Quantum Mpemba effect

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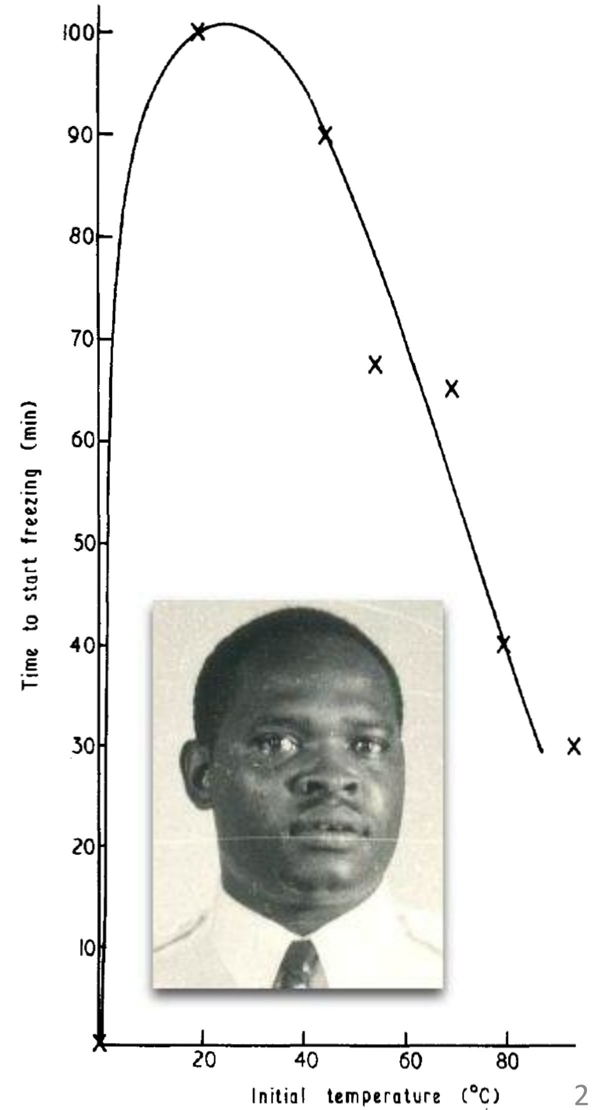
Refs: PRL**131**, 032901 (2021)=Editors' Suggestion and arXiv:2311.01347.

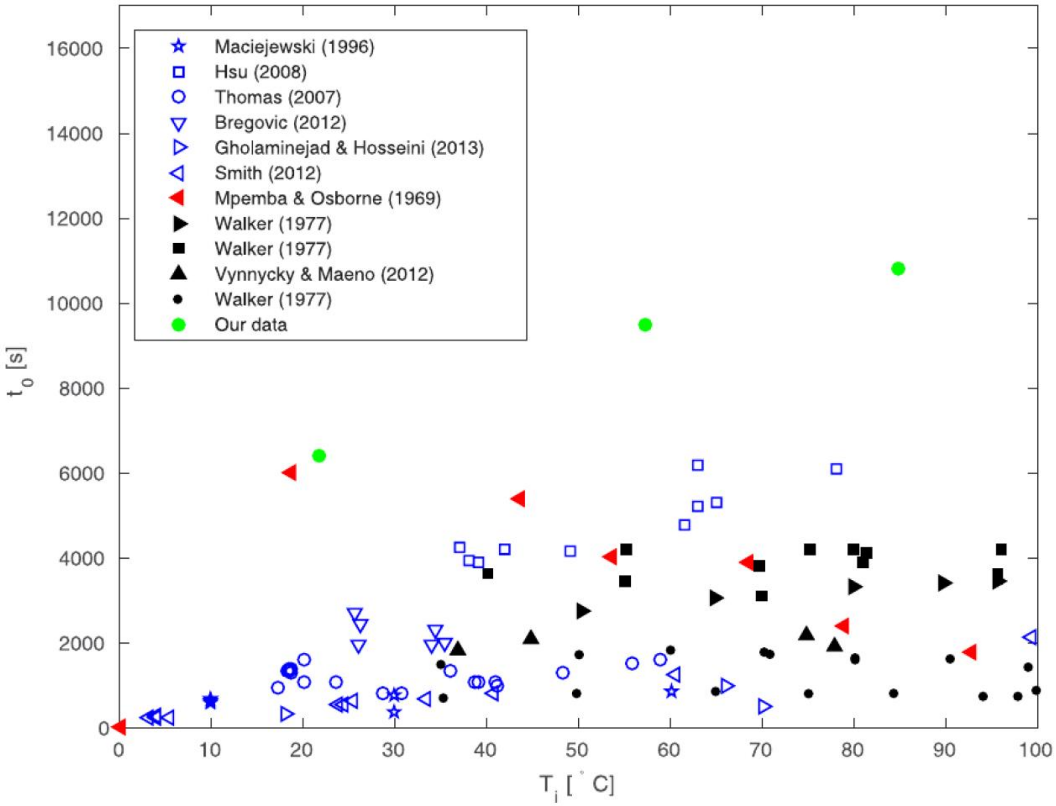
What is the Mpemba effect?

- What is Mpemba effect?
  - Erasto B. Mpemba found that some hot suspensions (*ice cream mix*) can freeze faster than cold (1963).
  - With the help of D. G. Osborne he has published a scientific paper (1969).



Debates

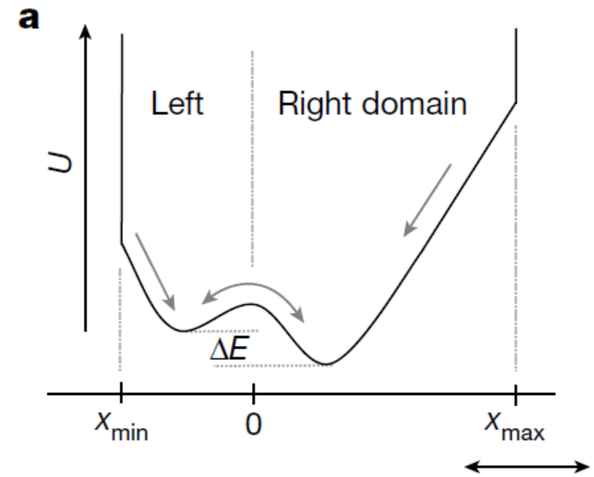
- Poor reproducibility
- The right figure is one counter example of Mpemba effect.
- However, people believe the existence of Mpemba-like phenomena.

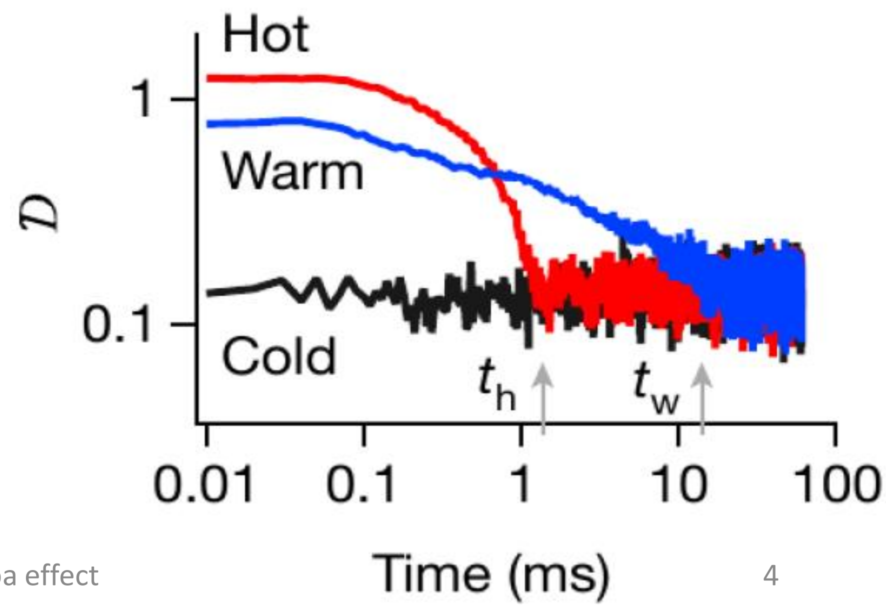


Burridge and Linden, Sci. Rep. **6**, 37665 (2016).

Experimental confirmation

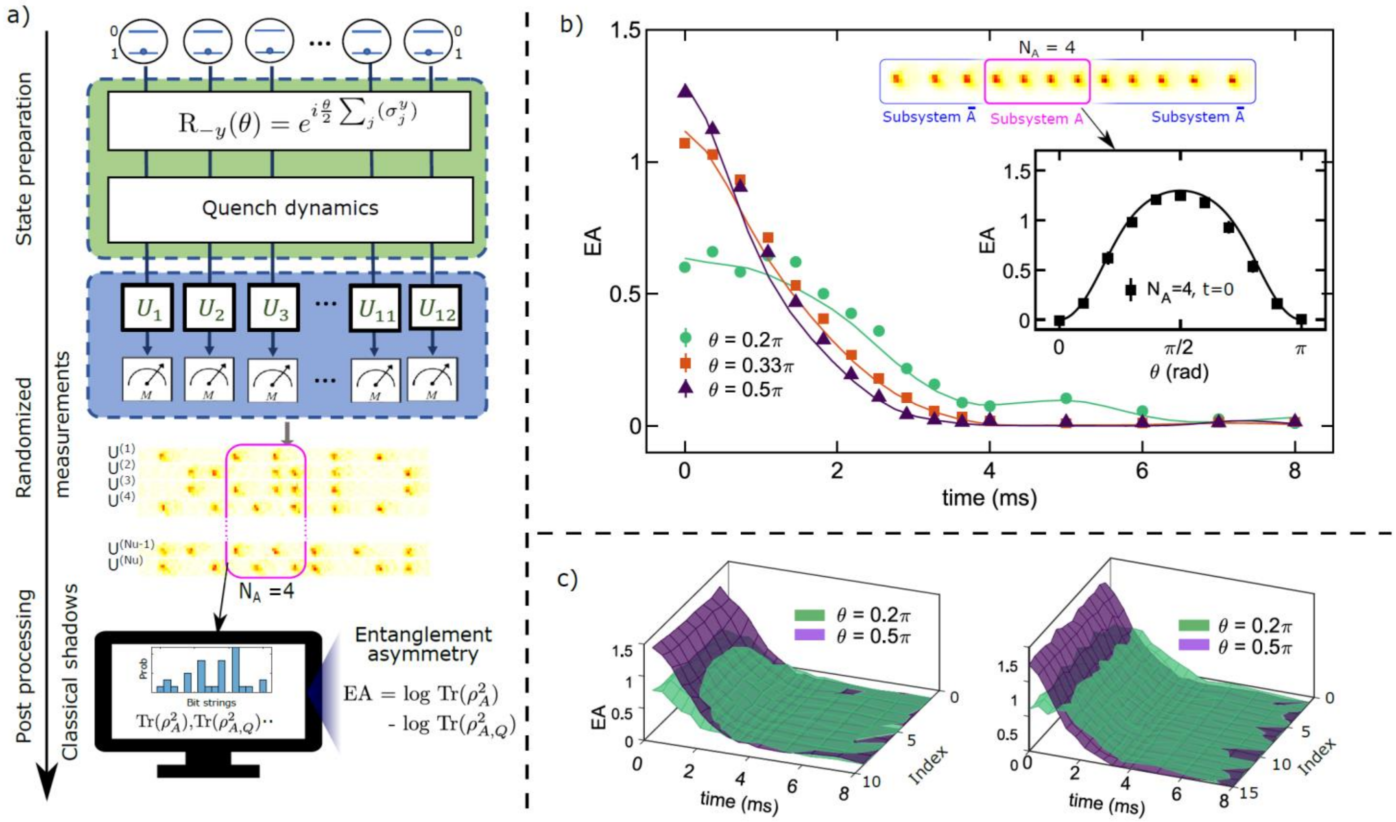
- Kumar & Bechhoeffer, Nature **584**, 64 (2020).
- They have analyzed trapped colloids in a double well potential.
- They observed the distance between the distribution and equilibrium one.



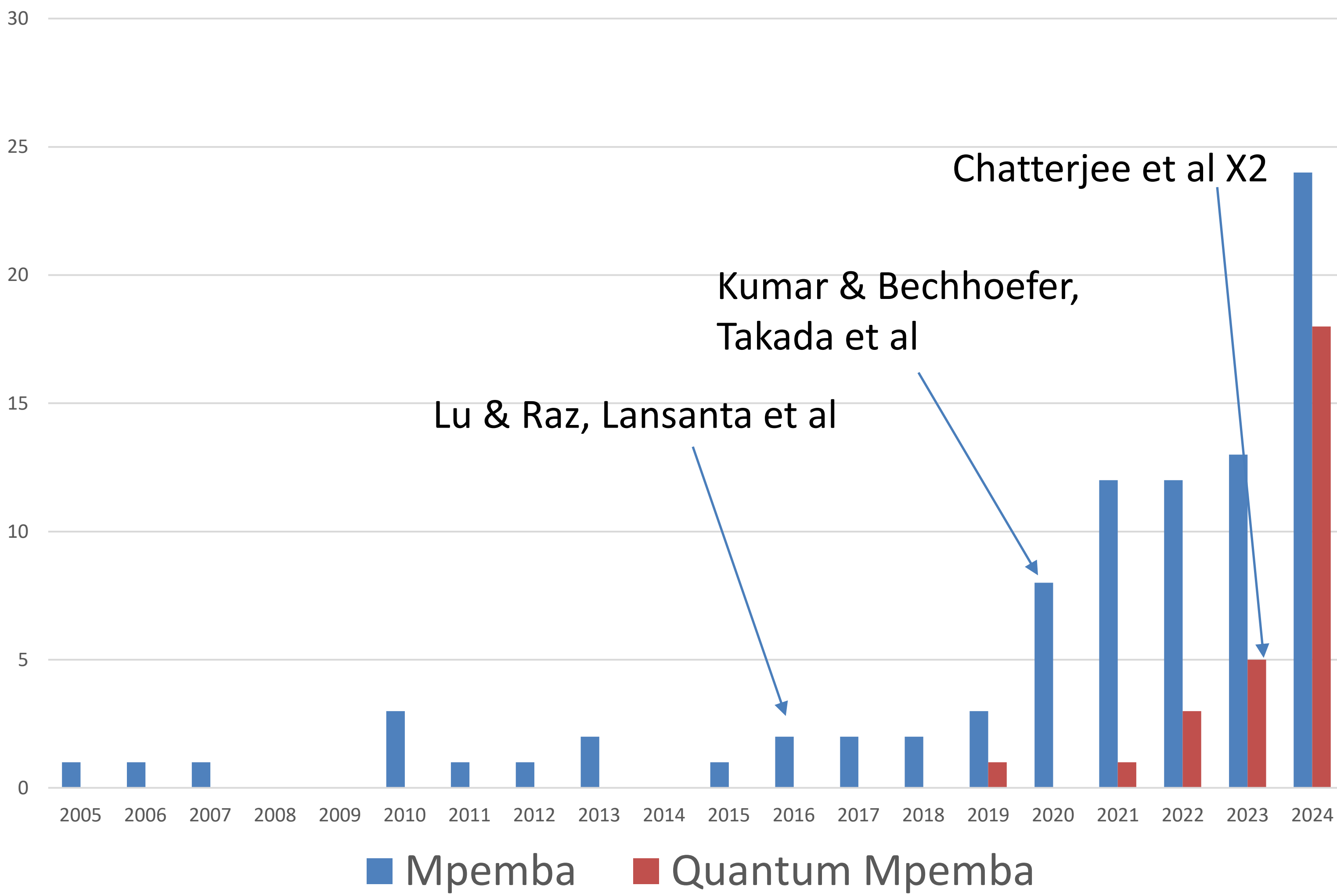


Experimental QMPE

- The first experimental report on QMPE exists this year (arXiv:2401.04270).
- This is observed in a trapped quantum simulator.

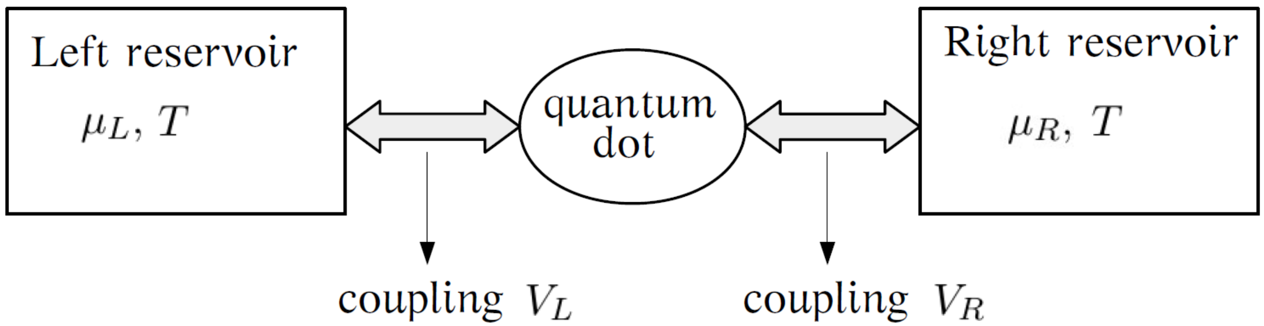


Papers on Mpemba effect in arXiv



Quench dynamics of Anderson model

A single quantum dot connected to two reservoirs



Total Hamiltonian:

$$\hat{H}^{tot} = \hat{H}^{s} + \hat{H}^{r} + \hat{H}^{int}$$

System Hamiltonian ($\hat{H}^{s}$)

Reservoir Hamiltonian ($\hat{H}^{r}$)

System-reservoirs interaction Hamiltonian ($\hat{H}^{int}$)

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$$\hat{H}^s = \sum_{\sigma} \epsilon_0 \hat{d}_\sigma^\dagger \hat{d}_\sigma + U \hat{n}_\uparrow \hat{n}_\downarrow$$

$$\hat{H}^r = \sum_{\gamma,k,\sigma} \epsilon_k \hat{a}_{\gamma,k,\sigma}^\dagger \hat{a}_{\gamma,k,\sigma}$$

$$\hat{H}^{int} = \sum_{\gamma,k,\sigma} V_\gamma \hat{d}_\sigma^\dagger \hat{a}_{\gamma,k,\sigma} + \text{h.c.}$$

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ϵ_0: energy of electron in quantum dot

ϵ_k: energy of electron corresponding to wave number k in reservoirs

U: electron-electron interaction in quantum dot

V_L, V_R: coupling strength between quantum dot and reservoirs

$\hat{d}^\dagger, \hat{d}$: creation and annihilation operators in quantum dot

$\hat{a}^\dagger, \hat{a}$: creation and annihilation operators in reservoirs

$\hat{n}$: number operator $(= \hat{d}^\dagger \hat{d})$

γ: reservoir indices L, R σ: up-spin $(\uparrow)$ or down-spin $(\downarrow)$

Quantum Master equation:

The time evolution of the density matrix (column vector) is given by

$$\frac{d}{dt}|\hat{\rho}(t)\rangle = \hat{K}|\hat{\rho}(t)\rangle$$

with the following Lindbladian (or, rate matrix)

$$\hat{K} = \begin{pmatrix} -2f_-^{(1)} & f_+^{(1)} & f_+^{(1)} & 0 \\ f_-^{(1)} & -f_-^{(0)} - f_+^{(1)} & 0 & f_+^{(0)} \\ f_-^{(1)} & 0 & -f_-^{(0)} - f_+^{(1)} & f_+^{(0)} \\ 0 & f_-^{(0)} & f_-^{(0)} & -2f_+^{(0)} \end{pmatrix}$$

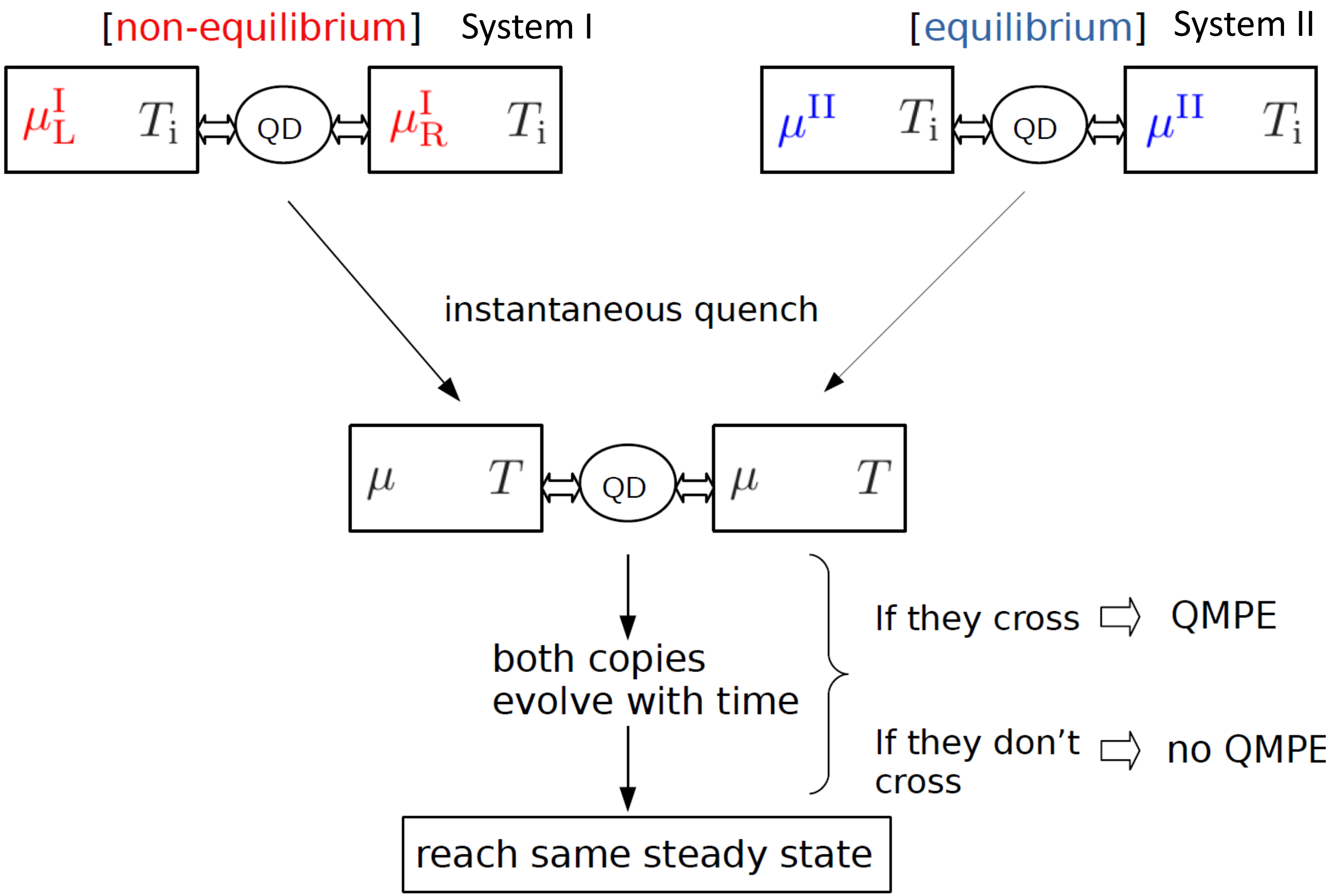
where

$$f_+^{(j)} := f_L^{(j)}(\mu_L, U) + f_R^{(j)}(\mu_R, U) \qquad \text{and} \qquad f_-^{(j)} = 2 - f_+^{(j)}$$

with the Fermi-Dirac distribution:

$$f_\gamma^{(j)}(\mu_\gamma, U) = \frac{1}{1+e^{(\epsilon_0 + jU - \mu_\gamma)/T}}$$

Protocol



[non-equilibrium] System I
μ_L^I T_i QD μ_R^I T_i
[equilibrium] System II
μ^{II} T_i QD μ^{II} T_i
instantaneous quench
μ T QD μ T
both copies
evolve with time
If they cross ⇨ QMPE
If they don't cross ⇨ no QMPE
reach same steady state

QMPE in density matrix

- a_2 is zero $\Longrightarrow$ No contribution from slowest relaxation mode

- To show QMPE in density matrix elements:

$$\Delta\rho_\alpha(\tau) := \rho_\alpha^{\mathrm{I}}(\tau) - \rho_\alpha^{\mathrm{II}}(\tau), \qquad \alpha = 1,2,3,4 \quad (\equiv \uparrow\downarrow, \uparrow, \downarrow, \text{vacant})$$

$$= e^{\lambda_3\tau}\hat{R}_{\alpha,4}\Delta a_4\left[S_\alpha + e^{-(\lambda_3-\lambda_4)\tau}\right]$$

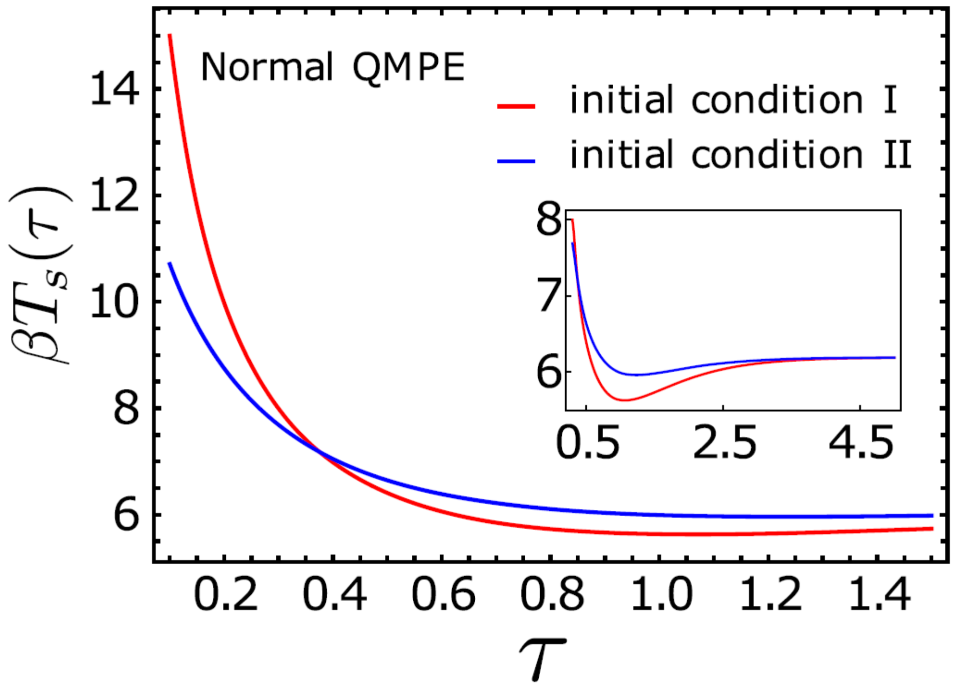
Necessary criterion for QMPE: $S_\alpha < 0 \;\; \& \;\; |S_\alpha| < 1$

$$S_\alpha := (\hat{R}_{\alpha,3}\Delta a_3)/(\hat{R}_{\alpha,4}\Delta a_4)$$

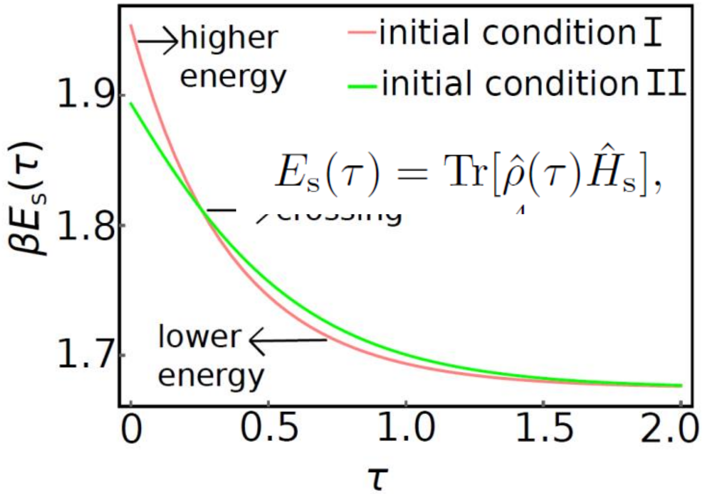
combined role of the faster relaxation modes on QMPE

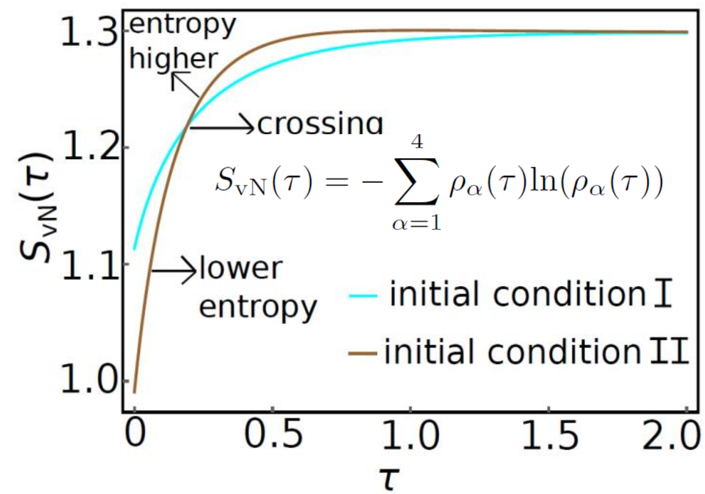
Thermal Mpemba effect

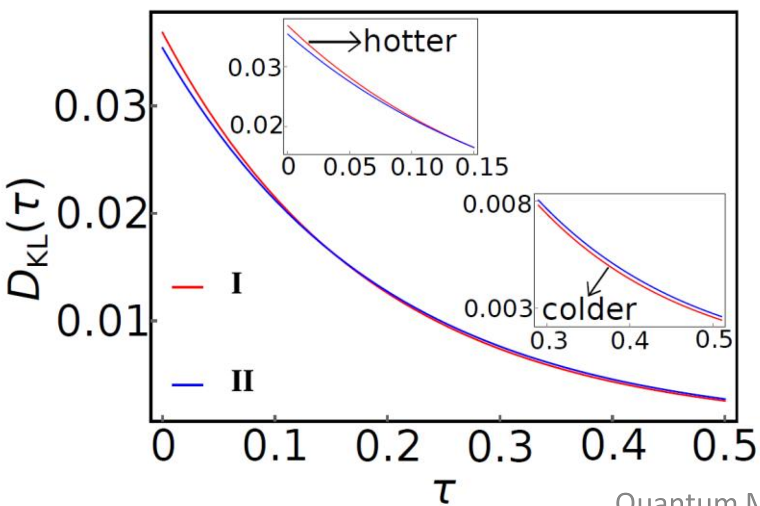
$$T_s(\tau) := \frac{\partial E_s(\tau)}{\partial S_{\mathrm{vN}}(\tau)} = \frac{\partial E_s(\tau)}{\partial \tau} \Big/ \frac{\partial S_{\mathrm{vN}}(\tau)}{\partial \tau} \qquad S_{\mathrm{vN}}(\tau) = -\sum_\alpha \rho_\alpha(\tau) \ln[\rho_\alpha(\tau)]$$



Mpemba effect in the other observables







$$D_{\mathrm{KL}}(\tau) = \sum_{\alpha=1}^{4} \rho_\alpha(\tau) \ln\left(\frac{\rho_\alpha(\tau)}{\rho_{\mathrm{ss},\alpha}}\right)$$

Summary of quantum Mpemba effect

- We have demonstrated the existence of Mpemba-like phenomena after a sudden change of system.
- Such effects can be observed in the density matrix elements, von Neumann , energy and temperature.
- Mpemba effect may be useful to speed-up to get a desired state.

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Multiple Mpemba effect using exceptional points

- The previous model is quasi-classical because off-diagonal elements of the density matrix do not play any roles.
- We need to know the effect of entanglements.
- The model of open quantum systems may have exceptional points.
- The minimum model to satisfy the above requirement is Hatano’s model.

Model

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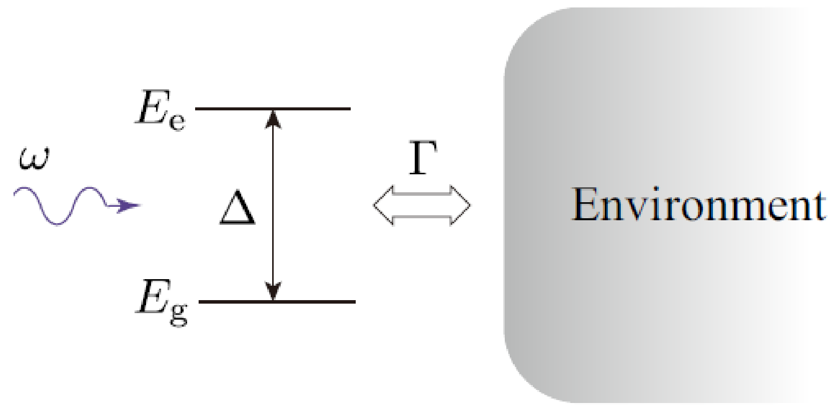
- We consider the Lindblad equation for a two-level open quantum system.
- N. Hatano, Mol. Phys. **117**, 2121 (2019).

$$i\dot{\rho}_{\mathrm{eg}} = \delta\rho_{\mathrm{eg}} - \frac{d}{2}(\rho_{\mathrm{ee}} - \rho_{\mathrm{gg}}) - \frac{i}{2}\Gamma\rho_{\mathrm{eg}},$$

$$i\dot{\rho}_{\mathrm{ge}} = -\delta\rho_{\mathrm{ge}} + \frac{d}{2}(\rho_{\mathrm{ee}} - \rho_{\mathrm{gg}}) - \frac{i}{2}\Gamma\rho_{\mathrm{ge}},$$

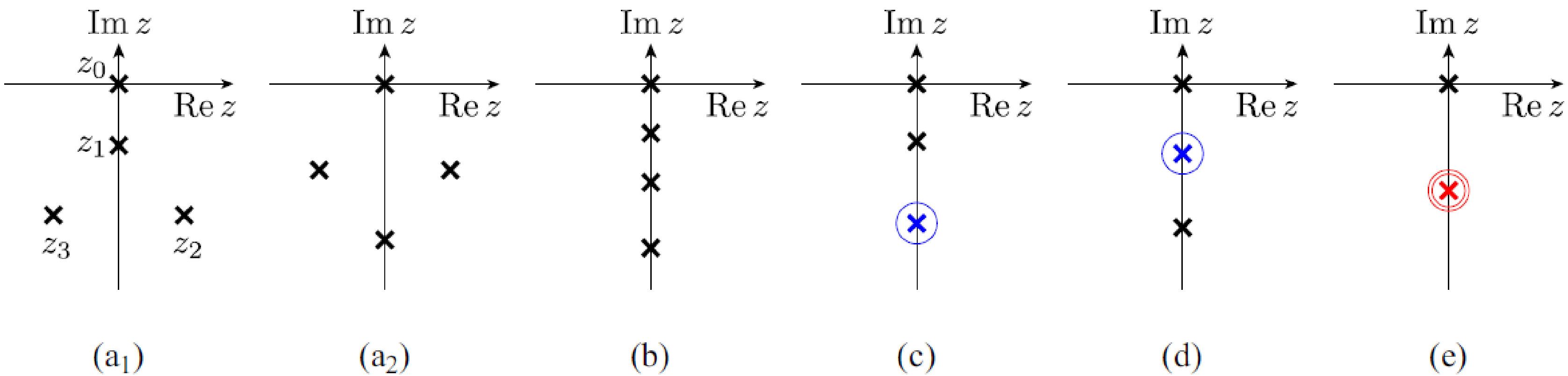
$$i\dot{\rho}_{\mathrm{ee}} = -\frac{d}{2}(\rho_{\mathrm{eg}} - \rho_{\mathrm{ge}}) - i\Gamma\rho_{\mathrm{ee}},$$

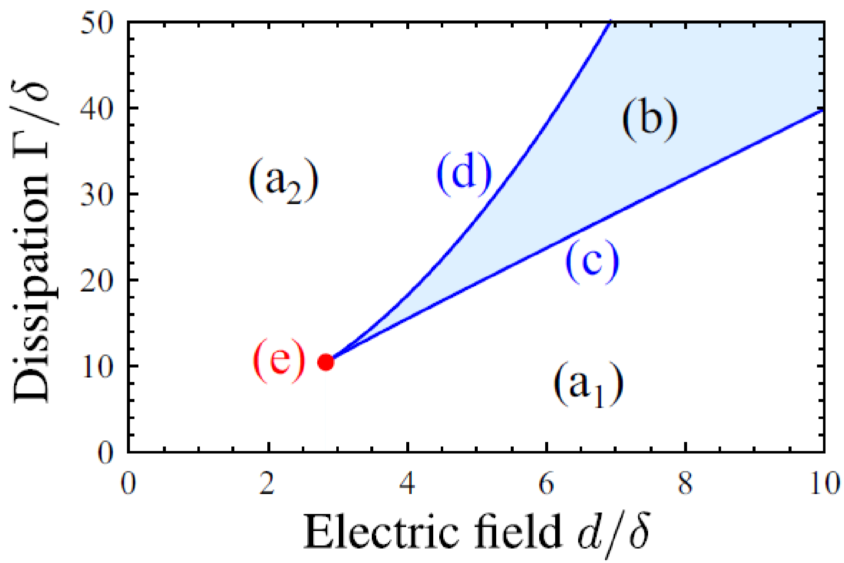
$$i\dot{\rho}_{\mathrm{gg}} = \frac{d}{2}(\rho_{\mathrm{eg}} - \rho_{\mathrm{ge}}) + i\Gamma\rho_{\mathrm{ee}}.$$



d: parameter related to electric field

Eigenvalues & phase diagrams





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Setup

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- To clarify the role of exceptional points, we consider quenches to the exceptional point.

initial condition I — quench → EP

initial condition II — quench → EP

$$i\frac{d}{dt}|\widehat{\rho}(t)\rangle = \widehat{\mathcal{L}}|\widehat{\rho}(t)\rangle.$$

<u>At EP</u>

$$|\hat{\rho}(t)\rangle = \begin{pmatrix} \rho_1(t) \\ \rho_2(t) \\ \rho_3(t) \\ \rho_4(t) \end{pmatrix} \equiv \begin{pmatrix} \rho_{\mathrm{eg}}(t) \\ \rho_{\mathrm{ge}}(t) \\ \rho_{\mathrm{ee}}(t) \\ \rho_{\mathrm{gg}}(t) \end{pmatrix} = \begin{pmatrix} \rho_{\mathrm{eg}}(t) \\ \rho^*_{\mathrm{eg}}(t) \\ 1-\rho_{\mathrm{gg}}(t) \\ \rho_{\mathrm{gg}}(t) \end{pmatrix} = \begin{pmatrix} \rho_{\mathrm{re}}(t) + \mathrm{i}\,\rho_{\mathrm{im}}(t) \\ \rho_{\mathrm{re}}(t) - \mathrm{i}\,\rho_{\mathrm{im}}(t) \\ 1-\rho_{\mathrm{gg}}(t) \\ \rho_{\mathrm{gg}}(t) \end{pmatrix}.$$

contribution contribution

Evolution of density matrix

- The density matrix is given by

$$\rho_j(t) = \sum_{k=1}^{4} e^{-\lambda_k t} r_{k,j} a_k - \mathrm{i}t e^{-\lambda_2 t} r_{2,j} a_3,$$
$$a_k = \sum_{n=1}^{4} \ell_{k,n} \rho_n(0),$$

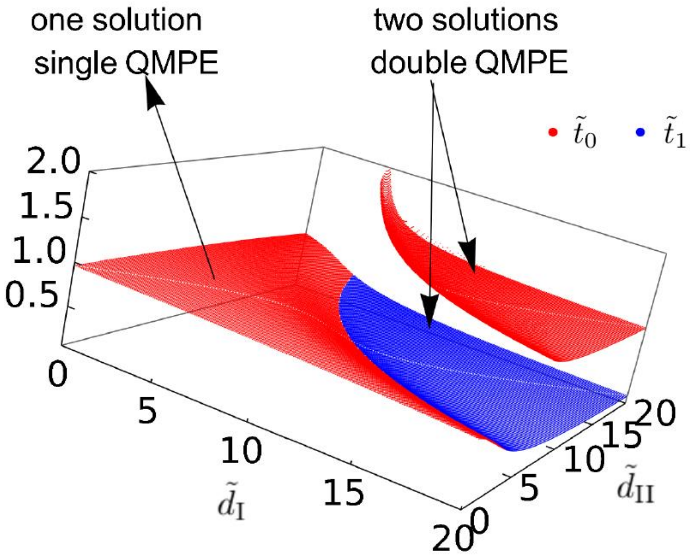
- The difference of density element in two copies $$\Delta\rho_{\mathrm{gg}}(t) = -e^{-\lambda_2 t}\left[\alpha_1 e^{-(\lambda_4-\lambda_2)t} + t\,\alpha_2 + \alpha_3\right],$$

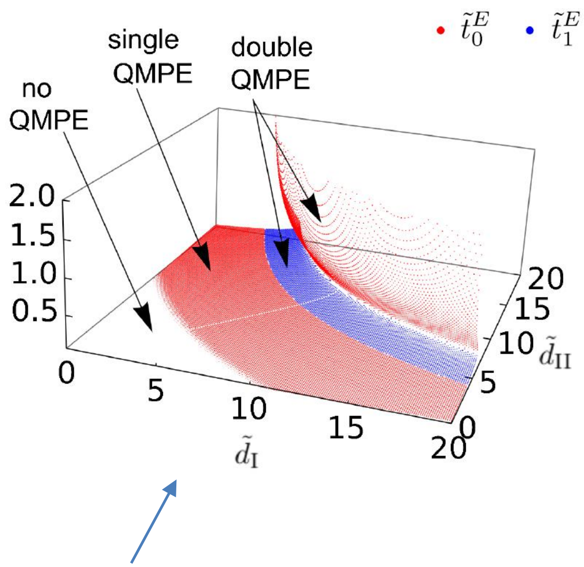
$$\alpha_1 = a_4^{\mathrm{I}} - a_4^{\mathrm{II}},\ \ \alpha_2 = -\mathrm{i}(a_3^{\mathrm{I}} - a_3^{\mathrm{II}}),\ \ \alpha_3 = a_2^{\mathrm{I}} - a_2^{\mathrm{II}}.$$

It is not difficult to get the condition of $\Delta\rho_{gg}$=0.

Time of intersections

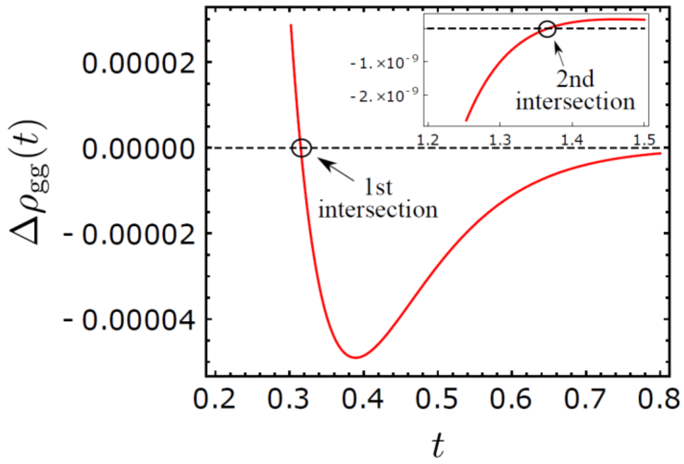
- We obtain the exact time for the intersection:



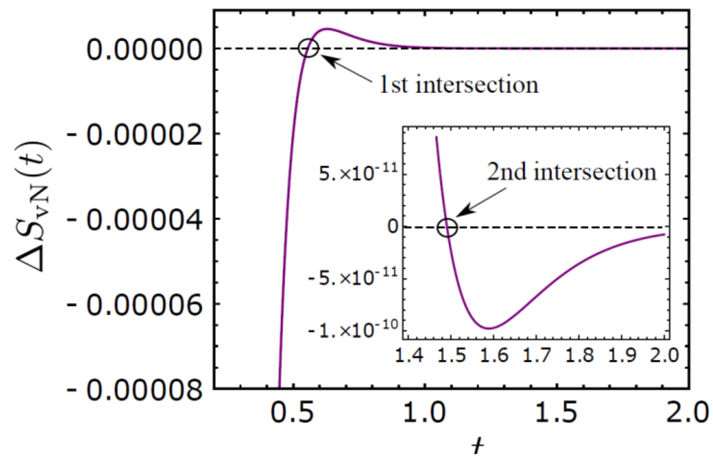


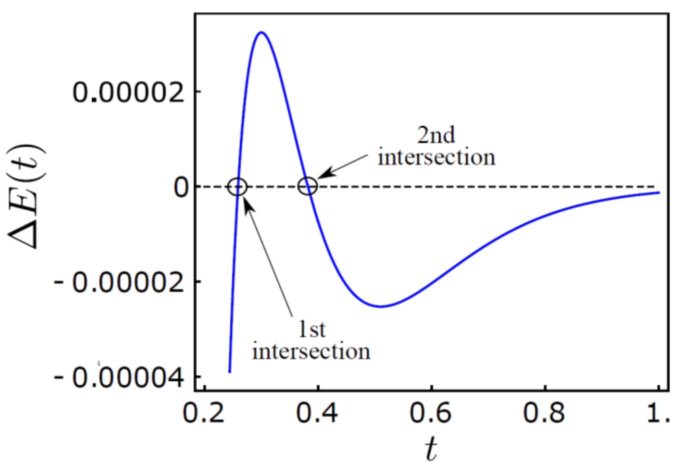
Intersection time for the energy

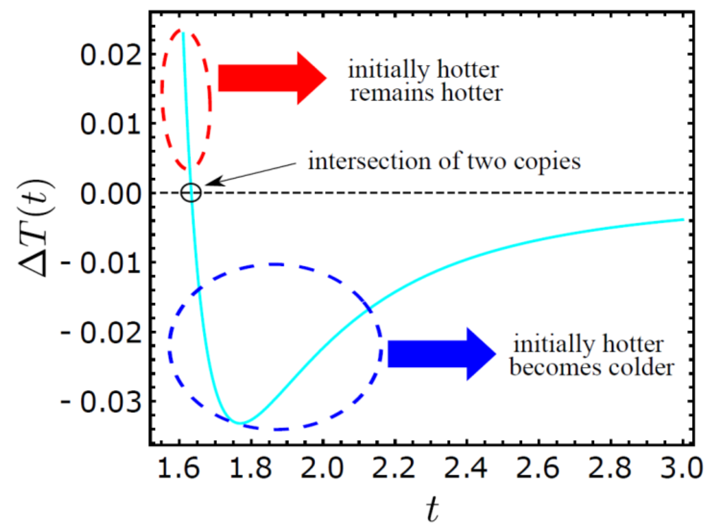
QMPE for various variables



(a)

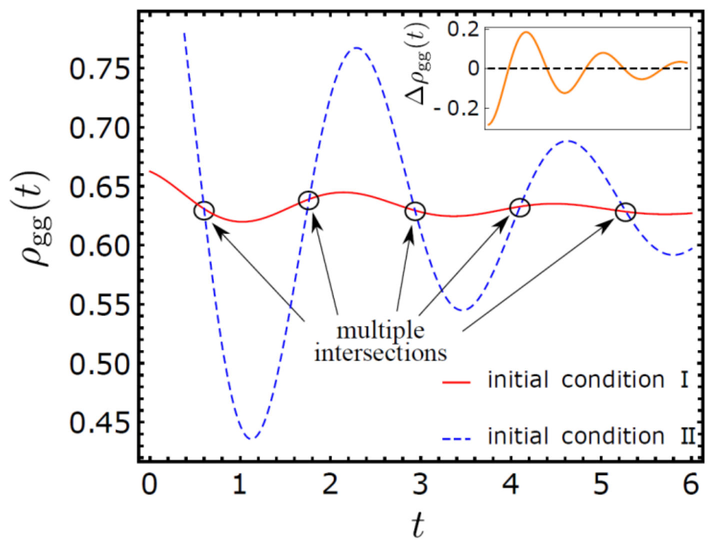


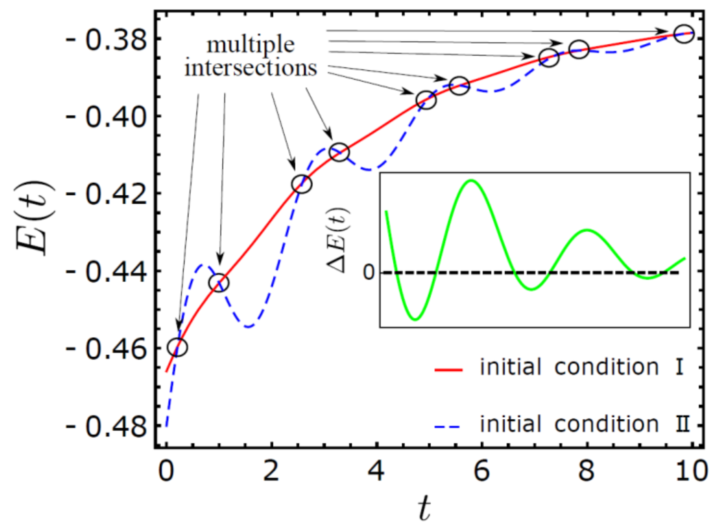


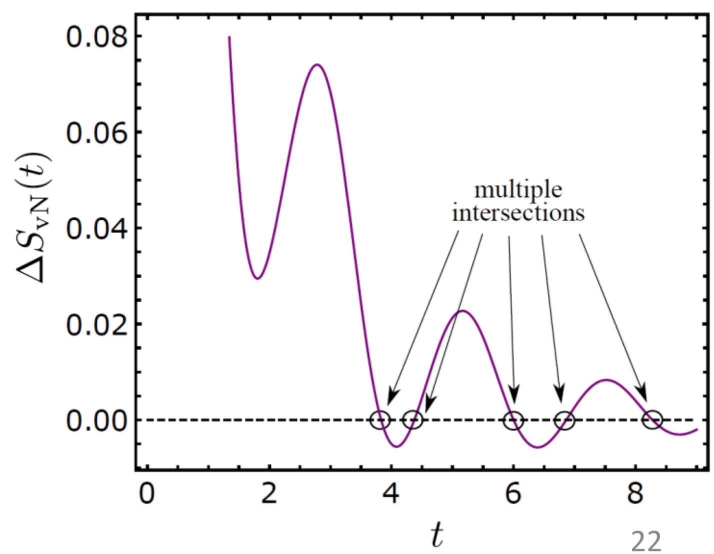


Multiple Mpemba effect in (a_1)

- The region (a_1) has complex eigenvalues.=>Oscillations

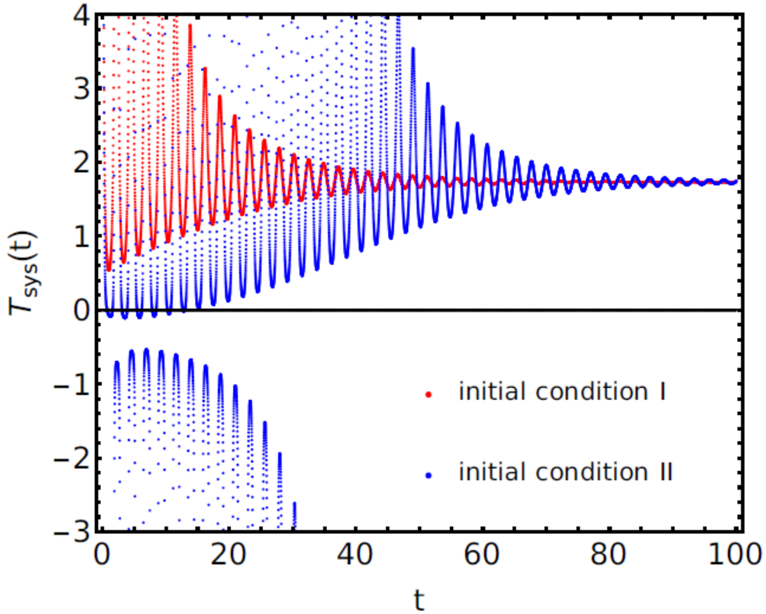


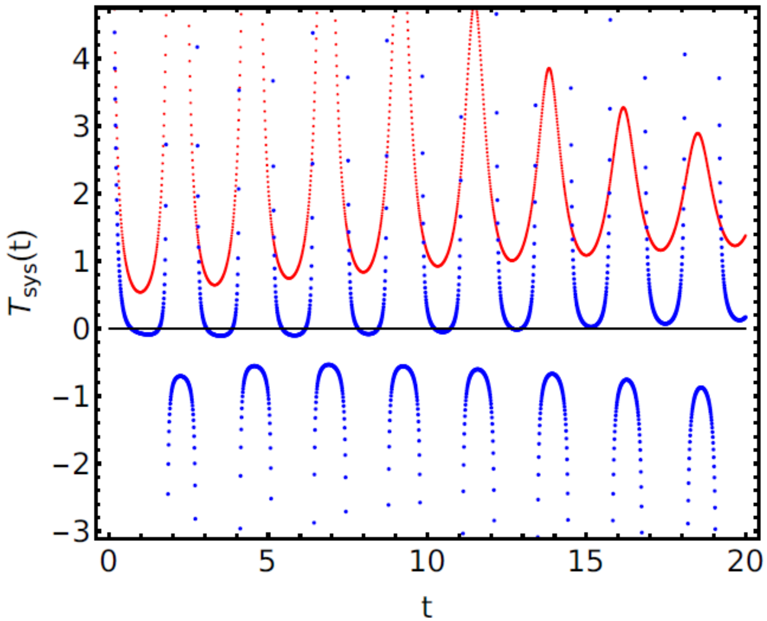




Multiple Thermal Mpemba effect

- If the system has complex eigenvalues, the behavior can be oscillate.
- Then, multiple Mpemba effect in region (a_1) can be observed.





Discussion

- It is not difficult to generate MPE by the control of initial condition.
  - Nonequilibrium initial conditions have lower symmetries than that in equilibrium (Ares et al. 2023).
  - We can eliminate the slowest eigenmode by the unitary transformation of the initial condition (Carollo et al, 2021).
- What is the best protocol to get the fastest relaxation?
  - Connection to the speed-limit problem?

Summary

- We demonstrate the occurrence of quantum Mpemba effect (QMPE) in Anderson model and Hatano's model.
  - Thermal QMPE can be observed easily.
  - The slow modes are not always important.
  - Difference of the relaxation rate between equilibrium and nonequilibrium initial conditions is important.
- QMPE is generic.
- If there exist exceptional points, the observation of QMPE is easier than that in the absence of EP.
- Multiple QMPE can be observed easily.