(Non-fluctuating) Hydrodynamics in strongly correlated quantum gases

Ultracold atoms

Keisuke Fujii

Advances in Fluctuating Hydrodynamics: Bridging the Micro and Macro Scales

27 June 2024

✔Collaborators

- Yusuke Nishida (Tokyo Tech)
- Tilman Enss (Heidelberg)

KF & T. Enss, Ann. Phys. **453**, 169296 (2023)
KF & Y. Nishida, PRA **103**, 053320 (2021)
KF & Y. Nishida, PRA **102**, 023310 (2020)
KF & Y. Nishida, PRA **98**, 063634 (2018)

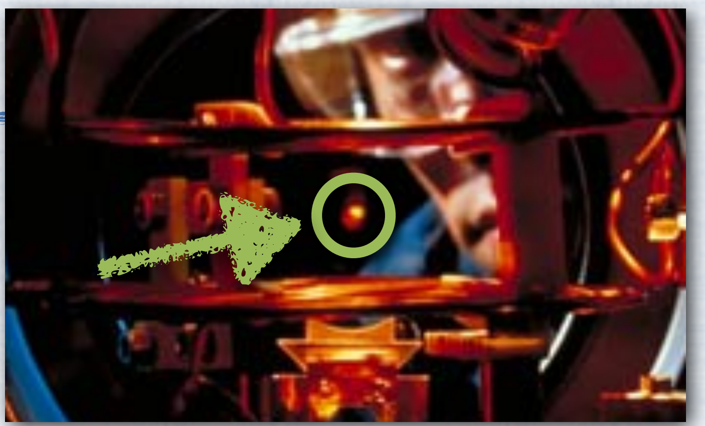
(Non-fluctuating) Hydrodynamics in strongly correlated quantum gases

Ultracold atoms

The main purpose of my talk :

Introducing transport phenomena in ultracold atoms,
in particular, strongly interacting fermions,
with latest, state-of-the-art experiments

Ultracold atoms



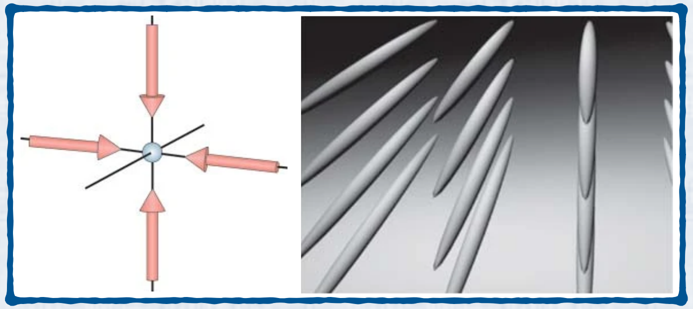
https://physics.aps.org/story/v21/st11

Quantum many-body system cooled down to nano-Kelvin

- ▸ Cold $T \sim 50\,\mathrm{nK} \quad (T/T_F \sim 0.1)$
- ▸ Dilute $n \sim 5 \times 10^{12}\,\mathrm{m}^{-3}$ (cf. $n_0 = 2.69 \times 10^{25}\,\mathrm{m}^{-3}$ for ideal gas at $0\,C^{\circ}, 1\,\mathrm{atm}$)

☑ **High Experimental Tunability**

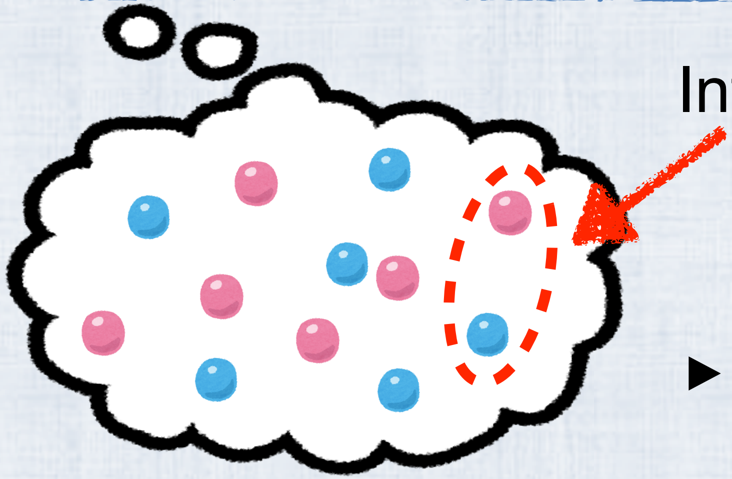
- **Quantum statistics** by changing species of trapping atoms
- **Spatial dimension** by laser confinement
- **Interaction strength** via the Feshbach resonance
- **Trapping potential** by digital micro-mirror device



1D system by laser confinement
I. Bloch, Nat. Phys. (2005)

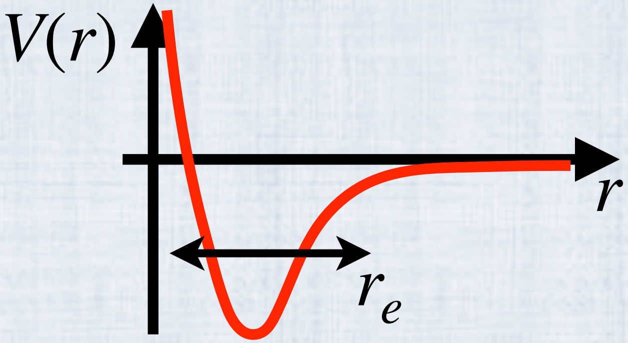
➡ Nice platform to study many-body physics

Interacting fermions (spin-1/2)



Interaction potential : $V(r)$

- Scattering length : a
- ~~Potential radius~~ : r_e



▸ Cold & dilute : $r_e \ll \lambda_T,\ n^{-1/3},\ a$

(thermal de Broglie wavelength $\lambda_T \propto T^{-1/2}$)

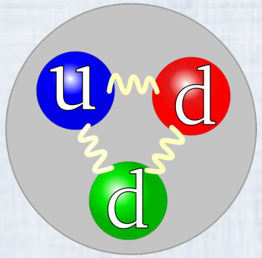
➡ $r_e \to 0$ The interaction is characterized only by the scattering length

experimentally tunable

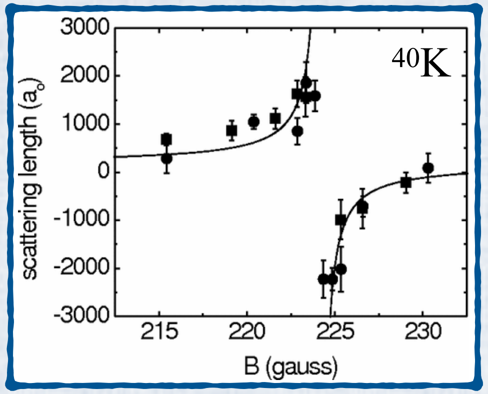
✓ Universal !! : independent of the detail of $V(r)$

E.g.

- Neutrons: $|a|/r_e \sim 18$



- Ultracold atoms: **tunable**



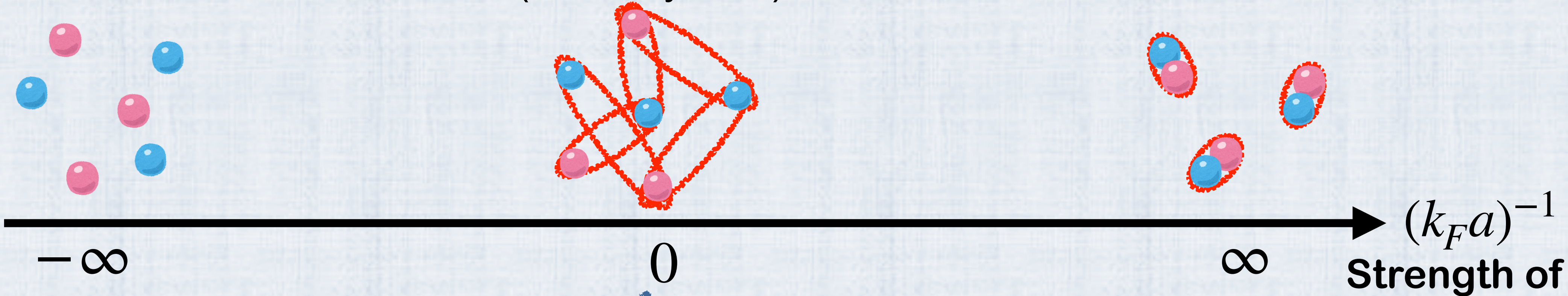
Tuning of scattering length via an applied magnetic field

C. A. Regal & D. S. Jin, PRL (2003)

Attractive interacting fermions

- Changing the attractive interaction strength, i.e., the scattering length a

Free Fermions | Unitary Fermi gas (Unitarity limit) | Free Molecular Bosons



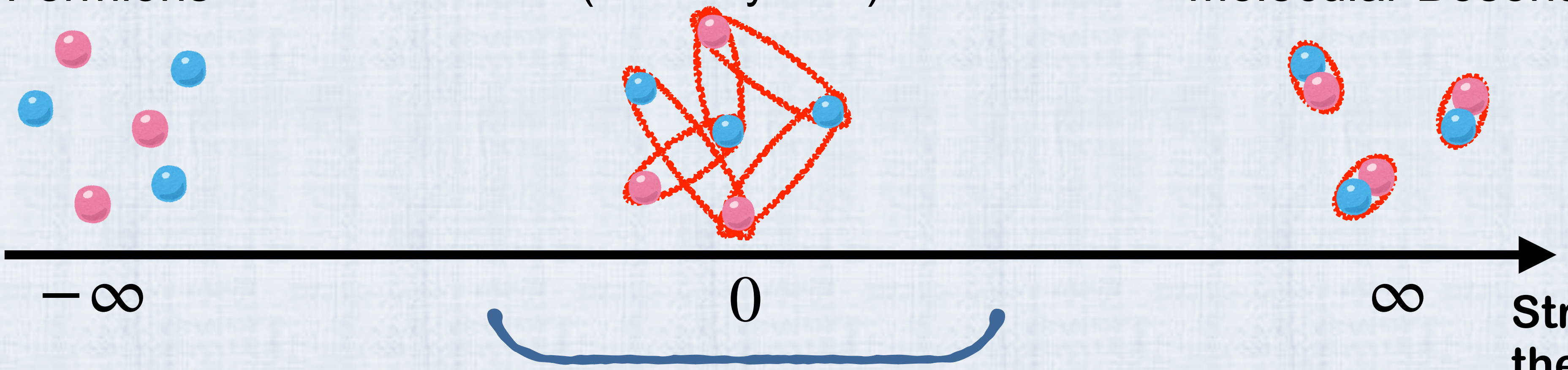
Strength of the attractive interaction

Critical magnitude of the attraction that creates a two-particle bound state

→ **Emergence of conformal invariance** ✓Yusuke's talk

Attractive interacting fermions

- Changing the attractive interaction strength, i.e., the scattering length a



Strong-correlated regime

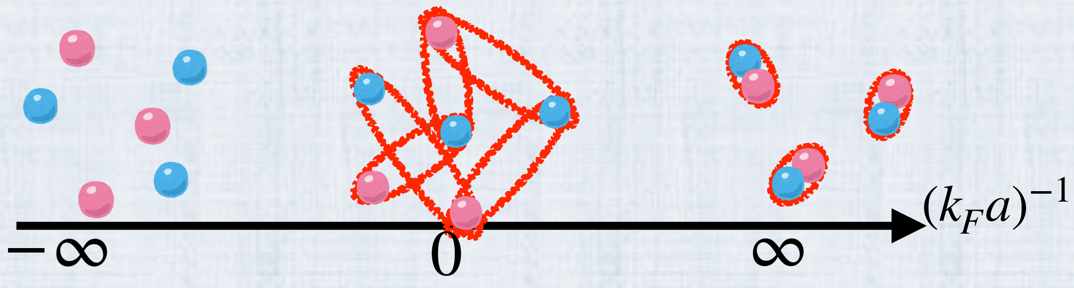
→ Still fascinating and challenging!!

- Transport properties in this system
  - Universal lower bound on the shear viscosity $\dfrac{\eta}{s} \geq \dfrac{\hbar}{4\pi k_B}$ P. K. Kovtun, D. T. Son, & A. O. Starinets (2005)
    In the unitary Fermi gas, $\eta/s\,[\hbar/k_B] \lesssim 0.5$
  - Vanishing bulk viscosity in the unitary limit $\zeta = 0 \quad @ \ |a| = \infty$ D. T. Son (2007)

Plan of this talk

1. Introduction : Ultracold atoms & Attractively interacting fermions

- **Universality** of Fermi gases & its appearance in transport



2. Transport coefficients

2. 1 Shear viscosity

2. 2 Thermal conductivity

2. 3 Bulk viscosity

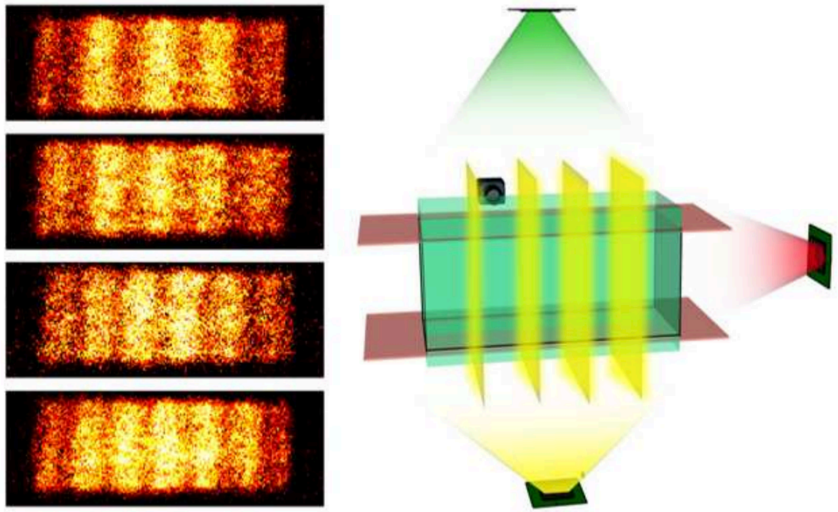
3. For fluctuating hydrodynamics

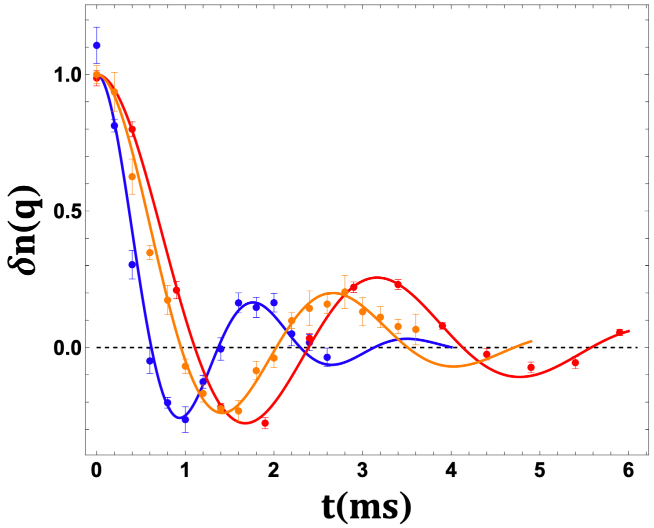
4. Summary & Outlook

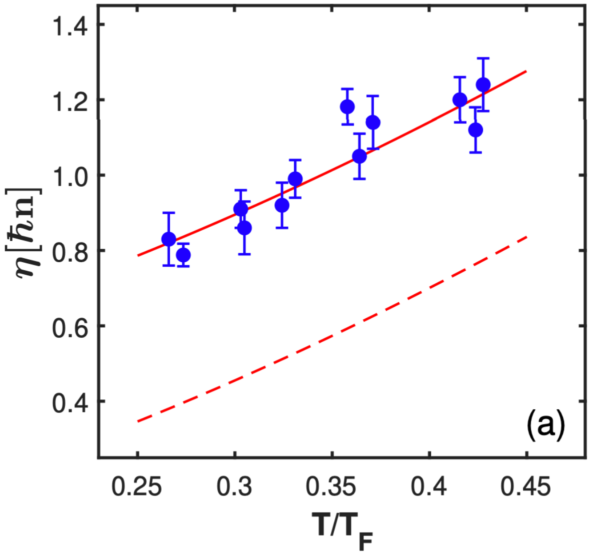
Latest measurement of the shear viscosity

Relaxation dynamics from spatially modulated initial states in the unitary Fermi gas

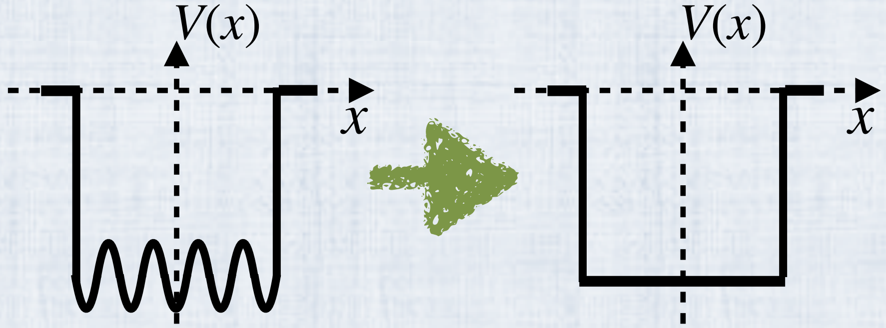
Xiang Li, J. Huang, & J. E. Thomas, [arXiv:2402.14104] (2024)







1. Initially, preparing spatially modulated box potential
2. Removing spatial modulation and observing relaxations



Latest measurement of the shear viscosity

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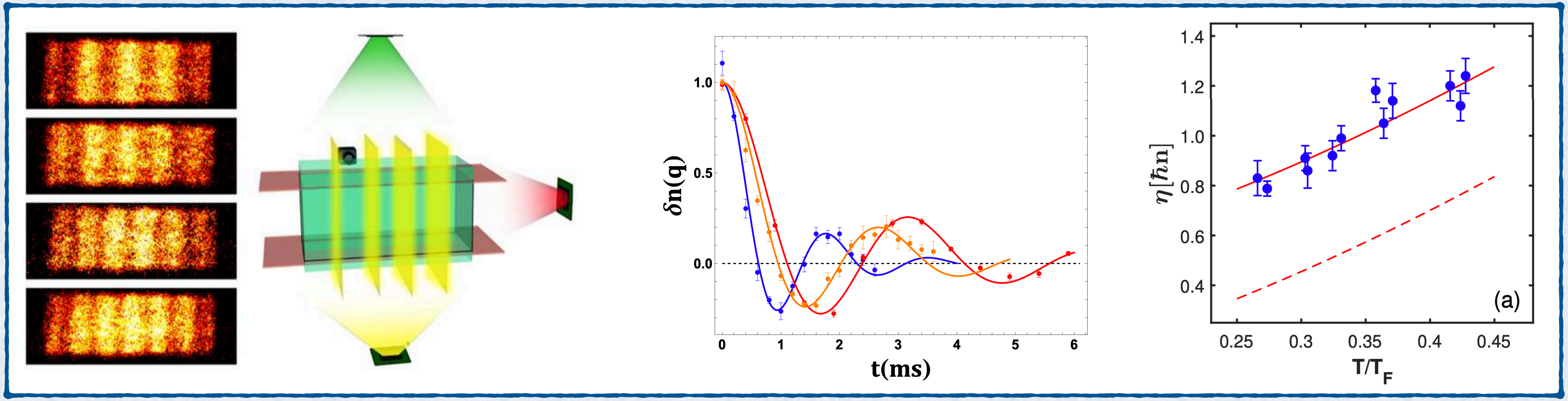

- The shear viscosity is well expressed in this form up to the critical temperature.

$$\eta = \frac{15}{32\sqrt{\pi}} \frac{(m k_B T)^{3/2}}{\hbar^2} + \alpha \hbar n$$

Critical temperature : $T_c \sim 0.23 T_F$

Dimensionless fit parameter : $\alpha = 0.45(04)$

High-temperature limit result

Universal correction
to the high-temperature result

High-temperature expansion

In the high-temperature limit with fixed particle number density

- Thermal de Broglie wavelength $\lambda_T = \sqrt{2\pi\hbar^2/(mk_BT)} \to 0$

When $\lambda_T \ll n^{-1/3}$ (i.e., mean particle distance), **the inter-particle interaction is effectively weak** even around the unitary limit.

➡ Systematic expansion w.r.t the fugacity $z \equiv \exp(\beta\mu)$

In the high-temperature limit, $z \ll 1$ $(\because \mu \to -\infty)$

✓ The leading-order transport coefficients can be computed from the Boltzmann eq.

KF & Y. Nishida, PRA **103**, 053320 (2021)

$$\left[\frac{\partial}{\partial t} + \frac{\vec{p}}{m}\cdot\frac{\partial}{\partial \vec{r}}\right] f = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}$$

with two-body transition rate $\mathcal{W} = \left|\frac{4\pi}{m}\frac{1}{a^{-1} - i\sqrt{(\vec{p}-\vec{q})^2/4}}\right|^2$

characterized only by the scattering length

➡ $$\eta = \frac{15}{32\sqrt{\pi}}\frac{(mk_BT)^{3/2}}{\hbar^2} + O(z)$$

The experiment claims that the higher-order corrections are almost uniquely determined by the particle number density n.

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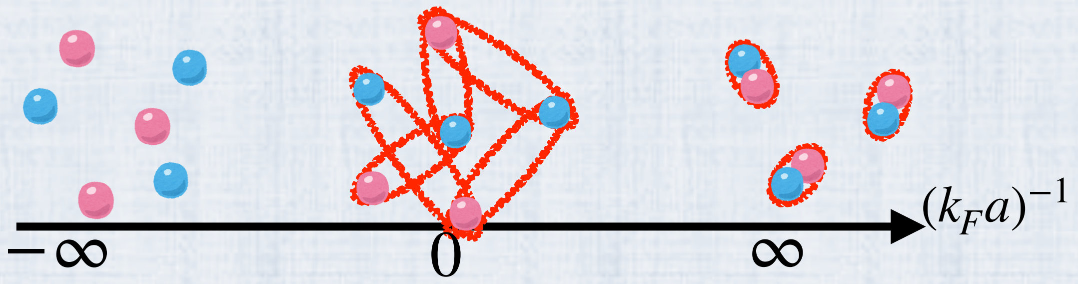
   - **Universality** of Fermi gases & its appearance in transport



2. **Transport coefficients**
   - 2. 1 Shear viscosity
   - **2. 2 Thermal conductivity**
   - 2. 3 Bulk viscosity

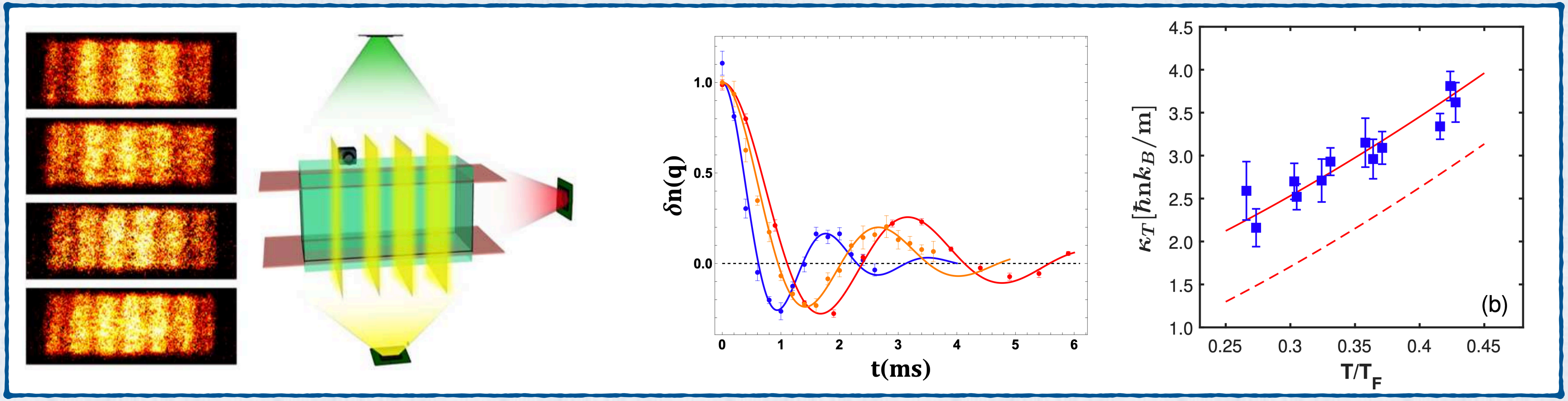
3. For fluctuating hydrodynamics

4. Summary & Outlook

Thermal conductivity via relaxation

Relaxation dynamics from spatially modulated initial states in the unitary Fermi gas

Xiang Li, J. Huang, & J. E. Thomas, [arXiv:2402.14104] (2024)



- The thermal conductivity is also expressed in the same form

$$T\kappa = \frac{225}{128\sqrt{\pi}}\frac{m^{1/2}(k_BT)^{5/2}}{\hbar^2} + \alpha'\frac{k_BT}{m}\hbar n$$

high-temperature limit result

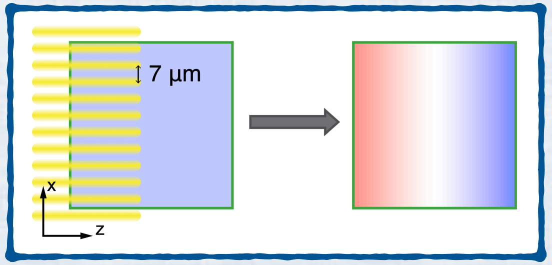
Critical temperature : $T_c \sim 0.23T_F$

Dimensionless fit parameter : $\alpha' \approx 0.825$

Creation of temperature gradient

Thermography in the unitary Fermi gas

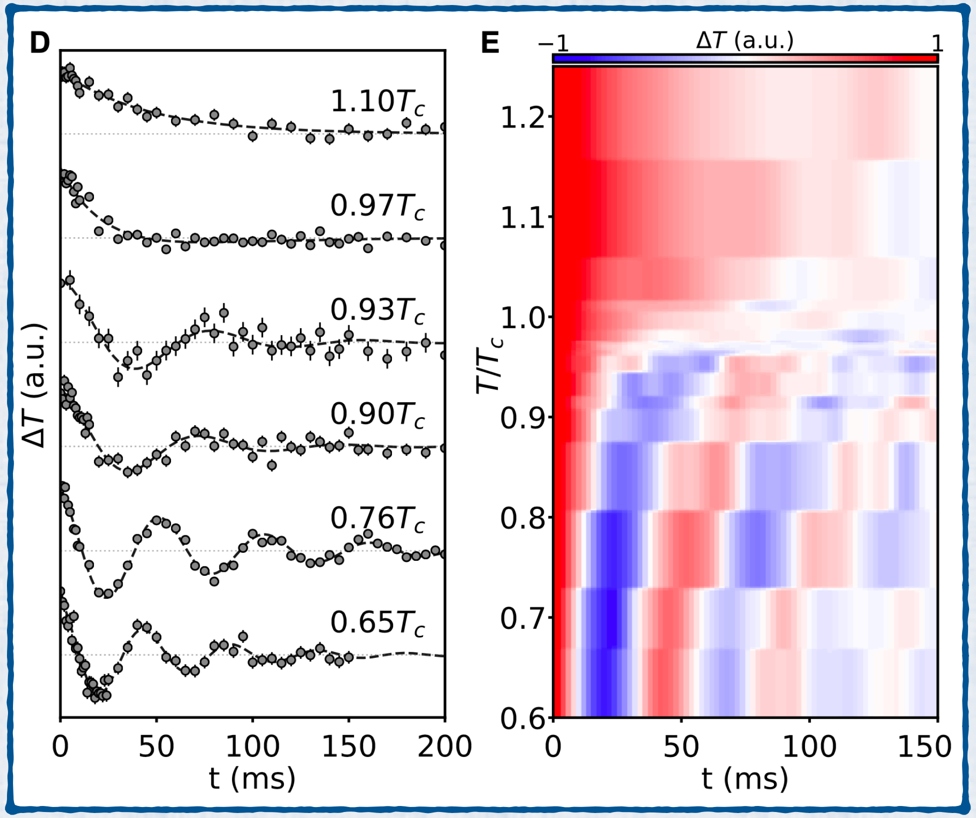
M. Zwierlein's exp. Science **383**, 629 (2024)



Temperature gradient can be created by exposing an intensity-modulated light.

Below T_c, the temperature gradient propagates.

→ **Second sound in superfluids**

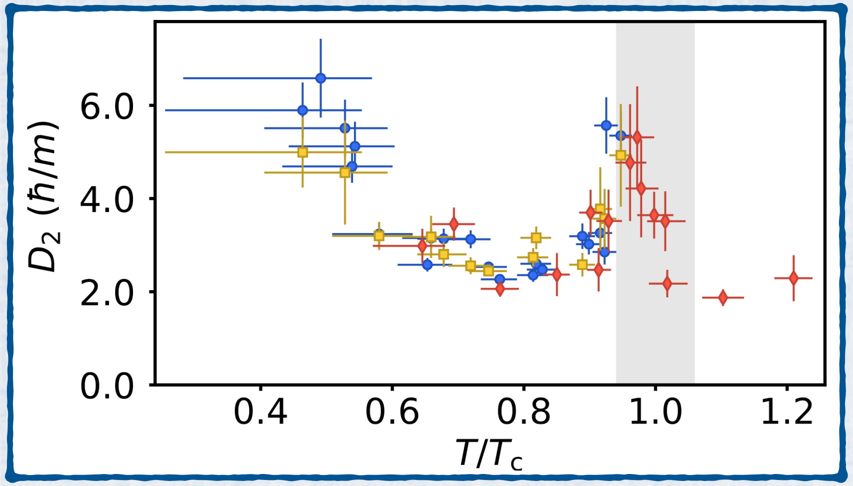


Model F universality?

Second sound diffusivity :

$$D_2 \propto |T - T_c|^{-\nu/2} \quad \text{with } \nu \approx 0.672$$

In the exp., qualitative accuracy is still hard to achieve.



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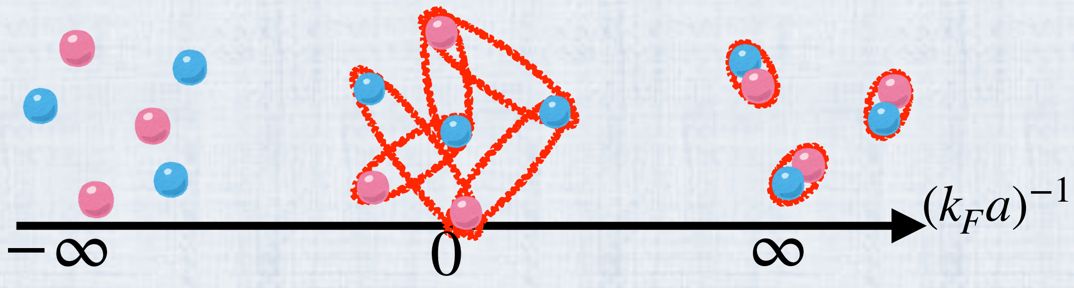
    - **Universality** of Fermi gases & its appearance in transport



2. Transport coefficients
    - 2. 1 Shear viscosity
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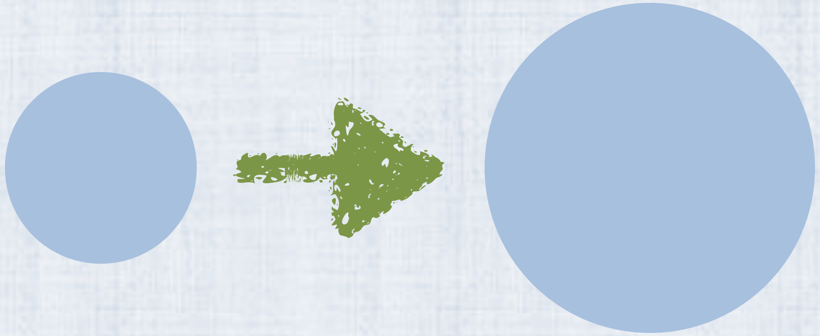
Isotropic expansion @ $|a| = \infty$

- Vanishing bulk viscosity in the unitary limit $\zeta = 0$ @ $|a| = \infty$ D. T. Son (2007)

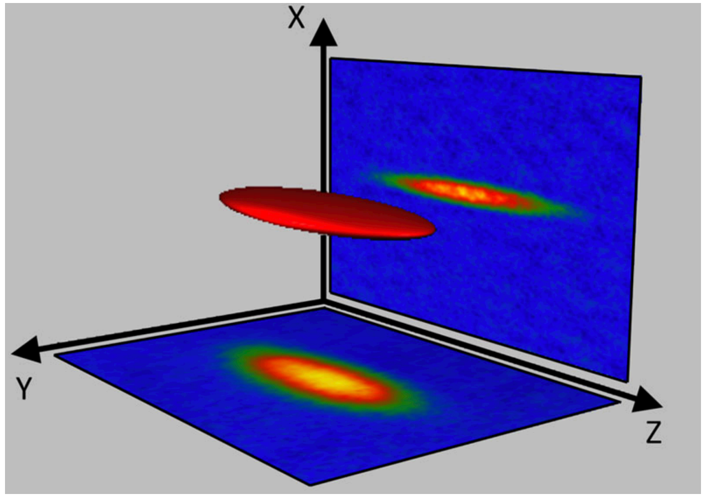


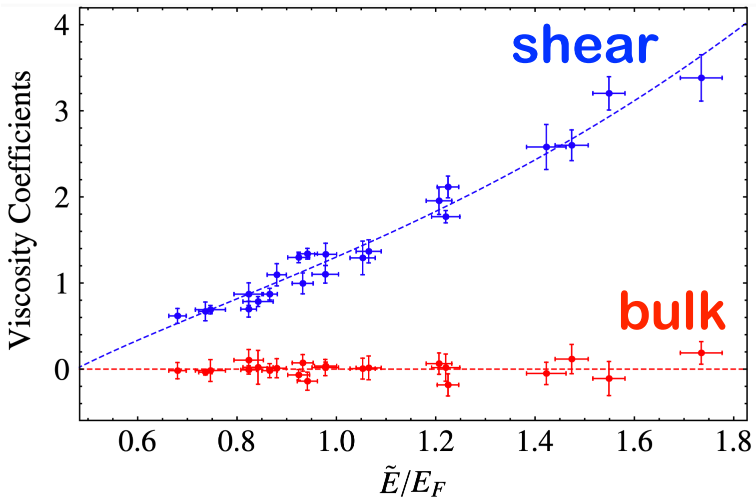
Isotropic expansion = Scale transformation

→ In scale-invariant fluids,
NO entropy production during isotropic expansions.

✓Yusuke's talk
for a more rigorous symmetry-based argument

▶ Fluid expansions from cigar-shaped trap E. Elliott, J. A. Joseph, & J. E. Thomas, PRL **112**, 040405 (2014).

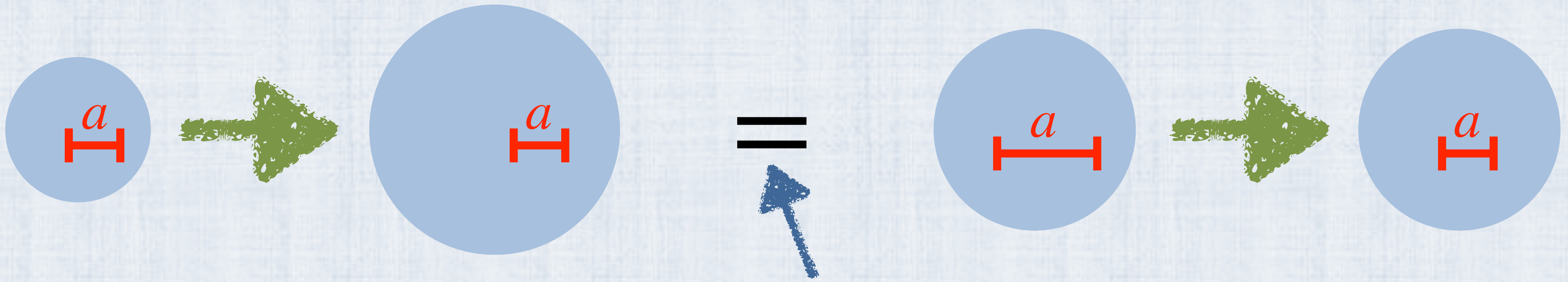




$\tilde{E}$: interaction-indep. initial energy
~ temperature

Isotropic expansion @ $|a| \neq \infty$

Isotropic expansion with fixed scattering length ≠ Scale transformation



Relative size change between the fluid size & the scattering length

▸ Stress tensor in hydrodynamics with spacetime-dependent scattering length

KF & Y. Nishida, PRA **98**, 063634 (2018)

$$\Pi_{ij} = p\delta_{ij} + \rho v_i v_j - \sigma_{ij} \qquad \sigma_{ij} = \zeta V_a(t,\vec{x}) + \text{shear}$$

dissipative correction includes derivatives of $a(t,\vec{x})$

Bulk strain rate tensor with $a(t,\vec{x})$

$$V_a(t,\vec{x}) = \nabla\cdot\vec{v} - 3[\partial_t \ln a + \vec{v}\cdot\nabla \ln a]$$

uniquely determined from the 2nd law of local thermodynamics

Isotropic expansion @ $|a| \neq \infty$

- Stress tensor in hydrodynamics with spacetime-dependent scattering length

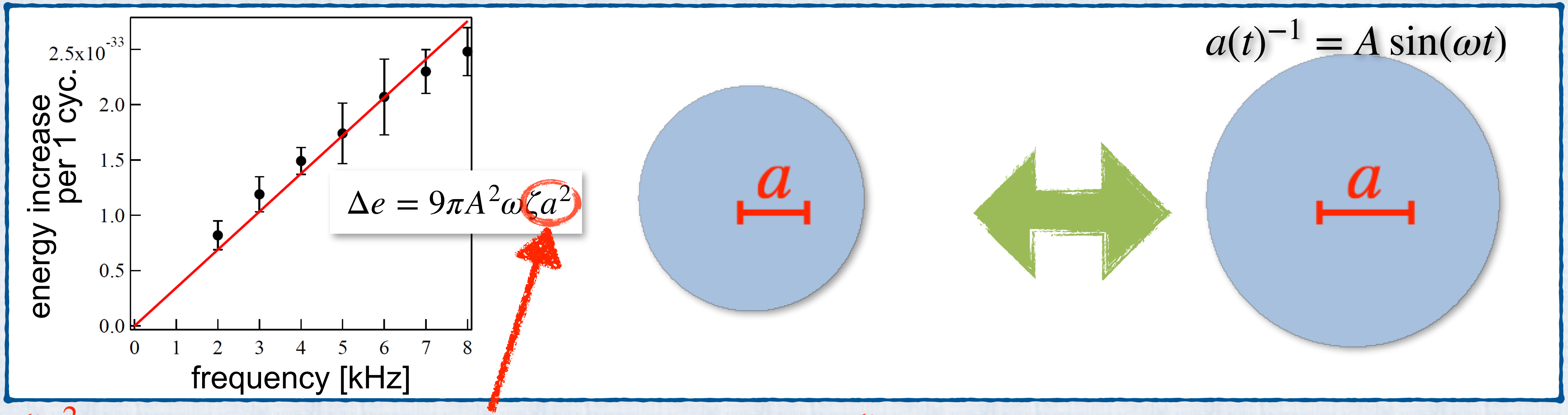
KF & Y. Nishida, PRA **98**, 063634 (2018)

Bulk strain rate tensor with $a(t, \vec{x})$

$$V_a(t, \vec{x}) = \nabla \cdot \vec{v} - 3[\partial_t \ln a + \vec{v} \cdot \nabla \ln a]$$

- Time-periodic oscillation of the scattering length

T. Tsumori, M. Horikoshi, *et al.*, (Ongoing exp.)



ζa^2 **is non-zero even in the unitary limit, although ζ itself vanishes.**

- High-temperature expansion calculation of ζa^2

KF & Y. Nishida, PRA **102**, 023310 (2020);
KF & T. Enss, Ann. Phys. **453**, 169296 (2023)

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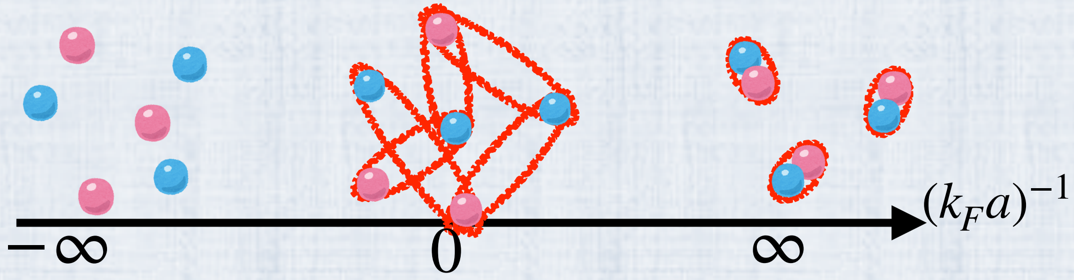
   - ▶ **Universality** of Fermi gases & its appearance in transport



2. Transport coefficients
   - 2. 1 Shear viscosity
   - 2. 2 Thermal conductivity
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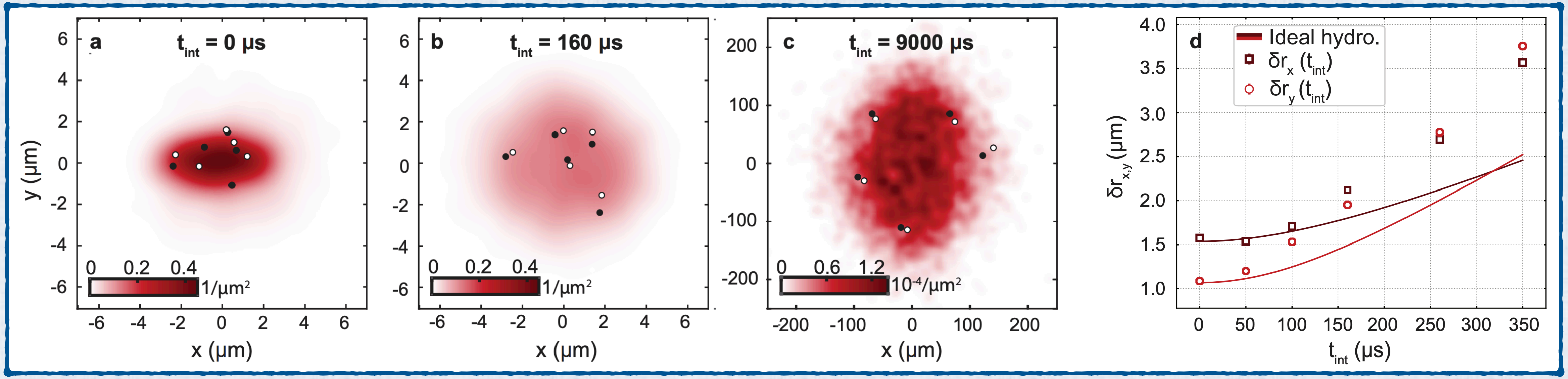
3. **For fluctuating hydrodynamics**

4. Summary & Outlook

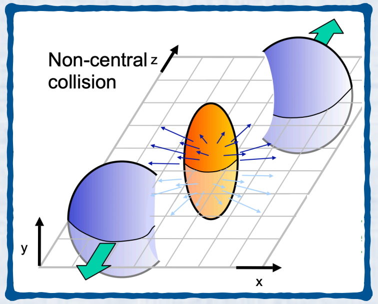
Mesoscopic systems

Anisotropic expansion in 2D 5+5 particles

S. Jochim's exp., [arXiv:2308.09699] (2023);
"Emergent hydrodynamic behaviour of few strongly interacting fermions"



- $1 + 1 \sim$ many-body systems can be realized. ➡ **Hydrodynamic fluctuation?**
- Inversion of the aspect ratio ➡ **Analogy of heavy-ion collisions**



E. Iancu, CERN-2014-003

Applications of fluctuating hydro. to heavy-ion collisions

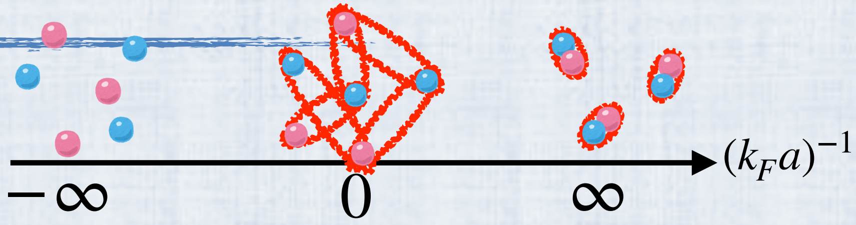
✓Keiji's overview talk & Yukinao's talk

☑ Advantages of ultracold atoms

- Time-resolved measurement of various quantities
- Experimental tunability

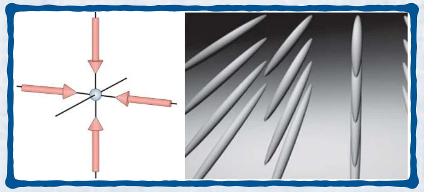
Summary & Outlook

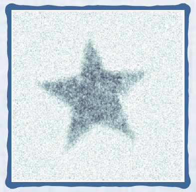
Hydrodynamics in strongly correlated Fermi gases



✓ **Experimental tunability**

- Spatial dim., interaction strength, trapping potential, and so on



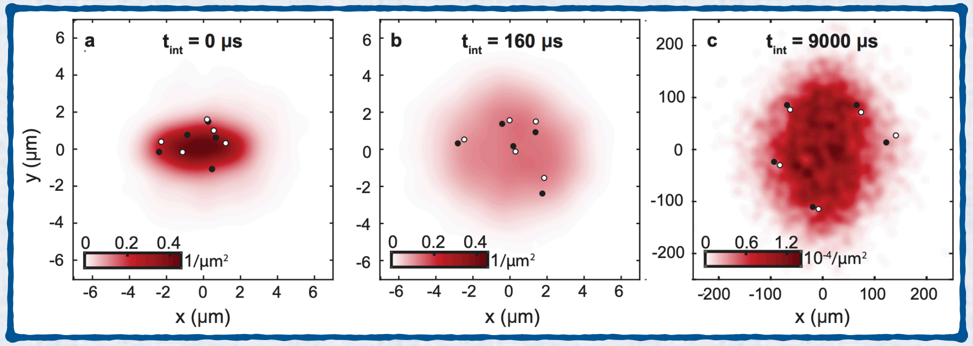


✓ **Universality appearing in transport**

- Various experiments to measure transport using state-of-the art techniques
- Theoretical challenge to strongly correlated regime
  - high-temperature expansion, conformal symmetry, etc.

☑ **Application of Fluctuating hydro.?**

- Mesoscopic systems can be realized in a well-controlled manner.



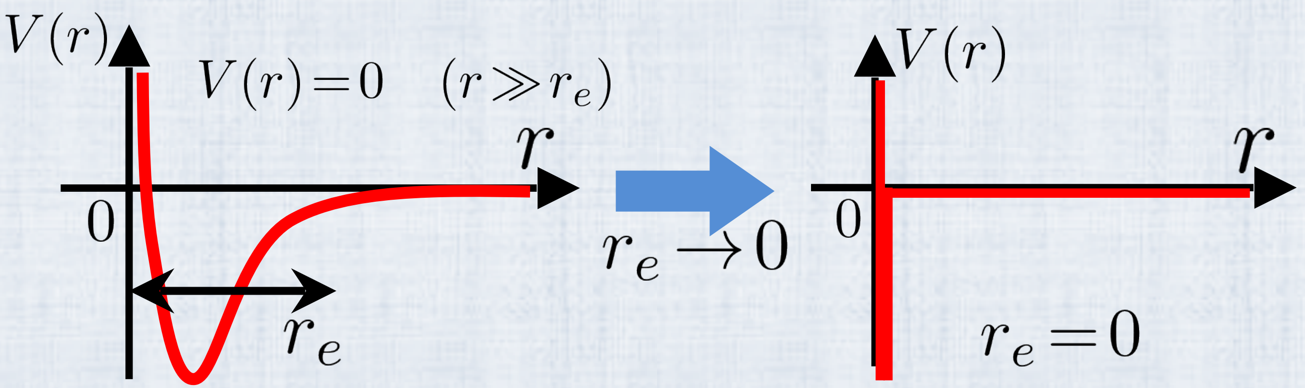
Let's discuss the application with you!!

Thank you!! & Danke schön!!

Model Hamiltonian

Interaction potential : $V(r)$

- Scattering length : a
- ~~Potential radius~~ : r_e sufficiently small



"Contact Interaction"

Fermions with contact interaction

$$\hat{H} = \sum_{\sigma=\uparrow,\downarrow} \int \mathrm{d}\boldsymbol{x}\, \hat{\psi}^\dagger_\sigma(\boldsymbol{x}) \frac{-\nabla^2}{2m} \hat{\psi}_\sigma(\boldsymbol{x}) + \frac{g}{2} \sum_{\sigma,\rho} \int \mathrm{d}\boldsymbol{x}\, \hat{\psi}^\dagger_\sigma(\boldsymbol{x}) \hat{\psi}^\dagger_\rho(\boldsymbol{x}) \hat{\psi}_\rho(\boldsymbol{x}) \hat{\psi}_\sigma(\boldsymbol{x})$$

Cutoff regularization

$$g = \frac{\Omega_{d-1}}{m} \frac{d-2}{a^{2-d} - \frac{\Lambda^{d-2}}{\Gamma(\frac{d}{2})\Gamma(2-\frac{d}{2})}}$$

$$\Omega_{d-1} \equiv \frac{(4\pi)^{d/2}}{2\Gamma(2-d/2)}$$

✓ Scattering amplitude

$$f(k) = \begin{cases} -\dfrac{1}{a^{-1} + ik} & \text{(3D)} \\[2ex] -\dfrac{2\pi}{\ln(ka) - i\frac{\pi}{2}} & \text{(2D)} \end{cases}$$

Significance of ζa^2

Kubo formula for the bulk viscosity

$$\zeta = \lim_{\omega \to 0} \frac{\mathrm{Im}[\mathcal{R}_{\Pi\Pi}(\omega)]}{\omega}$$

$$\hat{\Pi} \equiv \frac{1}{3}\sum_i \hat{\Pi}_{ii} - \left(\frac{\partial p}{\partial \mathcal{N}}\right)_{\mathcal{E}} \hat{\mathcal{N}} - \left(\frac{\partial p}{\partial \mathcal{E}}\right)_{\mathcal{N}} \hat{\mathcal{H}}$$

Pressure operator

▶ Pressure relation $\frac{1}{3}\sum_i \hat{\Pi}_{ii} = \frac{2}{3}\hat{\mathcal{H}} + \frac{\hat{\mathcal{C}}}{12\pi m a}$

Breaking of the conformal invariance

Non-relativistic counterpart of the traceless condition for the stress tensor due to conformality

✓ Pair correlation

$$\zeta a^2 \simeq \frac{1}{(4\pi m)^2 \omega} \mathrm{Im} \int_0^\infty \mathrm{d}t\, e^{i\omega t} \int \mathrm{d}^3\vec{x}\, \langle [\hat{\mathcal{C}}(t,\vec{x}), \hat{\mathcal{C}}(0,\vec{0})] \rangle$$

$$\hat{\mathcal{C}}(t,\vec{x}) = \hat{\Delta}^\dagger \hat{\Delta}(t,\vec{x})$$

$\hat{\Delta} = m g_0 \hat{\psi}_\downarrow \hat{\psi}_\uparrow$ (fermion pair field)

$$-4\pi m \left(\frac{\partial \langle \hat{\mathcal{C}}(t,\vec{x})\rangle}{\partial a^{-1}(0,\vec{0})}\right)_{\mathcal{S}/\mathcal{N}}$$

: Response to $a(0,\vec{0})$

✓ ζa^2 is finite even at unitarity and shows a strong signature at the superfluid phase transition

driven by pair fluctuations

T. Enss, PRL **123**, 205301 (2019)