Rotation of the CMB polarisation by foreground lensing

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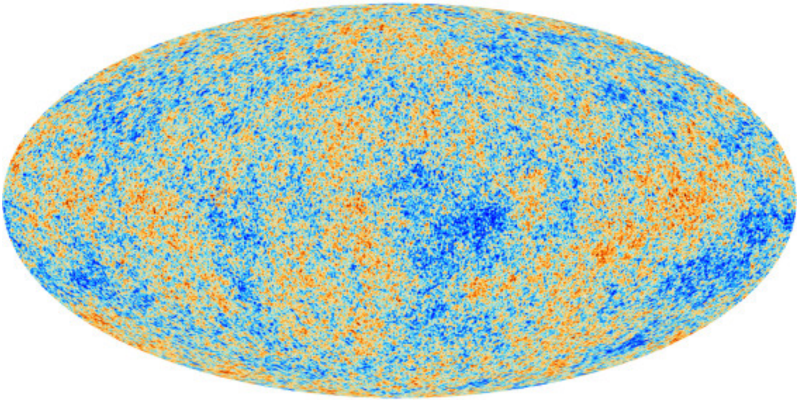
Context

Some time ago there have been several papers on CMB lensing at second order and beyond:

1. Pratten and Lewis [arXiv:1605.05662],
2. Marozzi, Di Dio, Fanizza and Durrer [arXiv:1605.08761],
3. Lewis and Pratten [arXiv:1608.01263],
4. Marozzi, Di Dio, Fanizza and Durrer [arXiv:1612.07263],
5. Marozzi, Di Dio, Fanizza and Durrer [arXiv:1612.07650],
6. Lewis, Hall and Challinor [arXiv:1706.02673].
7. Di Dio, Durrer, Fanizza and Marozzi, [arXiv:1905.12573]

Papers 4 & 5 disagree with 3 & 6 in saying that at second order lensing, the parallel propagation of polarisation leads to rotation. Furthermore they find that this effect is relatively large and therefore will be observable with future CMB experiments (S4). Papers 3 & 6 find no rotation and obtain unobservabley small changes from lensing beyond the Born approximation in both, temperature and polarisation.

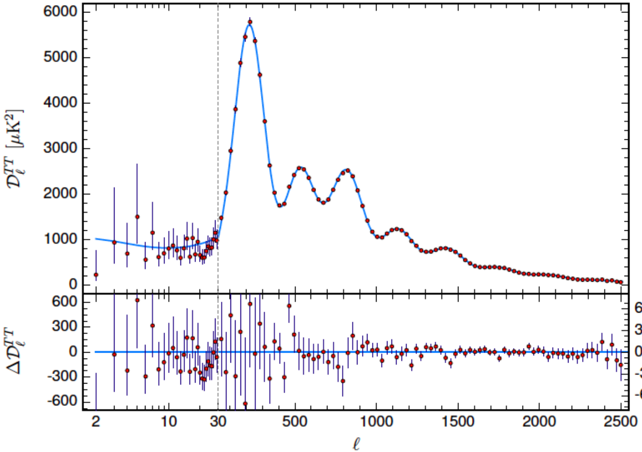
Outline



(ESA/Planck Collaboration)

The Cosmic Microwave Background (CMB) is our most precious dataset in cosmology.

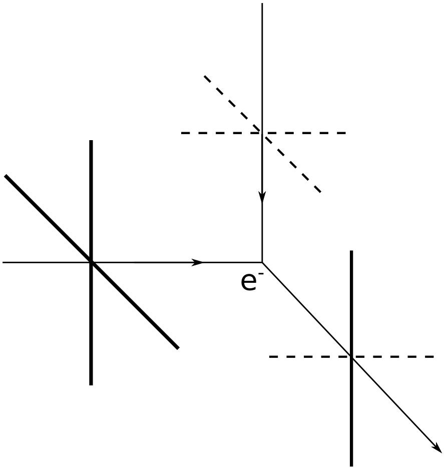
The CMB spectrum



The Planck Collaboration, 2018

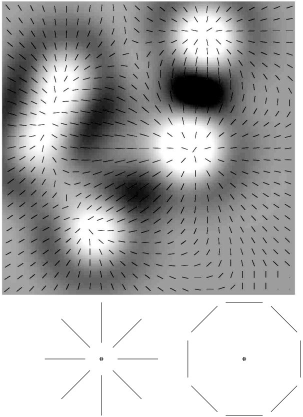
$$\Delta T(\mathbf{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m} \qquad C_\ell = \langle a_{\ell m} a^*_{\ell m} \rangle \qquad \mathcal{D}_\ell = \ell(\ell+1)C_\ell/(2\pi)$$

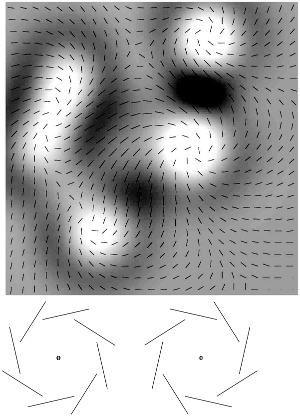
Thomson scattering depends on the polarisation direction relative to the outgoing photon. An incoming intensity quadrupole leads to an outgoing polarisation.



CMB polarization

Polarisation which forms a gradient field is so called E-mode polarisation while a curl field is B-mode polarisation.
At first order, only vector and tensor perturbations generate B-modes





A rotation by $\pi/4$ turns pure E-modes into pure B-modes and vice versa.

In principle polarisation is a helicity-2 tensor on the sphere and has to be expanded in spin-2 spherical harmonics, ${}_{\pm 2}Y_{\ell m}$. For simplicity here we discuss it in the flat sky approximation which is sufficient to describe a small portion of the sky and therefore lensing which is mainly relevant on small angular scales.

$$\mathbf{n} = \mathbf{n}_0 + \mathbf{x}, \qquad (\mathbf{n}_0 \cdot \mathbf{x}) = 0\,.$$

To define the Stokes parameters we introduce a basis $\mathbf{e}_1$, $\mathbf{e}_2$ in the sky normal to $\mathbf{n}_0$. The corresponding helicity basis is $\mathbf{e}_\pm = (\mathbf{e}_1 \pm i\mathbf{e}_2)/\sqrt{2}$ and

$$\mathcal{P} \equiv Q + iU = \mathcal{P}_{++} \quad \mathcal{P}^* \equiv Q - iU = \mathcal{P}_{--}\,.$$

$$Y_{\ell m}(\mathbf{n}) \mapsto \frac{1}{2\pi}\exp(i\boldsymbol{\ell}\cdot\mathbf{x})\,, \quad {}_{\pm 2}Y_{\ell m}(\mathbf{n}) \mapsto -\frac{\ell^2}{2\pi}\exp(i\boldsymbol{\ell}\cdot\mathbf{x})e^{\mp 2i\phi_\ell}$$

The CMB polarization spectra

In the flat sky approximation the polarisation correlation function between two points $\mathbf{x}$ and $\mathbf{x}'$ with $\mathbf{r} = \mathbf{x}' - \mathbf{x}$ can be written as

$$\begin{aligned}
\xi_+(r) &= \langle \mathcal{P}_r(\mathbf{x})\mathcal{P}_r^*(\mathbf{x}')\rangle = \langle Q_r(\mathbf{x})Q_r(\mathbf{x}')\rangle + \langle U_r(\mathbf{x})U_r(\mathbf{x}')\rangle = \langle \mathcal{P}(\mathbf{x})\mathcal{P}^*(\mathbf{x}')\rangle \\
\xi_-(r) &= \langle \mathcal{P}_r(\mathbf{x})\mathcal{P}_r(\mathbf{x}')\rangle = \langle Q_r(\mathbf{x})Q_r(\mathbf{x}')\rangle - \langle U_r(\mathbf{x})U_r(\mathbf{x}')\rangle = \langle e^{-4i\phi_r}\mathcal{P}(\mathbf{x})\mathcal{P}(\mathbf{x}')\rangle \,.
\end{aligned}$$

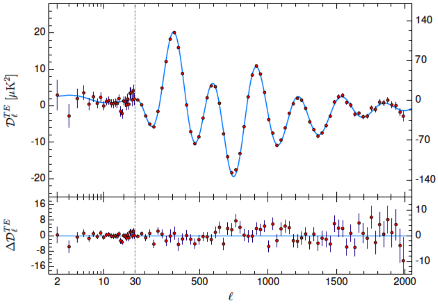
Here $\mathcal{P}_r = \exp(-2i\phi_r)\mathcal{P}$ is the polarisation with respect to the direction of $\mathbf{e}_1 \equiv \hat{\mathbf{r}}$. This can be expressed in terms of the polarisation power spectra,

$$\begin{aligned}
\xi_+(r) &= \frac{1}{2\pi}\int_0^\infty \ell d\ell [C_\ell^{\mathcal{E}} + C_\ell^{\mathcal{B}}] J_0(\ell r) \\
\xi_-(r) &= \frac{1}{2\pi}\int_0^\infty \ell d\ell [C_\ell^{\mathcal{E}} - C_\ell^{\mathcal{B}}] J_4(\ell r)
\end{aligned}$$

The inverse relation gives

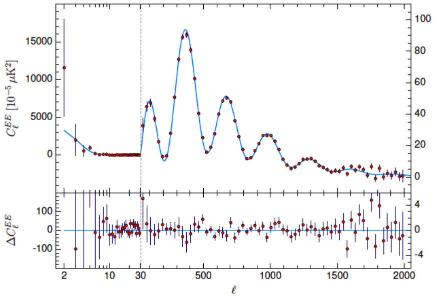
$$\begin{aligned}
C_\ell^{\mathcal{E}} &= \pi\int_0^\infty r dr [\xi_+(r) J_0(\ell r) + \xi_-(r) J_4(\ell r)] \\
C_\ell^{\mathcal{B}} &= \pi\int_0^\infty r dr [\xi_+(r) J_0(\ell r) - \xi_-(r) J_4(\ell r)]
\end{aligned}$$

(see Challinor & Lewis, 2006 or Durrer, 2008 for details)



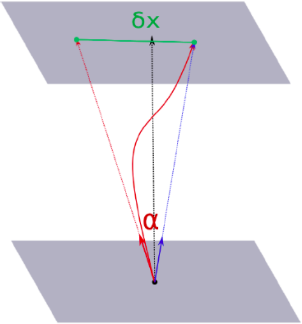
T-E correlation

$$\mathcal{D}_\ell^{TE} = \frac{\ell(\ell+1)}{2\pi} C_\ell^{TE}$$



E-E spectrum

The presence of foreground structure deflects photon geodesics from a straight line.



$$\mathbf{x} \mapsto \mathbf{x} + \delta\mathbf{x}$$

$$\mathbf{n} = (\theta^1, \theta^2) \mapsto \mathbf{n} + \boldsymbol{\alpha}$$

The Jacobian of this lens map is given by

$$\begin{aligned} A_{ab} &= \delta_{ab} + \nabla_b \alpha_a \\ &= \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 + \omega \\ -\gamma_2 - \omega & 1 - \kappa + \gamma_1 \end{pmatrix} \end{aligned}$$

κ describes magnification, γ is the shear and ω a rotation.

In a 'quasi Newtonian' situation with gravitational potential Φ, the deflection angle is the gradient of the lensing potential,

$$\psi(\mathbf{n}, t_*) = -2\int_0^{\lambda_*} d\lambda \frac{\lambda_* - \lambda}{\lambda_* \lambda} \Phi(\lambda\mathbf{n}, t_0 - \lambda) \qquad A_{ab} = \delta_{ab} + \nabla_b \nabla_a \psi \quad \Rightarrow \quad \omega = 0\,.$$

CMB lensing

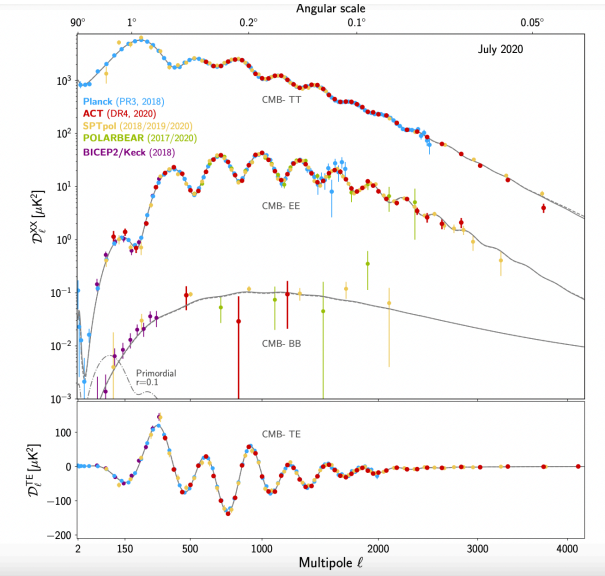
Due to the foreground gravitational potential (inhomogeneities in the matter distribution) the CMB temperature anisotropies and polarisation are lensed:

$$\tilde{T}(\mathbf{n}) = T(\mathbf{n}+\boldsymbol{\alpha}), \qquad \tilde{\mathcal{P}}(\mathbf{n}) = \mathcal{P}(\mathbf{n}+\boldsymbol{\alpha}), \qquad \boldsymbol{\alpha} = \nabla\psi\,,$$

$$\psi(\mathbf{n}) = -2\int_0^{r_*} dr \frac{(r_*-r)}{r_* r}\Phi(r\mathbf{n}, \tau_0 - r)$$

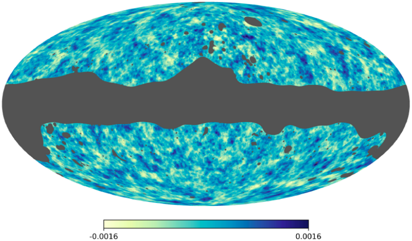
- Lensing of the CMB is a second order effect.
- Lensing *E* polarisation induces *B* polarisation.

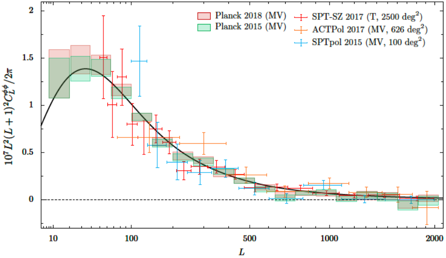
Lensing B modes



Atacama Cosmology Telescope [arXiv:2007.07289]

$$\psi(\mathbf{n}) = -2\int_0^{r_*} dr \frac{(r_* - r)}{r_* r}\Phi(r\mathbf{n}, t_0 - r)$$





Planck 2018

Cosmological parameters

- Even though dark energy was originally postulated by SN observations, at presence its value is best determined by the CMB.
- Thanks to the degeneracy breaking CMB-lensing observations, we do not even need SN observations any more to infer dark energy.
- The CMB data are consistent with a simple, spatially flat cosmological model containing mainly a cosmological constant Λ and CDM together with a nearly scale invariant spectrum of purely scalar fluctuations, a 5(6) parameter model.

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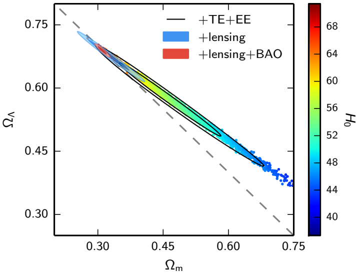
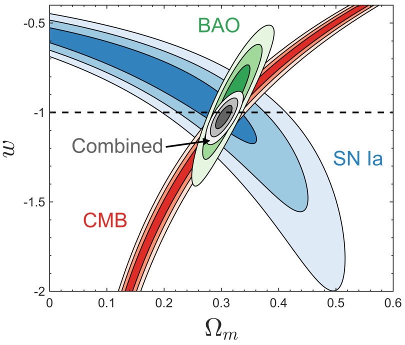
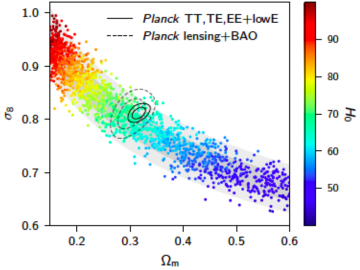


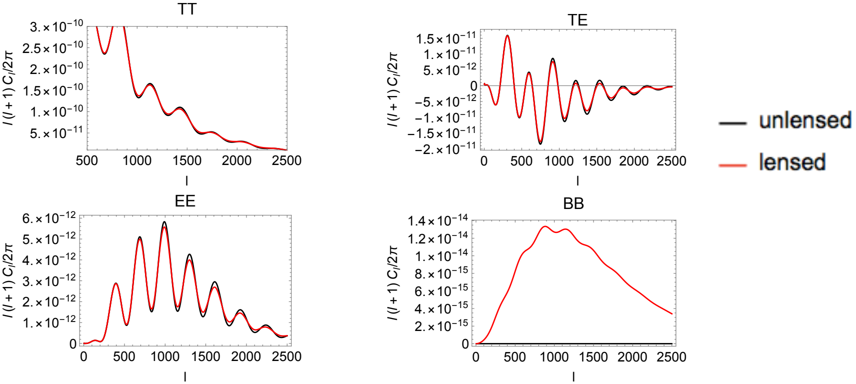




(Huterer et al., 2017)

(← Planck Collaboration, 2015/18)

CMB lensing of scalar perturbations



The effect of lensing is clearly visible, it becomes more than 10% for $\ell \gtrsim 1000$.

Since lensing is so large $> 10\%$, might it be necessary to include effects beyond the leading order? In the present calculations in CLASS or Camb, this is already done to some extent: For $\mathbf{r} = \mathbf{x} - \mathbf{x}'$, $r = |\mathbf{r}|$ the lensed correlation function $\tilde{\xi}(r)$ is given by

$$\begin{aligned}\tilde{\xi}(r) &= \langle \tilde{T}(\mathbf{x})\tilde{T}(\mathbf{x}')\rangle = \langle T(\mathbf{x}+\boldsymbol{\alpha})T(\mathbf{x}'+\boldsymbol{\alpha}')\rangle \\ &= \int \frac{d^2\boldsymbol{\ell}}{2\pi}\int \frac{d^2\boldsymbol{\ell}'}{2\pi}\langle e^{-i\boldsymbol{\ell}\cdot(\mathbf{x}+\boldsymbol{\alpha})}e^{i\boldsymbol{\ell}'\cdot(\mathbf{x}'+\boldsymbol{\alpha}')}\rangle\langle T(\boldsymbol{\ell})T(\boldsymbol{\ell}')\rangle \\ &= \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2}C_\ell e^{-i\boldsymbol{\ell}\mathbf{r}}\left\langle e^{i\boldsymbol{\ell}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})}\right\rangle .\end{aligned}$$

For the second equal sign we use that temperature anisotropies can be considered uncorrelated with the deflection angle. Assuming that primordial pertubations are Gaussian, $\boldsymbol{\alpha}$ is a Gaussian variable. Applying Wick's theorem for products of $\boldsymbol{\alpha}$'s we find

$$\langle e^{i\boldsymbol{\ell}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})}\rangle = \exp\left(-\frac{1}{2}\left\langle\left[\boldsymbol{\ell}\cdot(\boldsymbol{\alpha}'-\boldsymbol{\alpha})\right]^2\right\rangle\right) = \ell^2\left[A_0(0) - A_0(r) + A_2(r)\cos(2\phi_\ell)\right] .$$

$$A_n(r) = \int_0^\infty \frac{d\ell\,\ell^3}{2\pi}C_\ell^\psi J_n(r\ell) .$$

It is well known that this resummation is relevant for present high precision measurement of CMB temperature anisotropies at high ℓ.

In the above resummation, the deflection angle $\boldsymbol{\alpha}$ is always computed to first order.

What about other non-linearities of the lens map itself? At higher orders we have e.g.

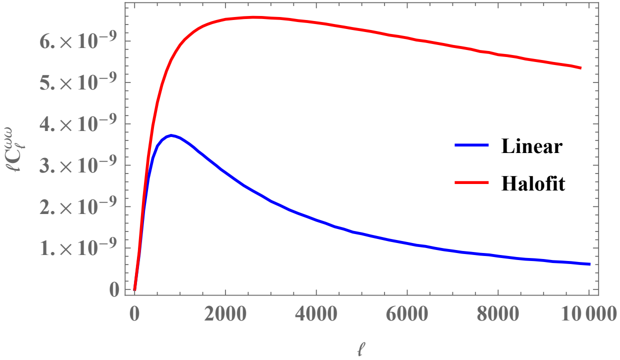
$$\begin{aligned}\phi(\mathbf{n}) &= -2\int_0^{r_*} dr\frac{(r_*-r)}{r_*r}\Phi(r(\mathbf{n}+\boldsymbol{\alpha}(r)),\tau_0-r) \\ &= -2\int_0^{r_*} dr\int_0^{r} dr'\,\frac{(r_*-r)}{r_*r}\,\frac{(r-r')}{rr'}\nabla^a\Phi(r'\mathbf{n},\tau_0-r')\nabla_a\Phi(r\mathbf{n},\tau_0-r)\,,\end{aligned}$$

a so called post Born term.

At second order, also in a 'quasi-Newtonian' situation ω no longer vanishes. More precisely one finds

$$\omega(\mathbf{n}) = 2\epsilon^{ab}\int_0^{r_*} dr\,\frac{r_*-r}{r_*\,r}\left[\nabla_a\nabla^c\Phi(r\mathbf{n})\int_0^{r} dr'\,\frac{r-r'}{r\,r'}\nabla_b\nabla_c\Phi(r'\mathbf{n})\right]\,.$$

$$\omega^{(2)}(\boldsymbol{\ell}) = \frac{2}{(2\pi)}\int_0^{r_s} dr \frac{r_s - r}{r_s r}\int_0^{r} dr_1 \frac{r - r_1}{r r_1}\int d^2\ell_1 \,(\boldsymbol{\ell}\wedge\boldsymbol{\ell}_1)\left(\boldsymbol{\ell}_1\cdot\boldsymbol{\ell} - \ell_1^2\right) \times \Phi_W(z(r),\boldsymbol{\ell}_1)\,\Phi_W(z(r_1),\boldsymbol{\ell}-\boldsymbol{\ell}_1)\ .$$



The controversy: Does polarisation rotate?

For a photon geodesic k^μ and an emitter/observer velocity field u^μ we define the photon direction n^μ by

$$k^\mu = k(u^\mu + n^\mu)\,, \qquad u^2 = -1\,, \quad n^2 = 1\,, \quad n^\mu u_\mu = 0\,.$$

On the sphere normal to n and u we define the Sachs basis

$$e_1^\mu\,,\; e_2^\mu \quad \text{via} \;\; e_a \cdot e_b = \delta_{ab} \;\; \text{and} \; e_a \cdot n = e_a \cdot u = 0\,.$$

If u where parallel transported along k, hence $\nabla_k u = 0$, we could request also $\nabla_k e_a = 0$ for the transport of e_a, but since in general $\nabla_k u \neq 0$, we can only request that

$$\Pi^\mu_\sigma \nabla_k e^\mu_a = 0\,, \qquad \Pi^\mu_\sigma = \delta^\mu_\sigma + u_\sigma u^\mu - n_\sigma n^\mu\,. \qquad (\star)$$

where Π^μ_σ denotes the projection into the subspace normal to u and n.
The helicity basis is as before, $e_\pm = (e_1 \pm ie_2)/\sqrt{2}$ and $\mathcal{P}(\mathbf{n}) = \mathcal{P}_{++}$.
Under a rotation by an angle β of the Sachs basis, the polarisation rotates by $\mathcal{P} \mapsto e^{2i\beta}\mathcal{P}$.
We therefore just have to investigate whether the Sachs basis rotates under its propagation law $(\star)$.

The controversy: Newtonian coordinates (NG)

Lewis and collaborators have done the calculation in Newtonian gauge,

$$ds^2 = -e^{2\Phi}dt^2 + e^{-2\Phi}\delta_{ij}dx^i dx^j\,.$$

They found $\beta^{(2)} = 0$.

One can actually show that under parallel transport to all orders e_a^μ only acquires components along itself, u and k in this gauge, so that no rotation from e_1^μ into e_2^μ can occur. This is not surprising since the spatial part of the metric is conformally flat.

The controversy: Geodesic light cone (GLC) coordinates

We, Marozzi and collaborators used GLC coordinates. These coordinates are the observer proper time τ, a null coordinate w labelling lightcones and two angular coordinates labelling incoming photon directions θ^a ($a = 1, 2$). The GLC line-element depends on six arbitrary functions (Υ, $U^a, \gamma_{ab} = \gamma_{ba}$), and takes the form

$$ds^2 = \Upsilon^2 dw^2 - 2\Upsilon dw d\tau + \gamma_{ab}(d\theta^a - U^a dw)(d\theta^b - U^b dw)$$

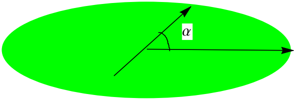
which can be expressed in terms of Φ for purely scalar quasi-Newtonian perturbations (see Ben Dayen et al. 2012). We have computed the rotation of the Sachs basis in this gauge and found up to second order

$$\beta^{(2)}|_{GLC} = -\omega^{(2)}|_{NG} \,.$$

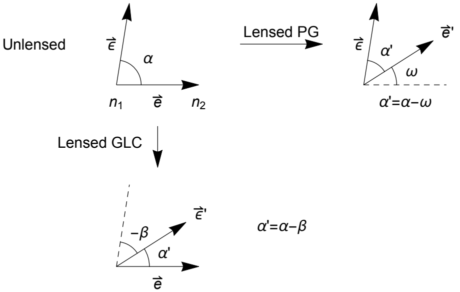
What does this mean? Which calculation is correct?

Resolution

What do we really observe? Rotation wrt what? We must define two physical directions and determine whether the angle between them changes.
Toy example Consider an ellipse of, say, slightly higher temperature and with fixed polarisation direction.



What happens to these directions under lensing?



In Newtonian coordinates $\beta_{NG} = 0$. In GCL coordinates $\omega_{GCL} = 0$ by definition. Furthermore $\omega_{NG} = -\beta_{GCL}$.
But in general

$$\alpha - \alpha' = \omega - \beta$$

Hence the gauge invariant, physical angle is $\omega - \beta$ which is the same for both coordinate systems.

Resolution

But what about the power spectra, C_ℓ^{XY} which are measureable quantities and can be computed in any coordinate system?
To address the question we consider only rotation (neglecting other effects from lensing). In GLC gauge the photon position is then not affected and we obtain for the lensed polarisation

$$\tilde{\mathcal{P}}_r = \exp(+2i\beta)\mathcal{P}$$

purely from the rotation of the polarisation wrt our coordinates.
In NG polarisation is not rotated but $\tilde{P}(\mathbf{x}) = P(R(\omega)\mathbf{x})$, where $R(\omega)$ around the flat sky direction $\mathbf{n}_0$. Hence the lensed spectrum relates to the unlensed one via

$$\tilde{\mathcal{P}}_r = \exp(-2i\omega)\mathcal{P}_r$$

and since $\omega = -\beta$ the lensed polarisation tensor expressed in the basis given by the connecting vector $\mathbf{r}$ is modified in the same way by the rotation induced by second order lensing.

Resolution

Since the spectra can be expressed purely in terms of $\mathcal{P}_r$, this implies

$$\tilde{C}_\ell^{\mathcal{E}}|_{NG} \equiv \tilde{C}_\ell^{\mathcal{E}}|_{GLC}$$

and equivalently with $\tilde{C}_\ell^{\mathcal{B}}$.

The modification of the CMB spectra by lensing rotation comes entirely from the rotation of the image in NG coordinates while it comes entirely from the rotation of the Sachs basis in GLC coordinates, where the incoming photon directions are coordinates.

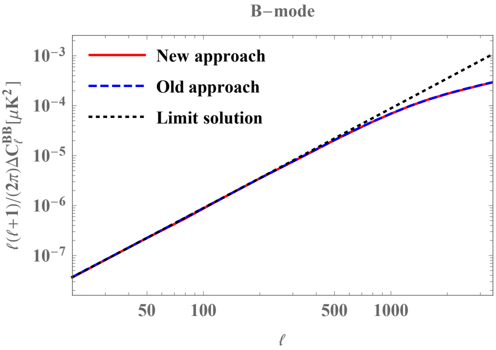
The rotation by ω describes the rotation of infinitesimal images (in the chosen coordinate system) which are Lie transported.

The rotation by β describes the rotation of the Sachs basis under parallel transport.

Only the relative rotation, $\omega - \beta$ is physical and therefore observable. Both ω and β depend on the chosen coordinate system.

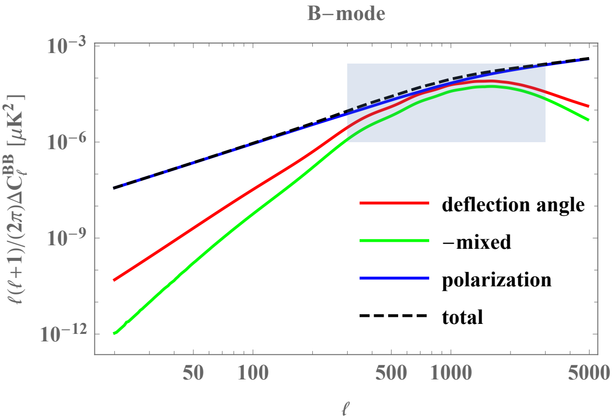
Results

$$\Delta \tilde{C}^B_\ell = 4\int \frac{d^2\ell'}{(2\pi)^2} C^{\alpha}_{|\boldsymbol{\ell}-\boldsymbol{\ell}'|} C^E_{\ell'} \Bigg[\cos^2(2\varphi_{\boldsymbol{\ell},\boldsymbol{\ell}'}) - \frac{\ell\ell'}{|\boldsymbol{\ell}-\boldsymbol{\ell}'|^2} \sin(4\varphi_{\boldsymbol{\ell},\boldsymbol{\ell}'}) \sin(\varphi_{\boldsymbol{\ell},\boldsymbol{\ell}'}) + \frac{(\ell\ell')^2}{|\boldsymbol{\ell}-\boldsymbol{\ell}'|^4} \sin^2(2\varphi_{\boldsymbol{\ell},\boldsymbol{\ell}'}) \sin^2(\varphi_{\boldsymbol{\ell},\boldsymbol{\ell}'}) \Bigg].$$



(figure from D Dioi et al. [1905.12573]).

Results



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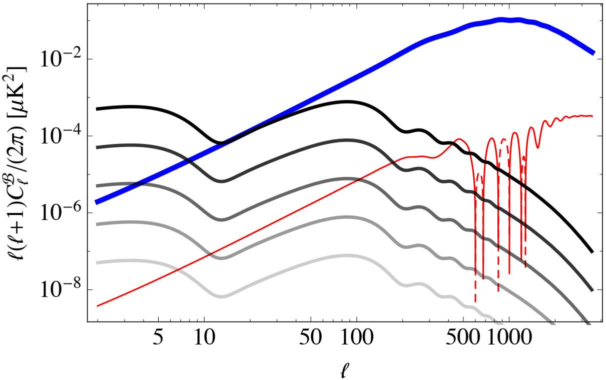
Not correctly subtracted B-modes from lensing (incorrect 'delensing') can mimic primordial B-modes from inflation.



Blue: first order lensing B-modes. Red: B-modes from higher order lensing (rotation). Grey-tones: primordial B-modes from inflation, $r = (10^{-2}, 10^{-3}, 10^{-4}, 10^{-5}, 10^{-6})$. (figure from Marozzi et al. [1612.07263])

Conclusions

- The CMB is the most precise cosmological dataset.
- We hope to still increase significantly the precision of CMB polarisation measurements and to measure primordial B-modes from inflation if $r \gtrsim 10^{-3}$, i.e. the energy scale of inflation is roughly 10^{16}GeV.
- For this, we have to correctly subtract the contribution from lensing beyond the leading order.
- We have to develop new delensing techniques since the higher order signal is non-Gaussian.
- But even if $r \ll 10^{-3}$, with S4 CMB experiments we shall discover the rotation of CMB polarisation which is a frame dragging effect in the CMB. This is a formidable test of GR on cosmological scales.

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