Gravitational positivity bounds from finite energy sum rules

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based on: [JHEP06(2023)032 T. Noumi, **JT**]

(see also [JHEP11(2020)054 **JT**, K. Aoki, S. Hirano])

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Introduction

- Many pheno models have been studied in cosmology.

  e.g.) Inflation models, Modification of GR, …

➢ non-renormalizable interactions are often included.

- (perturbative) unitarity is broken at high-E.

⇒ "**Low-energy Effective Field Theory (EFT)**"

which will be replaced by UV complete theory at high-E.

Introduction

Energy

Microscopic Theory

High-E

… Unknown, consistent at UV.

Unitarity, Causality,
Locality, Lorentz inv

e.g.) renormalizable theory

Embedded appropriately.

Low-E

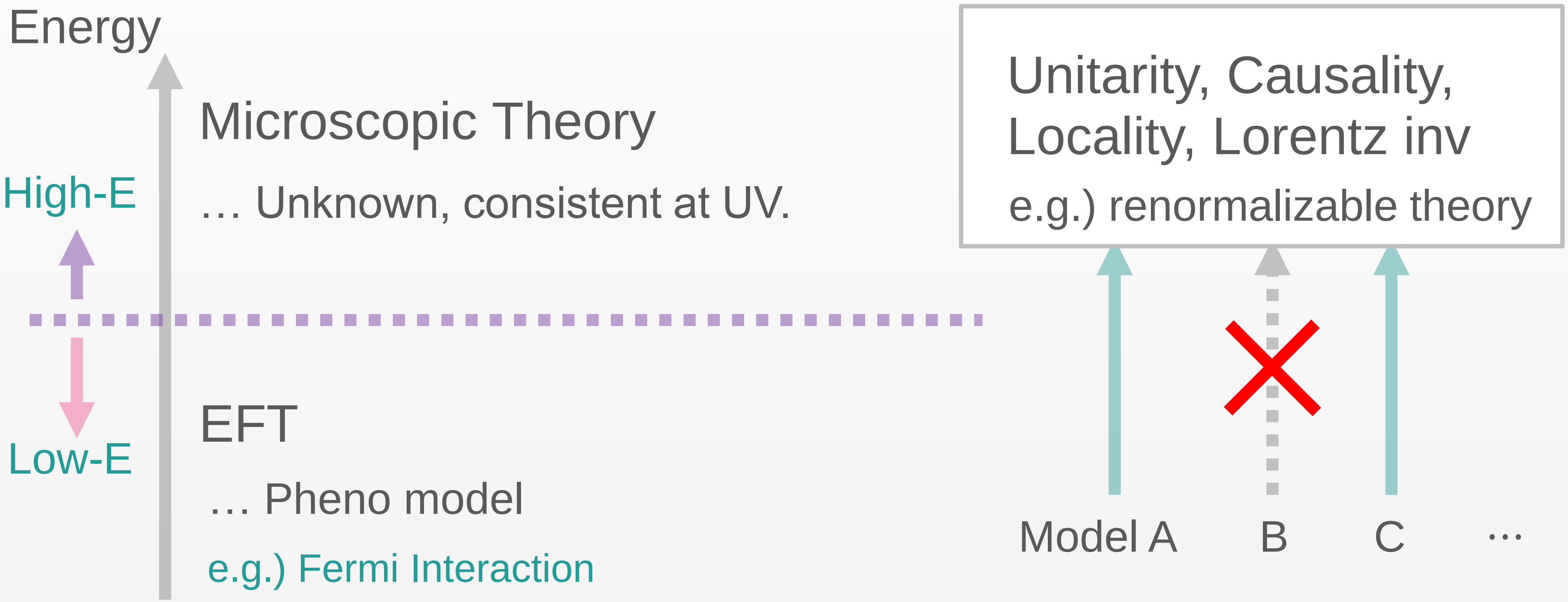
EFT

… Pheno model

e.g.) Fermi Interaction ⇒ Embedded into SM

Model A B C ⋯

Introduction



- There exist “**bad EFTs**” which are inconsistent with unitarity etc. at UV.

  … quantified by 2→2 scattering: **Positivity Bounds** [Adams + (‘06), Pham (’85)]

- **Important for model building and probing UV physics.**
  - ➢ If “bad EFT” is favored by experiments ⇒ insights on UV physics!

Our works:

- Positivity bounds have been formulated in **non-gravity** context.

[Adams + (‘06), Pham (’85)]

➡ *Can we formulate positivity for gravitational theories?*

- Today’s Talk:
  - **Review** positivity bounds without gravity.

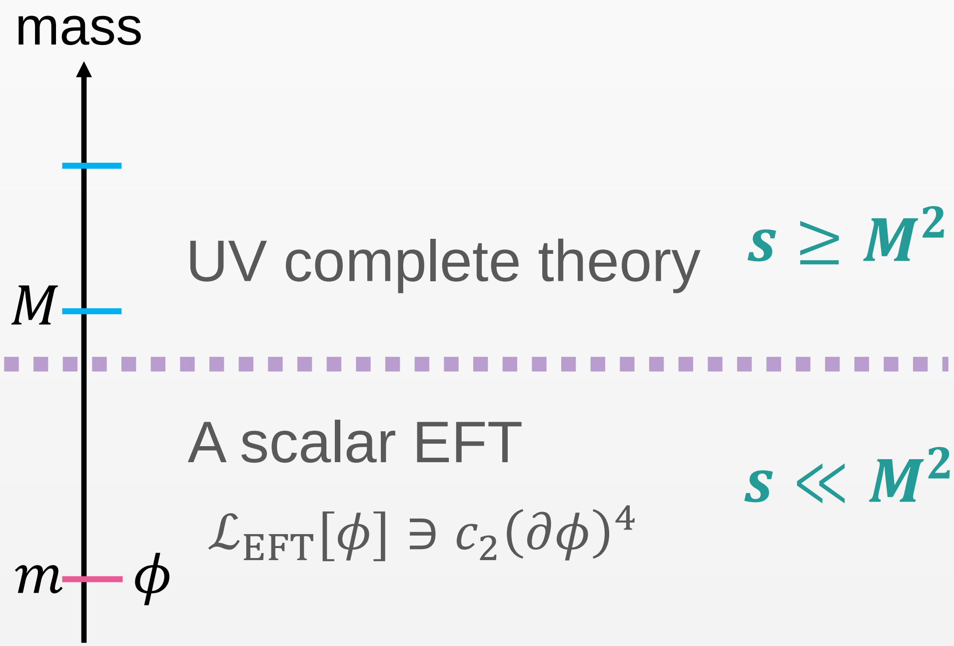
    - basics & (personal) intuitive understanding

  - **Formulate** gravitational positivity bounds.

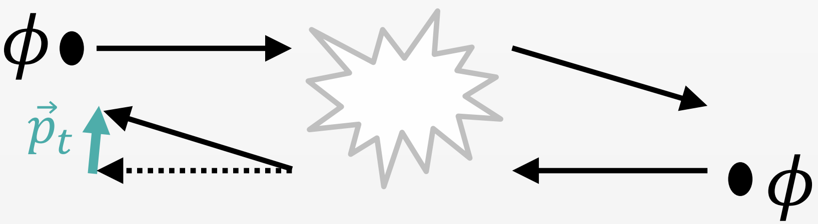
    - excitation of graviton (massless spin-2)

Positivity bound (without gravity)

- Let's consider a scalar EFT and focus on $\phi\phi \to \phi\phi$ amplitude $\mathcal{M}(s,t)$.



➢ In the CM frame

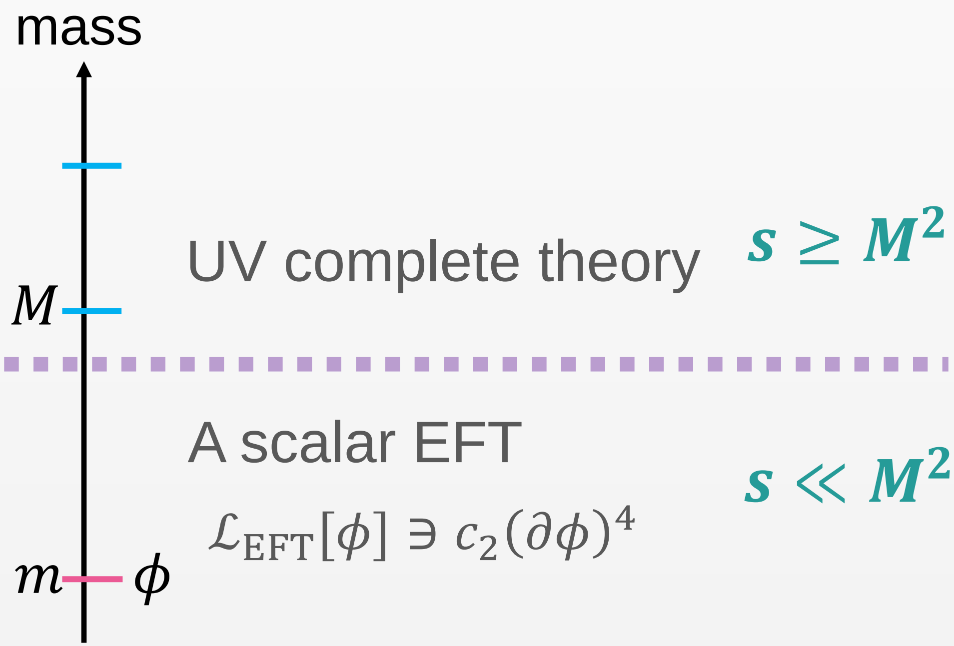


s = (CM energy)^2
$t = -|\vec{p}_t|^2$
~(momentum transfer)^2

- We can view an EFT amplitude as the low-s limit of full amplitude $\mathcal{M}(s,t)$.

Positivity bound (without gravity)

- Let's consider a scalar EFT and focus on $\phi\phi \to \phi\phi$ amplitude $\mathcal{M}(s,t)$.



UV complete theory $\quad$ $\boldsymbol{s \geq M^2}$ $\quad$ $\mathcal{M}(s,t) = ???$

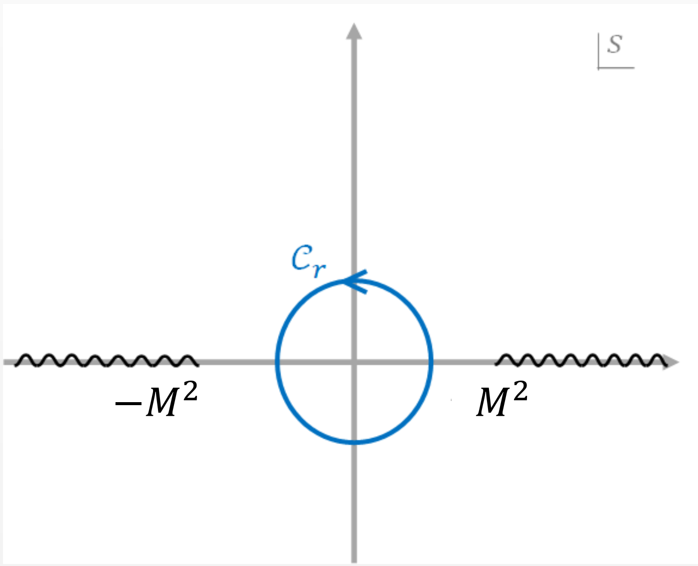
A scalar EFT

$\mathcal{L}_{\mathrm{EFT}}[\phi] \ni c_2(\partial\phi)^4$ $\quad$ $\boldsymbol{s \ll M^2}$ $\quad$ $\mathcal{M}(s,t)\big|_{\mathrm{low}} \sim c_2 s^2 + \cdots$

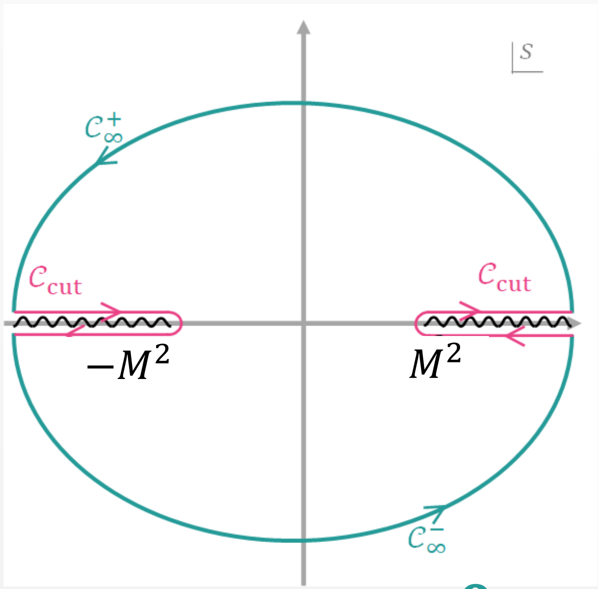
- We can view an EFT amplitude as the low-s limit of full amplitude $\mathcal{M}(s,t)$.
- **Key**: **A few general properties of UV theory** (e.g., unitarity) highly constrain the allowed behavior of $\mathcal{M}(s,t)$, and also $\mathcal{M}(s,t)|_{\mathrm{low}}$.
- We focus on how heavy physics generates c_2. We ignore loops of ϕ.

Positivity bound (without gravity)

- Analytic structure of $\mathcal{M}(s,0)$. Consider a complex integral of $\mathcal{M}(s,0)/s^3$:



Contour deformation



$$\int_{\mathcal{C}_r} \frac{\mathrm{d}s}{2\pi i}\frac{\mathcal{M}(s,0)}{s^3} = \frac{2}{\pi}\int_{M^2}^{\infty} \mathrm{d}s \frac{\operatorname{Im}\mathcal{M}(s,0)}{s^3} + \oint_{\mathcal{C}_\infty} \frac{\mathrm{d}s}{2\pi i}\frac{\mathcal{M}(s,0)}{s^3} > \mathbf{0}.$$

$\mathbf{0}$

$= c_2$, EFT calculable

> 0 (Unitarity)

$\lim_{|s|\to\infty} \left|\mathcal{M}(s,0)/s^2\right| = 0\,.$

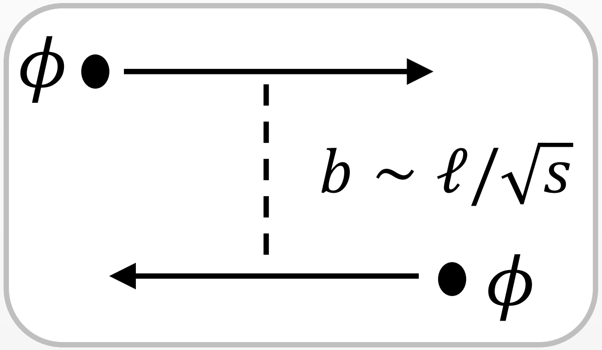
"Locality"(Froissart-Martin)

$\mathcal{M}(s,0) \sim c_0 s^0 + \boldsymbol{c_2} s^2 + \cdots$

Intuition

- **Unitarity & locality** ⇒ mild high-energy behavior:

$$\mathcal{M} \sim \sum_{\ell=0}^{\infty} (2\ell+1) f_\ell(s) P_\ell(\cos\theta) \quad < s^2$$



- **Unitarity:** $|f_\ell(s)| \leq 1$
- **Locality:** effective interaction radius $< M^{-1} \to |f_\ell(s)| \sim e^{-Mb}$

- **Causality** implies **analyticity** (maybe more nontrivial)
  - "**Signal model**": we have an initial signal $f_{\rm in}(t)$ and an out-signal $f_{\rm out}(t)$,

[Camanho-Edelstein-Maldacena-Zhiboedov+('14)]

$$f_{\rm out}(t) = \int_{-\infty}^{\infty} {\rm d}t'\, S(t-t') f_{\rm in}(t').$$

$$\Leftrightarrow \quad \tilde{f}_{\rm out}(\omega) = \tilde{S}(\omega)\tilde{f}_{\rm in}(\omega), \quad S(t) = \int_{-\infty}^{\infty} \frac{{\rm d}\omega}{2\pi} \tilde{S}(\omega) e^{-i\omega t}.$$

S-matrix element

- Causality: $S(t<0) = 0 \;\Rightarrow\; \tilde{S}(\omega)$ is analytic in UHP.

Positivity bound (without gravity)

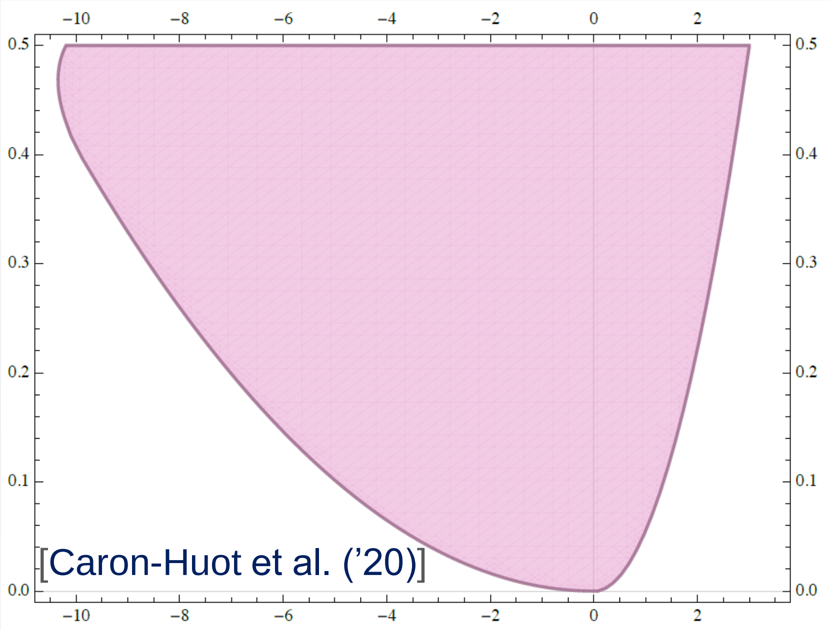
- A sum rule for c_2: $$\boldsymbol{c_2} = \frac{4}{\pi}\int_{M^2}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\,\mathcal{M}(s,0)}{s^3} > \mathbf{0}.$$

$$\mathcal{M}(s,t)\Big|_{\mathrm{low}} = (\mathrm{poles}) + \lambda + \underline{c_2}(s^2+t^2+u^2) + \underline{c_3}(stu) + \underline{c_4}(s^2+t^2+u^2)^2 + \cdots$$

- It is also possible to constrain $c_3, c_4, \cdots$ using crossing symmetries.

[Arkani-Hamed et al, ('21), Caron-Huot et al. ('20), Tollley et al.('20), Bellazzini et al.('20),…]

e.g.) Bounds in $\left(\frac{M^2 c_3}{c_2}, \frac{M^4 c_4}{c_2}\right)$-plane



- $\left|\frac{M^2 c_3}{c_2}\right|, \left|\frac{M^4 c_4}{c_2}\right| \lesssim \mathcal{O}(1) - \mathcal{O}(10).$
- c_2 cannot be much smaller than higher-order coeffs.

e.g.)
massive Galileon model: $M \sim m$.

[Tollley et al.('20]

Intuition for bounds

- The presence of such bounds are expected.

$$\mathcal{M}(s,t)\Big|_{\text{low}} = (\text{poles}) + \lambda + c_2(s^2+t^2+u^2) + c_3(stu) + c_4(s^2+t^2+u^2)^2 + \cdots$$
$$= (\text{poles}) + \lambda + c_2 s^2\left(1 - \frac{c_3}{c_2}t + \frac{4c_4}{c_2}t^2 + \cdots\right) + \cdots$$

- The process mediated by heavy states of mass $\sim M$ should be localized in the domain $L \lesssim M^{-1}$ in position space.

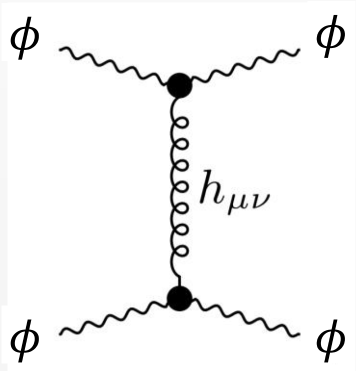
c.f.) $\widetilde{\mathcal{M}}(s,b) \sim e^{-Mb}$ @ $b \gtrsim M^{-1}$

- The form of derivative expansion: $\sim (L^2\partial^2)^{\#} \Leftrightarrow (L^2 t)^{\#}$

Extension to Gravity?

- Quantum gravity S-matrix: **not fully understood.** c.f.) [Häring+('22)]
- Feature: t-channel graviton exchange **grows as fast as s^2,**

$$\mathcal{M}(s,t) \ni$$



$$\sim \frac{s^2}{M_{\rm pl}^2 t}$$

- **Assume the mild UV behavior**: $\lim_{|s|\to\infty} \left|\mathcal{M}(s,t)s^{-2}\right| = 0$ for $t < 0$

  c.f.) tree-level string: $\mathcal{M} \sim s^{2+\alpha' t}/M_{\rm pl}^2 t$

- We can then derive a sum rule for c_2.

  Note: We assume the weakly-coupled UV completion of gravity so that we can ignore graviton loops and work up to $\mathcal{O}\left(M_{\rm pl}^{-2}\right)$.

Positivity bounds **with** Gravity (1/2)

- The sum rule for c_2 contains **graviton t-channel pole:**

$$c_2 = \lim_{t\to 0^-}\left\{\int_{M^2}^{\infty} ds \frac{\mathrm{Im}\,\mathcal{M}(s,t)}{s^3} + \frac{\mathbf{1}}{\boldsymbol{M_{\mathrm{pl}}^2 t}}\right\} = \quad "\infty - \infty" \overset{?}{>} \mathbf{0}$$

$$\mathcal{M}(s,t) \sim (s,t,u\ \text{poles}) + c_2\, s^2 + \cdots \quad \ni \left[\frac{-1}{M_{\mathrm{pl}}^2 t} + c_2\right] s^2$$

- One solution: consider sum rules **away from $t = 0$ limit.**

[Caron-Huot+ ('21)]

✓ Perform smearing in t to localize the sum rule in the domain $t \gtrsim -M^2$. Equivalent to work in impact parameter $b \lesssim M^{-1}$.

✓ IR div in D=4: $c_2 > -50\left(M_{\mathrm{pl}} M\right)^{-2} \log(0.3 M b_{\mathrm{IR}})$.

Positivity bounds **with** Gravity (1/2)

- The sum rule for c_2 contains **graviton t-channel pole:**

$$c_2 = \lim_{t\to 0^-} \left\{ \int_{M^2}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\,\mathcal{M}(s,t)}{s^3} + \frac{\mathbf{1}}{\boldsymbol{M_{\mathrm{pl}}^2 t}} \right\} = \quad \text{"}\infty - \infty\text{"} \overset{?}{>} 0$$

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- One solution: consider sum rules **away from $t = 0$ limit.**

[Caron-Huot+ ('21)]

- Another solution: **The graviton t^{-1} pole is canceled due to the Regge behavior** $\mathrm{Im}\,\mathcal{M} \simeq f(t) s^{2+\alpha' t+\alpha'' t^2/2\cdots} @\ s \gg M^2$.

[JT-Aoki-Hirano ('20)]

Positivity bounds with Gravity (1/2)

- The sum rule for c_2 contains **graviton t-channel pole:**

$$c_2 = \lim_{t\to 0^-}\left\{\int_{M^2}^{\infty} ds \frac{\mathrm{Im}\,\mathcal{M}(s,t)}{s^3} + \frac{\mathbf{1}}{\boldsymbol{M_{\mathrm{pl}}^2 t}}\right\} = \quad \text{"}\infty-\infty\text{"} \overset{?}{>} 0$$

$$\int_{s_*\gg M^2}^{\infty} ds \frac{\mathrm{Im}\,\mathcal{M}(s,t)}{s^3} \sim f(t)\int_{s_*\gg M^2}^{\infty} ds \frac{s^{2+\alpha' t+\cdots}}{s^3} = \frac{\boldsymbol{-f(0)s_*}}{\boldsymbol{\alpha' t}} + \mathcal{O}(t^0).$$

- Another solution: **The graviton t^{-1} pole is canceled due to the Regge behavior** $\mathrm{Im}\,\mathcal{M} \simeq f(t)s^{2+\alpha' t+\alpha'' t^2/2\cdots}$ @ $s \gg M^2$.

[JT-Aoki-Hirano ('20)]

Positivity bounds **with** Gravity (2/2)

(Related discussions: [Hamada+('18 '23)] [Herrero-Valea+('20)] [Bellazzini+('19)] [Alberte+('20,'21)])

- **Gravitational positivity bound:** [JT-Aoki-Hirano ('20)]

$$c_2 = \underbrace{\int_{M^2}^{s_*} \mathrm{d}s \frac{\mathrm{Im}\,\mathcal{M}(s,0)}{s^3}}_{>0\text{ (unitarity)}} + \frac{1}{M_{\rm pl}^2}\left[\underbrace{-\frac{2\partial_t f(t)|_{t=0}}{f(0)}}_{<0\text{ (unitarity!)}} + \frac{\alpha''}{\alpha'}\right]$$

- **Negative term: depends on details of the Regge behavior.**
- $c_2 = 0$ **is allowed.**
- **Can we constrain the size of negativity ?**

➡ **Yes!** (by using the "finite energy sum rule") [Noumi-JT ('22)]

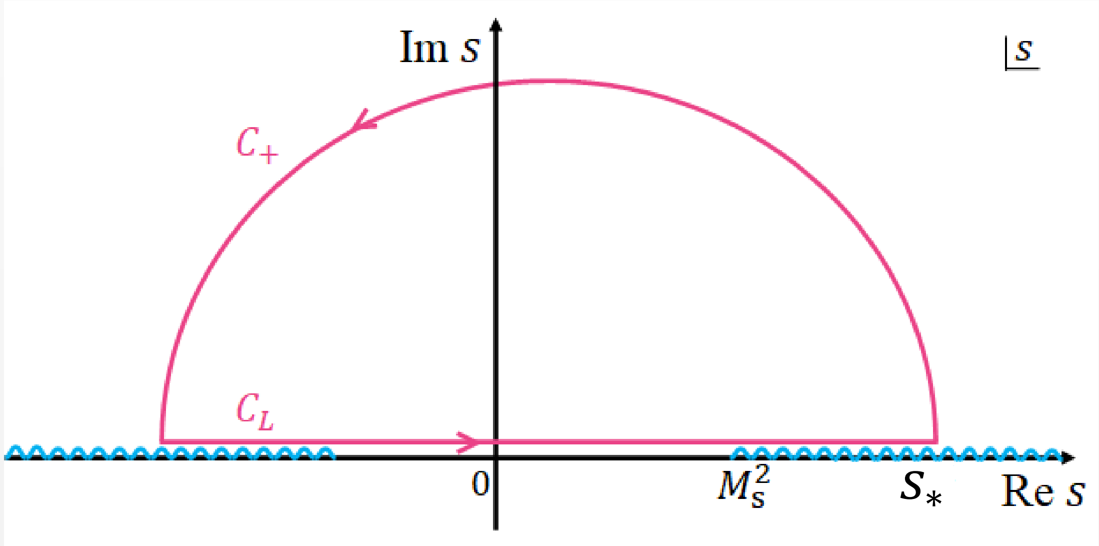
Intuition: t-dependence $\Rightarrow f'/f \sim \mathcal{O}(M^{-2})$.

c.f.) [de Rham + ('22)] which appeared on arXiv in the same week, also discusses FESR!

Finite energy sum rules

[Noumi-JT ('22)]

- We consider the scattering of massless identical scalar.
- We have $\int_{C_+ + C_L} \frac{\mathrm{d}s}{2\pi i} (s+t/2)^{2n+1} \mathcal{M}(s,t) = 0 \quad (n = 0,1,2,\ldots)$.



* We ignored s, u-channel poles of light particles.

- We assume $\int_{C_+} \frac{\mathrm{d}s}{2\pi i} (\cdots) \mathcal{M}(s,t) \simeq \int_{C_+} \frac{\mathrm{d}s}{2\pi i} (\cdots) \mathcal{M}_{\mathrm{R}}(s,t)$,

$$\mathcal{M}_{\mathrm{R}} = \frac{-f(t)\left[e^{-i\pi\alpha(t)} + 1\right]}{\sin \pi\alpha(t)} (s/s_*)^{\alpha(t)}, \quad \alpha(t) = 2 + \alpha' t + \alpha'' t^2/2 + \cdots.$$

Finite energy sum rules

[Noumi-JT ('22)]

- **Finite energy sum rules (FESRs)** [for $n = 0,1,2, \ldots$]

$$\frac{f(t)}{\alpha(t)+2n+2} = \underbrace{\frac{1}{(s_* + t/2)^{2n+2}} \int_{M_s^2}^{s_*} \mathrm{d}s\, (s+t/2)^{2n+1}\, \mathrm{Im}\, \mathcal{M}(s,t)}_{=: S_{2n+1}(t)} .$$

- FESRs directly connect Regge parameters with "infrared" physics $s \leq s_*$!

✓ We can derive FESRs for $f(t)$ and $\alpha(t)$. For instance,

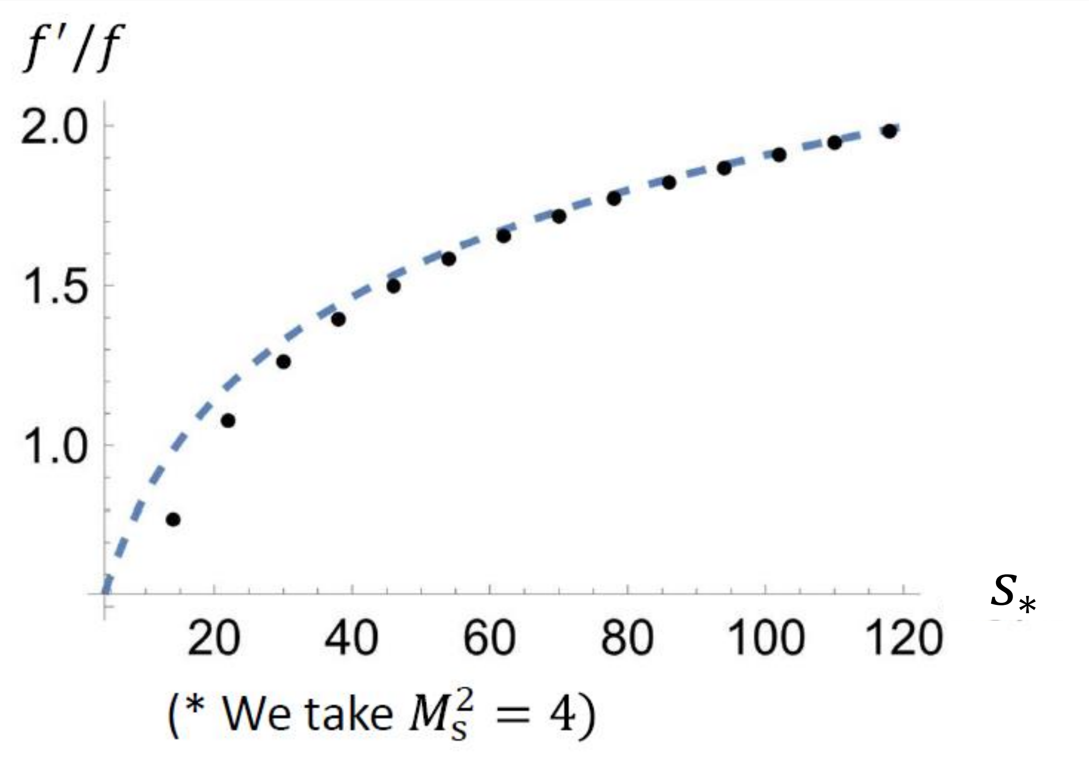
$$f'(0) = \frac{2}{n-m}\left[(n+2)^2 S'_{2n+1}(0) - (m+2)^2 S'_{2m+1}(0)\right] \quad (n, m = 0,1,2, \ldots).$$

FESR test: examples

[Noumi-JT ('22)]

$$\mathrm{Im}\,\mathcal{M}_{\text{type II}}(s,t)|_{s\gg 4,\, t\sim 0} \simeq \frac{256}{[\Gamma(1+t/4)]^2}\left(\frac{s+t/2}{4}\right)^{2+t/2}.$$

$$f(t) = \frac{256}{\left[\Gamma\left(1+\frac{t}{4}\right)\right]^2}\left(\frac{s_* + t/2}{4}\right)^{2+t/2}$$



Key Idea

[Noumi-JT ('22)]

$$\frac{f(t)}{\alpha(t)+2n+2} = \frac{1}{(s_*+t/2)^{2n+2}}\int_{M_S^2}^{s_*} \mathrm{d}s\,(s+t/2)^{2n+1}\,\mathrm{Im}\,\mathcal{M}(s,t)\,.$$

- FESRs were useful in the context of strong interactions.

  [Igi (1962), Dolen+(1967,68), Ademollo+(1967,68)…]

  e.g.) ・Experimental inputs for the RHS ⇒ Constraints on LHS.

- Our case: no experimental input for $\mathrm{Im}\,\mathcal{M}(s,0)$ with $s > M_S^2$.

- But, we have a theoretical input !! **"Null constraints"**

  … implied by **crossing symmetry.**

  [Arkani-Hamed+('19, '21), Bellazzini+ ('20), Tolley+ ('20), Caron-Huot+('20)]

Null constraints

[Noumi-JT ('22)]

- Low-energy expansion of $\mathcal{M}(s,t)$ reads

$$\mathcal{M}(s,t) - (\text{poles}) = \lambda + c_2(s^2+t^2+u^2) + c_3(stu) + c_4(s^2+t^2+u^2)^2 + \cdots$$
$$= \lambda + 2c_2(s^2+st+t^2) - c_3(s^2t+st^2) + c_4(s^4+2s^3t+3s^2t^2+\cdots) + \cdots$$

➡ **(s^4-coefficient) = 3 × (s^2t^2-coefficient)**

- This is nontrivial if both sides are written in terms of sum rules.

e.g.) $0 = \int_{M_s^2}^{\infty} \frac{\mathrm{d}s}{s^8} \sum_{\text{even } J} (2J+1)\rho_J(s) G(\mathcal{J}^2), \quad G(\mathcal{J}^2) := \mathcal{J}^2(\mathcal{J}^2-6)(2\mathcal{J}^2-49).$

* $\rho_J \geq 0$: imaginary part of partial wave amplitude

* $\mathcal{J}^2 := J(J+1)$.

J	0	2	4	$J \geq 6$
$G(\mathcal{J}^2)$	0	0	-2520	> 0

" (Low-J contributions) = (Large-J contributions) "

Null constraints

[Noumi-JT ('22)]

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- How can we implement the null constraints to FESRs?

✓ FESRs: $f' = \int_{M_s^2}^{s_*}(\cdots)\,\mathrm{d}s$, Null constraints: $0 = \int_{M_s^2}^{\infty}(\cdots)\,\mathrm{d}s$.

IR part of null constraints

[Noumi-JT ('22)]

- Focus on the following null constraint.

$$0 = \int_{M_s^2}^{\infty} \frac{\mathrm{d}s}{s^8} \sum_{\text{even } J} (2J+1)\rho_J(s) G(\mathcal{J}^2), \quad G(\mathcal{J}^2) := \mathcal{J}^2(\mathcal{J}^2 - 6)(2\mathcal{J}^2 - 49).$$

J	0	2	4	$J \geq 6$
$G(\mathcal{J}^2)$	0	0	-2520	> 0

- (IR part) + (UV part) = 0

$$(\text{IR part}) := \int_{M_s^2}^{s_*} \frac{\mathrm{d}s}{s^8} \sum_{\text{even } J} (2J+1)\rho_J(s) G(\mathcal{J}^2),$$

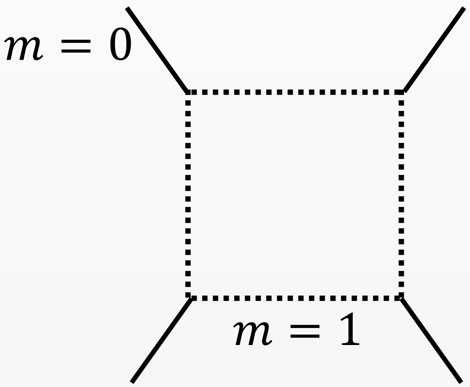
- IR part: only low-J states can be excited for which $G(\mathcal{J}^2) < 0$. So, we will have

$$(\text{IR part}) \leq 0, \qquad (\text{UV part}) \geq 0.$$

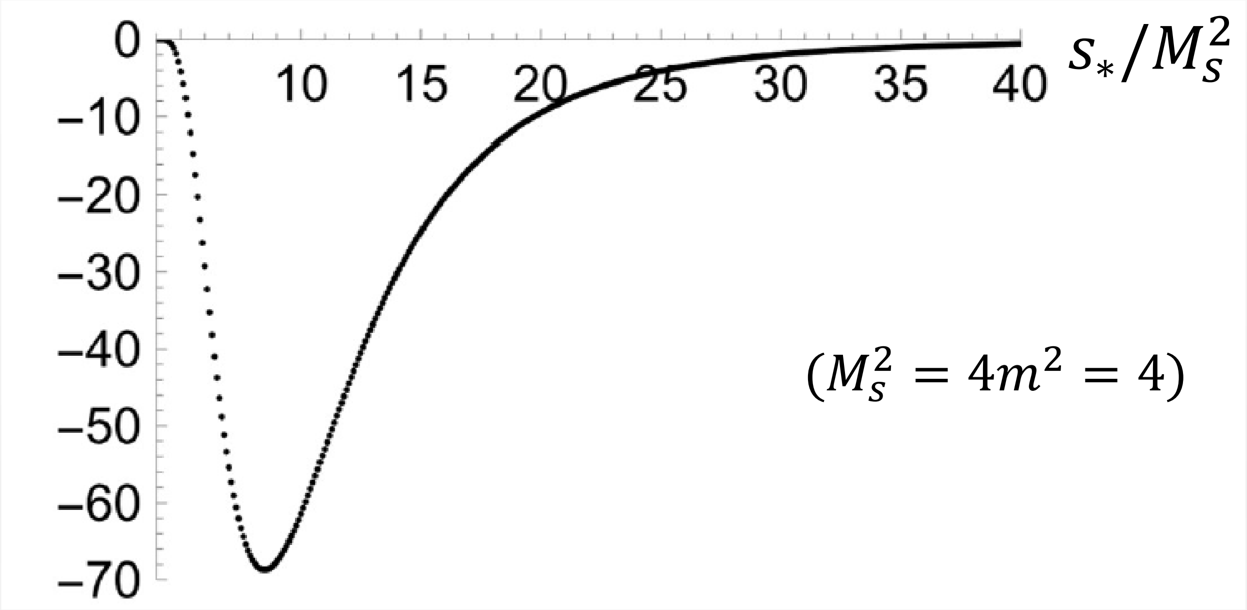
IR part of null constraints

[Noumi-JT ('22)]

- Example: scalar box amplitude.
  - ✓ External: massless scalar
  - ✓ Internal: massive scalar with mass $m = 1$



$10^9 \times$ (IR part)



IR part of null constraints

[Noumi-JT ('22)]

- Another derivation:

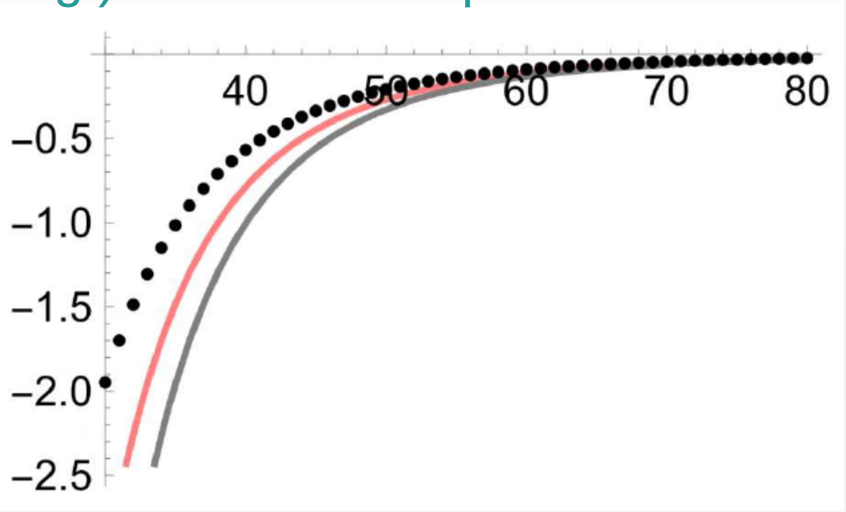
$$(\text{IR part}) = \int_{M_S^2}^{s_*} \frac{\mathrm{d}s}{s^8}(\cdots) = -\int_{s_*}^{\infty} \frac{\mathrm{d}s}{s^8} \sum_{n=1}^{3} c_{3,n}(s\partial_t)^n \,\mathrm{Im}\mathcal{M}(s,t)\Big|_{t=0}$$

$$(c_{3,3}, c_{3,2}, c_{3,1}) = (12, -90, 180).$$

- For sufficiently large s_*, n=3 term will be dominant:

$$(\text{IR part}) \simeq -c_{3,3}\int_{s_*}^{\infty} \frac{\mathrm{d}s}{s^8}(s\partial_t)^3 \mathrm{Im}\mathcal{M}(s,t)\Big|_{t=0} \leq 0.$$

e.g.) scalar box amplitude.



Use of IR part of null constraints

[Noumi-JT ('22)]

- Add the IR part of null constraint to derive an upper bound.

$$f' = \int_{M_S^2}^{s_*} ds \frac{1}{s_*^2}\left[18\left(\frac{s}{s_*}\right)^2 - 8\right]\sum_{\text{even } J}(2J+1)\rho_J(s)\mathcal{J}^2$$

$$\leq \int_{M_S^2}^{s_*} ds \frac{1}{s_*^2}\left[18\left(\frac{s}{s_*}\right)^2 - 8\right]\sum_{\text{even } J}(2J+1)\rho_J(s)\mathcal{J}^2 - \beta \times (\text{IR part}) \qquad [\beta > 0]$$

$$= \sum_{\text{even } J}(2J+1)\int_{M_S^2}^{s_*}\frac{ds}{s}\rho_J(s)\left(\frac{s}{s_*}\right)^2\left\{\left[18\left(\frac{s}{s_*}\right)^2 - 8\right]\frac{\mathcal{J}^2}{s} - \frac{\beta}{s^8}\left(\frac{s_*}{s}\right)^2 G(\mathcal{J}^2)\right\}$$

$\leq I(\beta)$ $(\because G(\mathcal{J}^2) \approx 2\left(\mathcal{J}^2\right)^3$ at $J \gg 1)$

$$\leq I(\beta)\sum_{\text{even } J}(2J+1)\int_{M_S^2}^{s_*}\frac{ds}{s}\rho_J(s)\left(\frac{s}{s_*}\right)^2 = I(\beta)\frac{f}{4}$$

$= f/4 \Leftarrow$ (FESR for f)

- We choose β to optimize the bound.

Results

[Noumi-JT ('22)]

- We confirm that the Regge parameters $f(t)$ **and** $\alpha(t)$ **are governed by the scales of higher-spin tower** M_s and α', ignoring loops of light particles.

$$f'/f < 9.1 \times 10^2 M_s^{-2} \qquad \alpha''/\alpha' > -2f'/f - 2.4 \times 10^5 M_s^{-4}/\alpha'$$

$$c_2 > -M_{\rm pl}^{-2} M_s^{-2} \left[3.7 \times 10^3 + 2.4 \times 10^5 \left(M_s^2 \alpha'\right)^{-1}\right]$$

* We choose $s_* = 10M_s^2$ as a benchmark point in this talk.

- **IR finite grav. positivity bounds in D=4** dimensions!
- **New bounds** on gravitational Regge parameters.
- An extension to higher-D is straightforward.

Comparisons

[Noumi-JT ('22)]

- Our bounds are easily satisfied by string amplitude.

$$\frac{f'}{f} < \frac{10^2}{M_s^2} \times \{3.0, 2.2, 1.8, 1.5, 1.4, 1.3, 1.2\} \qquad (D = 4,5,6,\cdots,10)$$

$$\left.\frac{f'}{f}\right|_{\text{type-II}} \simeq \frac{5.86}{M_s^2}$$

- In general, we expect $c_2 > \frac{-\mathcal{O}(1-10)}{M_{\text{pl}}^2 M_s^2}$. This is proven in [Caron-Huot+('21)] in higher dimensions $D > 4$.
- However, the finite bound on c_2 in $D = 4$ was not known.
- Stronger bound on c_2 and bounds on other parameters such as c_3, c_4 will exist: ***future work.***

Summary & Prospects

- Positivity bounds are useful to reveal UV-IR correlations.

  - In non-grav case, EFT parameters must be within compact regions.

- We derived **gravitational positivity bounds** based on FESR.

  - A finite bound in $\boldsymbol{D = 4}$ spacetime dims.

- Future works include:

  - Bounds on other EFT parameters, stronger bound.

  - Application to modified gravity models, curved background effects.

  - Bounds without using Regge behavior (as done in [Caron-Huot+(‘21)]).