1 List of References for lectures by Luca Delacretaz

Fluctutation corrections to hydrodynamics have been appreciated since their discovery in molecular dynamics numerics long ago [1], see e.g. Refs. [2, 3, 4, 5, 6]. In particular, Ref. [7] provides a detailed study in a mode coupling approximation.

More recent efforts were able to formulate fluctuating hydrodynamics as a systematic effective field theory, for both classical and quantum systems [8, 9, 10, 11, 12]. The first part of these lectures most closely follows a slightly different perspective that emphasises symmetry breaking [13] (see also [14, 15] for a more quantum information perspective on this point). These methods can be generalized to open systems as well, with density matrix evolving according to the Lindblad equation: see [16, 17, 18] and [19] for a review.

The second part of these lectures studies power law corrections to diffusion, following [20]. Consequences of these corrections include the diffuson cascade [21]. Depending on time, we may also discuss operator matching equations [21, 22].

References

[1] B. J. Alder and T. E. Wainwright, Decay of the Velocity Autocorrelation Function, Phys. Rev. A **1**, 18–21, 1970.

[2] M. H. Ernst, E. H. Hauge and J. M. J. van Leeuwen, Asymptotic time behavior of correlation functions, Phys. Rev. Lett. **25**, 1254–1256, 1970.

[3] P. C. Martin, E. D. Siggia and H. A. Rose, Statistical Dynamics of Classical Systems, Phys. Rev. A **8**, 423–437, 1973.

[4] D. Forster, D. R. Nelson and M. J. Stephen, Large-distance and long-time properties of a randomly stirred fluid, Phys. Rev. A **16**, 732–749, 1977.

[5] A. Andreev, Corrections to the hydrodynamics of liquids, **48**, Sov. Phys. JETP, 1978.

[6] V. Morozov, Nonlocal hydrodynamics and dispersion of transport coefficients in simple fluid, Physica A: Statistical Mechanics and its Applications **117**, 511–530, 1983.

[7] M. H. Ernst and J. R. Dorfman, Nonanalytic dispersion relations for classical fluids, Journal of Statistical Physics **12**, 311–359, 1975.

[8] M. Crossley, P. Glorioso and H. Liu, Effective field theory of dissipative fluids, JHEP **09**, 095, 2017, [arXiv:1511.03646 [hep-th]].

[9] K. Jensen, R. Marjieh, N. Pinzani-Fokeeva and A. Yarom, A panoply of Schwinger-Keldysh transport, SciPost Phys. **5**, 053, 2018, [arXiv:1804.04654 [hep-th]].

[10] F. M. Haehl, R. Loganayagam and M. Rangamani, Effective Action for Relativistic Hydrodynamics: Fluctuations, Dissipation, and Entropy Inflow, JHEP **10**, 194, 2018, [arXiv:1803.11155 [hep-th]].

[11] X. Chen-Lin, L. V. Delacrétaz and S. A. Hartnoll, Theory of diffusive fluctuations, Phys. Rev. Lett. **122**, 091602, 2019, [arXiv:1811.12540 [hep-th]].

[12] A. Jain and P. Kovtun, Late Time Correlations in Hydrodynamics: Beyond Constitutive Relations, Phys. Rev. Lett. **128**, 071601, 2022, [arXiv:2009.01356 [hep-th]].

[13] C. O. Akyuz, G. Goon and R. Penco, The Schwinger-Keldysh coset construction, JHEP **06**, 004, 2024, [arXiv:2306.17232 [hep-th]].

[14] O. Ogunnaike, J. Feldmeier and J. Y. Lee, Unifying Emergent Hydrodynamics and Lindbladian Low-Energy Spectra across Symmetries, Constraints, and Long-Range Interactions, Phys. Rev. Lett. **131**, 220403, 2023, [arXiv:2304.13028 [cond-mat.str-el]].

[15] L. A. Lessa, R. Ma, J.-H. Zhang, Z. Bi, M. Cheng and C. Wang, Strong-to-Weak Spontaneous Symmetry Breaking in Mixed Quantum States, 2024, [arXiv:2405.03639 [quant-ph]].

[16] E. Altman, L. M. Sieberer, L. Chen, S. Diehl and J. Toner, Two-dimensional superfluidity of exciton-polaritons requires strong anisotropy, 2013, [arXiv:1311.0876 [cond-mat.stat-mech]].

[17] Y. Minami and Y. Hidaka, Spontaneous symmetry breaking and nambu-goldstone modes in dissipative systems, Phys. Rev. E **97**, 012130, 2018.

[18] M. Hongo, S. Kim, T. Noumi and A. Ota, Effective Lagrangian for Nambu-Goldstone modes in nonequilibrium open systems, Phys. Rev. D **103**, 056020, 2021, [arXiv:1907.08609 [hep-th]].

[19] L. M. Sieberer, M. Buchhold, J. Marino and S. Diehl, Universality in driven open quantum matter, 2023, [arXiv:2312.03073 [cond-mat.stat-mech]].

[20] A. A. Michailidis, D. A. Abanin and L. V. Delacrétaz, Corrections to diffusion in interacting quantum systems, 2023, [arXiv:2310.10564 [cond-mat.stat-mech]].

[21] L. V. Delacretaz, Heavy Operators and Hydrodynamic Tails, SciPost Phys. **9**, 034, 2020, [arXiv:2006.01139 [hep-th]].

[22] P. Glorioso, L. V. Delacrétaz, X. Chen, R. M. Nandkishore and A. Lucas, Hydrodynamics in lattice models with continuous non-Abelian symmetries, SciPost Phys. **10**, 015, 2021, [arXiv:2007.13753 [cond-mat.stat-mech]].

A Imposing KMS

We found during lectures that the hydro EFT is given by

$$Z[A^a_\mu, A^r_\mu] = \int D\phi^a D\mu^r e^{iS_{\rm eff}[\partial_\mu\phi^a + A^a_\mu, \mu^r, F^r_{\mu\nu}]} \tag{1}$$

We would now like to impose KMS. KMS transforms the sources to

$$\begin{aligned} A^1(t) &\to A^1(-t + i\beta/2) \\ A^2(t) &\to A^2(-t - i\beta/2)\,. \end{aligned} \tag{2}$$

A sufficient condition for Z to be invariant under this transformation is for the action to be invariant under the same transformation for the fields. If we were in the superfluid phase, with symmetry breaking pattern $U(1)_1 \times U(1)_2 \to 1$, this transformation would be

$$\begin{aligned} \phi^1(t) &\to -\phi^1(-t + i\beta/2) \simeq -\phi^1(-t) - \frac{i}{2}\beta\dot\phi^1(-t) + \cdots\,, \\ \phi^2(t) &\to -\phi^2(-t - i\beta/2) \simeq -\phi^2(-t) + \frac{i}{2}\beta\dot\phi^2(-t) + \cdots\,. \end{aligned} \tag{3}$$

We expanded in $\beta\partial_t$, because we will ultimately be interested in the late time / low frequency physics. For the a/r fields this would read

$$\begin{aligned} \phi^a(t) &\to -\phi^a(-t) - i\beta\dot\phi^r(-t) + \cdots\,, \\ \phi^r(t) &\to -\phi^r(-t) - \frac{i}{4}\beta\dot\phi^a(-t) + \cdots\,. \end{aligned} \tag{4}$$

In the normal (diffusive) phase, ϕ^r is absent and instead μ^r plays the role of $D_0\phi^r = \dot\phi^r + A^r_0$. The transformation laws are thus (some minus signs disappeared due to time derivatives)

$$\mu^r \to \mu^r(-t) + \frac{i}{4}\partial_t D_0\phi^a(-t) \tag{5a}$$

$$D_0\phi^a = \partial_0\phi^a + A^a_0 \to D_0\phi^a(-t) + i\beta\dot\mu^r(-t) \tag{5b}$$

$$D_i\phi^a = \partial_i\phi^a + A^a_i \to -\left[D_i\phi^a(-t) + i\beta\partial_i\mu^r(-t) + i\beta E^r_i(-t)\right]\,, \tag{5c}$$

with $E^r_i \equiv F^r_{0i}$. To derive these relations, start by using Eq. (4) and then replace $D_0\phi^r \to \mu^r$ on the RHS. Note the appearance of the electric field in the last equation.

We are now ready to build KMS invariant building blocks. First, the term

$$\chi \int dt d^d x\, \mu^r D_0 \phi^a \tag{6}$$

is invariant, after removing a total derivative. This holds more generally for

$$\int dt d^d x\, n(\mu^r) D_0 \phi^a \,, \tag{7}$$

for any smooth function $n(\mu)$.

Now let's turn to the dissipative terms. First note that at the order we are working (and recalling the scaling $\mu^r \sim \phi^a$), we can approximate (5a) by $\mu^r \to \mu^r(-t)$. Then: the combination

$$D_i \phi^a \left[D_i \phi^a + i\beta \partial_i \mu^r + i\beta E_i^r \right] \tag{8}$$

transforms to itself (evaluated at $-t$) under KMS, so that any power of it multiplied by a function of μ^r, is KMS invariant:

$$\int dt d^d x\, F(\mu^r) \, \left(D_i \phi^a \left[D_i \phi^a + i\beta \partial_i \mu^r + i\beta E_i^r \right] \right)^N \,. \tag{9}$$

One can proceed in a similar manner to higher order in derivatives and ϕ^a and find further KMS invariants (see, e.g., Ref. [12]).