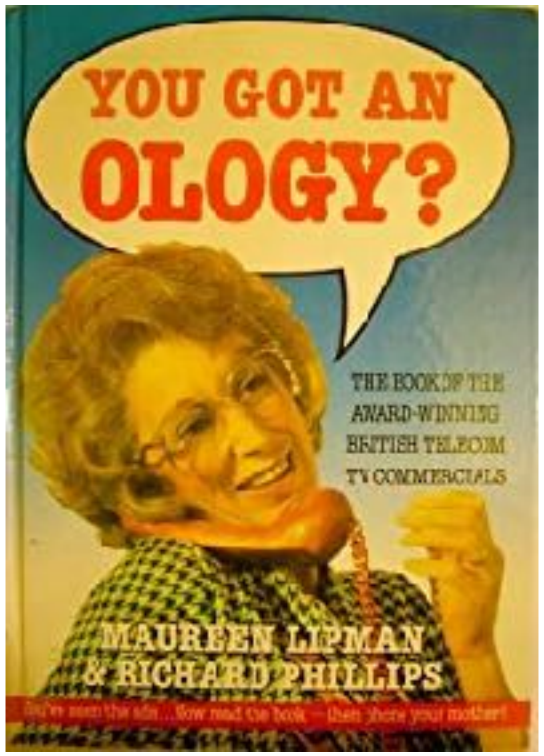
$$\omega = \ell^2 k^2$$

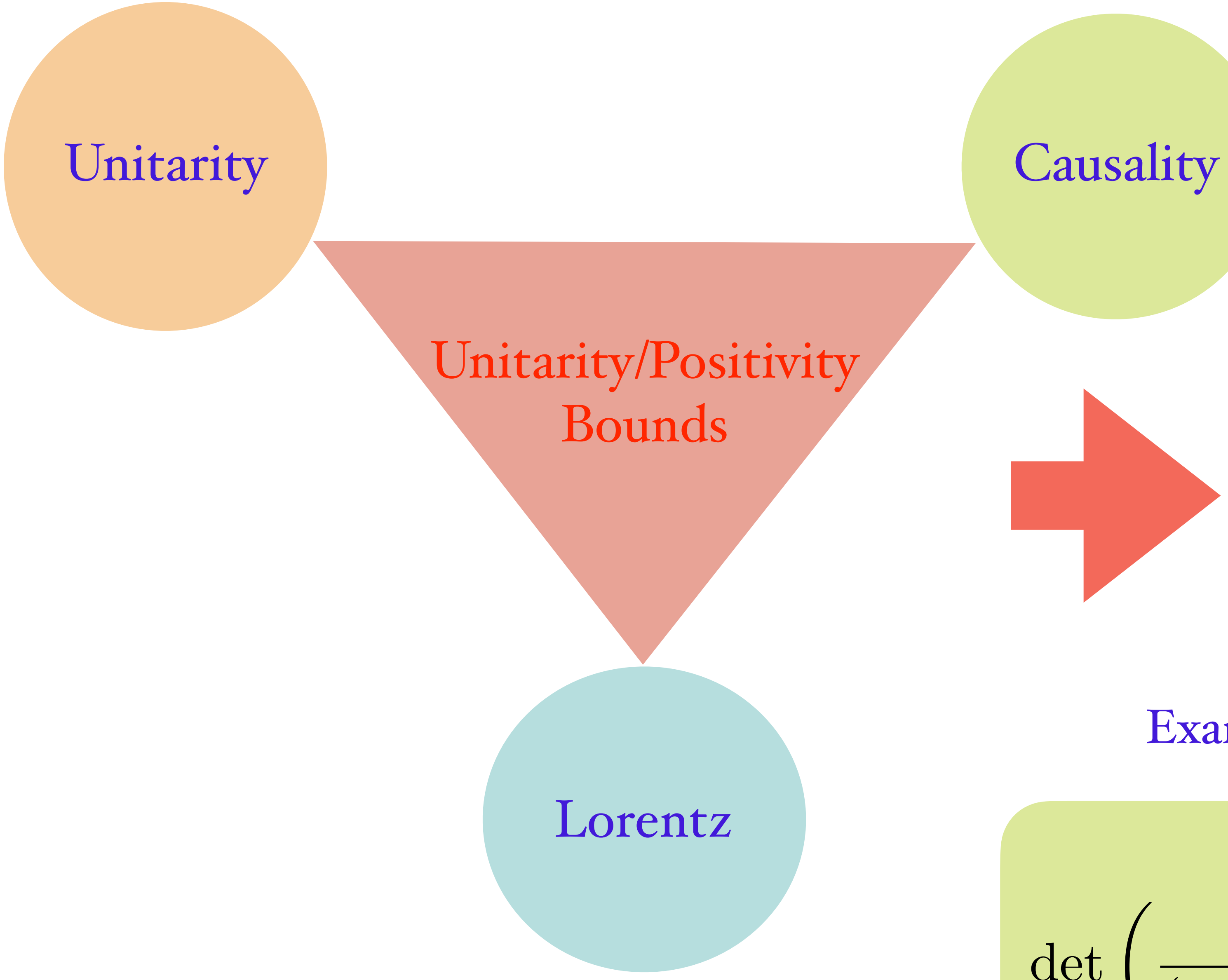
UV insensitivity

- Check we are insensitive to diffusion terms; $\ell^2 = 2^n \times 10^{-3}$



S-Matrix Bootstrapology





Unitarity

Causality

Unitarity/Positivity
Bounds

Lorentz

Implies Infinite number of
Nonlinear constraints
on low energy scattering amplitudes

Example:

$$\det_{pq}\left(\frac{1}{(M+p+q)!}\frac{d^{M+p+q}}{ds^{M+p+q}}\mathcal{A}'_s(2m^2-t/2,t)\right)>0$$

Even M+p+q $\qquad$ $0\leq t<4m^2$

Tremendous recent progress....

AJT, Wang, Zhou 2020

New positivity bounds from full crossing symmetry

Positivity + Crossing Symmetry

$$A(s,t) = F(\alpha)\frac{s^{1/2}}{(s-4m^2)^{\alpha}}\sum_{\ell=0}^{\infty}(2\ell+2\alpha)C_{\ell}^{(\alpha)}(\cos\theta)a_{\ell}(s), \quad \alpha = \frac{D-3}{2}$$

$$\langle\langle X(\mu,l)\rangle\rangle = \frac{\sum_{\ell}\int \mathrm{d}\mu\rho_{\ell,\alpha}(\mu)X(\mu,l)}{\sum_{\ell}\int \mathrm{d}\mu\rho_{\ell,\alpha}(\mu)}$$

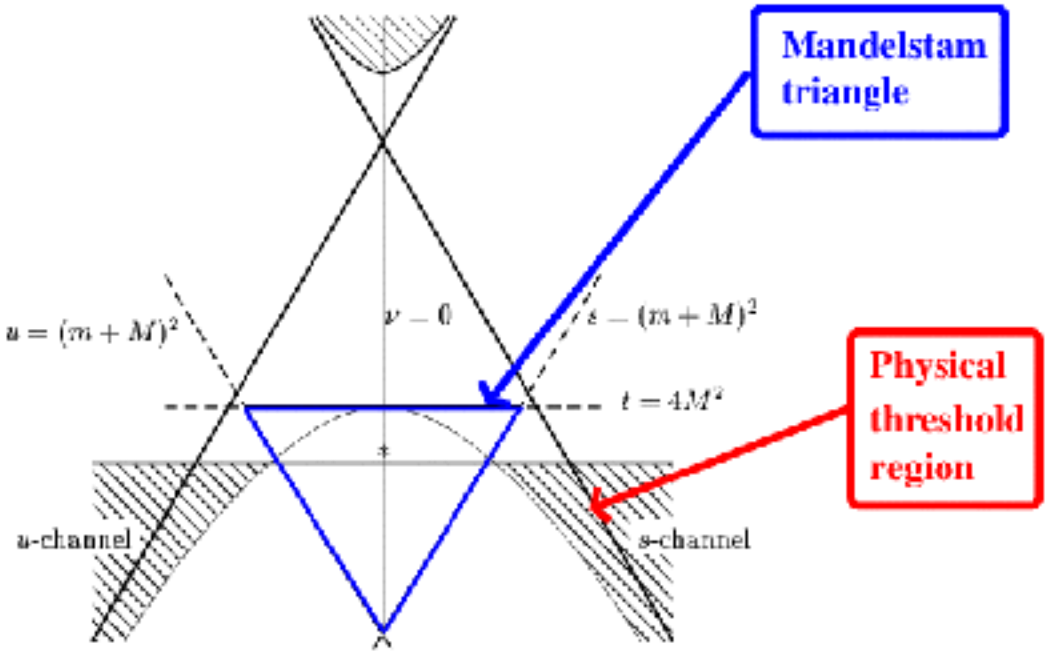
$$\boxed{\frac{1}{2(2N+2)!}\partial_t^M\partial_s^{2N+2}\mathcal{A}'(s,t)|_{s=t=0} = \langle\langle\frac{1}{\mu^{2N}}\rangle\rangle > 0}$$

Caron-Huot+Van Duong Extremal Effective Field Theories 2011.02957

Arkani-Hamed, Huang, Huang, EFT-Hedron, 2012.15849

Bellazzini et al, Positive Moments for Scattering Amplitudes, 2011.0003

Crossing Symmetry



What is impact of FULL crossing symmetry?

$$a(t) + \int_{4m^2}^{\infty} \frac{\mathrm{d}\mu}{\pi(\mu-\mu_p)^2} \left[\frac{(s-\mu_p)^2}{\mu - s} + \frac{(u-\mu_p)^2}{\mu - u} \right] \mathrm{Im} A(\mu, t)$$
$$= a(s) + \int_{4m^2}^{\infty} \frac{\mathrm{d}\mu}{\pi(\mu-\mu_p)^2} \left[\frac{(t-\mu_p)^2}{\mu - t} + \frac{(u-\mu_p)^2}{\mu - u} \right] \mathrm{Im} A(\mu, s)$$

$$0 = \mathcal{A}(s,t) - \mathcal{A}(t,s) = \sum_{\ell} \int d\mu \rho_{\ell,\alpha}(\mu) \left[\frac{2H_{D,\ell}\, st(s^2 - t^2)}{(D-2)D\mu^2} + \ldots \right]$$

$$\langle\!\langle X(\mu, l) \rangle\!\rangle = \frac{\sum_\ell \int \mathrm{d}\mu \rho_{\ell,\alpha}(\mu) X(\mu, l)}{\sum_\ell \int \mathrm{d}\mu \rho_{\ell,\alpha}(\mu)}$$

NULL CONSTRAINTS

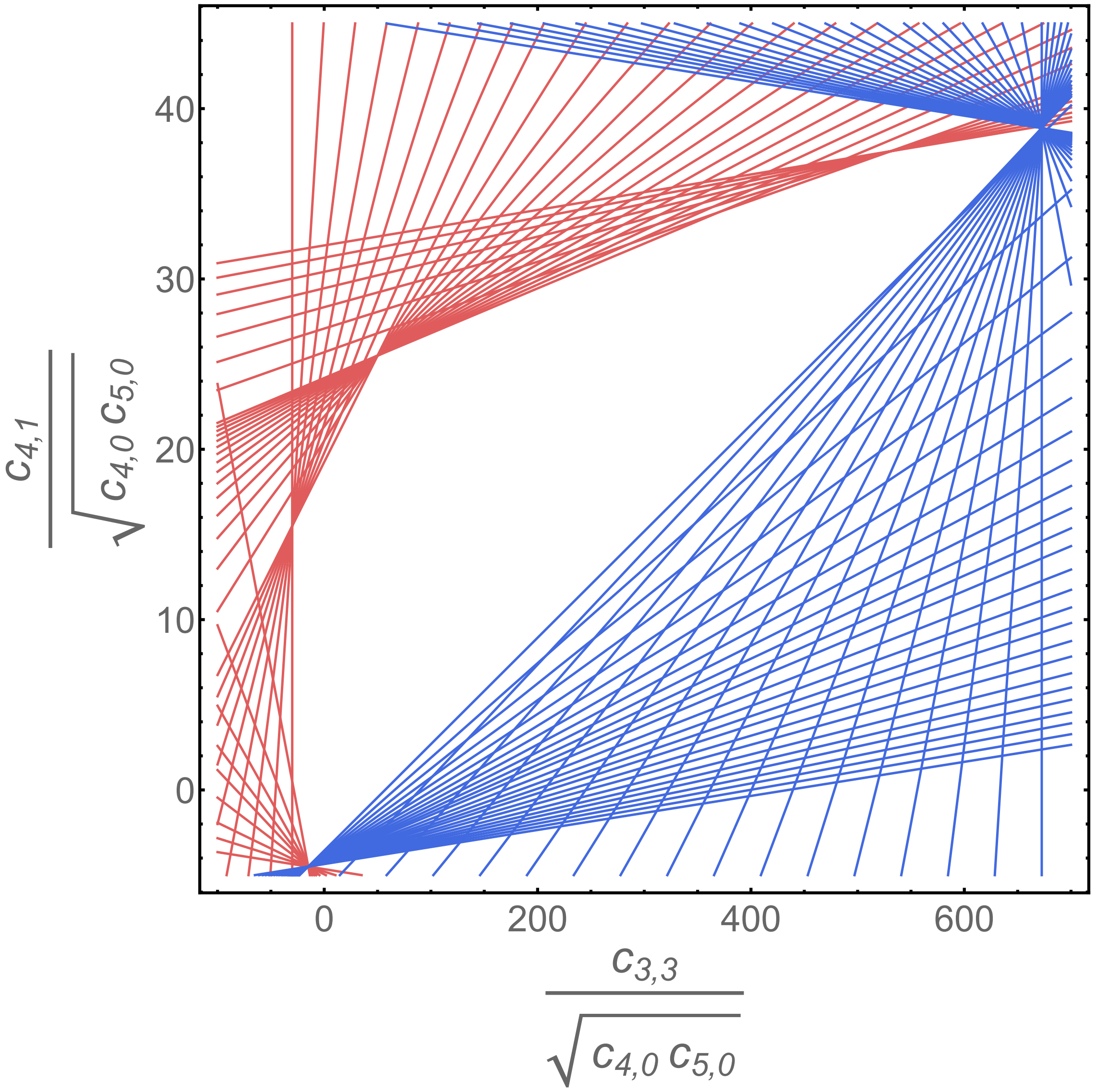
$$\langle\langle \frac{H_{D,\ell}}{\mu^2} \rangle\rangle = 0$$

$$H_{D,\ell} = \ell(\ell + D - 3)[4 - 5D - 2(3-D)\ell + 2\ell^2]$$

Upper and Lower Bounds on Wilson Coefficients

‘Islands of Positivity’

$$\mathcal{A}'(s,t) = \sum_{p.q=0}^{\infty} c_{p,q} w^p t^q \qquad w = -(s-2m^2)(u-2m^2)$$



AJT, Wang, Zhou 2020
“New positivity bounds from full crossing symmetry”

Caron-Huot+Van Duong 2020
“Extremal Effective Field Theories”

UV Constraints on IR Symmetries

e.g. scattering amplitude for weakly broken Galileon

$$\frac{m^2}{\Lambda^2} \ll 1$$

$$y = stu$$
$$x = s^2 + t^2 + u^2$$

$$\mathcal{A}'(s,t) \sim \frac{1}{\Lambda^{D-4}} \left(\frac{m^2}{\Lambda^6} x + \frac{1}{\Lambda^6} y + \frac{1}{\Lambda^8} x^2 + \dots \right) \sim -c_{11} y + c_{10} \left(\frac{1}{2} x - t^2 \right) + c_{20} \left(\frac{1}{2} x - t^2 \right)^2$$

Contradiction!!!

$$-\frac{3}{2\Lambda^2} c_{10} < c_{11} < \frac{5D-4}{(D-2)\Lambda^2} c_{10} \quad \Rightarrow \quad -\mathcal{O}(1) \frac{m^2}{\Lambda^D} < \frac{1}{\Lambda^{D-2}} < \mathcal{O}(1) \frac{m^2}{\Lambda^D}$$

1. No strict local Wilsonian UV completion for Galileons
2. Conformal Galileon allowed provided $\Lambda \sim l$
3. DBI, AdS-DBI are allowed

Compact positivity bounds and causality

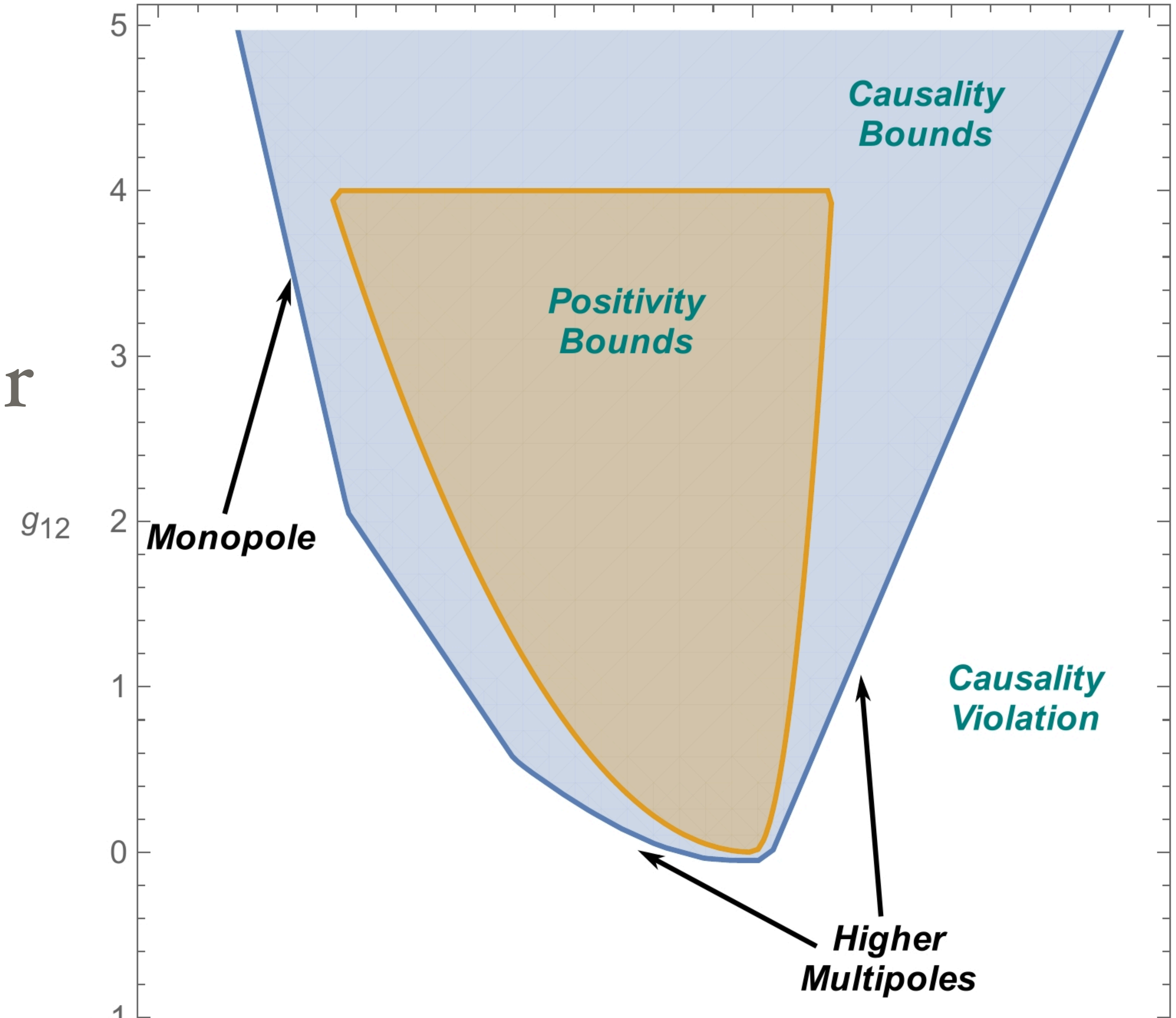
Carrillo Gonzalez, de Rham, Pozsgay, AJT ‘Causal Effective Field Theories’ 2023

For Goldstone model:

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 + \frac{g_8}{\Lambda^4}(\partial\phi)^4 + \frac{g_{10}}{\Lambda^6}(\partial\phi)^2\left[(\phi_{,\mu\nu})^2 - (\Box\phi)^2\right] + \frac{g_{12}}{\Lambda^8}((\phi_{,\mu\nu})^2)^2 - g_{\text{matter}}\phi J$$

Causality = positivity of Eisenbud-Wigner scattering time delay

$$\Delta T_\ell = 2\frac{\partial\delta_\ell}{\partial\omega}\Big|_\ell \quad \gtrsim -\omega^{-1}$$



Positivity of EFT of Gravity

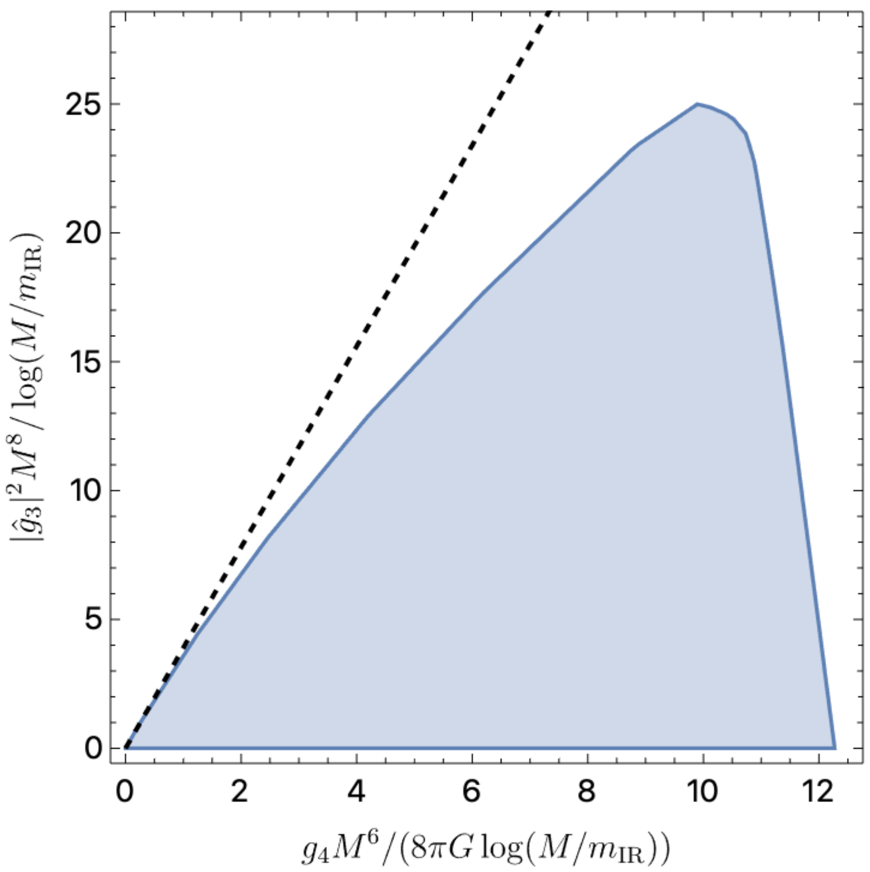
'Causality constraints on corrections to Einstein gravity'
Caron-Huot, Li, Parra-Martinez, Simmons-Duffin 2022
'Graviton partial waves and causality in higher dimensions'
Caron-Huot, Li, Parra-Martinez, Simmons-Duffin 2022
'Crossing Symmetric Spinning S-matrix Bootstrap: EFT bounds'
Chowdhury, Ghosh, Holder, Raman, Sinha 2022
'Constraints on Regge behaviour from IR physics'
de Rham, Jailty, AJT 2023

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R - \frac{1}{3!}\left(\alpha_3 R^{(3)} + \tilde{\alpha}_3 \tilde{R}^{(3)}\right) + \frac{1}{4}\left(\alpha_4 (R^{(2)})^2 + \alpha_4' (\tilde{R}^{(2)})^2 + 2\tilde{\alpha}_4 R^{(2)}\tilde{R}^{(2)}\right) + \dots\Big] + S_{\text{matter}}$$

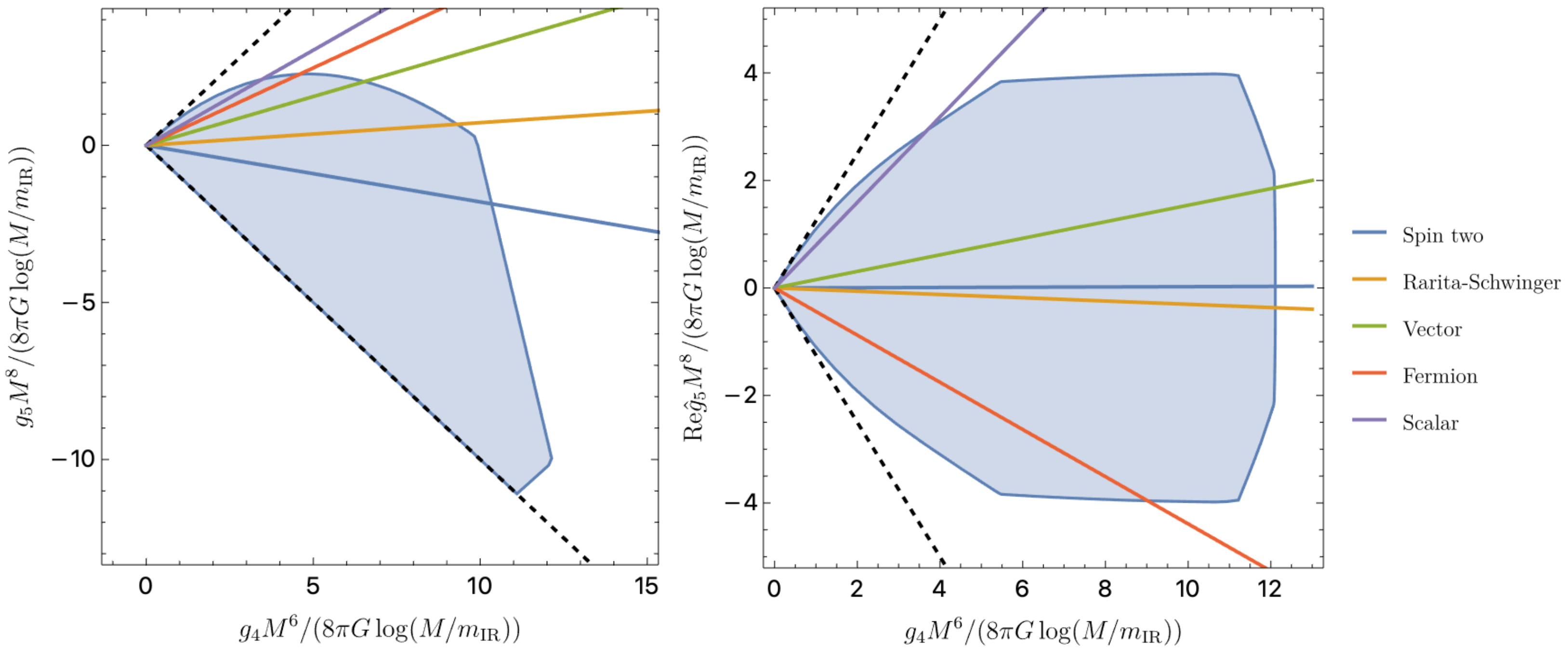
$$R^{(2)} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}\,, \qquad \tilde{R}^{(2)} = R_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}\,, \qquad \tilde{R}_{\mu\nu\rho\sigma} \equiv \tfrac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}R_{\alpha\beta\rho\sigma}\,,$$

$$R^{(3)} = R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}R_{\alpha\beta}{}^{\mu\nu}\,, \qquad \tilde{R}^{(3)} = R_{\mu\nu}{}^{\rho\sigma}R_{\rho\sigma}{}^{\alpha\beta}\tilde{R}_{\alpha\beta}{}^{\mu\nu}\,.$$

$$\widehat{g}_3 = \alpha_3 + i\tilde{\alpha}_3, \qquad g_4 = 8\pi G(\alpha_4 + \alpha_4')\,, \qquad \widehat{g}_4 = 8\pi G(\alpha_4 - \alpha_4' + i\tilde{\alpha}_4)$$



Caron-Huot, Li, Parra-Martinez, Simmons-Duffin 2022




S-matrix Positivity


Local field theory

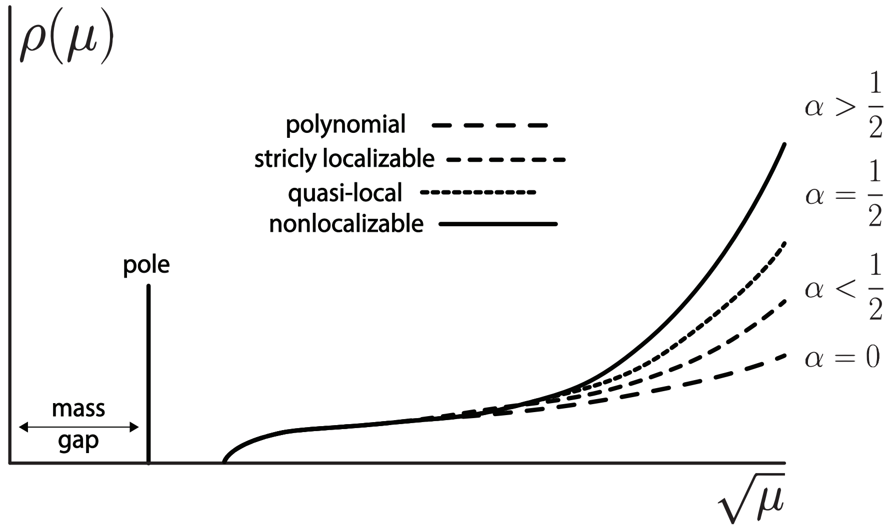
Locality in Quantum gravity?

Aharony and Banks '98

In field theory with UV fixed point at finite volume, density of states for most operators grows as

$$\rho(E) \sim e^{c' V^{1/d} E^{(d-1)/d}}$$

with preferential operators polynomial $\rho(E) \sim E^p$



In Quantum Gravity we expect high energy properties to be dominated by production of black holes

$$\rho(E) \sim e^{S_{\text{BH entropy}}} = e^{c(E/M_{\text{Pl}})^{\frac{d-2}{d-3}}} \qquad \rho(E) \sim e^{E r_*(E)}$$

where $r_*(E)$ is Schwarzschild radius

This implies Giddings-Lippert 2001 locality bound. e.g. local correlation functions only exist for

$$b > r_*(E)$$

$$\mathcal{A}(s,t) \sim e^{r_*(\sqrt{s})\sqrt{t}} \qquad t > 0$$

Giddings-Porto 2009

An exactly solvable case: dRGT Massive Gravity in 2D .....

AJT, de Rham, Gabadadze 2010

... Or why two zweibeins are better than one

$$S_\lambda = \int d^dx \left[-\frac{1}{2\lambda}(Tr[K^2]-(Tr[K])^2)\right] + S_{matter}(g,\varphi) + S_{EH}$$

$$K^\mu_\nu = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\gamma_{\alpha\nu}}$$

$$\Downarrow$$

$$S_\lambda = \int d^2x \frac{1}{2\lambda}\epsilon_{ab}\epsilon^{\mu\nu}(e-f)^a_\mu(e-f)^b_\nu + S_{\text{matter}}(e,\varphi) + S_{EH}$$

Ghost-free nonlinear Fierz-Pauli mass

Topological

Unique structure for which Boulware-Deser ghost is removed

$$\lambda = 1/m^2$$

A tale of two T's

AJT 2019

$$S_\lambda = \int d^2x \frac{1}{2\lambda} \epsilon_{ab} \epsilon^{\mu\nu} (e-f)^a_\mu (e-f)^b_\nu + S_{\text{matter}}(e,\varphi) + S_{EH}$$

Stress energy for **f** $\quad \det f \, T^\mu_a = \frac{\delta S_\lambda}{\delta f^a_\mu(x)} = -\frac{1}{\lambda} \epsilon_{ab} \epsilon^{\mu\nu} (e^b_\nu - f^b_\nu)$

Equation of motion for **e** $\quad \frac{1}{\lambda} \epsilon_{ab} \epsilon^{\mu\nu} (e^b_\nu - f^b_\nu) + \det e \, T_{M\,a}^{\ \ \mu} = 0$

Equivalent to $T\bar{T}$ deformation!!

$$\boxed{\begin{aligned} \frac{\partial S_\lambda}{\partial \lambda} &= -\int d^2x \frac{1}{2\lambda^2} \epsilon_{ab} \epsilon^{\mu\nu} (e-f)^a_\mu (e-f)^b_\nu \\ &= -\int d^2x \frac{1}{2} \epsilon_{ab} T^a_\mu T^b_\nu \\ &= -\int d^2x \det T \end{aligned}}$$

f stress energy

Massive Gravity in Two dimensions

How do we describe a massive gravity theory in two dimensions?

Massive Gravity = Diffeomorphisms Spontaneously Broken

$$Diff(M) \to Isom(M)$$

Three ingredients:

1. Dynamical metric describing spacetime $g_{\mu\nu}$
2. Fixed reference metric (acts as VEV of Higgs field) $\gamma_{\mu\nu}$
3. Stueckelberg Fields (Goldstone Modes) Φ^A

$$\gamma_{\mu\nu} = \hat{\gamma}_{AB}(\Phi)\partial_\mu \Phi^A \partial_\nu \Phi^B$$

Field Dependent Diffeomorphisms

The undeformed theory is not diff invariant, hence the diffeomorphism symmetry in Stuckelberg form is a **redundancy**. We can gauge fix to define the theory - however different gauge fixings lead to different formulations which are related by field dependent diffeomorphisms

Unitary gauge - $\Phi^a = x^a$

Unitary gauge

Generic gauge - $\Phi^a(x) = x^a + \pi^a(x)$

Transformation of scalar - $\tilde{S}(\Phi^a) = S(x^a) = \tilde{S}(x^a + \pi^a(x))$

Generic gauge

Transformations:

Perturbatively local - non-perturbatively non-local

Quantum equivalence

AJT 2019

Quantum deformation is defined by path integral flow

$$i\frac{\partial Z_\lambda}{\partial\lambda} = -\frac{1}{2}\nabla_e^2 Z_\lambda = -\frac{1}{2}\int d^2x\epsilon_{\mu\nu}\epsilon^{ab}\frac{\delta^2 Z_\lambda}{\delta e_\mu^a(x)\delta e_\nu^b(x)}$$

- Zweibein superspace measure equivalent to Polyakov measure

$$\delta s^2 = -\int d^2x\,\epsilon^{\mu\nu}\epsilon_{ab}\delta e_\mu^a(x)\delta e_\nu^b(x) = -2\int d^2x\ \det(\delta e_\mu^a(x))$$

$$\delta s^2 = \int d^2x\sqrt{-g}\left[2\delta\omega^2 + \frac{1}{4}\left(g^{\mu\nu}g^{\alpha\beta}\delta g_{\mu\alpha}\delta g_{\nu\beta}\right) - \frac{1}{4}\left(g^{\mu\nu}\delta g_{\mu\nu}\right)^2\right]$$

$$\delta e_\mu^a(x) = \eta^{ac}\epsilon_{cd}\delta\omega(x)e_\mu^d(x) + \delta h_\mu^a(x)$$

Polyakov measure used in quantizing string!!

Solution:

$T\bar{T}$ deformation

Undeformed Seed theory

$$Z_\lambda(f) = \int De(x)\,e^{i\int d^2x\frac{1}{2\lambda}\epsilon^{\mu\nu}\epsilon_{ab}(e-f)_\mu^a(e-f)_\nu^b}Z_0(e)$$

Topological property for flat metric

Fixing $f^a = d\Phi^a(x)$ then Φ^a e.o.m.s impose $e^a = dX^a(x)$

Hence

$$\boxed{\begin{aligned} S_{\text{mass}} &= \int d^2x \frac{1}{2\lambda}\epsilon_{ab}\epsilon^{\mu\nu}(d(\Phi - X))^a_\mu (d(\Phi - X))^b_\nu \\ &= \int dx_\mu \frac{1}{2\lambda}\epsilon_{ab}\epsilon^{\mu\nu}(\Phi^a - X^a)(d(\Phi - X))^b_\nu \end{aligned}}$$

Noted by Cardy via less transparent means 2018

At the S-matrix level, the deformation corresponds to a Castillejo-Dalitz-Dyson (CDD) factor

$$S(\{p_i\}) \rightarrow \left[\Pi_{i<j} e^{i\frac{1}{2}\lambda\epsilon^{ab}p^i_a p^j_b}\right] S(\{p_i\})$$

e.g. integrable theory maps to an integrable theory!!

S-matrix growth

S-matrix satisfies:

$$\hat{S}(\{p\}) = S(\{p\})e^{i\frac{\lambda}{2}\sum_{i<j}\epsilon_{ab}p_i^a p_j^b}$$

- Lorentz Invariant
- Analyticity (Causality)
- Crossing symmetry
- Unitarity

e.g. 2-2 scattering:

$$e^{2i\delta(s)} = e^{i\frac{1}{2}\lambda s} \qquad Im(s) > 0$$

but violates:

- Polynomial/exponential boundedness **(locality)**

by comparison, a local 2D field theory looks like which is polynomiall
bounded

$$e^{2i\delta(s)} = \Pi_j\left(\frac{\mu_j + s}{\mu_j - s}\right) \qquad Im(s) > 0$$

Deformation of a CFT = (Non-) Critical String Theory

Now assume seed theory is classically conformal

AJT 2019

$$S_{CFT}(\Omega^2 g, \{\Omega^{-\Delta_I}\varphi_I\}) = S_{CFT}(g, \{\varphi_I\})$$

Mass term breaks Conformal symmetry - Introduce conformal Stueckelberg fields via

$$g \to \hat{\Omega}^2 g \quad \varphi_I \to \hat{\Omega}^{-\Delta_I}\varphi_I$$

Integrating out the conformal Stueckelberg field gives

$$S_\lambda = \int d^2x \left[\frac{1}{2\lambda}\sqrt{-\det \partial_\mu \Phi^A \partial_\nu \Phi^B \hat{\gamma}_{AB}(\Phi)} - \frac{1}{4\lambda}\sqrt{-g}g^{\mu\nu}\partial_\mu\Phi^A\partial_\nu\Phi^B\hat{\gamma}_{AB}(\Phi)\right] + S_{CFT}[g, \varphi]$$

For example for $S_{CFT}[g,\varphi] = \int d^2x - \frac{1}{2}G_{IJ}(\varphi)\partial_\mu\varphi^I\partial_\nu\varphi^J$

deformed theory is a worldsheet string with target space metric

$ds^2_{\text{target}} = \frac{1}{2\lambda}\hat{\gamma}_{AB}(\Phi)d\Phi^A d\Phi^B + G_{IJ}(\varphi)d\varphi^I d\varphi^J$ in a non-zero B-field $B_{+-}(\Phi) = \hat{\gamma}_{+-}(\Phi)$

$$S_\lambda = \int d^2x\sqrt{-g}\left[-\frac{1}{4\lambda}\hat{\gamma}_{AB}(\Phi)g^{\mu\nu}\partial_\mu\Phi^A\partial_\nu\Phi^B - \frac{1}{2}G_{IJ}g^{\mu\nu}\partial_\mu\varphi^I\partial_\nu\varphi^B - \frac{1}{4\lambda}B_{AB}(\Phi)\epsilon^{\mu\nu}\partial_\mu\Phi^A\partial_\nu\Phi^B\right]$$

Massive gravity coupled to a CFT with central charge c=24 is
equivalent to critical bosonic string with nonzero B field

Locality bound

Given a wavepacket of energy or momentum E, the minimum distance over which it may be localized is

$$L \sim E\lambda \quad \longrightarrow \quad \Delta x_R \Delta x_L > \lambda$$

If interpreted as a time delay/advance, associated phase shift is

$$\delta(E) \sim LE \sim E^2\lambda \sim \lambda s$$

CDD factors!

At any finite order in perturbation theory, correlation functions are local (tempered distrubutio polynomially bounded) - Non-perturbatively they resum to a Jaffe non-localizable behaviour (e.g Cardy 2019)

$$G(k) \sim e^{\lambda k^2}$$

If bootstrap/positivity bounds were applied to scattering on string world sheet - they would conclude that it has no UV completion!!

Be wary of assumptions! in

Landscape versus Swampland