Interacting massless gravitons from extended minimal theory of bigravity

Zhibang Yao
zhbyao@rikkyo.ac.jp

Department of Physics
Rikkyo University

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GC2024@YITP, Kyoto University

Based on 2403.XXXX collaborated with Xian Gao (SYSU) and Shinji Mukohyama

Outline

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Extended minimal theory of bigravity with auxiliary constraints

Interacting two copies of GR through auxiliary constraints

Summary & Outlook

Backgroud

The no-go theorems: Interacting spin-2 particles

- The Weinberg theorem[1]: All particles interact with the massless spin-2 field (graviton) with equal strength at a long-distance limit.
- The Weinberg-Witten theorem[2] [3]: Massless particles interacting with graviton in a Minkowski background are with at most spin-2.
- A corollary: The only massless spin-2 particle interacts with (massless) graviton is the graviton itself.

$$\text{graviton} + \text{spin-2} + \text{interaction} + \text{massless} + \text{Lorentz} = \text{no go}$$

$$\Rightarrow \text{graviton} + \text{spin-2} + \text{interaction} + \text{massive} + \text{Lorentz} = \text{way out}$$

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[1] [S. Weinberg, Phys. Rev., 1964]
[2] [S. Weinberg and E. Witten, Phys. Lett. B, 1980]
[3] [M. Porrati, Phys. Rev. D, 2008]

Backgroud

Development of Massive gravity:

1939	Fierz–Pauli[1]	linear level, $2_t + 2_v + 1_s$ DOF
2000	DGP[2]	five dimension, $2_t + 2_v + 1_s$ DOF
2011	dRGT[3]	ghost-free, $2_t + 2_v + 1_s$ DOF
2012	Hassan-Rosen[4]	bigravity, $4_t + 2_v + 1_s$ DOF
2016	MTMG[5]	minimal theory, 2_t DOF
2021	MTBG[6]	Lorentz breaking, 4_t DOF

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[1] [M. Fierz and W. Pauli, PRSL, 1939]

[2] [G. Dvali, G. Gabadadze and M. Porrati, PLB, 2000]

[3] [C. de Rham, G. Gabadadze and A. Tolley, PRL, 2011]

[4] [S. Hassan and R. Rosen, JHEP, 2012]

[5] [A. De Felice and S. Mukohyama, PLB, 2016]

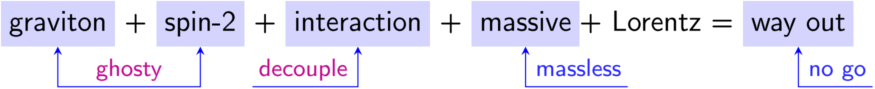
[6] [A. De Felice, F. Larrouturou, S. Mukohyama and M. Oliosi, JCAP, 2021]

Backgroud

Question:

Can we have interacting massless gravitons from the massless phase of massive gravity?

Answer[1]:



graviton + spin-2 + interaction + massive + Lorentz = way out

Another way out: Lorentz violation

For the purpose of constructing a theory for the interacting massless gravitons, we have to break the Lorentz symmetry.

graviton + spin-2 + interaction + massless + Lorentz breaking = way out

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[1] [N. Boulanger, T. Damour, L. Gualtieri and M. Henneaux, NPB, 2001]

Backgroud

Minimal theory of bigravity[1]: MTBG

Hamil. constr. momen. constr.

$$H_{\mathrm{T}} \equiv \int \mathrm{d}^3x \Big[N\,\mathcal{H}_0 + \tilde{N}\,\tilde{\mathcal{H}}_0 + N^i\,\mathcal{H}_i + \tilde{N}^i\,\tilde{\mathcal{H}}_i + \lambda\pi + \tilde{\lambda}\tilde{\pi} + \lambda^i\pi_i + \tilde{\lambda}^i\tilde{\pi}_i$$
$$+\mu_1\left(\mathcal{C}_0 - \tilde{\mathcal{C}}_0\right) + \nu^i\left(\mathcal{C}_i - \beta\tilde{\mathcal{C}}_i\right) + \mu_2\left(\sqrt{h}\nabla^2\frac{\mathcal{C}_0}{\sqrt{h}} + \sqrt{\tilde{h}}\tilde{\nabla}^2\frac{\tilde{\mathcal{C}}_0}{\sqrt{\tilde{h}}}\right)\Big], \qquad (1)$$

where

sqrt. matr.

$$\mathcal{H}_0 \equiv \mathcal{H}_0^{(\mathrm{GR})} - \mathcal{M}_0\left(\mathfrak{m}^i{}_j\right), \quad \mathfrak{m}^i{}_l\,\mathfrak{m}^l{}_j = h^{ik}\tilde{h}_{kj}, \qquad (2)$$

$$\mathcal{C}_0 \equiv -m^2\left(\pi^i_j - \frac{\pi}{2}\delta^i_j\right)\mathcal{U}^i_j\left(\mathfrak{m}^i{}_j\right), \quad \mathcal{C}_i \equiv \frac{m^2 M_{\mathrm{Pl}}^2}{2}\sqrt{h}\nabla_j\mathcal{U}^j_i\left(\mathfrak{m}^i{}_j\right), \qquad (3)$$

$$\#_{\mathrm{dof}} = \frac{1}{2}\left(40 - 11\times 2 - 10\right) = 4. \qquad (4)$$

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[1] [A. De Felice, F. Larrouturou, S. Mukohyama and M. Oliosi, JCAP, 2021]

Backgroud

MTBG: corresponding action

$$\begin{aligned} S \;=\; & \frac{M_{\mathrm{Pl}}^2}{2}\int \mathrm{d}t\mathrm{d}^3x\Big\{\sqrt{g}\,{}^4R - m^2\sqrt{h}\Big[N\mathcal{M}_0 + \mathcal{U}^i{}_j\nabla_i\nu^j + \left(\mu_1+\nabla^2\mu_2\right)\mathcal{U}^i{}_jK^j{}_i \\ & + \frac{m^2\left(\mu_1+\nabla^2\mu_2\right)^2}{4N}\left(\mathcal{U}^i{}_j - \frac{\mathcal{U}^k{}_k}{2}\delta^i_j\right)\mathcal{U}^j{}_i\Big] + (\alpha,\beta,\sim)\Big\}, \end{aligned} \tag{5}$$

tilde sector

Quadratic action: tensor perturbation

$$\begin{aligned} S_{\mathrm{T}}^{(2)} \;=\; & \frac{M_{\mathrm{Pl}}^2}{8}\int \mathrm{d}t\mathrm{d}^3x\Big[Na^3\left(\left(\dot{\gamma}_{ij}/N\right)^2 - \left(\partial_k\gamma_{ij}/a\right)^2\right) \\ & + \alpha^2\tilde{N}\tilde{a}^3\left(\left(\dot{\tilde{\gamma}}_{ij}/\tilde{N}\right)^2 - \left(\partial_k\tilde{\gamma}_{ij}/\tilde{a}\right)^2\right) - m^2\mu_{\mathrm{T}}^2\left(\gamma_{ij}-\tilde{\gamma}_{ij}\right)^2\Big]. \end{aligned} \tag{6}$$

massless: $m = 0$

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The minimally modified gravity with auxiliary constraints

The MMG with auxiliary constraints[1] [2]:

aux. constr.

$$H_{\mathrm{T}} = \int \mathrm{d}^3x \left(\mathscr{H} + N^i \mathcal{H}_i + \lambda^i \pi_i + \mu_{\mathrm{n}} \mathcal{S}^{\mathrm{n}} \right) , \quad n = 1, 2, \cdots, \mathcal{N} \, (\mathcal{N} \leq 4) , \tag{7}$$

and the number of DOF is counted in the following way

2nd-class ACs

$$\begin{aligned} \#_{\mathrm{dof}} &= \frac{1}{2} \left(\#_{\mathrm{var}} \times 2 - \#_{\mathrm{1st}} \times 2 - \#_{\mathrm{2nd}} \right) \\ &= \frac{1}{2} \Big[(2_{\mathrm{t}} + 4_{\mathrm{v}} + 4_{\mathrm{s}}) \times 2 \\ &\quad - \underbrace{(1_{\mathrm{s}} + 2_{\mathrm{v}}) \times 2}_{\mathcal{H}_i \approx 0_i \& \pi_i \approx 0_i} \times 2 - \#^{\mathrm{s}}_{\mathrm{1st}} \times 2 - \#^{\mathrm{s}}_{\mathrm{2nd}} \Big] \\ &= 2_{\mathrm{t}} + \frac{1}{2} \underbrace{\left(4_{\mathrm{s}} - \#^{\mathrm{s}}_{\mathrm{1st}} \times 2 - \#^{\mathrm{s}}_{\mathrm{2nd}} \right)}_{=0}, \end{aligned} \tag{8}$$

1st-class ACs

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[1] [Z. Yao, M. Oliosi, X. Gao and S. Mukohyama, PRD, 2021]

[2] [Z. Yao, M. Oliosi, X. Gao and S. Mukohyama, PRD, 2023]

The minimally modified gravity with auxiliary constraints

# ACs	Minimalizing cond.	Symmetrizing cond.	Classifications	Ident. key	Examples
$\mathcal{N} = 4$	none	none	$\#^{\mathrm{s}}_{\mathrm{1st}} = 0,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 4$	IV-0-4	Mixed Traces
$\mathcal{N} = 3$	$[S^1, S^{\mathrm{n}}]$	$[S^1, \mathscr{H}]$	$\#^{\mathrm{s}}_{\mathrm{1st}} = 1,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 2$	III-1-2	unknown
		none	$\#^{\mathrm{s}}_{\mathrm{1st}} = 0,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 4$	III-0-4	unknown
$\mathcal{N} = 2$	$[S^1, S^{\mathrm{n}}] \& [S^2, S^2]$	$[S^1, \mathscr{H}] \ \& \ [S^2, \mathscr{H}]$	$\#^{\mathrm{s}}_{\mathrm{1st}} = 2,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 0$	II-2-0	unknown
		$\left[S^2, \dot{S}^1\right] \ \& \ [S^2, \mathscr{H}]$	$\#^{\mathrm{s}}_{\mathrm{1st}} = 1,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 2$	II-1-2b	unknown
		none	$\#^{\mathrm{s}}_{\mathrm{1st}} = 0,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 4$	II-0-4b	Linear AC
	$[S^1, S^{\mathrm{n}}] \ \& \ \left[S^1, \dot{S}^1\right]$	$\left[\dot{S}^1, H_{\mathrm{P}}\right]$	$\#^{\mathrm{s}}_{\mathrm{1st}} = 1,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 2$	II-1-2a	4DEGB
		none	$\#^{\mathrm{s}}_{\mathrm{1st}} = 0,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 4$	II-0-4a	unknown
$\mathcal{N} = 1$	$[S^1, S^1]\,,\ \left[S^1, \dot{S}^1\right]$ $\&\ \left[\dot{S}^1, \dot{S}^1\right]$	$\left[\dot{S}^1, \mathscr{H}\right]$	$\#^{\mathrm{s}}_{\mathrm{1st}} = 2,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 0$	I-2-0	GR & $f(\mathscr{H})$
		$\left[\dot{S}^1, \ddot{S}^1\right]$	$\#^{\mathrm{s}}_{\mathrm{1st}} = 1,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 2$	I-1-2b	unknown
	$[S^1, S^1]\,,\ \left[S^1, \dot{S}^1\right]$ $\&\ \left[S^1, \ddot{S}^1\right]$	$\left[\ddot{S}^1, \mathscr{H}\right]$	$\#^{\mathrm{s}}_{\mathrm{1st}} = 1,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 2$	I-1-2a	Cuscuton & QEC
		none	$\#^{\mathrm{s}}_{\mathrm{1st}} = 0,\ \#^{\mathrm{s}}_{\mathrm{2nd}} = 4$	I-0-4	unknown

Extended MTBG with auxiliary constraints

Building blocks:

$$\underbrace{\{g_{\mu\nu};\tilde{g}_{\mu\nu}\}}_{\text{Bimetric}} \Rightarrow \underbrace{\left\{\begin{array}{cccccc} N, & N^i, & h_{ij}; & \tilde{N}, & \tilde{N}^i, & \tilde{h}_{ij} \\ \pi, & \pi_i, & \pi^{ij}; & \tilde{\pi}, & \tilde{\pi}_i, & \tilde{\pi}^{ij} \end{array}\right\}}_{\text{Phase space: 40-dim.}} \tag{9}$$

Canonical transformations:

$$\begin{bmatrix} M \\ \tilde{M} \end{bmatrix} \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} \alpha & \tilde{\alpha} \\ \beta & \tilde{\beta} \end{bmatrix} \begin{bmatrix} N \\ \tilde{N} \end{bmatrix}, \quad [p,\tilde{p}] \equiv \sqrt{2} \begin{bmatrix} \alpha & \tilde{\alpha} \\ \beta & \tilde{\beta} \end{bmatrix}^{-1} [\pi,\tilde{\pi}], \tag{10}$$

$$\begin{bmatrix} M^i \\ \tilde{M}^i \end{bmatrix} \equiv \frac{1}{\sqrt{2}} \begin{bmatrix} \zeta & \tilde{\zeta} \\ \eta & \tilde{\eta} \end{bmatrix} \begin{bmatrix} N^i \\ \tilde{N}^i \end{bmatrix}, \quad [p_i,\tilde{p}_i] \equiv \sqrt{2} \begin{bmatrix} \zeta & \tilde{\zeta} \\ \eta & \tilde{\eta} \end{bmatrix}^{-1} [\pi_i,\tilde{\pi}_i], \tag{11}$$

Extended MTBG with auxiliary constraints

Hamiltonian with spatial diffeomorphism:

$$H_\mathrm{T} \equiv \int \mathrm{d}^3x \left(\overset{\text{free fun.}}{\mathscr{H}} + M^i \overbrace{\mathcal{H}_i^{\mathrm{tot}} + \lambda^i p_i}^{\text{3d-diff. constr.}} + \mu_\mathrm{n} \overbrace{\mathcal{S}^\mathrm{n} + \nu_\mathrm{m}^i \mathcal{V}_i^\mathrm{m}}^{\text{aux. constr.}} \right) \tag{12}$$

where $\mathscr{H}$, $\mathcal{S}^\mathrm{n}$, $\mathcal{V}_i^\mathrm{m}$ are general functions of the canonical variables and the total momentum constraint is defined by

$$\begin{aligned} \underset{\uparrow\ \text{generator: 1st-class}}{\mathcal{H}_i^{\mathrm{tot}}} \quad &\equiv \quad \pi \nabla_i N + \pi_j \nabla_i N^j + \sqrt{h} \nabla_j \frac{\pi_i N^j}{\sqrt{h}} - 2\sqrt{h} \nabla_j \frac{\pi_i^j}{\sqrt{h}} \\ & \quad + \tilde{\pi} \tilde{\nabla}_i \tilde{N} + \tilde{\pi}_j \tilde{\nabla}_i \tilde{N}^j + \sqrt{\tilde{h}} \tilde{\nabla}_j \frac{\tilde{\pi}_i \tilde{N}^j}{\sqrt{\tilde{h}}} - 2\sqrt{\tilde{h}} \tilde{\nabla}_j \frac{\tilde{\pi}_i^j}{\sqrt{\tilde{h}}} . \end{aligned} \tag{13}$$

Extended MTBG with auxiliary constraints

Number of the auxiliary constraints:

Assuming the ACs are the 2nd-class, the number of DOF is counted by

$$\begin{aligned}\#_{\rm dof} &= \frac{1}{2}\left(10\times 2\times 2-\#_{\rm 1st}\times 2-\#_{\rm 2nd}\right)\\ &= \frac{1}{2}\Big[\underbrace{(4_{\sf s}+4_{\sf v}+2_{\sf t})\times 2}_{g_{\mu\nu}\&\tilde{g}_{\mu\nu}}\times 2-\underbrace{(1_{\sf s}+2_{\sf v})\times 2}_{\mathcal{H}_i^{\sf tot}\approx 0\&p_i\approx 0}\times 2\\ &\quad -\underbrace{1_{\sf s}\times\mathcal{N}}_{\mathcal{S}^{\sf n}\approx 0}-\underbrace{(1_{\sf s}+2_{\sf v})\times\mathcal{M}}_{\mathcal{V}_i^{\sf m}\approx 0_i}\Big]\\ &= 4_{\sf t}+\underbrace{(4_{\sf v}-\mathcal{M}_{\sf v})}_{=0}+\frac{1}{2}\underbrace{(12_{\sf s}-\mathcal{N}_{\sf s}-\mathcal{M}_{\sf s})}_{=0},\end{aligned} \tag{14}$$

and we solve $\mathcal{M}=4$ and $\mathcal{N}=8$ which means there should be no more than four $\mathcal{V}_i^{\sf m}\approx 0$ and eight $\mathcal{S}^{\sf n}\approx 0$.

Extended MTBG with auxiliary constraints

Hamiltonian with linear lapses and shifts:

$$
\begin{aligned}
H_{\mathrm{T}} \equiv \int \mathrm{d}^3x \Big(& \overbrace{\mathcal{V} + M\,\mathcal{H} + \tilde{M}\,\tilde{\mathcal{H}} + \tilde{M}^i\,\tilde{\mathcal{H}}_i}^{\mathscr{H}:\ \text{free fun.}} + M^i \mathcal{H}_i^{\mathrm{tot}} + \lambda^i p_i \\
& + \tilde{\lambda}^i \underset{\mathcal{V}_i^2 \uparrow}{\tilde{p}_i} + \lambda \underset{\mathcal{S}^3 \uparrow}{p} + \tilde{\lambda} \underset{\mathcal{S}^4 \uparrow}{\tilde{p}} + \mu_1 \mathcal{S}^1 + \mu_2 \mathcal{S}^2 + \nu^i \mathcal{V}_i^1 \Big),
\end{aligned}
\tag{15}
$$

where the undetermined parts are the arbitrary functions of $\left(h_{ij}, \pi^{ij}, \tilde{h}_{ij}, \tilde{\pi}^{ij}; \nabla_i, \tilde{\nabla}_i\right)$

$$
\begin{aligned}
\#_{\mathrm{dof}} = \frac{1}{2}\Big[& (4_{\mathrm{s}} + 4_{\mathrm{v}} + 2_{\mathrm{t}}) \times 2 \times 2 \\
& - \Big(\underbrace{1_{\mathrm{s}} + 1_{\mathrm{s}} + 2_{\mathrm{v}}}_{p \approx 0 \,\&\, p_i \approx 0_i} + \underbrace{1_{\mathrm{s}} + 1_{\mathrm{s}} + 2_{\mathrm{v}}}_{\tilde{p} \approx 0 \,\&\, \tilde{p}_i \approx 0_i} + \underbrace{1_{\mathrm{s}} + 2_{\mathrm{v}}}_{\mathcal{H}_i^{\mathrm{tot}} \approx 0_i} \Big) \times 2 \\
& - \Big(\underbrace{1_{\mathrm{s}} + 2_{\mathrm{v}}}_{\tilde{\mathcal{H}}_i \approx 0_i} + \underbrace{1_{\mathrm{s}} + 1_{\mathrm{s}}}_{\mathcal{H} \approx 0 \,\&\, \tilde{\mathcal{H}} \approx 0} + \underbrace{1_{\mathrm{s}} + 1_{\mathrm{s}}}_{\mathcal{S}^1 \approx 0 \,\&\, \mathcal{S}^2 \approx 0} + \underbrace{1_{\mathrm{s}} + 2_{\mathrm{v}}}_{\mathcal{V}_i \approx 0_i} \Big) \Big] = 4_{\mathrm{t}}.
\end{aligned}
\tag{16}
$$

Extended MTBG with auxiliary constraints

The undetermined functions:

$$\left\{\mathcal{V},\mathcal{H},\tilde{\mathcal{H}},\tilde{\mathcal{H}}_i,\mathcal{S}^1,\mathcal{S}^2,\mathcal{V}_i^1\right\}, \tag{17}$$

and we first choose $\tilde{\mathcal{H}}_i$ in the following "anti-symmetric" form

$$\begin{aligned}\tilde{\mathcal{H}}_i\to\tilde{\mathcal{H}}_i^{\mathrm{anti}} \approx\;& \pi\nabla_i N+\pi_j\nabla_i N^j+\sqrt{h}\nabla_j\frac{\pi_i N^j}{\sqrt{h}}-2\sqrt{h}\nabla_j\frac{\pi_i^j}{\sqrt{h}}\\ &-\tilde{\pi}\tilde{\nabla}_i\tilde{N}-\tilde{\pi}_j\tilde{\nabla}_i\tilde{N}^j-\sqrt{\tilde{h}}\tilde{\nabla}_j\frac{\tilde{\pi}_i\tilde{N}^j}{\sqrt{\tilde{h}}}+2\sqrt{\tilde{h}}\tilde{\nabla}_j\frac{\tilde{\pi}_i^j}{\sqrt{\tilde{h}}}.\end{aligned} \tag{18}$$

which satisfies the following property

$$\left[\int \mathrm{d}^3x\xi^i\tilde{\mathcal{H}}_i^{\mathrm{anti}},\int \mathrm{d}^3y\zeta^B\left(\psi_B\left(\phi^J,\pi_J;\nabla_j\right)+\tilde{\psi}_B\left(\tilde{\phi}^J,\tilde{\pi}_J;\tilde{\nabla}_j\right)\right)\right]\approx 0, \tag{19}$$

Extended MTBG with auxiliary constraints

Traces variables:

Next we restrict

$$\left(h_{ij}, \pi^{ij}, \nabla_i; \tilde{h}_{ij}, \tilde{\pi}^{ij}, \tilde{\nabla}_i\right) \to \left(\mathscr{R}^{\mathrm{i}}, \mathit{\Pi}^{\mathrm{i}}, \mathscr{Q}^{\mathrm{i}}, \mathscr{U}^{i}, \mathscr{W}^{i}, \mathscr{M}^{\mathrm{i}}; \sim\right), \tag{20}$$

where

$$\underset{\text{potentials}}{\boxed{\mathscr{R}^{\mathrm{i}}}} \equiv \left\{R^i_i, R^i_j R^j_i, R^i_j R^j_k R^k_i\right\}, \quad \underset{\text{kinetic terms}}{\boxed{\mathit{\Pi}^{\mathrm{i}}}} \equiv \left\{\pi^i_i, \pi^i_j \pi^j_i, \pi^i_j \pi^j_k \pi^k_i\right\}$$

$$\underset{\text{mixed terms}}{\boxed{\mathscr{Q}^{\mathrm{i}}}} \equiv \left\{R^i_j \pi^j_i, R^i_j \pi^j_k \pi^k_i, R^i_j R^j_k \pi^k_i\right\}$$

$$\underset{\text{interactions}}{\boxed{\mathscr{W}^{i}}} \equiv \left\{R^i_j \mathbb{m}^i{}_j, R^i_j \mathbb{m}^j{}_k \mathbb{m}^k{}_i, R^i_j R^j_k \mathbb{m}^k{}_i\right\} \tag{21}$$

$$\underset{\text{mass terms}}{\boxed{\mathscr{U}^{i}}} \equiv \left\{\mathbb{m}^i{}_j \pi^j_i, \mathbb{m}^i{}_j \pi^j_k \pi^k_i, \mathbb{m}^i{}_j \mathbb{m}^j{}_k \pi^k_i\right\}$$

$$\underset{\text{mass terms}}{\boxed{\mathscr{M}^{\mathrm{i}}}} \equiv \left\{\mathbb{m}^i{}_i, \mathbb{m}^i{}_j \mathbb{m}^j{}_i, \mathbb{m}^i{}_j \mathbb{m}^j{}_k \mathbb{m}^k{}_i\right\}, \mathbb{m}^i{}_l \mathbb{m}^l{}_j = h^{ik} \tilde{h}_{kj}. \tag{22}$$

Extended MTBG with auxiliary constraints

Flat FLRW background:

$$\mathrm{d}s_g^2 = -M^2\left(t\right)\mathrm{d}t^2 + a^2\left(t\right)\mathfrak{g}_{ij}\mathrm{d}x^i\mathrm{d}x^j, \quad \mathrm{d}s_{\tilde{g}}^2 = -\tilde{M}^2\left(t\right)\mathrm{d}t^2 + \tilde{a}^2\left(t\right)\tilde{\mathfrak{g}}_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{23}$$

Traces variables: 2nd order

$$\mathscr{R}^1 \to \gamma^{ij}\frac{\Delta}{a^2}\gamma_{ij}, \quad \mathscr{R}^2 \to \gamma^{ij}\frac{\Delta^2}{a^4}\gamma_{ij}, \quad \mathscr{R}^3 \to 0, \tag{24}$$

$$\mathscr{M}^1\&\mathscr{M}^2\&\mathscr{M}^3 \to \left(\gamma_{ij}\gamma^{ij}, \tilde{\gamma}_{ij}\tilde{\gamma}^{ij}, \gamma_{ij}\tilde{\gamma}^{ij}\right), \tag{25}$$

$$\mathscr{W}^1\&\mathscr{W}^2 \to \left(\gamma^{ij}\frac{\Delta}{a^2}\gamma_{ij}, \underbrace{\boxed{\tilde{\gamma}^{ij}\frac{\Delta}{a^2}\gamma_{ij}}}_{\text{interaction term}}\right), \quad \mathscr{W}^3 \to \gamma^{ij}\frac{\Delta^2}{a^4}\gamma_{ij}, \tag{26}$$

Extended MTBG with auxiliary constraints

Determine the free functions:

$$\mathcal{H} \to \mathcal{H}_0\left(\mathscr{R}^{\mathrm{i}}, \varPi^{\mathrm{i}}, \mathscr{M}^{\mathrm{i}}\right) \approx 0, \quad \tilde{\mathcal{H}} \to \tilde{\mathcal{H}}_0\left(\tilde{\mathscr{R}}^{\mathrm{i}}, \tilde{\varPi}^{\mathrm{i}}, \tilde{\mathscr{M}}^{\mathrm{i}}\right) \approx 0, \tag{27}$$

$$\mathcal{S}^1 \to \mathcal{C}_0\left(\mathscr{Q}^{\mathrm{i}}, \mathscr{W}^{i}, \mathscr{U}^{i}, \mathscr{M}^{\mathrm{i}}\right) - \tilde{\mathcal{C}}_0\left(\tilde{\mathscr{Q}}^{\mathrm{i}}, \tilde{\mathscr{W}}^{\mathrm{i}}, \tilde{\mathscr{U}}^{\mathrm{i}}, \tilde{\mathscr{M}}^{\mathrm{i}}\right) \approx 0, \tag{28}$$

$$\mathcal{S}^2 \to \sqrt{h}\nabla^2 \frac{\mathcal{C}_0}{\sqrt{h}} + \sqrt{\tilde{h}}\tilde{\nabla}^2 \frac{\tilde{\mathcal{C}}_0}{\sqrt{\tilde{h}}} \approx 0, \tag{29}$$

$$\mathcal{V}_i^1 \to \sqrt{h}\nabla_j \mathcal{U}^i{}_j\left(\mathscr{M}^i{}_j\right) - \sqrt{\tilde{h}}\tilde{\nabla}_j \tilde{\mathcal{U}}_i{}^j\left(\tilde{\mathscr{M}}_i{}^j\right) \equiv \mathcal{C}_i - \tilde{\mathcal{C}}_i \approx 0, \tag{30}$$

and

$$\mathcal{V} \to \mathcal{V}_0\left(\mathscr{R}^{\mathrm{i}}, \varPi^{\mathrm{i}}, \mathscr{Q}^{\mathrm{i}}, \mathscr{W}^{i}\right) + \tilde{\mathcal{V}}_0\left(\tilde{\mathscr{R}}^{\mathrm{i}}, \tilde{\varPi}^{\mathrm{i}}, \tilde{\mathscr{Q}}^{\mathrm{i}}, \tilde{\mathscr{W}}^{\mathrm{i}}\right). \tag{31}$$

Extended MTBG with auxiliary constraints

The extended MTBG:

$$\begin{aligned} H_{\mathrm{T}} \equiv & \int \mathrm{d}^3x \Big[\mathcal{V}_0 + \tilde{\mathcal{V}}_0 + M\mathcal{H}_0 + \tilde{M}\tilde{\mathcal{H}}_0 + M^i\mathcal{H}_i^{\mathrm{tot}} + \tilde{M}^i\tilde{\mathcal{H}}_i^{\mathrm{anti}} \\ & +\lambda^i p_i + \tilde{\lambda}^i \tilde{p}_i + \lambda p + \tilde{\lambda}\tilde{p} + \mu_1 \left(\mathcal{C}_0 - \tilde{\mathcal{C}}_0\right) \\ & +\nu^i \left(\mathcal{C}_i - \tilde{\mathcal{C}}_i\right) + \mu_2\Big(\sqrt{h}\nabla^2\frac{\mathcal{C}_0}{\sqrt{h}} + \sqrt{\tilde{h}}\tilde{\nabla}^2\frac{\tilde{\mathcal{C}}_0}{\sqrt{\tilde{h}}}\Big)\Big], \end{aligned} \tag{32}$$

where

$$\left\{\mathcal{V}_0, \mathcal{H}_0, \mathcal{C}_0, \mathcal{C}_i; \tilde{\mathcal{V}}_0, \tilde{\mathcal{H}}_0, \tilde{\mathcal{C}}_0, \tilde{\mathcal{C}}_i\right\}, \tag{33}$$

are promote to be arbitrary functions of the traces variables.

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Massless phase of the extended MTBG:

We simply remove the dependence of mass terms $\mathscr{U}^i$ and $\mathscr{M}^{\mathrm{i}}$ in the Hamiltonian and obtain the massless phase of the extended MTBG

$$\begin{aligned} H_{\mathrm{T}} \equiv & \int \mathrm{d}^3x \Big[\mathcal{V}_0 + \tilde{\mathcal{V}}_0 + M\mathcal{H}_0 + \tilde{M}\tilde{\mathcal{H}}_0 + M^i\mathcal{H}_i^{\mathrm{tot}} + \tilde{M}^i \underset{\uparrow\ \text{1st-class}}{\boxed{\tilde{\mathcal{H}}_i^{\mathrm{anti}}}} \\ & + \lambda^i p_i + \tilde{\lambda}^i \tilde{p}_i + \lambda p + \tilde{\lambda}\tilde{p} + \mu_1\left(\mathcal{C}_0 - \tilde{\mathcal{C}}_0\right) \\ & + \nu^i\cancel{(\mathcal{C}_i - \tilde{\mathcal{C}}_i)} + \mu_2\Big(\sqrt{h}\nabla^2\frac{\mathcal{C}_0}{\sqrt{h}} + \sqrt{\tilde{h}}\tilde{\nabla}^2\frac{\tilde{\mathcal{C}}_0}{\sqrt{\tilde{h}}}\Big)\Big], \end{aligned} \tag{34}$$

and the number of DOF counts

$$\begin{aligned} \#_{\mathrm{dof}} = & \frac{1}{2}\Big[(4_{\mathrm{s}} + 4_{\mathrm{v}} + 2_{\mathrm{t}}) \times 2 \times 2 - \Big(\underbrace{1_{\mathrm{s}} + 1_{\mathrm{s}}}_{\mathcal{H}\approx 0 \& \tilde{\mathcal{H}}\approx 0} + \underbrace{1_{\mathrm{s}} + 1_{\mathrm{s}}}_{\mathcal{S}^1\approx 0 \& \mathcal{S}^2\approx 0}\Big) \\ & - \Big(\underbrace{1_{\mathrm{s}} + 1_{\mathrm{s}} + 2_{\mathrm{v}}}_{p\approx 0 \& p_i\approx 0_i} + \underbrace{1_{\mathrm{s}} + 1_{\mathrm{s}} + 2_{\mathrm{v}}}_{\tilde{p}\approx 0 \& \tilde{p}_i\approx 0_i} + \underbrace{1_{\mathrm{s}} + 2_{\mathrm{v}}}_{\mathcal{H}_i^{\mathrm{tot}}\approx 0_i} + \underbrace{1_{\mathrm{s}} + 2_{\mathrm{v}}}_{\tilde{\mathcal{H}}_i^{\mathrm{anti}}\approx 0_i}\Big) \times 2\Big] = 4_{\mathrm{t}}. \end{aligned} \tag{35}$$

Interacting two copies of GR through ACs

A concrete model: Interacting two copies of GR through ACs

$$
\begin{aligned}
H_{\mathrm{T}} \;\equiv\; & \int \mathrm{d}^3x \Big[M\mathcal{H}_0^{(\mathrm{GR})} + \tilde{M}\tilde{\mathcal{H}}_0^{(\mathrm{GR})} + M^i\mathcal{H}_i^{\mathrm{tot}} + \tilde{M}^i\tilde{\mathcal{H}}_i^{\mathrm{anti}} + \lambda^i p_i + \tilde{\lambda}^i\tilde{p}_i \\
& + \lambda p + \tilde{\lambda}\tilde{p} + \mu_1\left(\mathcal{C}_0 - \tilde{\mathcal{C}}_0\right) + \mu_2\Big(\sqrt{h}\nabla^2\frac{\mathcal{C}_0}{\sqrt{h}} + \sqrt{\tilde{h}}\tilde{\nabla}^2\frac{\tilde{\mathcal{C}}_0}{\sqrt{\tilde{h}}}\Big)\Big], \qquad (36)
\end{aligned}
$$

where we choose

$$
\mathcal{H}_0 \to \mathcal{H}_0^{(\mathrm{GR})} = \frac{1}{\sqrt{h}}\left(\mathit{\Pi}^2 - \frac{1}{2}\mathit{\Pi}^1\mathit{\Pi}^1\right) - \sqrt{h}\mathscr{R}^1, \quad \mathcal{C}_0 \to \mathcal{C}_0\left(\mathscr{W}^{\mathrm{i}}\right), \qquad (37)
$$

$$
\tilde{\mathcal{H}}_0 \to \tilde{\mathcal{H}}_0^{(\mathrm{GR})} = \frac{1}{\sqrt{\tilde{h}}}\left(\tilde{\mathit{\Pi}}^2 - \frac{1}{2}\tilde{\mathit{\Pi}}^1\tilde{\mathit{\Pi}}^1\right) - \sqrt{\tilde{h}}\tilde{\mathscr{R}}^1, \quad \tilde{\mathcal{C}}_0 \to \tilde{\mathcal{C}}_0\left(\tilde{\mathscr{W}}^{\mathrm{i}}\right). \qquad (38)
$$

Interacting two copies of GR through ACs

The corresponding action:

$$S = \int \mathrm{d}t\mathrm{d}^3x \left(\pi^{ij} \frac{\delta \overline{\mathcal{E}}}{\delta \pi^{ij}} - \mathcal{E} + \tilde{\pi}^{ij} \frac{\delta \overline{\tilde{\mathcal{E}}}}{\delta \tilde{\pi}^{ij}} - \tilde{\mathcal{E}} \right), \tag{39}$$

where we denote

$$\mathcal{E}\left(\pi^{ij}\right) \equiv M \mathcal{H}_0^{(\mathrm{G.R})} + \mu_1 \mathcal{C}_0 + \mu_2 \sqrt{h} \nabla^2 \frac{\mathcal{C}_0}{\sqrt{h}}, \tag{40}$$

$$\tilde{\mathcal{E}}\left(\tilde{\pi}^{ij}\right) \equiv \tilde{M} \tilde{\mathcal{H}}_0^{(\mathrm{G.R.})} - \mu_1 \tilde{\mathcal{C}}_0 + \mu_2 \sqrt{\tilde{h}} \tilde{\nabla}^2 \frac{\tilde{\mathcal{C}}_0}{\sqrt{\tilde{h}}}. \tag{41}$$

and the π^{ij} and $\tilde{\pi}^{ij}$ should be understood as the solution of the canonical equations

$$\frac{\delta \overline{\mathcal{E}}}{\delta \pi^{ij}} = \dot{h}_{ij} - \mathscr{L}_{\vec{M}} h_{ij} - \mathscr{L}_{\vec{\tilde{M}}} h_{ij}, \quad \frac{\delta \overline{\tilde{\mathcal{E}}}}{\delta \tilde{\pi}^{ij}} = \dot{\tilde{h}}_{ij} - \mathscr{L}_{\vec{M}} \tilde{h}_{ij} + \mathscr{L}_{\vec{\tilde{M}}} \tilde{h}_{ij}, \tag{42}$$

Interacting two copies of GR through ACs

The tensor perturbation:

$$
\begin{aligned}
S_{\mathrm{T}}^{(2)} \quad = \quad & \int \mathrm{d}t\mathrm{d}^3x \frac{1}{4}\Big(\mathcal{G}_0\left(t\right)\dot{\gamma}_{ij}\dot{\gamma}^{ij} + \mathcal{W}_0\left(t\right)\gamma_{ij}\frac{\Delta}{a^2}\gamma^{ij} - \mathcal{W}_2\left(t\right)\gamma_{ij}\frac{\Delta^2}{a^4}\gamma^{ij} \\
& + \tilde{\mathcal{G}}_0\left(t\right)\dot{\tilde{\gamma}}_{ij}\dot{\tilde{\gamma}}^{ij} + \tilde{\mathcal{W}}_0\left(t\right)\tilde{\gamma}_{ij}\frac{\Delta}{\tilde{a}^2}\tilde{\gamma}^{ij} - \tilde{\mathcal{W}}_2\left(t\right)\tilde{\gamma}_{ij}\frac{\Delta^2}{\tilde{a}^4}\tilde{\gamma}^{ij} \\
& + \left(\mathcal{U}_0\left(t\right) + \tilde{\mathcal{U}}_0\left(t\right)\right)\gamma_{ij}\frac{\Delta}{a\tilde{a}}\tilde{\gamma}^{ij}\Big),
\end{aligned}
\tag{43}
$$

interaction

where $\chi \equiv a/\tilde{a}$ and

$$
\mathcal{G}_0\left(t\right) = \frac{a^3}{M}\left(t\right), \quad \mathcal{W}_0\left(t\right) = a^3 M\left(t\right) - \chi\left(2\frac{\partial\bar{\mathcal{C}}_0}{\partial\mathscr{W}^1} + 3\chi\frac{\partial\bar{\mathcal{C}}_0}{\partial\mathscr{W}^2}\right)\bar{\mu}_1\left(t\right), \tag{44}
$$

$$
\mathcal{W}_2\left(t\right) = \chi\frac{\partial\bar{\mathcal{C}}_0}{\partial\mathscr{W}^3}\bar{\mu}_1\left(t\right), \quad \mathcal{U}_0\left(t\right) = \chi^2\left(\frac{\partial\bar{\mathcal{C}}_0}{\partial\mathscr{W}^1} + 2\chi\frac{\partial\bar{\mathcal{C}}_0}{\partial\mathscr{W}^2}\right)\bar{\mu}_1\left(t\right). \tag{45}
$$

Outline

Backgroud

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Interacting two copies of GR through auxiliary constraints

Summary & Outlook

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Thank you very much for your attention!