Nonrelativistic conformality, contact correlation, Bose-Fermi duality in 1D

Yusuke Nishida (Tokyo Tech)

YITP-RIKEN iTHEMS International Molecule-type Workshop 2024

Advances in Fluctuating Hydrodynamics: Bridging the Micro and Macro Scales

June 17 - 28 (2024) @ YITP

Plan of this talk

1. Static bulk viscosity in 3D
   - Nonrelativistic conformal invariance
   - Its implications for hydrodynamics
2. Dynamic bulk viscosity in 1D
   - Bose-Fermi duality and integrability
   - Implications of integrability breaking …

K. Fujii & Y. Nishida,
PRA 98, 063634 (2018); 102, 023310 (2020); 103, 053320 (2021)

T. Tanaka & Y. Nishida,
PRE 106, 064104 (2022); PRL 129, 200402 (2022)

Y. Nishida, Ann. Phys. 410 (2019) 167949; PRA 106, 063317 (2022)

1. Nonrelativistic CFT

Y. Nishida & D. T. Son, PRD (2007); arXiv:1004.3597

Nonrelativistic CFT

Maximal spacetime symmetries of

U. Niederer, HPA (1972)
C. R. Hagen, PRD (1972)

$$S_{\text{free}} = \int dt\, d^d\vec{x}\; \psi^\dagger \left(i\partial_t + \frac{\vec{\nabla}^2}{2m} \right) \psi$$

- Translations in time and space
- Spatial rotations
- Galilean boosts
- Phase rotation
- Scale transformation

$$\vec{x} \to e^{-s}\vec{x}, \quad t \to e^{-2s}t, \quad \psi \to e^{(d/2)s}\psi$$

- Conformal transformation

$$\vec{x} \to \frac{\vec{x}}{1-ct}, \quad t \to \frac{t}{1-ct},$$

$$\psi \to (1-ct)^{d/2} \exp\left(i\frac{c}{1-ct}\frac{m}{2}\vec{x}^2 \right) \psi$$

Nonrelativistic CFT

- Scale transformation

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Nonrelativistic CFT

- **Scale transformation** (infinitesimal)

U. Niederer, HPA (1972)
C. R. Hagen, PRD (1972)

$$\delta_s \psi = s\left(\frac{d}{2} + \vec{x}\cdot\vec{\nabla} + 2t\partial_t\right)\psi$$

$$= -is[D - 2tH, \psi]$$

$$D \equiv \int d^d\vec{x}\; \vec{x}\cdot\psi^\dagger(-i\overleftrightarrow{\nabla})\psi$$

- **Conformal transformation**

$$\vec{x} \to \frac{\vec{x}}{1-ct}, \quad t \to \frac{t}{1-ct},$$

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$$D \equiv \int d^d\vec{x}\ \vec{x}\cdot\psi^\dagger(-i\overleftrightarrow{\nabla})\psi$$

- **Conformal transformation** (infinitesimal)

$$\delta_c\psi = c\left(i\frac{m}{2}\vec{x}^2 - t\frac{d}{2} - t\vec{x}\cdot\vec{\nabla} - t^2\partial_t\right)\psi$$
$$= -ic[C - tD + t^2H, \psi]$$

$$C \equiv \frac{m}{2}\int d^d\vec{x}\ \vec{x}^2\psi^\dagger\psi$$

~ mean square radius
~ harmonic potential

Nonrelativistic CFT

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U. Niederer, HPA (1972)
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$$\delta_s\psi = s\left(\frac{d}{2} + \vec{x}\cdot\vec{\nabla} + 2t\partial_t\right)\psi$$

$$= -is\,[D - 2tH, \psi] \equiv -is\,[D(t), \psi]$$

$$D \equiv \int d^d\vec{x}\; \vec{x}\cdot\psi^\dagger(-i\overleftrightarrow{\nabla})\psi$$

$\Rightarrow$

$$[D, H] = 2iH$$

$$\dot{D}(t) = 0$$

- **Conformal transformation** (infinitesimal)

$$\delta_c\psi = c\left(i\frac{m}{2}\vec{x}^2 - t\frac{d}{2} - t\vec{x}\cdot\vec{\nabla} - t^2\partial_t\right)\psi$$

$$= -ic\,[C - tD + t^2H, \psi] \equiv -ic\,[C(t), \psi]$$

$$C \equiv \frac{m}{2}\int d^d\vec{x}\; \vec{x}^2\psi^\dagger\psi$$

$\Rightarrow$

$$[C, H] = iD$$

$$\dot{C}(t) = 0$$

Nonrelativistic CFT

Generators (D, C, H) obey SO(2,1) Lie algebra

$$[D,H]=2iH,\quad [C,H]=iD,\quad [D,C]=-2iC$$

scale invariance $\quad \dot{n}=-\vec{\nabla}\cdot\vec{j}\quad$ always true

$$H=H_0+V(r)\quad \Rightarrow \quad H'=H_0+e^{-2s}V(e^{-s}r)$$

$$e^{-isD}He^{isD}=e^{2s}H'$$

r_0 & a $\qquad$ $e^s r_0$ & $e^s a$

H=H' for zero-range (r_0=0)
& infinite scattering length ($a=\infty$) interaction
(relevant to cold atom experiments)

2. Hydrodynamics

Y. Nishida & D. T. Son, PRD (2007); arXiv:1004.3597

K. Fujii & Y. Nishida, Phys. Rev. A 98, 063634 (2018)

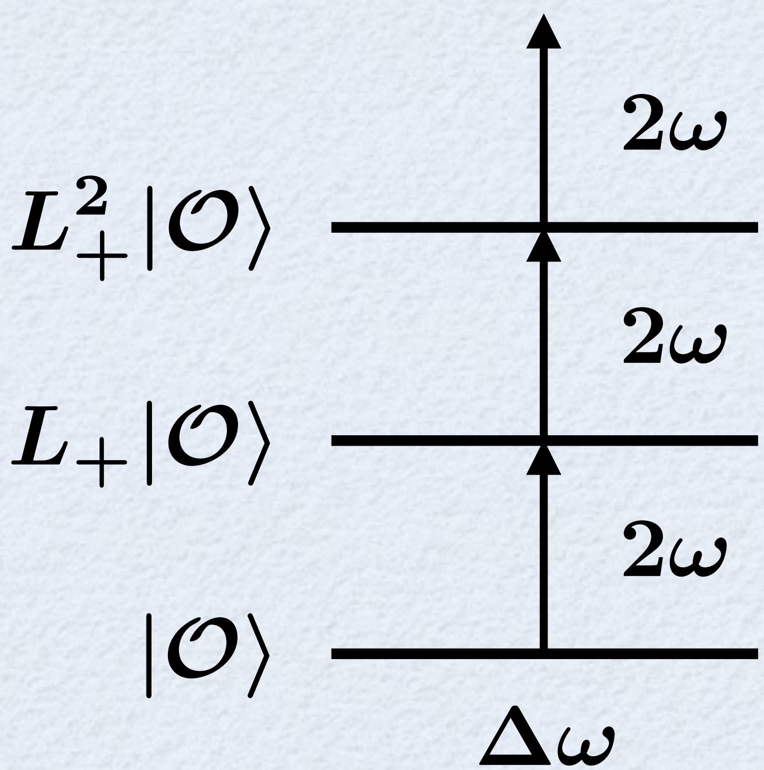
Operator-State correspondence

$$[D, H] = 2iH, \quad [C, H] = iD, \quad [D, C] = -2iC$$

$$H_\omega \equiv H + \omega^2 C, \quad L_\pm \equiv H - \omega^2 C \pm i\omega D$$

$$[H_\omega, L_\pm] = \pm 2\omega L_\pm, \qquad [L_+, L_-] = -4\omega H_\omega$$

raising & lowering operators



$$L_-|\mathcal{O}\rangle = 0$$

$$H_\omega L_+^n|\mathcal{O}\rangle = (\Delta\omega + 2n\omega)\, L_+^n|\mathcal{O}\rangle$$

Valid for
any scale invariant systems
confined by harmonic potential

Y. Nishida & D. T. Son, PRD (2007); arXiv:1004.3597
Y. Castin & F. Werner, PRA (2006); arXiv:1103.2851

Breathing mode

Arbitrary time-evolving state $|\Psi_t\rangle = e^{-iH_\omega t}|\Psi_0\rangle$

$$\langle C\rangle = \langle\Psi_0|e^{iH_\omega t}\,\frac{2H_\omega - L_+ - L_-}{4\omega^2}\,e^{-iH_\omega t}|\Psi_0\rangle$$

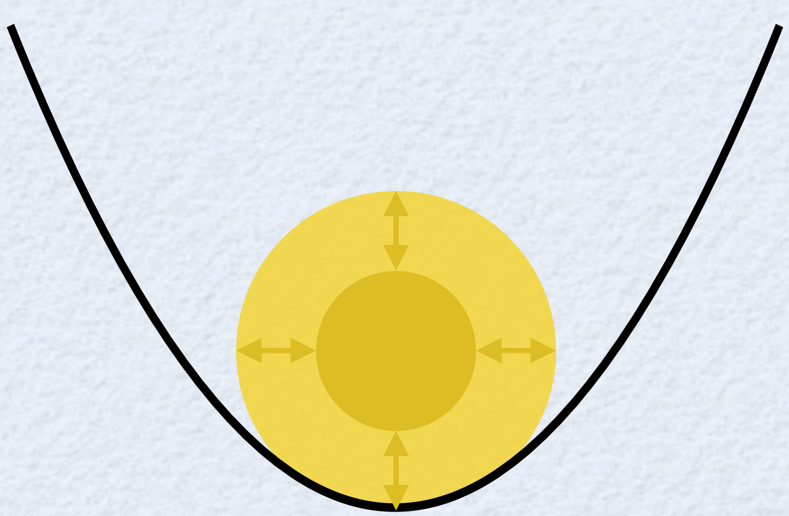
$$= \langle\Psi_0|\frac{2H_\omega - e^{i2\omega t}L_+ - e^{-i2\omega t}L_-}{4\omega^2}|\Psi_0\rangle$$

$$= \frac{\langle\Psi_0|H_\omega|\Psi_0\rangle - \cos(2\omega t + \varphi)|\langle\Psi_0|L_+|\Psi_0\rangle|}{2\omega^2}$$

$$\left(C \equiv \frac{m}{2}\int d^d\vec{x}\;\vec{x}^2\psi^\dagger\psi\right)$$

Mean square radius

$$\langle\vec{x}^2\rangle = A + B\cos\left(2\omega t + \varphi\right)$$



Undamped "breathing mode" with frequency right at 2ω

L. P. Pitaevskii & A. Rosch, PRA (1997)

Breathing mode

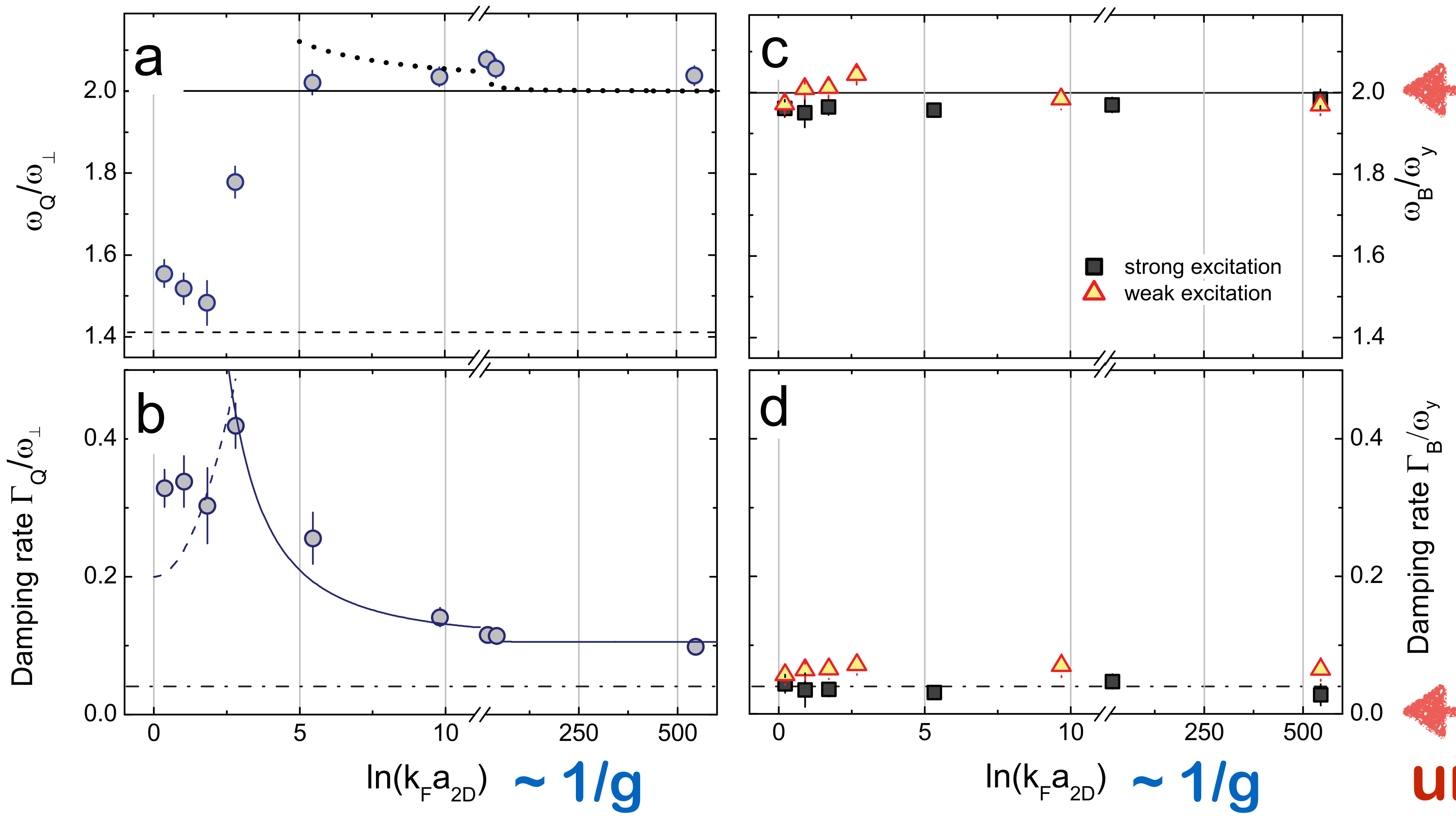
H is scale invariant for $V = -g\,\delta^2(\vec{r})$ in 2D ??

Tunable via Feshbach resonance with ultracold atoms

T ~ 0.4 T_F

M. Köhl's group, PRL (2012)

Quadrupole mode | Breathing mode



~ 1/g   ~ 1/g

2ω

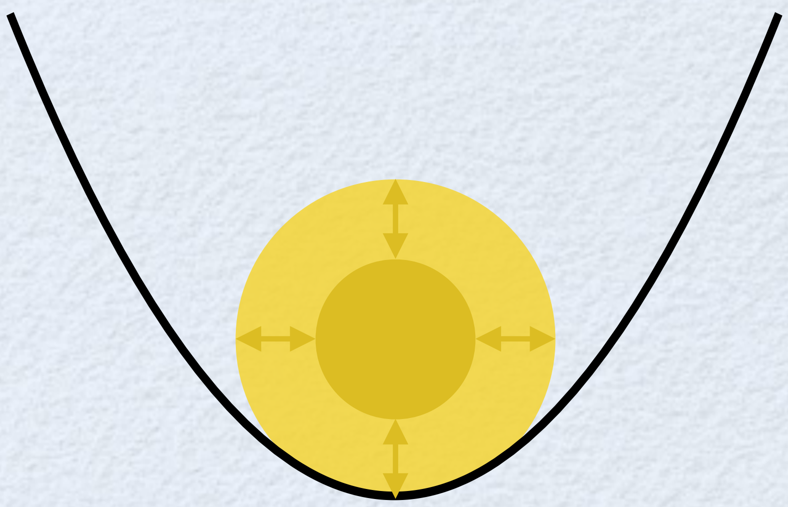
undamped

Bulk viscosity @ a=infinite

D. T. Son, PRL (2007)

Undamped "breathing mode"

for any scale invariant systems confined by harmonic potential



⇒ Vanishing bulk viscosity !?

When coupled with external gauge field & metric

$$S = \int dt\, d^d\vec{x} \sqrt{g} \left(i\psi^\dagger \overleftrightarrow{D}_t \psi - \frac{g^{ij}}{2m} \vec{D}_i \psi^\dagger \vec{D}_j \psi + \mathcal{L}_{\text{int}} \right)$$

is invariant under

D. T. Son & M. Wingate, Ann Phys (2006)

- Gauge transformation $\psi \to e^{i\chi(\vec{x},t)}\psi$
- General coordinate transformation $\vec{x} \to \vec{x}'(\vec{x},t)$
- Conformal transformation $t \to t'(t)$

Bulk viscosity @ a=infinite

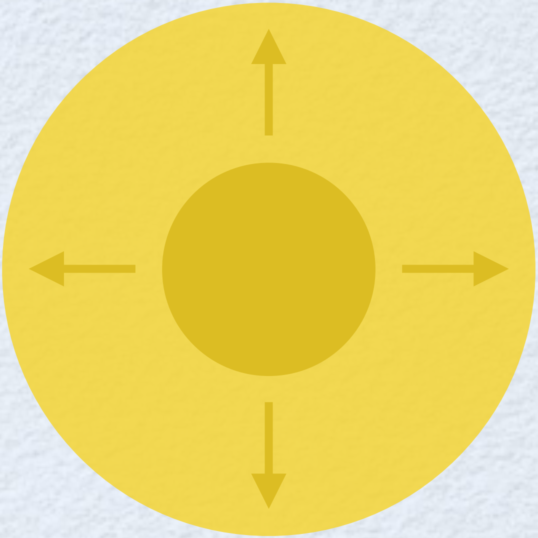
D. T. Son, PRL (2007)

Microscopic symmetries must be inherited by hydrodynamics

Viscous stress tensor coupled with metric

$$\pi_{ij} = \zeta\,\delta_{ij}\,\partial_k v^k + \text{shear}$$

fluid expansion



$$\pi_{ij} = \cancel{\zeta}\,g_{ij}\left(\nabla_k v^k + \partial_t \ln\sqrt{g}\right) + \text{shear}$$

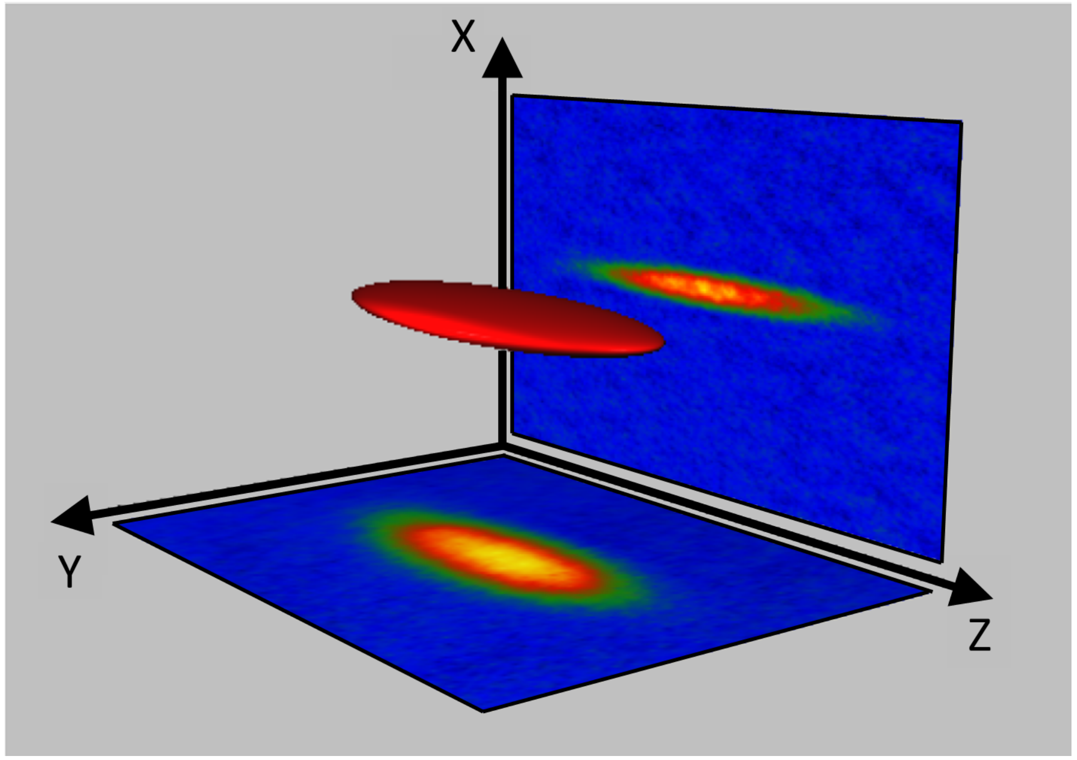
volume expansion

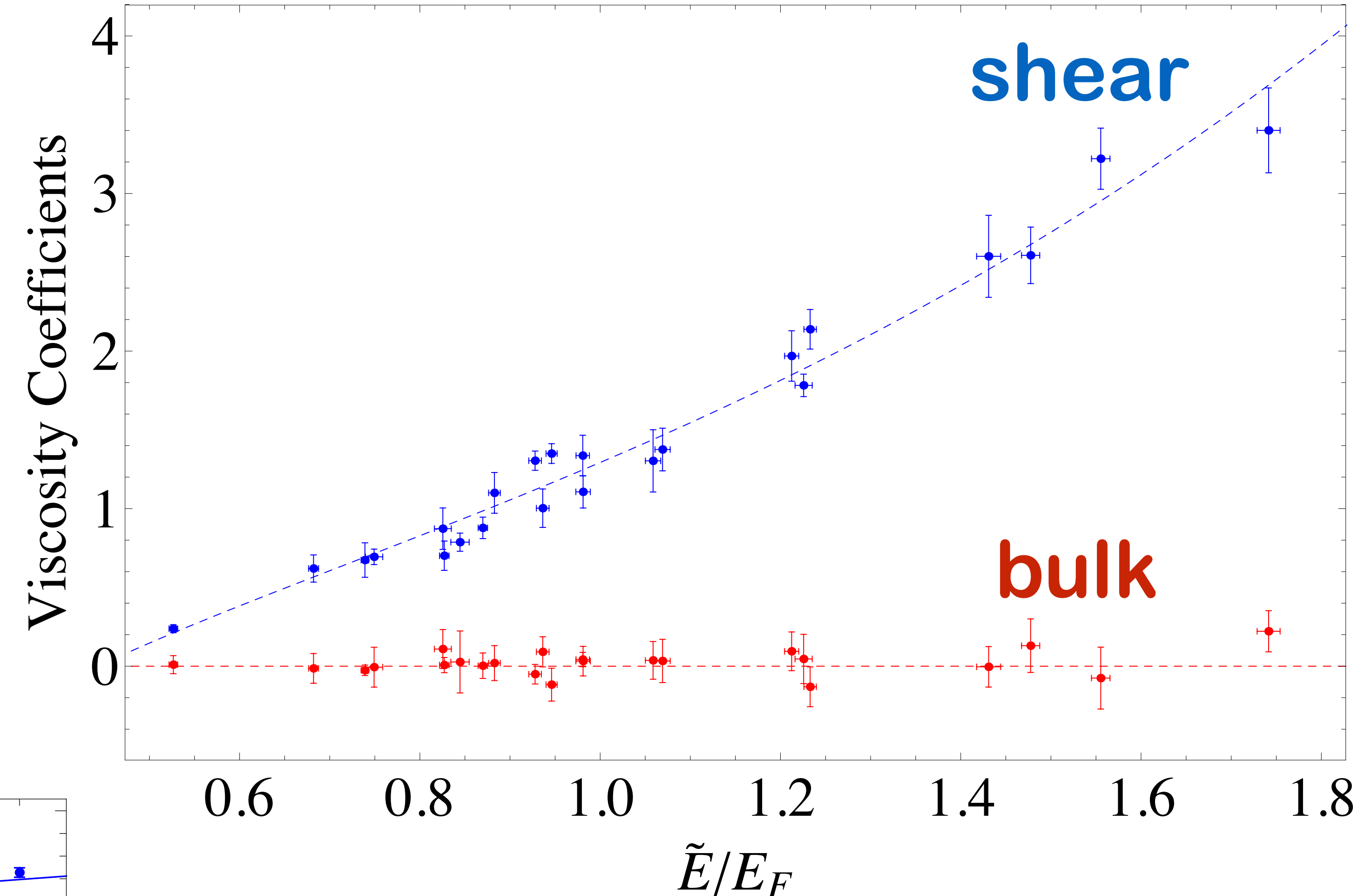
is invariant under general coordinate transformation

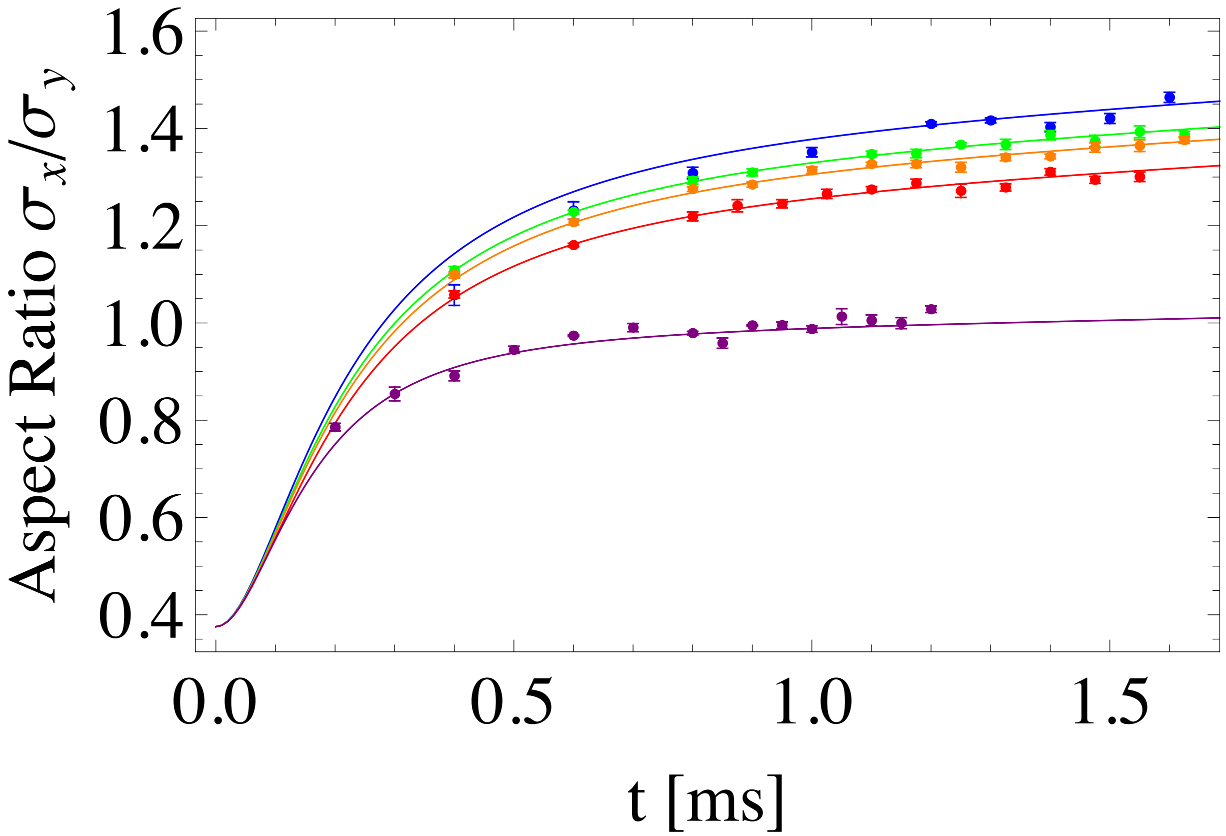
but is NOT under conformal transformation

Vanishing bulk viscosity @ a=infinite

Bulk viscosity @ a=infinite







$$\frac{1}{N}\int d^3\vec{x}\,(\eta,\zeta)$$

$$T/T_{\mathrm{F}} = 0.2 \sim 0.6$$

J. E. Thomas's group, PRL (2014)

Bulk viscosity @ a=finite

Scattering length explicitly breaks scale invariance

because $S(a) \to S(e^s a)$

$$\left(\vec{x} \to e^{-s}\vec{x}, \quad t \to e^{-2s}t, \quad \psi \to e^{(d/2)s}\psi\right)$$

Scale invariance is "formally" recovered if

$$a(\vec{x}, t) \to a'(\vec{x}', t') = e^{-s} a(x, t)$$

spurion field (spacetime-dependent)

Microscopic symmetries must be

inherited by hydrodynamics

$$\pi_{ij}^{\text{bulk}} = \zeta\, g_{ij}\left[\left(\nabla_k v^k + \partial_t \ln\sqrt{g}\right)\right]$$

is NOT invariant under conformal transformation

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K. Fujii & Y. Nishida, PRA (2018)

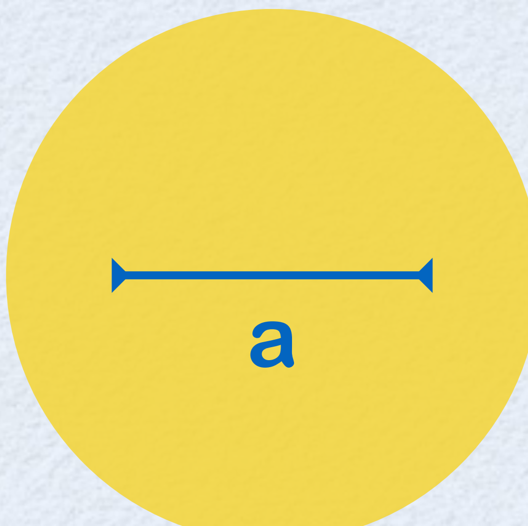
$$\pi_{ij}^{\text{bulk}} = \zeta\, g_{ij} \left[\left(\nabla_k v^k + \partial_t \ln\sqrt{g} \right) - d \left(\partial_t \ln a + v^k \partial_k \ln a \right) \right]$$

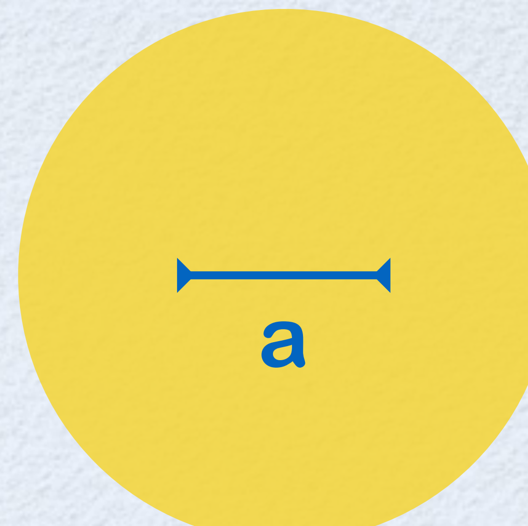
is ~~NOT~~ invariant under conformal transformation

Bulk viscosity @ a=finite

$$\pi_{ij}^{\text{bulk}} = \zeta\, g_{ij} \left[\left(\nabla_k v^k + \partial_t \ln \sqrt{g} \right) - d \left(\partial_t \ln a + v^k \partial_k \ln a \right) \right]$$

expansion of fluid ⟷ contraction of scattering length





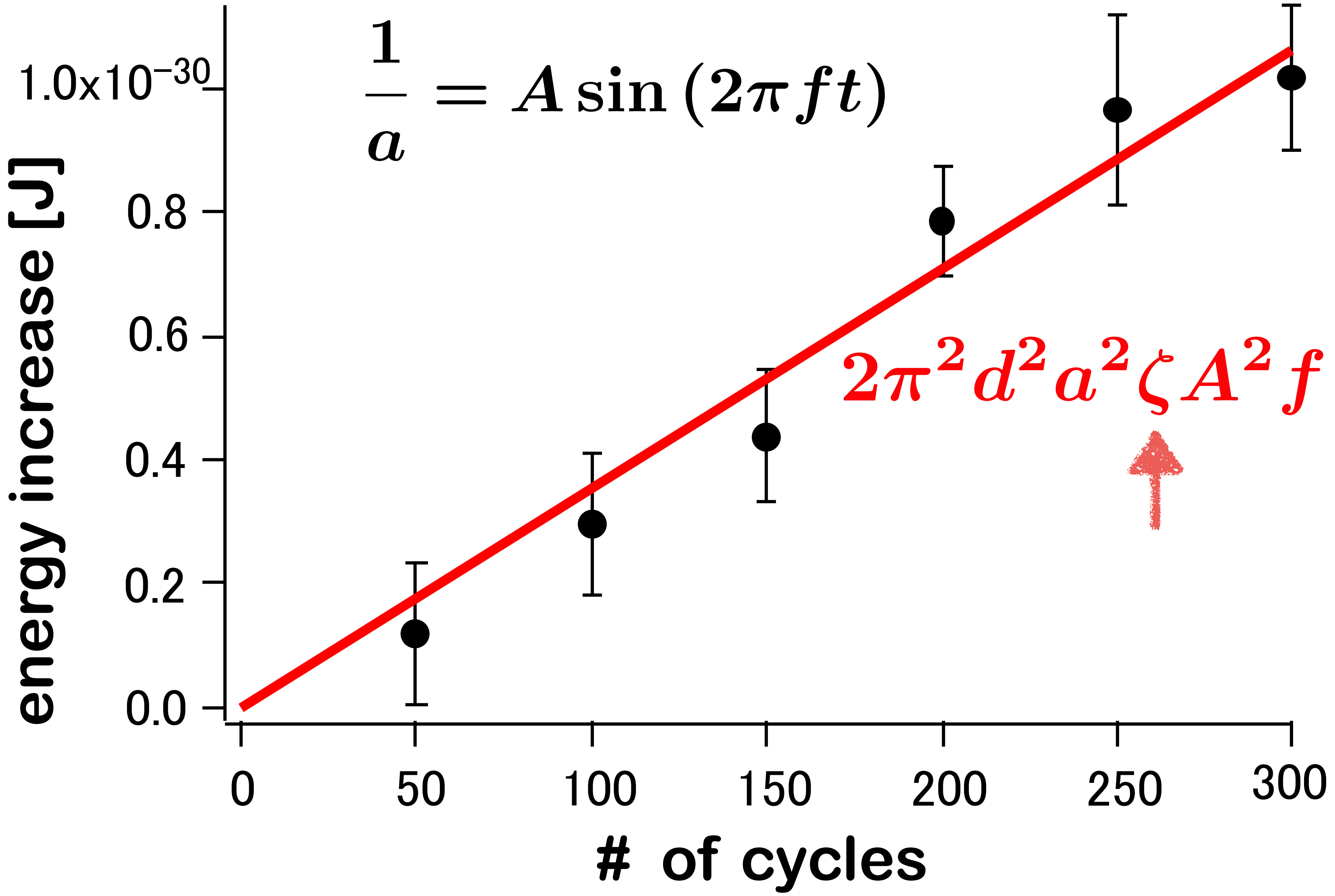
Entropy & energy production even in stationary systems

K. Fujii & Y. Nishida, PRA (2018)

$$T\dot{\mathcal{S}} = \frac{d^2\zeta}{a^2}\dot{a}^2 + O(\dot{a}^3)$$

$$\dot{\mathcal{E}} = \frac{\mathcal{C}_{\text{eq}}}{m\Omega_{d-1}a^{d-1}}\dot{a} + \frac{d^2\zeta}{a^2}\dot{a}^2 + O(\dot{a}^3)$$

Bulk viscosity @ a=finite



Tsumori, Horikoshi, et al. (U. Tokyo)

➡ **Ongoing experiment (DAMOP 2019) toward extraction of bulk viscosity**

$$\dot{\mathcal{E}} = \frac{\mathcal{C}_{\text{eq}}}{m\Omega_{d-1}a^{d-1}}\dot{a} + \frac{d^2\zeta}{a^2}\dot{a}^2 + O(\dot{a}^3)$$

3. 1D Bose gas

T. Tanaka & Y. Nishida, PRL 129, 200402 (2022)

Y. Nishida, PRA 106, 063317 (2022)

Cold atom realization

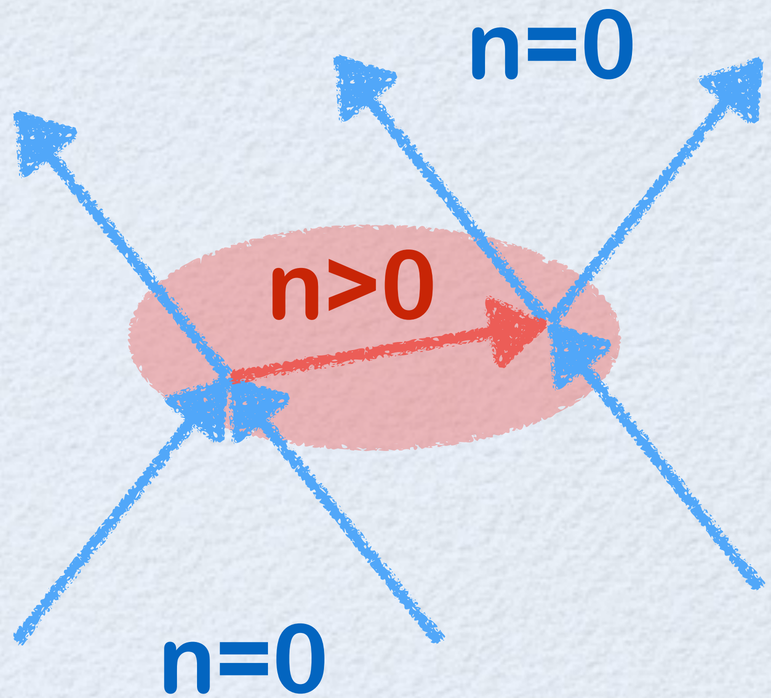
1D Bose gas is realized

by confining 3D bosons into a tight 1D waveguide

$$\hat{\mathcal{H}}_{3\mathrm{D}} = \hat{\Phi}^\dagger \left(-\frac{\nabla^2}{2m} + \frac{y^2+z^2}{2ml_\perp^4} \right) \hat{\Phi} + \frac{2\pi a_{3\mathrm{D}}}{m} (\hat{\Phi}^\dagger \hat{\Phi})^2$$

$$T, \mu \ll \hbar\omega_\perp, \quad |a_{3\mathrm{D}}| \ll l_\perp$$

$$\hat{\mathcal{H}}_{1\mathrm{D}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m} \hat{\phi} + \frac{g_2}{2} (\hat{\phi}^\dagger \hat{\phi})^2$$



effective 3-body interaction

I. E. Mazets & J. Schmiedmayer, NJP (2010)

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$$T, \mu \ll \hbar\omega_\perp, \quad |a_{3\mathrm{D}}| \ll l_\perp$$

$$\hat{\mathcal{H}}_{1\mathrm{D}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m} \hat{\phi} + \frac{g_2}{2} (\hat{\phi}^\dagger \hat{\phi})^2 + \frac{g_3}{6} (\hat{\phi}^\dagger \hat{\phi})^3 + \cdots$$

with $g_2 = 2\frac{a_{3\mathrm{D}}}{ml_\perp^2}, \quad g_3 = -12\ln(4/3)\frac{a_{3\mathrm{D}}^2}{ml_\perp^2}$

effective 3-body interaction

I. E. Mazets & J. Schmiedmayer, NJP (2010)

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integrable non-integrable

⇒ Dynamic bulk viscosity for **Lieb-Liniger model**

$$\hat{\mathcal{H}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m} \hat{\phi} + \frac{g_B}{2} (\hat{\phi}^\dagger \hat{\phi})^2 \qquad g_B = -\frac{2}{ma}$$

Kubo formula

Dynamic bulk viscosity

J. M. Luttinger, Phys. Rev. (1964)
H. Mori, PTP (1962)

$$\zeta(\omega) = \underline{\beta K_{\pi\pi}(w)} + \frac{\mathcal{N}}{iw}\left(\frac{\partial p}{\partial \mathcal{N}}\right)_{\mathcal{S}/\mathcal{N}}\Bigg|_{w\to\omega+i0^+}$$

Kubo's canonical correlation

Stress operator from momentum continuity equation

$$\hat{\pi} = 2\hat{\mathcal{H}} + \frac{\hat{\mathcal{C}}}{ma} - \frac{\partial_x^2(\hat{\phi}^\dagger\hat{\phi})}{4m}$$

$$\hat{\mathcal{C}} = \hat{\phi}^\dagger\hat{\phi}^\dagger\hat{\phi}\hat{\phi}$$

contact density

$$\zeta(\omega) = \frac{1}{iw}\underline{\frac{R_{\mathcal{CC}}(w)}{(ma)^2}} + \frac{1}{iw}\frac{1}{m}\underline{\left(\frac{\partial \mathcal{C}}{\partial a}\right)_{\mathcal{N},\mathcal{S}}}$$

contact correlation thermodynamics

⇒ computable at high temperature, weak coupling,
& strong coupling due to Bose-Fermi duality

Bose-Fermi duality

Lieb-Liniger (LL) model

E. H. Lieb & W. Liniger, Phys. Rev. (1963)

$$\hat{\mathcal{H}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m}\hat{\phi} + \frac{g_B}{2}(\hat{\phi}^\dagger\hat{\phi})^2 \qquad g_B = -\frac{2}{ma}$$

Its energy eigenfunction satisfies contact condition

$$\lim_{x_i \to x_j} \Phi_E(x_1, \ldots, x_N) \propto |x_i - x_j| - a$$

⇒ **Bose-Fermi mapping** M. Girardeau , J. Math. Phys. (1960)

$$\Psi_E(x_1, \ldots, x_N) \equiv \prod_{i<j} \mathrm{sgn}(x_i - x_j)\Phi_E(x_1, \ldots, x_N)$$

Fermionic eigenfunction with contact interaction

Cheon-Shigehara (CS) model

T. Cheon & T. Shigehara, PRL (1999)

$$\hat{\mathcal{H}} = -\hat{\psi}^\dagger \frac{\partial_x^2}{2m}\hat{\psi} + \frac{g_F}{2}|\hat{\psi}(\partial_x\hat{\psi})|^2 \qquad \frac{1}{g_F} = -\frac{m\Lambda}{\pi} + \frac{m}{2a}$$

Bose-Fermi duality

Lieb-Liniger (LL) model

E. H. Lieb & W. Liniger, Phys. Rev. (1963)

$$\hat{\mathcal{H}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m}\hat{\phi} + \frac{g_B}{2}(\hat{\phi}^\dagger\hat{\phi})^2 \qquad g_B = -\frac{2}{ma}$$

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$-g_B$

- $g_B \sim 1/g_F$ ⇒ Weak-strong duality
- 3-body term ~ $|\hat{\psi}(g_F\hat{\psi}\partial_x\hat{\psi})|^2$ Y. Sekino & Y. Nishida, PRA (2021)
  is needed for complete correspondence
- Duality holds for thermodynamics at the same “a”
  but does not for general correlations
  with exceptions of density and contact correlations

Bose-Fermi duality

Dynamic bulk viscosity

J. M. Luttinger, Phys. Rev. (1964)
H. Mori, PTP (1962)

$$\zeta(\omega) = \underline{\beta K_{\pi\pi}(w)} + \frac{\mathcal{N}}{iw}\left(\frac{\partial p}{\partial \mathcal{N}}\right)_{\mathcal{S}/\mathcal{N}}\Bigg|_{w\to\omega+i0^+}$$

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Stress operator from momentum continuity equation

$$\hat{\pi} = 2\hat{\mathcal{H}} + \frac{\hat{\mathcal{C}}}{ma} - \frac{\partial_x^2(\hat{\psi}^\dagger\hat{\psi})}{4m}$$

$$\hat{\mathcal{C}} = |(mg_F/2)\hat{\psi}\partial_x\hat{\psi}|^2$$

contact density

$$\zeta(\omega) = \frac{1}{iw}\underline{\frac{R_{\mathcal{CC}}(w)}{(ma)^2}} + \frac{1}{iw}\frac{1}{m}\underline{\left(\frac{\partial \mathcal{C}}{\partial a}\right)_{\mathcal{N},\mathcal{S}}}$$

contact correlation — thermodynamics

⇒ Formally the same expression as LL model

Bose-Fermi duality

Dynamic bulk viscosity

$$\zeta(\omega) = \frac{1}{iw}\frac{R_{\mathcal{CC}}(w)}{(ma)^2} + \frac{1}{iw}\frac{1}{m}\left(\frac{\partial \mathcal{C}}{\partial a}\right)_{\mathcal{N},\mathcal{S}}$$

with $$R_{\mathcal{CC}}(w) = -\frac{L}{Z}\sum_{n,n'}\frac{e^{-\beta E_n} - e^{-\beta E_{n'}}}{w + E_n - E_{n'}}|\langle n|\hat{\mathcal{C}}|n'\rangle|^2$$

$$\langle n|\hat{\mathcal{C}}|n'\rangle_B = N(N-1)\int_{x_3,\ldots,x_N}\Phi_n^*(x,x,x_3,\ldots,x_N)\Phi_{n'}(x,x,x_3,\ldots,x_N)$$

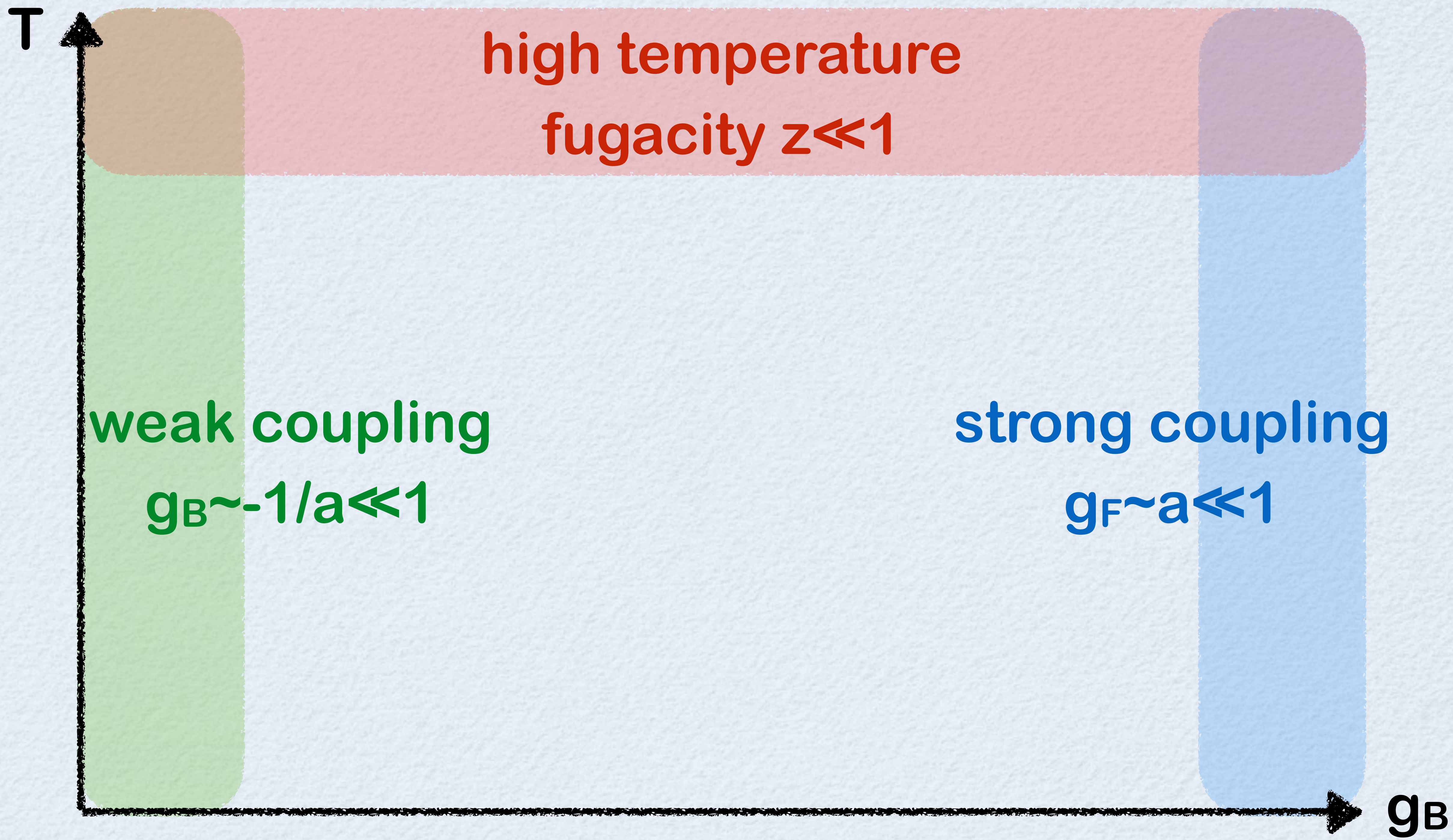
Bose-Fermi mapping $\Psi_E(*) \equiv \prod_{i<j}\mathrm{sgn}(x_i - x_j)\Phi_E(*)$

$$\langle n|\hat{\mathcal{C}}|n'\rangle_F = N(N-1)\int_{x_3,\ldots,x_N}\Psi_n^*(x,x,x_3,\ldots,x_N)\Psi_{n'}(x,x,x_3,\ldots,x_N)$$

⇒ Bose-Fermi duality holds for dynamic bulk viscosity !

Dynamic bulk viscosity

Perturbations are possible at three limits

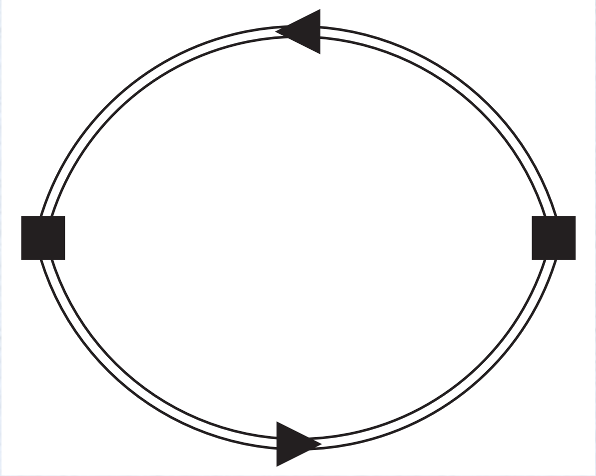


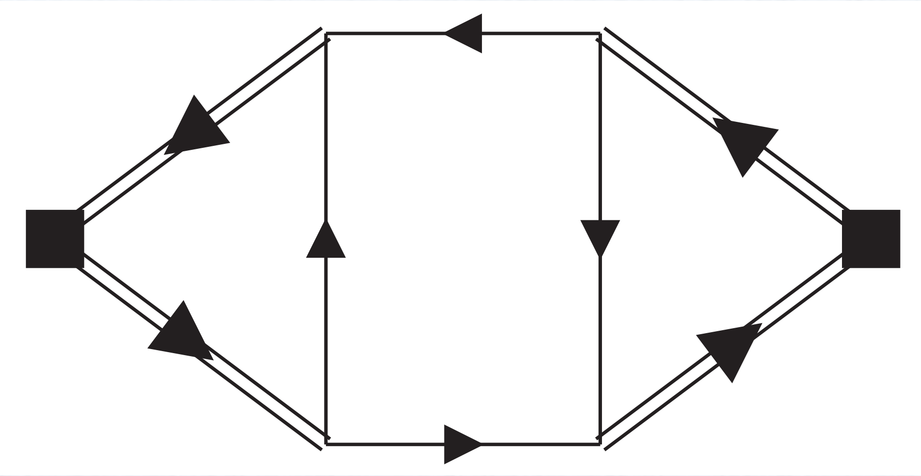
Dynamic bulk viscosity

Perturbations are possible at three limits

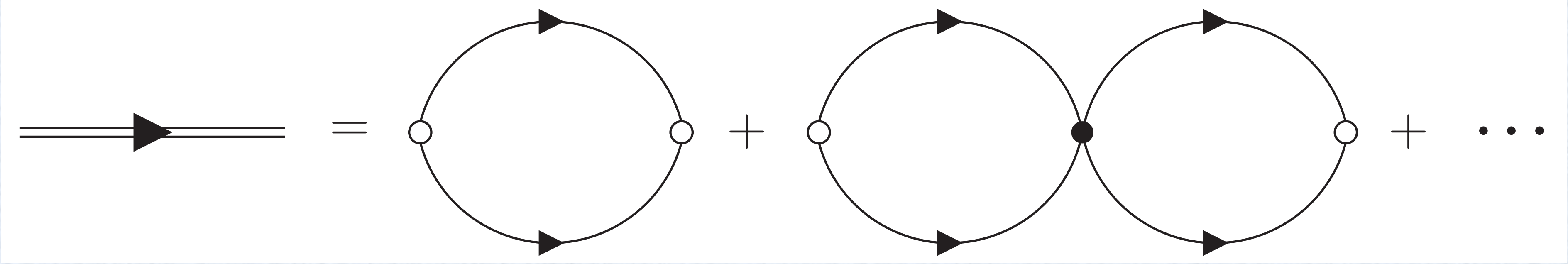
$$\zeta(\omega) = \zeta_{\text{reg}}(\omega) + \frac{iD}{\omega + i0^+}$$

regular part   Drude peak





contact correlation diagrams



Dynamic bulk viscosity

Perturbations are possible at three limits

$$\zeta(\omega) = \zeta_{\text{reg}}(\omega) + \frac{iD}{\omega + i0^+}$$

regular part		Drude peak		
$O(z^2)$		$O(z^3)$		high temperature
$O(a^{-2})$	$\gg$	$O(a^{-4})$	@	weak coupling
$O(a^2)$		$O(a^6)$		strong coupling

Consistent with

vanishing at any frequency for conformal systems

D. T. Son, PRL (2007); E. Taylor and M. Randeria, PRA (2010)

divergence at zero frequency for integrable systems

K. A. Matveev & M. Pustilnik, PRL (2017)

Dynamic bulk viscosity

Perturbations are possible at three limits

$$\zeta(\omega) = \zeta_{\mathrm{reg}}(\omega) + \frac{iD}{\omega + i0^+} + \frac{D}{\omega^2 \tau} + \cdots \quad \to \zeta \quad (\omega \to 0)$$

regular part   Drude peak   → $\dfrac{iD}{\omega + i/\tau}$

Drude peak remains because of integrability,
i.e., 2-body interaction does not yield relaxation in 1D
under energy & momentum conservations

If integrability is broken by 3-body interaction,
it does yield relaxation time of $\tau \sim g_3^{-2}$
and higher-order terms have stronger singularities

Their resummation leads to finite transport in 3D

K. Fujii & T. Enss, Ann. Phys. (2023)

Dynamic bulk viscosity

Non-integrable systems in 3D

$$\zeta(\omega) = \zeta_{\mathrm{reg}}(\omega) + \frac{iD}{\omega + i0^+} + \frac{D}{\omega^2\tau} + \cdots \quad \to \zeta \quad (\omega \to 0)$$

regular part   Drude peak

$$\Rightarrow \frac{iD}{\omega + i/\tau}$$

understood microscopically (diagrammatically) with Kubo formula in some perturbative limits

Non-integrable systems in 1D

$$\zeta(\omega) \sim \omega^{-1/3} \quad (\omega \to 0)$$ by hydrodynamic fluctuations

- How can it be understood microscopically ?
- What class of diagrams need to be resumed ?
- How does coefficient vanish in conformal limits ?

Summary of this talk

Static and dynamic bulk viscosity

- Measure of conformality breaking
- Naturally couples with spacetime-dependent scattering length as it simulates fluid expansion
- Bose-Fermi duality in 1D to access strong coupling
- Some microscopic understanding for 1D integrable & 3D non-integrable systems but may not for 1D non-integrable systems (?)

Hydrodynamic fluctuation …

- How can it be understood microscopically ?
- What class of diagrams need to be resumed ?
- How does coefficient vanish in conformal limits ?

Experimental measure

Dynamic bulk viscosity

$$\zeta(\omega) = \frac{1}{iw}\frac{R_{\mathcal{C}\mathcal{C}}(w)}{(ma)^2} + \frac{1}{iw}\frac{1}{m}\left(\frac{\partial \mathcal{C}}{\partial a}\right)_{\mathcal{N},\mathcal{S}}\Big|_{w\to\omega+i0^+}$$

contact correlation

$\hat{\mathcal{C}}$

$$\hat{\mathcal{H}} = -\hat{\phi}^\dagger \frac{\partial_x^2}{2m}\hat{\phi} + \frac{g_B}{2}(\hat{\phi}^\dagger\hat{\phi})^2 \qquad g_B = -\frac{2}{ma}$$

Linear response theory under $a(t) = a + \delta a \sin(\omega t)$

$$\mathcal{C}(t) - \bar{\mathcal{C}}[a(t)] = \mathrm{Im}\left[\frac{R_{\mathcal{C}\mathcal{C}}(\omega) - R_{\mathcal{C}\mathcal{C}}(0)}{ma^2}\,\delta a\, e^{-i\omega t}\right] + O(\delta a^2)$$

$$\Rightarrow \quad \dot{\mathcal{E}}(t) = \frac{\mathcal{C}(t)}{ma^2(t)}\,\dot{a}(t), \quad T\dot{\mathcal{S}}(t) = \frac{\mathcal{C}(t) - \bar{\mathcal{C}}[a(t)]}{ma^2(t)}\,\dot{a}(t)$$

Experimental measure

Dynamic bulk viscosity

$$\zeta(\omega) = \frac{1}{iw}\frac{R_{\mathcal{CC}}(w)}{(ma)^2} + \frac{1}{iw}\frac{1}{m}\left(\frac{\partial \mathcal{C}}{\partial a}\right)_{\mathcal{N},\mathcal{S}}\Bigg|_{w\to\omega+i0^+}$$

contact correlation

Contact correlation can be extracted by measuring contact, energy, or entropy densities

Linear response theory under $a(t) = a + \delta a \sin(\omega t)$

$$\mathcal{C}(t) - \bar{\mathcal{C}}[a(t)] = \mathrm{Im}\left[\frac{R_{\mathcal{CC}}(\omega) - R_{\mathcal{CC}}(0)}{ma^2}\,\delta a\, e^{-i\omega t}\right] + O(\delta a^2)$$

$$\Rightarrow \quad \dot{\mathcal{E}}(t) = \frac{\mathcal{C}(t)}{ma^2(t)}\dot{a}(t), \quad T\dot{\mathcal{S}}(t) = \frac{\mathcal{C}(t) - \bar{\mathcal{C}}[a(t)]}{ma^2(t)}\dot{a}(t)$$