Operational Estimation Method for Bare Viscosity in Fluctuating Hydrodynamics (Role of Ultraviolet Cutoff)

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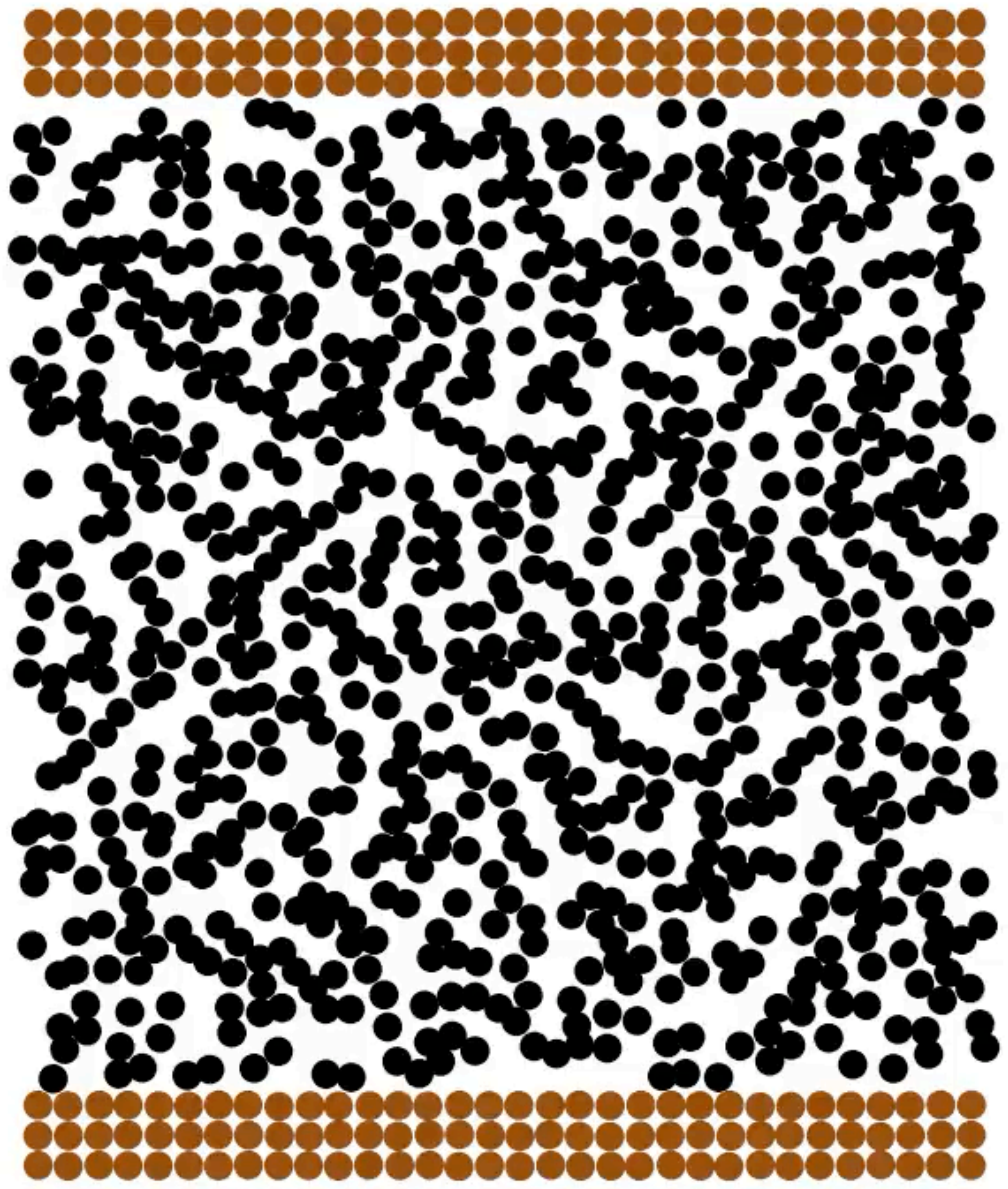
In collaboration with Yuki Minami (Gifu Univ.), and Keiji Saito (Kyoto Univ.)

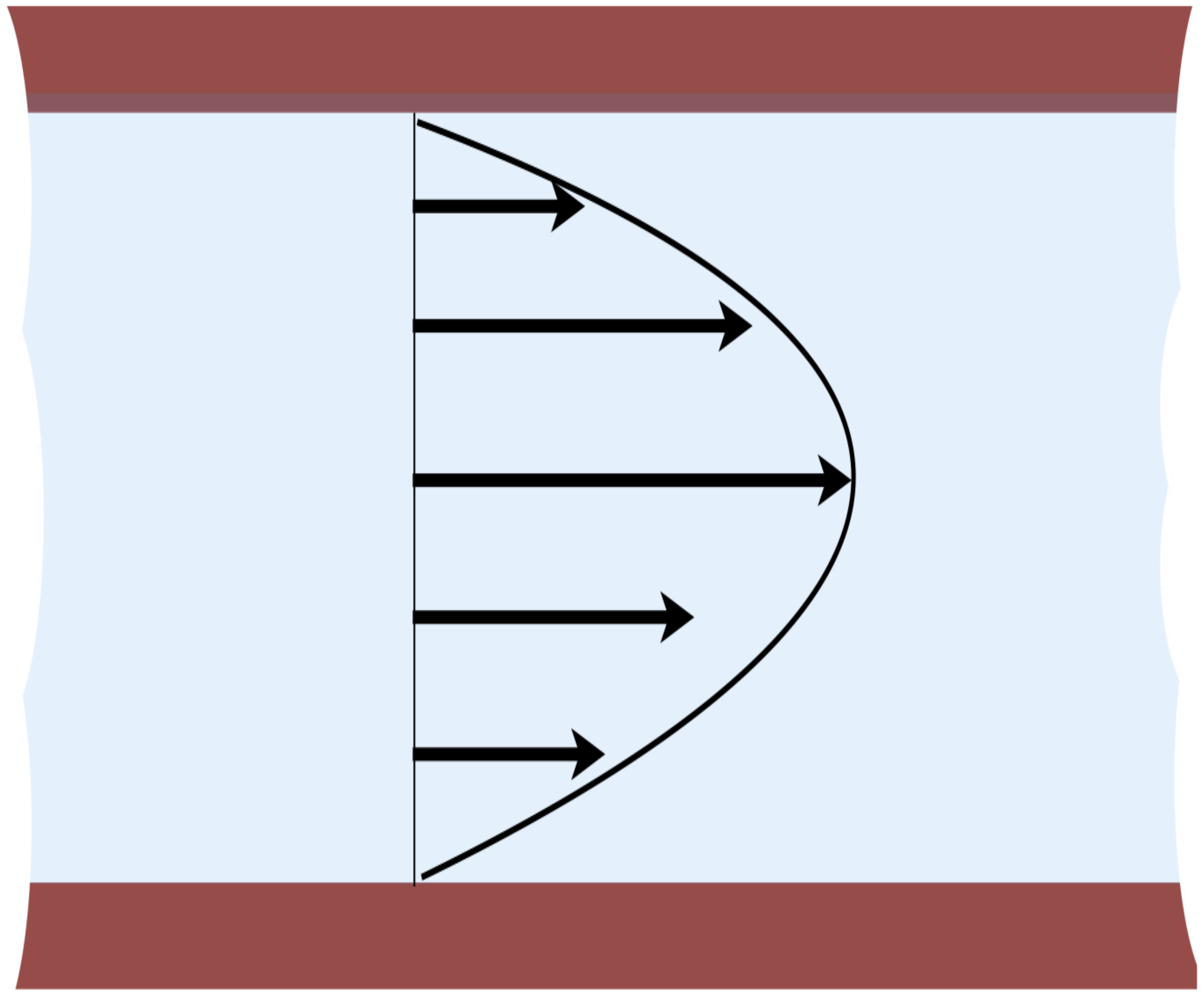
Theme of this talk : Background

Microscopic system



coarse-graining

Hydrodynamic description



microscopic particles obeying classical mechanics

The motion of the system can be described by hydrodynamic theory at a macroscopic scale

Theme of this talk : Fluctuating hydrodynamics

▶ Hydrodynamics describes slow motion of conserved variables.

We consider a situation where mass and momentum are conserved quantities.

Mass conservation $$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho v)$$

Momentum Conservation $$\frac{\partial (\rho v^a)}{\partial t} = -\nabla_b \mathbf{\Pi}^{ab}$$

density field (ρ), velocity field (v), momentum flux ($\mathbf{\Pi}^{ab}$)

The specific properties of fluids are given by a constitutive equation.

Fluctuating constitutive eq.

$$\mathbf{\Pi}^{ab}(\boldsymbol{r}, t) := \underbrace{p\delta_{ab}}_{\text{pressure}} + \underbrace{\rho v^a v^b}_{\text{advection}} \underbrace{- \eta_0\left(\nabla_a v^b + \nabla_b v^a - \frac{2}{d}\delta_{ab}\nabla^c v^c \right) - \zeta_0 \delta_{ab} \nabla^c v^c}_{\text{viscous dissipation}} \underbrace{+ \mathbf{\Pi}_R^{ab}}_{\text{noise}}$$

$$\left\langle \Pi_R^{ab}(\boldsymbol{r}_1, t_1) \Pi_R^{cd}(\boldsymbol{r}_2, t_2) \right\rangle = 2k_B T \delta^2(\boldsymbol{r}_1 - \boldsymbol{r}_2)\delta(t_1 - t_2)\left[\eta_0\left(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{cb}\right) + \left(\zeta_0 - \eta_0\right)\delta_{ab}\delta_{cd}\right]$$

■ Fluctuating hydrodynamics includes a noise term arising from thermal motion of atoms.

Theme of this talk : Goal

▶ Bare viscosity

It is a parameter included in the constitutive eq. of fluctuating hydrodynamics.

Fluctuating constitutive eq.

$$\boldsymbol{\Pi}^{ab}(\boldsymbol{r},t) := p\delta_{ab} + \rho v^a v^b - \eta_0\left(\nabla_a v^b + \nabla_b v^a - \frac{2}{d}\delta_{ab}\nabla^c v^c\right) - \zeta_0\delta_{ab}\nabla^c v^c + \boldsymbol{\Pi}_R^{ab}$$

Transport coefficient η_0 **: shear viscosity,** ζ_0 **: bulk viscosity,**

■ Here, bare viscosity is defined for a given UV cutoff length.

(I will explain this point in detail later)

The goal of this talk

- **Demonstrating that bare viscosity η_0 is observable.**
- **Specifying the value of bare viscosity η_0 from atomic systems.**

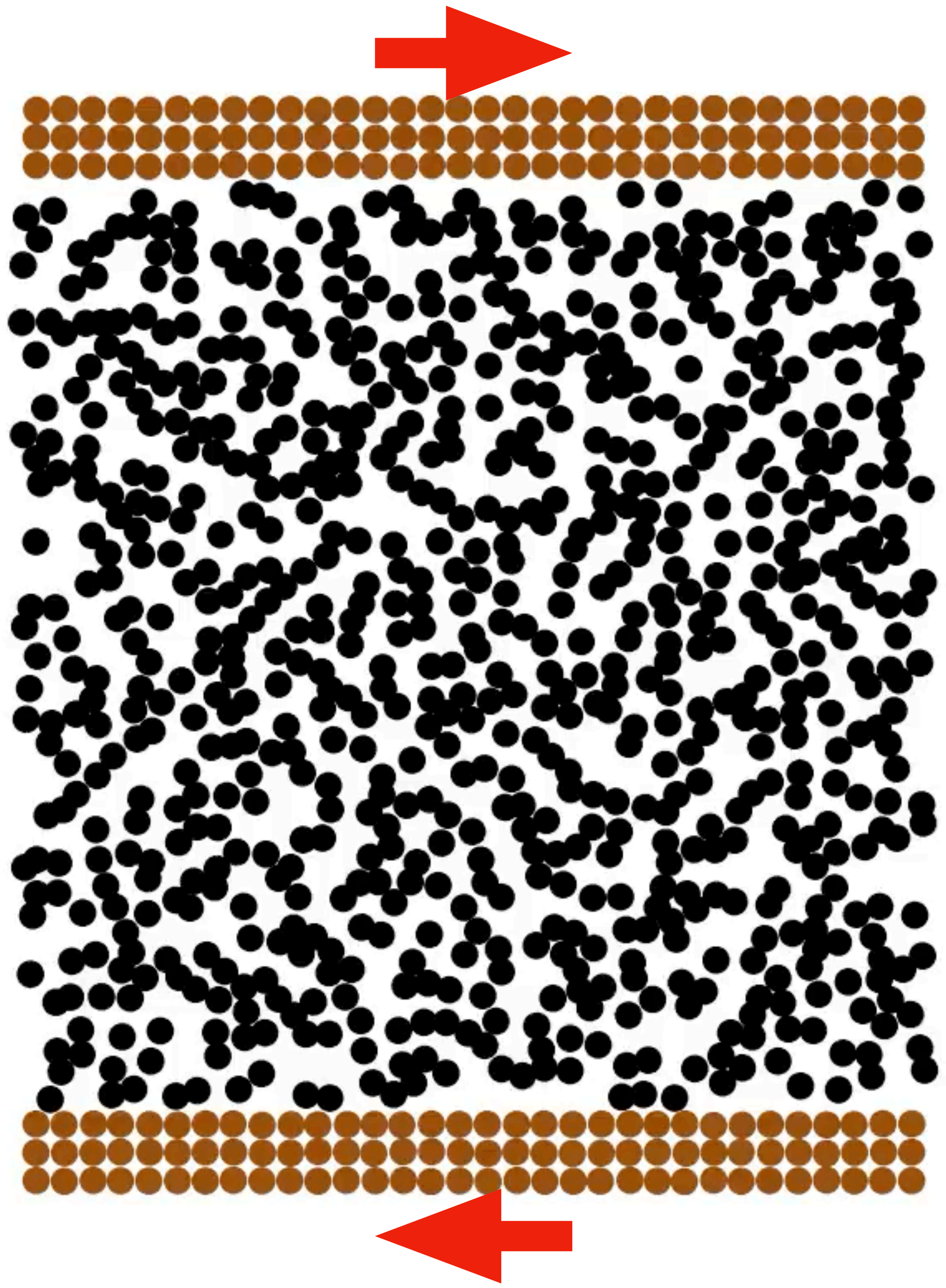
Contents in this talk

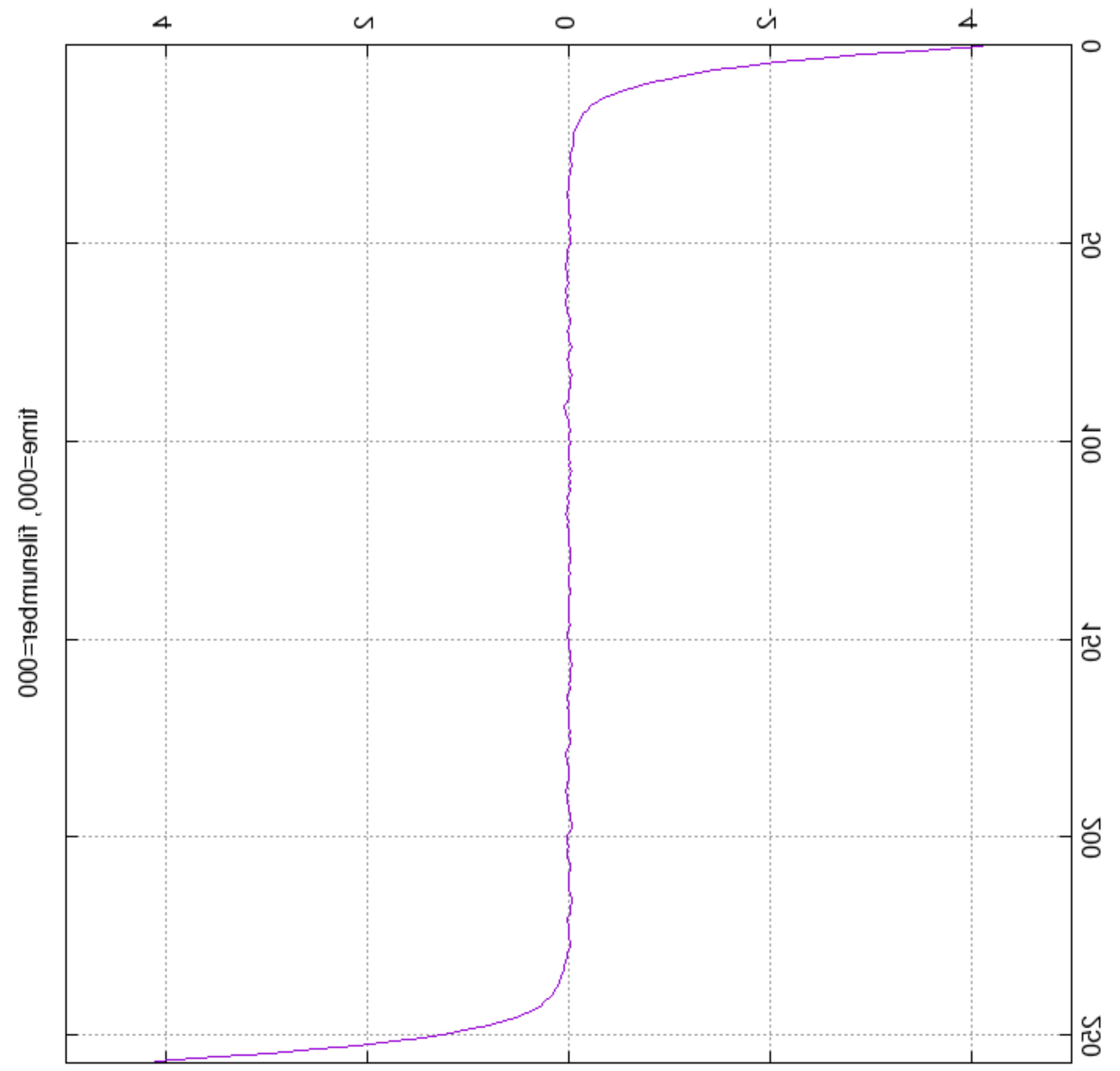
1.1 Why is observing bare viscosity difficult?

Standard method to measure viscosity

▶ We consider a practical method for measuring the shear viscosity.

1. We create the Couette geometry by moving two parallel walls in opposite directions.
   In the steady state, the simplest flow pattern, known as shear flow is realized.



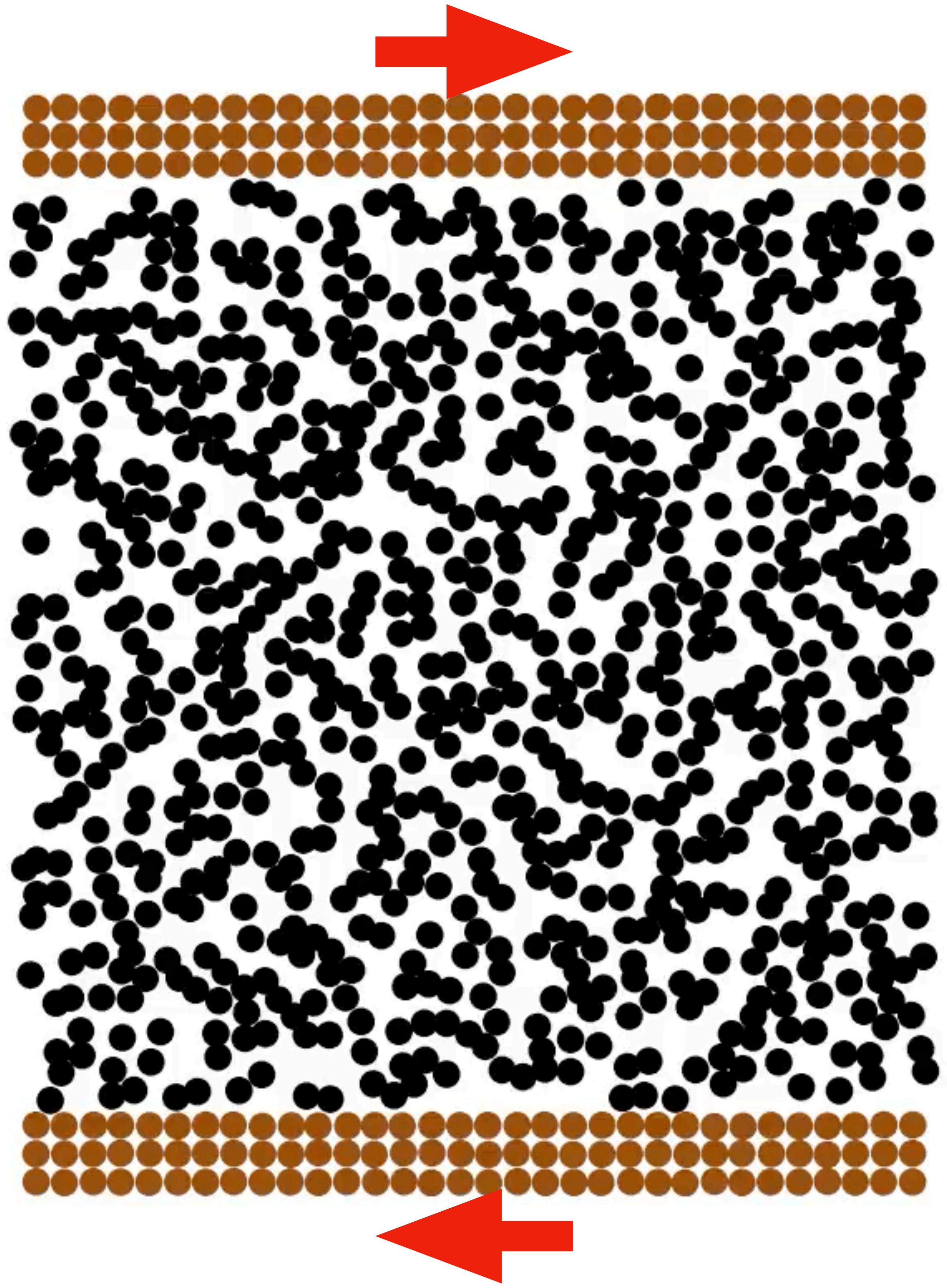


Velocity field $v^x(y)$

Standard method to measure viscosity

▶ We consider a practical method for measuring the shear viscosity.

2. We measure the noise-averaged velocity field and momentum flux in the steady state.



Noise (or Time)-averaged velocity field $\langle v^a(\boldsymbol{r}) \rangle_{\mathrm{ss}}^{\dot\gamma}$

Noise (or Time)-averaged momentum flux $\langle \Pi^{ab}(\boldsymbol{r}) \rangle_{\mathrm{ss}}^{\dot\gamma}$

Standard method to measure viscosity

- We consider a practical method for measuring the shear viscosity.

3. We calculate the viscosity through the relationship between the velocity and momentum flux.

XY component of Fluctuating momentum flux

$$\Pi^{xy}(\boldsymbol{r}, t) := \underbrace{\rho v^x v^y}_{\text{advection}} - \underbrace{\eta_0\left(\nabla_x v^y + \nabla_y v^x\right)}_{\text{shear viscosity}} \underbrace{+\Pi_R^{xy}}_{\text{noise}}$$

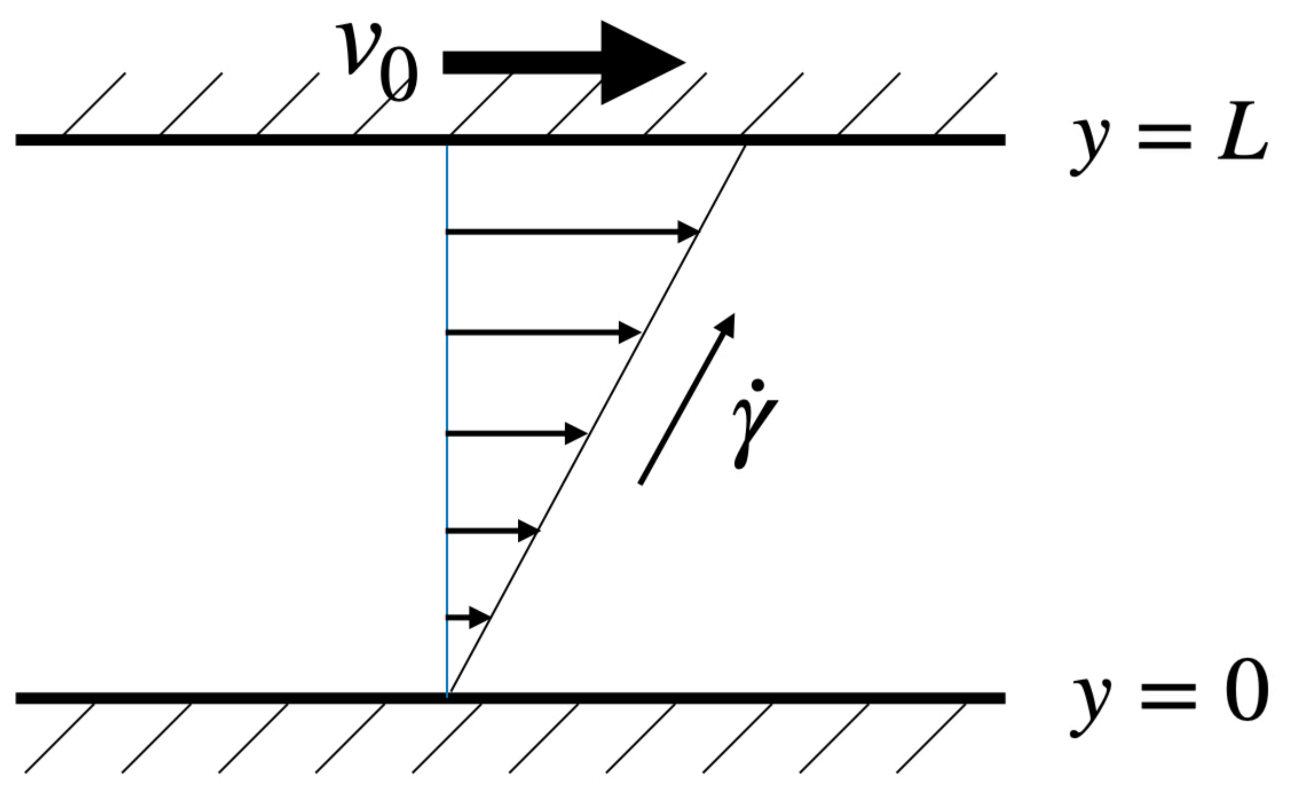
When velocity field is fluctuating,

the fluctuating shear flow is given as

$$v^x(t) = \dot{\gamma} y + \delta v^x(t)$$

$$v^y(t) = 0 + \delta v^y(t)$$

$$\langle \delta v^x \rangle_{\mathrm{ss}}^{\dot{\gamma}} = \langle \delta v^y \rangle_{\mathrm{ss}}^{\dot{\gamma}} = \langle \Pi_R^{xy}(\boldsymbol{r}, t) \rangle_{\mathrm{ss}}^{\dot{\gamma}} = 0$$



➡ The noise-averaged momentum flux under shear flow is calculated as

$$\langle \Pi^{xy}(\boldsymbol{r}, t) \rangle_{\mathrm{ss}}^{\dot{\gamma}} := \underbrace{\langle \rho \delta v^x \delta v^y \rangle_{\mathrm{ss}}^{\dot{\gamma}}}_{\text{advection}} - \eta_0 \dot{\gamma}$$

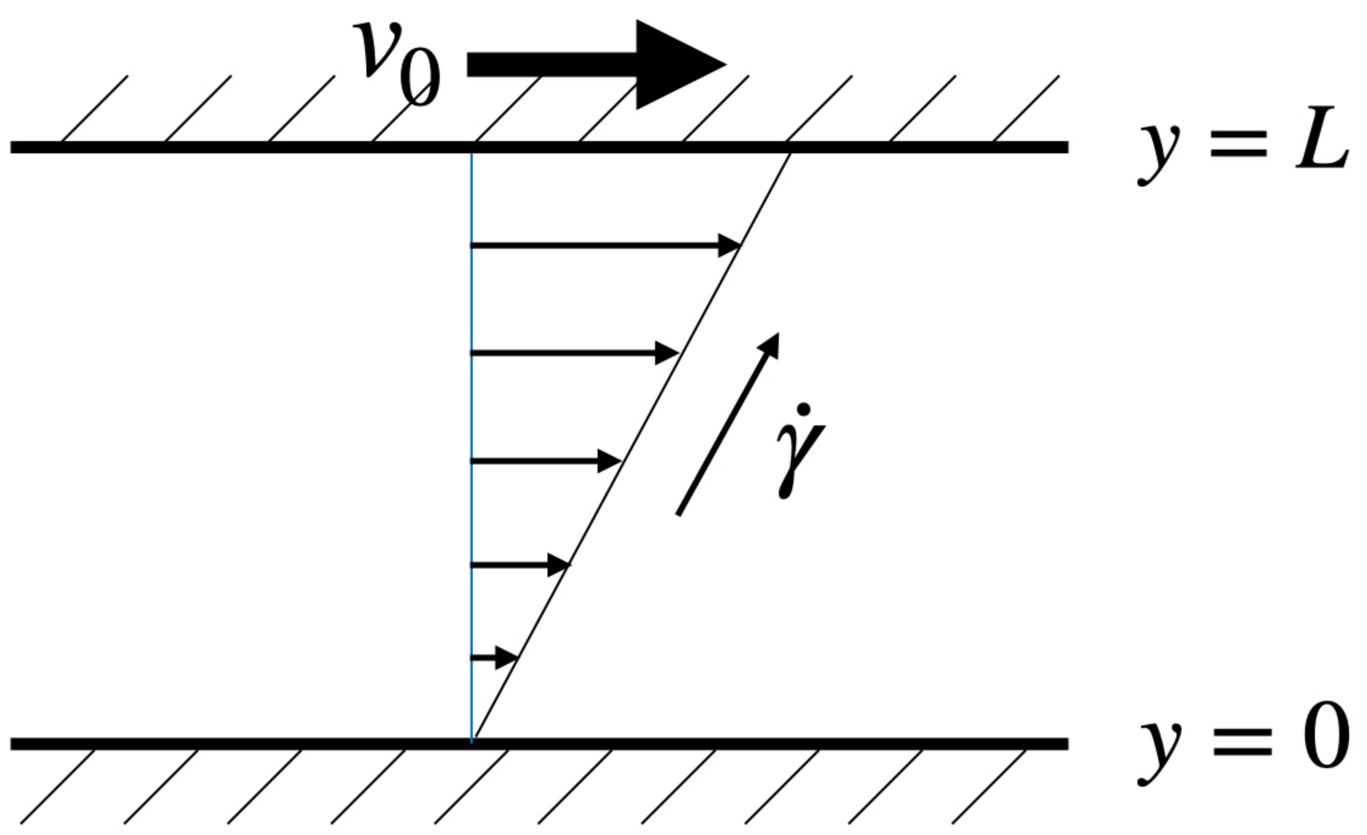
Standard method to measure viscosity

▶ We consider a practical method for measuring the shear viscosity.

3. We calculate the viscosity through the relationship between the velocity and momentum flux.

Observed viscosity η $\neq$ Bare viscosity η_0

$$\eta := \frac{-\langle \Pi^{xy}(\boldsymbol{r},t)\rangle^{\dot{\gamma}}_{\mathrm{ss}}}{\dot{\gamma}} = \eta_0 - \frac{\langle \rho \delta v^x \delta v^y \rangle^{\dot{\gamma}}_{\mathrm{ss}}}{\dot{\gamma}}$$



The nonlinear coupling of fluctuations is NOT zero,
because $\delta v^x(t)$ and $\delta v^y(t)$ can interact due to advection with shear flow $\dot{\gamma}$.

When we try to observe the viscosity from the ratio of momentum flux to velocity gradient, the resulting viscosity always includes the correction arising from the fluctuations.

Observing bare viscosity is difficult !!

1.2 How different are η and η_0 ?
Why do we aim to observe bare viscosity ?

How different are η_0 and η ?

- In the 1970s, the difference between η_0 and η was calculated based on the fluctuating hydrodynamics.

incompressible condition $\nabla \cdot \boldsymbol{v} = 0$

Fluctuating Navier-Stokes eq.

$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right] = -\nabla p + \eta_0 \nabla^2 \boldsymbol{v} + \nabla \boldsymbol{\Pi}_R$$

$$\left\langle \Pi_R^{ab}(\boldsymbol{r}_1, t_1)\Pi_R^{cd}(\boldsymbol{r}_2, t_2)\right\rangle = 2k_B T\eta_0 \delta^2(\boldsymbol{r}_1 - \boldsymbol{r}_2)\delta(t_1 - t_2)\left[\left(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{cb}\right) - \delta_{ab}\delta_{cd}\right]$$

$$\eta = \eta_0 + C_{2d}\log\left(\frac{L}{a_{\mathrm{uv}}}\right)$$

in two dimensions

Anomalous transport in low-dimensional fluids

$$\eta = \eta_0 + C_{3d}\left(\frac{1}{L} - \frac{1}{a_{\mathrm{uv}}}\right)$$

in three dimensions

UV cutoff dependence of fluctuating hydrodynamics

Observed viscosity η depends on the system size L and the ultraviolet (UV) cutoff length a_{uv}

UV cutoff dependence

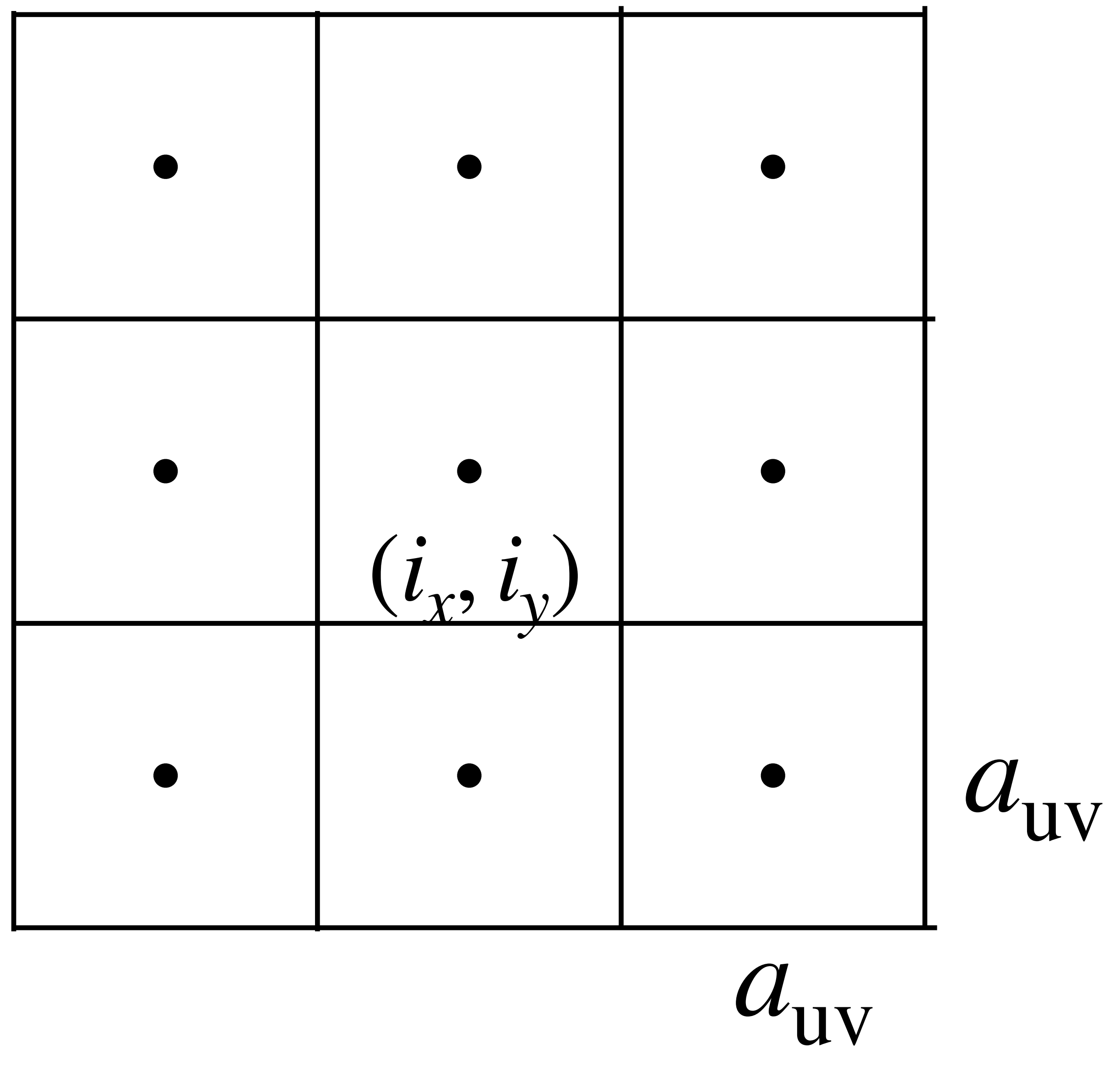
▶ When solving continuum theory numerically or analytically, it is necessary to introduce a spatial discretization width, known as the ultraviolet (UV) cutoff length.

Simple understanding of UV cutoff length

$$\left.\frac{\partial \boldsymbol{v}}{\partial x}\right|_{(i_x,i_y)} \simeq \frac{\boldsymbol{v}_{(i_x+1,i_y)} - \boldsymbol{v}_{(i_x,i_y)}}{a_{\mathrm{uv}}}$$

$$\eta = \eta_0 + C_{2d}\log\left(\frac{L}{a_{\mathrm{uv}}}\right)$$ in two dimensions

$$\eta = \eta_0 + C_{3d}\left(\frac{1}{L} - \frac{1}{a_{\mathrm{uv}}}\right)$$ in three dimensions

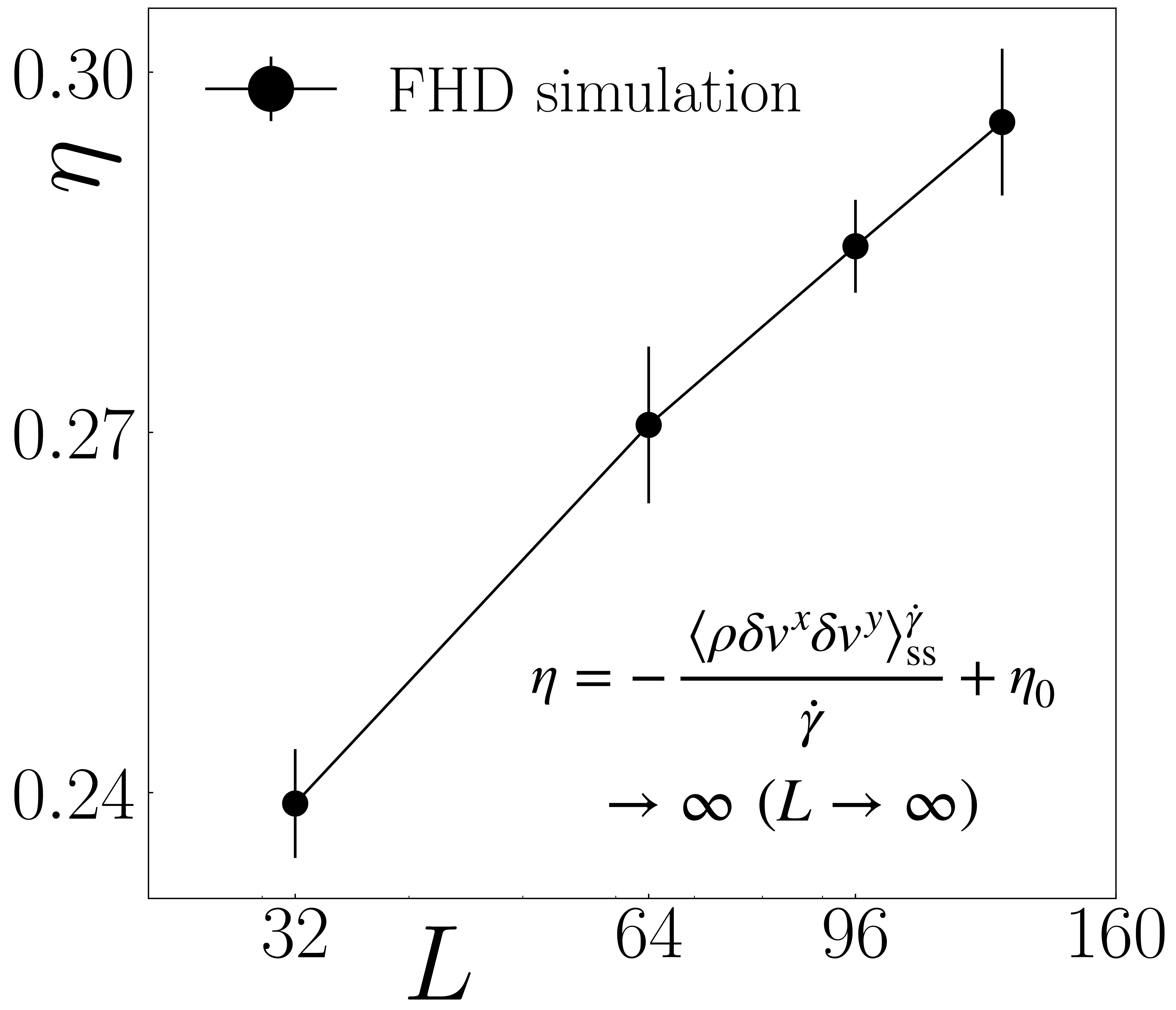


Spatial resolution

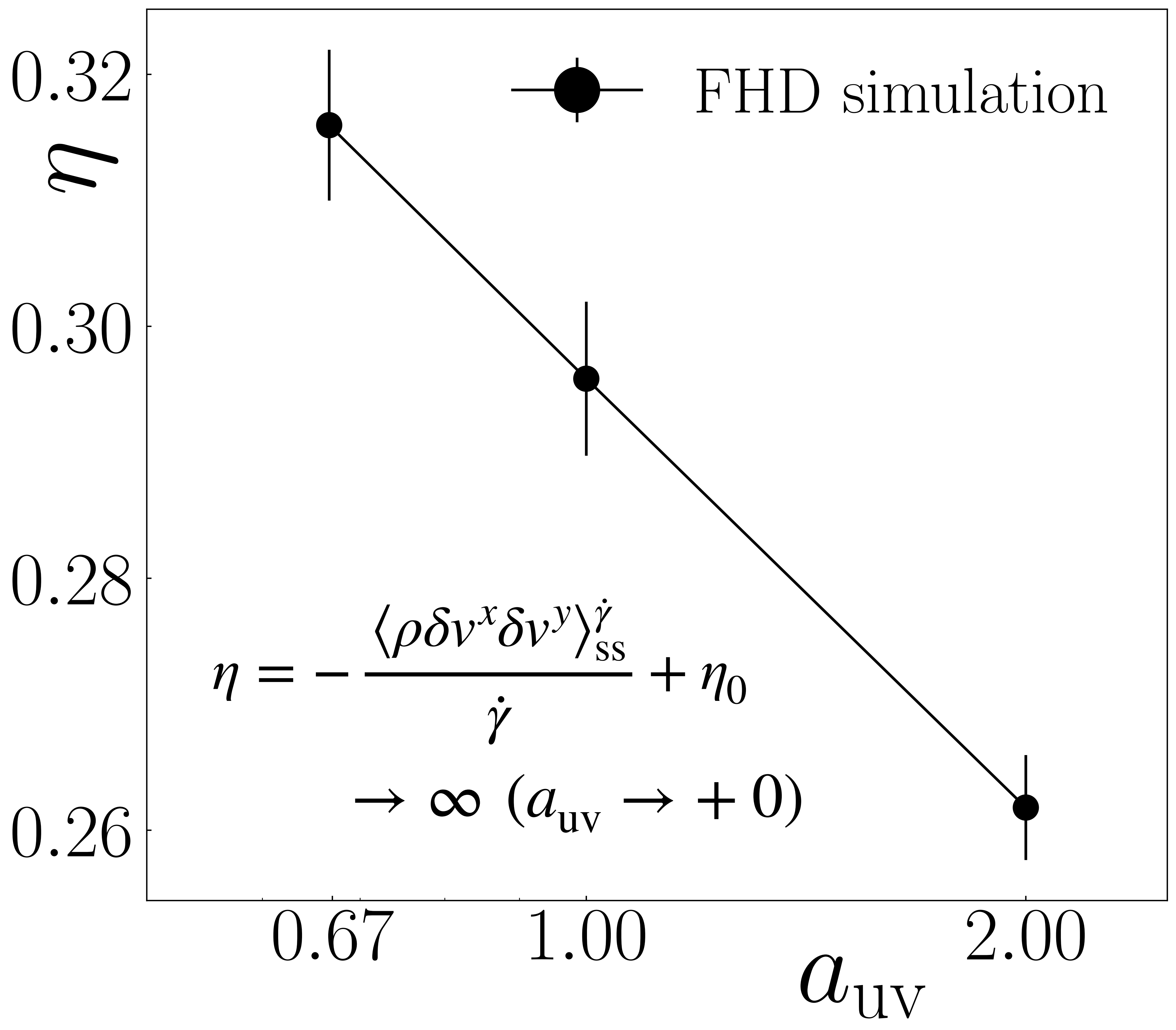
Physical quantities calculated within the framework of fluctuating hydrodynamics generally depend on the UV cutoff length above two dimensions
(when the UV cutoff length changes while all parameters are fixed to the same values)

Why do we aim to observe bare viscosity?

Anomalous transport in low-dimensional systems

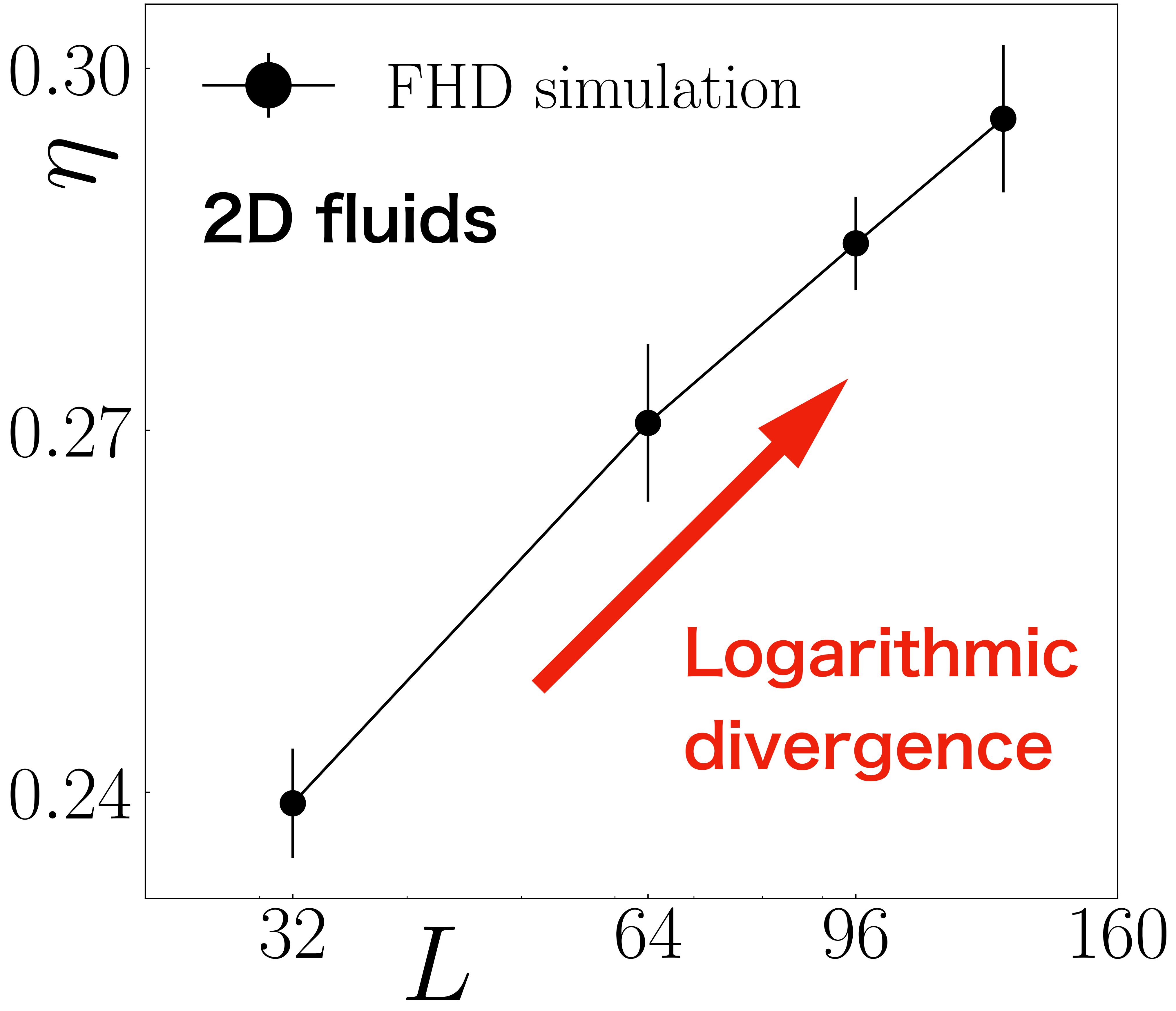


UV cutoff dependence of fluctuating hydrodynamics



Anomalous transport in 2D

- In low-dimensional systems, it is well known that transport coefficients can diverge with the system size.



$$\eta = -\underline{\frac{\langle \rho \delta v^x \delta v^y \rangle_{\mathrm{ss}}^{\dot{\gamma}}}{\dot{\gamma}}} + \eta_0$$

$$\to \infty \ (L \to \infty)$$

The divergence of shear viscosity can be understood from the divergence of the nonlinear coupling of fluctuations.

Long-time tail problem

Formula in nonequilibrium steady state

$$\eta = -\frac{\langle \rho \delta v^x \delta v^y \rangle_{\mathrm{ss}}^{\dot\gamma}}{\dot\gamma} + \eta_0$$

$$\rightarrow \infty \ (L \rightarrow \infty)$$

Green-Kubo formula and Long-time tail

$$\eta = \frac{1}{k_B T L^2} \int_{a_{\mathrm{uv}}/v_0}^{L/v_0} dt \left\langle \mathbf{\Pi}^{xy}(t) \mathbf{\Pi}^{xy}(0) \right\rangle_{\mathrm{eq}}$$

$$\left\langle \mathbf{\Pi}^{xy}(t) \mathbf{\Pi}^{xy}(0) \right\rangle_{\mathrm{eq}} \sim t^{-d/2}$$

⟺

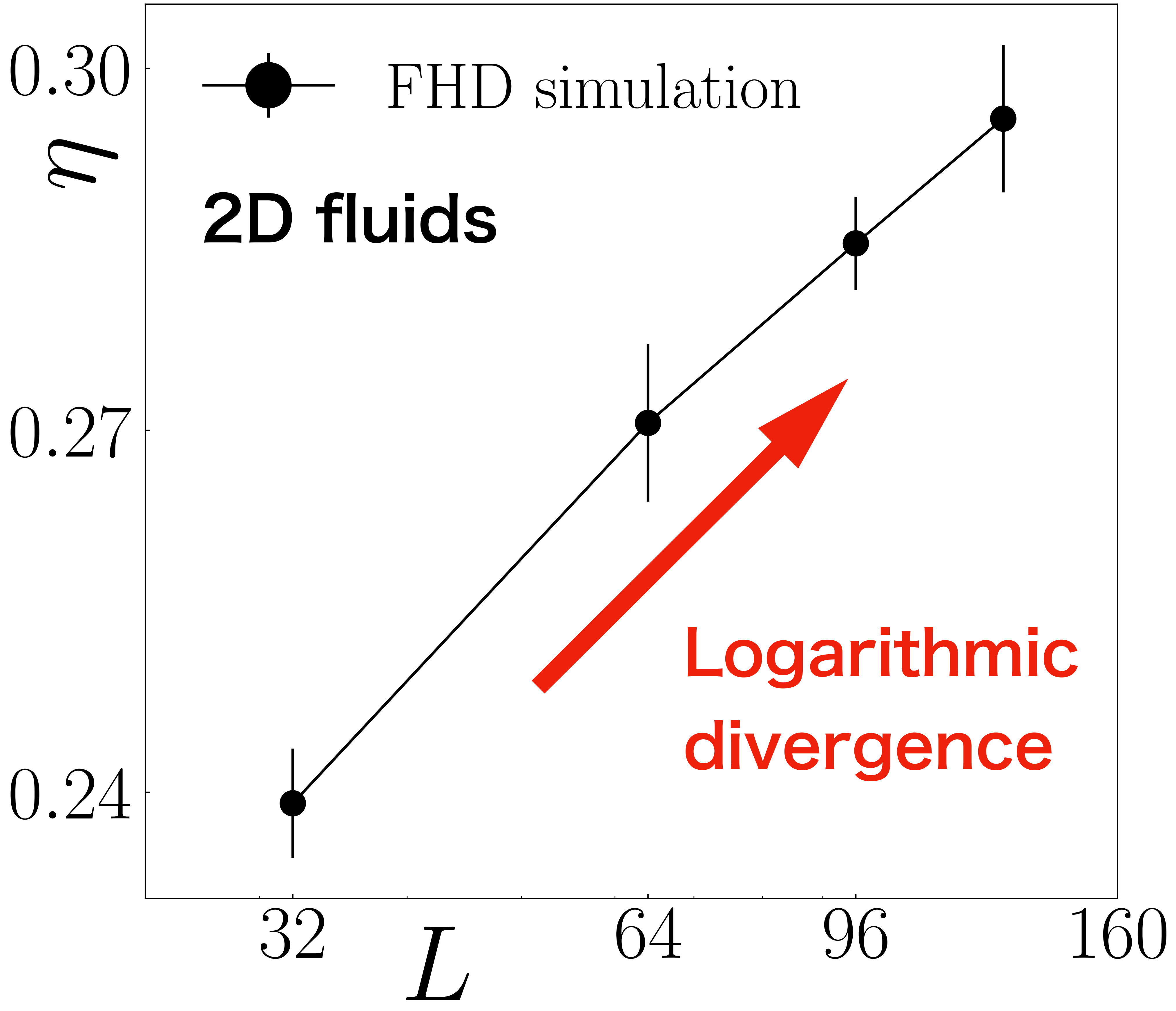
These two frameworks are equivalent.

▶ The existence of long-time tails in the correlation functions leads to a divergence in shear viscosity in 2D.

$$\eta = \frac{1}{k_B T L^2} \int_{a_{\mathrm{uv}}/v_0}^{L/v_0} dt \left\langle \sigma^{xy}(t) \sigma^{xy}(0) \right\rangle_{\mathrm{eq}} \sim \int_{a_{\mathrm{uv}}/v_0}^{L/v_0} dt \frac{1}{t} \sim \log \frac{L}{a_{\mathrm{uv}}}$$

Anomalous transport in 2D

- In low-dimensional systems, it is well known that transport coefficients can diverge with the system size.



$$\eta = -\frac{\langle \rho \delta v^x \delta v^y \rangle_{\mathrm{ss}}^{\dot{\gamma}}}{\dot{\gamma}} + \eta_0$$

$$\to \infty \ (L \to \infty)$$

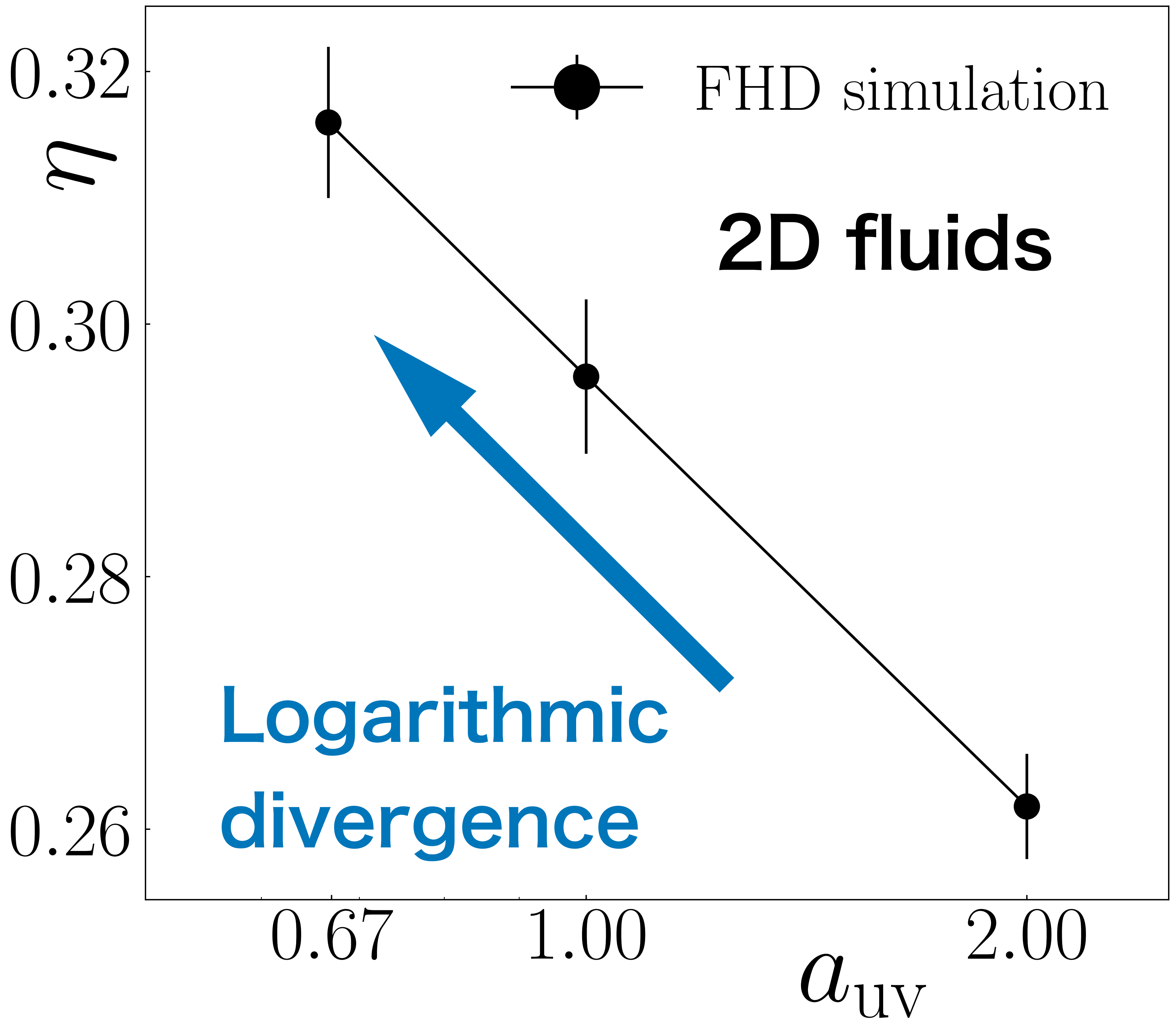
The nonlinear coupling of fluctuations includes the long-time tail effect, which leads to the system size dependence.

We address this problem!!

Observing bare viscosity ➡ Removing the long-time tail effects completely.

UV cutoff dependence

- By changing only the UV cutoff length a_{uv} while keeping all other parameters fixed, the predictions of fluctuating hydrodynamics change above two dimensions.
- In fluctuating hydrodynamics, observed viscosity (or momentum flux) diverges as $a_{\mathrm{uv}} \to +0$



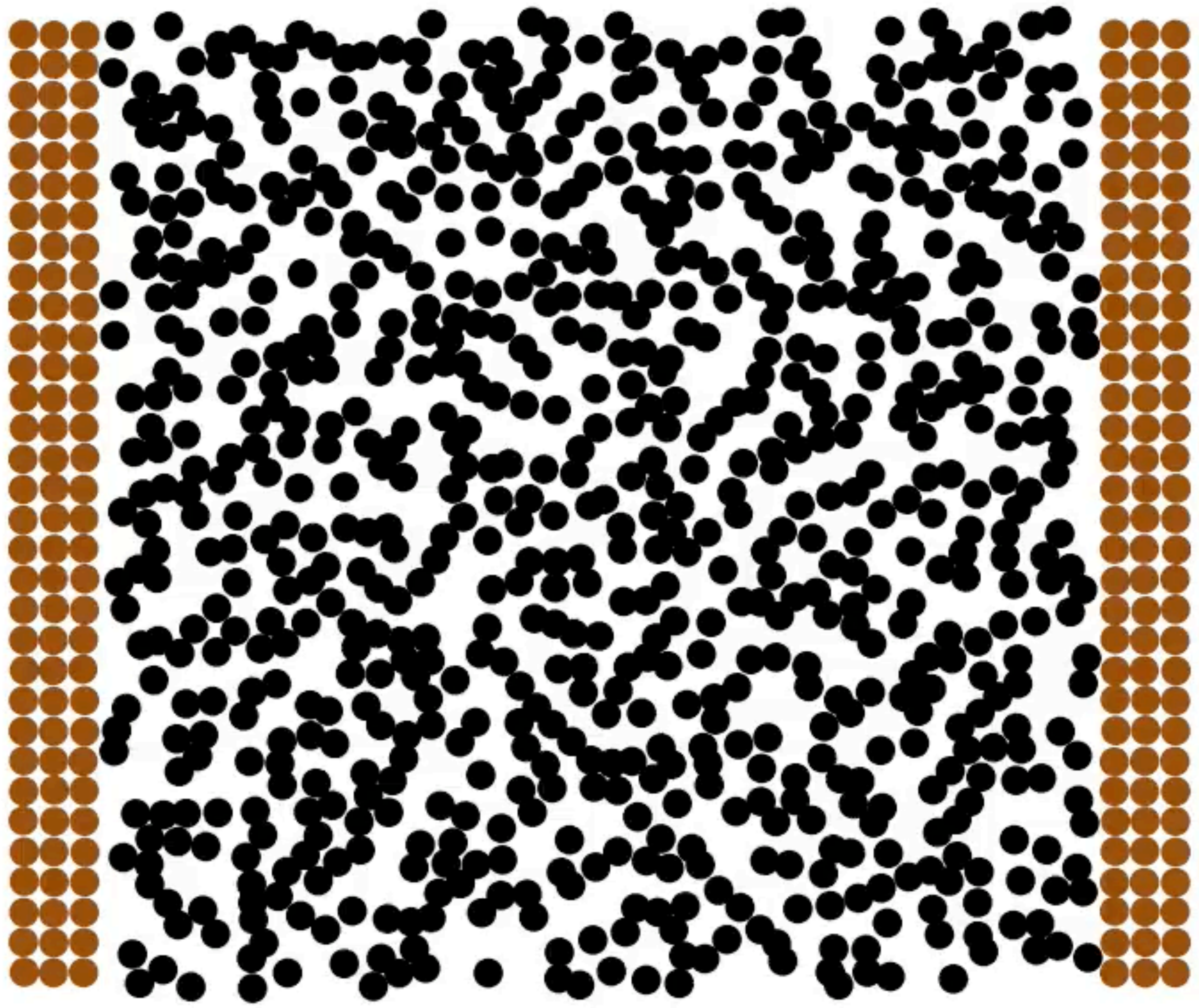
$$\eta = -\frac{\langle \rho \delta v^x \delta v^y \rangle_{\mathrm{ss}}^{\dot{\gamma}}}{\dot{\gamma}} + \eta_0$$

$$\to \infty \ (a_{\mathrm{uv}} \to +0)$$

The UV cutoff length a_{uv} is also the parameter to determine the prediction of fluctuating hydrodynamics uniquely.

UV cutoff dependence

▶ In atomic systems, the concept of UV cutoff length does not exist !!
(because the classical Hamiltonian dynamics is not continuum theory.)



We also address this problem!!

$$\eta = -\frac{\langle \rho \delta v^x \delta v^y \rangle_{\mathrm{ss}}^{\dot{\gamma}}}{\dot{\gamma}} + \eta_0$$

$$\to \infty \ (a_{\mathrm{uv}} \to +0)$$

The UV cutoff length a_{uv} is also the parameter to determine the prediction of fluctuating hydrodynamics uniquely.

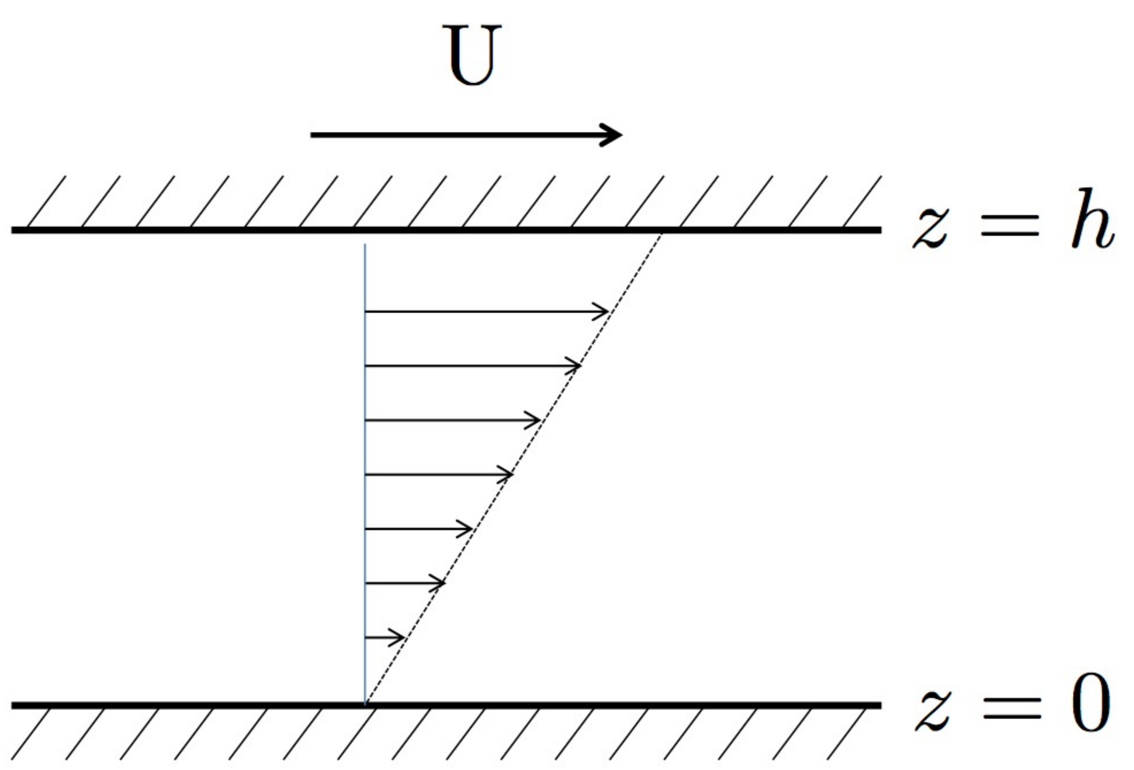
We also address this problem!!

Observing bare viscosity ➡ **Determining the value of UV cutoff length**

Summary of Introduction

Observed viscosity η $\neq$ Bare viscosity η_0

$$\eta := \frac{-\langle \Pi^{xy}(\boldsymbol{r}, t)\rangle_{\rm ss}^{\dot{\gamma}}}{\dot{\gamma}} = \eta_0 - \frac{\langle \rho \delta v^x \delta v^y \rangle_{\rm ss}^{\dot{\gamma}}}{\dot{\gamma}}$$



Including the effects of long-time tail

Observing bare viscosity is the challenging problem.

Observing bare viscosity → **Removing the long-time tail effects completely.**

Observing bare viscosity → **Determining the value of UV cutoff length**

To address these issues, we focus on **two-dimensional dense fluids**.

2.1 Main idea of our study

Preliminary simulation

▶ We consider solving fluctuating hydrodynamics numerically

2d fluid

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \boldsymbol{v}) \qquad p(\rho) = C_{\mathrm{press}}\rho \qquad c_T := \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_T} = \sqrt{C_{\mathrm{press}}}$$

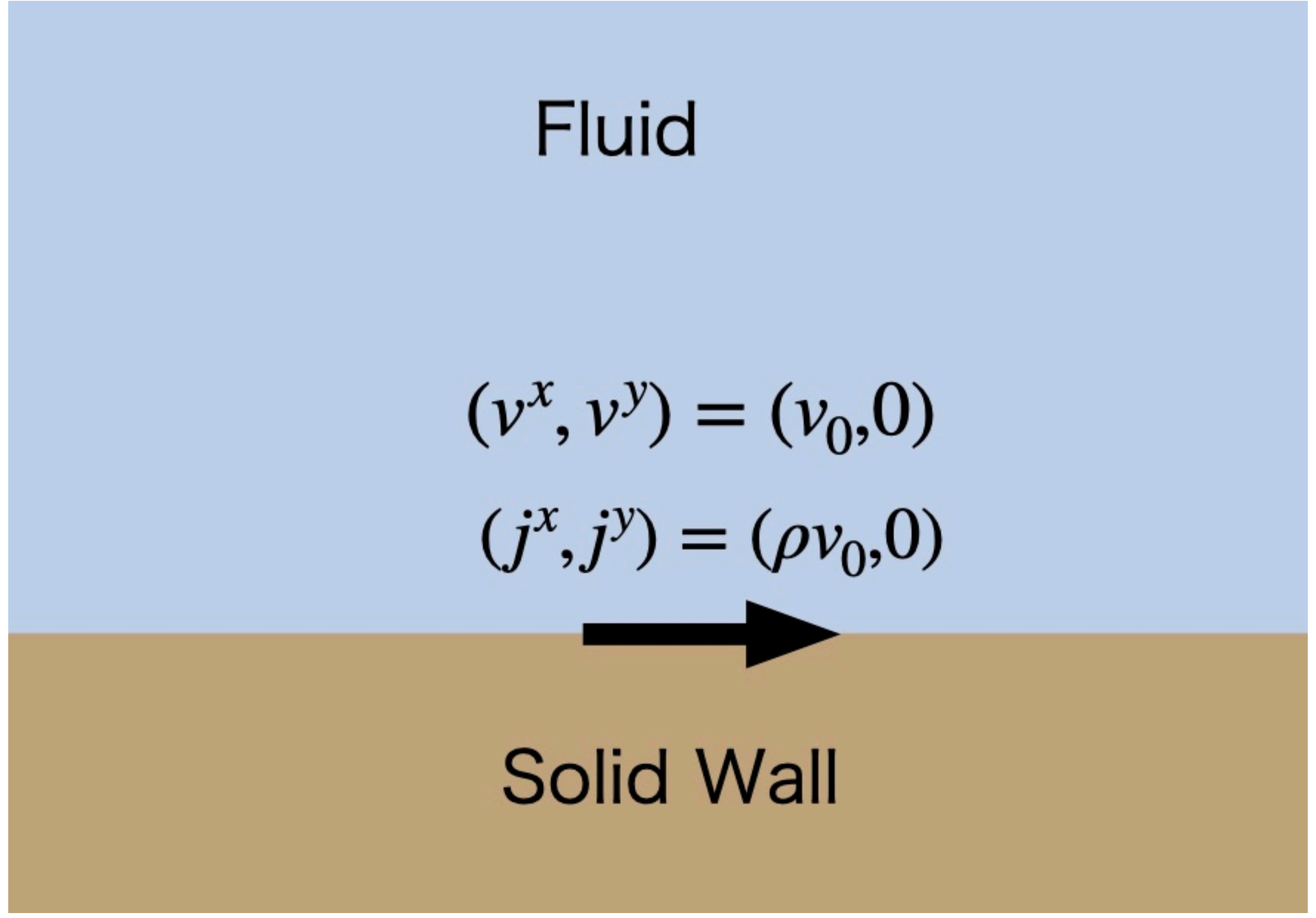
$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right] = -\nabla p + \eta_0 \nabla^2 \boldsymbol{v} + \zeta_0 \nabla(\nabla\cdot\boldsymbol{v}) + \nabla \boldsymbol{\Pi}_R$$

The sufficiently large C_{press} yields nearly incompressible fluids

We apply a common boundary condition in fluid dynamics

1. The velocity field at the wall is $(v^x, v^y) = (v_0, 0)$
2. The momentum density field at the wall is $(j^x, j^y) = (\rho v_0, 0)$

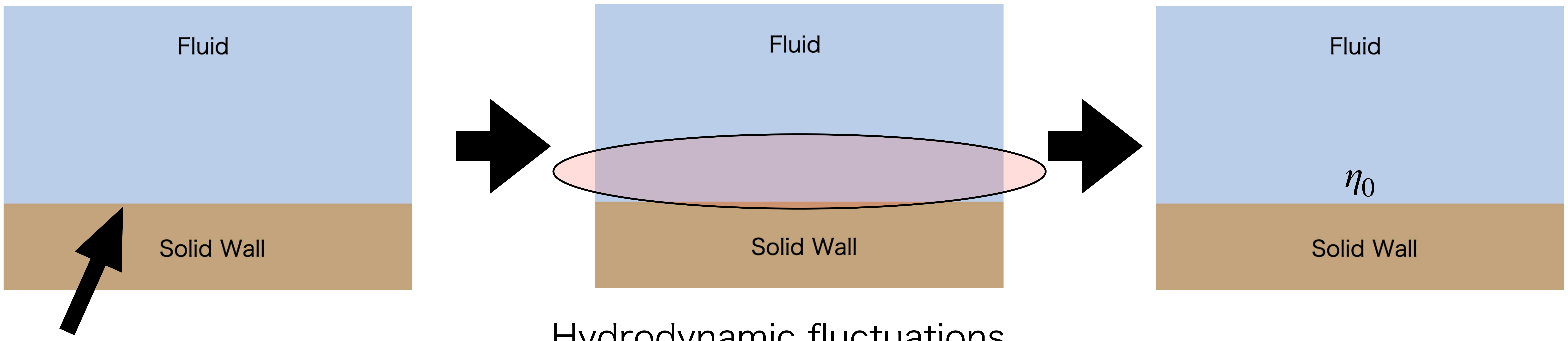
The fluid does not fluctuate at all at the solid walls



Main idea of our study

The motivation for this simulation comes from the expectation that bare viscosity can be observed near non-fluctuating walls.

▶ Main idea of our study : focusing on fluids near solid walls.



Fluid does not fluctuate at walls

Hydrodynamic fluctuations (or long-time tail) does not develop near walls

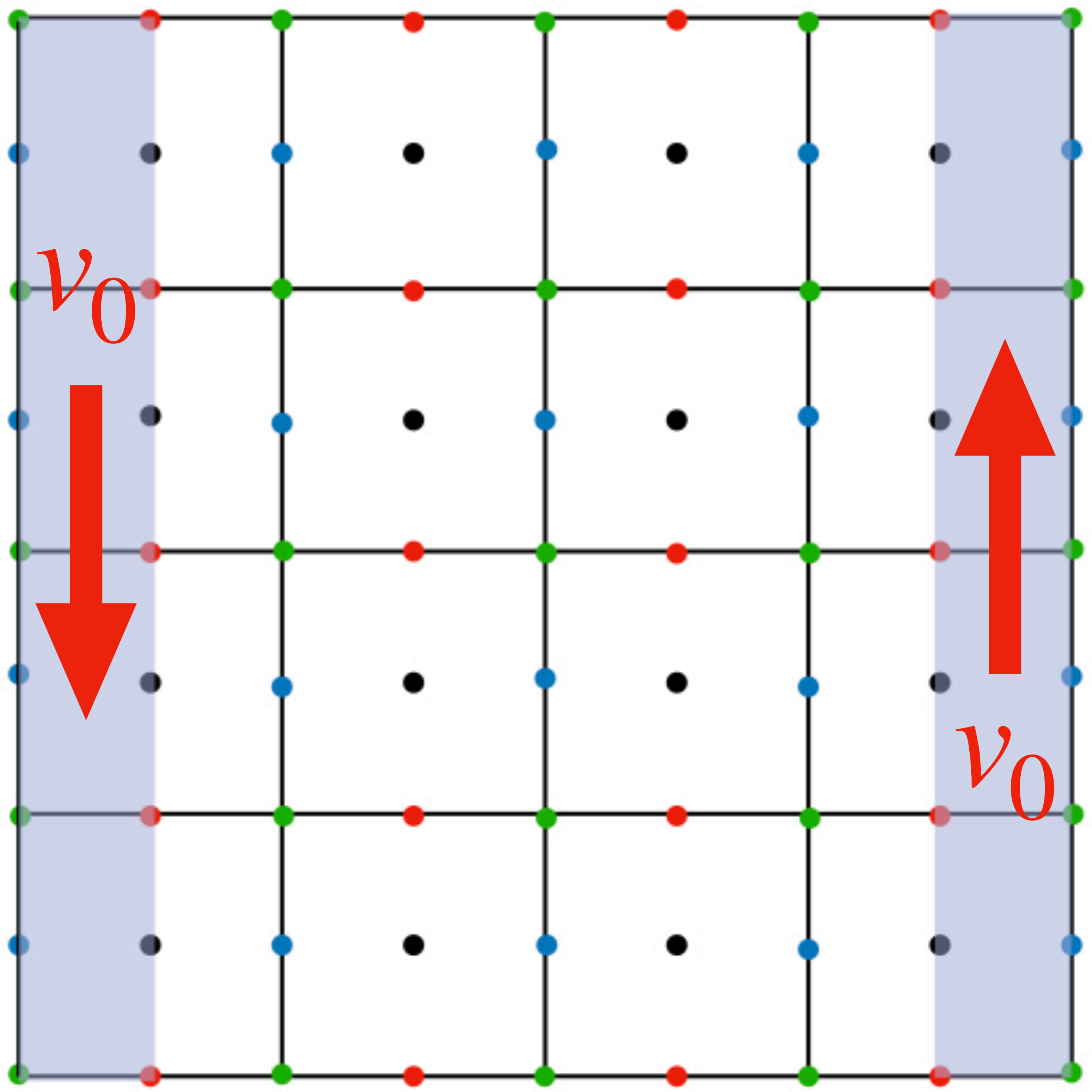
Bare viscosity appear near walls ?

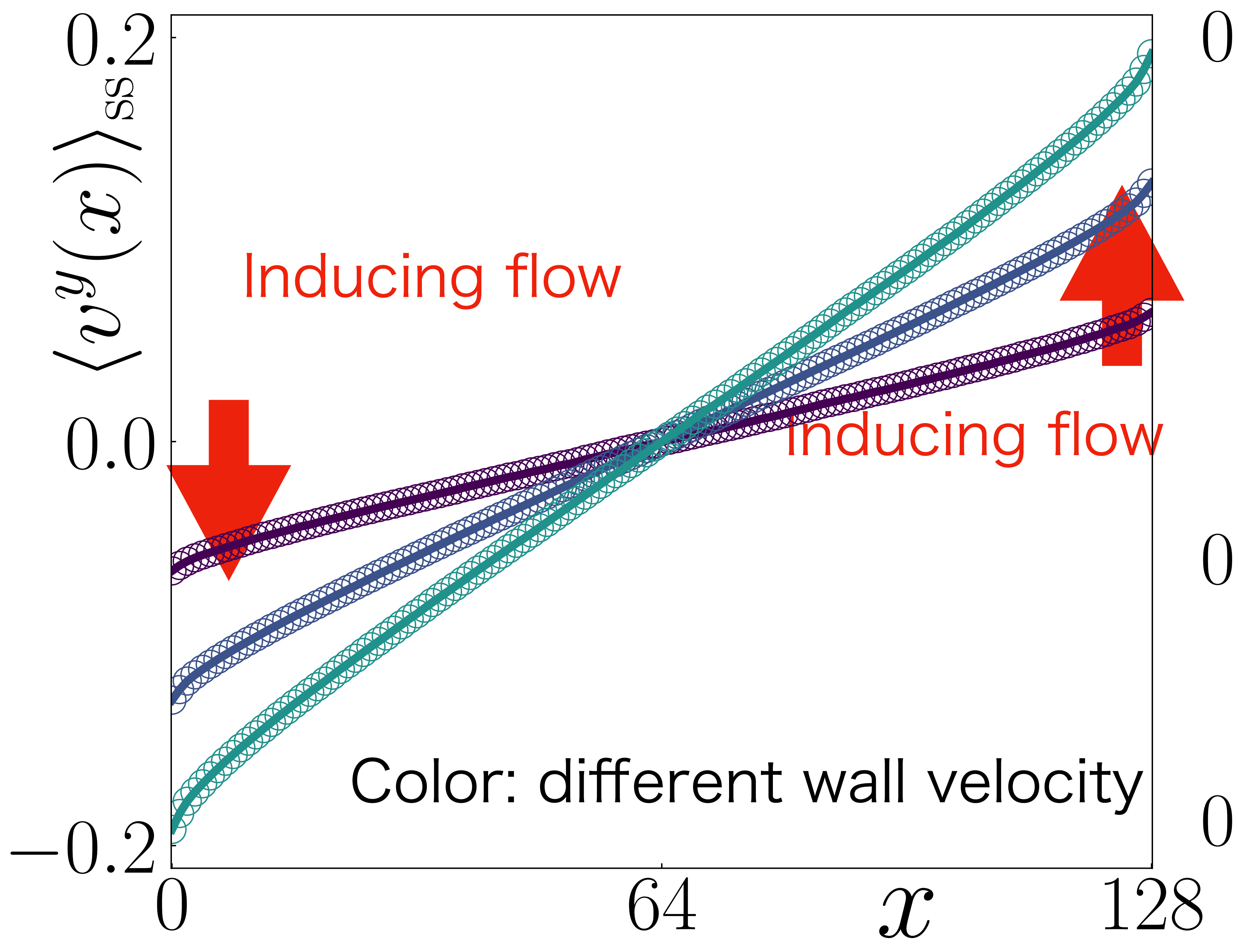
Steady state profile in fluctuating hydro

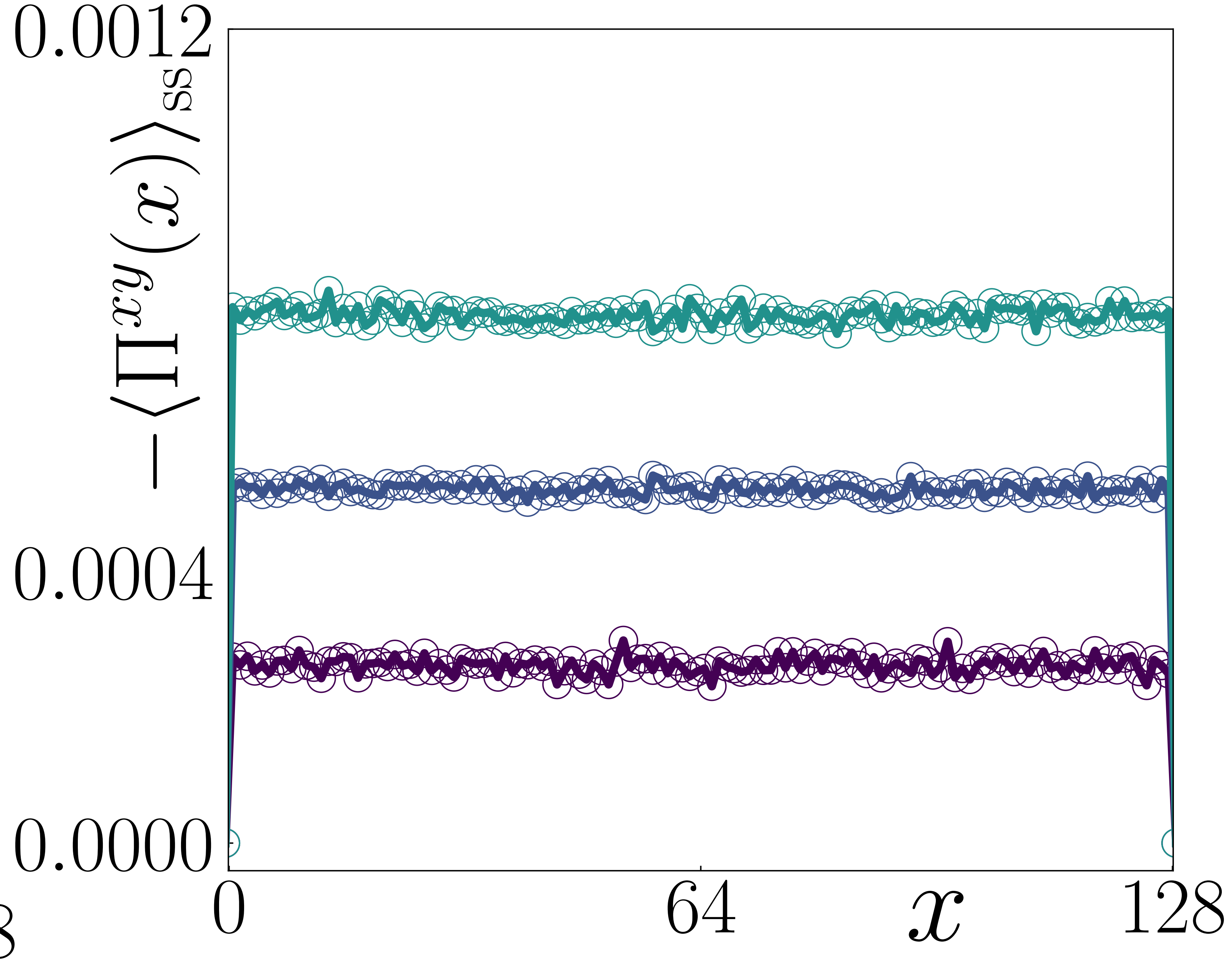
$\eta_0 = 0.1,\ a_{\rm uv} = 1.0,\ \rho_0 = 0.765,\ k_B T = 1.0,\ L = 128$

(I will explain the units for physical quantities later)

- We observe the velocity profile and momentum flux profile in the steady state.
- We add wall velocity v_0 of three different magnitudes.

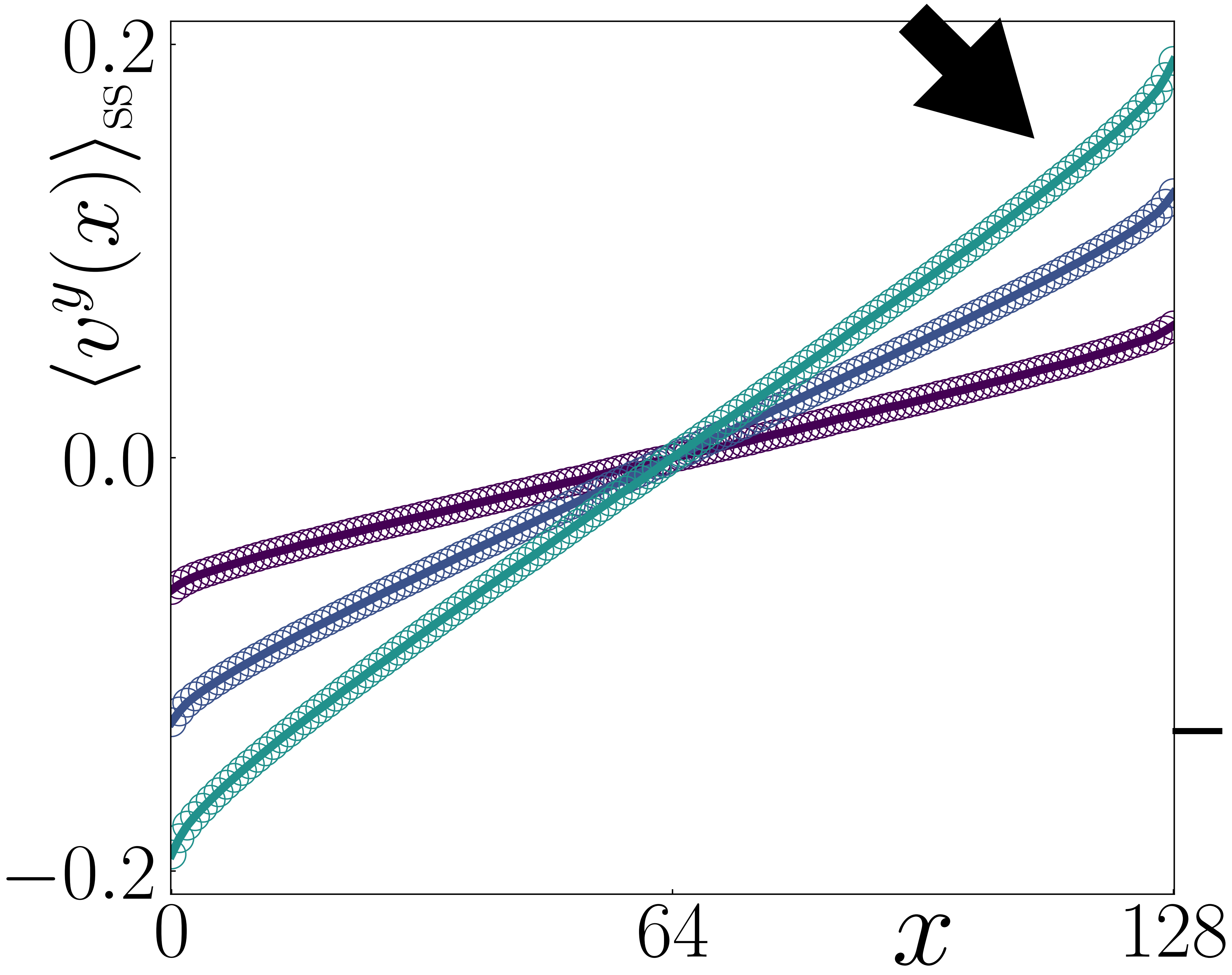




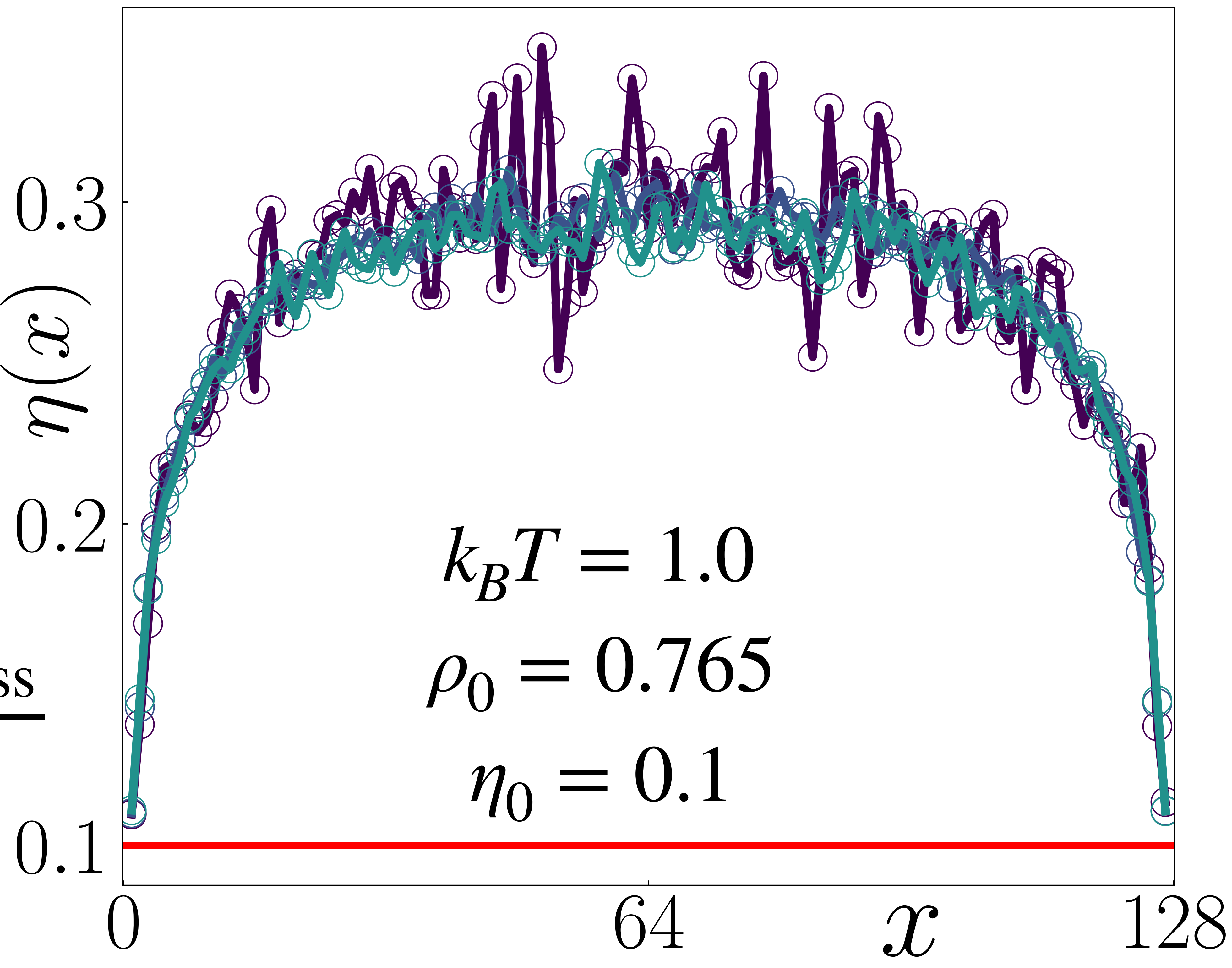


Observation of bare viscosity

Velocity gradient is not spatially uniform



$$-\langle \Pi^{xy}\rangle_{ss} = \eta(x)\frac{\partial\langle v^y\rangle_{ss}}{\partial x}$$

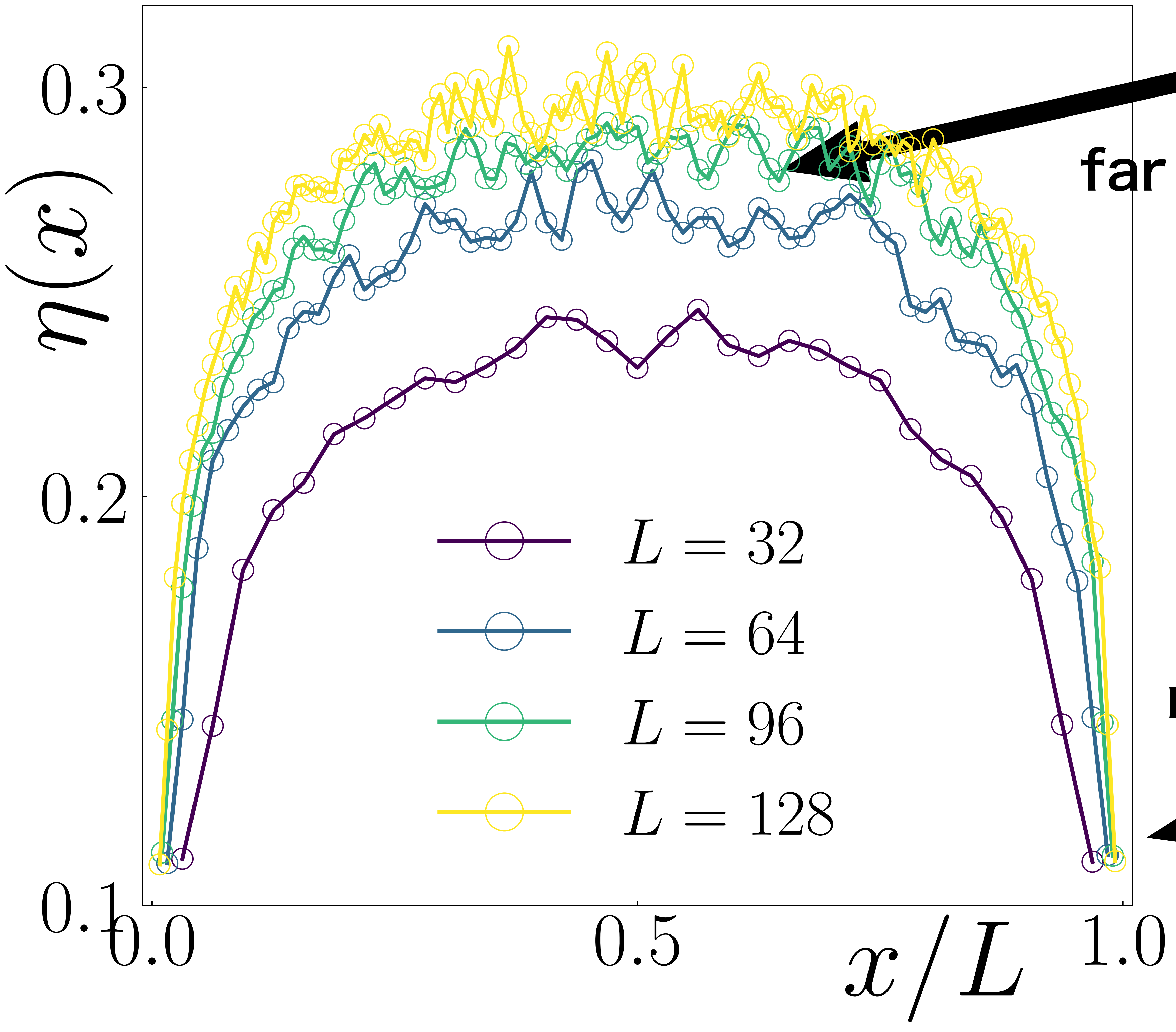


The expected results are obtained!!

- Bare viscosity η_0 is observed near solid walls.
- As we move away from the solid wall, it deviates from bare viscosity η_0

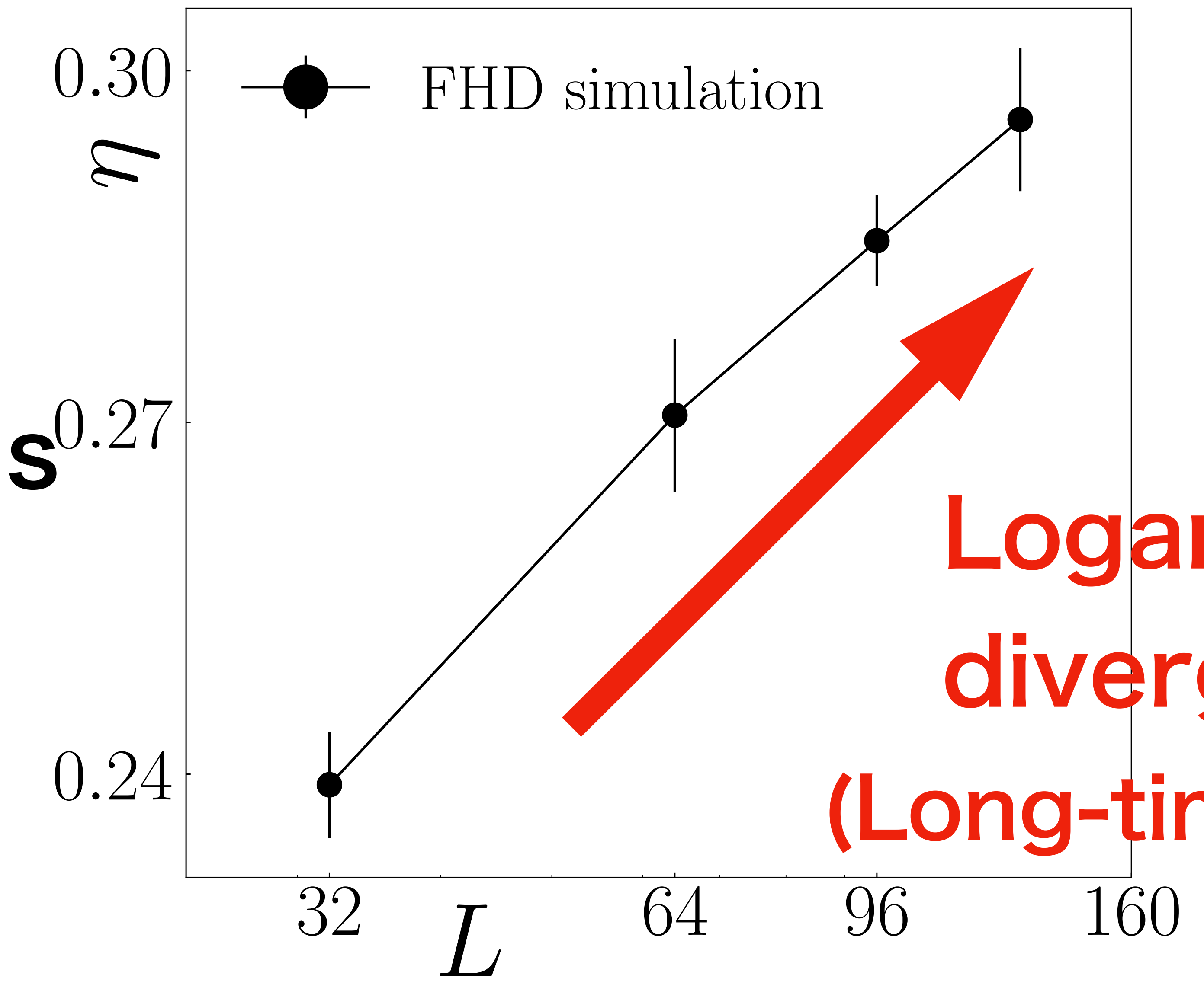
Observation of bare viscosity

- We change the system size L while fixing to the other parameters.

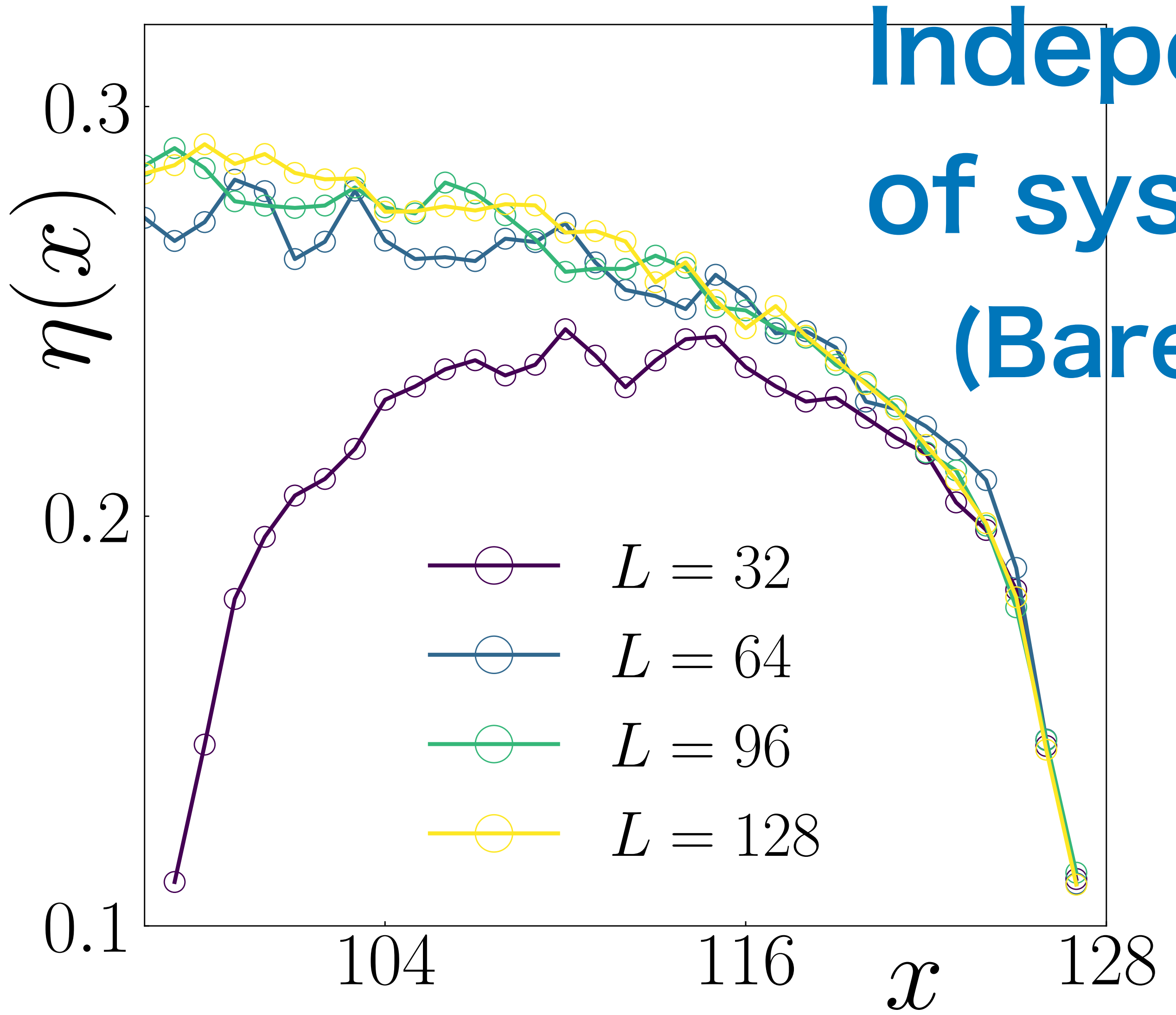


far from walls

near walls



Logarithmic divergence (Long-time tail)



Independent of system size (Bare viscosity)

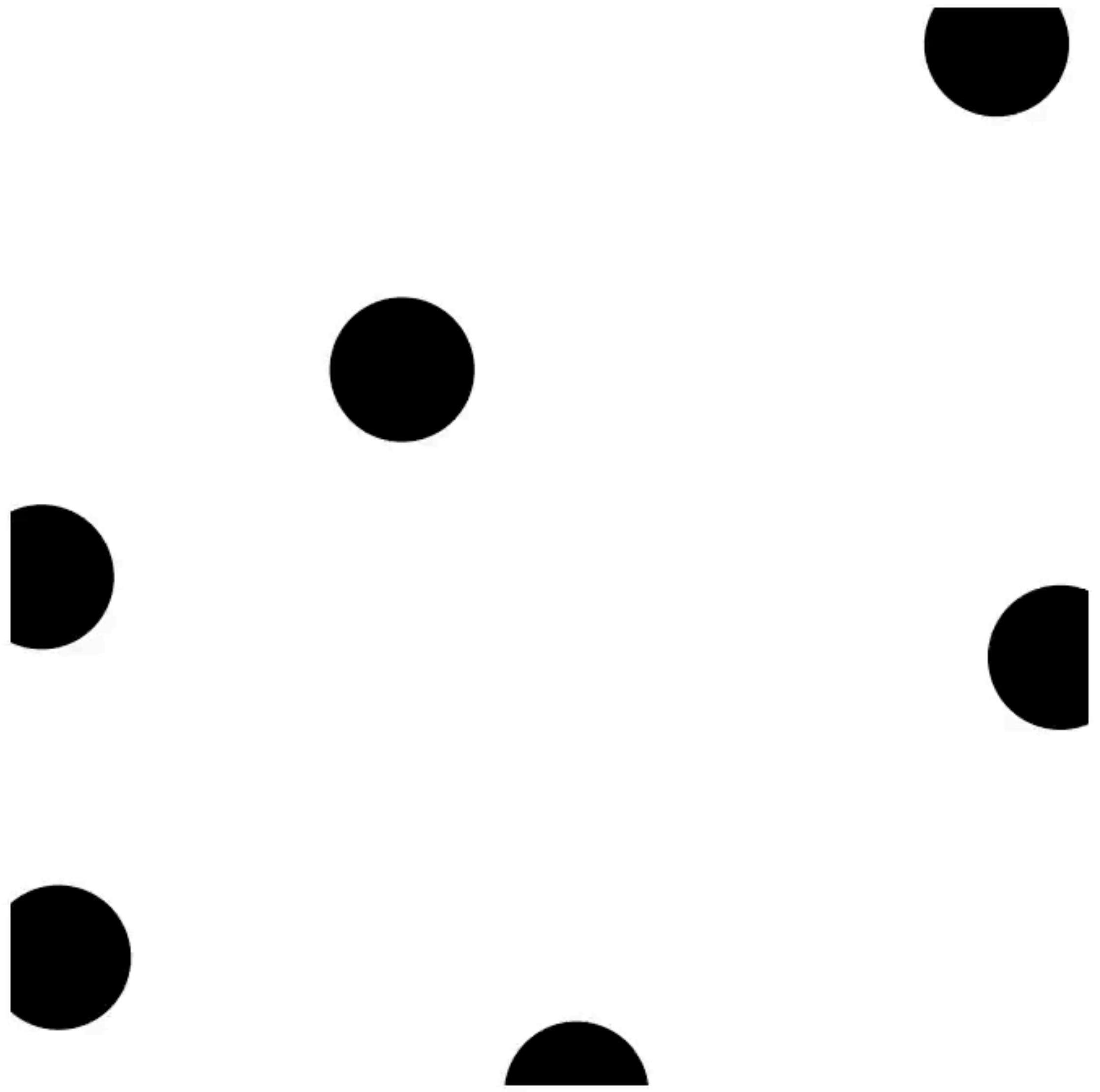
Main results

The idea of examining the behaviors near walls seems good!

2.2 Is this result robust for more realistic walls

Molecular dynamics (MD) simulation

▶ We perform molecular dynamics (MD) simulations.



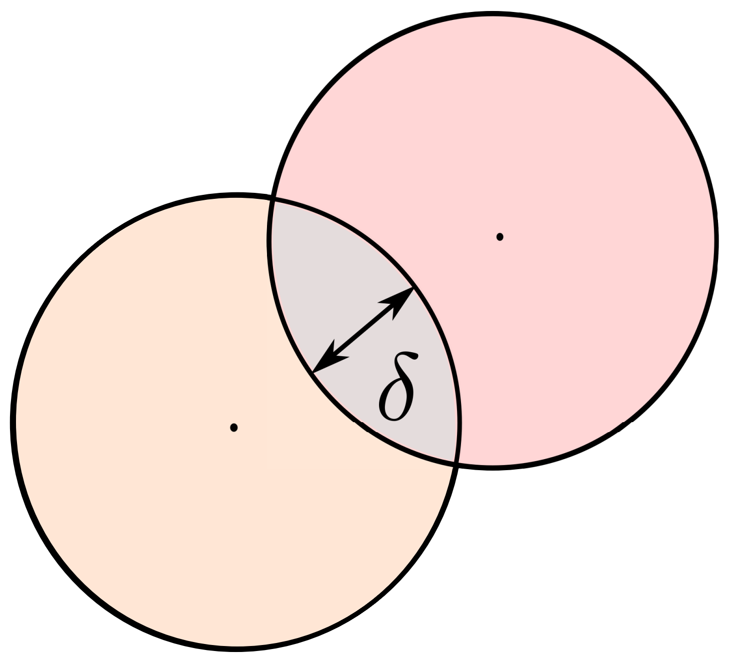
In MD simulations, molecules are represented as particles that follow the classical Hamiltonian dynamics.

$$\frac{d\boldsymbol{r}_i}{dt} = \frac{\boldsymbol{p}_i}{m} \qquad \frac{d\boldsymbol{p}_i}{dt} = -\frac{\partial V}{\partial \boldsymbol{r}_i}$$

▷ simple repulsive potential

$$V(r) = 10\delta^{\alpha} \quad \text{for } \delta > 0$$

$$V(r) = 0 \quad \text{for } \delta < 0$$



Units for the MD simulation

atomic diameter σ

atomic mass m

thermal velocity $v_{th} := \sqrt{k_B T/m}$

(or microscopic time $\tau = \sigma/v_{th}$)

Implementation of solid walls

- Solid walls are implemented as a collection of particles.

1. Solid particles are trapped using an on-site potential.

$$V_{\rm onsite}(\boldsymbol{q}) = V_0\left[\sin(2\pi q_x) + \sin(2\pi q_y)\right] \qquad V_0 = 50$$

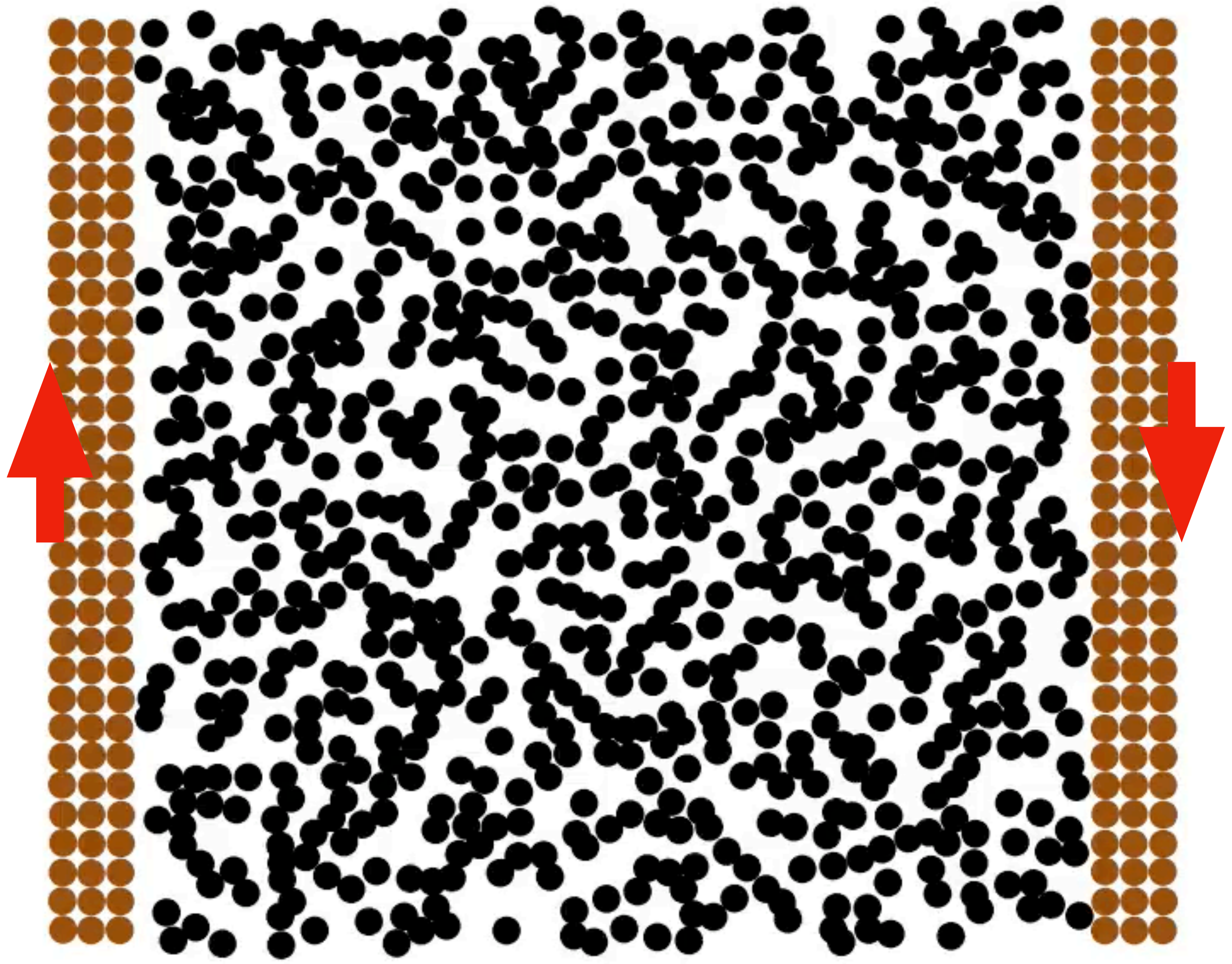
2. Solid particles are thermalized using the Langevin thermostat.

$$\frac{d\boldsymbol{q}_j}{dt} = \frac{\boldsymbol{p}^w}{m}$$

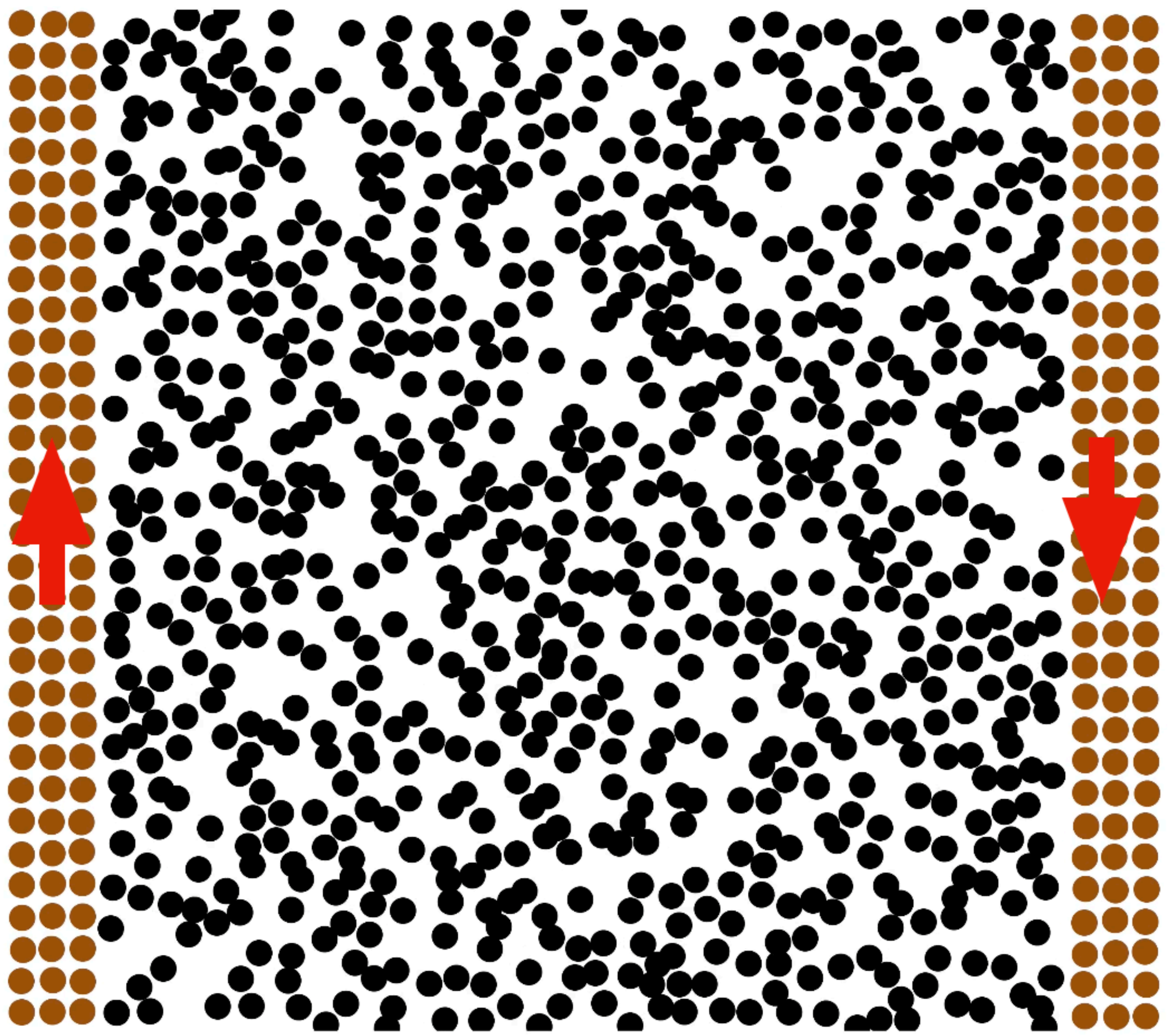
$$\frac{d\boldsymbol{p}^w_j}{dt} = -\frac{\partial V_{\rm onsite}(\boldsymbol{q}_j - v_0 t \boldsymbol{e}_x)}{\partial \boldsymbol{q}_j} - \sum_{i=1}^{N} \frac{\partial V_{\rm wf}(|\boldsymbol{r}_i - \boldsymbol{q}_j|)}{\partial \boldsymbol{q}_j} - \gamma \boldsymbol{p}^w_j + \xi_j(t)$$

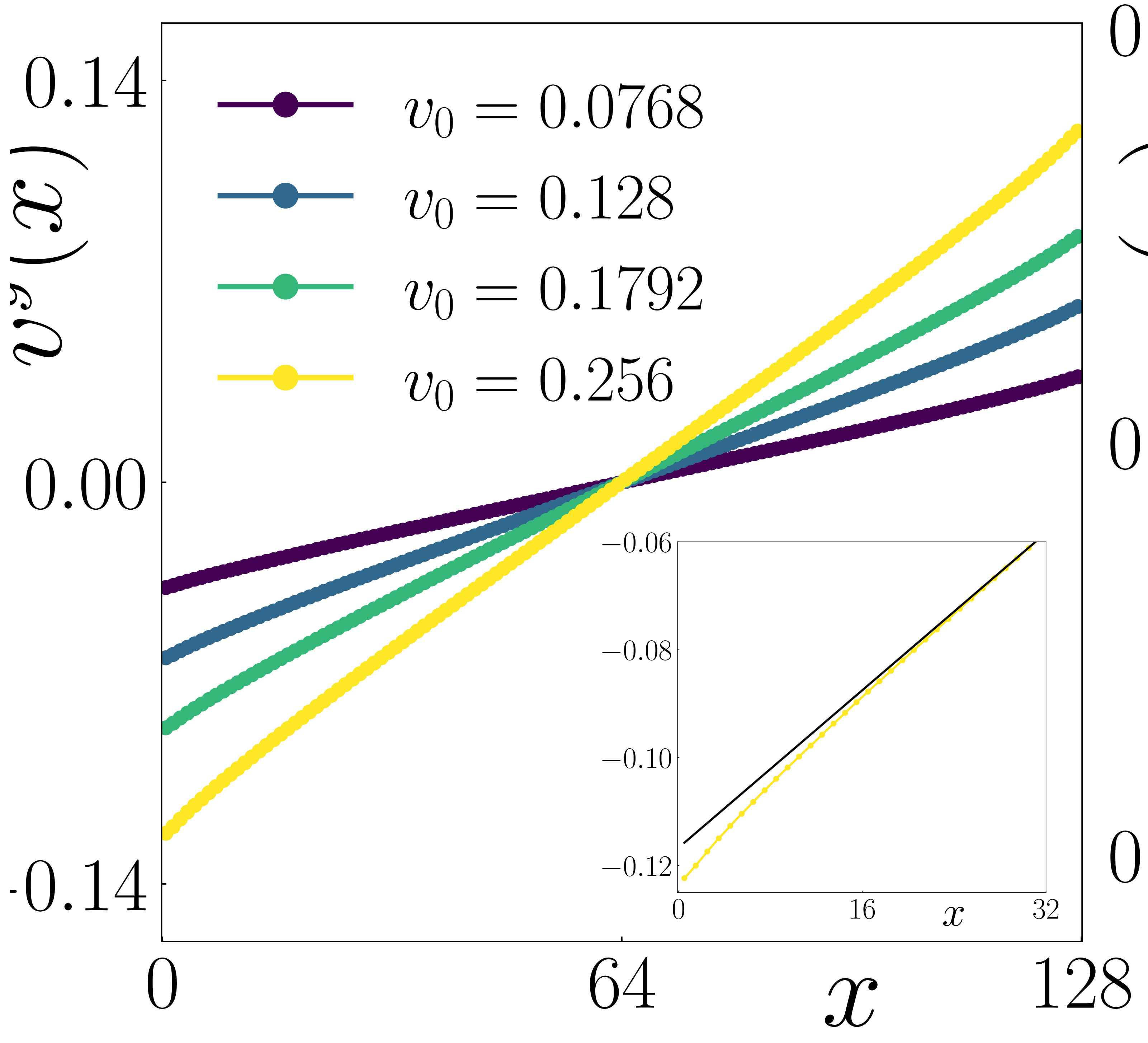
3. Fluid particles interact with solid particles.

4. The motion of the walls is simulated by moving the solid particles collectively at a velocity v_0.

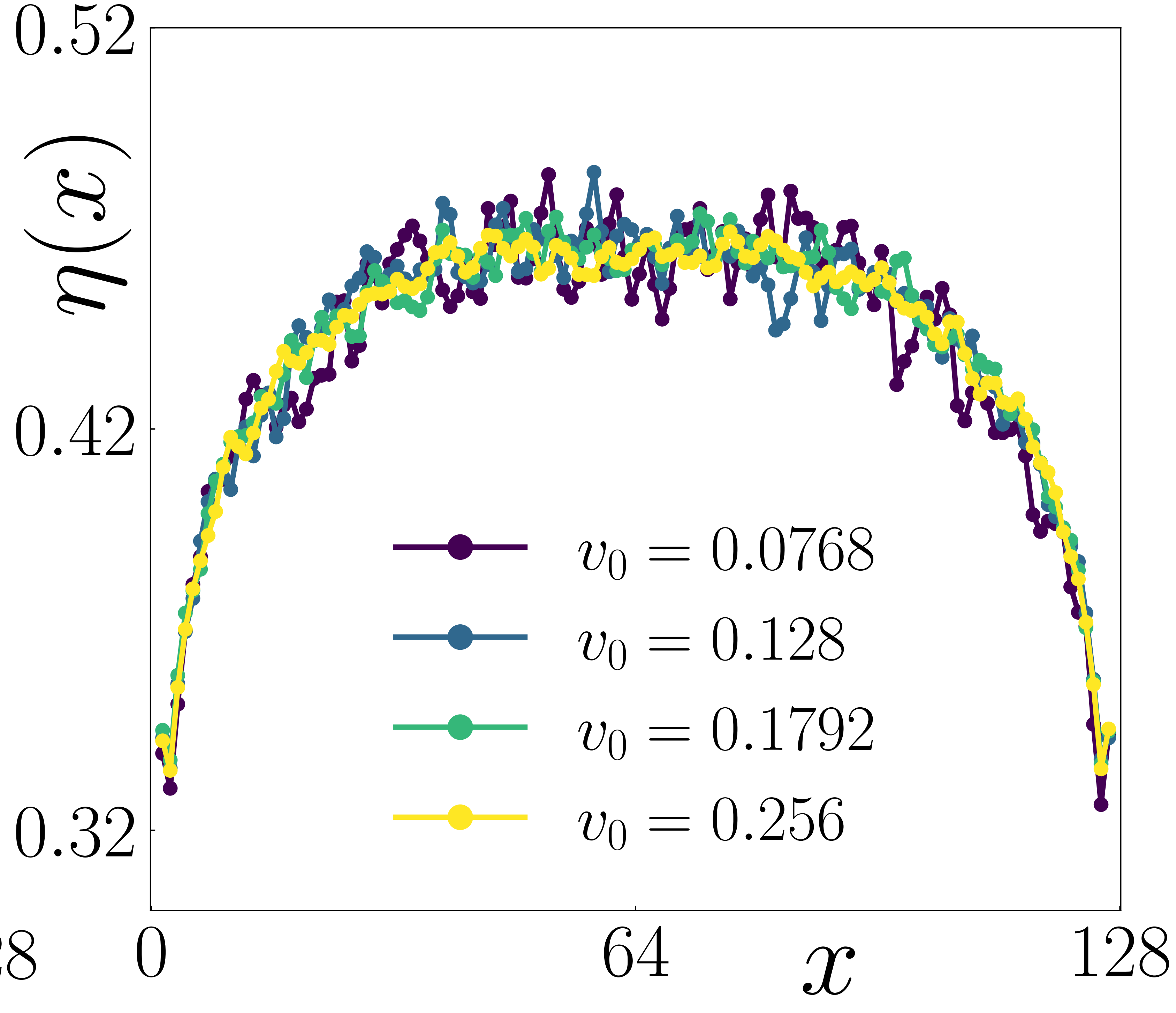


MD simulation results





Color: different wall velocities

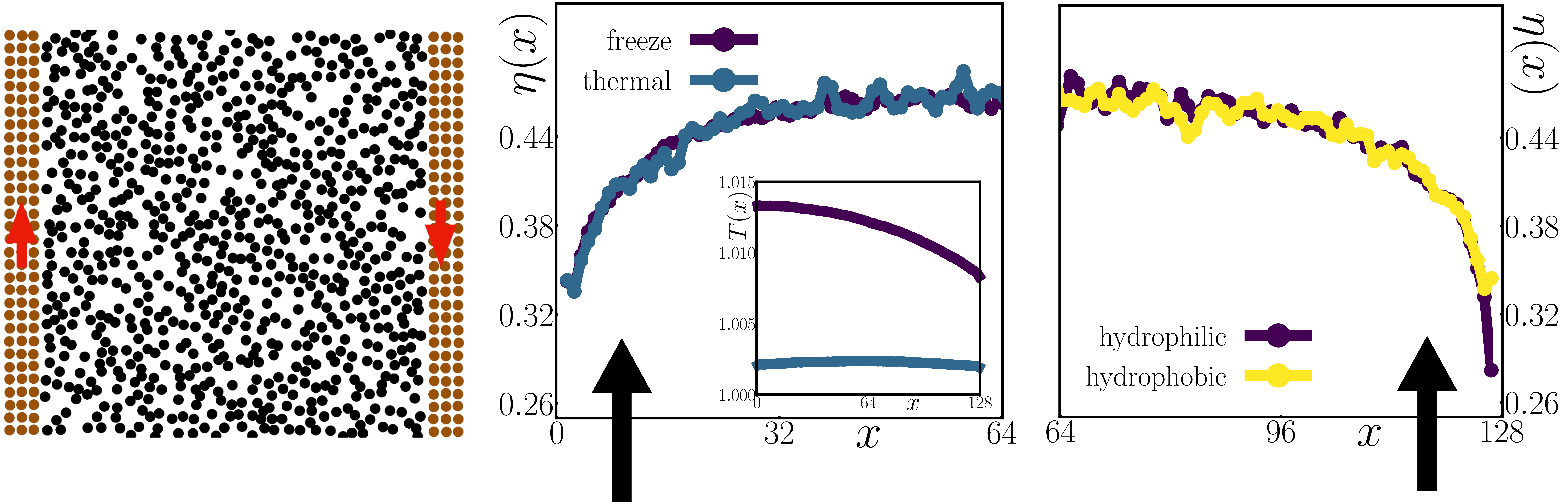


$$-\langle \Pi^{xy}(x) \rangle_{\rm ss}^{\dot{\gamma}} = \eta(x) \frac{\partial v^y}{\partial x}$$

▶ The local viscosity is observed in the same way as in the fluctuating hydrodynamics.

The observed viscosity decreases near solid walls, which is consistent with the behavior in fluctuating hydrodynamics.

Changing the wall types



Freeze: Set the wall temperature to 0.

Thermal: Set the wall temperature to a finite value.

hydrophilic: Use attractive solid-fluid interactions (LJ).

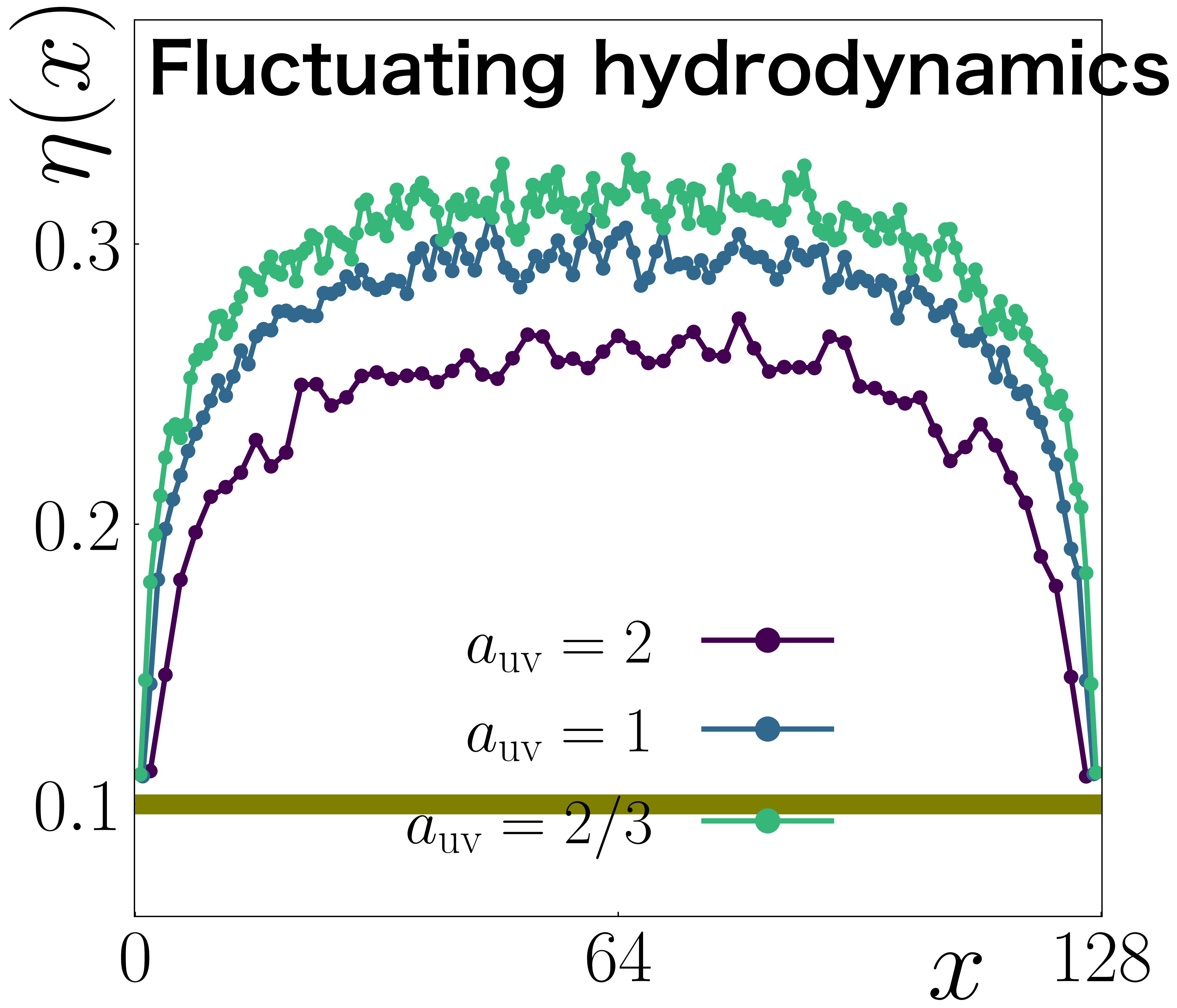
hydrophobic: Use only repulsive solid-fluid interactions (WCA

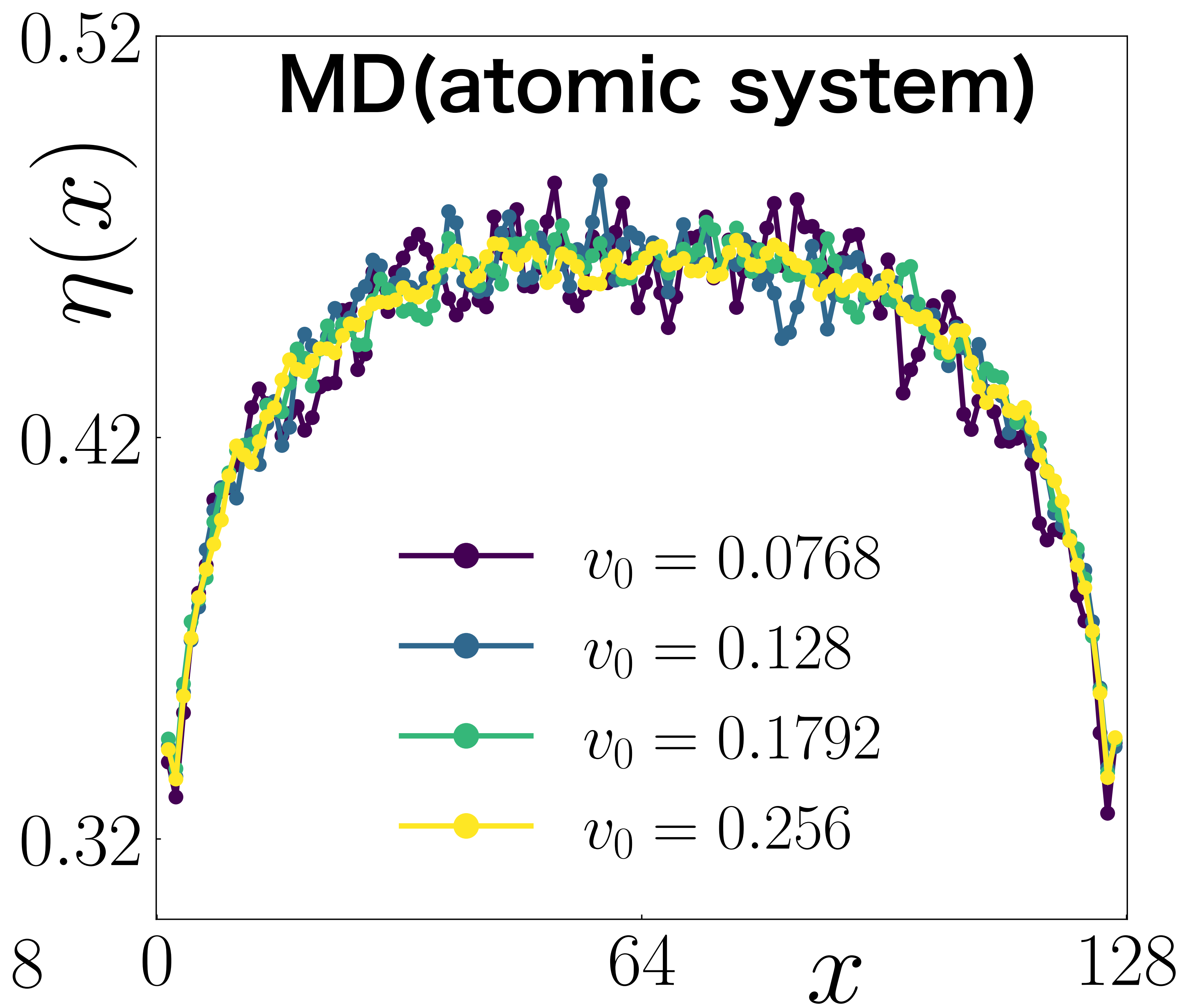
■ The microscopic properties of walls do not affect the results at the quantitative level.

This suggests the robustness of the results of the fluctuating hydrodynamics simulations.

2.3 The determination of bare viscosity

Quantitative comparison





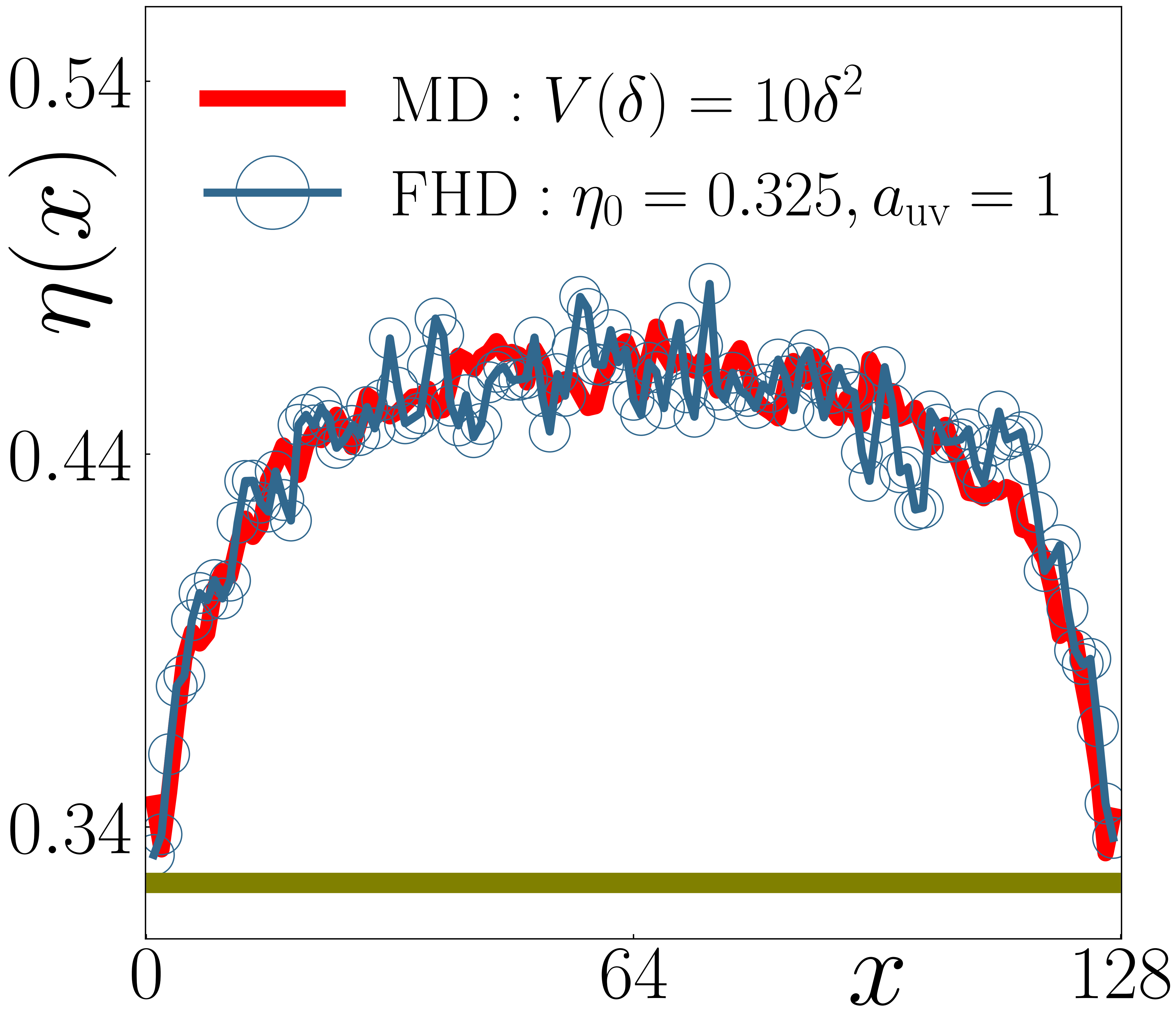
▶ We determine the bare viscosity of the atomic system by quantitatively comparing the results of two models.

Set the same system size, density, and temperature to match the units of both models.

Use bare viscosity η_0 and UV cutoff a_{uv} as fitting parameters.

Quantitative agreement

▶ We fix the UV cutoff length to $a_{\rm uv} = 1.0$ (i.e atomic diameter) and use only bare viscosity η_0 as a adjustable parameter.



The fluctuating hydrodynamics with $\eta_0 = 0.325,\ a_{\rm uv} = 1.0$ reproduces the local viscosity $\eta(x)$ of the MD simulation with high accuracy.

The agreement between the two models is observed even at the atomic diameter scale.

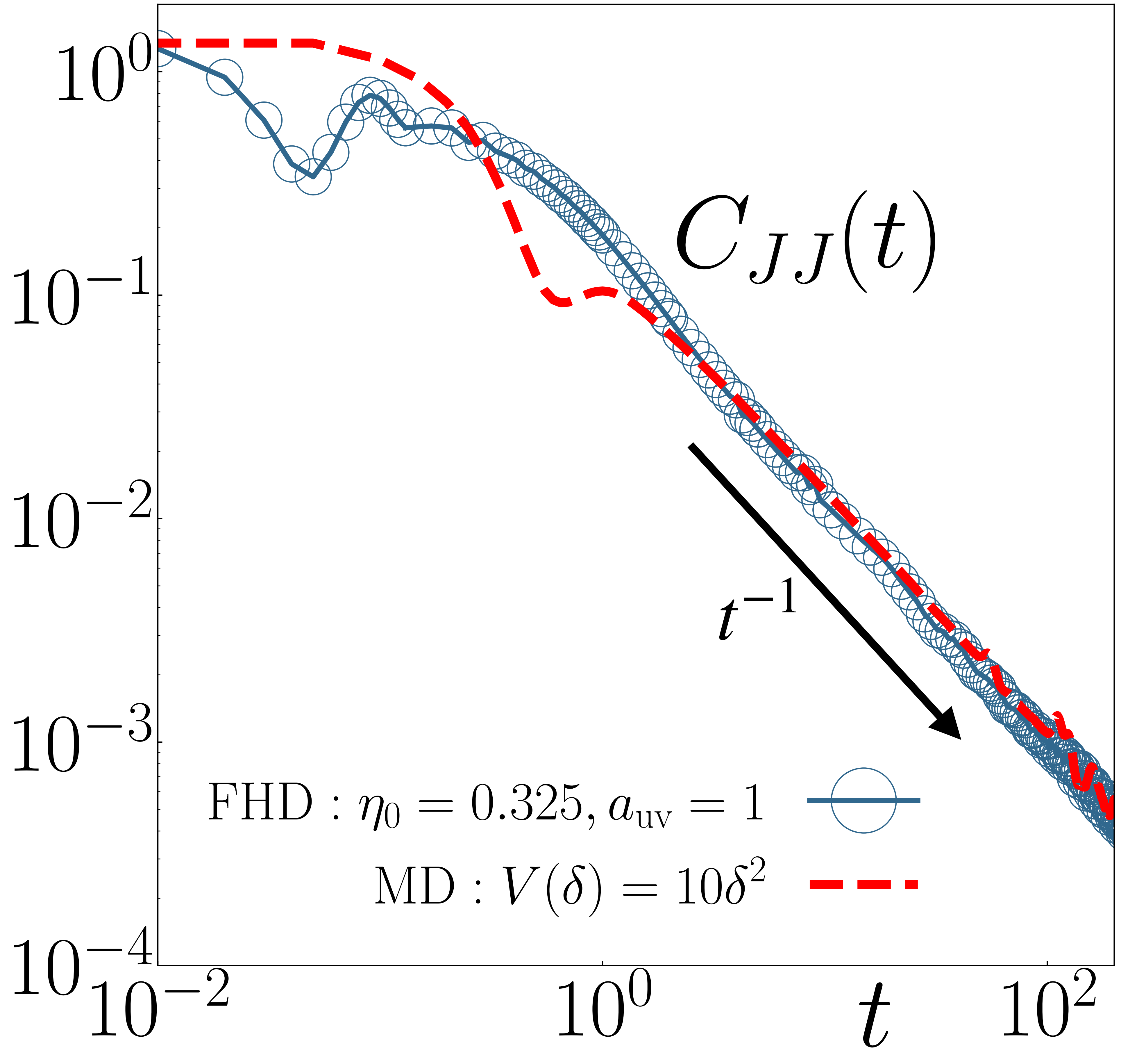
Consistency check : long-time tail in EQ

▶ To validate our estimate of bare viscosity, we compare the time correlation of the momentum density field **in equilibrium**.

$$C_{JJ}(t) := \frac{1}{2}\langle \boldsymbol{j}(\boldsymbol{r},t)\cdot \boldsymbol{j}(\boldsymbol{r},0)\rangle_{\rm eq} \qquad \boldsymbol{j} := \rho \boldsymbol{v}$$

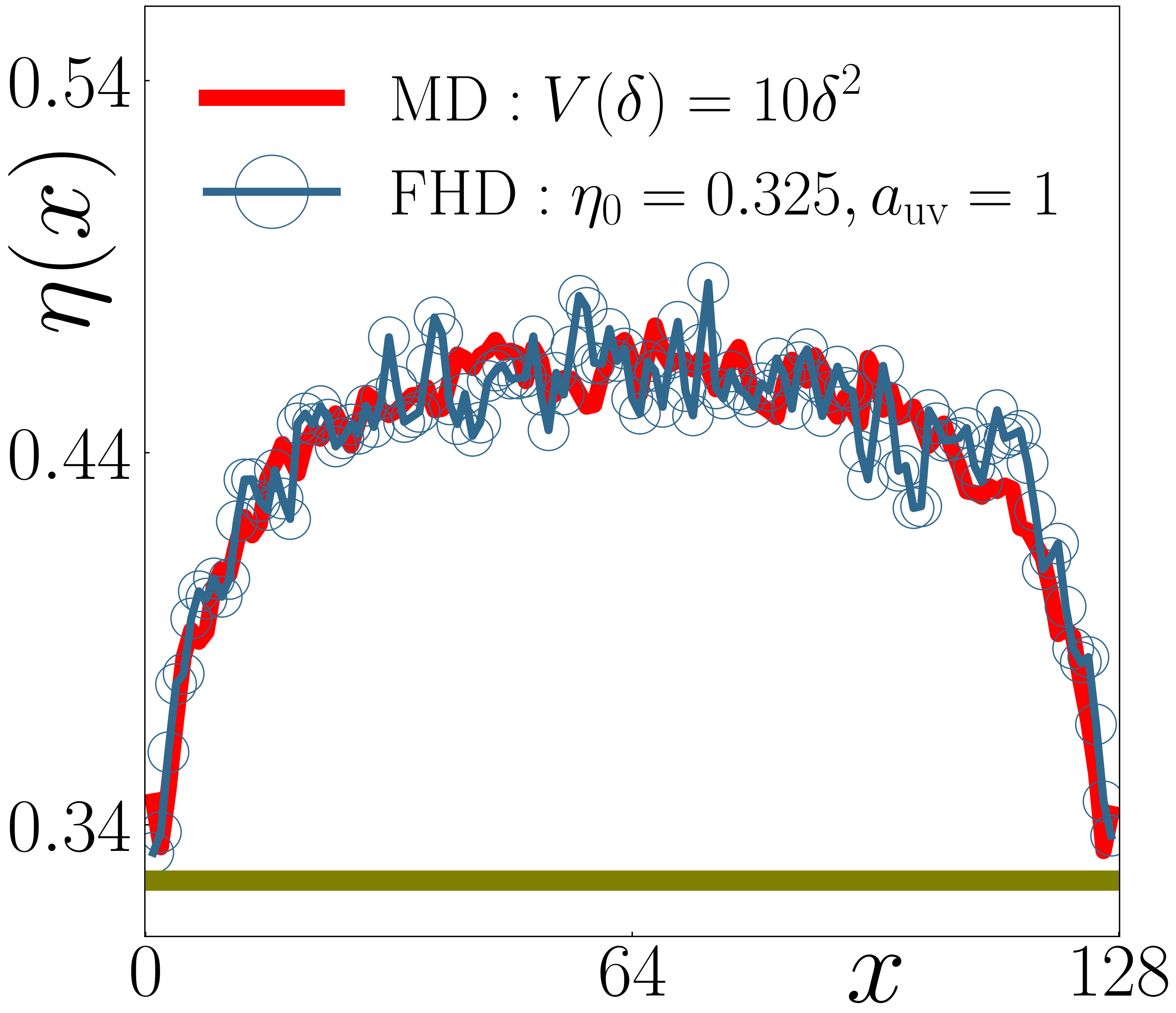
The fluctuating hydrodynamics with $\eta_0 = 0.325$ reproduces the long-time tail $C_{JJ}(t)$ of the MD simulation with high accuracy.

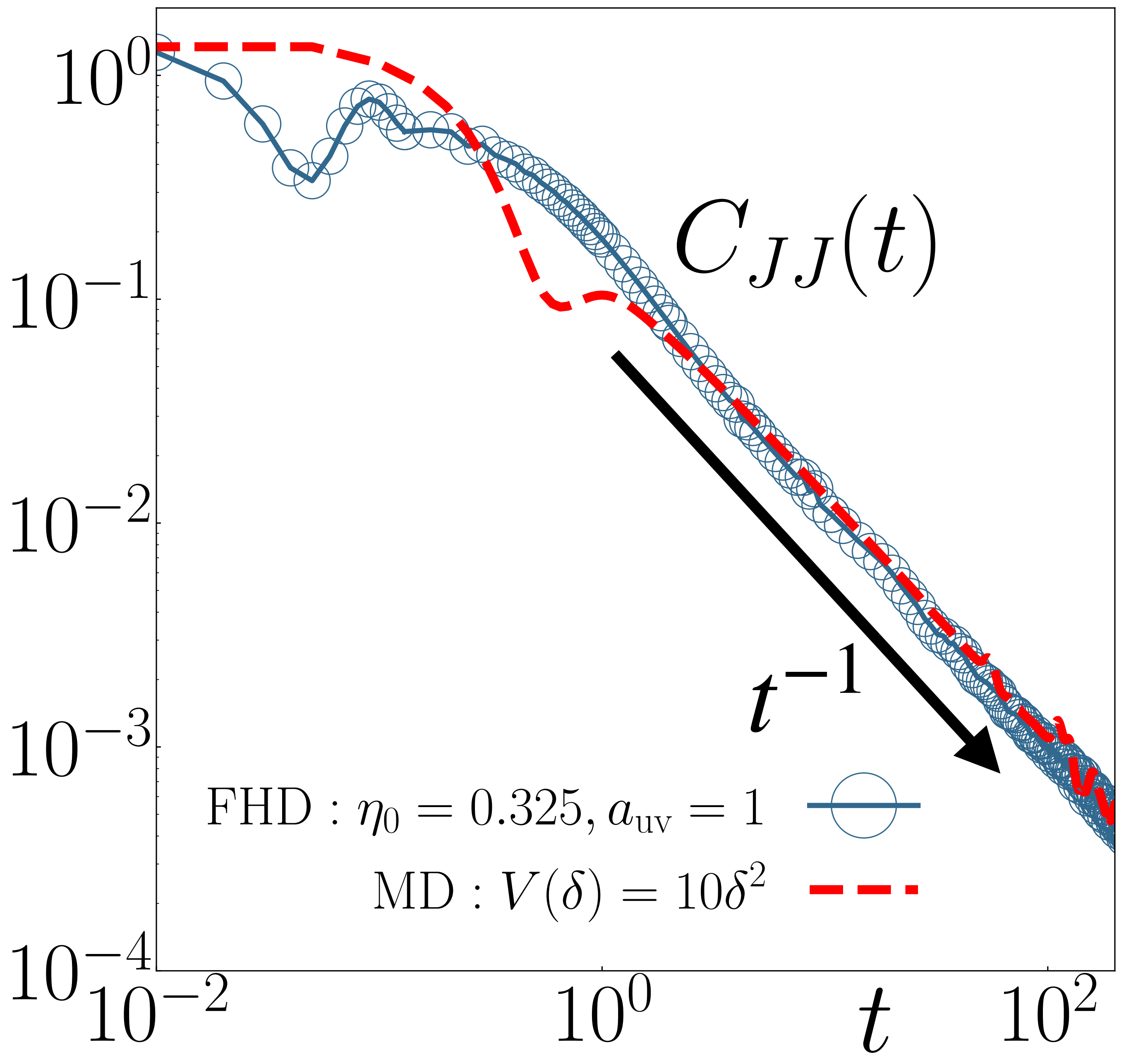
The agreement between the two models is observed even at the atomic time scale.

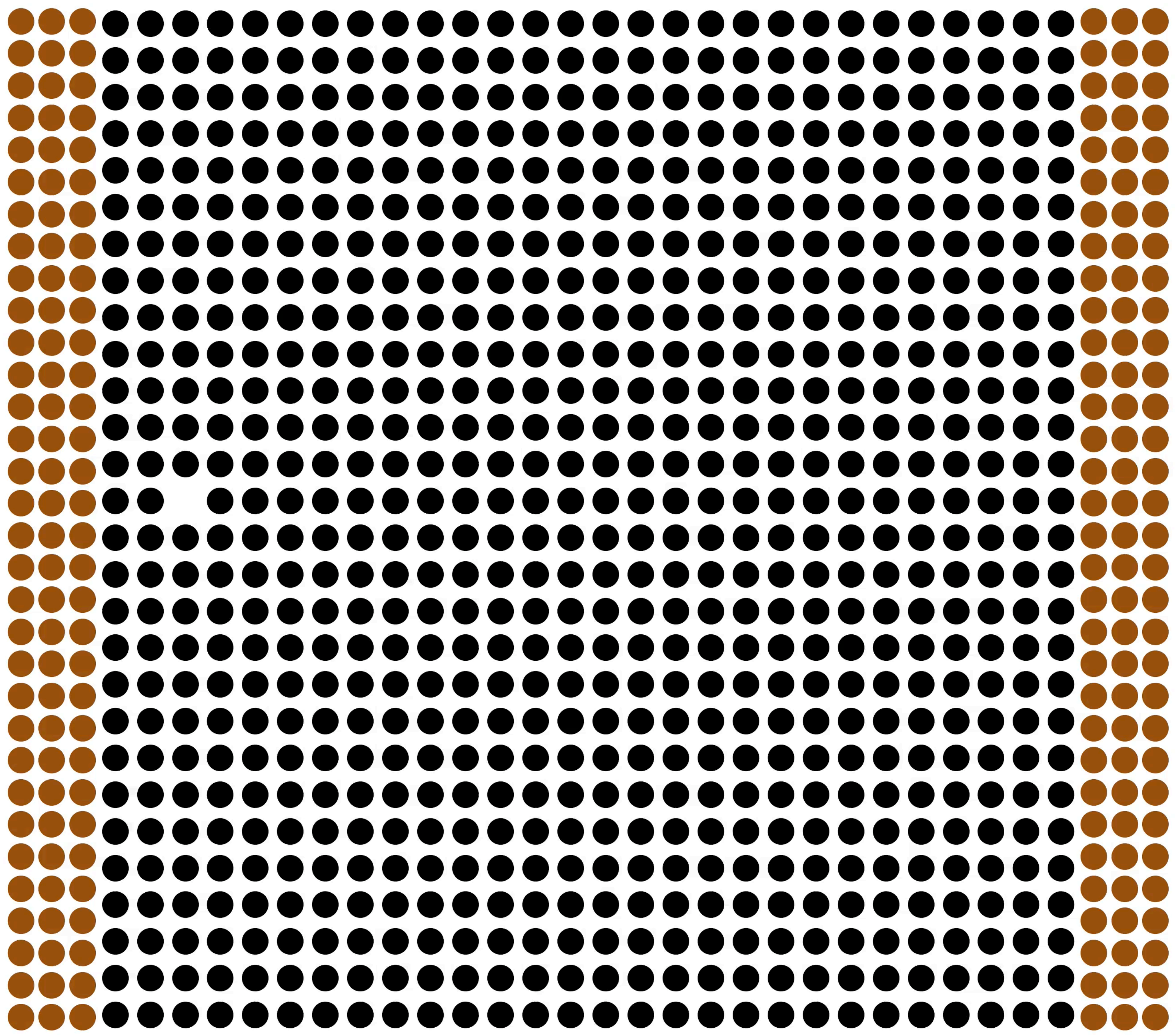


Description ability of atomic scale behavior

Fluctuating hydrodynamics can reproduce MD results down to the atomic scale.





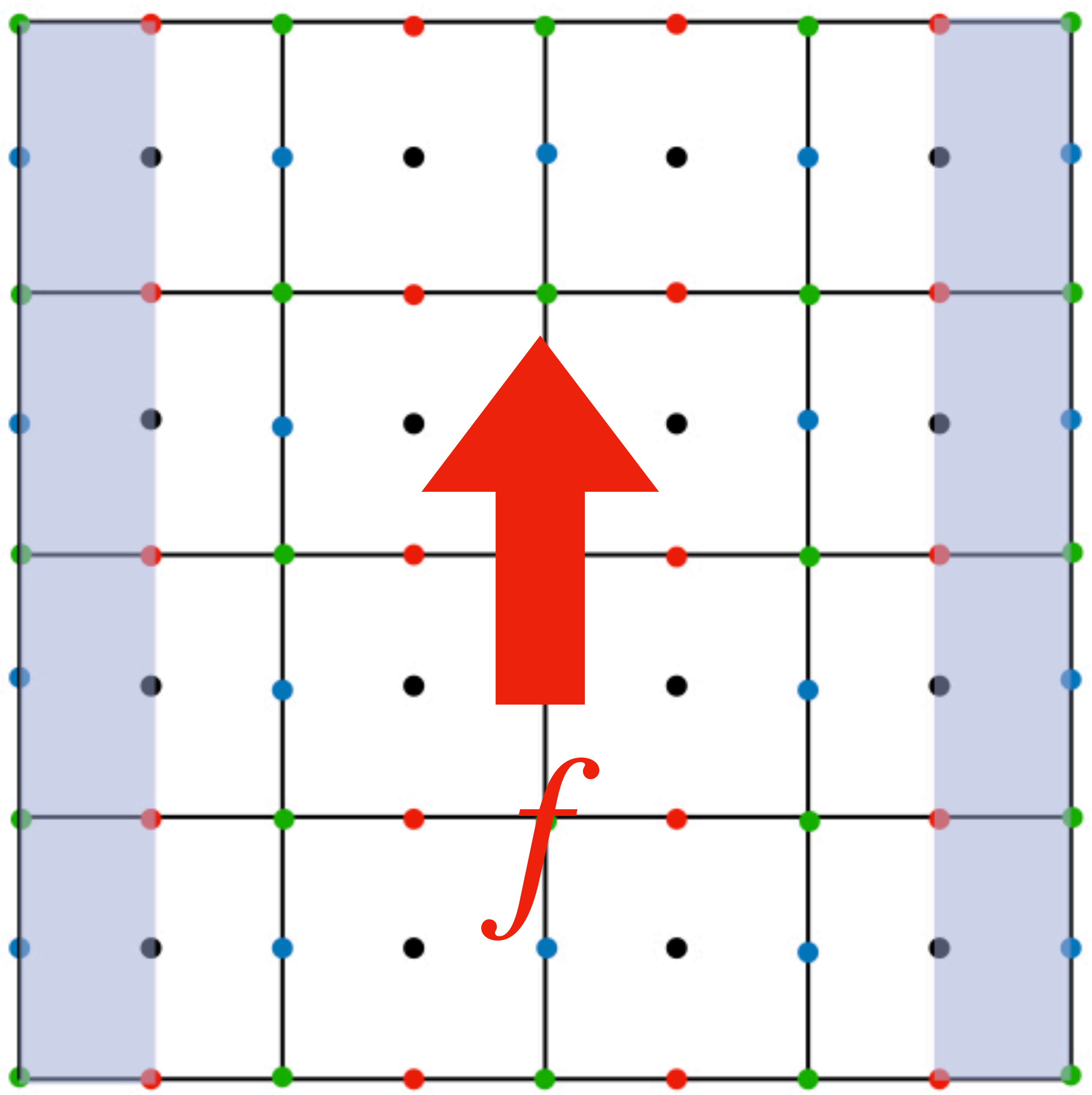


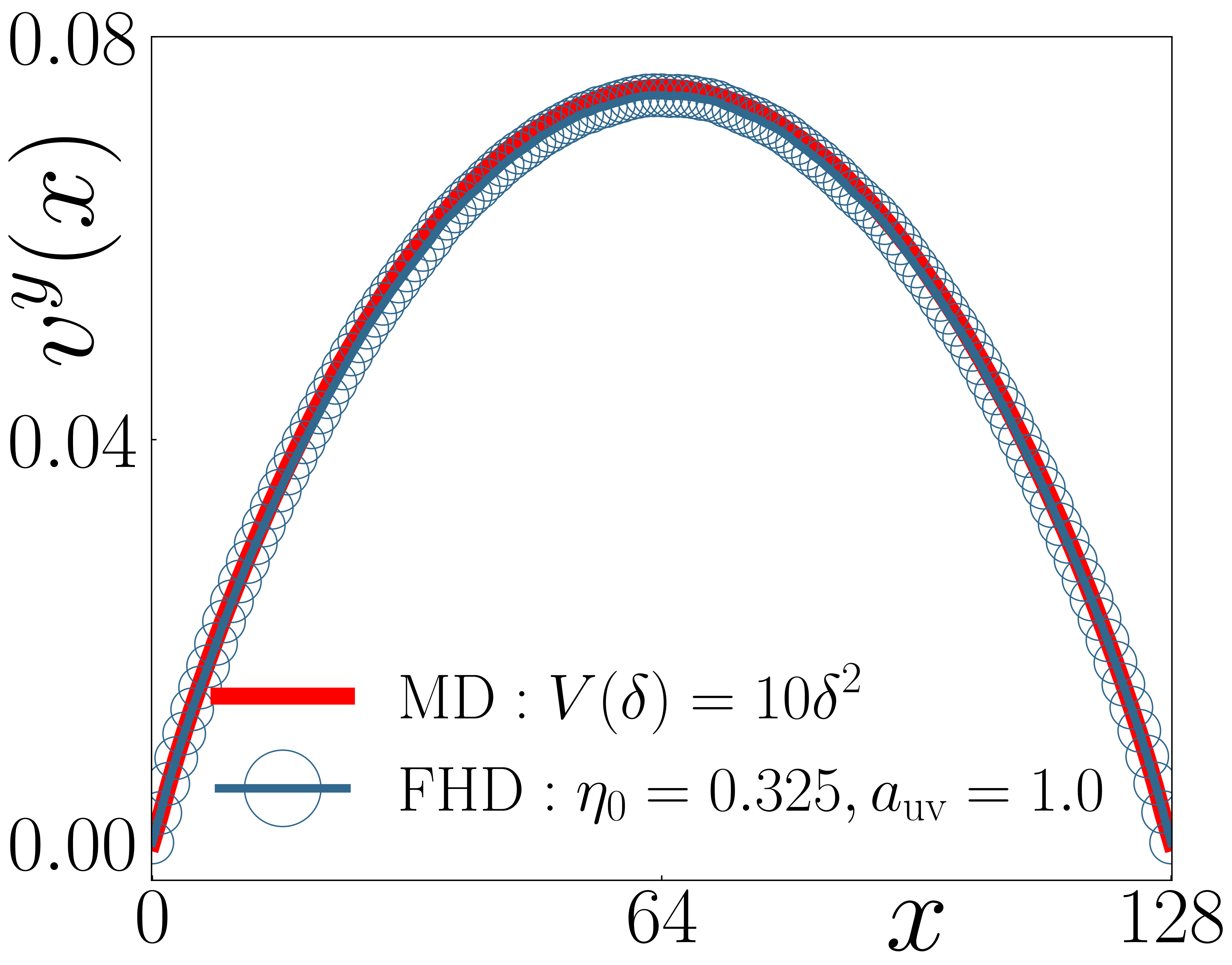
The mean free path of our system is roughly a few atomic diameters.

Our results suggest that fluid description is possible even at the atomic scale in such dense systems.

Consistency check : Poiseuille flow

- As another consistency test, we perform the simulation of the Poiseuille flow.



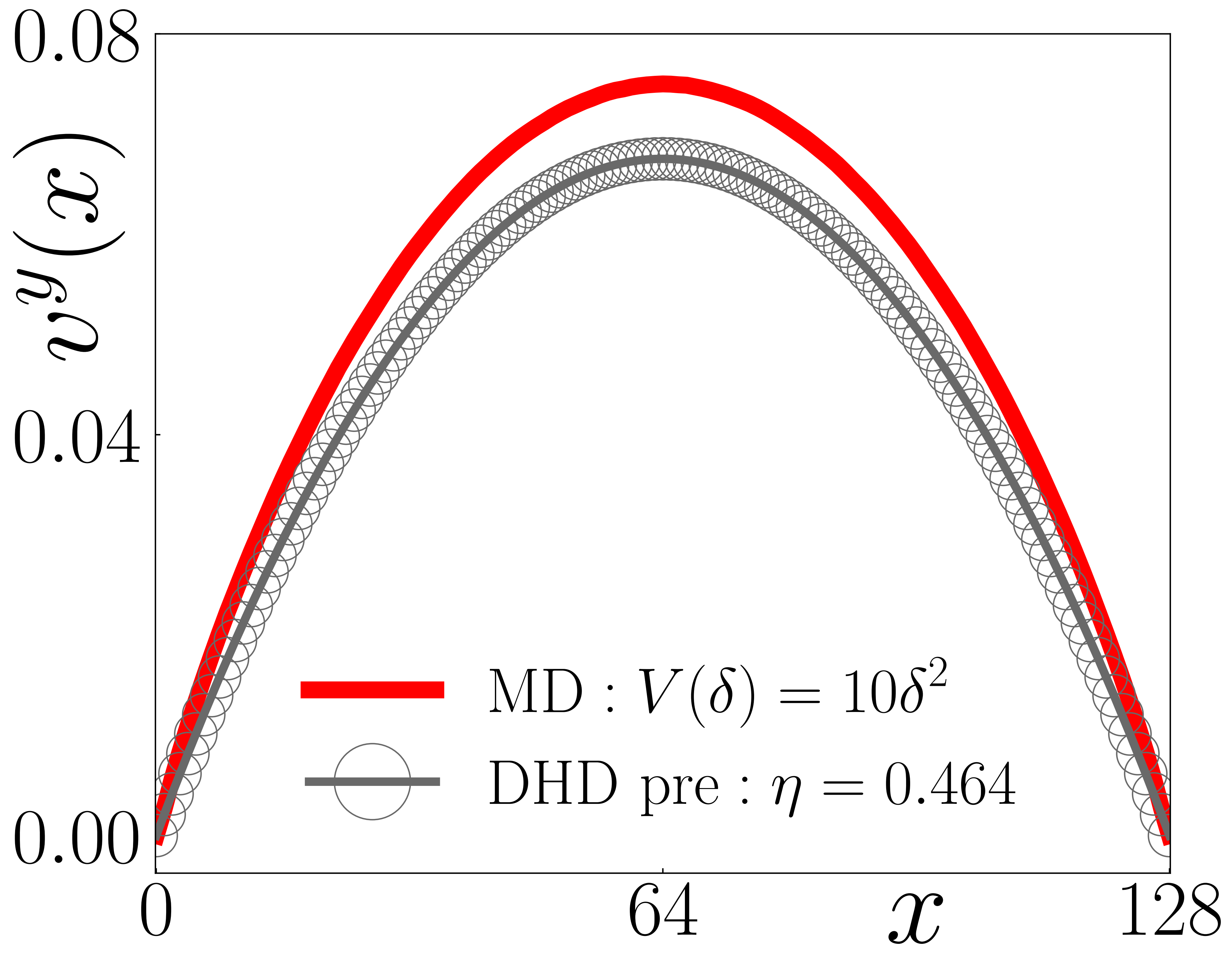


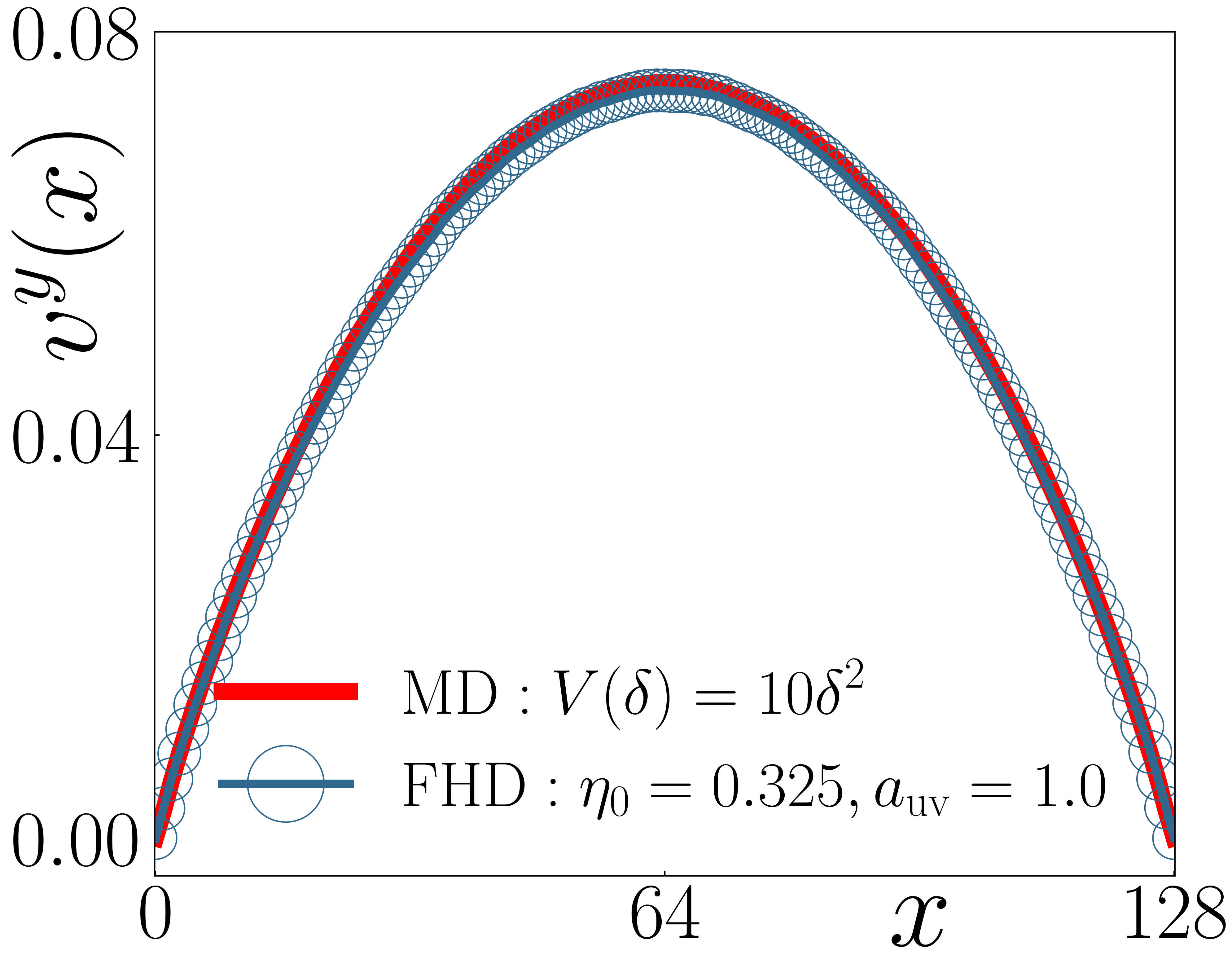
The Poiseuille flow is realized by adding a constant force to entire fluids and imposing periodic boundary condition in the flow direction.

Even for this setup, the predictions of fluctuating hydrodynamics and MD simulations are in good agreement.

Consistency check : Poiseuille flow

- As another consistency test, we perform the simulation of the Poiseuille flow.





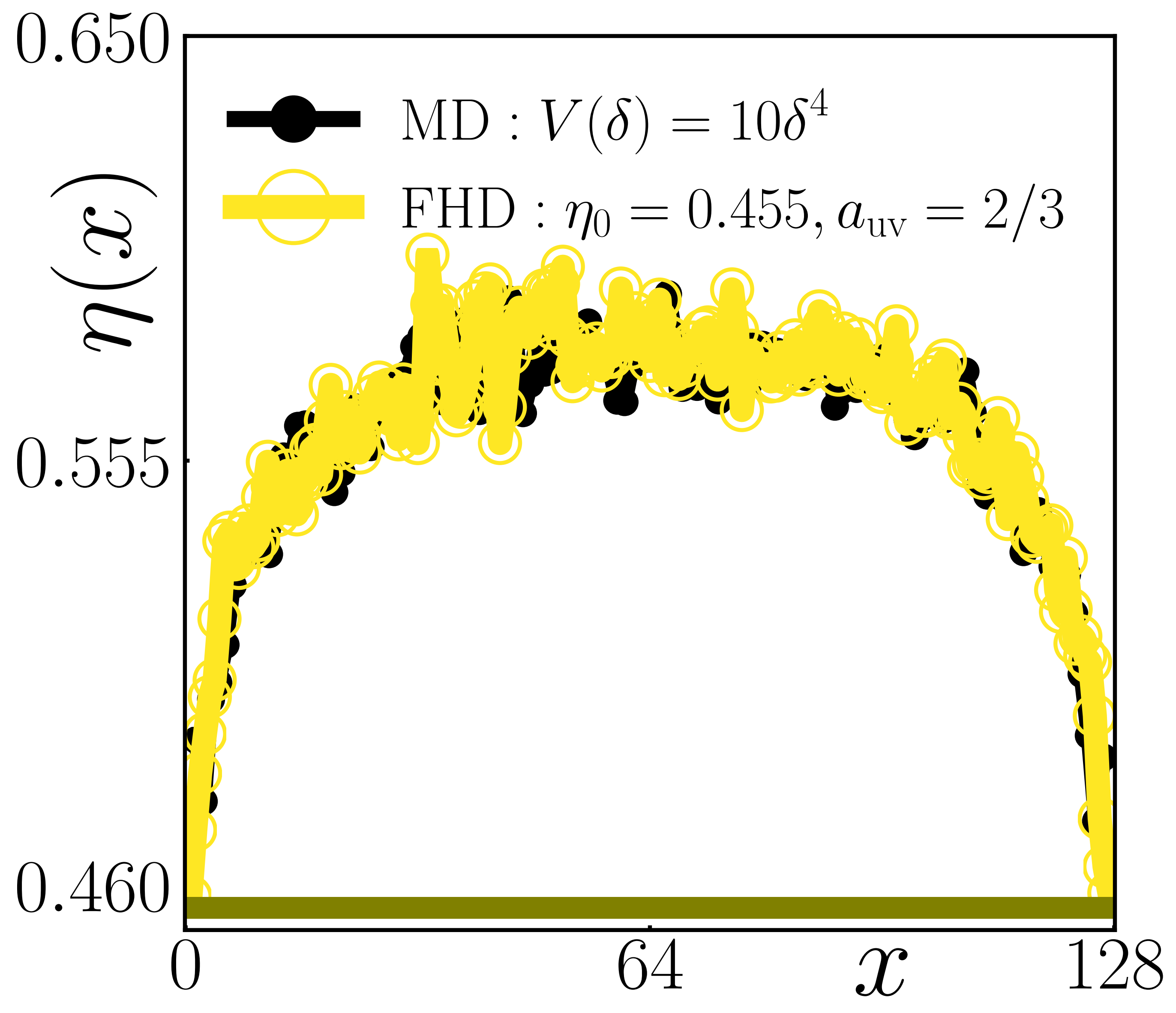
- The deterministic hydrodynamics with the viscosity observed in bulk region ($\eta = 0.464$) cannot reproduce the results of MD simulations.

Fluctuating hydrodynamics with bare viscosity is necessary to describe fluids near walls (at least in low-dimensional systems).

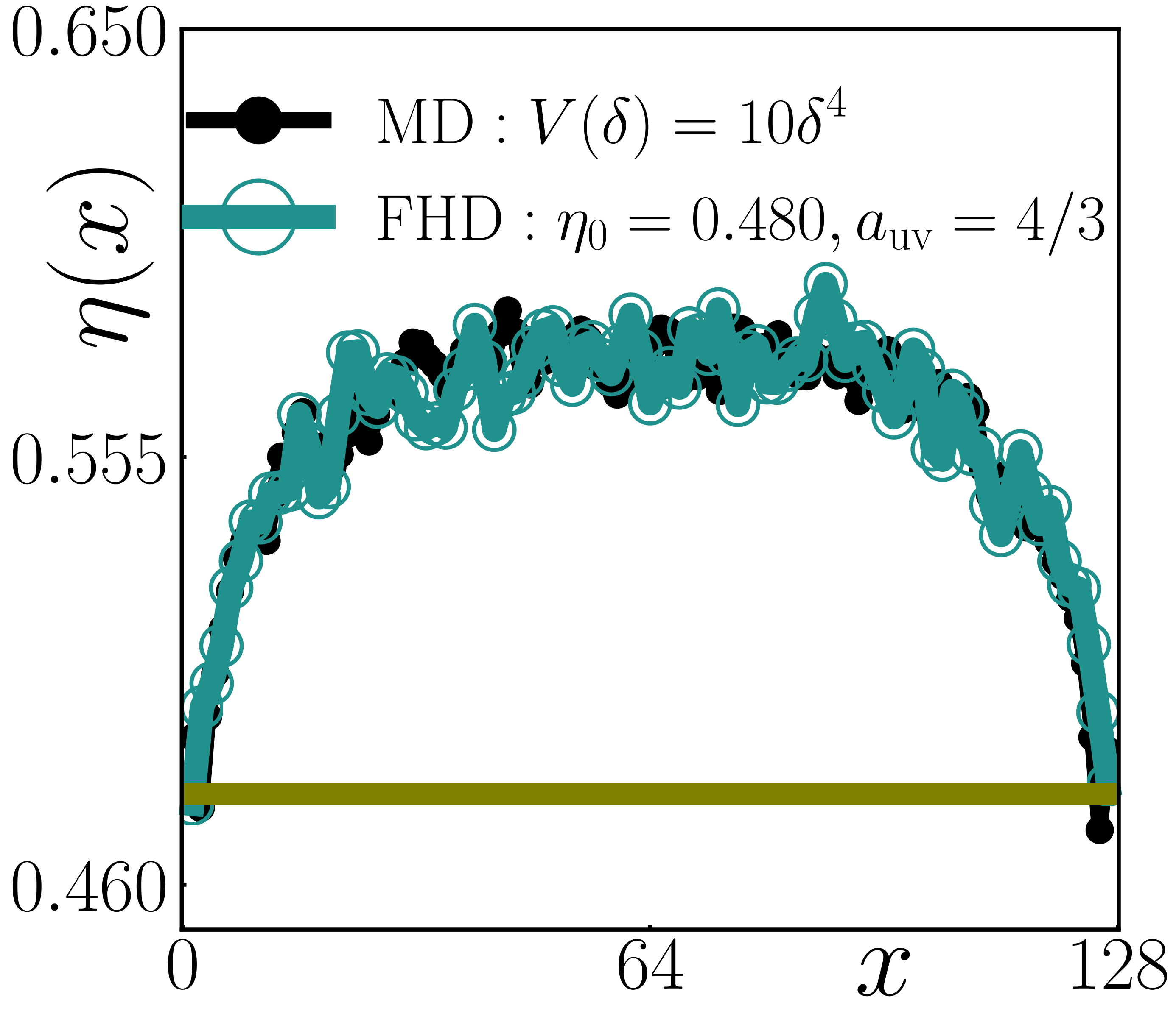
2.4 Effects of varying UV cutoff length

Best value of UV cutoff length

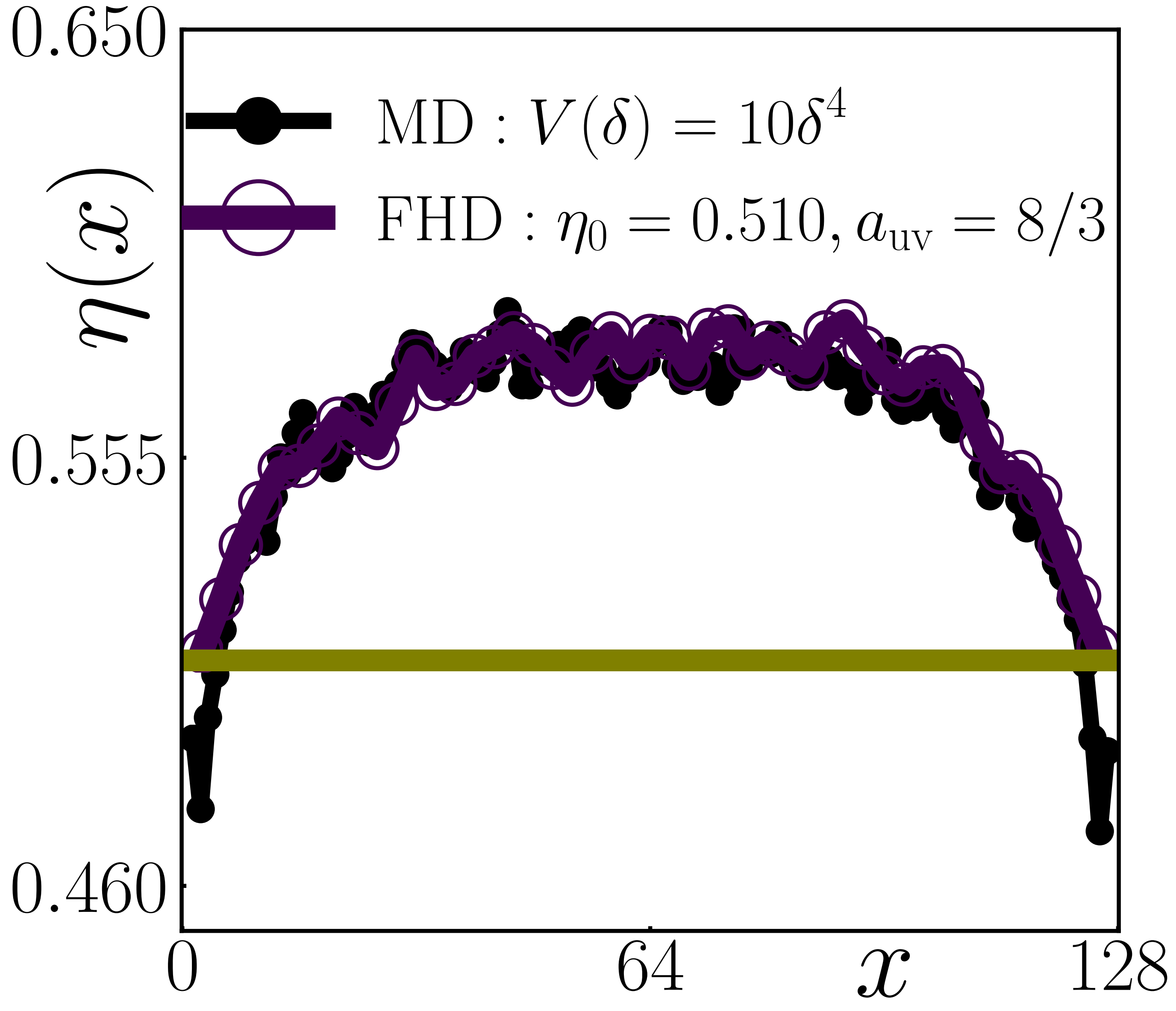
▶ We investigated the best-fit bare viscosity for different UV cutoff lengths.



$\eta_0 = 0.455$ for $a_{uv} = 2/3$



$\eta_0 = 0.480$ for $a_{uv} = 4/3$



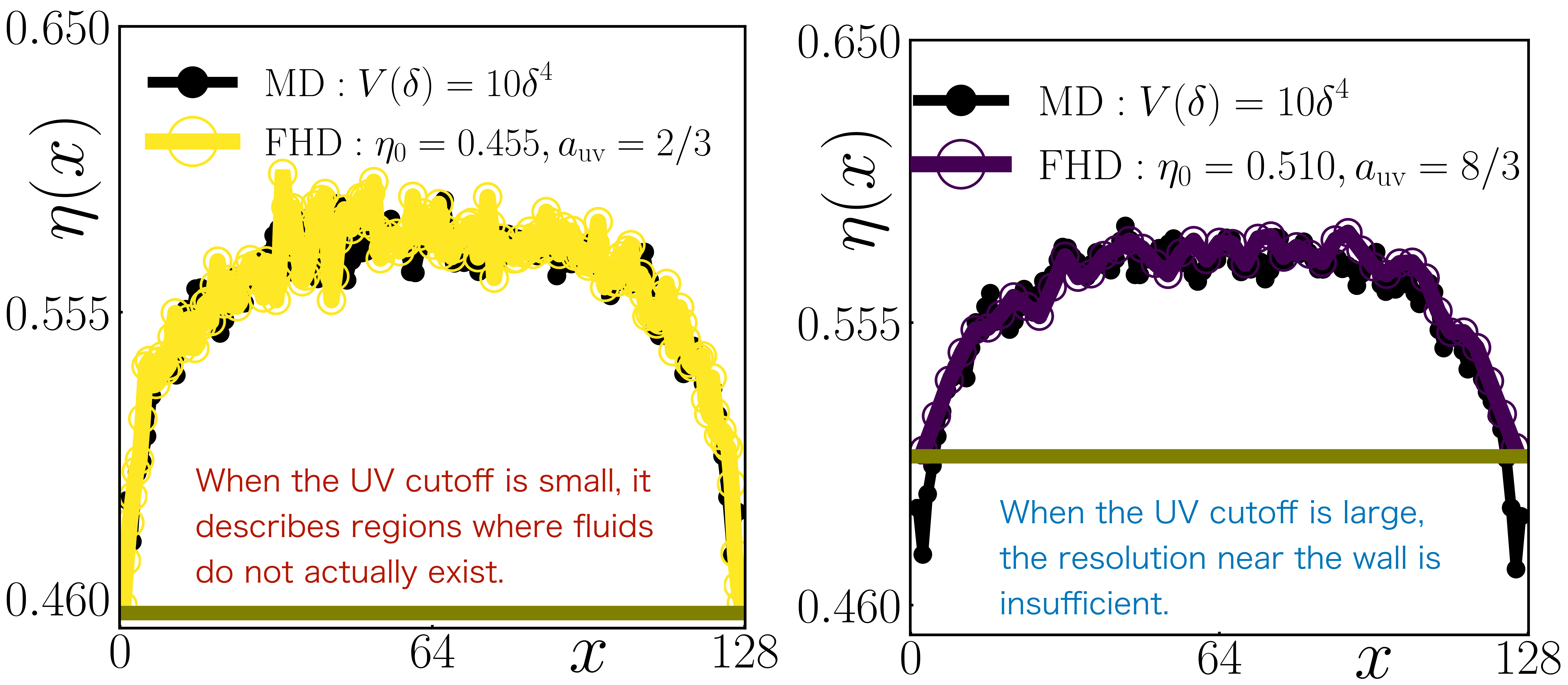
$\eta_0 = 0.510$ for $a_{uv} = 8/3$

Black: MD
(all the same data)

Colored: FHD
(with different (η_0, a_{uv}))

By carefully choosing η_0, the MD results can be well reproduced for any a_{uv}.

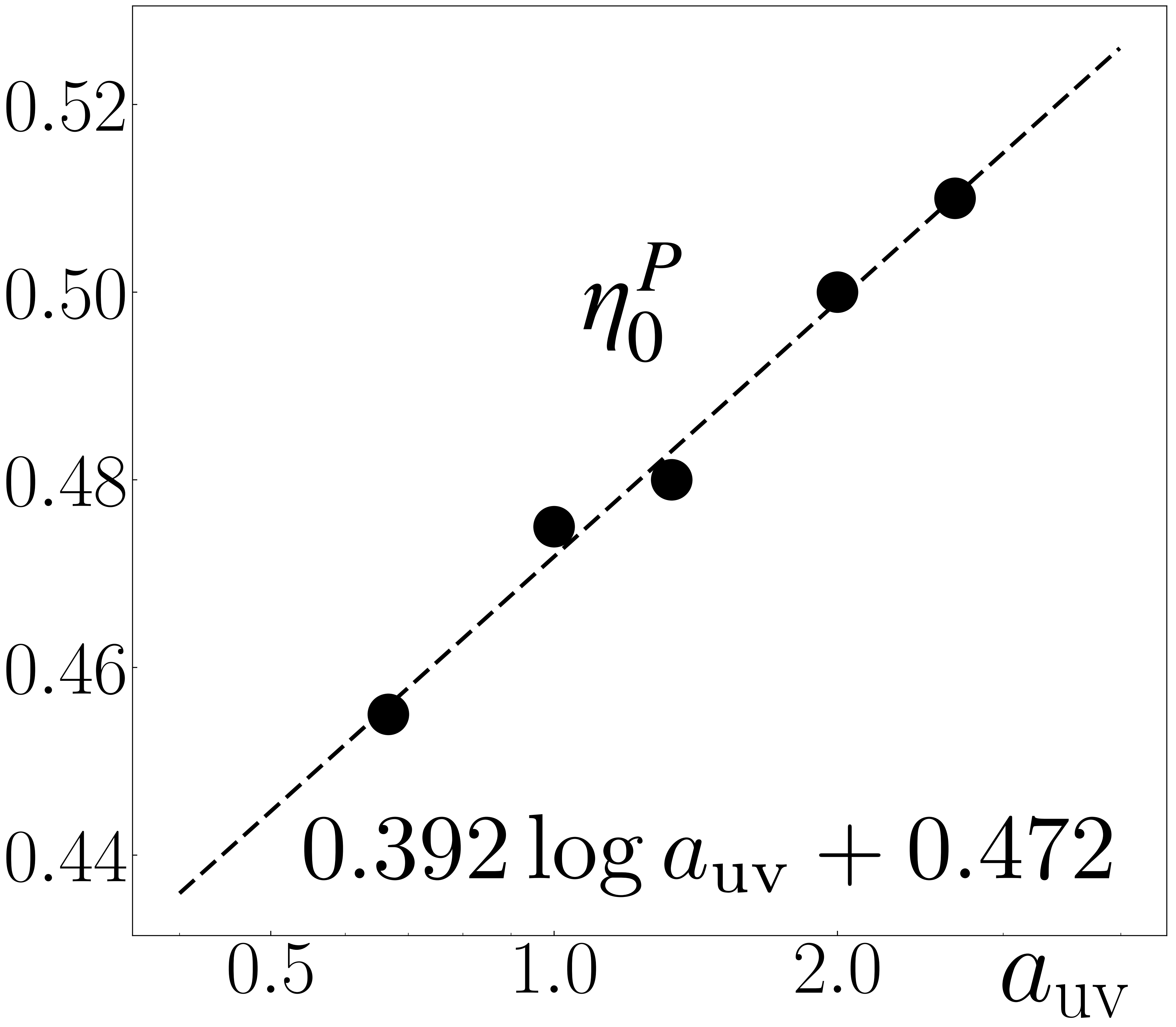
Best value of UV cutoff length



The best value of UV cutoff length is about atomic diameter!!

Physical value of bare viscosity

▶ We define the "bare viscosity" that reproduces the behavior in the atomic system as the physical bare viscosity η_0^P .



In practice, the physical bare viscosity η_0^P is determined to satisfy

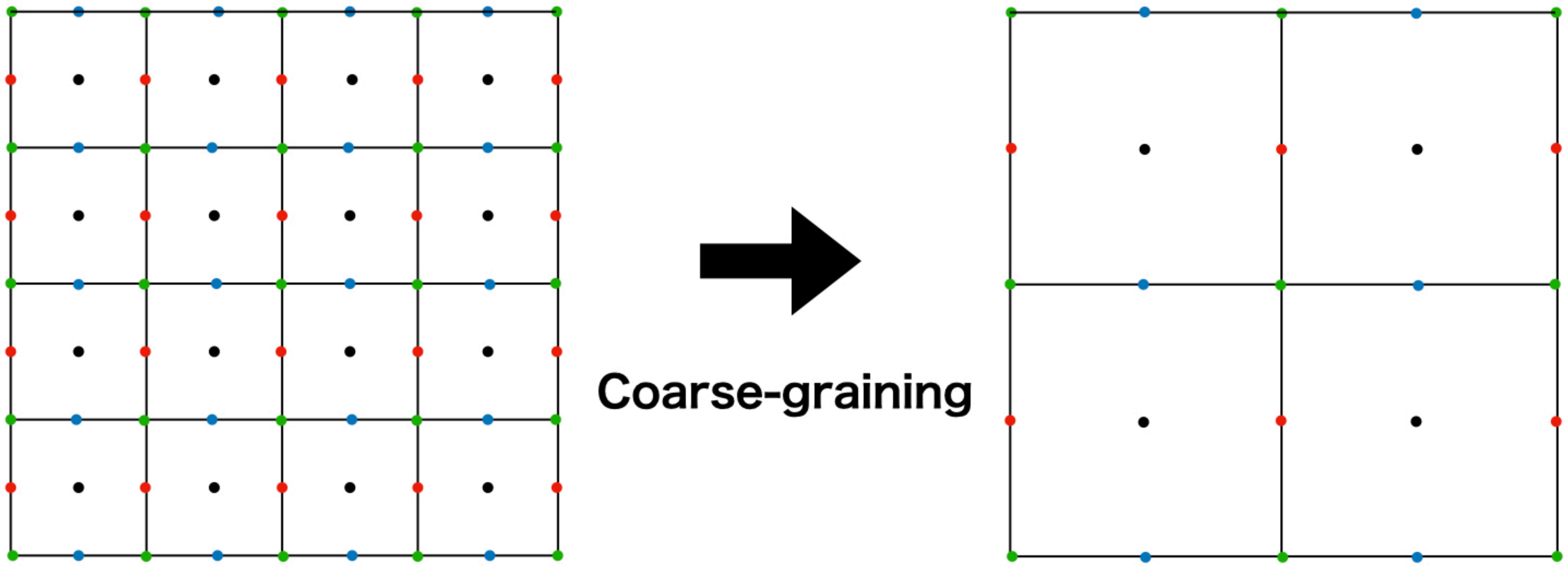
$$\eta_{\rm MD}(x) = \eta_{\rm FH}(x : \eta_0^P, a_{\rm uv})$$

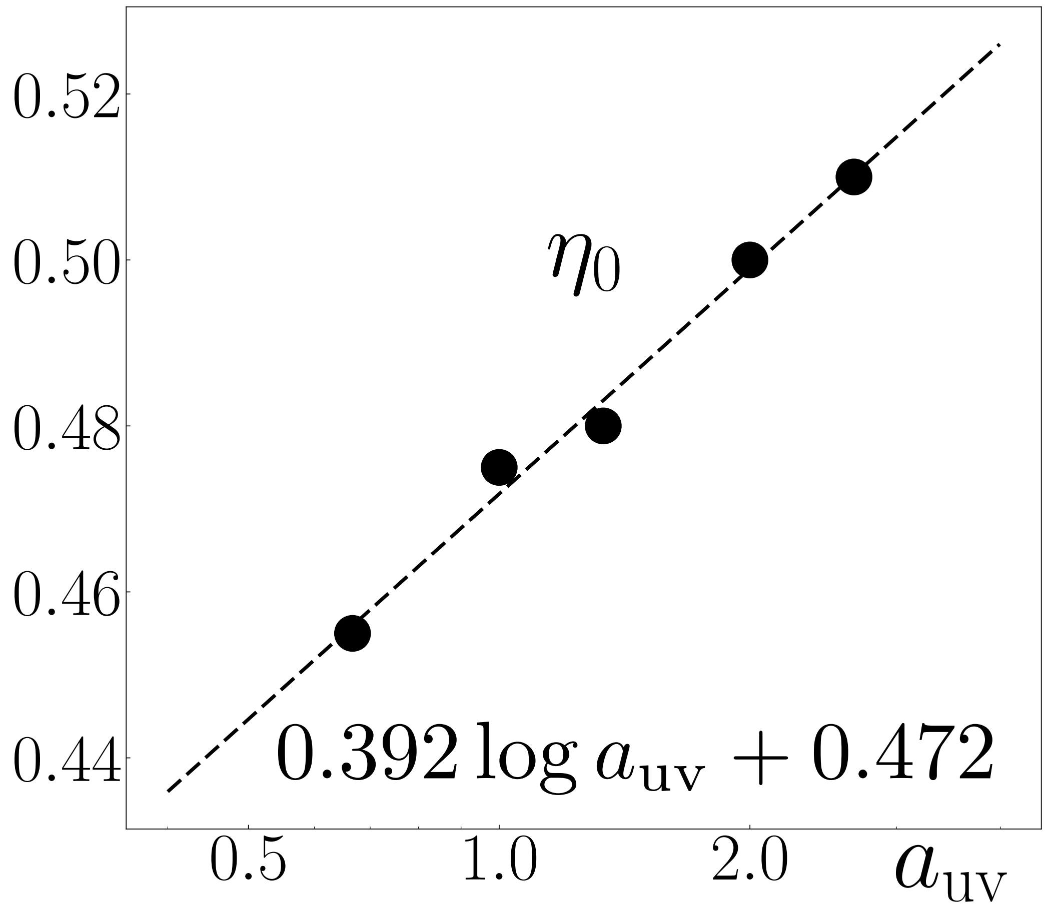
The physical bare viscosity η_0^P depends on the UV cutoff length.

If $(\eta_0, a_{\rm uv})$ lie on this relationship, any pair will reproduce the macroscopic phenomena well.

Renormalization group

- Changing the UV cutoff length is related to coarse-graining using the renormalization group.





Any parameters along the renormalization group flow can accurately reproduce the phenomena.

2.5 Simple estimation method for bare viscosity

Analytical expression of local viscosity

▶ We can calculate the theoretical expression for the noise-averaged Couette flow.

incompressible condition $\nabla \cdot \boldsymbol{v} = 0$

Fluctuating Navier-Stokes eq.

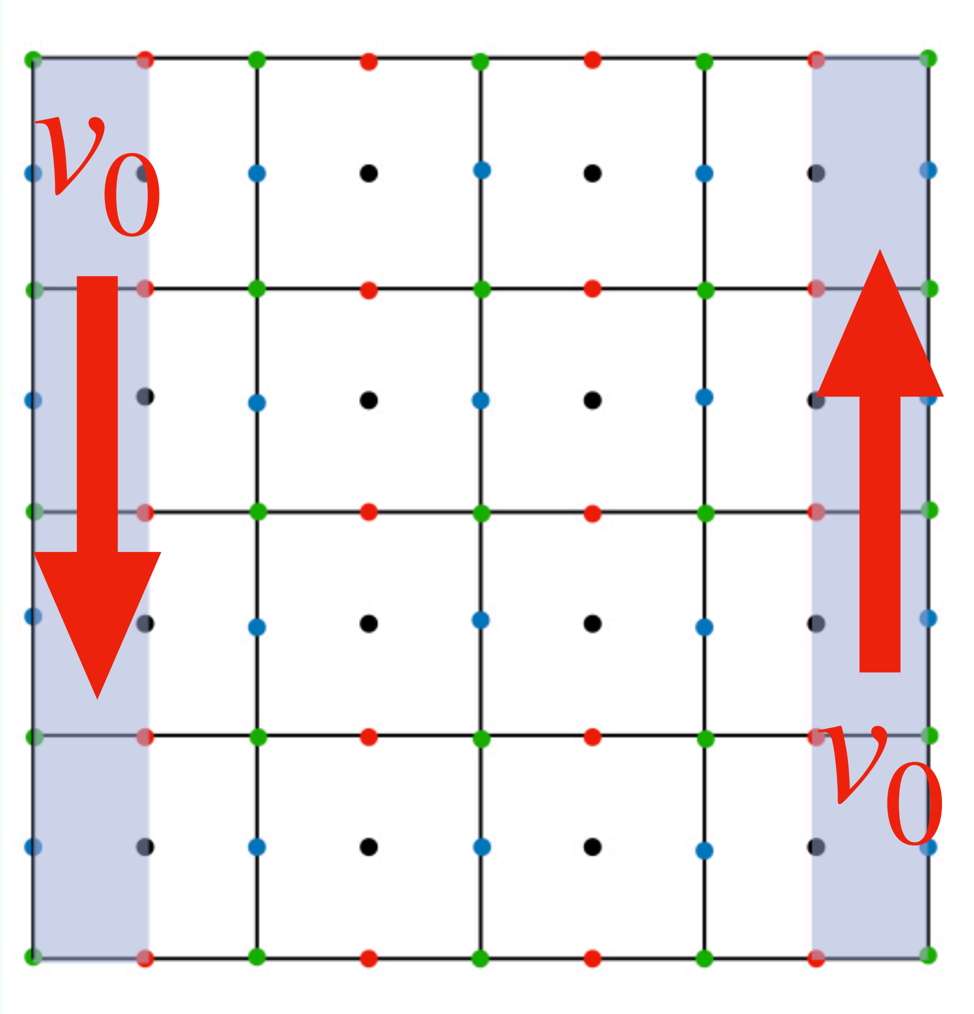
$$\rho\left[\frac{\partial \boldsymbol{v}}{\partial t} + \epsilon(\boldsymbol{v}\cdot\nabla)\boldsymbol{v}\right] = -\nabla p + \eta_0 \nabla^2 \boldsymbol{v} + \nabla \boldsymbol{\Pi}_R$$

$$\left\langle \Pi_R^{ab}(\boldsymbol{r}_1, t_1)\Pi_R^{cd}(\boldsymbol{r}_2, t_2)\right\rangle = 2k_B T \eta_0 \delta^2(\boldsymbol{r}_1 - \boldsymbol{r}_2)\delta(t_1 - t_2)\left[\left(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{cb}\right) - \delta_{ab}\delta_{cd}\right]$$

Boundary condition

$$v^x\Big|_{y=0} = 0 \qquad v^y\Big|_{y=0} = -v_0$$

$$v^x\Big|_{y=L} = 0 \qquad v^y\Big|_{y=L} = v_0$$



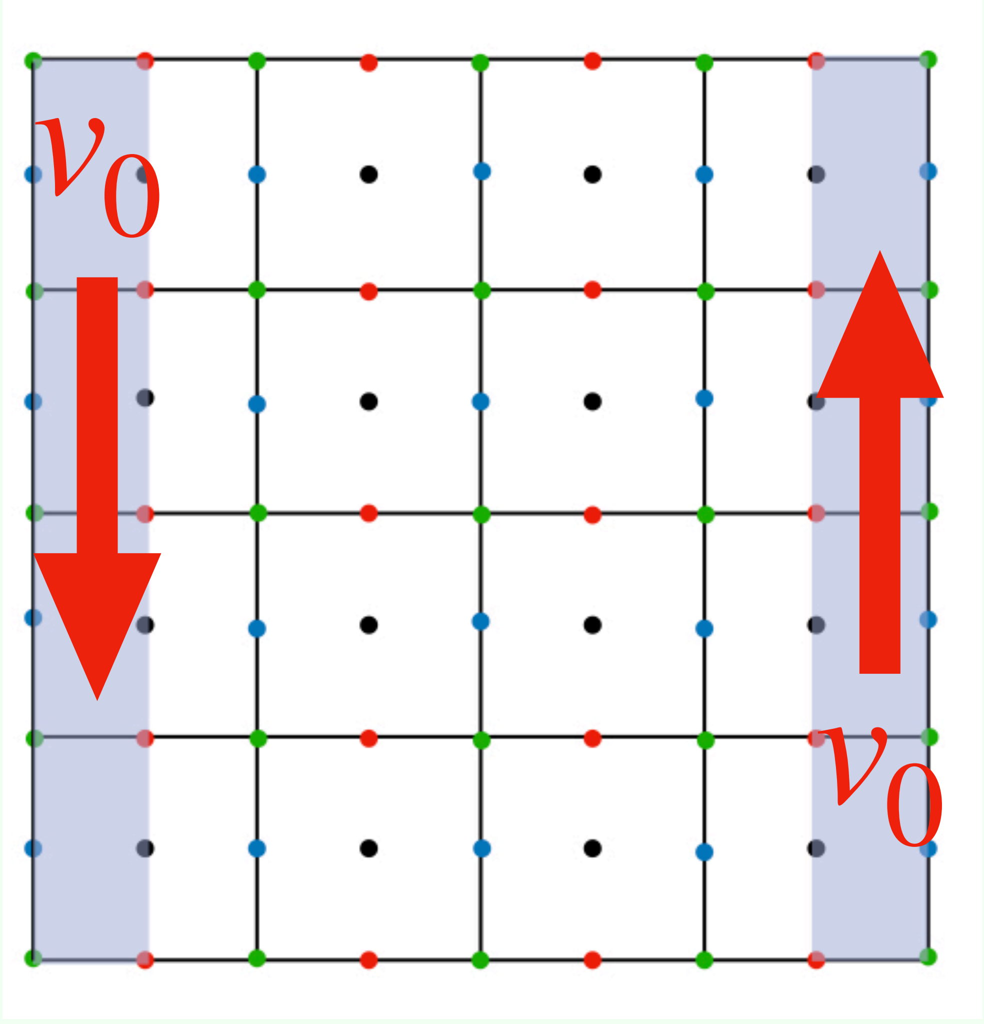
This calculation can be done using a perturbative expansion in ϵ.

(We adopted some approximations but the details are omitted here.)

Analytical expression of local viscosity

▶ We obtained the theoretical expression for the noise-averaged Couette flow.

$$\langle v^y(x)\rangle = \dot{\gamma}x - \epsilon^2\frac{\dot{\gamma}A}{L}\sum_{k_x}\frac{1}{k_x}\left(x - \frac{\sin(2k_x x)}{2k_x}\right) \quad k_x := \frac{\pi}{L}n$$



$\dot{\gamma}$: velocity gradient at $x = 0, L$

A : numerical factor depending on density, temperature⋯.

$$\eta(x) = \eta_0\left(1 + \epsilon^2\frac{A}{L}\sum_{k_x}\frac{1}{k_x}\sin^2(k_x x)\right)$$

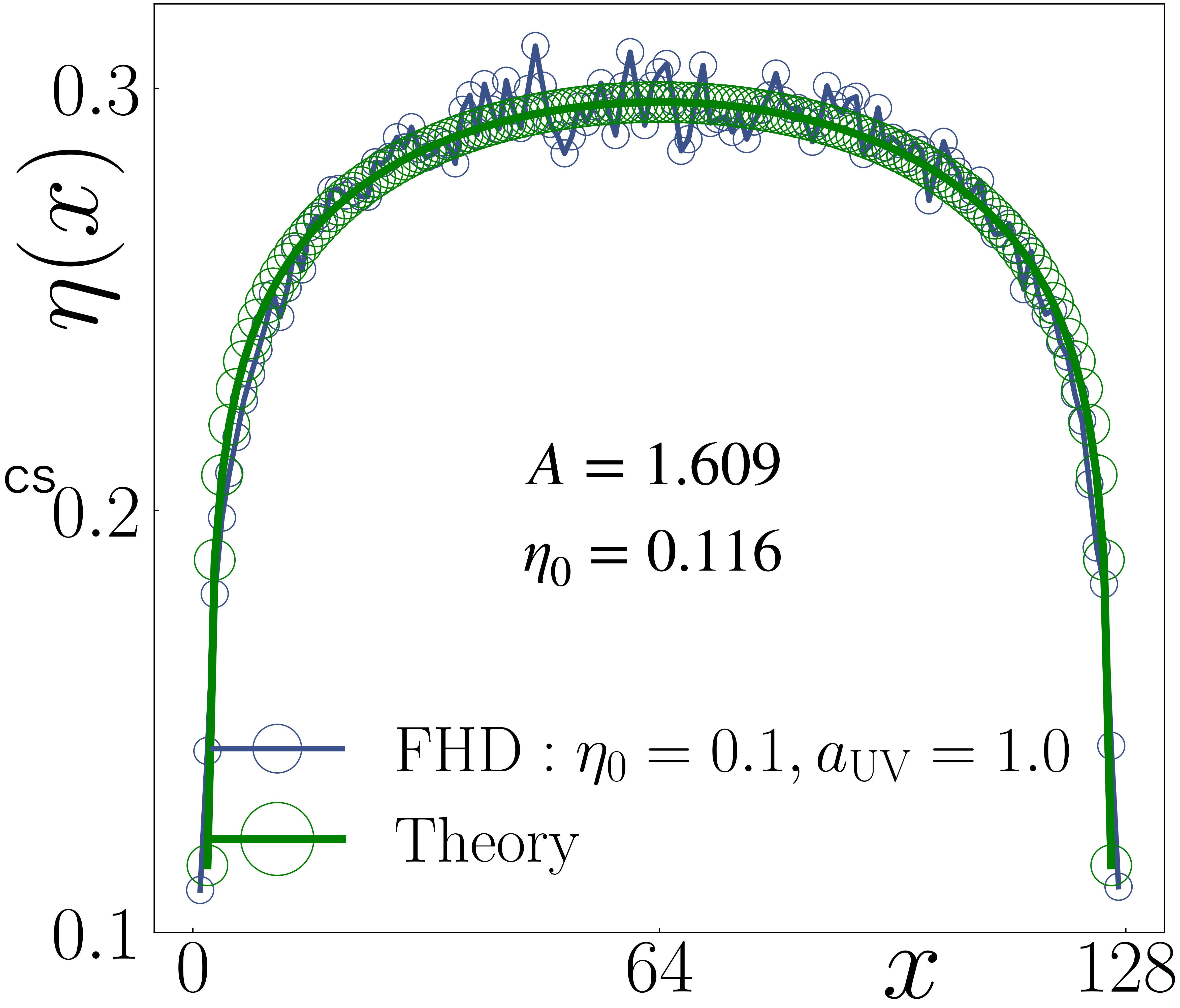
The information of a_{uv} is included within the range of the sum.

Validity of Analytical expression

▶ We confirmed that this analytical expression accurately reproduces the simulation results.

$$\eta(x) = \eta_0\left(1 + \frac{A}{L}\sum_{k_x}\frac{1}{k_x}\sin^2(k_x x)\right)$$

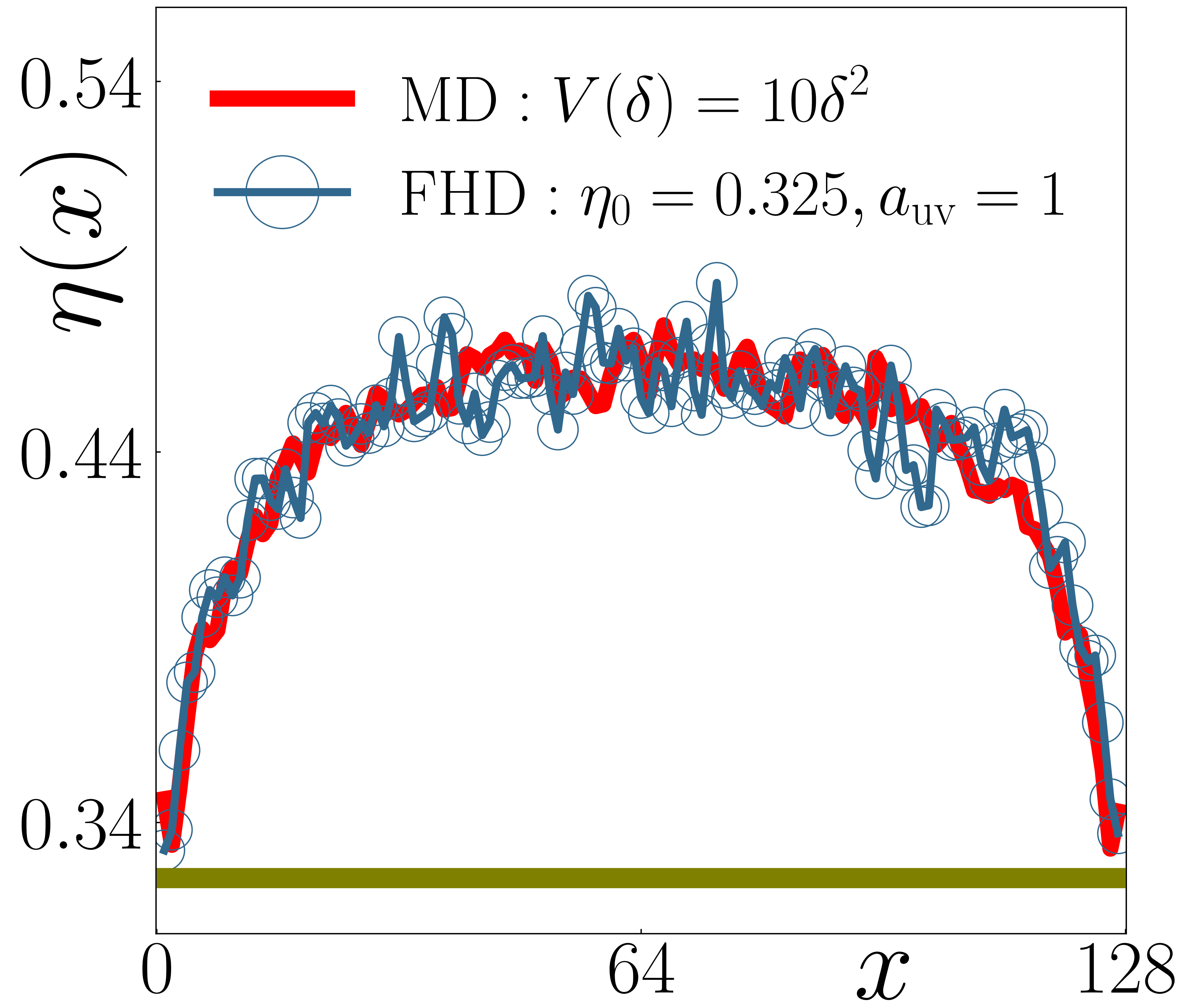
We fixed $a_{\mathrm{uv}} = 1.0$ and compared the numerical solution (full order) of the fluctuating hydrodynamics to this analytical expression for the local viscosity.



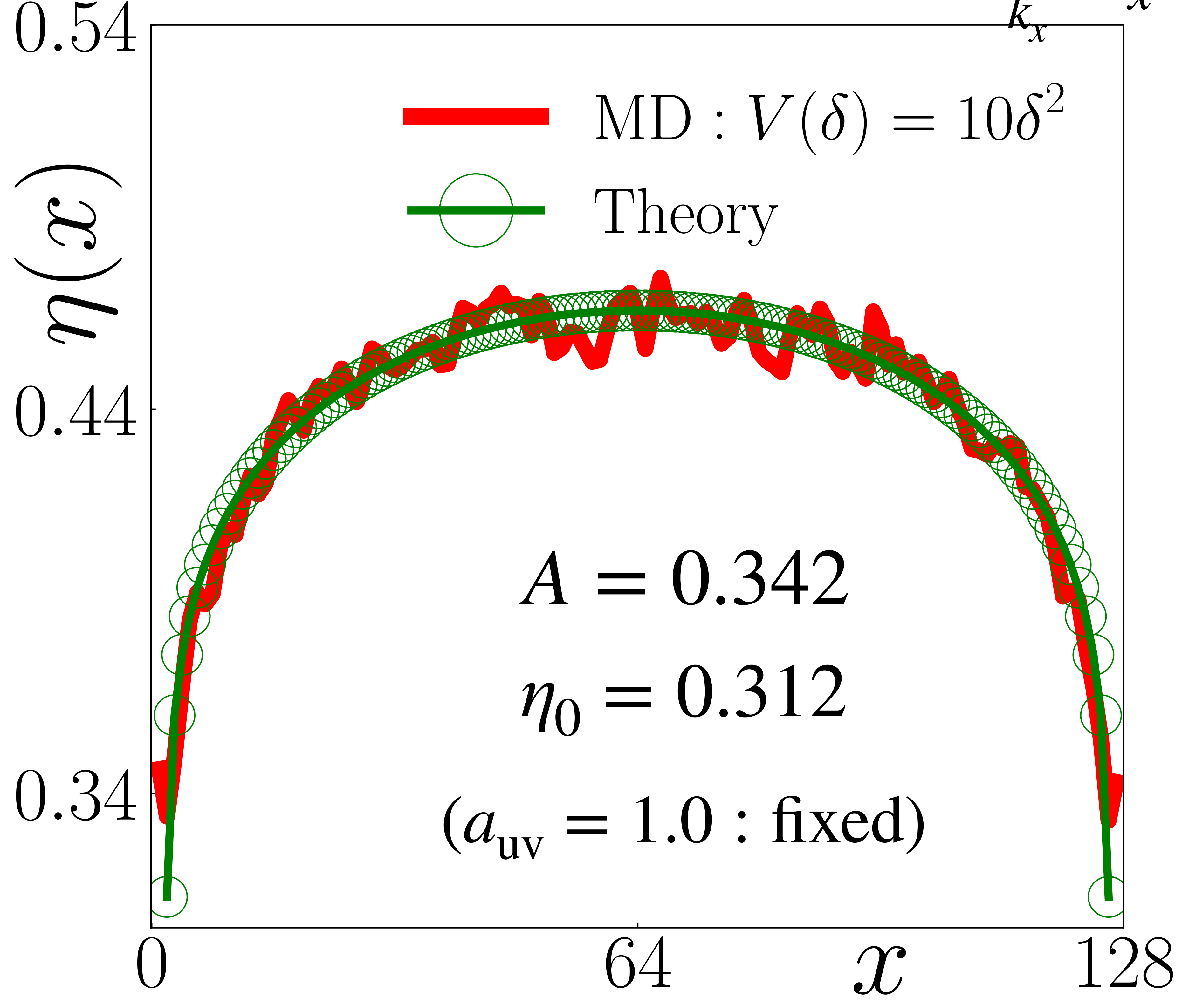
Simple estimation method for local viscosity

- We proceed to fit the MD results to estimate the bare viscosity.

$$\eta(x) = \eta_0\left(1 + \frac{A}{L}\sum_{k_x}\frac{1}{k_x}\sin^2(k_x x)\right)$$



(previously presented)
solving fluctuating hydrodynamics numerically



fitting the MD results using the analytical expression

We can easily estimate the bare viscosity using this analytical expression.

Summary

Main Message

▶Bare viscosity is directly observed near solid walls.

We believe that this result is robust even for real solid walls.

Discussion

▶ Fluctuating hydrodynamics and the renormalization group are strongly connected.
Can we understand this connection from a microscopic perspective and is there a concept of the best choice for the UV cutoff length from a microscopic perspective?
(for example, mean free path?)

▶ What happens when the mean free path is large?
While this study focused on dense liquids, how might the breakdown of fluctuating hydrodynamics appear in dilute gases?