INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

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Primordial Black Holes from Inflation

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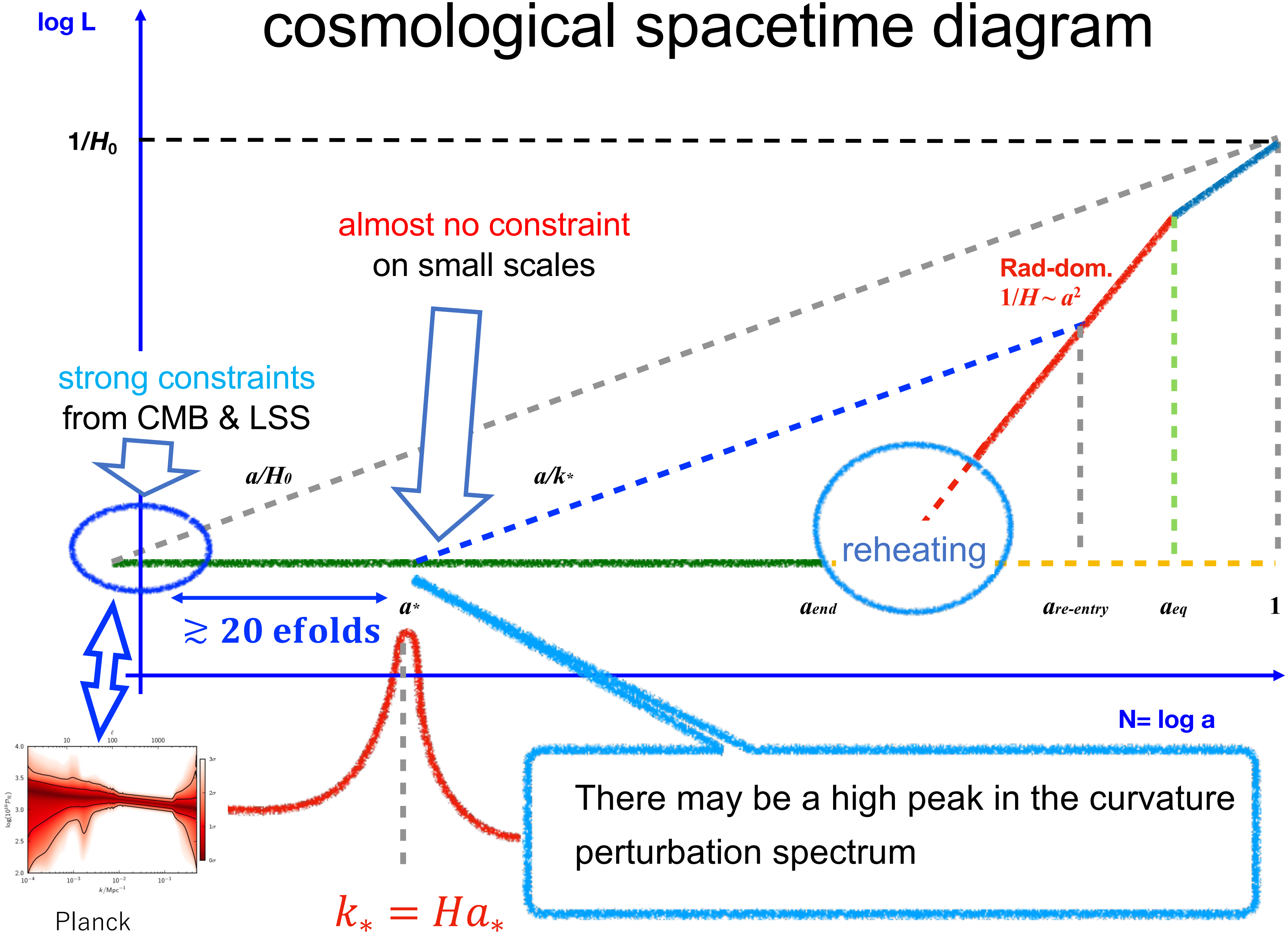
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

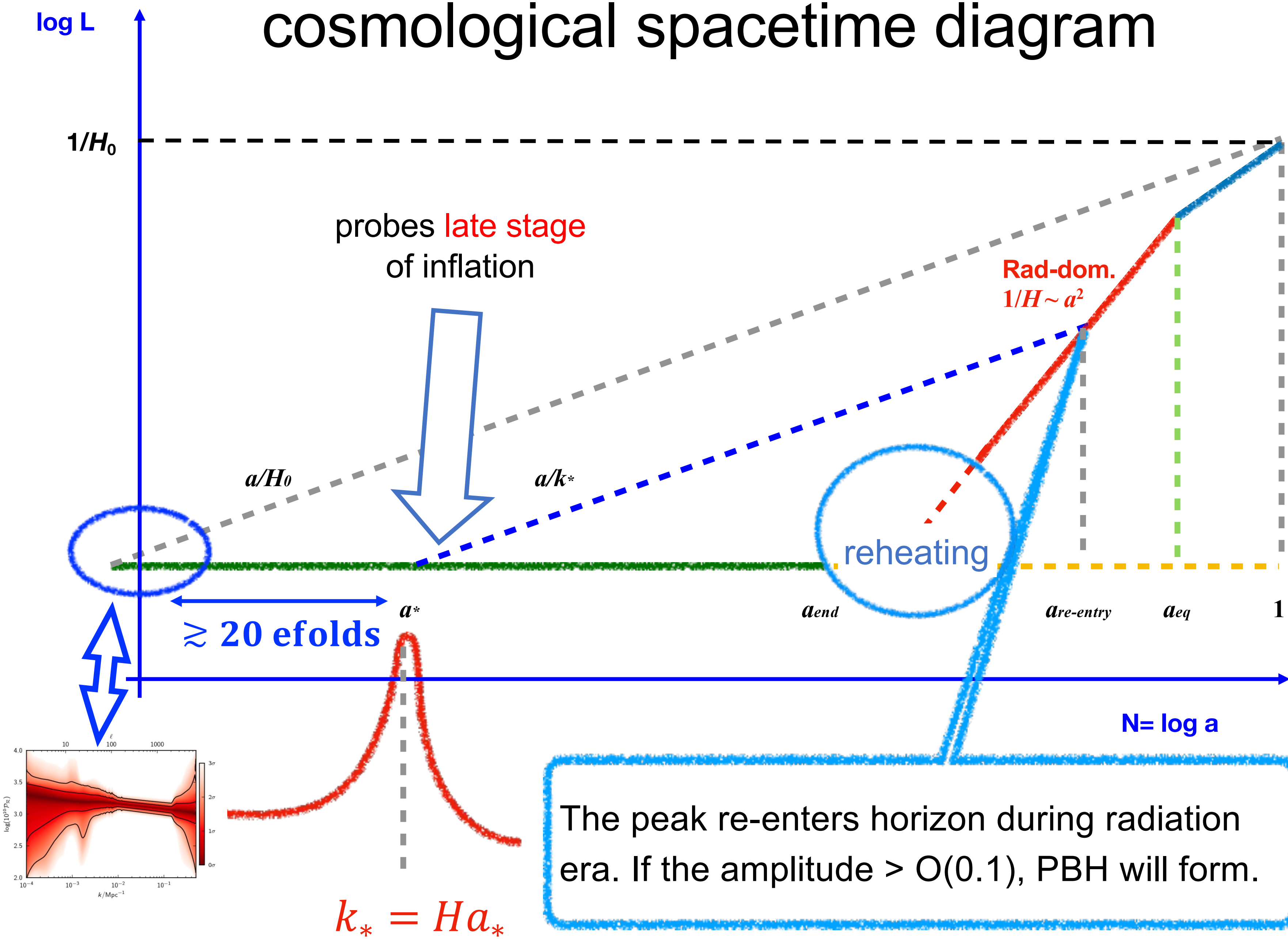
PBH formation

- conventional scenario -

cosmological spacetime diagram



cosmological spacetime diagram

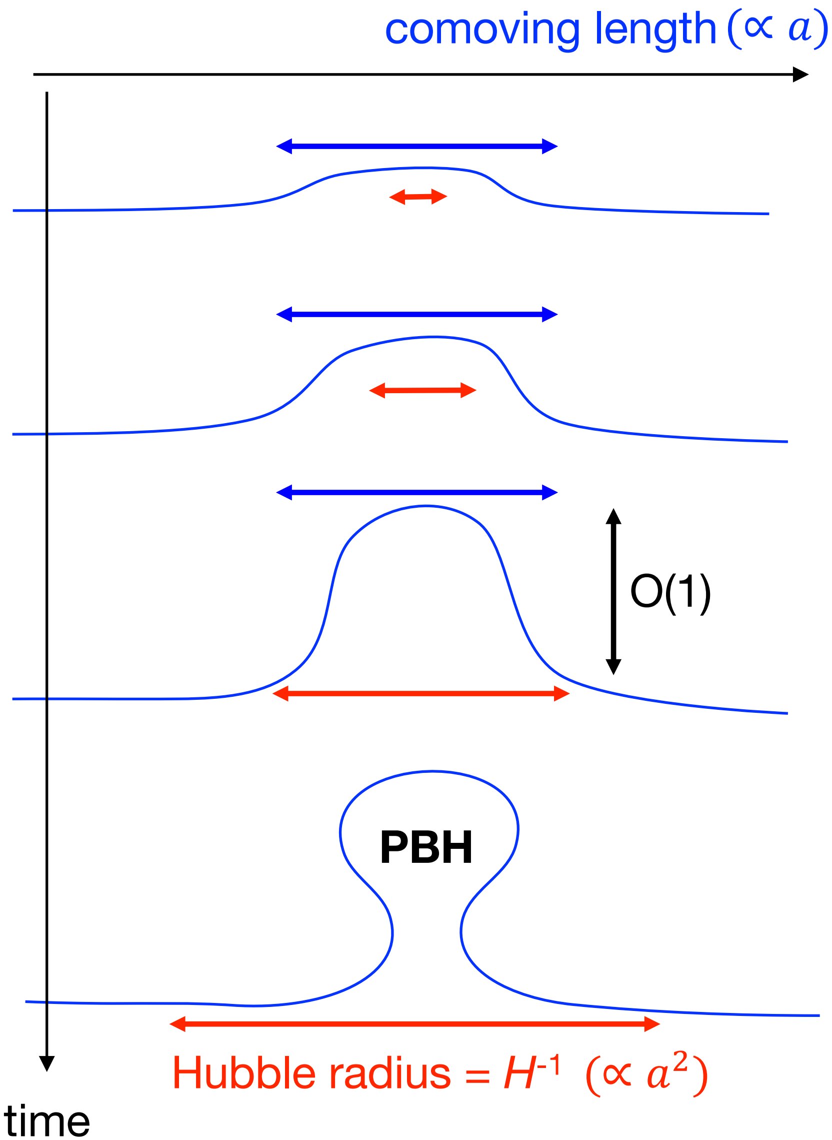


Conventional PBH formation in a nutshell

- Primordial Black Holes (PBHs) are those formed in the very early universe, conventionally when the universe was radiation-dominated.
- Presumably they originate from a large positive curvature perturbation produced during inflation (which hence should be a rare event).
- For a BH to form during radiation dominance, the perturbation must be O(1) on the Hubble horizon scale.

$$M_{\rm PBH} \sim M_{\rm horizon}$$

$$\sim \left(\frac{100\,\mathrm{MeV}}{T}\right)^2 M_\odot \sim \left(\frac{\ell}{1\,\mathrm{pc}}\right)^2 M_\odot$$



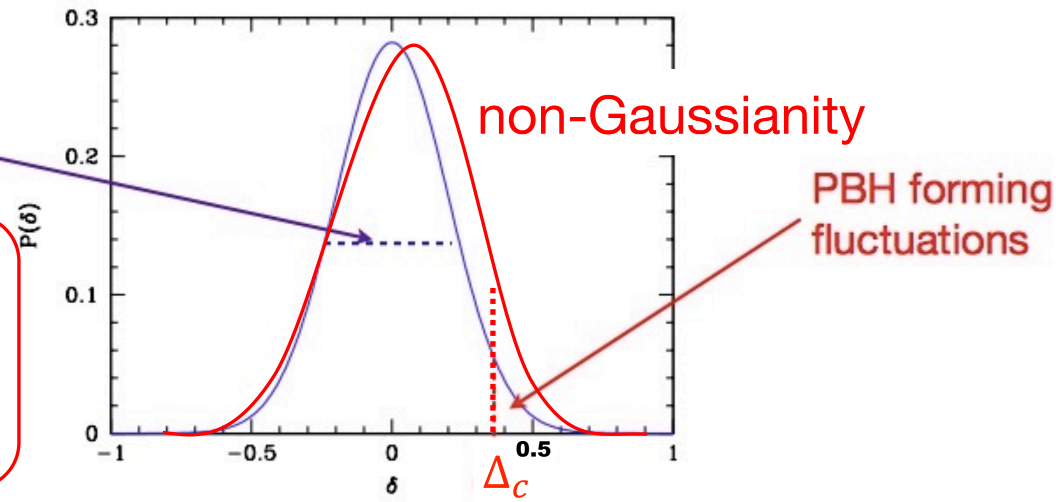
fraction β that turns into PBHs

for Gaussian probability distribution

σ(M_H) (mass variance)
typical size of fluctuations

$$ {}^{(3)}R \approx 6H^2 \frac{\delta\rho}{\rho} $$

$$ {}^{(3)}R = O(H^2) \quad \Longleftrightarrow \quad \frac{\delta\rho}{\rho} = O(1) $$



- When $\sigma_M \ll \Delta_c$, β can be approximated by exponential:

$$ \beta \approx \sqrt{2/\pi}\,\frac{\sigma_M}{\Delta_c}\exp\left(-\frac{\Delta_c^2}{2\sigma_M^2}\right) \qquad \Delta_c \equiv \left(\frac{\delta\rho_c}{\rho}\right)_{\rm crit} \sim 0.4 $$

Carr, ApJ 201, 1 (1975), …

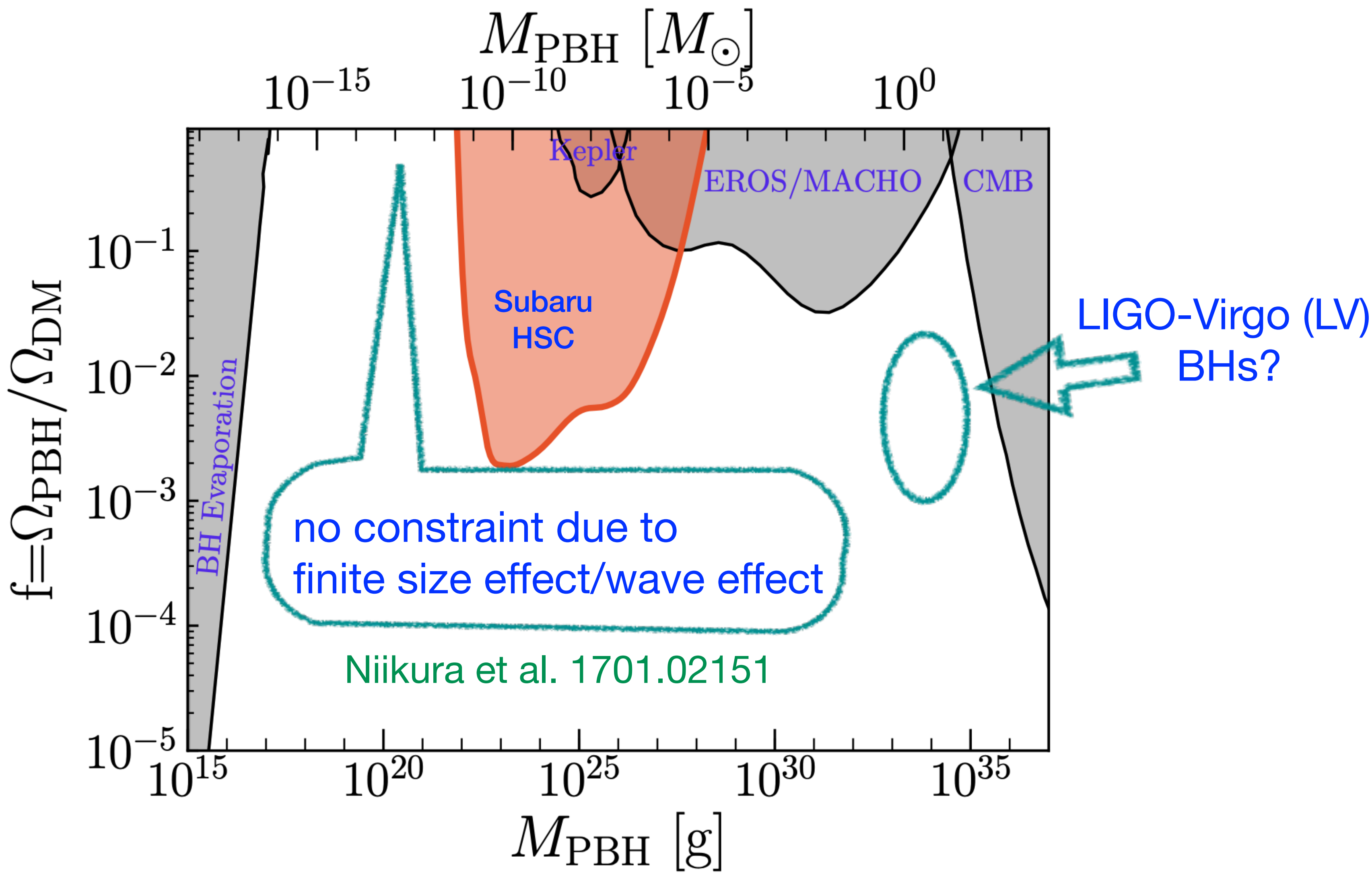
- Recent studies indicates enhanced production: $\Delta_c \sim 0.2$
  using peak theory

Yoo, Harada, Garriga & Kohri, 1805.03946

- Non-Gaussianity may significantly affect β

Young, Regan & Byrnes, 1512.07224, ….

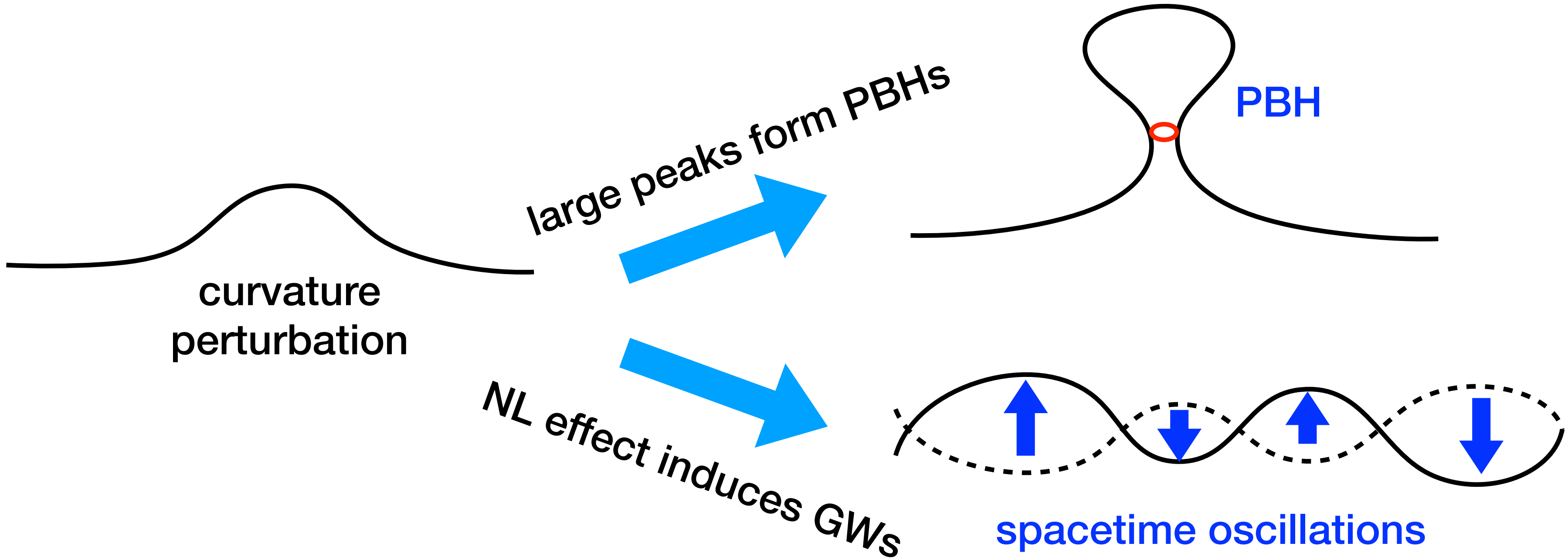
observational constraints



big window at $M_{\rm PBH} \approx 10^{17} - 10^{22}$g ⟷ $T_{\rm re\text{-}entry} \sim 10^4$ -10^8 GeV

induced GWs

GWs can capture PBHs!



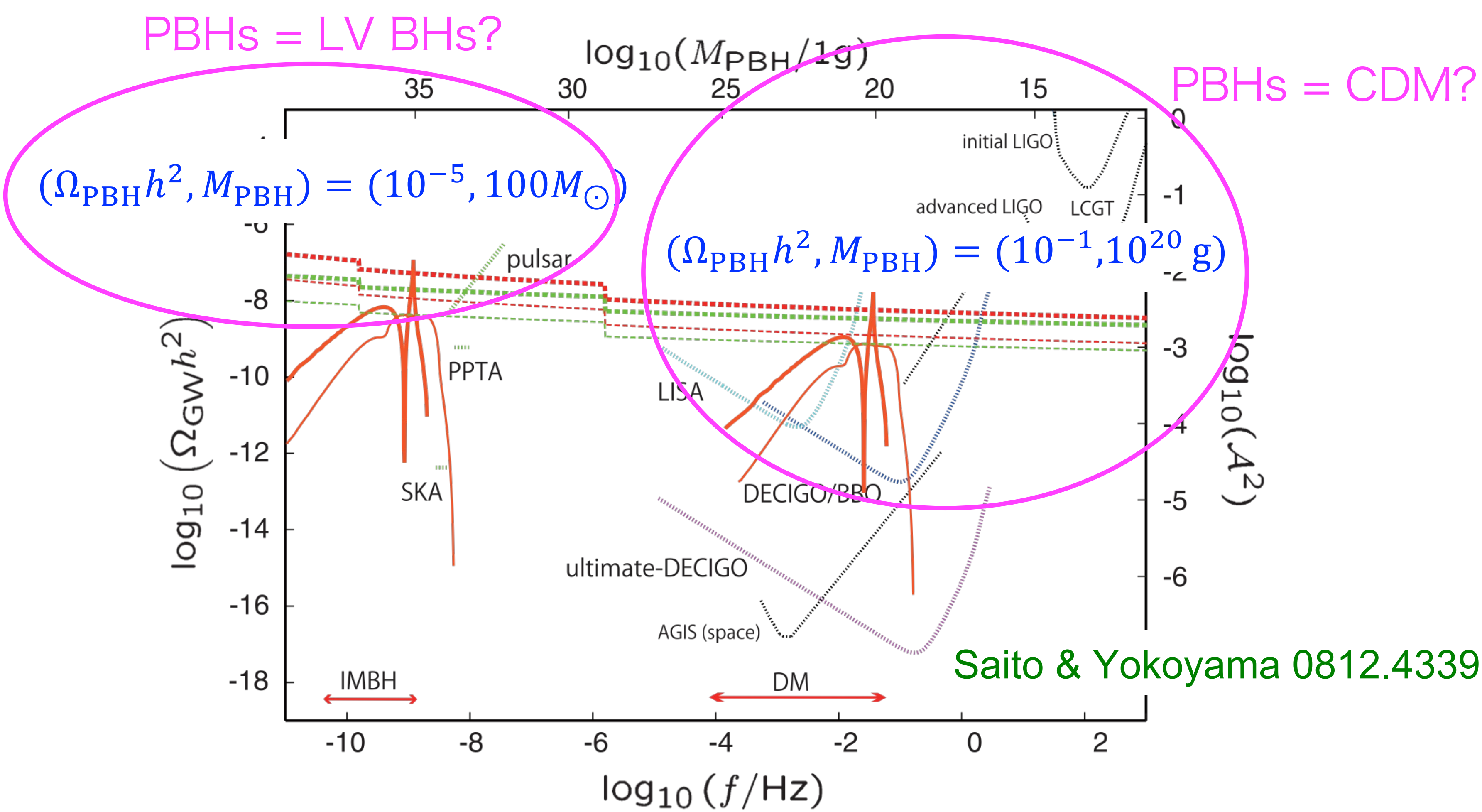
PBHs = CDM with $M_{PBH} \sim 10^{21}$g
generates GWs with f~10^{-3} Hz

⇨ Background GWs
at LISA band

PBHs =LV BHs with $M_{PBH} \sim 10 M_{\odot}$
generates GWs with f~10^{-8} Hz

⇨ Background GWs
at PTA band

GWs can test PBH scenario!



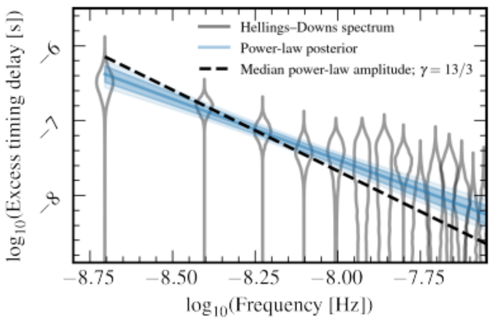
- **PBHs =LV BHs scenario is already constrained by PTA data**

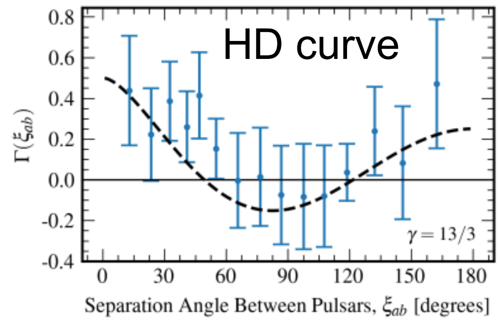
Cai, Pi, Wang & Yang 1907.06372

Recent News from NANOGrav + CPTA + EPTA

Evidence for Stochastic GW Background!?

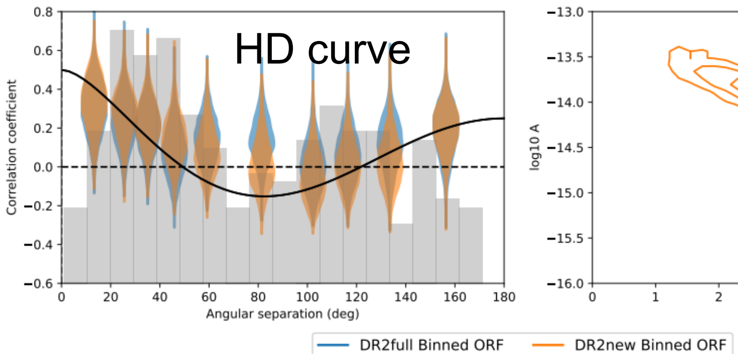
NANOGrav: 2306.16213, EPTA: 2306.16214, CPTA: 2306.16216

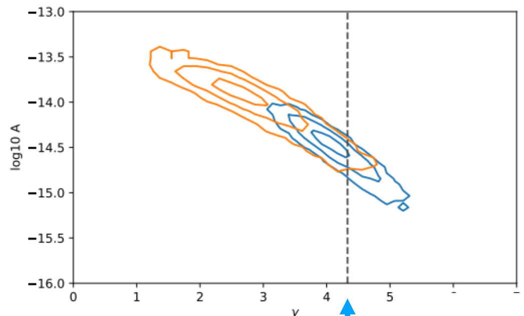




NANOGrav 15 yr data

γ=13/3: expected index if SMBH mergers



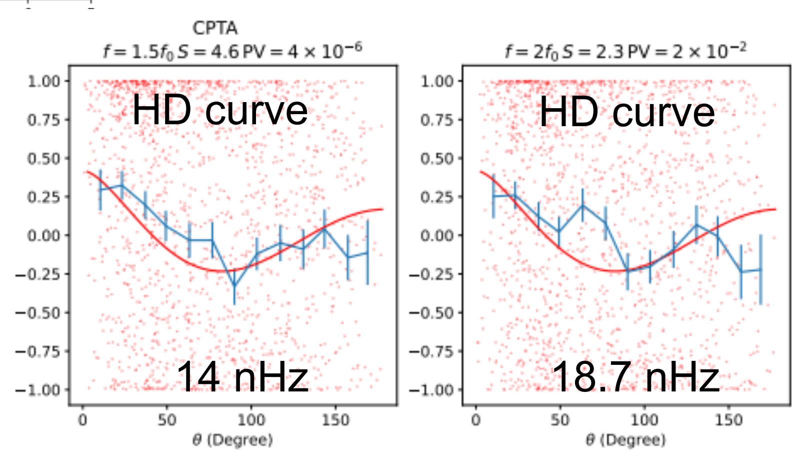


γ=13/3

EPTA 25 yr/10 yr data

CPTA 3 yr data

frequency dependence

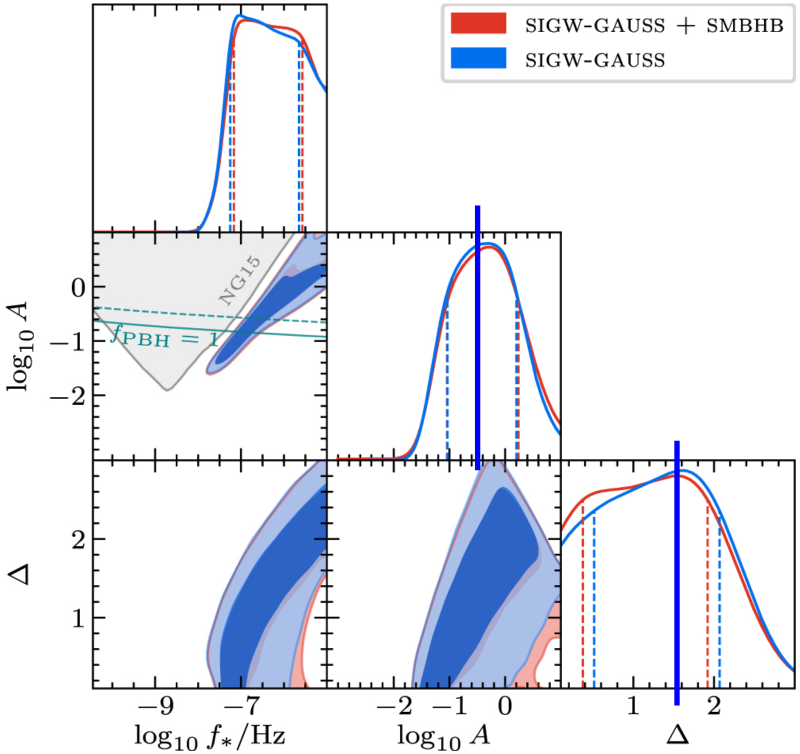


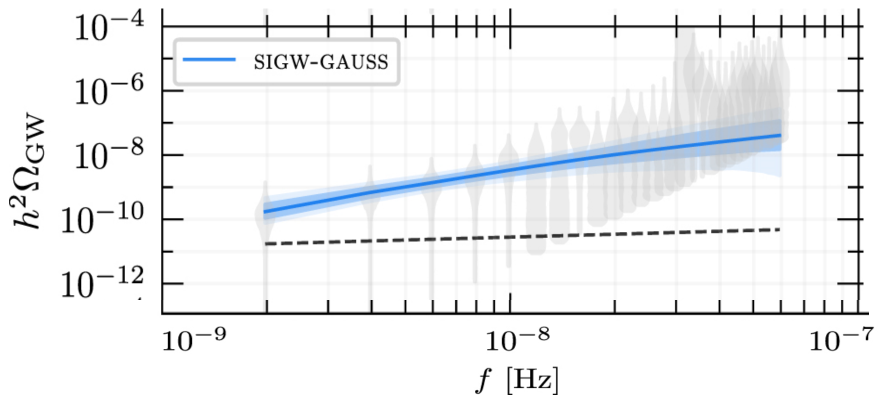
induced GWs?

NANOGrav: 2306.16219

If NANOGrav Data = iGWs, with curvature perturbation spectrum,

$$\mathcal{P}_R(k) = \frac{A}{\sqrt{2\pi}\,\Delta}\exp\left[-\frac{1}{2}\left(\frac{\ln k - \ln k_*}{\Delta}\right)^2\right]$$





$A \sim 10^{-0.4\pm0.6}$

$\Delta \sim 1.6^{+0.4}_{-1.0}$

$\Rightarrow$ $\dfrac{A}{\sqrt{2\pi}\Delta} \sim 0.1$

too large!?

Inflation models

constraints on single-field case

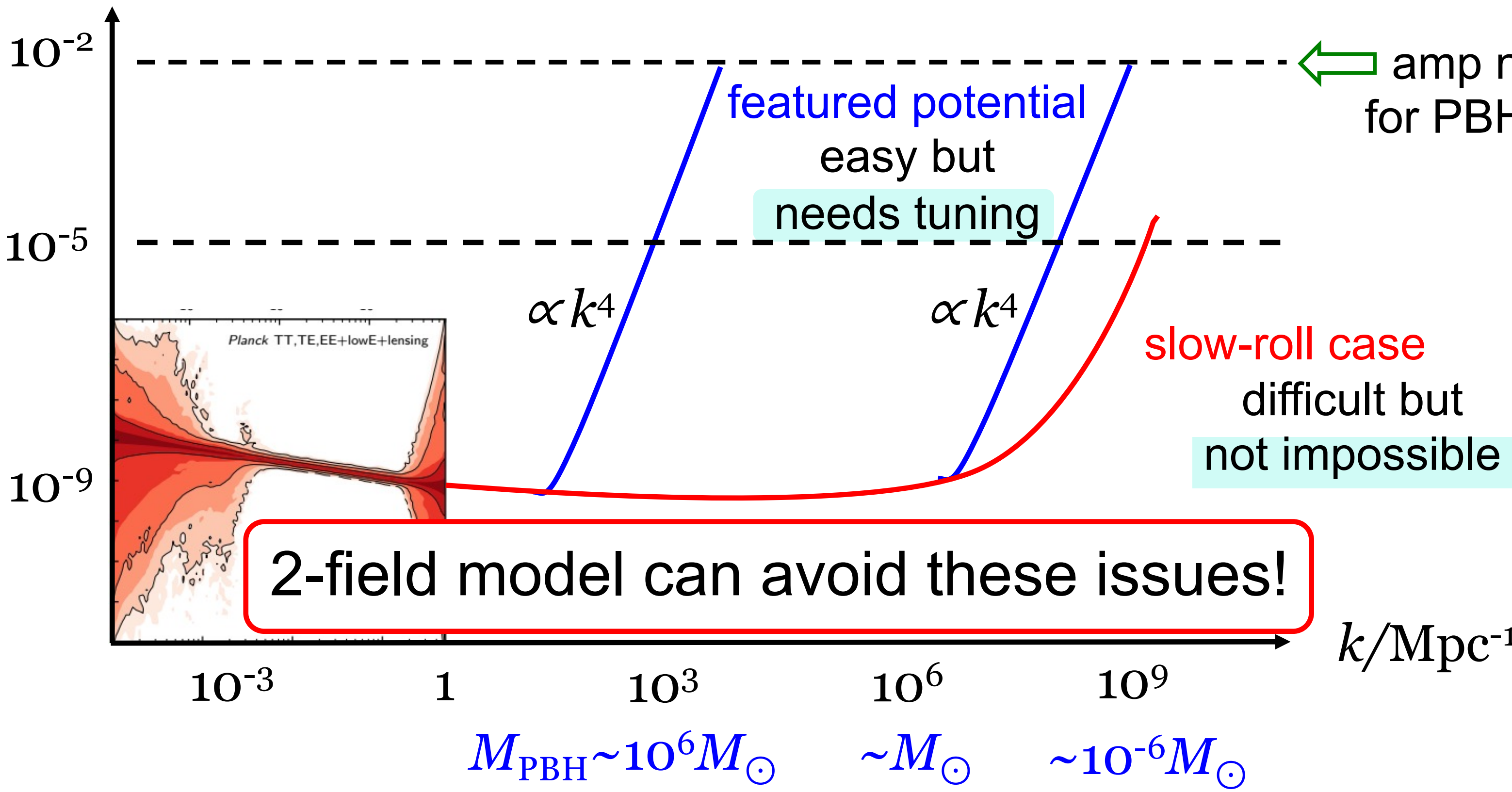
- slow-roll case — Kohri, Lyth & Melchiorri, 0711.5006

$$P_{\mathcal{R}}(k) = P_{\mathcal{R}}(k_*)\exp[(n_s-1)\ln(k/k_*) + n_s' \ln^2(k/k_*)]$$

$n_s \approx 0.9649,\ n_s' \lesssim 0.013$ Planck 2018 X, 1807.06211

- a feature in the potential leads to the spectrum $\propto k^4$

Leach, MS, Wands & Liddle ('01), …



Two-field model 1: 2-stage inflation

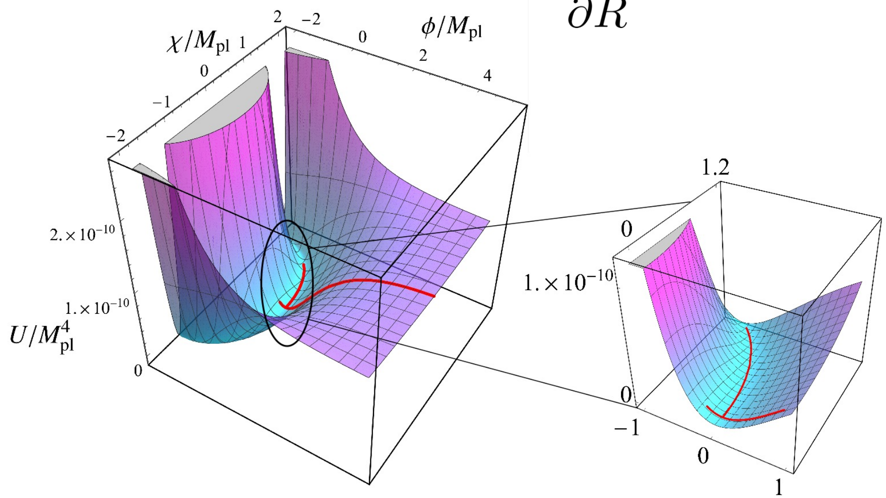
$$S=\int d^4x\sqrt{-g}\left[\frac{M_{pl}^2}{2}f(R)-\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi-V(\chi)\right]$$

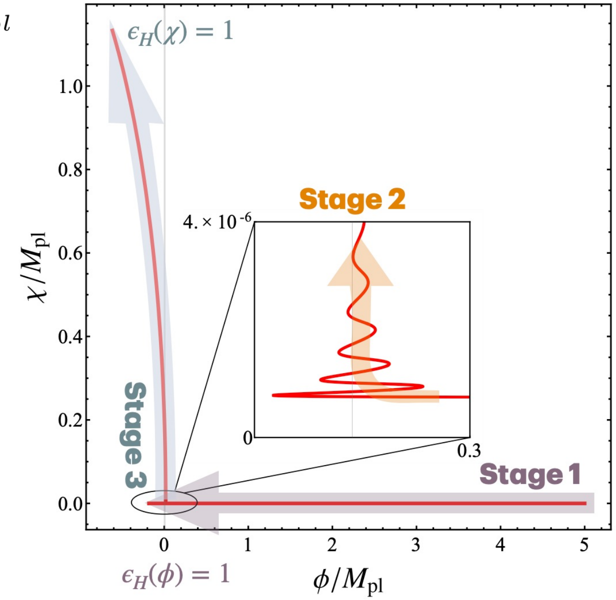
Pi, Zhang, Huang & MS, 1712.09896
Wang, Zhang & MS, 2302.xxxx

$$f(R)=R+\frac{R^2}{6M^2}-\frac{\xi R}{M_{pl}^2}(\chi-\chi_0)^2$$

~ Starobinsky (scalaron) + curvaton

$$\frac{\partial f(R)}{\partial R}\equiv e^{\sqrt{2/3}\,\phi/M_{pl}}$$





- scalaron ϕ becomes massive at the end of Stage 1
- many 2-stage models can lead to PBH formation

eg, Kawasaki et al., 1606.07631

Superhorizon enhancement of curvature perturbation

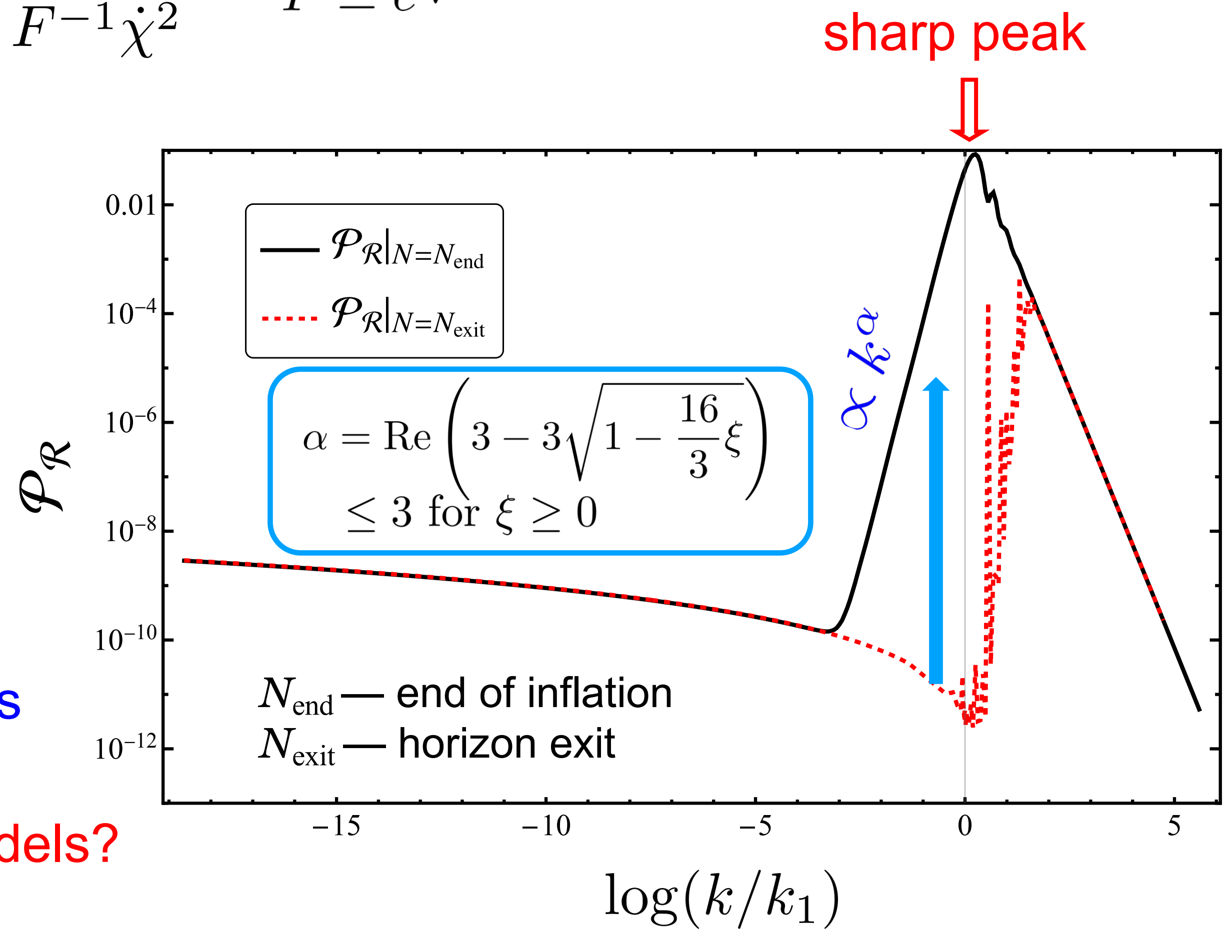
curvature perturbation on comoving slices

$$\mathcal{R} = H\frac{\dot{\phi}\delta\phi + F^{-1}\dot{\chi}\delta\chi}{\dot{\phi}^2 + F^{-1}\dot{\chi}^2} \qquad F \equiv e^{\sqrt{2/3}\,\phi/M_{pl}}$$

- $\mathcal{R}$ is enhanced due to contribution from decaying $\delta\chi$ during 1st stage

- non-Gaussianity seems small

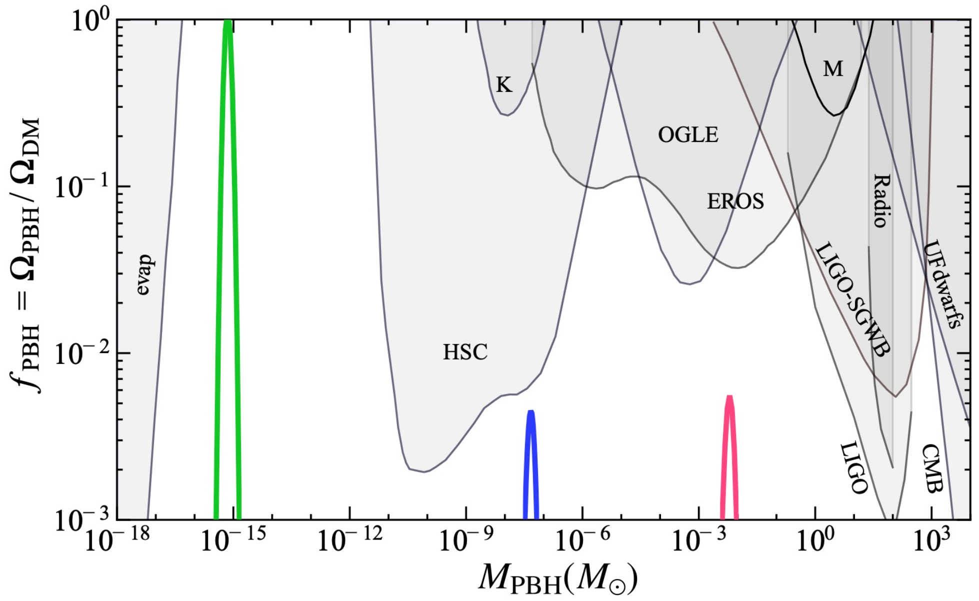
common in 2-stage models?



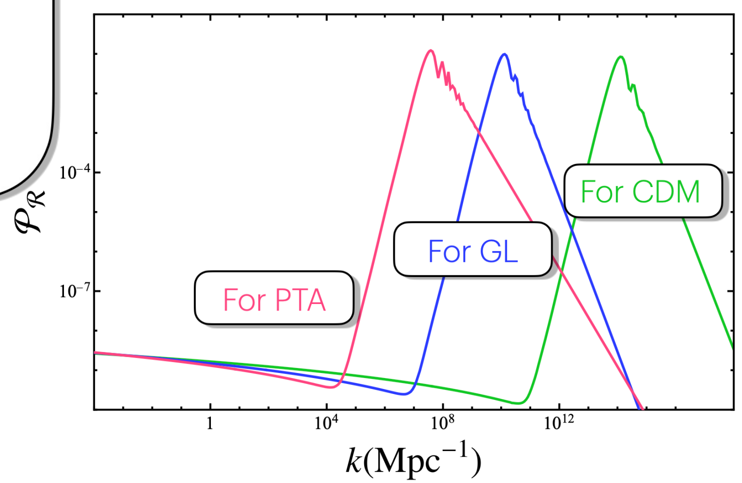
PBH mass function

with criterion based on Press-Schechter formalism

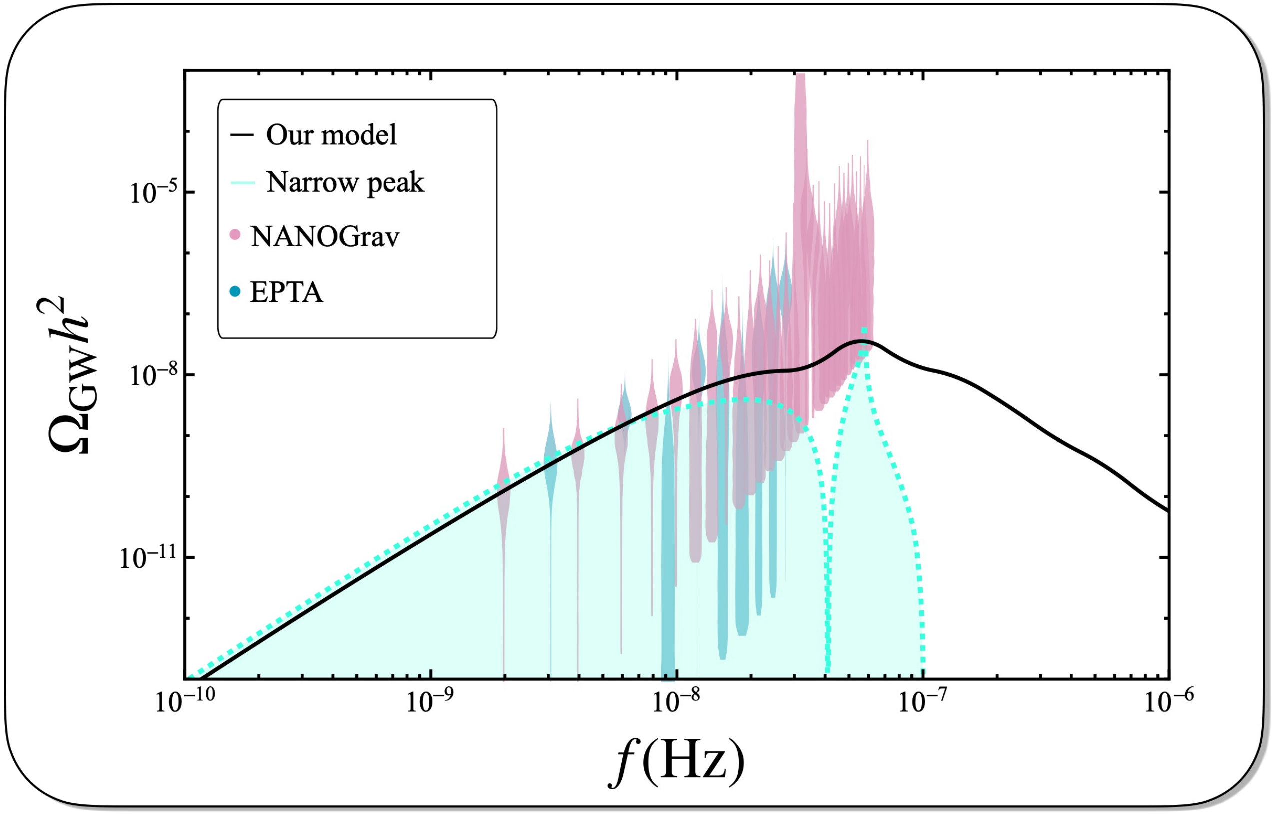
$$\left(\frac{\delta\rho}{\rho}\right)_{cr} \simeq 0.4$$



scalaron+c model can realize PBH=CDM scenario with monochromatic mass function!



can our model explain PTA result?



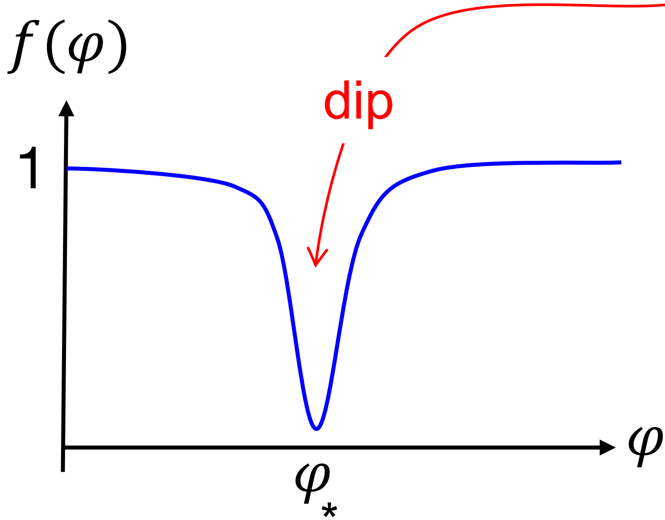
seems possible…

but this is NOT based on statistical analysis

Two-field model 2: non-minimal curvaton

Pi & MS, 2112.12680

$$\mathscr{L} = -\frac{1}{2}(\partial\varphi)^2 - V(\varphi) - \frac{1}{2}\underline{f(\varphi)}^2(\partial\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 \quad (m_\chi^2 \ll H^2)$$

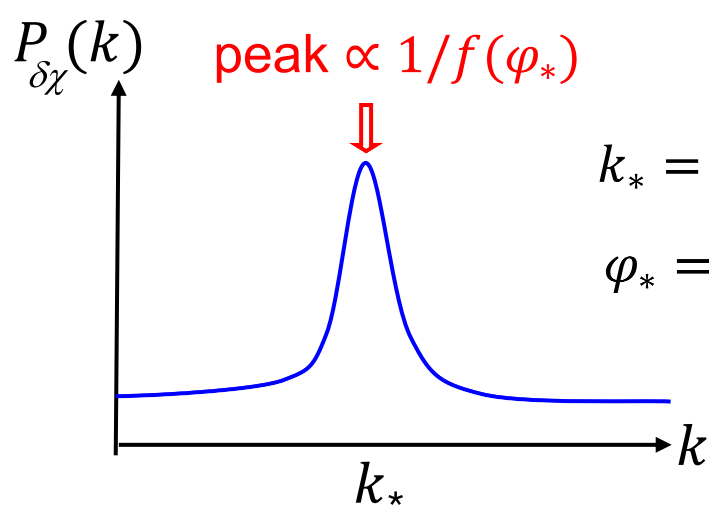


- assume f <<1 when $\varphi \sim \varphi_*$
- vacuum fluctuation: $f(\varphi)\delta x = \dfrac{H}{2\pi}$

$\Rightarrow$ $\delta x = \dfrac{H}{2\pi f}$

- $\delta\chi$ is enhanced at $\varphi = \varphi_*$

$\Rightarrow$ leads to PBH formation



$k_* = Ha(t_*)$

$\varphi_* = \varphi(t_*)$

Highly non-Gaussian curvature perturbation

$$e^{4\zeta} - \left[\frac{4r}{3+r}\left(1+\frac{\delta\chi}{\chi}\right)^2\right]e^{\zeta} + \left[\frac{3r-3}{3+r}\right] = 0$$

MS, Valiviita & Wands, astro-ph/0607627

ζ = curvature perturbation on uniform density slices

$r = \rho_\chi/\rho_{\rm tot}$ at epoch of curvaton decay

- Criterion $\zeta > \zeta_{\rm cr}$~0.5 gives a highly nonlinear expression in $\delta \equiv \delta\chi/\chi$
- For $\delta \equiv \delta\chi/\chi \gg 1,\ r \ll 1,$

$$e^{3\zeta} \approx \frac{4r}{3}\delta^2 \Rightarrow P(\zeta) \approx \frac{1}{r^{1/2}\sigma}\exp\left[-\frac{3e^{3\zeta}}{8r\sigma^2}+\frac{3}{2}\zeta\right]; \quad \sigma^2 = \langle\delta^2\rangle$$

PDF tail is highly non-Gaussian for $\sigma^2 r \gg 1$

important for induced GWs

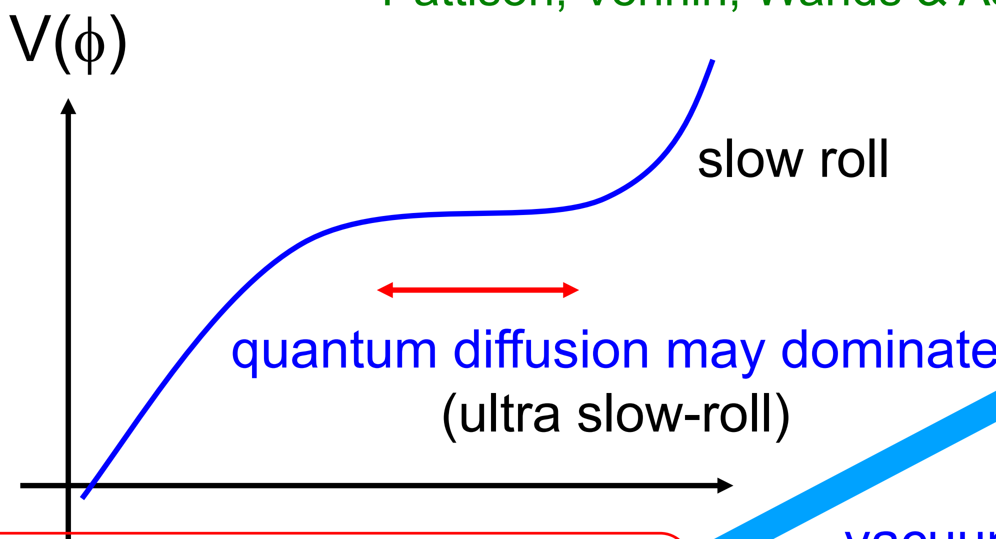
Power spectrum is still perturbative for $r \ll 1$

$$P_{\delta\chi}(k_*) \longrightarrow \zeta = \frac{\delta\chi}{\chi} + \frac{3}{4r}\left(\frac{\delta\chi}{\chi}\right)^2 \text{ for r<<1} \longrightarrow P_\zeta(k_*)$$

single-field with featured potential

Single-field model 1: potential w/ inflection point

Pattison, Vennin, Wands & Assadullahi, 2101.05741, …



$$\mathcal{R} = \delta N = -\frac{H}{\dot{\phi}}\,\delta\phi \sim O(1)$$

$$\delta\phi \sim \frac{\dot{\phi}}{H}$$

vacuum fluctuations $\frac{H}{2\pi}$

classical background motion

tail is exponentially enhanced: exp(- $c\mathcal{R}$) instead of exp(- $c\mathcal{R}^2$)

➡ fully non-Gaussian PBH formation

exponential tail is actually quite common in potential with a feature

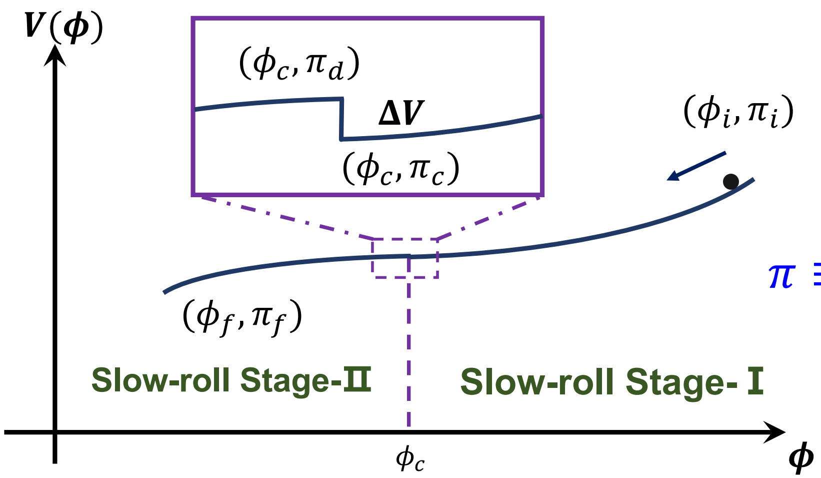
↳ $N \sim \log(a\phi + b)$

Pi & MS, 2211.13932

Single-field model 2: Upward step

- One Small Step for an Inflaton, One Giant Leap for Inflation -

Cai, Ma, MS, Wang & Zhou, 2112.13836



$$\pi \equiv \frac{d\phi}{Hdt}$$

- energy conservation at the step:

$$\pi_d = -\sqrt{\pi_c^2 - 6\Delta V/V}$$

(in $M_{\rm Planck} = 1$ units)

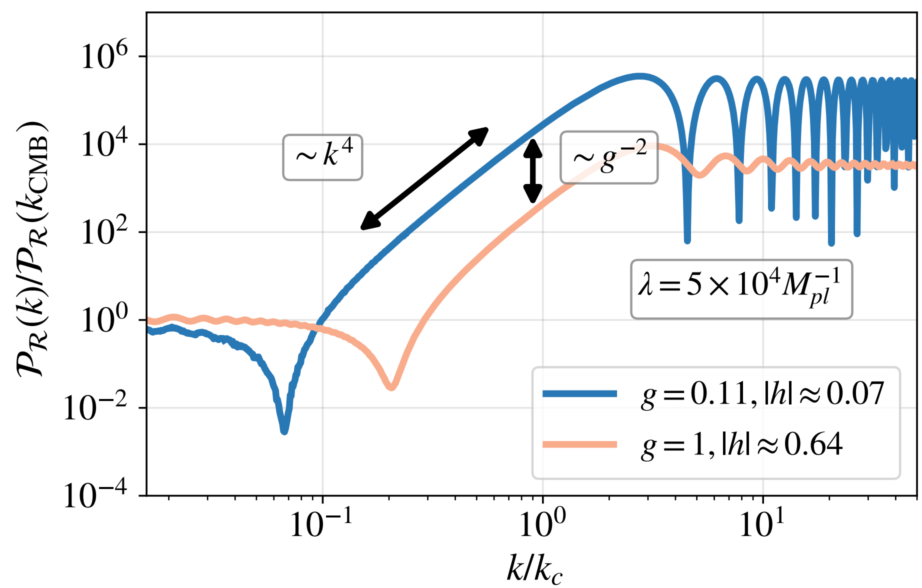
$$\pi_c = -\sqrt{2\epsilon_c}$$

ϵ_c: SR parameter at $\phi = \phi_c$

even for a tiny step, $\Delta V << V$, $P_R(k)$ is enhanced by $1/g^2$ if

$$g \equiv \frac{\pi_d}{\pi_c} \ll 1$$

$$1 - \Delta V/(V\epsilon_c) \ll 1$$



non-perturbative non-Gaussian tail

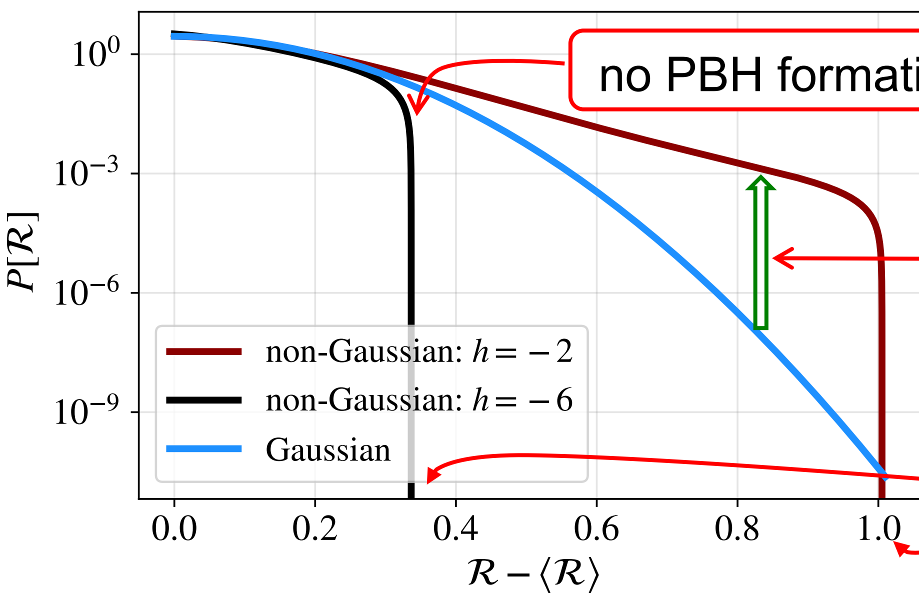
- perturbative non-Gaussian parameters are small if $-h \equiv \dfrac{6\sqrt{2\varepsilon_V}}{|\pi_d|} \ll 1$

$$\mathcal{R} = \mathcal{R}_G + \frac{|h|}{4}\mathcal{R}_G^2 + \frac{|h|^2}{8}\mathcal{R}_G^3 + \cdots \implies \mathcal{P}(k) \approx \mathcal{P}_G(k)$$

power spectrum is given by Gaussian part

- tail of distribution is extremely non-Gaussian

$$\left|\frac{d\mathcal{R}_G}{d\mathcal{R}}\right| \qquad P[\mathcal{R}] = \frac{2-|h|\mathcal{R}}{\Omega}\exp\left[-\frac{\mathcal{R}^2(4-|h|\mathcal{R})^2}{32\sigma_{\mathcal{R}}^2}\right]$$

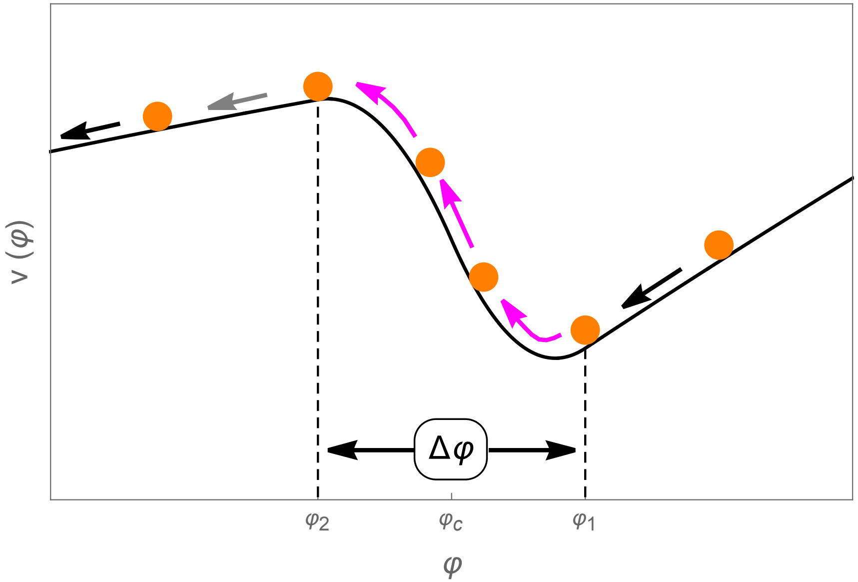


no PBH formation if $\mathcal{R}_{\rm cut} < \mathcal{R}_{\rm crit} \sim 0.5$

exponential enhancement for $|h| < O(1)$

$\mathcal{R}_{\rm cut} \equiv \dfrac{2}{|h|}$ $\cdots$ ∃cutoff

Finite-width upward step



Kawaguchi, Fujita & MS, 2305.18140

$$\omega_{s2} \simeq \frac{\sqrt{2}\,|\pi_1|}{\Delta\varphi}$$ width parameter

zero width limit: $\omega_{s2} \to \infty$

$$\pi \equiv \frac{d\phi}{Hdt}$$

➢ finite width gives rise to exponential tail

$$\mathcal{R} = -\frac{\delta\varphi}{\pi} + \frac{\kappa g}{3}\left(1-\sqrt{1+\frac{2\gamma}{g^2}\delta\varphi+\frac{\gamma^2}{g^2}\delta\varphi^2}\right) - \frac{1}{2\omega_{s2}}\log\left(1+\frac{2\gamma}{g^2}\delta\varphi+\frac{\gamma^2}{g^2}\delta\varphi^2\right)$$

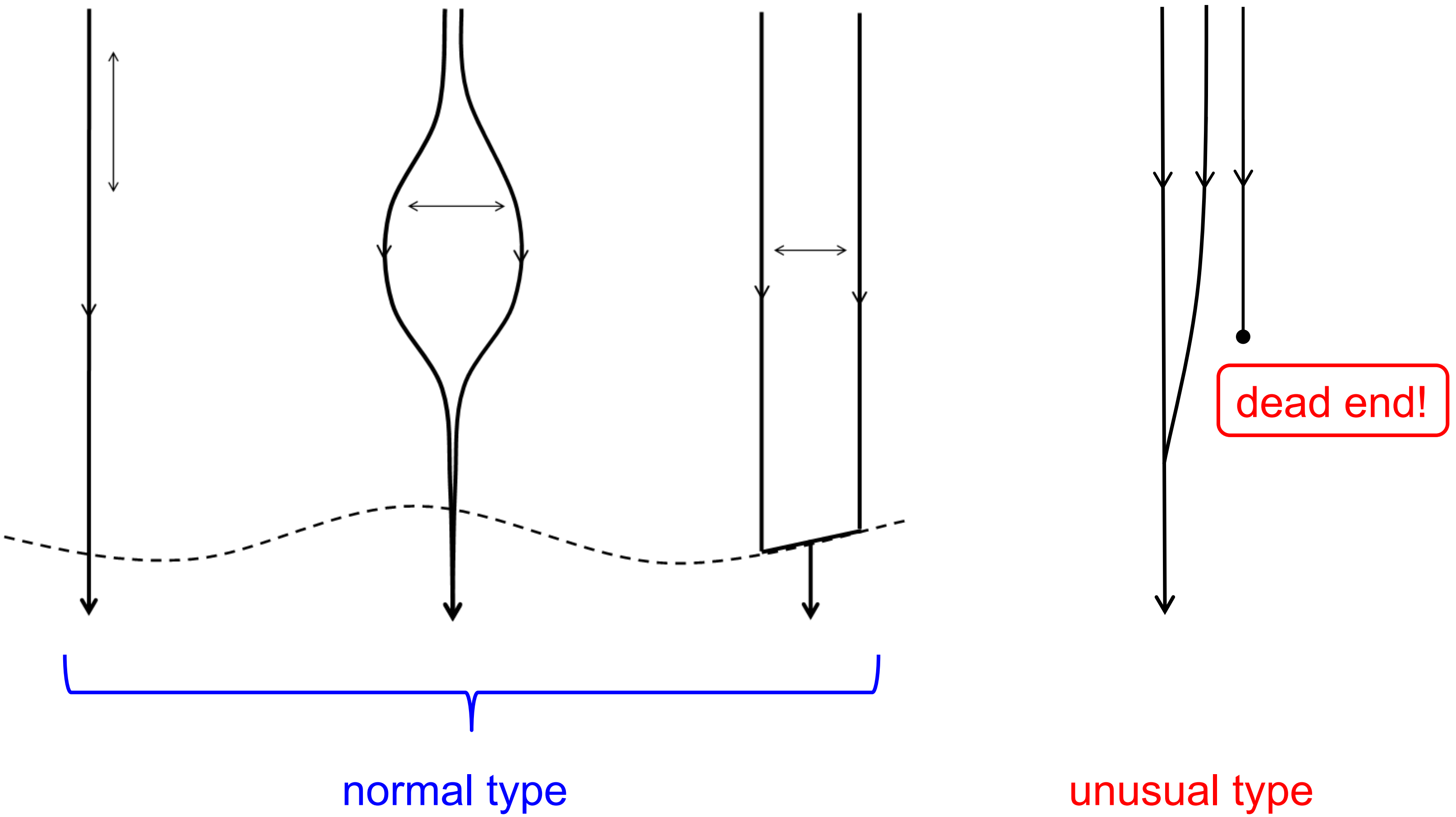
$$\left|\frac{d\delta\varphi}{d\mathcal{R}}\right| \propto \exp[-2\omega_{s2}\mathcal{R}]$$

$$\kappa \equiv \sqrt{\frac{\epsilon_{V1}}{\epsilon_{V2}}}$$

$$g \equiv \frac{\pi_2}{\pi_1} < 1 \qquad \gamma \propto -V''$$

PDF: $P(\mathcal{R}) = P(\delta\phi)\left|\frac{d\delta\phi}{d\mathcal{R}}\right|$

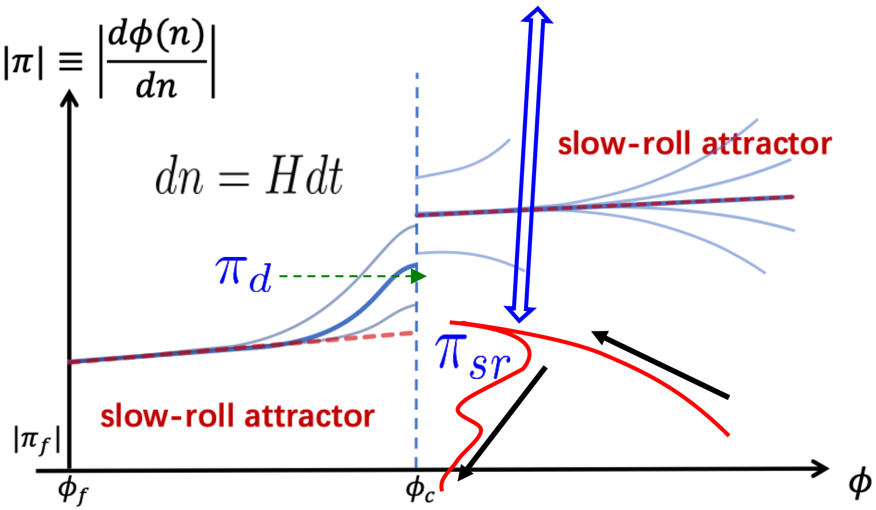
unusual type of field space trajectories

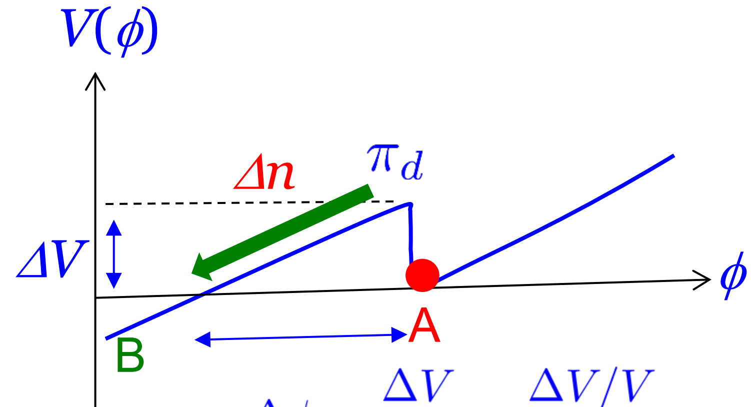


PBH formation during inflation

$$\mathcal{R} \simeq \frac{2}{|h|}\left(1-\sqrt{1-|h|\mathcal{R}_G}\right)$$ PDF cutoff at $\mathcal{R} = \mathcal{R}_{\rm cut} \equiv \frac{2}{|h|}$

⇨ trajectories that can't climb the step





$$\Delta\phi = \frac{\Delta V}{V'} = \frac{\Delta V/V}{\sqrt{2\epsilon}}$$

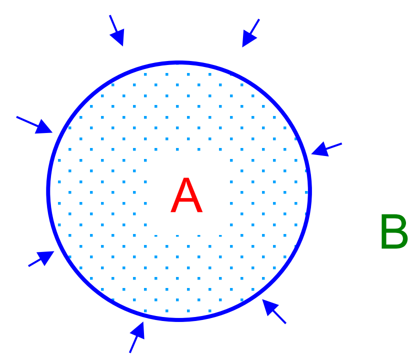
$$\Delta n \simeq \frac{\Delta\phi}{|\pi_{sr}|} = \frac{\Delta V/V}{\pi_{sr}^2}$$

⇧ # of e-folds region A expands

region stuck at $\phi=\phi_c$ will become PBH!

$$\text{Prob}\sim\exp[-(2\sigma_R^2 h^2)^{-1}]$$

region A expands until $V(\phi)$ surrounding it becomes smaller than $V(\phi_A)=V_0$



$$M_{\rm BH} \simeq (M_{pl}/H)e^{\alpha\Delta n}$$

$\alpha = 2 \sim 3$ depending on EOS after inflation

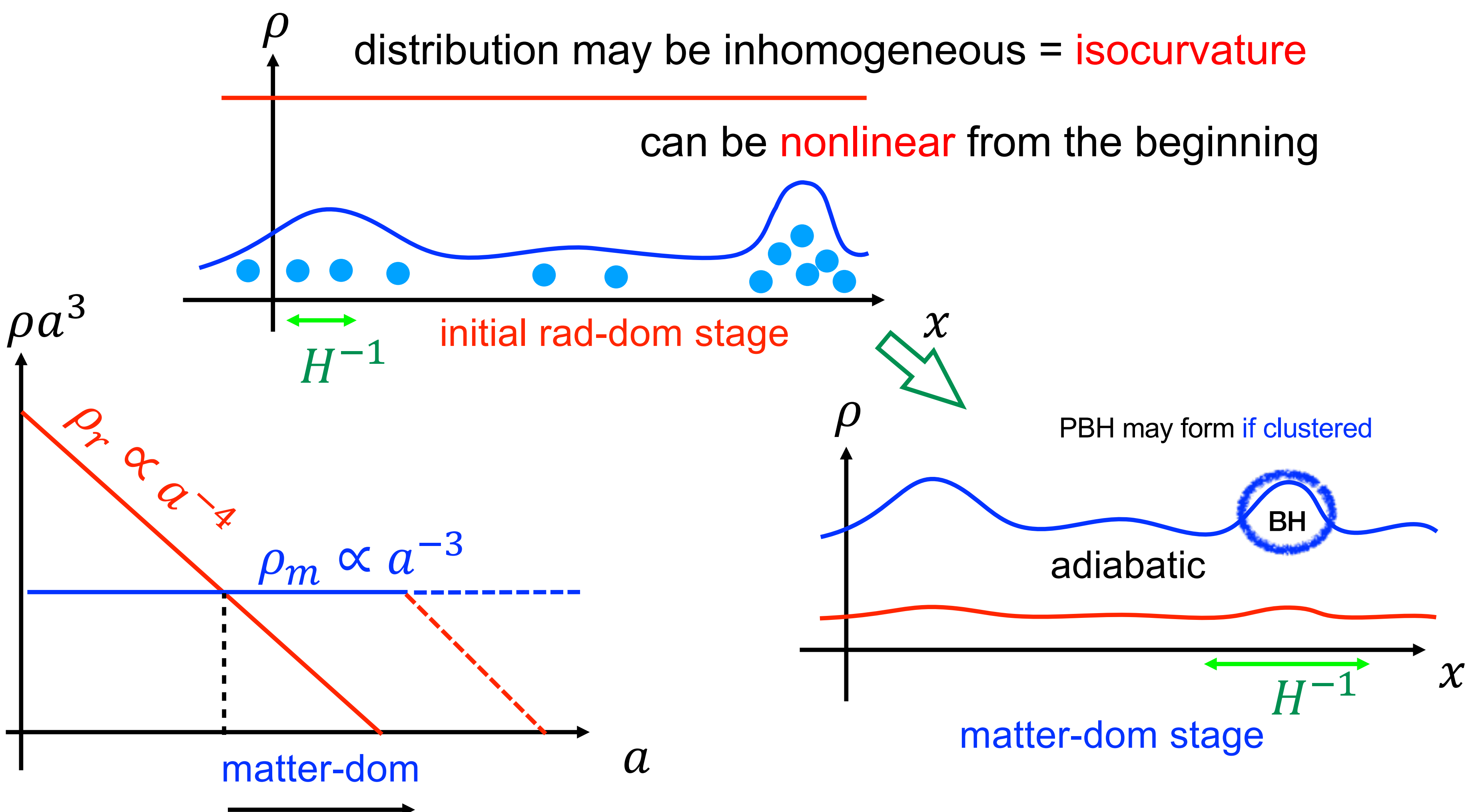
Deng & Vilenkin,1710.02865

Isocurvature

PBHs from Isocurvature Perturbation

eg, E. Cotner, A. Kusenko, MS & V. Takhistov,1907.10613

non-grav formation of compact objects/Q-balls/etc inside horizon.



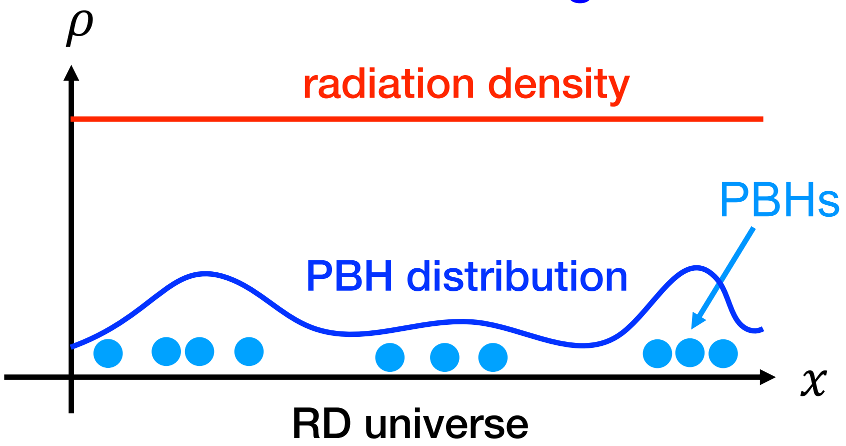
rad-dom again if objects eventually decay

What if formed objects are PBHs?

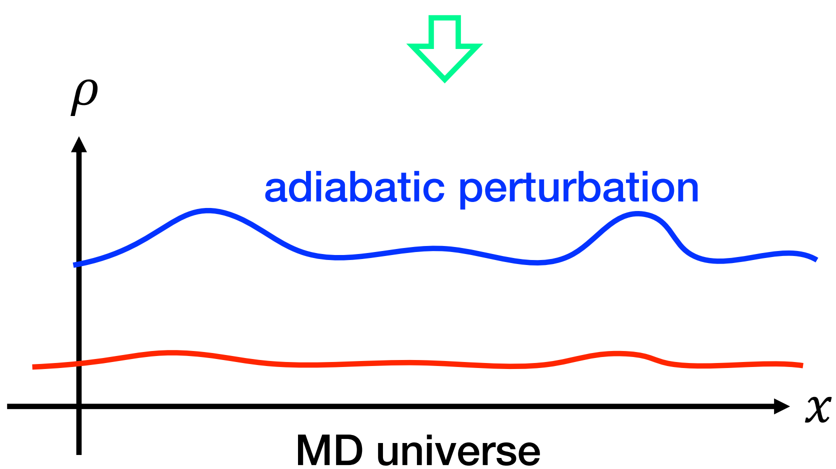
Papanikolaou, Vennin & Langlois, 2010.11573
Domenech, Lin & MS, 2012.08151

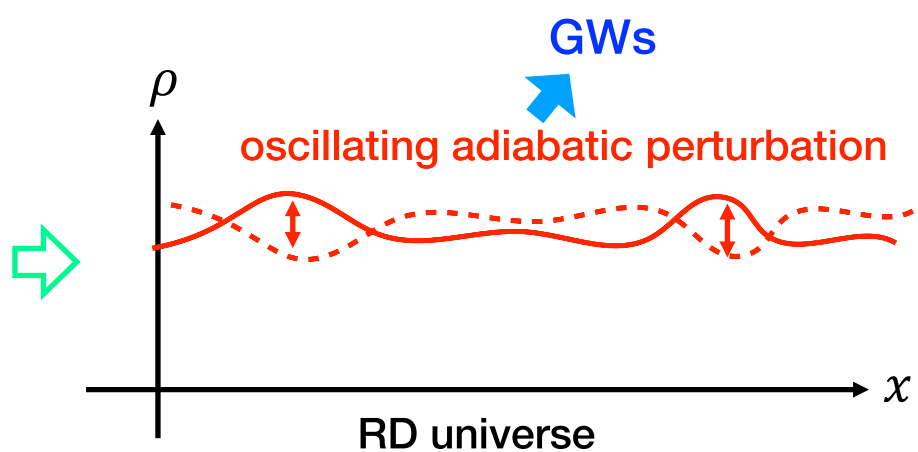
- PBHs may be formed from curvature perturbation or by alternative strong force

Flores & Kusenko, 2008.12456



inhomogeneous PBH distribution may induce GWs when the universe is reheated by PBH evaporation





Constraints on early PBH dominated universe

Domenech, Lin & MS, 2012.08151
Domenech, Takhistov & MS, 2105.06816

- Assumptions
  - Monochromatic mass function for PBHs.
  - Poisson distribution for $\delta n_{\rm PBH}/n_{\rm PBH}$:

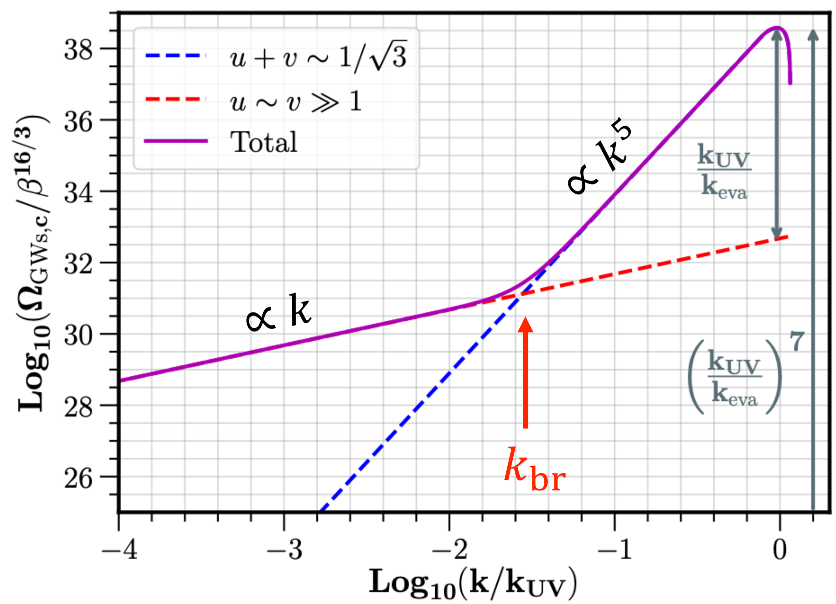
$$\mathcal{P}_S(k) = \frac{2}{3\pi}(k/k_{\rm UV})^3;\, k < k_{\rm UV} = n_{\rm PBH}^{-1/3}$$

- Resulting spectrum
  - sharp rise ~ k^5 near the peak.
  - Peak value:

$$\left(\frac{\Omega_{GW,max}}{\Omega_{r,0}}\right) \approx 5\times10^{34}\beta^{16/3}\left(\frac{M}{10^4\mathrm{g}}\right)^{14/3}$$

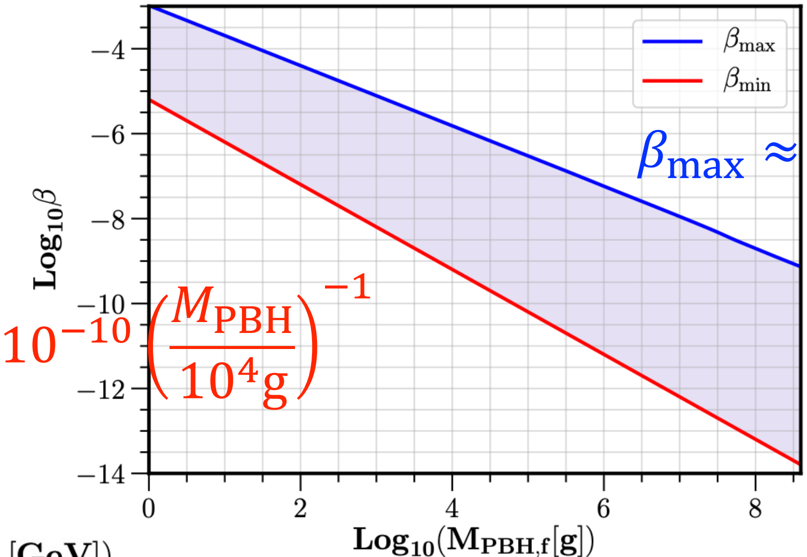
β: PBH fraction at formation

constraints on β



$$k_{\rm br} \approx 0.04 k_{\rm UV}(M_{\rm PBH}/10^4\mathrm{g})^{-1/6}$$

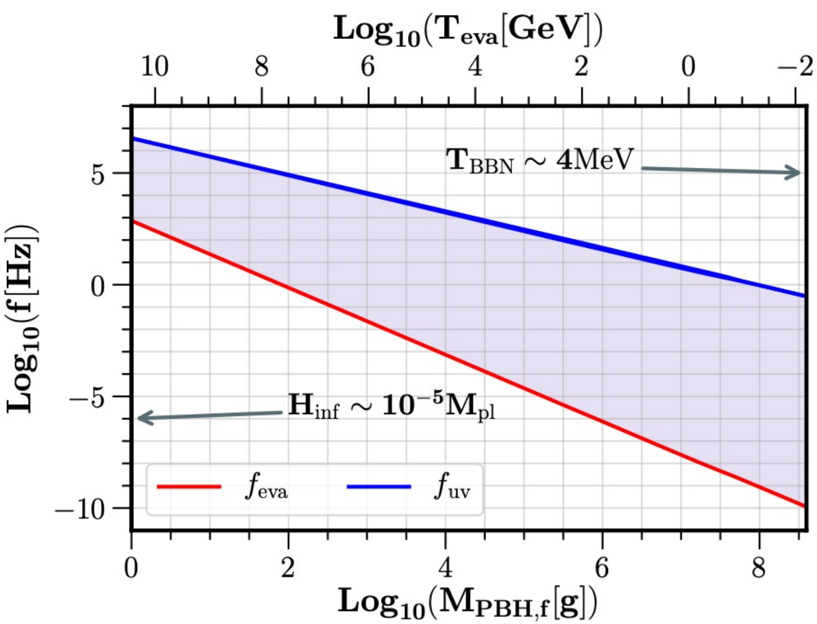
Constraints on β and frequencies



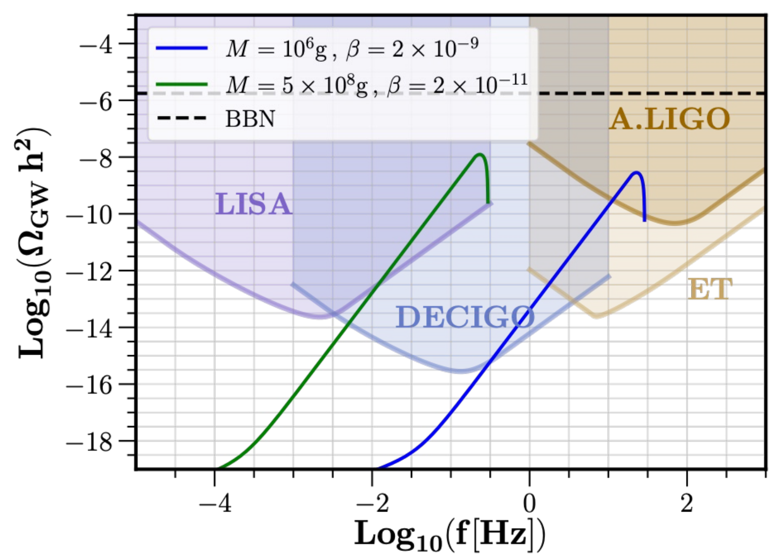
$$\beta_{\max} \approx 1\times10^{-6}\left(\frac{M_{\mathrm{PBH}}}{10^4\mathrm{g}}\right)^{-17/24}$$

$$\beta_{\min} \approx 6\times10^{-10}\left(\frac{M_{\mathrm{PBH}}}{10^4\mathrm{g}}\right)^{-1}$$

allowable range of β



frequency range vs MPBH



GW detectors sensitivity curves

summary

- various inflation models can lead to PBH formation
- late stage of inflation can be probed by PBHs and the associated secondary/induced GWs
- (nonlinear) isocurvature perturbations may play important roles in PBH cosmology
- PBHs may play central roles in GW cosmology

PBH-GW Cosmology!