Thermal Transport in 2D Rotor Model

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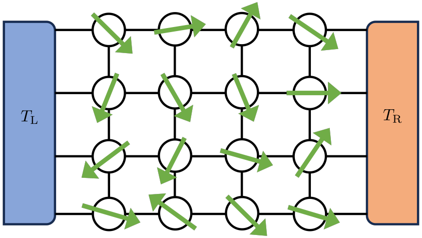
Microscopic model: Hamiltonian XY (Rotor) model



- XY spins on a square lattice
- Phase variable θ_i
- Conjugate momentum p_i

- Hamiltonian

$$H = \sum_{i} \frac{p_i^2}{2} + \frac{1}{2} \sum_{i \neq j} (1 - \cos(\theta_i - \theta_j))$$

- Time evolution

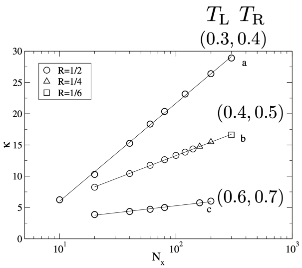
$$\dot{\theta}_i = p_i$$

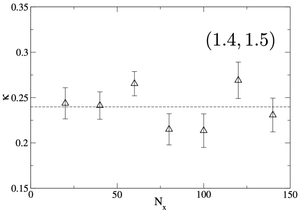
$$\dot{p}_i = - \sum_{j \in \text{NN of } i} \sin(\theta_i - \theta_j)$$

- Momentum and Energy are conserved.
- Static properties:
  - High temperature: **disordered phase**
  - Low temperature: **KT phase**

Numerical observations [Delfini-Lepri-Livi 2005]

Size-dependent heat conductivity $\kappa(L)$





$$\kappa(L) \sim \ln L \quad (?)$$

$$\kappa(L) \sim L^0$$

Phase transition from anomalous to normal at KT point?

Hydrodynamic theory?

Hydrodynamics at low temperatures

- Slow variables: phase θ, phase velocity $v = \partial_t \theta$, energy density e
- Entropy functional $\mathcal{S}[\theta, v, e] = \ln P_{\text{eq}}$:

$$\mathcal{S}[\theta, v, e] = \int_{\boldsymbol{r}} s(u), \quad u = e - \frac{v^2}{2} - \frac{\rho_0}{2}(\nabla\theta)^2$$

- Constraints
  - Velocity v and energy e are **conserved**.
  - **“Internal Galilean symmetry”**: $\theta \mapsto \theta + v_0 t$, $v \mapsto v + v_0$, $e \mapsto e + v_0 v + \frac{v_0^2}{2}$
- **Fluctuating hydrodynamics**

$$\begin{aligned}
\partial_t \theta &= v, \\
\partial_t v &= -\nabla \cdot \left(-\rho_0 \nabla\theta - \gamma_0 \beta \nabla v + \sqrt{2\gamma_0}\boldsymbol{\xi}^v \right), \\
\partial_t e &= -\nabla \cdot \left(\left(-\rho_0 \nabla\theta - \gamma_0 \beta \nabla v + \sqrt{2\gamma_0}\boldsymbol{\xi}^v \right) v + \lambda_0 \nabla\beta + \sqrt{2\lambda_0}\boldsymbol{\xi}^e \right)
\end{aligned}$$

Hydrodynamics at low temperatures (cont')

$$\partial_t \theta = v,$$

$$\partial_t v = -\nabla \cdot \left(-\rho_0 \nabla\theta \; -\gamma_0 \beta \nabla v \; +\sqrt{2\gamma_0}\boldsymbol{\xi}^v \right),$$

$$\partial_t e = -\nabla \cdot \left(\left(-\rho_0 \nabla\theta \; -\gamma_0 \beta \nabla v \; +\sqrt{2\gamma_0}\boldsymbol{\xi}^v \right) v + \lambda_0 \nabla \beta \; +\sqrt{2\lambda_0}\boldsymbol{\xi}^e \right)$$

- Reversible current induced by spin-wave stiffness (spin-wave stiffness ρ_0)
- Dissipative current ($\beta = \partial s/\partial u$, transport coefficients γ_0, λ_0)
- Thermal noise (white Gaussian noises $\boldsymbol{\xi}^v$, $\boldsymbol{\xi}^e$)

Linear analysis

- Most probable homogeneous configuration with total energy density e_0

$$\theta(\boldsymbol{r}) = \theta_0, \quad v(\boldsymbol{r}) = 0, \quad e(\boldsymbol{r}) = e_0$$

- Expand around this configuration

$$\partial_t \theta = v, \quad \partial_t v = \rho_0 \triangle \theta + \frac{\gamma_0}{T_0} \triangle v + \cdots, \quad \partial_t \delta e = \frac{\lambda_0}{c_0 T_0^2} \triangle \delta e + \cdots$$

  - **Two damped sound modes** $\omega_\pm(\boldsymbol{k}) = -i \frac{\gamma_0}{2T_0} \boldsymbol{k}^2 \mp \sqrt{\rho_0} |\boldsymbol{k}| + O(\boldsymbol{k}^3)$
  - **One diffusion mode** $\omega_0(\boldsymbol{k}) = -i \frac{\lambda_0}{c_0 T_0^2} \boldsymbol{k}^2$

Anomalous heat transport

We treat θ as a scalar field.

Bare perturbation theory: Energy current density $\boldsymbol{j}^e \sim -\rho_0 v \nabla\theta + \cdots$

$$\kappa_{\text{sing}}(L) \sim \int \langle v\nabla\theta v\nabla\theta \rangle \sim \int^L dt \int d^d\boldsymbol{k}\; e^{-i(\omega_+(\boldsymbol{k})+\omega_-(\boldsymbol{k}))t} \sim \int^L dt\; t^{-d/2} \sim L^{1-d/2}$$

RG analysis: Up to one-loop,

$$\kappa_{\text{R}}(\boldsymbol{k}=\boldsymbol{0},\omega) \sim \begin{cases} \kappa & d>2 \\ \ln\left(\frac{1}{\omega}\right) & d=2 \\ \omega^{-1+d/2} & d<2 \end{cases}$$

Summary

- Anomalous transport in BKT phase
- Use of hydrodynamics
- RG analysis is consistent with numerical simulations.

Discussions

- **Effect of vortices**
  - Numerical simulations suggests transition from anomalous to normal at the KT pt.
  - Neglecting periodicity of phase field is not allowed at high temperatures.
  - **Need defet-phase hydrodynamics?**
- **"Hamiltonian" $O(N)$ model for $N > 2$**
  - **KT phase is special? No anomaly for $N > 2$ even in $d \leq 2$?**