Holographic dark energy for inflationary cosmology and the trans-Planckian censorship conjecture

Main reference: Physical Review D **103**, 083505 (2021)

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Fukushima University

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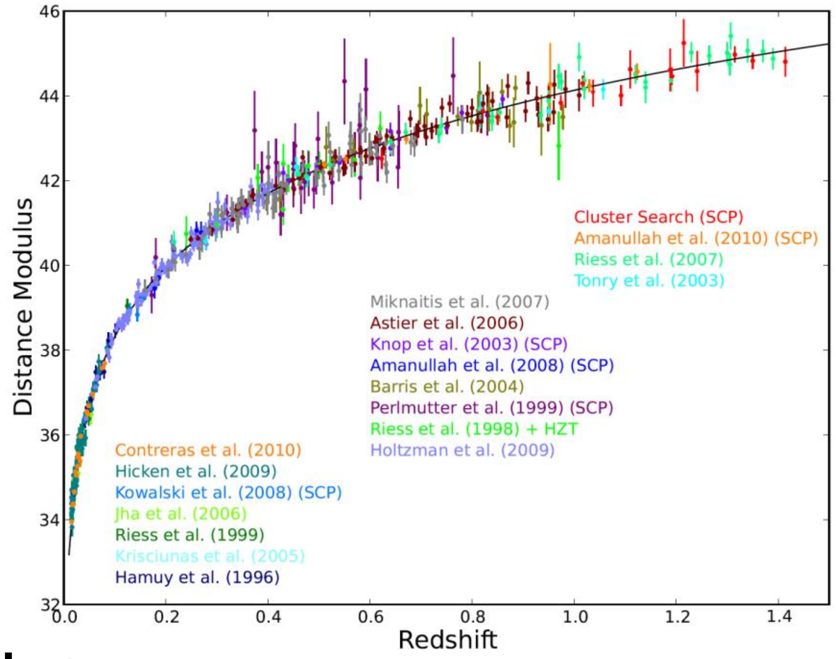
I. Introduction

- According to the observations of Type Ia Supernovae, the current expansion of the universe is accelerating ("Dark Energy Problem")



[N. Suzuki *et al*. [Supernova Cosmology Project Collaboration], Astrophys. J. **746**, 85 (2012)]

- The universe is considered to be spatially flat.

⟶ Two main approaches

(1) To introduce dark energy within General Relativity (GR)

(2) To extend gravity theories

Reviews: E.g., [Copeland, Sami and Tsujikawa, Int. J. Mod. Phys. D **15**, 1753 (2006)]

[Nojiri and Odintsov, Phys. Rept. **505**, 59 (2011)]

[KB, Capozziello, Nojiri, Odintsov, Astrophys. Space Sci. **342**, 155-228 (2012)]

Candidates of dark energy and modified gravity theories

(1) Candidates of dark energy

▪ Cosmological constant ▪ Scalar field (Quintessence) ▪ Fluid description

▪ Horographic dark energy

[Hsu, Phys. Lett. B **594**, 13 (2004)]
[Horvat, Phys. Rev. D **70**, 087301 (2004)]
[M. Li, Phys. Lett. B **603**, 1 (2004)]

Holographic principle:

Entropy of the system is measured by area and not volume.

[G. 't Hooft, Conf. Proc. C **930308**, 284 (1993), gr-qc/9310026]

(2) Candidates of modified gravity theories

▪ $f(R)$ gravity

$f(R)$: Arbitrary function of the Ricci scalar R

▪ DGP braneworld scenario

▪ Galileon gravity

▪ Horndeski theory ▪ DHOST

▪ Massive gravity ▪ Bimetric gravity

Beyond

GR

Relation between holographic entropy and dark energy

Generalization of black hole entropy

$$S = \gamma A^{\delta}$$

[Tsallis and Cirto, Eur. Phys. J. C **73**, 2487 (2013)]

M_p : Planck mass

$$A = 4\pi L^2$$

L : Cut-off scale of the long scale (IR)

γ : Constant

$\gamma = 1/(4G)$, $\delta = 1$ → Bekkenstein-Hawking entropy

Λ : Cut-off scale of the small scale (UV)

$$L^3\Lambda^3 \leq S^{3/4}$$

$$\Lambda^4 \leq \gamma\,(4\pi)^{\delta}\,L^{2\delta-4}$$

Λ^4 : Energy density of the vacuum

[Cohen, Kaplan, and Nelson, Phys. Rev. Lett. **82**, 4971 (1999)]

(i) **Application to entropy of the cosmic horizon**

(ii) $\rho_{DE} L^4 \leq S$

※ Λ^4 is taken to be the dark energy density ρ_{DE}.

[M. Li, Phys. Lett. B **603**, 1 (2004)]

[S. Wang, Y. Wang and M. Li, Phys. Rept. **696**, 1 (2017)]

⇨ **Dark energy density**

$$\rho_{DE} = BL^{2\delta-4}$$

$B \equiv \gamma\,(4\pi)^{\delta} c^2$ (Dimension : $M^{2\delta}$)

$\delta = 1 \rightarrow \rho_{DE} = 3c^2 M_p^2 L^{-2}$, $\delta = 2 \rightarrow \rho_{DE} = const.$ (Cosmological constant)

c : Speed of licht in the vacuum

Holographic dark energy

[Saridakis, KB, Myrzakulov, Anagnostopoulos, JCAP **1812**, 012 (2018)]

- **Dark energy density** $\rho_{DE} = BL^{2\delta-4}$

  B : constant（dimension : $[L]^{-2\delta}$）

- Homogeneous and isotropic universe

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

$a(t)$: Scale factor

$H \equiv \dot{a}/a$: Hubble parameter

（`dot’ : Time derivative）

- **Future event horizon**
  [M. Li, Phys. Lett. B **603**, 1 (2004)]

$$R_h \equiv a\int_t^\infty \frac{dt}{a} = a\int_a^\infty \frac{da}{Ha^2} \longrightarrow \rho_{DE} = BR_h^{2\delta-4}$$

- **Friedmann equation**

$$3M_p^2 H^2 = \rho_m + \rho_{DE}$$
$$-2M_p^2 \dot{H} = \rho_m + p_m + \rho_{DE} + p_{DE}$$

$\rho_m,\ p_m$: Matter energy density, pressure

p_{DE} : Dark energy pressure

- **Energy conservation**

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0\,, \quad \dot{\rho}_{DE} + 3H\rho_{DE}(1+w_{DE}) = 0$$

▪ Density parameters

$$\Omega_{DE} \equiv \frac{1}{3M_p^2H^2}\rho_{DE}\,, \quad \Omega_m \equiv \frac{1}{3M_p^2H^2}\rho_m$$

$$\rho_m = \rho_{m0}/a^3$$

$$a_0 = 1$$

$$\Omega_m = \Omega_{m0}H_0^2/(a^3H^2)$$

$$\Omega_m + \Omega_{DE} = 1$$

※`0’: present value

※Matter: dust

Future event horizon

$$\int_x^\infty \frac{dx}{Ha} = \frac{1}{a}\left(\frac{B}{3M_p^2H^2\Omega_{DE}}\right)^{\frac{1}{4-2\delta}} \qquad x \equiv \ln a$$

$$\frac{1}{Ha} = \frac{\sqrt{a(1-\Omega_{DE})}}{H_0\sqrt{\Omega_{m0}}}$$: Friedmann equation

$$\int_x^\infty \frac{dx}{H_0\sqrt{\Omega_{m0}}}\sqrt{a(1-\Omega_{DE})} = \frac{1}{a}\left(\frac{B}{3M_p^2H^2\Omega_{DE}}\right)^{\frac{1}{4-2\delta}}$$

※`prime’ : derivative with respect to x

⇨ **Differential equation of the cosmic evolution of dark energy**

$$\frac{\Omega'_{DE}}{\Omega_{DE}(1-\Omega_{DE})} = 2\delta - 1 + Q(1-\Omega_{DE})^{\frac{1-\delta}{2(2-\delta)}}(\Omega_{DE})^{\frac{1}{2(2-\delta)}}e^{\frac{3(1-\delta)}{2(2-\delta)}x}$$

$$Q \equiv 2(2-\delta)\left(\frac{B}{3M_p^2}\right)^{\frac{1}{2(\delta-2)}}\left(H_0\sqrt{\Omega_{m0}}\right)^{\frac{1-\delta}{\delta-2}}$$

Conservation of dark energy

$w_{DE} \equiv p_{DE}/\rho_{DE}$: Equation of state of dark energy

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1+w_{DE}) = 0$$

$$\dot{\rho}_{DE} = 2(\delta-2)BR_h^{2\delta-5}\dot{R}_h, \qquad \dot{R}_h = HR_h - 1$$

$$\longrightarrow \quad 2(\delta-2)B\left(\frac{\rho_{DE}}{B}\right)^{\frac{2\delta-5}{2(\delta-2)}}\left[H\left(\frac{\rho_{DE}}{B}\right)^{\frac{1}{2(\delta-2)}} - 1\right] + 3H\rho_{DE}(1+w_{DE}) = 0$$

⇒

Equation of state of dark energy

$$w_{DE} = \frac{1-2\delta}{3} - \frac{Q}{3}(\Omega_{DE})^{\frac{1}{2(2-\delta)}}(1-\Omega_{DE})^{\frac{\delta-1}{2(\delta-2)}}e^{\frac{3(1-\delta)}{2(\delta-2)}x}$$

Deceleration parameter

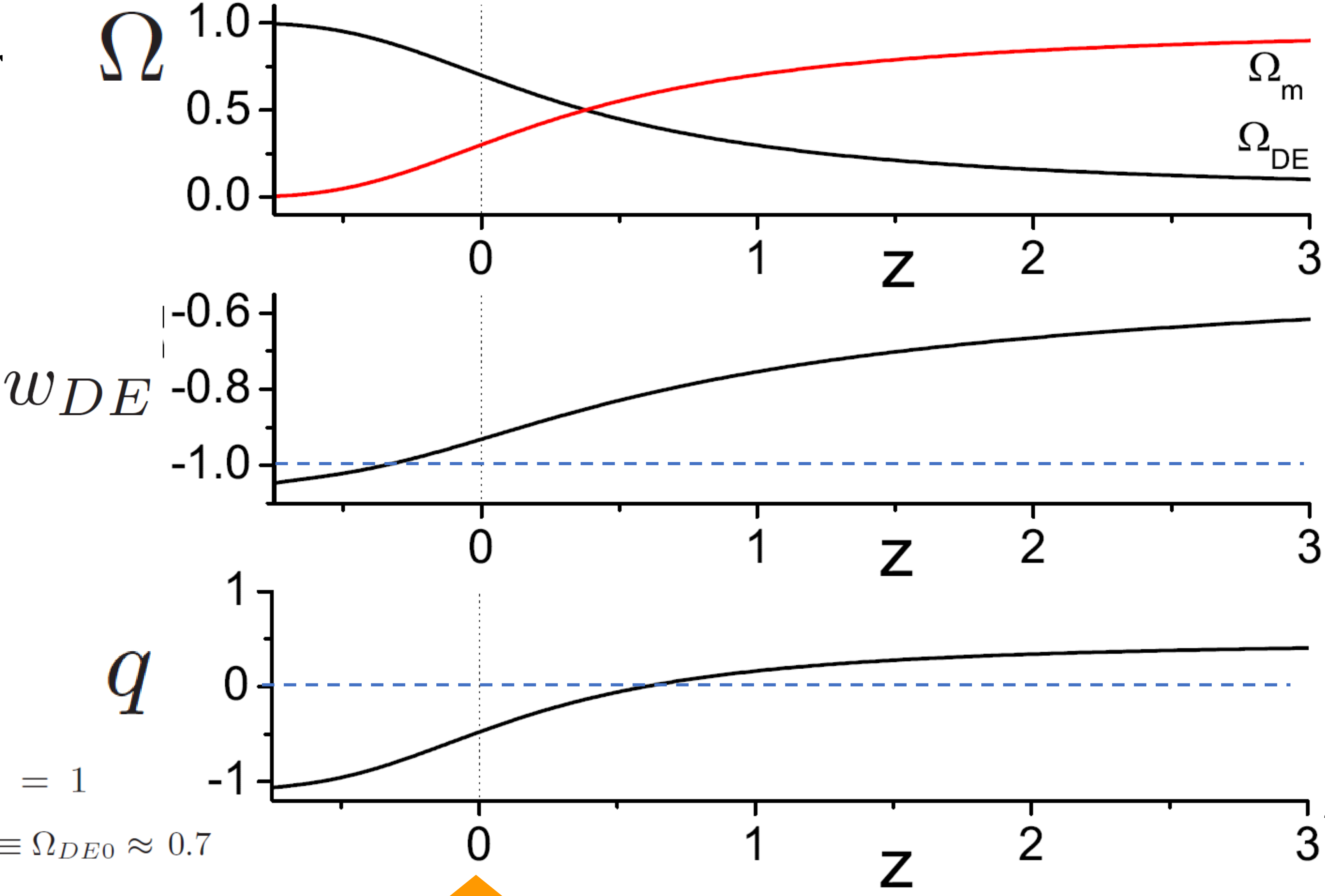
$$q \equiv -1 - \frac{\dot{H}}{H^2} = \frac{1}{2} + \frac{3}{2}(w_m\Omega_m + w_{DE}\Omega_{DE})$$

Evolution of cosmological parameters

- Density parameter
- Equation of state of dark energy
- Deceleration parameter

$\delta = 1.1, \ B = 3, \ M_p^2 = 1$

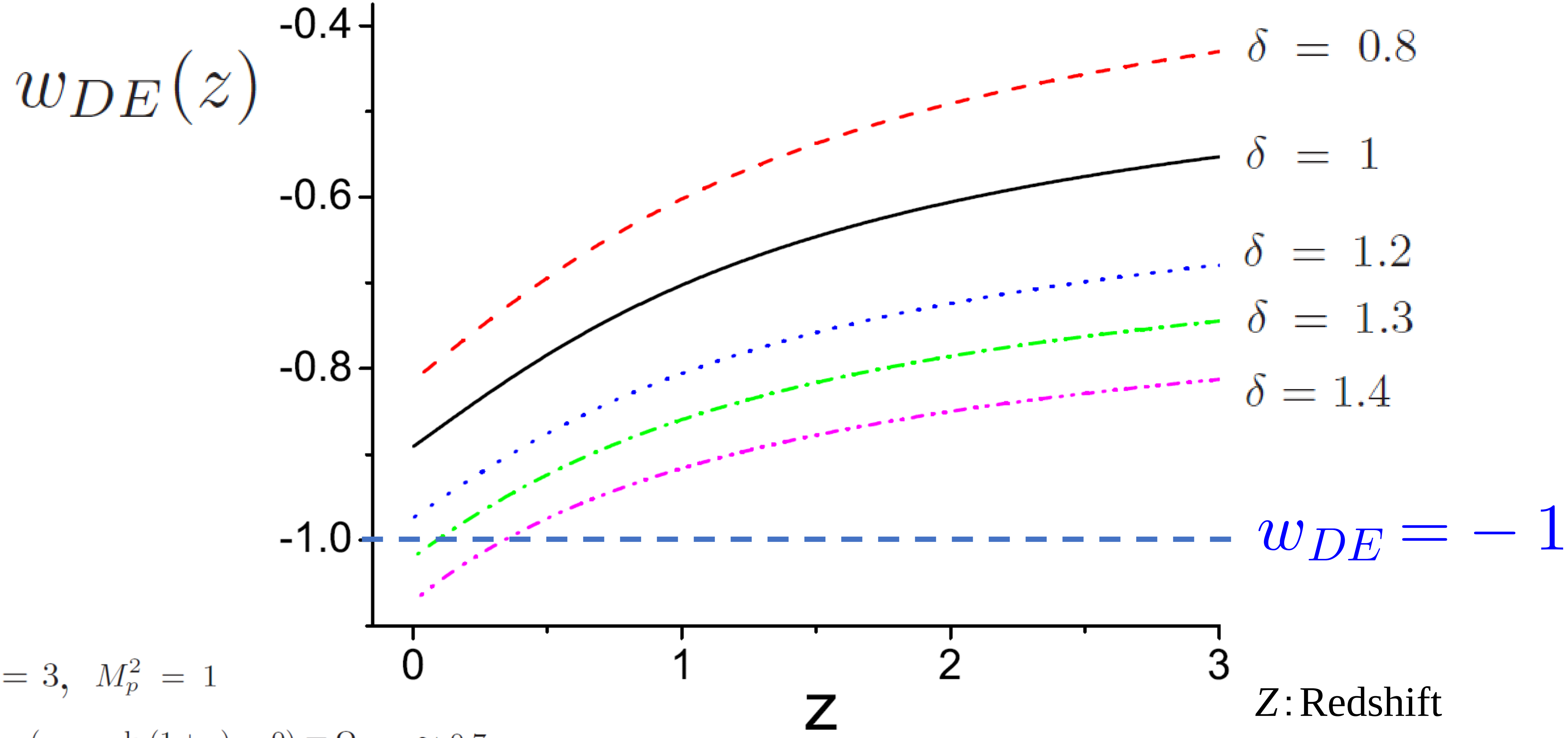
$\Omega_{DE}(x = -\ln(1+z) = 0) \equiv \Omega_{DE0} \approx 0.7$



Z : Redshift

The late-time cosmic acceleration is realized.

Evolution of the equation of state of dark energy



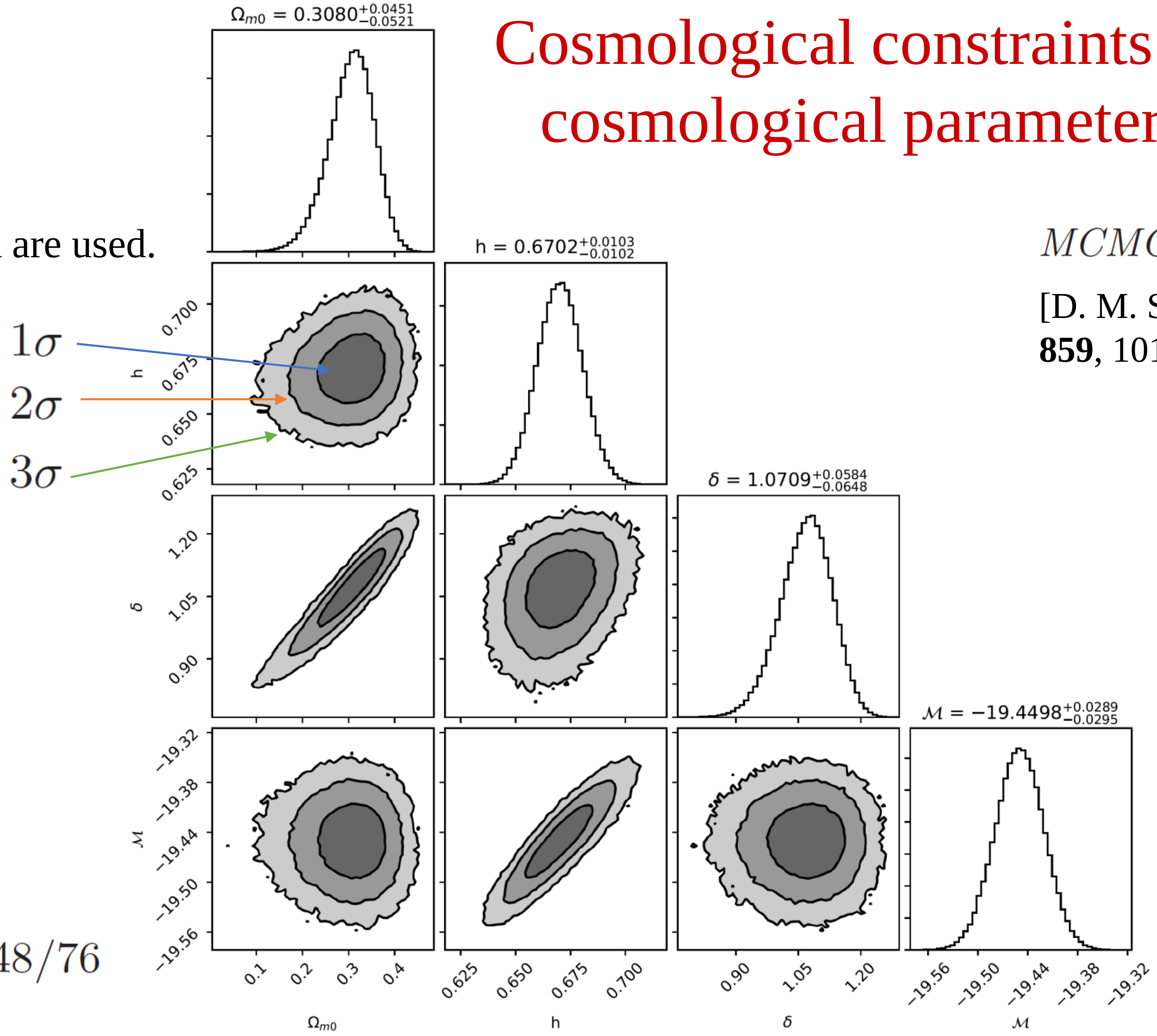
$B = 3, \; M_p^2 = 1$

$\Omega_{DE}(x = -\ln(1+z) = 0) \equiv \Omega_{DE0} \approx 0.7$

Z: Redshift

Cosmological constraints on cosmological parameters

$SNIa$, $H(z)$ data are used.



$$\chi^2_{min}/dof = 43.248/76$$

$MCMC\ chain$

[D. M. Scolnic et al., Astrophys. J. **859**, 101 (2018)]

Stability analysis

Squared sound speed

[Tavayef, Sheykhi, KB, Moradpour, Phys. Lett. B **781**, 195-200 (2018)]

$$v_s^2 = \frac{dP_D}{d\rho_D} = \frac{\dot{P}_D}{\dot{\rho}_D} = \frac{\rho_D}{\dot{\rho}_D}\ \dot{w}_D + w_D$$

$$\frac{\rho_D}{\dot{\rho}_D} = \frac{-1}{3H}\left(\frac{1-(2-\delta)\Omega_D}{2-\delta-(2-\delta)\Omega_D}\right)$$

: For the case that the IR cutoff is the Hubble horizon

$\longrightarrow$ $\rho_D = BH^{-2\delta+4}$

$$w_D = \frac{(2-\delta)(1-\delta)H\Omega_D'}{[1-(2-\delta)\Omega_D]^2}\ , \quad \dot{\Omega}_D = H\Omega_D'\ , \quad \Omega_D' = \frac{d\Omega_D}{d(\ln a)}$$

$$\Rightarrow \boxed{v_s^2 = \frac{(\delta-1)(\Omega_D-1)}{[1-(2-\delta)\Omega_D]^2}}$$

$$0 < \Omega_D < 1$$

$\delta \leq 1 \longrightarrow v_s^2 \geq 0$: **Stable against perturbations**

Stability analysis (2)

$$\rho_D = (\alpha H^2 + \beta \dot{H})^{-\delta+2}$$

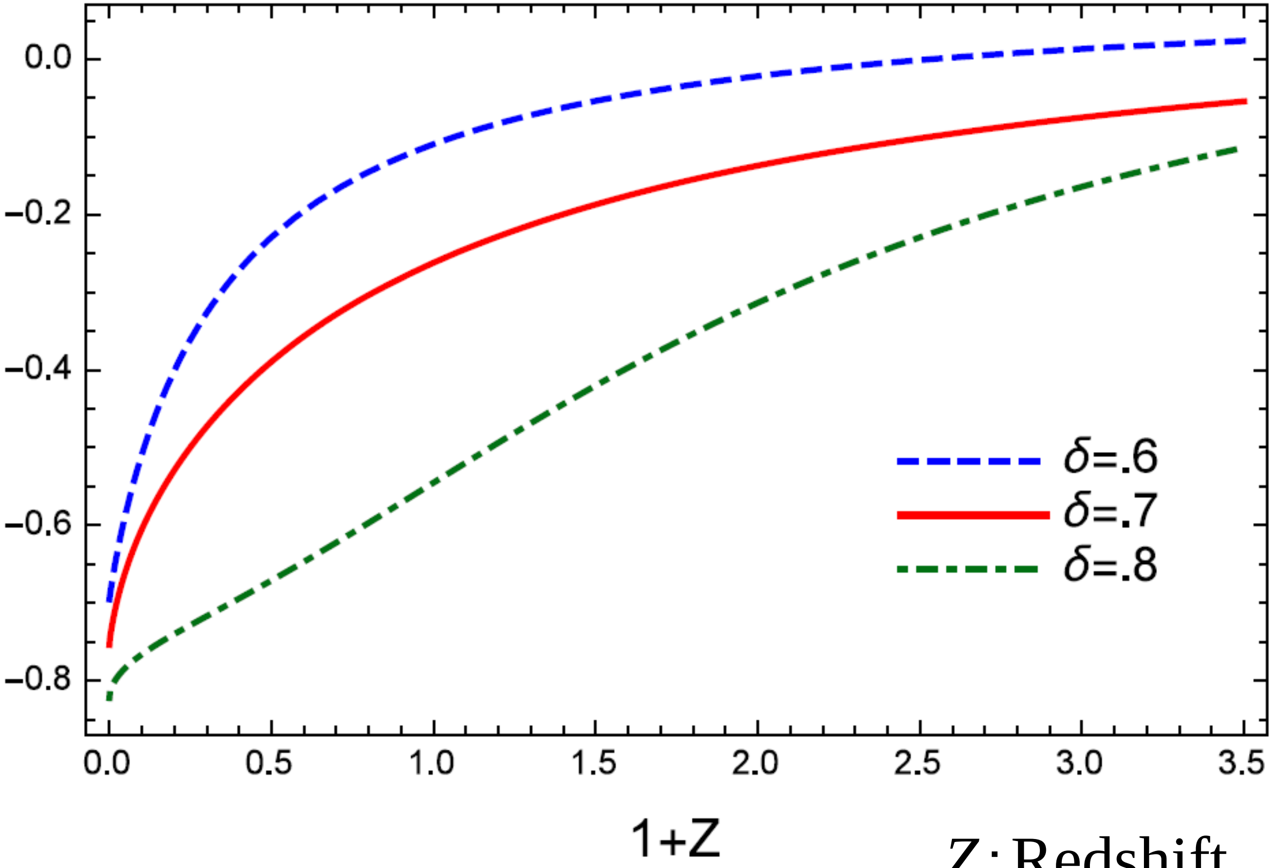
$\alpha,\ \beta$: dimensionless constant

[Granda and Oliveros, Phys. Lett. B **669**, 275 (2008); Phys. Lett. B **671**, 199 (2009)]

v_s^2

$$v_s^2 = -1 + \frac{2\alpha}{3\beta} + 2\frac{3^{\frac{-1+\delta}{2-\delta}}\ H^{\frac{2-2\delta}{-2+\delta}}\ \Omega_D{}^{\frac{-1+\delta}{2-\delta}}}{\beta(-2+\delta)} - \frac{(2\alpha-3\beta)\ H^{\frac{2\delta}{-2+\delta}}(-1+\Omega_D)}{-2*3^{\frac{1}{2-\delta}}\ H^{\frac{2}{-2+\delta}}\ \Omega_D{}^{\frac{1}{2-\delta}} + H^{\frac{2\delta}{-2+\delta}}[2\alpha-3\beta+3\beta\Omega_D]}$$

[Abdollahi Zadeh, Sheykhi, Moradpour, KB, Eur. Phys. J. C **78**, 940 (2018)]



Z: Redshift

$\Omega_D^0 = 0.73, \alpha = 0.8, \beta = 0.5,\ H_0 = 67$

Stability analysis (3)

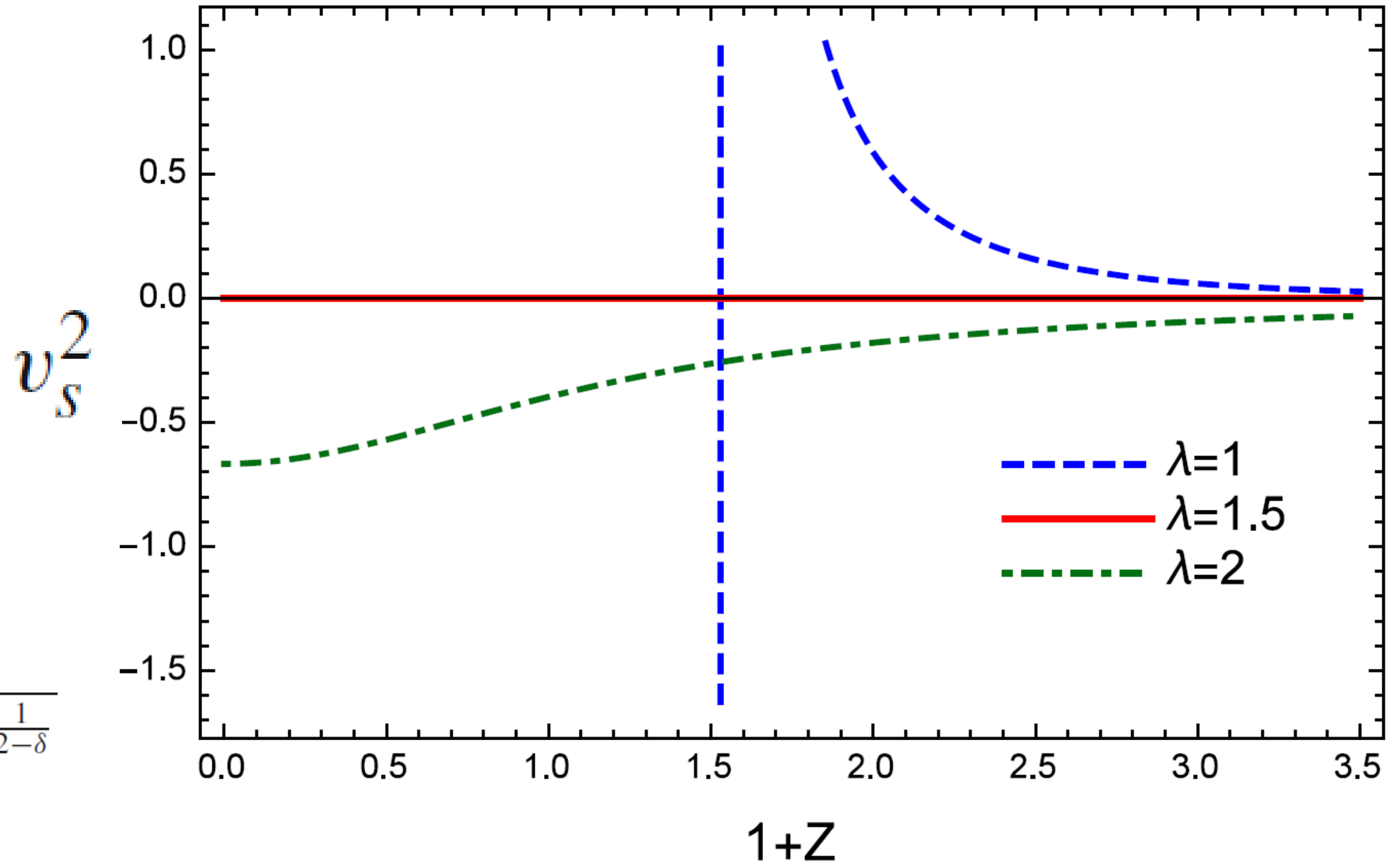
$$\rho_D = \lambda(2H^2 + \dot{H})^{-\delta+2}$$

λ : Constant

[Gao, Chen, Shen, Phys. Rev. D **79**, 043511 (2009)]

$$v_s^2 = \frac{1}{3} + 2\frac{3^{\frac{-1+\delta}{2-\delta}} H^{\frac{2-2\delta}{-2+\delta}} (\lambda^{-1}\Omega_D)^{\frac{1}{2-\delta}}}{\Omega_D(-2+\delta)} + \frac{(-1+\Omega_D)}{-1 - 3\Omega_D + 2 \times 3^{\frac{1}{2-\delta}} H^{\frac{2-2\delta}{-2+\delta}} (\lambda^{-1}\Omega_D)^{\frac{1}{2-\delta}}}$$

[Abdollahi Zadeh, Sheykhi, Moradpour, KB, Eur. Phys. J. C **78**, 940 (2018)]



Z : Redshift

$\Omega_D^0 = 0.73\,,\ \delta = 1\,,\ H_0 = 67$

Stability analysis (4)

$$\rho_D = \lambda(2H^2 + \dot{H})^{-\delta+2}$$

λ : Constant

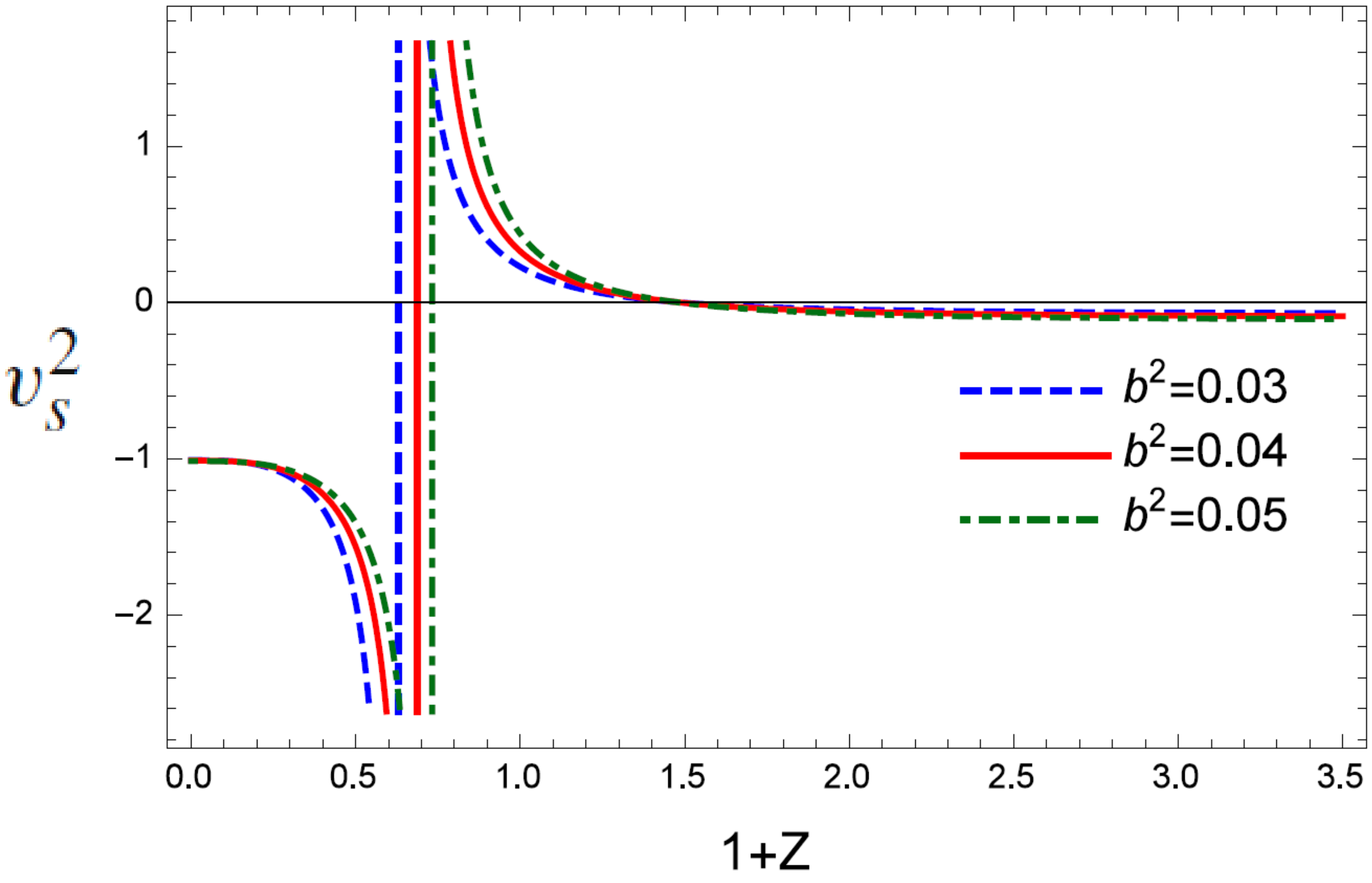
[Gao, Chen, Shen, Phys. Rev. D **79**, 043511 (2009)]

$$\dot{\rho}_m + 3H\rho_m = Q$$

$$\dot{\rho}_D + 3H(1+w_D)\rho_D = -Q$$

$$Q = 3b^2 qH\rho_D(1+u)\,, \qquad u = \frac{\rho_m}{\rho_D}$$

$$q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}$$



Z: Redshift

[Abdollahi Zadeh, Sheykhi, KB, Moradpour, Mod. Phys. Lett. A **35**, 2050053 (2019)]

$$\Omega_D^0 = 0.73\,,\ \delta = 1\,,\ \lambda = 1.5,\ \Omega_\sigma = .001\,,\ H_0 = 67$$

Purposes of this study

- We argue entropy based on the holography principle, which can be related to dark energy.

- Since the length scale is regarded as small enough during inflation, the energy density from the holographic principle can be expected to be large enough to drive inflation.

Cf. [Nojiri, Odintsov and Saridakis, Phys. Lett. B **797**, 134829 (2019)]
[Oliveros and Acero, EPL **128**, 59001 (2019)]

- We investigate the application of holographic principle to inflationary cosmology by analyzing inflationary observables. By comparing with the observations, we show a realistic parametric space.

- We also discuss the trans-Planckian censorship conjecture for the present scenario consistent with the Planck 2018 data.

Application to inflationary cosmology

[Mohammadi, Golanbari, KB, Lobo, Phys. Rev. D **103**, 083505 (2021)]

- **Energy density** $\rho = \sqrt{B}\, L_{IR}^{\delta-2}$

- **Infrared cut off**

※Dimensional analysis : [Granda and Oliveros, Phys. Lett. B **669**, 275 (2008); Phys. Lett. B **671**, 199 (2009)]

$$L_{IR}^{-2} = \alpha H^2 + \beta \dot{H}$$

α, β : dimensionless constant

Friedmann equation

$$H^2 = \frac{Bc^2}{3M_p^2}\left(\alpha H^2 + \beta \dot{H}\right)^{2-\delta}$$

→ Time derivative of the Hubble parameter

$$\dot{H} = \frac{H^2}{\beta}\left[\left(\frac{3M_p^2}{Bc^2}\right)^{\frac{1}{(2-\delta)}}\left(H^2\right)^{\frac{\delta-1}{2-\delta}} - \alpha\right]$$

$$N = \ln\left(a/a_i\right), \qquad dN = H dt$$

$$\dot{H} = \frac{1}{2}\frac{dH^2}{dN}$$

※`i': Beginning of inflation

$$\tilde{H} \equiv H/M_p$$

: dimensionless Hubble parameter

$$\tilde{H}_e^2 = \left(\frac{C}{\alpha - \beta}\right)^{\frac{\delta-2}{\delta-1}}$$

※`e': End of inflation

$$\epsilon_1 = 1$$

Taking integration in H in terms of the number of e-folds during inflationary stage

$$\Rightarrow \ln\left[\tilde{H}^2\left(C\left(\tilde{H}^2\right)^{\frac{\delta-1}{2-\delta}}\right)^{\frac{\delta-2}{\delta-1}} - \alpha\right]\Bigg|_{\tilde{H}_i}^{\tilde{H}_e} = \frac{-2\alpha N}{\beta}\,,$$

$$C \equiv \left(\frac{3M_p^2}{Bc^2}\right)^{\frac{1}{(2-\delta)}} M_p^{\frac{2(\delta-1)}{2-\delta}}$$

- **Slow-roll parameters** ($\leftarrow \dot{H}$)

$$\epsilon_{n+1} = d\ln(\epsilon_n)/dN$$

$$\epsilon_1 = \frac{-\dot{H}}{H} = \frac{-1}{\beta}\left[C\left(\tilde{H}^2\right)^{\frac{\delta-1}{2-\delta}} - \alpha\right]$$

$$\epsilon_2 = \frac{\dot{\epsilon}_1}{H\epsilon_1} = \frac{2C}{\beta}\left(\frac{\delta-1}{2-\delta}\right)\left(\tilde{H}^2\right)^{\frac{\delta-1}{2-\delta}}$$

$$\tilde{H}_\star^2 = \left[\frac{C}{\alpha}\left(1+\frac{\beta}{\alpha-\beta}\, e^{2\alpha N/\beta}\right)\right]^{\frac{\delta-2}{\delta-1}}$$

: Value of $\tilde{H}$ at the horizon crossing time ($\star$)

$\longrightarrow$ Inflationary observables

$$n_s = 1 - 2\epsilon_1 - 2\epsilon_2$$

: Spectral index of the scalar mode

$$r = 16\epsilon_1$$

: Tensor to scalar ratio

$$\mathcal{P}_s = H^2/8\pi^2\epsilon M_p^2$$

: Amplitude of the power spectrum of the scalar mode

$$\sim 10^{-9}$$

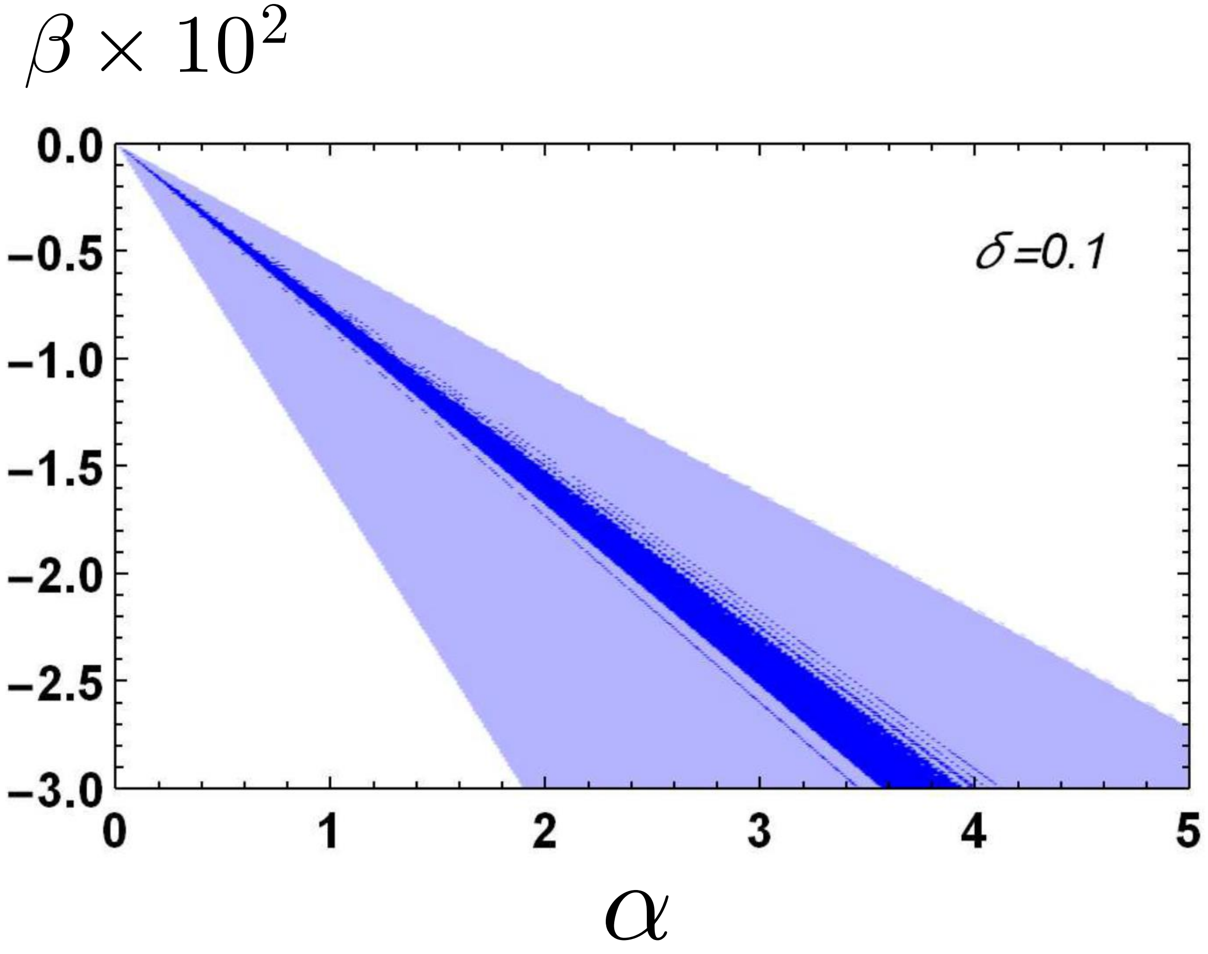
$\Rightarrow$

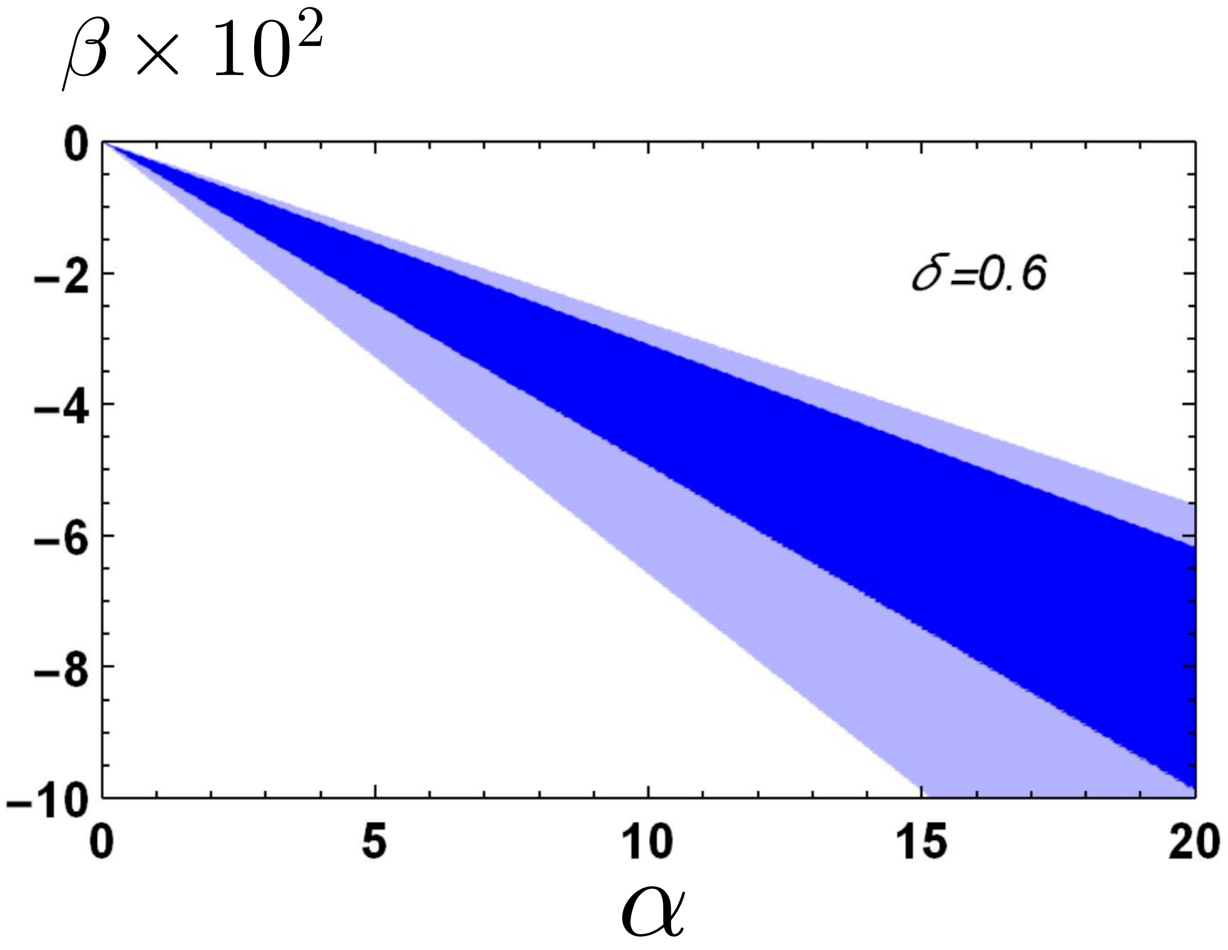
$$C = \frac{\alpha}{\left(1+\frac{\beta}{\alpha-\beta}\, e^{2\alpha N/\beta}\right)}\left(8\pi^2\epsilon\mathcal{P}_s\right)^{\frac{\delta-1}{\delta-2}}$$

Cf. [Nojiri, Odintsov and Saridakis, Phys. Lett. B **797**, 134829 (2019)]

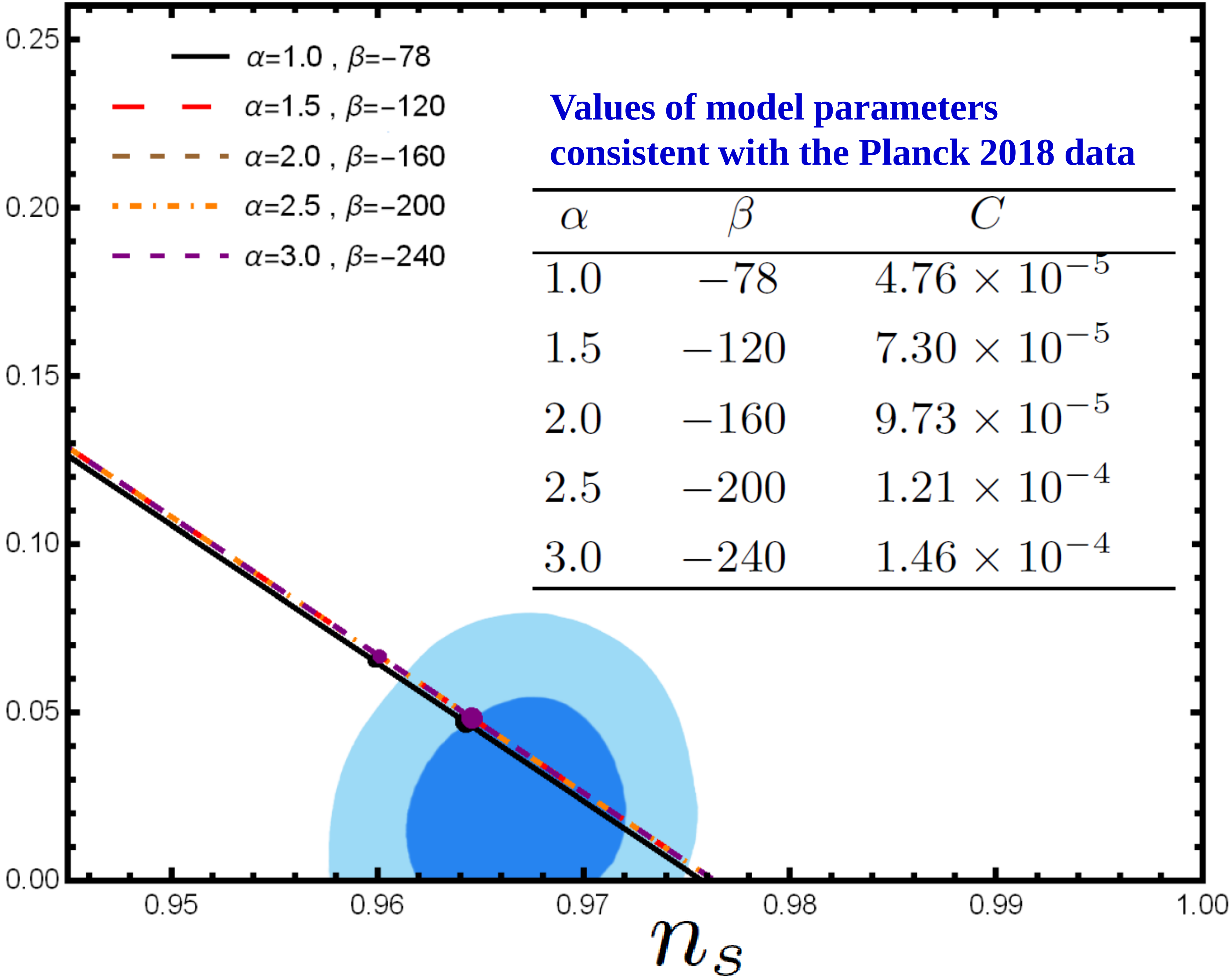
(α, β) space consistent with the Planck 2018 data

[Akrami *et al*. [Planck], Astron. Astrophys. **641**, A10 (2020)]





$r - n_s$ diagram



Values of model parameters consistent with the Planck 2018 data

α	β	C
1.0	-78	4.76×10^{-5}
1.5	-120	7.30×10^{-5}
2.0	-160	9.73×10^{-5}
2.5	-200	1.21×10^{-4}
3.0	-240	1.46×10^{-4}

[Akrami *et al*. [Planck], Astron. Astrophys. **641**, A10 (2020)]

IV. Correspondence between holographic dark energy and scalar field

→ We show that the behavior of inflation provided by the holographic dark energy approach into the dynamics of a scalar filed.

Cf. [Copeland, Sami and Tsujikawa, Int. J. Mod. Phys. D **15**, 1753-1936 (2006)]

(i) Canonical scalar field

$$S=\int d^4x\sqrt{-g}\left[-\frac{1}{2}(\nabla\phi)^2-V(\phi)\right] \quad (\nabla\phi)^2=g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$\rho_\phi=\frac{1}{2}\dot{\phi}^2+V(\phi),\quad p_\phi=\frac{1}{2}\dot{\phi}^2-V(\phi)$$

$$V(\phi)=\rho_{DE}-\frac{1}{2}\dot{\phi}^2=\left(\frac{1-w_{DE}}{2}\right)\rho_{DE}=\rho_{DE}+M_p^2\dot{H}$$

$$=Bc^2\left(\alpha H^2+\beta\dot{H}\right)^{2-\delta}+M_p^2\dot{H}$$

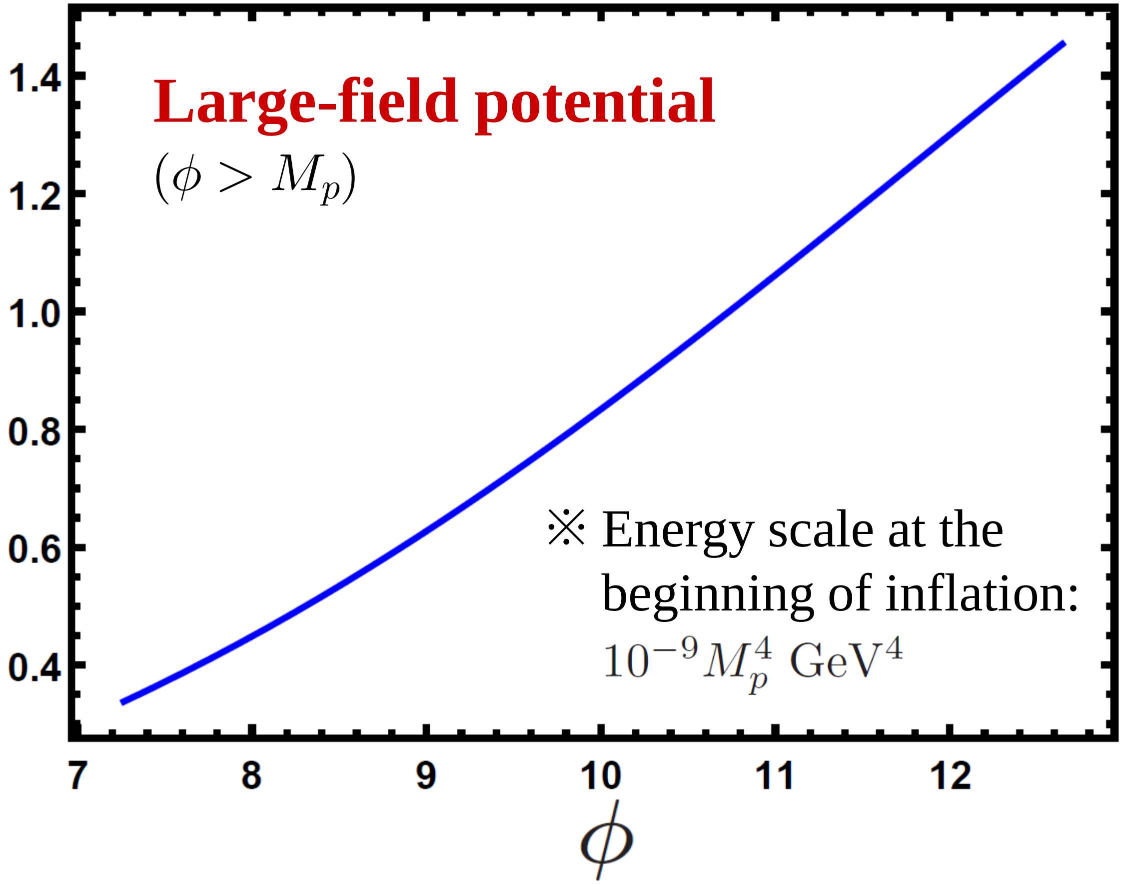
$$\phi'^2=-2M_p^2\dot{H}/H^2,\quad \phi'=d\phi/dN$$

→ With the definition of the 1st slow-roll parameter, the scalar field is obtained by the following integration

$$\Rightarrow\ \Delta\phi(N)=\sqrt{2}\,M_p\int_0^N\sqrt{\frac{-1}{\beta}\left[C\left(\tilde{H}^2\right)^{\frac{\delta-1}{2-\delta}}-\alpha\right]}\,dN$$

$N=0$: corresponds to the Horizon crossing of the perturbations

$V(\phi)$ $(\times 10^{-9})$



$\alpha=1,\ \beta=-78,\ \delta=0.1,\ M_p^2=1$

(ii) **Tachyon field**

$$S=-\int \mathrm{d}^4x\, V(\phi)\sqrt{-\det(g_{ab}+\partial_a\phi\partial_b\phi)}$$

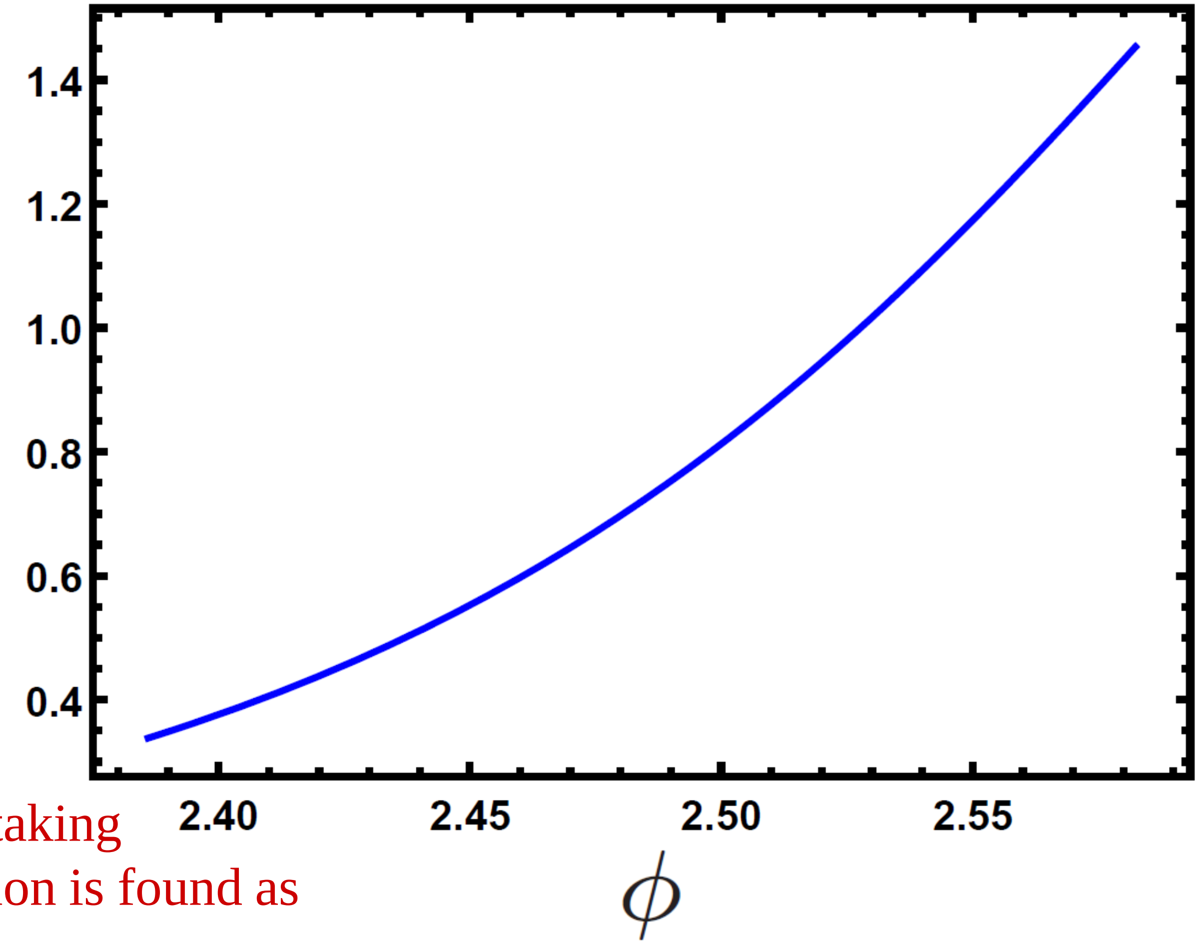
$$V(\phi)=\frac{V_0}{\cosh(\phi/\phi_0)}$$

$$\rho_\phi=\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}},\quad p_\phi=-V(\phi)\sqrt{1-\dot{\phi}^2}$$

$$w_{DE}=w_\phi=p_\phi/\rho_\phi=1-\dot{\phi}^2$$

$$V(\phi)=\rho_{DE}\sqrt{1-\dot{\phi}^2}$$

$V(\phi)\ (\times 10^{-9})$



→ With the definition of the 1st slow-roll parameter and taking integration, the change of the scalar field during inflation is found as

$$\phi'^2=\frac{2}{3M_p^2}\frac{\epsilon_1}{\tilde{H}^2}$$

$\alpha=1,\ \beta=-78,\ \delta=0.1,\ M_p^2=1$

$$\Rightarrow\quad \Delta\phi=\sqrt{\frac{2}{3M_p^2}}\int_0^N\sqrt{\frac{-1}{\beta\tilde{H}^2}\left[C\left(\tilde{H}^2\right)^{\frac{\delta-1}{2-\delta}}-\alpha\right]}\,dN$$

V. Trans-Planckian Censorship Conjecture

- **Trans-Planckian Problem**

If inflation lasted more than enough, it is possible to observe the length scale which would be originated on the scale smaller than the Planck length $l_p = m_p^{-1}$.

- **Trans-Planckian Censorship Conjecture**

[Bedroya, Brandenberger, Loverde and Vafa, Phys. Rev. D **101**, 103502 (2020)]

The question how we should treat the trans-Planckian mode (this is the question that does not arise in a consistent theory of quantum gravity).

※ H_f, a_f : Hubble parameter, scale factor at the end of inflation

$$\boxed{\frac{l_p}{a_i} < \frac{H_f^{-1}}{a_f}} \quad \Rightarrow \quad \left(\frac{C}{\alpha-\beta}\right)^{\frac{\delta-2}{\delta-1}} < \left(8\pi\, e^N\right)^2$$

$\mathcal{O}(10^{-14})$ $\quad$ $\mathcal{O}(10^{-54})$ ← Values of the present model consistent with the Planck 2018 data (P.19)

This relation could not be satisfied

Possible solutions [Dvali, Kehagias and Riotto, 2005.05146]

⇒

- The great part of the de Sitter fluctuations are produced with wavelength: $L \sim H^{-1}$
- Only the fluctuations with wavelength $L \ll H^{-1}$ are exponentially suppressed with a factor $e^{-1/LH}$.

VI. Conclusions

- It has been proposed that entropy based on the holography principle can be related to dark energy.
- Since the length scale is considered to be small enough during inflation, the energy density from the holographic principle can be expected to be large enough to drive inflation.
- We have explored the application of holographic principle to inflationary cosmology. The slow-roll parameters, scalar spectral index n_s, and tensor-to-scalar ratio r have been analyzed. By comparing with the Planck $r - n_s$ diagram, a parametric space for the constants of the model can be shown.

$\longrightarrow$ It has been discussed that for the present model consistent with the Planck 2018 data, the trans-Planckian censorship conjecture could not be satisfied.

Recent related study "The Area of a Rough Black Hole"

A B S T R A C T

[J. D. Barrow, Phys. Lett. B **808**, 135643 (2020)]

We investigate the consequences for the black hole area of introducing fractal structure for the horizon geometry. We create a three-dimensional spherical analogue of a 'Koch Snowflake' using a infinite diminishing hierarchy of touching spheres around the Schwarzschild event horizon. We can create a fractal structure for the horizon with finite volume and infinite (or finite) area. This is a toy model for the possible effects of quantum gravitational spacetime foam, with significant implications for assessments of the entropy of black holes and the universe, which is generally larger than in standard picture of black hole structure and thermodynamics, potentially by very considerable factors. The entropy of the observable universe today becomes $S \approx 10^{120(1+\Delta/2)}$, where $0 \leq \Delta \leq 1$, with $\Delta = 0$ for a smooth spacetime structure and $\Delta = 1$ for the most intricate. The Hawking lifetime of black holes is also reduced.

$$S_B = \left(\frac{A}{A_0}\right)^{1+\Delta/2}, \quad \rho_{DE} L^4 \leq S$$

$$0 \leq \Delta \leq 1$$

⇨ $$\rho_{DE} = C L^{\Delta - 2}$$

⇨ **Applications to cosmology**

[Saridakis, Phys. Rev. D **102**, 123525 (2020); JCAP **2007**, 031 (2020)]

[Mohammadi and Salehi, Phys. Lett. B **839**, 135643 (2023)]

Cf. Microscopic thermodynamic description for an arbitrary generalized entropy in terms of theparticle system
[Nojiri and Odintsov, Phys. Lett. B **845**, 138130 (2023)]

Back up slides

Barrow holographic dark energy

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We formulate Barrow holographic dark energy, by applying the usual holographic principle at a cosmological framework, but using the Barrow entropy instead of the standard Bekenstein-Hawking one. The former is an extended black-hole entropy that arises due to quantum-gravitational effects which deform the black-hole surface by giving it an intricate, fractal form. We extract a simple differential equation for the evolution of the dark energy density parameter, which possesses standard holographic dark energy as a limiting sub-case, and we show that the scenario can describe the universe thermal history, with the sequence of matter and dark energy eras. Additionally, the new Barrow exponent Δ significantly affects the dark energy equation of state, and according to its value it can lead it to lie in the quintessence regime, in the phantom regime, or experience the phantom-divide crossing during the evolution.

[Saridakis, Phys. Rev. D **102**, 123525 (2020)]

Modified cosmology through spacetime thermodynamics and Barrow horizon entropy

[Saridakis, JCAP **2007**, 031 (2020)]

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ABSTRACT: We present modified cosmological scenarios that arise from the application of the "gravity-thermodynamics" conjecture, using the Barrow entropy instead of the usual Bekenstein-Hawking one. The former is a modification of the black hole entropy due to quantum-gravitational effects that deform the black-hole horizon by giving it an intricate, fractal structure. We extract modified cosmological equations which contain new extra terms that constitute an effective dark-energy sector, and which coincide with the usual Friedmann equations in the case where the new Barrow exponent acquires its Bekenstein-Hawking value. We present analytical expressions for the evolution of the effective dark energy density parameter, and we show that the universe undergoes through the usual matter and dark-energy epochs. Additionally, the dark-energy equation-of-state parameter is affected by the value of the Barrow deformation exponent and it can lie in the quintessence or phantom regime, or experience the phantom-divide crossing. Finally, at asymptotically large times the universe always results in the de-Sitter solution.

[Saridakis, JCAP **2007**, 031 (2020)]

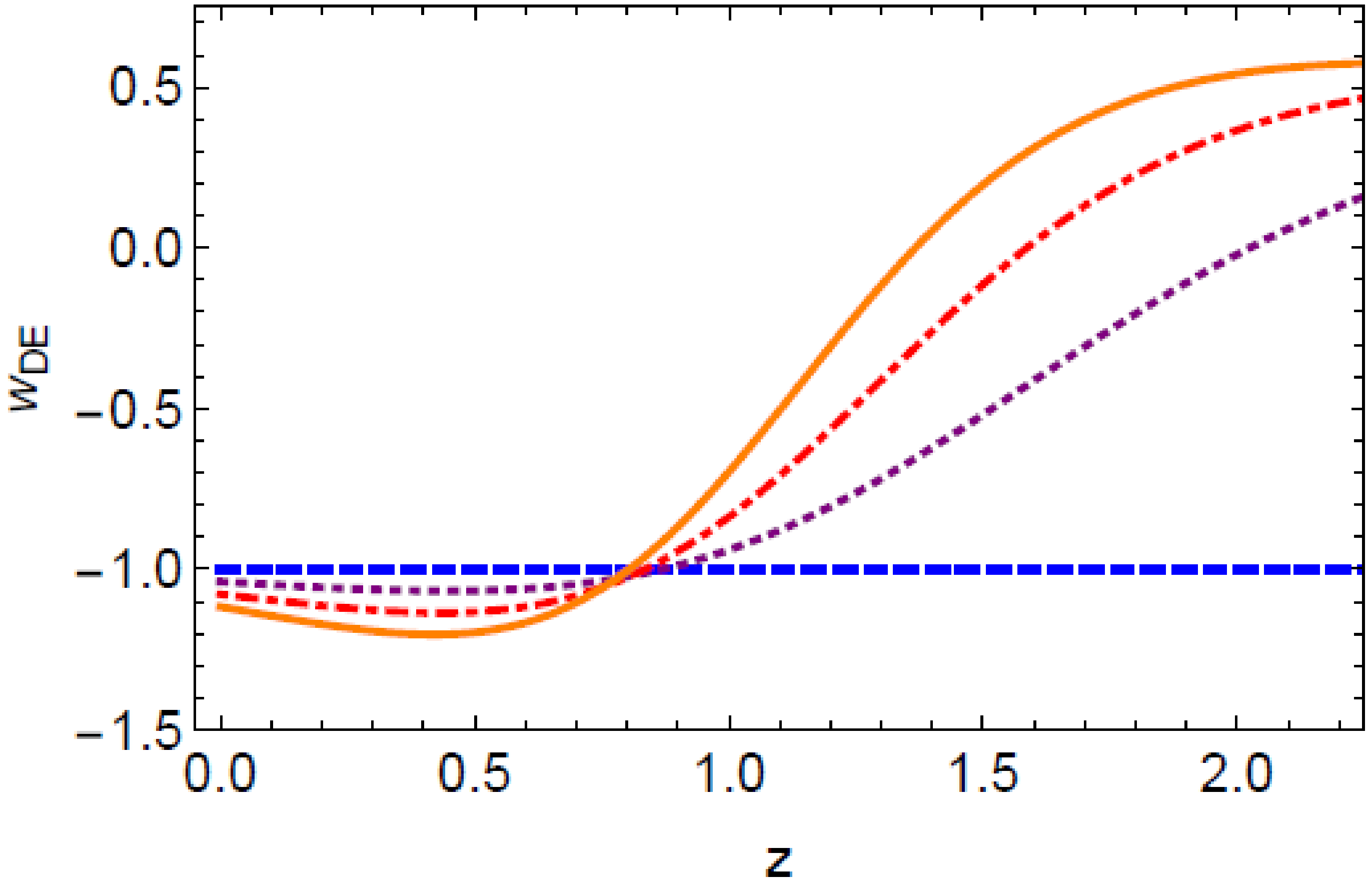

Figure 2. *The evolution of w_{DE} as a function of the redshift z, for $A_0 = 1$, and for $\Delta = 0$ (blue-dashed), $\Delta = 0.2$ (purple-dotted), $\Delta = 0.4$ (red-dashed-dotted), and $\Delta = 0.6$ (orange-solid). We have imposed $\Omega_{m0} \approx 0.3$ at present time.*

Infrared cut-off proposal for the Holographic density

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Abstract

We propose an infrared cut-off for the holographic the dark-energy, which besides the square of the Hubble scale also contains the time derivative of the Hubble scale. This avoids the problem of causality which appears using the event horizon area as the cut-off, and solves the coincidence problem.

[Granda and Oliveros, Phys. Lett. B **669**, 275 (2008)]

Trans-Planckian Problem

It is stated that with the ordinary renormalizable theories with Wilsonian UV-completion one could probe an arbitrary short distance. However, in Einstein theory of gravity the tracking only goes on until one reaches the Planck length scale. It is shown that the minimal localization radius is described by a classical gravitational radius which turns out to be larger than the Compton wavelength, thus indicating that the described object is classical. It is an intrinsic feature of the Einstein gravity that by further tracking beyond the Planck scale, the theory classicalizes and presents a black hole. Therefore, by tracking perturbations to the trans-Planckian time, we are actually scaling them back to their classicalization.

[G. Dvali and C. Gomez, 1005.3497]

Micro-canonical and canonical description for generalised entropy

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Few parameters dependent generalised entropy includes Tsallis entropy, Rényi entropy, Sharma-Mittal entropy, Barrow entropy, Kaniadakis entropy, etc as particular representatives. Its relation to physical systems is not always clear. In this paper, we propose the microscopic thermodynamic description for an arbitrary generalised entropy in terms of the particle system. It is shown that the change in the volume of the phase space of the particle system in the micro-canonical description or the difference in the integration measure in the phase space in the canonical description may lead to generalised entropy. Our consideration may help us understand the structure of quantum gravity.

[Nojiri and Odintsov, Phys. Lett. B **845**, 138130 (2023)]