Overview of Fluctuating hydrodynamics

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Content

1. Survey of hydrodynamics

2.Historical events and recent topics

3. Bare transport coefficient in heat conducting case

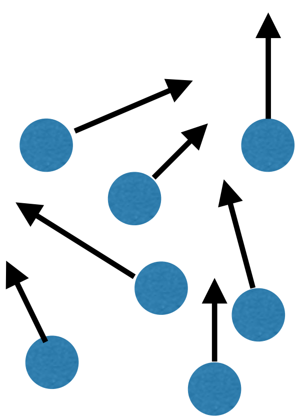
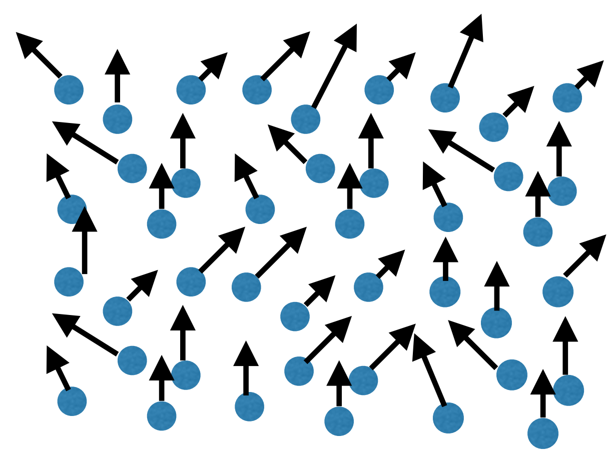
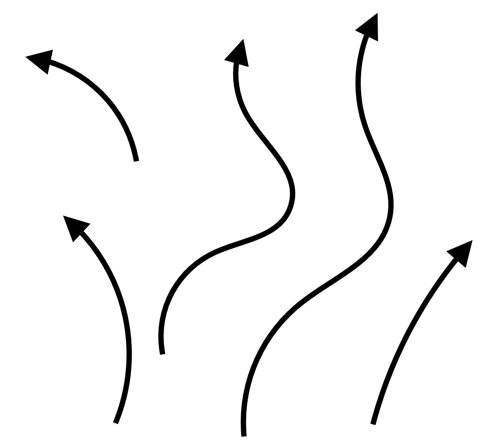
4. Several issues to be solved

5. Summary

1. Survey of fluctuating hydrodynamics

- Hierarchy on scales: Micro. $\rightleftarrows$ Meso. $\rightleftarrows$ Macro

scale →

Microscopic	Mesoscopic	Macroscopic
		
Deterministic Newton equation	Fluctuating hydrodynamics	Hydrodynamics Navier-stokes eq.

$$m_i \ddot{\boldsymbol{x}}_i = \boldsymbol{F}_i$$

$$\partial_t \boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla P + \nu \nabla^2 \boldsymbol{v}$$

$$\underbrace{\partial_t \boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v}}_{\text{Euler term}} + \nabla P = \nu_0 \nabla^2 \boldsymbol{v} + \boldsymbol{\xi}(\boldsymbol{r}, t)$$

Euler term

Bare transport coefficient (ν_0)

Noise ($\boldsymbol{\xi}(\boldsymbol{r}, t)$)

2. Historical and recent events

- Fluctuation in fluids — Bogoliubov (1946,1962)
- Fluctuating hydrodynamics (FHD) — Landau and Lifshitz (1958,1959)
- Projection method for FHD — Zwanzig (1961)
- Theory of diverging viscosity in low-dimensional fluids
  Numerics: Alder, Wainright (1970) — FHD theory:Forster, Nelson, Stephene (1977)
- Long-range correlation in nonequilibrium steady states
  FHD theory: Ronis, Procaccia (1982)
  Experiment: Law, Gammon, Sengers (1988)
- Diverging heat conductivity in low-dimensions
  Numerics: Lepri, Livi, Politi (1999) — FHD theory: Spohn (2014)
  Experiment: Chang et al. (2008)
- Phase transitions in active matters
  FHD theory: Tonner and Tu (1998)
- Applying to the QCD physics
  QCD phase boundaries, dynamics
  FHD theory: Hirano, Huovinen, Murase, Nara (2013)
  Akamatsu, Mazeliauskas, Taney (2017)

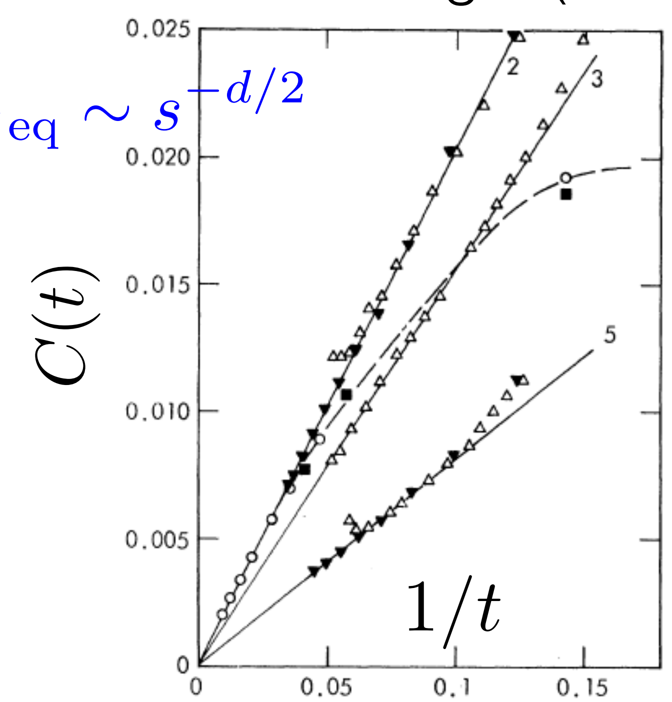
Diverging viscosity in 2D fluid

- Dynamical renormalization of macroscopic diffusion in fluids

$$\partial_t \boldsymbol{v} + (\boldsymbol{v}\cdot\nabla)\boldsymbol{v} + \nabla P = \nu_0 \nabla^2 \boldsymbol{v} + \boldsymbol{\xi}(\boldsymbol{r}, t)$$

Numerics: Alder-Wainright (1970)

$$\langle \boldsymbol{v}(s)\cdot\boldsymbol{v}(0)\rangle_{\text{eq}} \sim s^{-d/2}$$



One loop renormalization

Forster, Nelson, Stephen PR (1977)

2D: $C(\boldsymbol{x}, t) := \langle \boldsymbol{v}(\boldsymbol{x}, t)\cdot\boldsymbol{v}(\boldsymbol{0}, 0)\rangle$

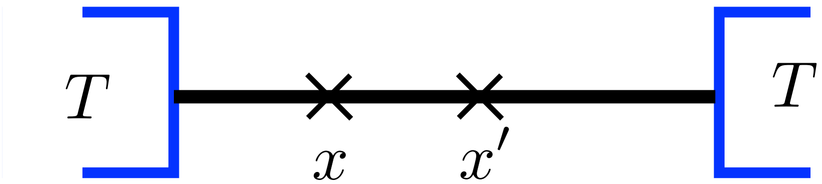
$$C(k,t) \sim \begin{matrix} \ln(1/k) & (\omega \to 0) \\ \ln(1/\omega) & (k \to 0) \end{matrix}$$

- No experiment

Long-range correlation

- Insulator case:

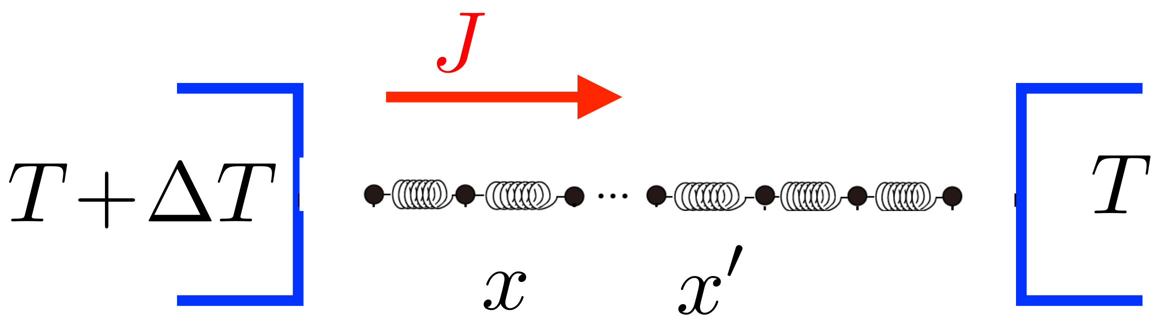
  Equilibrium



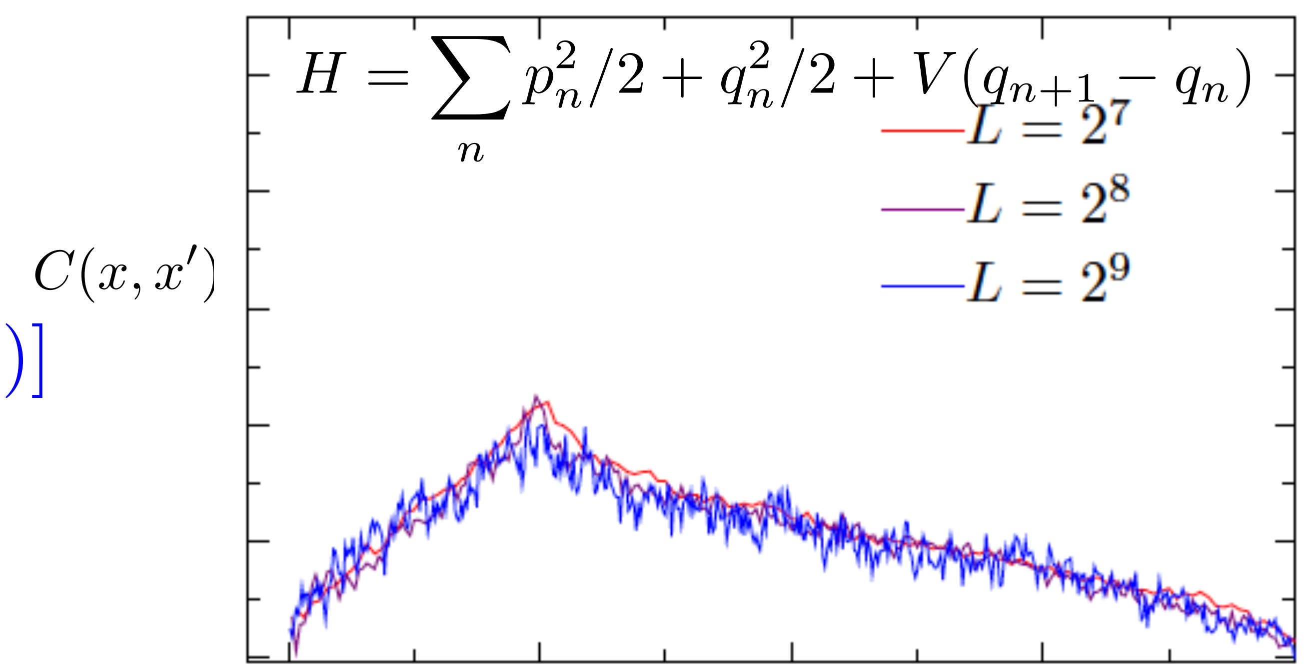
$$C(x,x') = \langle \delta\epsilon(x)\,\delta\epsilon(x')\rangle_{\rm eq} \;\sim 0 \;\; (x \neq x')$$

Nonequilibrium

$$C(x,x') = \langle \delta\epsilon(x)\,\delta\epsilon(x')\rangle_{\rm ss} \;\; \propto (\Delta T)^2 [\Delta^{-1}]_{x,x'}$$



$$\partial_t \epsilon(x,t) = -\partial_x[-(\kappa/c)\partial_x \epsilon(x,t) + \xi(x,t)]$$



- Fluid case: Light scattering for nonequilibrium liquid

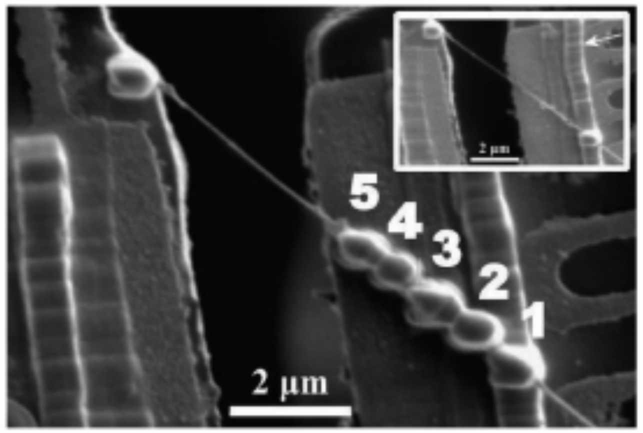
$$S_{\vec{k},\omega} \propto \frac{2\chi\gamma_T^2 \bar{T}^2 k^2}{C_p(\omega^2+\chi^2k^4)}\left[1 + \frac{\bar{T}}{\bar{T}^2}\frac{C_p}{\rho_0}\frac{\nu_t}{\chi}\frac{|\vec{\nabla} T|^2}{(\omega^2+\nu_t^2k^4)}\right]$$

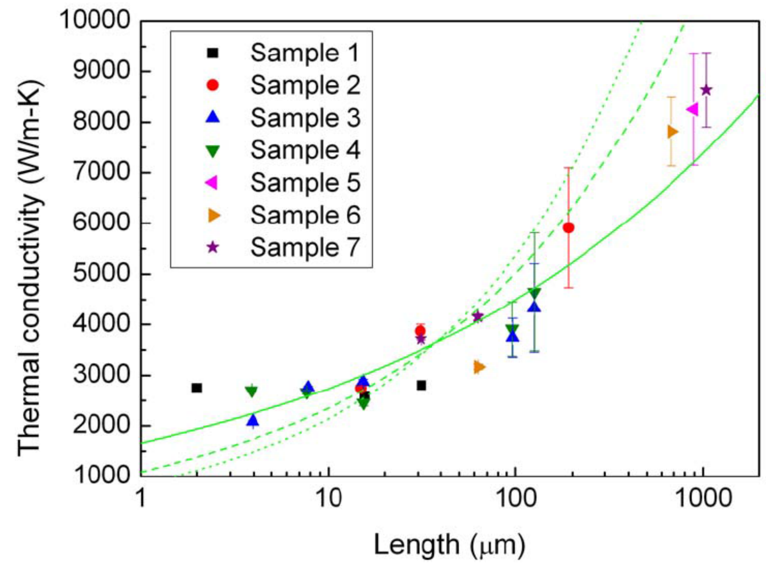
FHD theory: Ronis, Procaccia (1982)

Experiment: Law, Gammon, Sengers (1988)

3. Diverging heat conduction

- Experiment: Carbon nanotube





N^α

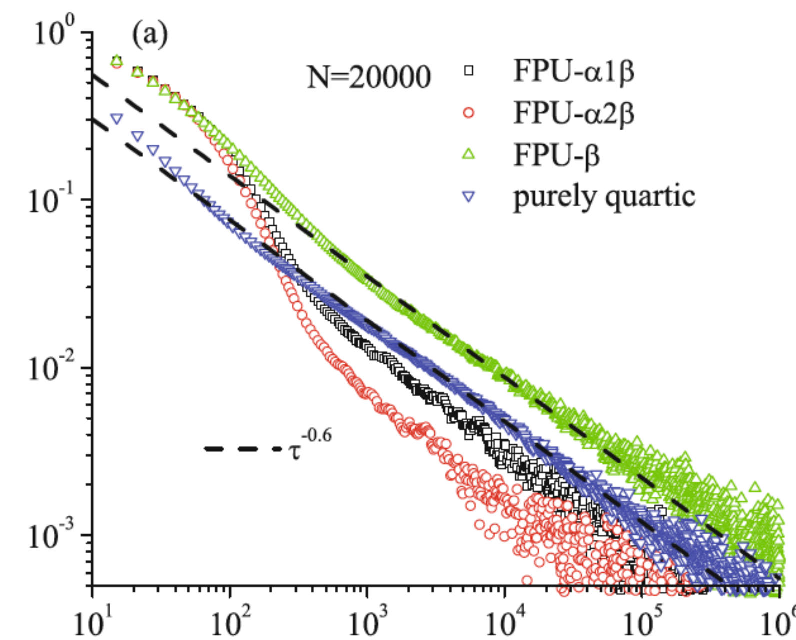
C. Chang et al., PRL (2008)
V. Lee et al., PRL (2017)

- Macroscopic transport coefficient (heat conductivity)

$$H = \sum_n p_n^2/2 + V(q_{n+1} - q_n)$$

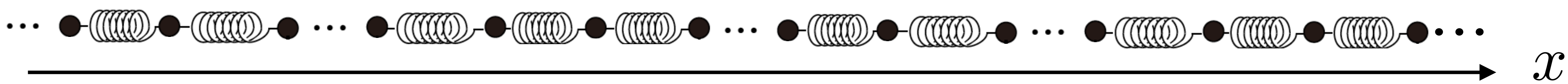
$$\kappa = \beta^2 \lim_{t\to\infty} \lim_{N\to\infty} \int_0^t ds \langle J(s)J(0)\rangle_{\mathrm{eq}}/N$$

$$\langle J(s)J(0)\rangle_{\mathrm{eq}} \sim s^{-a} \quad (0 < a < 1)$$



- Phenomenological FHD   Spohn (2014)

Macrocsopic divergence is explained by the FHD



- Phenomenological FHD H. Spohn, JSP (2014)

  Three conserved quantities: Total length, Momentum, Energy

Length

$$(\partial/\partial t)r(x,t) = -(\partial/\partial x)p(x,t)$$

Momentum

$$(\partial/\partial t)p(x,t) = -(\partial/\partial x)[\mathcal{J}_p(x,t) - D_{pp}(\partial/\partial x)p(x,t) + \xi_p(x,t)]$$

Energy

$$(\partial/\partial t)\varepsilon(x,t) = -(\partial/\partial x)[\mathcal{J}_\varepsilon(x,t) - D_{\varepsilon,r}(\partial/\partial x)r(x,t) - D_{\varepsilon,\varepsilon}(\partial/\partial x)\varepsilon(x,t) + \xi_\varepsilon(x,t)]$$

- Bare transport coefficient?

Micro — Macro

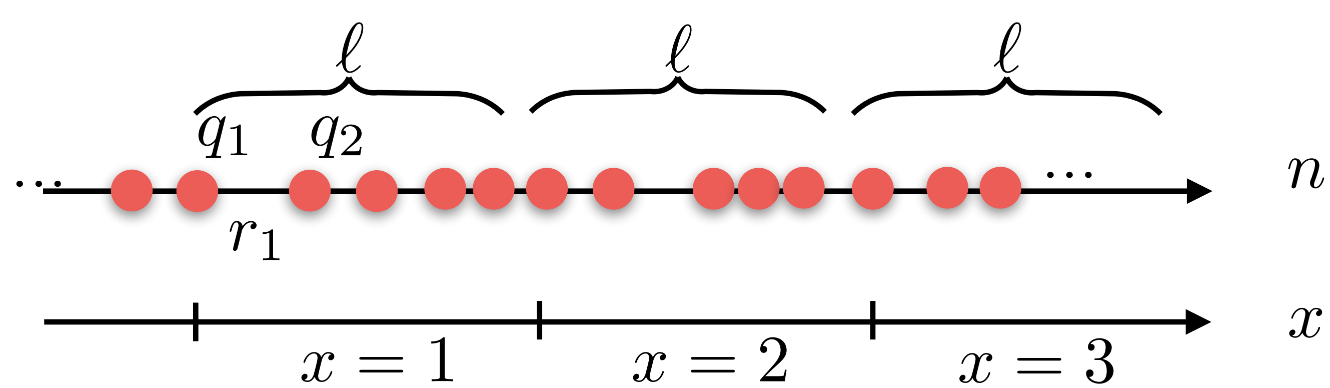
Finite $(D_{pp}, D_{\epsilon,r}, D_{\epsilon,\epsilon})$ ⇒ $\kappa \to \infty$

Fundamentals: What is the value of bare transport coefficient?

- Keywords to derive "hydrodynamics"

  "Coarse graining" "Projection"
  "Large deviation" (or Ensemble equivalence or GGE)

- "Coarse graining"

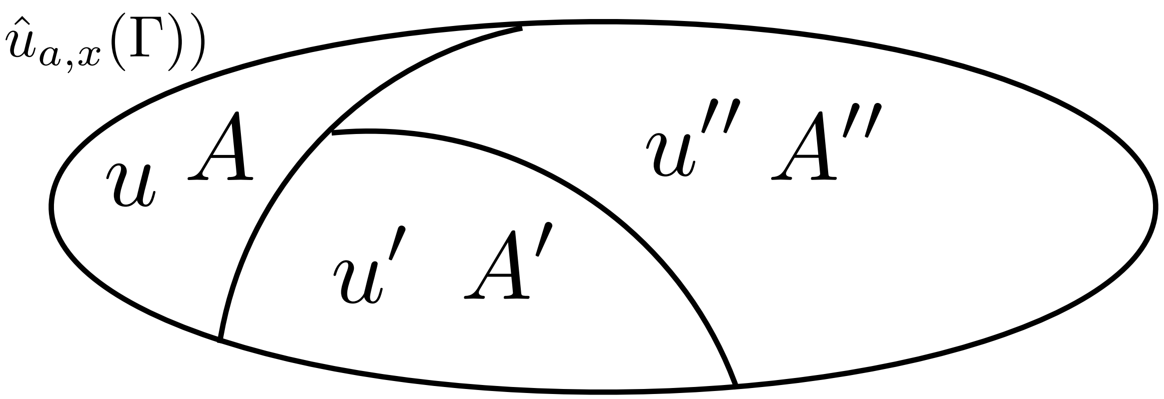


$$H = \sum_n p_n^2/2 + V(q_{n+1} - q_n)$$

- Projection: $2N \to 3\,(\text{conserved quantities})$ Zwanzig Phys. Rev. (1961)

$$\Gamma = (q_1, p_1, \cdots, q_N, p_N)$$

$$\mathcal{P}A(\Gamma) = \frac{1}{\Omega(\Gamma)} \int d\Gamma' A(\Gamma') \prod_{x=1}^{N/\ell} \prod_{a=r,p,\epsilon} \delta(\hat{u}_{a,x}(\Gamma') - \hat{u}_{a,x}(\Gamma))$$



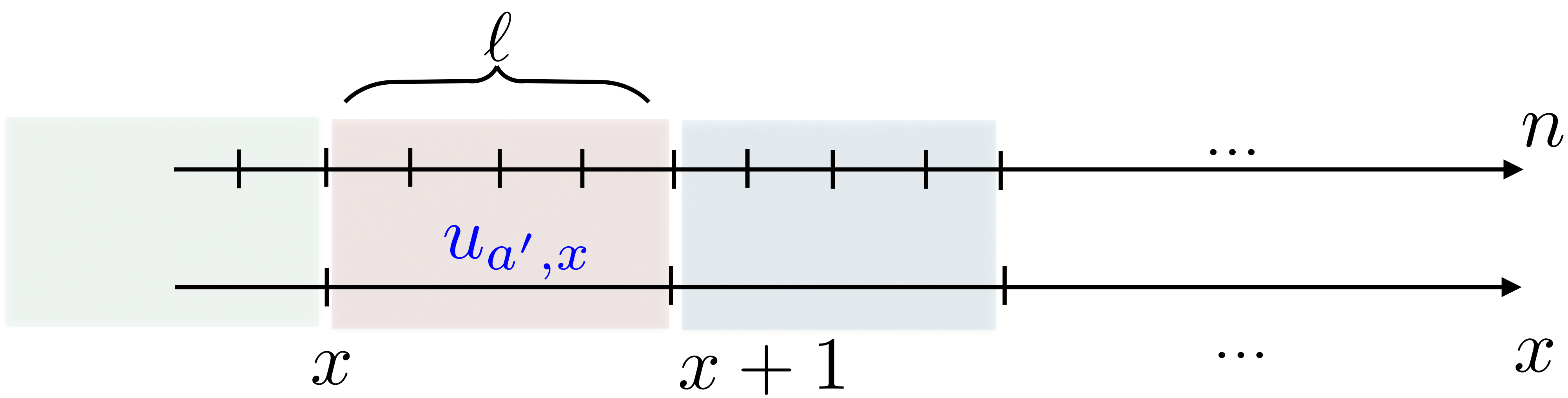
Fundamentals: What is the value of bare transport coefficient?

KS, Hongo, Dhar, Sasa, PRL (2021)

- Fokker-Planck equation

$$\partial_t f_t(\{u\}) = \frac{\delta}{\delta u_{a,x}}\left[\partial_x \langle \hat{J}_{a,x}\rangle^u_{\rm LM} f_t(\{u\})\right] + \frac{\delta}{\delta u_{a,x}}\Omega(u)(\partial_{x'}\partial_x K_{ax,a'x'})\frac{\delta}{\delta u_{a',x'}}\left(\frac{f_t(\{u\})}{\Omega(u)}\right)$$

Local microcanonical ensemble $\hat{\rho}_{\rm LM} = \prod_x \prod_{a'=r,p,\epsilon} \delta(\hat{u}_{a',x}(\Gamma) - u_{a',x})/\Omega(u)$



- Corresponding Langevin equation

$$\partial_t u_{a,x} = -\partial_x\left[\langle \hat{J}_{a,x}\rangle_{\rm LM;u} - K_{a,a}\partial_x\lambda_{a,x} + \xi_a\right]$$

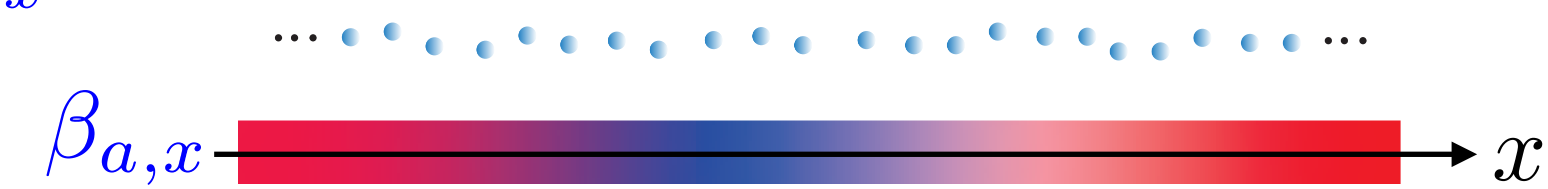
$$K_{a,a'} = \ell\int_0^\infty ds\, C_{a,a'}(s) \qquad C_{a,a'}(s) = (\ell/N)\langle(\sum_x \mathcal{Q}\hat{\mathcal{J}}_{a,x})(e^{s\mathbb{L}}\sum_{x'}\mathcal{Q}\hat{\mathcal{J}}_{a',x'})\rangle_{\rm eq}$$

Fundamentals: What is the value of bare transport coefficient?

KS, Hongo, Dhar, Sasa, PRL (2021)

- Ensemble-equivalence (GGE) Microcanonical = Gibbs

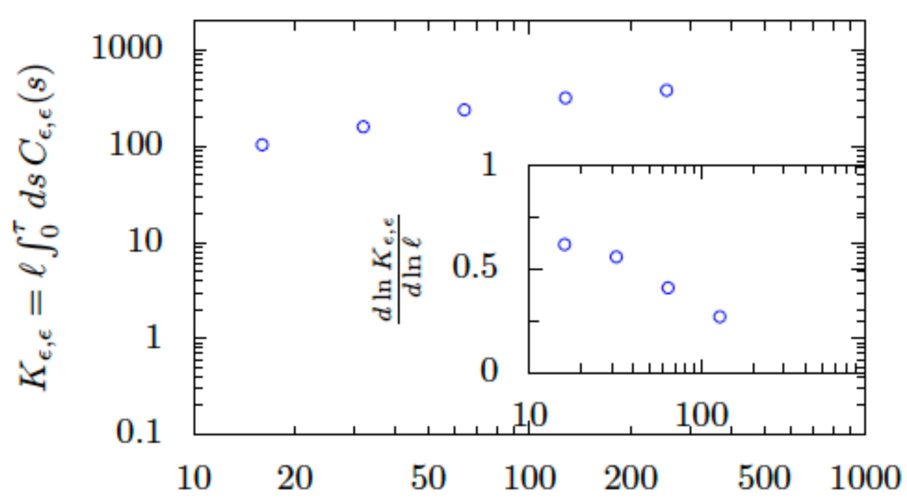
$$\hat{\rho}_{\mathrm{LM}} \simeq \hat{\rho}_{\mathrm{LG}} \qquad \hat{\rho}_{\mathrm{LG}} = \prod_x \hat{\rho}^{(x)}_{\mathrm{LG}}\,, \quad \hat{\rho}^{(x)}_{\mathrm{LG}} = e^{-\sum_{a=r,p,\epsilon} \lambda_{a,x}(t)\,\hat{u}_{a,x}}/Z_x$$



$$\rightarrow \langle \hat{J}_{a,x}\rangle^{u}_{\mathrm{LM}} \sim A_{a,a'}\delta u_{a',x} + (1/2) H^{a}_{a',a''}\delta u_{a',x}\delta u_{a'',x} + \cdots$$

- Fermi-Pasta-Ulam-Tsingou model

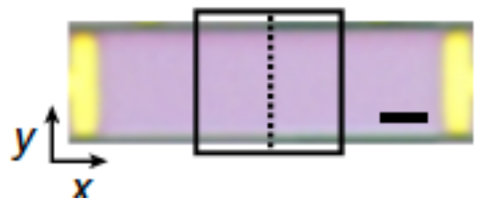
$$H = \sum_{n=1}^{N} p_n^2/2 + (1/2)\, r_n^2 + (k_3/3)\, r_n^3 + (k_4/4)\, r_n^4 \qquad (r_n = q_{n+1} - q_n)$$



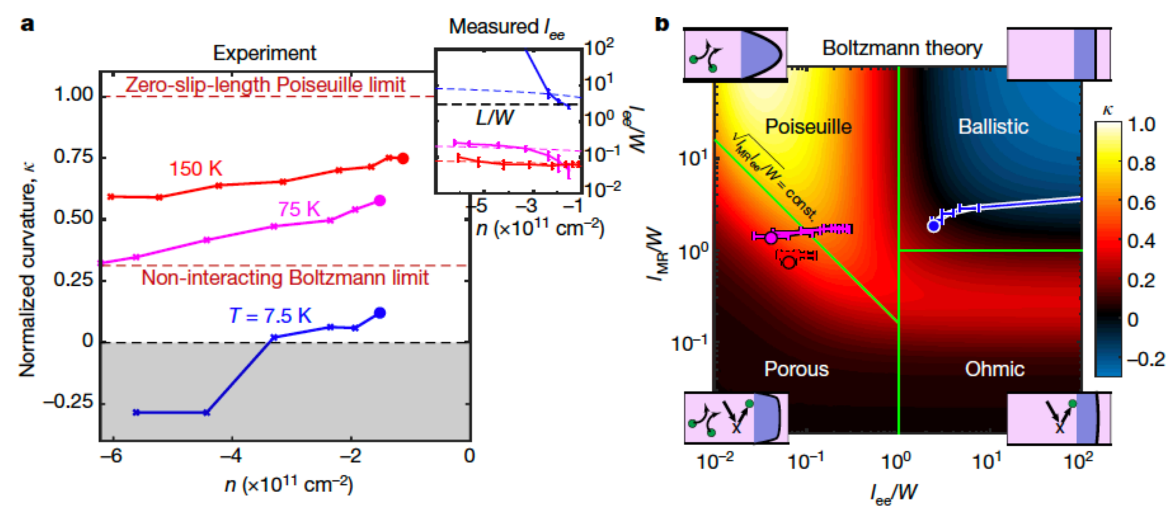
4. List of several open issues: missing parts

- Experiments on diverging viscosity in 2D fluids
- Looking at the bare viscosity in 2D fluids

  Graphene: 2D material ?

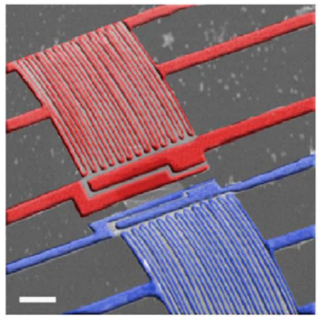


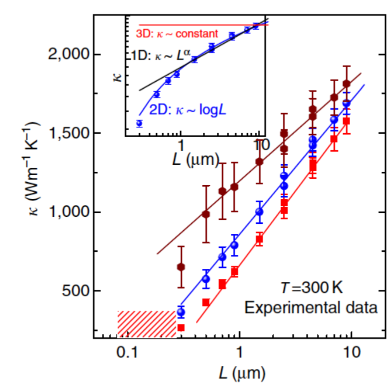
Sulpizio, Nature (2019)



- Fluctuating hydro. in 2D and 3D heat conduction

  Graphene:





Xu et al., Nature com. (2014)

- Fluctuating hydro. for long-range interacting systems

  Ion trapped systems, Rydberg atoms

- Fluctuating hydro. in quantum systems: Keldysh is enough?

5. Summary

- Overviewed history and recent events on the fluctuating hydro.

- List to be clarified

  Experiments on diverging viscosity in 2D fluids
  Finding the value of the bare viscosity in 2D fluids

  Fluctuating hydro. in higher dimensions, long-range interactions

  Fluctuating hydro. in quantum regime

- Let's discuss the FHD in the workshop!

Historical and recent events

- Turbulence (Forster et al. (1977); DeDominicis & Martin
- (1979); Fournier & Frisch (1983); Yakhot & Orszag (1986)).