Hyperuniformity in Chiral Active Fluids

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Advances in Fluctuating Hydrodynamics

Outline

- Introduction
  - Active Matter
  - Fluctuations in Active Matter
  - Hyperuniformity in Chiral Active Fluids
- Theory for HU in 2D Chiral Active Fluid
- Density Fluctuations in 3D Chiral Active Fluid
  - Microscopic Model and Hydrodynamic Equations
  - Numerical Simulation
- Summary

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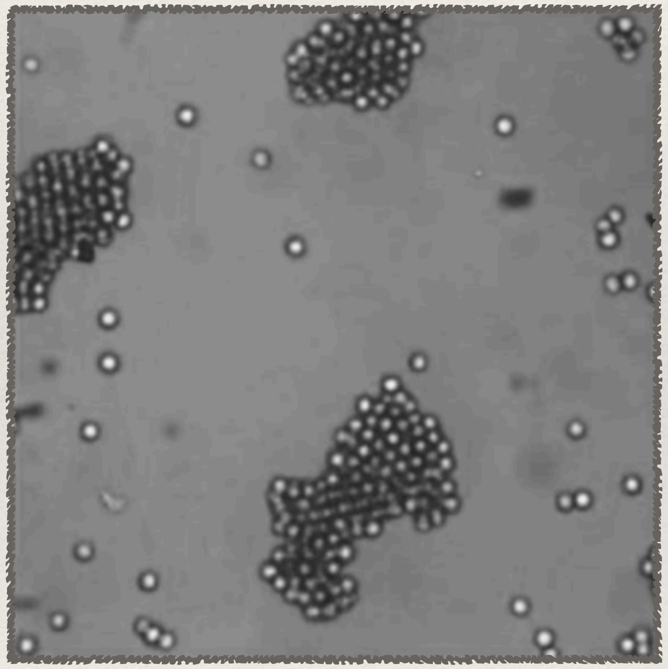
Introduction

What is Active Matter ?

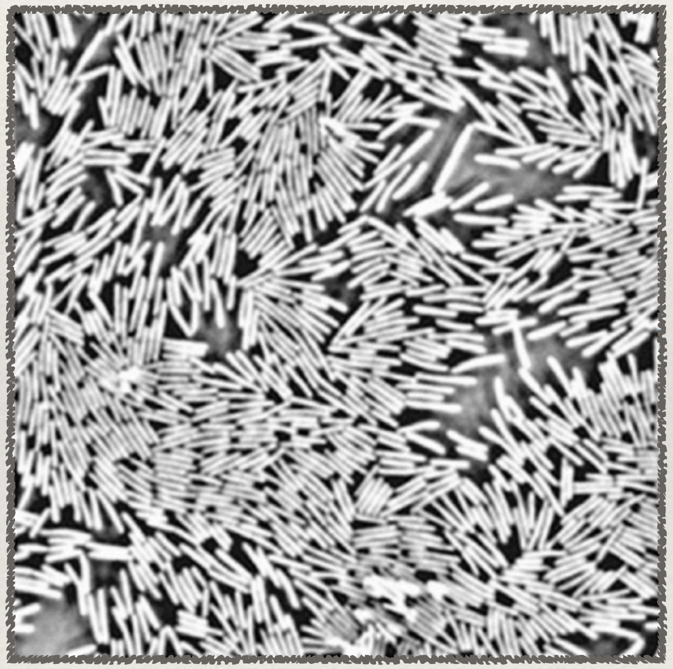
Active matter ≒ any collection of self-propelled objects

Janus particle



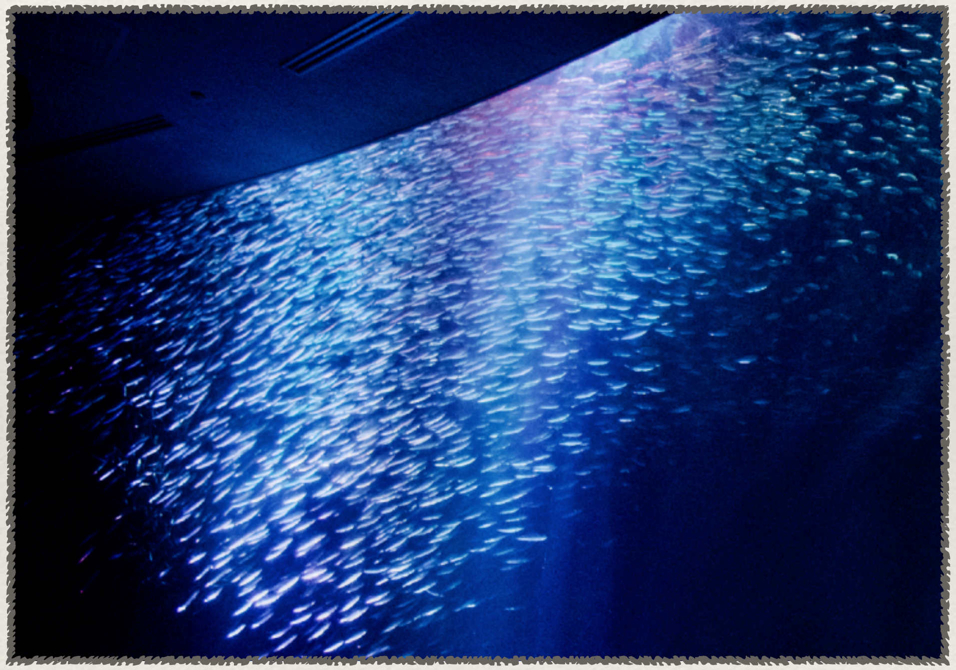
I. Buttinoni, *et al*., (2015)

Bacteria



H. P. Zhang, *et al*., (2010)

School of fish

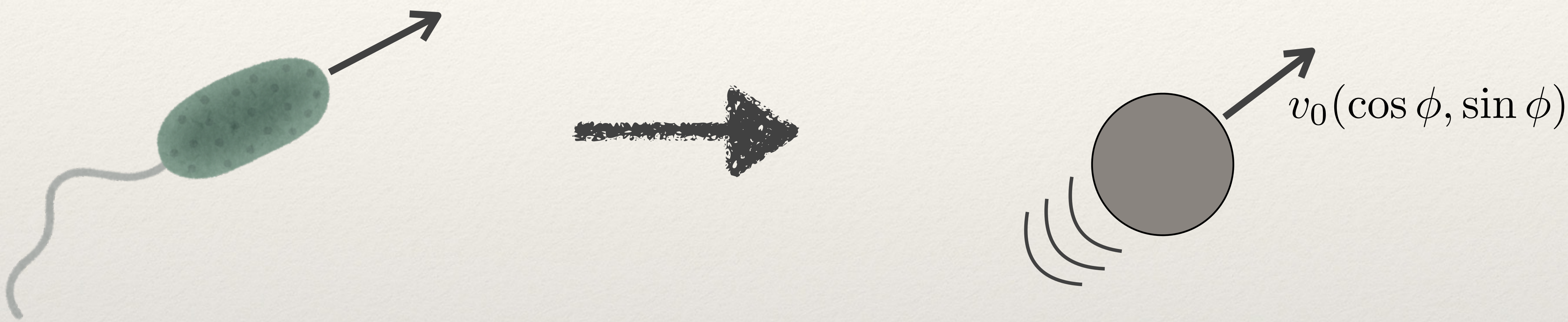


Taken at Nagoya public aquarium

Introduction

Modeling of Active Matter

- Mostly active particles are modeled as straight swimmers



Active Brownian particle

$$\dot{\boldsymbol{r}}_j(t) = \underbrace{\mu \boldsymbol{F}_j^{\mathrm{int}}}_{\text{interaction}} + \underbrace{v_0 \boldsymbol{e}(\phi_j(t))}_{\text{self-propelling}}$$

$$\dot{\phi}_j(t) = \sqrt{2D}\underbrace{\eta_j(t)}_{\text{noise}}$$

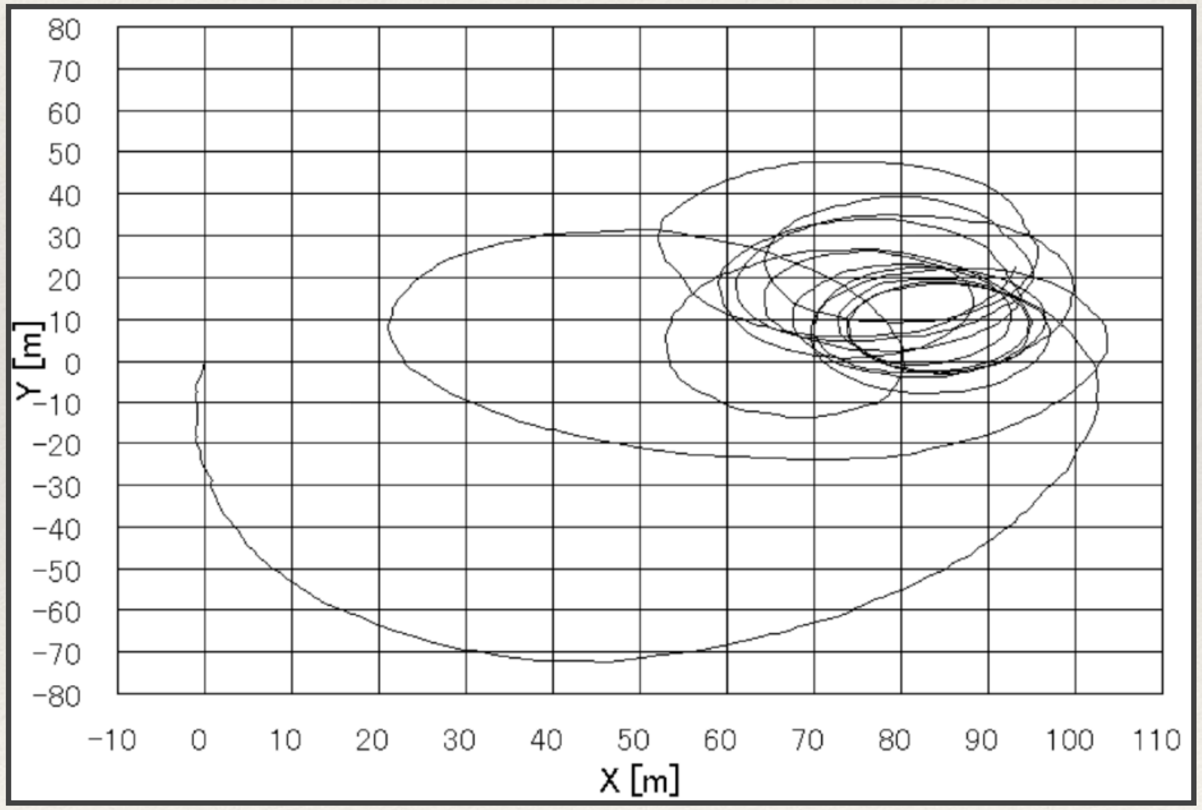
- ABP performs almost straight motion for the time scale $1/D$

Introduction

Chiral Active Matter

☑ Most active objects *can't* go straight!

A walking guy who can't see anything draws a circular trajectory



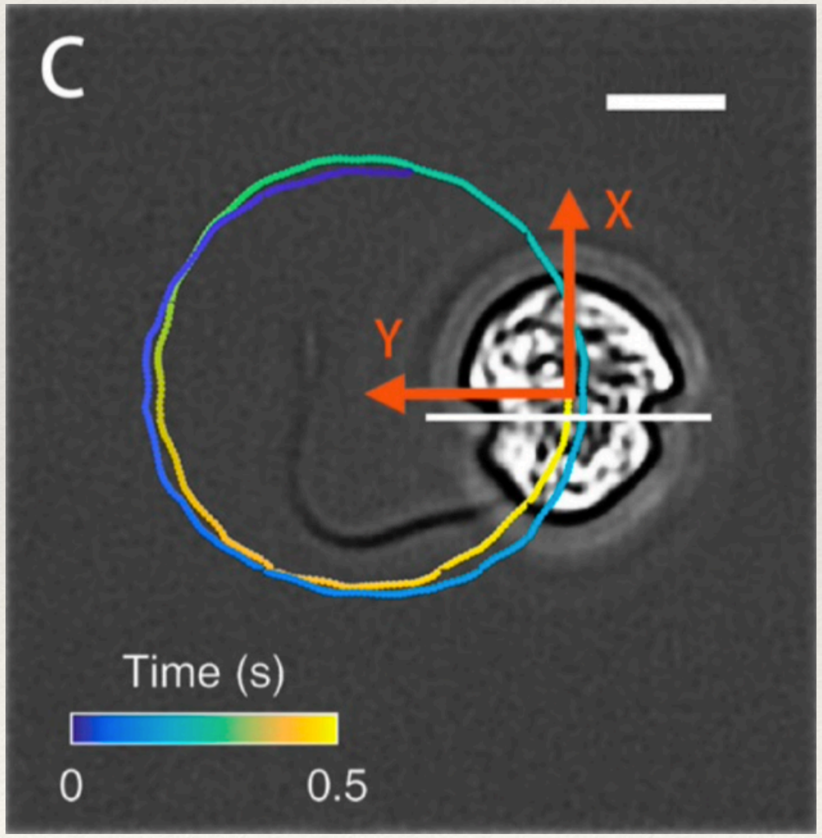
T. Obata *et al.*,(2005)

Introduction

Chiral Active Matter

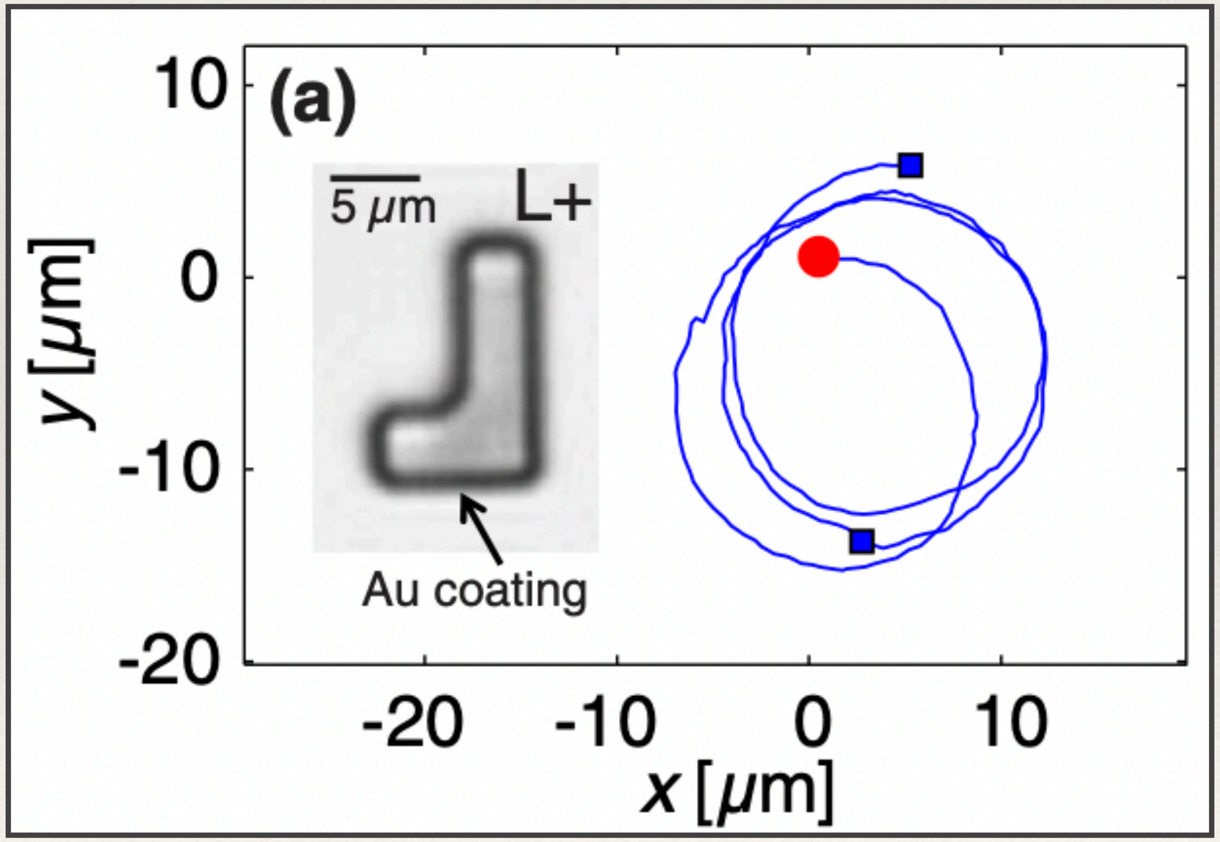
Chiral active matter:
A collection of swimmers whose motion don't have mirror symmetry

Marine algae



M. Huang *et al.*,(2021)

L-shaped particle



F. Kümmel *et al*. (2013)

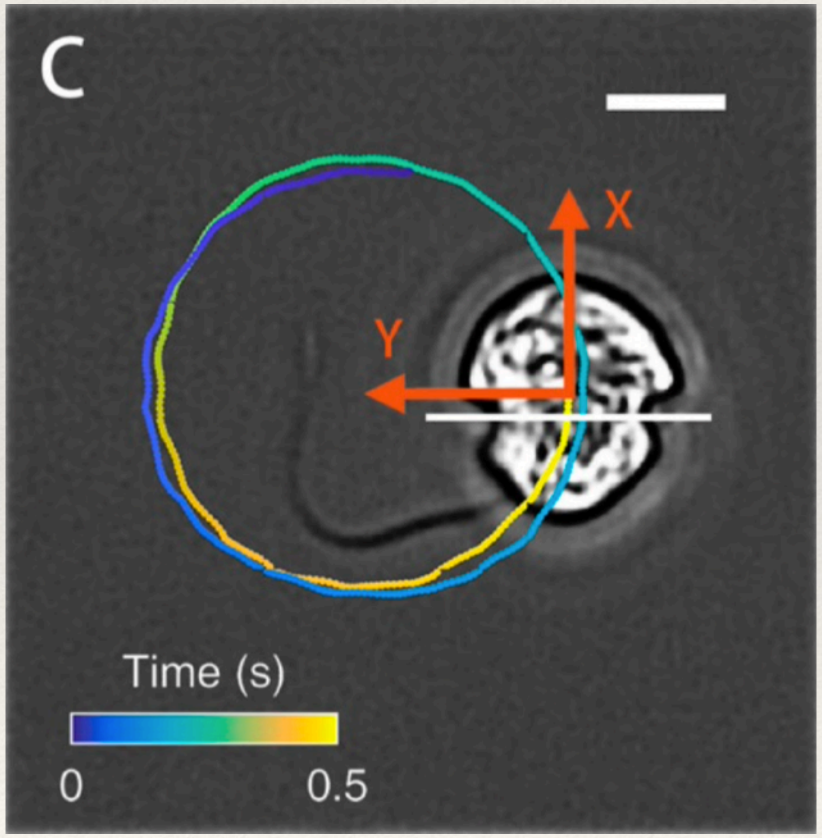
Introduction

Chiral Active Matter

Collective phenomenon we're interested in here:

Density fluctuations in chiral active fluids

Marine algae



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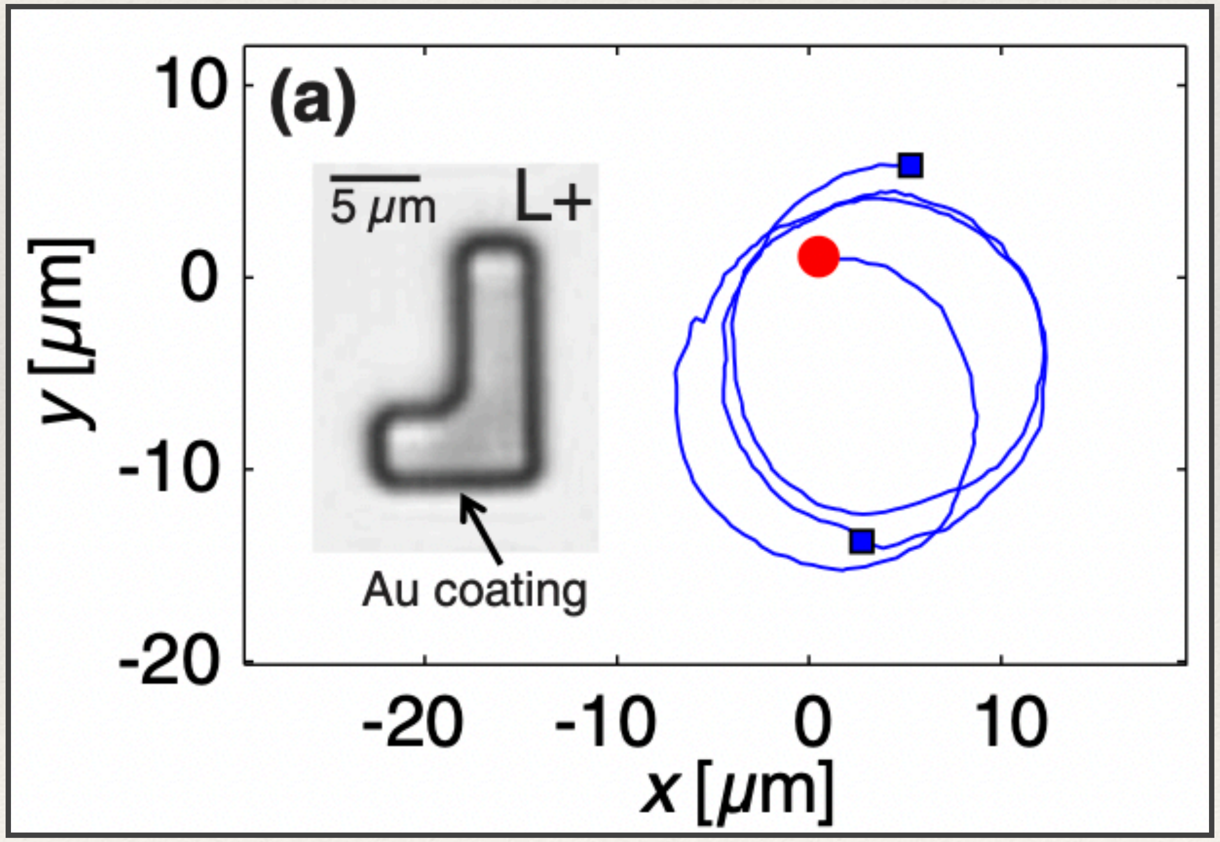


F. Kümmel *et al*. (2013)

Introduction

Fluctuations in Active Matter: Giant Number Fluctuations (GNF)

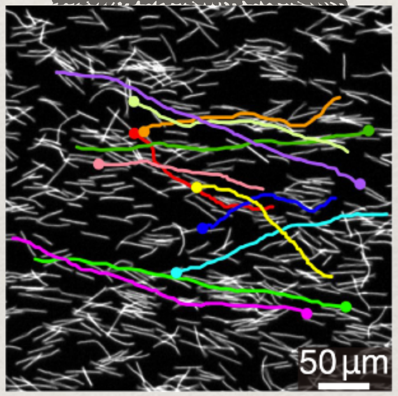
- **Number fluctuation** $\Delta N = \sqrt{\langle (N - \langle N \rangle)^2 \rangle}$

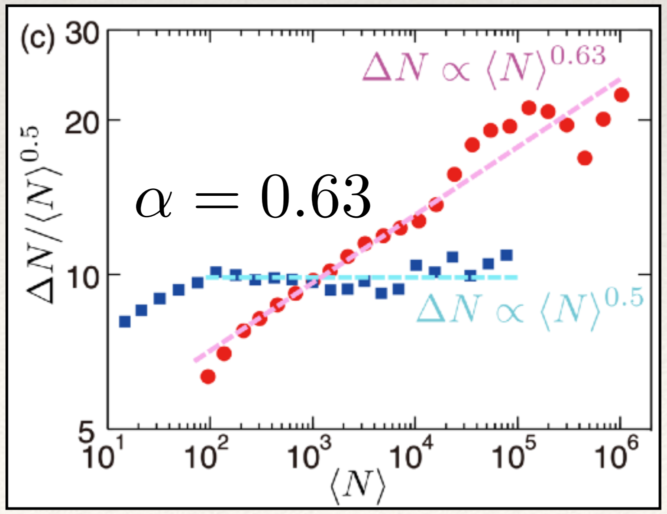
☑ **GNF:** $\Delta N \propto \langle N \rangle^{\alpha}$ with $\alpha > 0.5$ **(in equilibrium systems,** $\alpha = 0.5$ **)**

→ $$S(q) = \frac{1}{N} \langle \delta\rho(\boldsymbol{q}) \delta\rho^*(\boldsymbol{q}) \rangle \sim \frac{1}{q^{\beta}} \text{ with } \beta = 4\alpha - 2$$

Increases in density fluctuations!

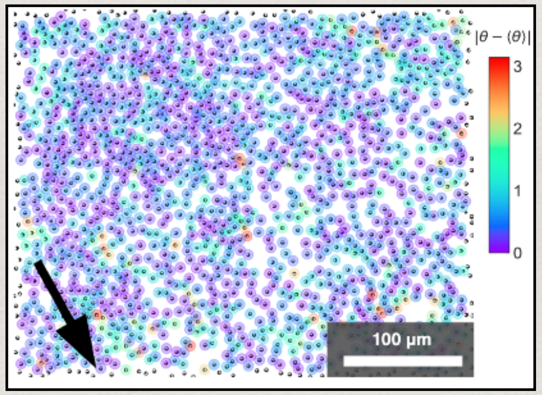
Bacteria

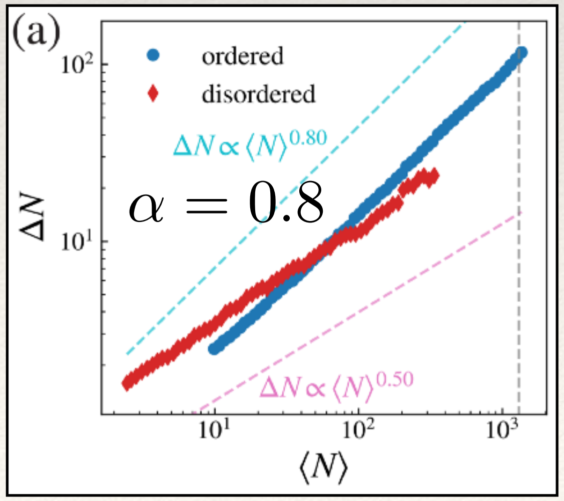




D. Nishiguchi *et al.*,(2017)

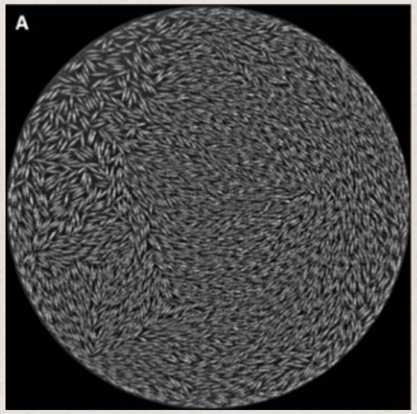
Janus particles

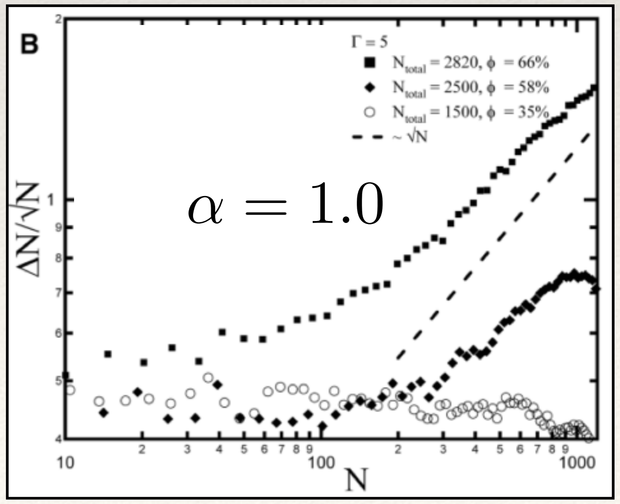




J. Iwasawa *et al.*,(2021)

Shaken rods





V. Narayan *et al.*,(2007)

Introduction

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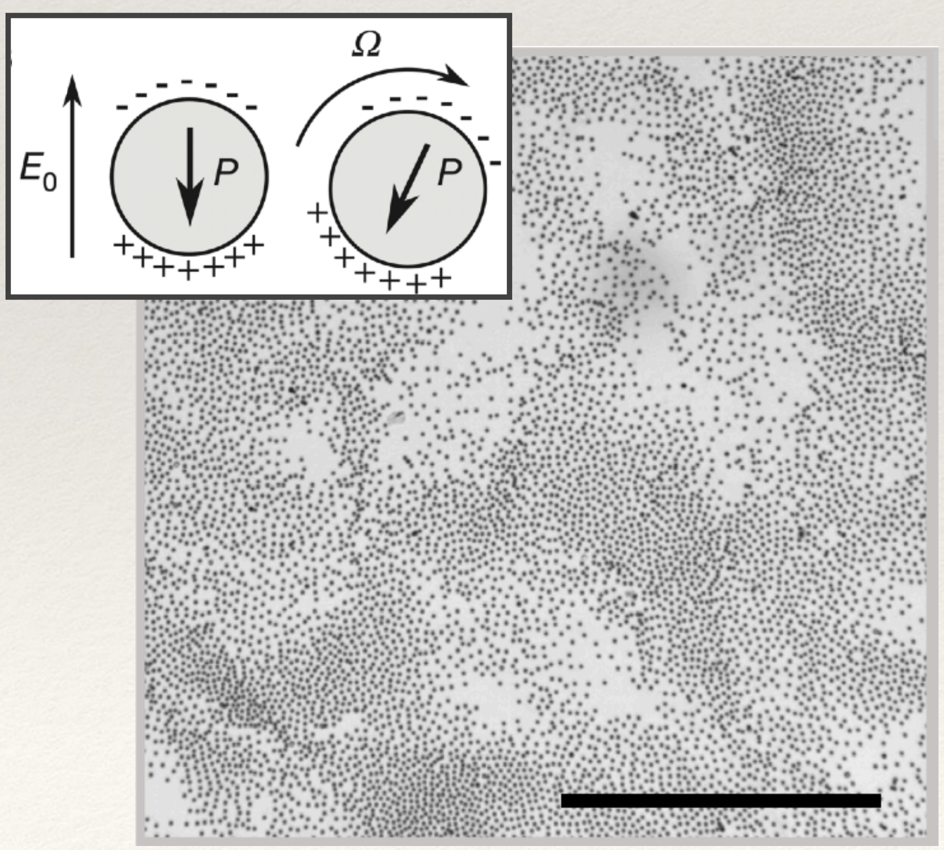
GNF: $\Delta N \propto \langle N \rangle^{\alpha}$ with $\alpha > 0.5$ **(in equilibrium systems,** $\alpha = 0.5$ **)**

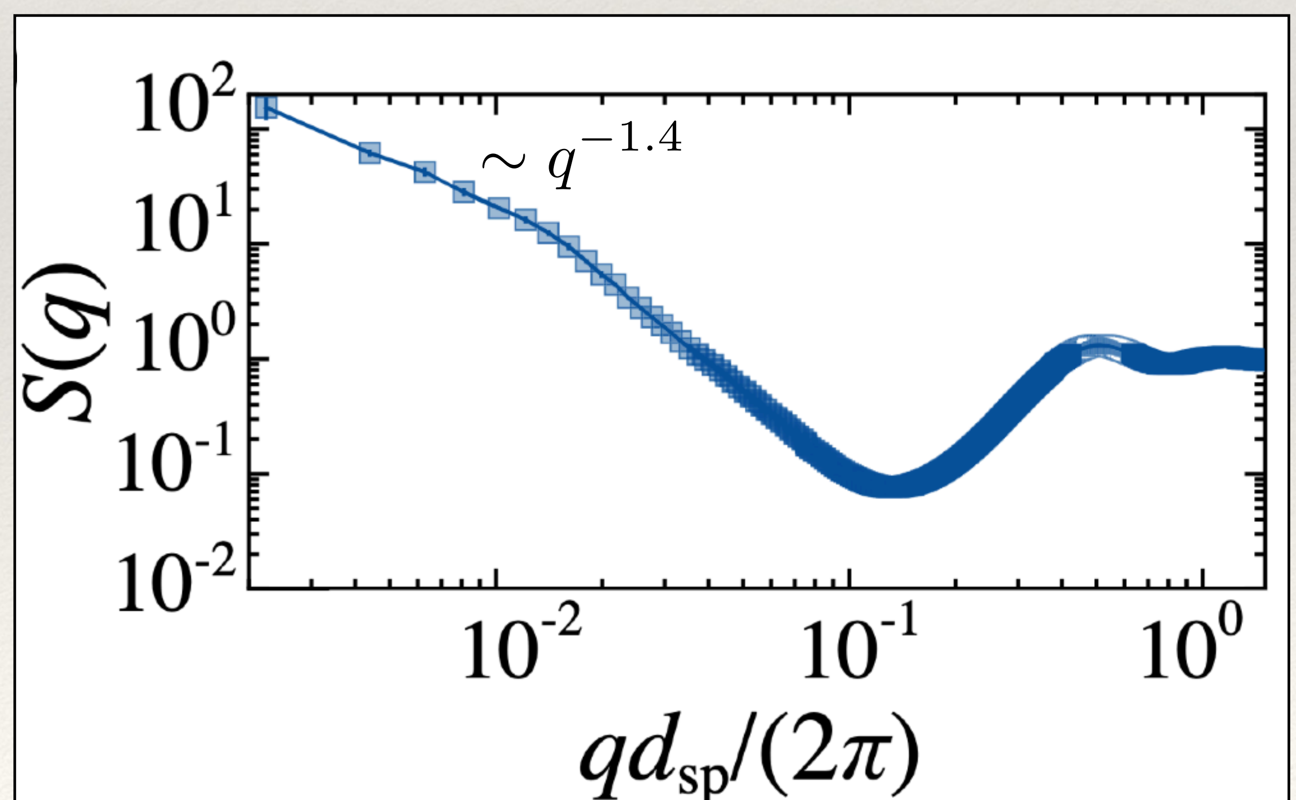
$$S(q) = \frac{1}{N} \langle \delta\rho(\boldsymbol{q}) \delta\rho^*(\boldsymbol{q}) \rangle \sim \frac{1}{q^{\beta}} \text{ with } \beta = 4\alpha - 2$$

Increases in density fluctuations!

Flocking state of the Quincke rollers

B. Zhang *et al.*,(2022)



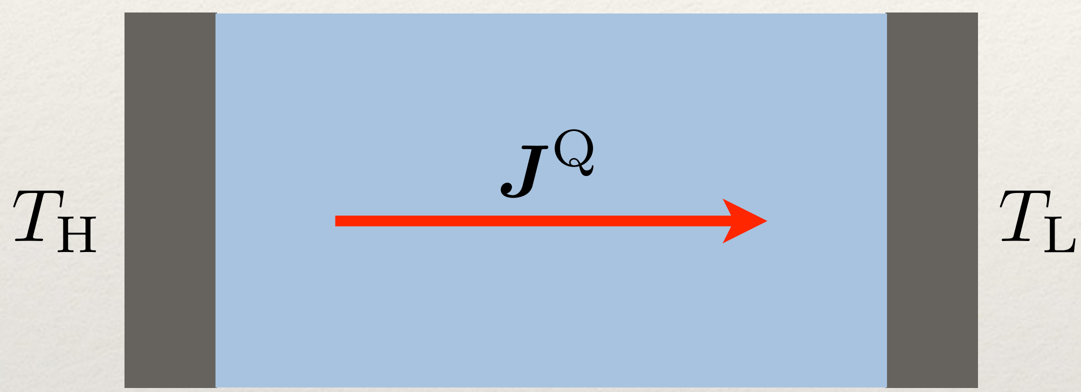


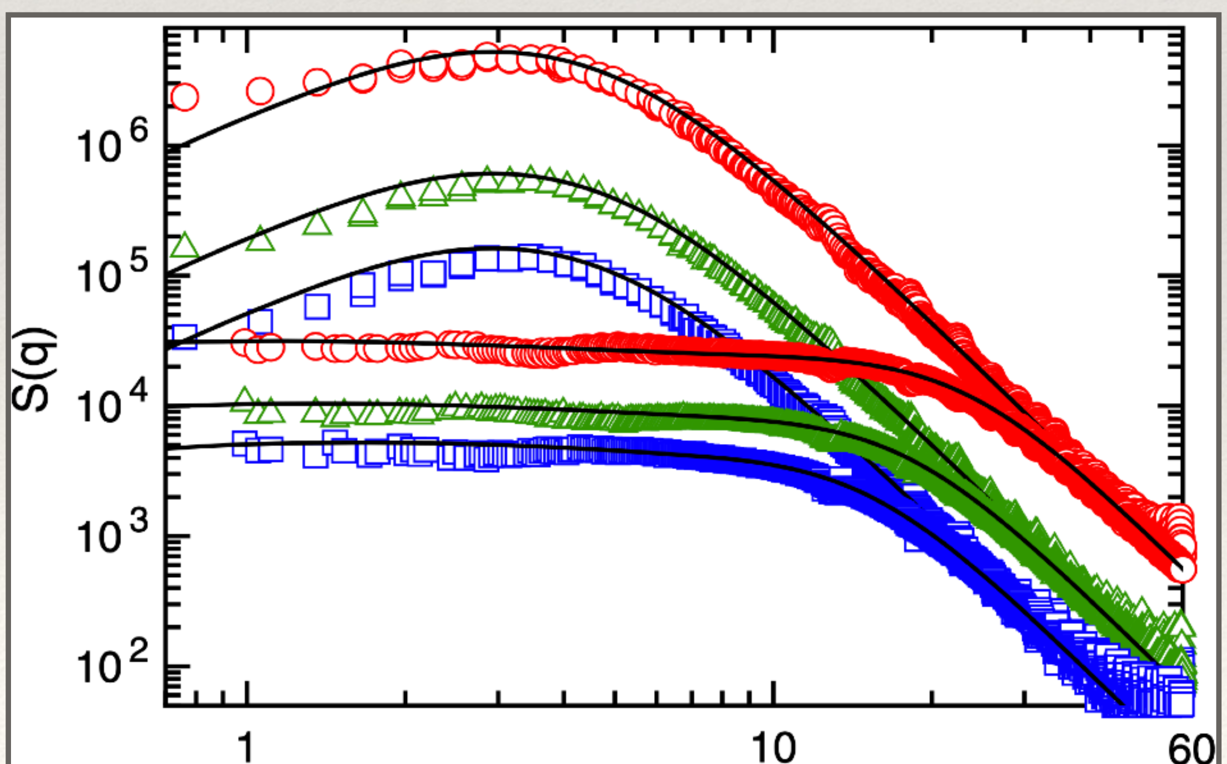
Introduction

Large Fluctuations in Non-equilibrium Systems

- Large fluctuations are abundant in systems out of equilibrium

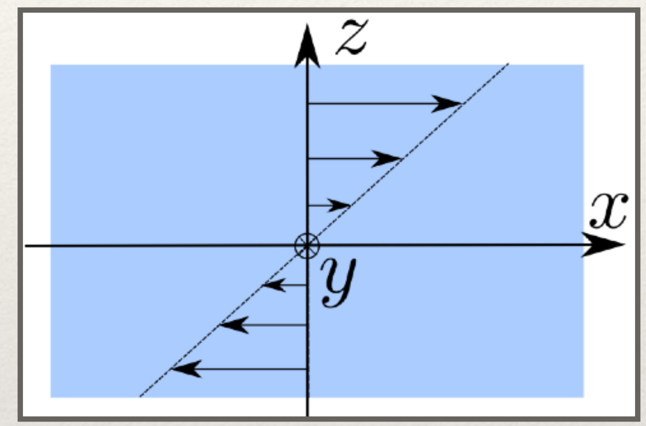
Temperature gradient

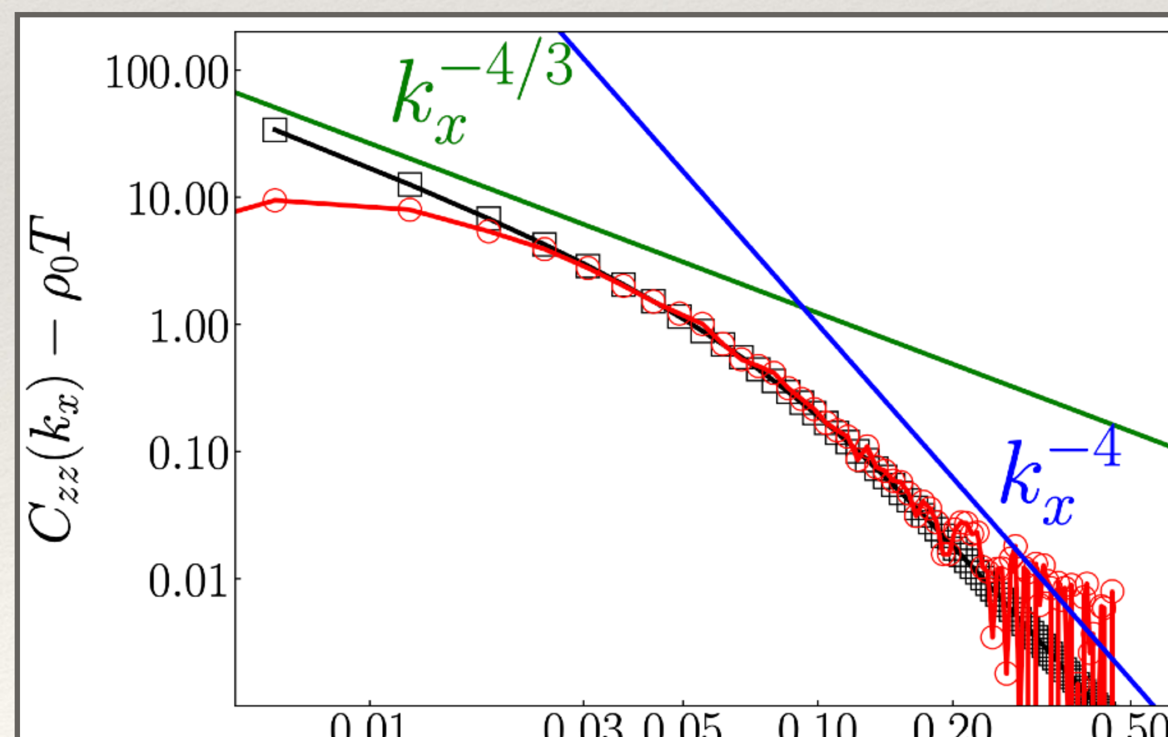




C. J. Takacs, *et al.*,(2011)

Steady shear flow





Nakano & Miami (2022)

Introduction

Hyperuniformity in 2D Chiral Active Fluid

Q. What about 2D chiral active fluids?

A. They show ***hyperuniformity***, not GNF

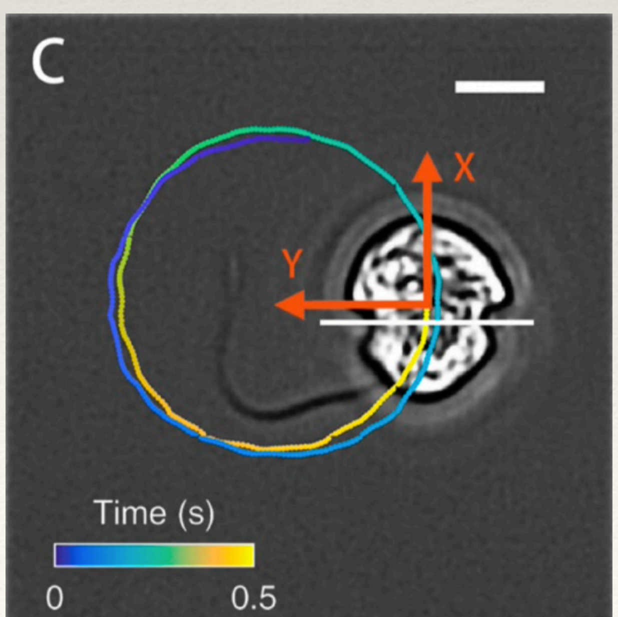
cf.) GNF means $S(q) \propto \frac{1}{q^{\beta}}$

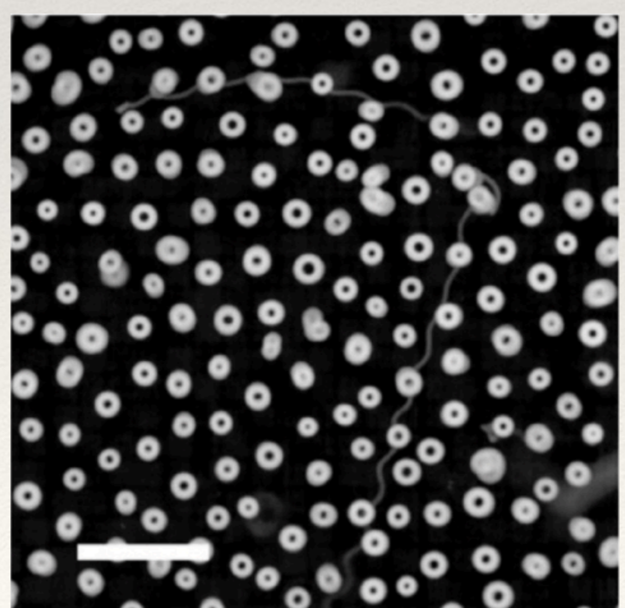
Hyperuniformity (HU): $S(q) \propto q^{\gamma}$ with $\gamma > 0$

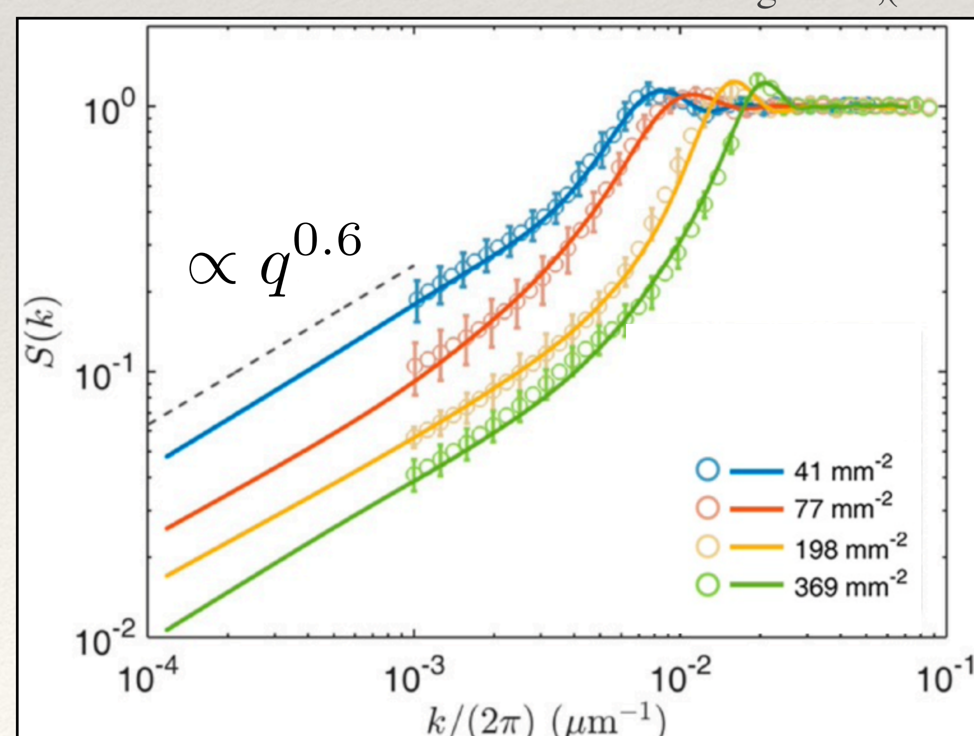
→ Anomalous **suppression** of density fluctuations (opposite to GNF!)

Experiment: Algae system

M. Huang *et al*.,(2021)







Introduction

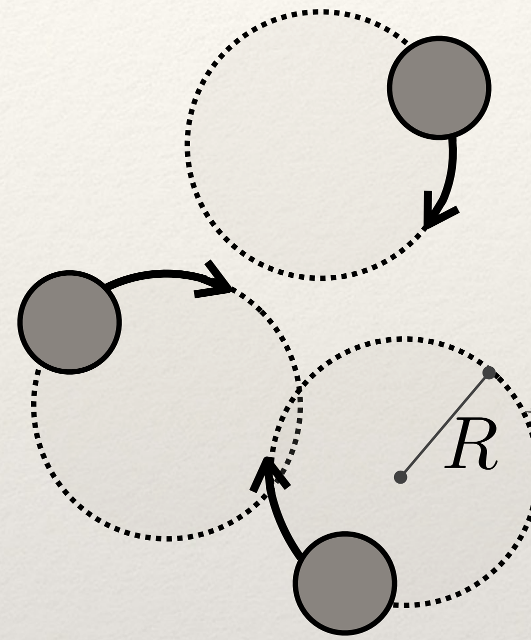
A Numerical Study on Hyperuniformity in 2D Chiral Active Fluid

☑ **HU in chiral active fluids has been observed numerically**

Chiral active Brownian particles

$$\dot{\boldsymbol{r}}_j(t) = \mu \underbrace{\boldsymbol{F}_j^{\text{int}}}_{\text{interaction}} + \underbrace{v_0 \boldsymbol{e}(\phi_j(t))}_{\text{self-propelling}}$$

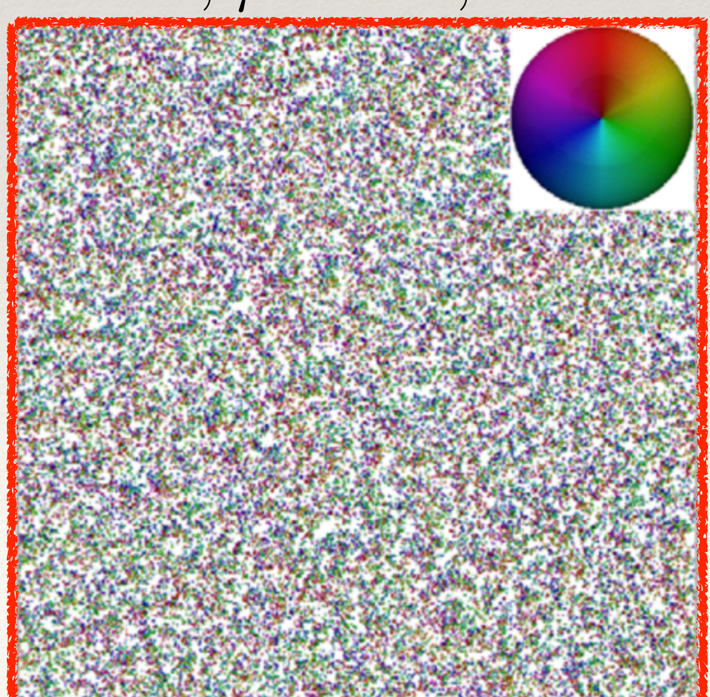
$$\dot{\phi}_j(t) = \underbrace{\Omega}_{\text{torque}} + \underbrace{\sqrt{2D}\eta_j(t)}_{\text{noise}}$$

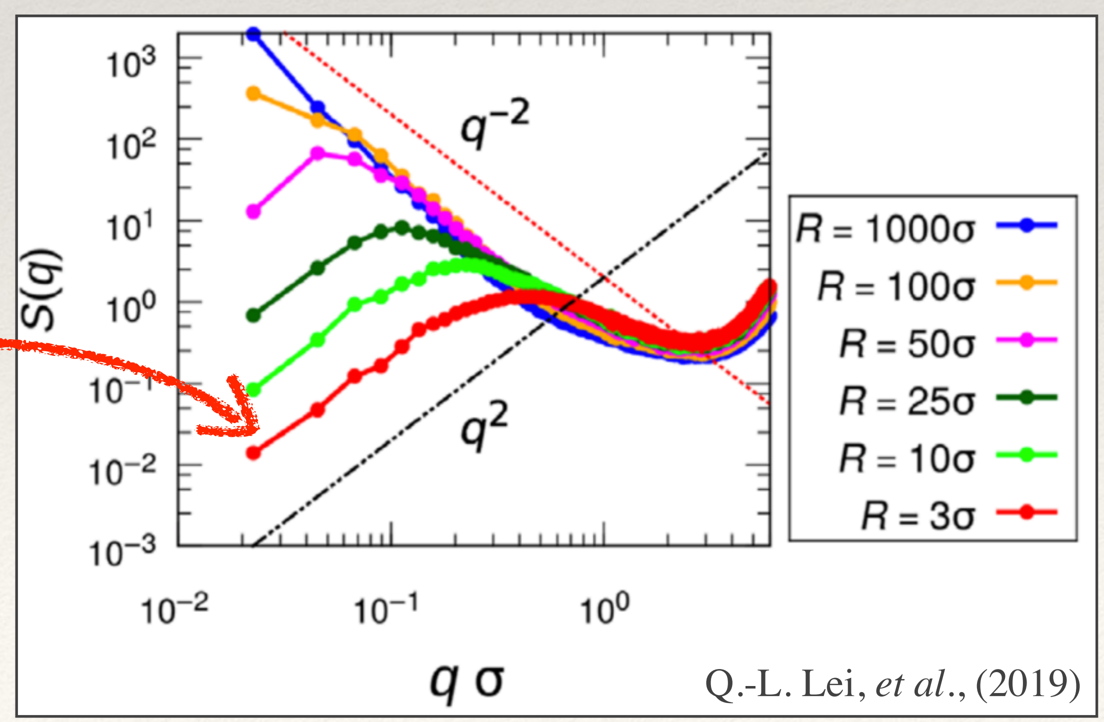


Orbital radius

$$R = \frac{v_0}{\Omega}$$

$D = 0, \varphi = 0.4, R = 3\sigma$





Q.-L. Lei, *et al.*, (2019)

Hyperuniformity characterized by

$$S(q) \propto q^2$$

Introduction

Summary of Backgrounds

- Most active matter systems (w/o chirality) show increases in density fluctuations at large scales

- 2D chiral active fluids exhibit the suppression of density fluctuations called hyperuniformity (observed numerically & experimentally)

Questions

Q. Can we theoretically understand hyperuniformity in 2D chiral active fluids from a microscopic point of view?

→ Yes!

Q. What about density fluctuations in 3D chiral active fluid?

→ Emergence of a singular correlation

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Theory for HU in 2D Chiral Active Fluid

Fluctuating Hydrodynamic Equations

$$\dot{\boldsymbol{r}}_j(t) = -\mu \sum_{k=1}^{N} \nabla_j U(\boldsymbol{r}_j - \boldsymbol{r}_k) + v_0 \boldsymbol{e}(\phi_j)$$

$$\dot{\phi}_j(t) = \Omega + \sqrt{2D}\eta_j(t)$$

$\rho(\boldsymbol{r},t) = \sum_{j=1}^{N} \delta(\boldsymbol{r} - \boldsymbol{r}_j(t))$ **:Density**

$\boldsymbol{p}(\boldsymbol{r},t) = \sum_{j=1}^{N} \boldsymbol{e}(\phi_j(t))\delta(\boldsymbol{r} - \boldsymbol{r}_j(t))$ **:Polarization**

$$\partial_t \rho(\boldsymbol{r},t) = -\nabla \cdot \boldsymbol{J}(\boldsymbol{r},t)$$

$$\partial_t \boldsymbol{p}(\boldsymbol{r},t) = -\nabla \cdot \left(\frac{\boldsymbol{J}(\boldsymbol{r},t)\boldsymbol{p}(\boldsymbol{r},t)}{\rho(\boldsymbol{r},t)} \right) - D\boldsymbol{p}(\boldsymbol{r},t) + \boldsymbol{\Omega} \times \boldsymbol{p}(\boldsymbol{r},t) + \sqrt{D\rho(\boldsymbol{r},t)}\boldsymbol{\Upsilon}(\boldsymbol{r},t)$$

$$\boldsymbol{J}(\boldsymbol{r},t) = -\mu\nabla P(\boldsymbol{r},t) + v_0 \boldsymbol{p}(\boldsymbol{r},t)$$

"Pressure": $\nabla P(\boldsymbol{r},t) := \rho(\boldsymbol{r},t)\nabla \dfrac{\delta \mathcal{F}[\rho(\cdot,t)]}{\delta \rho(\boldsymbol{r},t)}$ $\quad \mathcal{F}[\rho(\cdot,t)] = \dfrac{1}{2}\displaystyle\int_V \mathrm{d}^2\boldsymbol{r} \int_V \mathrm{d}^2\boldsymbol{r}' \; \rho(\boldsymbol{r},t)\rho(\boldsymbol{r}',t)U(\boldsymbol{r}-\boldsymbol{r}')$

Theory for HU in 2D Chiral Active Fluid

Fluctuating Hydrodynamic Equations

$$\partial_t \rho(\boldsymbol{r},t) = -\nabla \cdot \boldsymbol{J}(\boldsymbol{r},t)$$

$$\partial_t \boldsymbol{p}(\boldsymbol{r},t) = -\nabla \cdot \left(\frac{\boldsymbol{J}(\boldsymbol{r},t)\boldsymbol{p}(\boldsymbol{r},t)}{\rho(\boldsymbol{r},t)} \right) - D\boldsymbol{p}(\boldsymbol{r},t) + \boldsymbol{\Omega} \times \boldsymbol{p}(\boldsymbol{r},t) + \sqrt{D\rho(\boldsymbol{r},t)}\boldsymbol{\Upsilon}(\boldsymbol{r},t)$$

$$\boldsymbol{J}(\boldsymbol{r},t) = -\mu \nabla P(\boldsymbol{r},t) + v_0 \boldsymbol{p}(\boldsymbol{r},t)$$

Linearization:

$$\rho(\boldsymbol{r},t) = \rho + \delta\rho(\boldsymbol{r},t)$$

$$\boldsymbol{p}(\boldsymbol{r},t) = \boldsymbol{0} + \delta\boldsymbol{p}(\boldsymbol{r},t)$$

Assumption:

$$\nabla P(\boldsymbol{r},t) \simeq \frac{1}{\rho\chi} \nabla \delta\rho(\boldsymbol{r},t)$$

$$\chi^{-1} := \rho\, \partial P / \partial \rho \big|_{\rho(\boldsymbol{r})=\rho}$$

$$\partial_t \delta\rho(\boldsymbol{r},t) = b\nabla^2 \delta\rho(\boldsymbol{r},t) - v_0 \nabla \cdot \delta\boldsymbol{p}(\boldsymbol{r},t)$$

※ $b := \mu/(\rho\chi)$

$$\partial_t \delta\boldsymbol{p}(\boldsymbol{r},t) = -D\delta\boldsymbol{p}(\boldsymbol{r},t) + \boldsymbol{\Omega} \times \delta\boldsymbol{p}(\boldsymbol{r},t) + \sqrt{D\rho}\boldsymbol{\Upsilon}(\boldsymbol{r},t)$$

Theory for HU in 2D Chiral Active Fluid

Static Structure Factor

$$\partial_t \delta\rho(\boldsymbol{r},t) = b\nabla^2\delta\rho(\boldsymbol{r},t) - v_0\nabla\cdot\delta\boldsymbol{p}(\boldsymbol{r},t)$$

$$\partial_t \delta\boldsymbol{p}(\boldsymbol{r},t) = -D\delta\boldsymbol{p}(\boldsymbol{r},t) + \boldsymbol{\Omega}\times\delta\boldsymbol{p}(\boldsymbol{r},t) + \sqrt{D\rho}\,\boldsymbol{\Upsilon}(\boldsymbol{r},t)$$

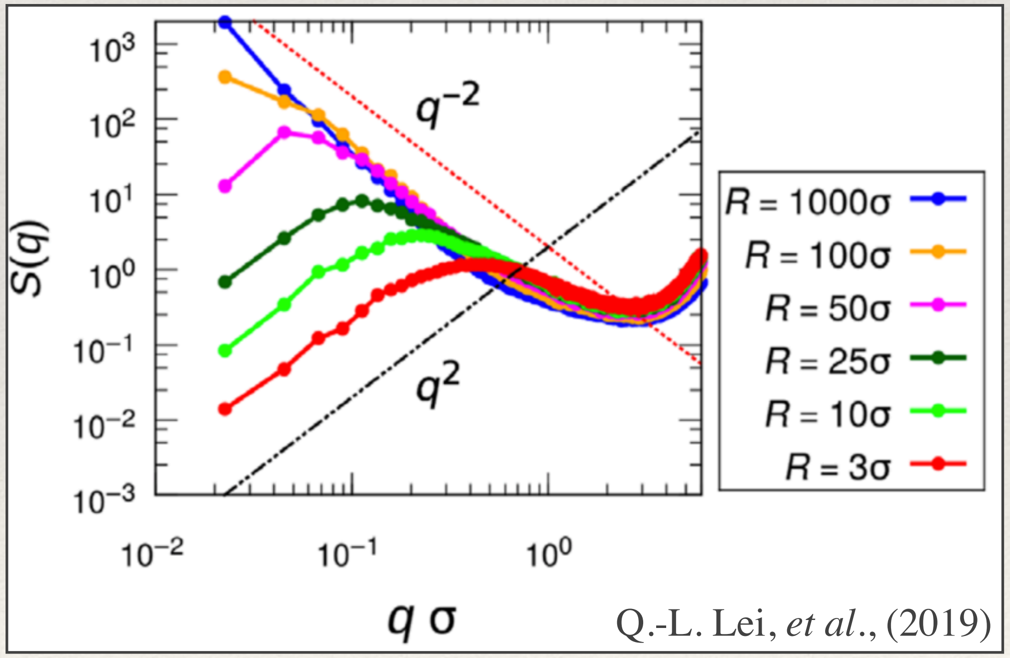
- **Static structure factor:** $S(q) = \frac{1}{N}\left\langle|\delta\rho(\boldsymbol{q},t=0)|^2\right\rangle = \frac{v_0^2}{2b}\cdot\frac{D+bq^2}{\Omega^2+(D+bq^2)^2}$

☑ **In the limit** $D\to 0,$

$$S(q) = \frac{v_0^2}{2\Omega^2}\cdot\frac{q^2}{1+b^2q^4/\Omega^2}$$

$$= \frac{1}{2}(Rq)^2 + O(q^6) \qquad R = v_0/\Omega$$

Hyperuniformity!



Q.-L. Lei, *et al.*, (2019)

☑ **We succeeded in deriving HU in 2D chiral active fluids!**

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