Relativistic distortions in galaxy density-ellipticity correlations

Shohei SAGA

(Institut d'Astrophysique de Paris)

Based on S.Saga, T.Okumura, A.Taruya, T.Inoue (MNRAS 2022)

5−9 Dec. 2022, New Frontiers in Cosmology with the Intrinsic Alignments of Galaxies

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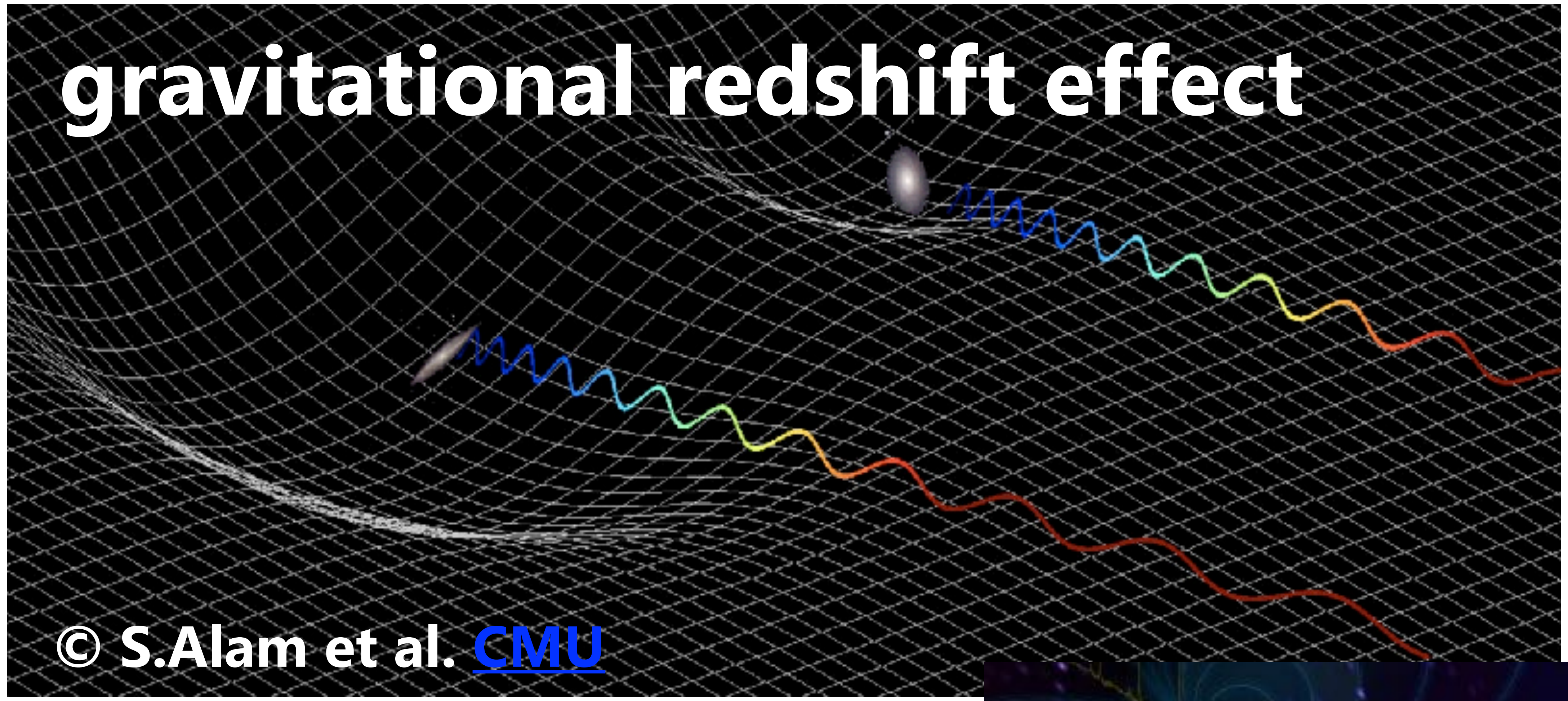
1. Relativistic effects on the observed redshift

Observed redshift : cosmological redshift (Hubble flow) + Doppler effect

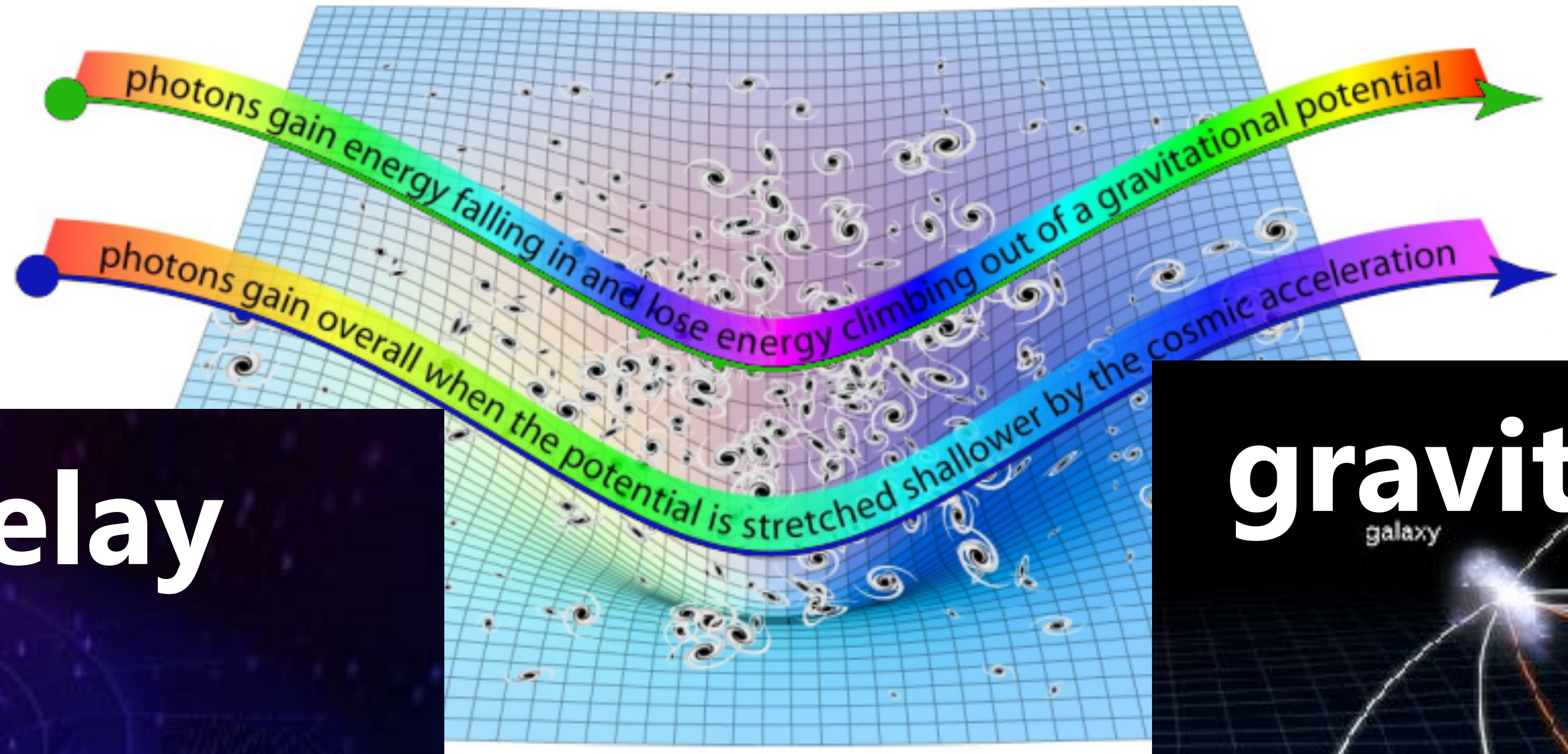
$$z_{\mathrm{obs}} = z_{\mathrm{cosmological}} + \boldsymbol{z}_{\mathbf{Doppler}}$$

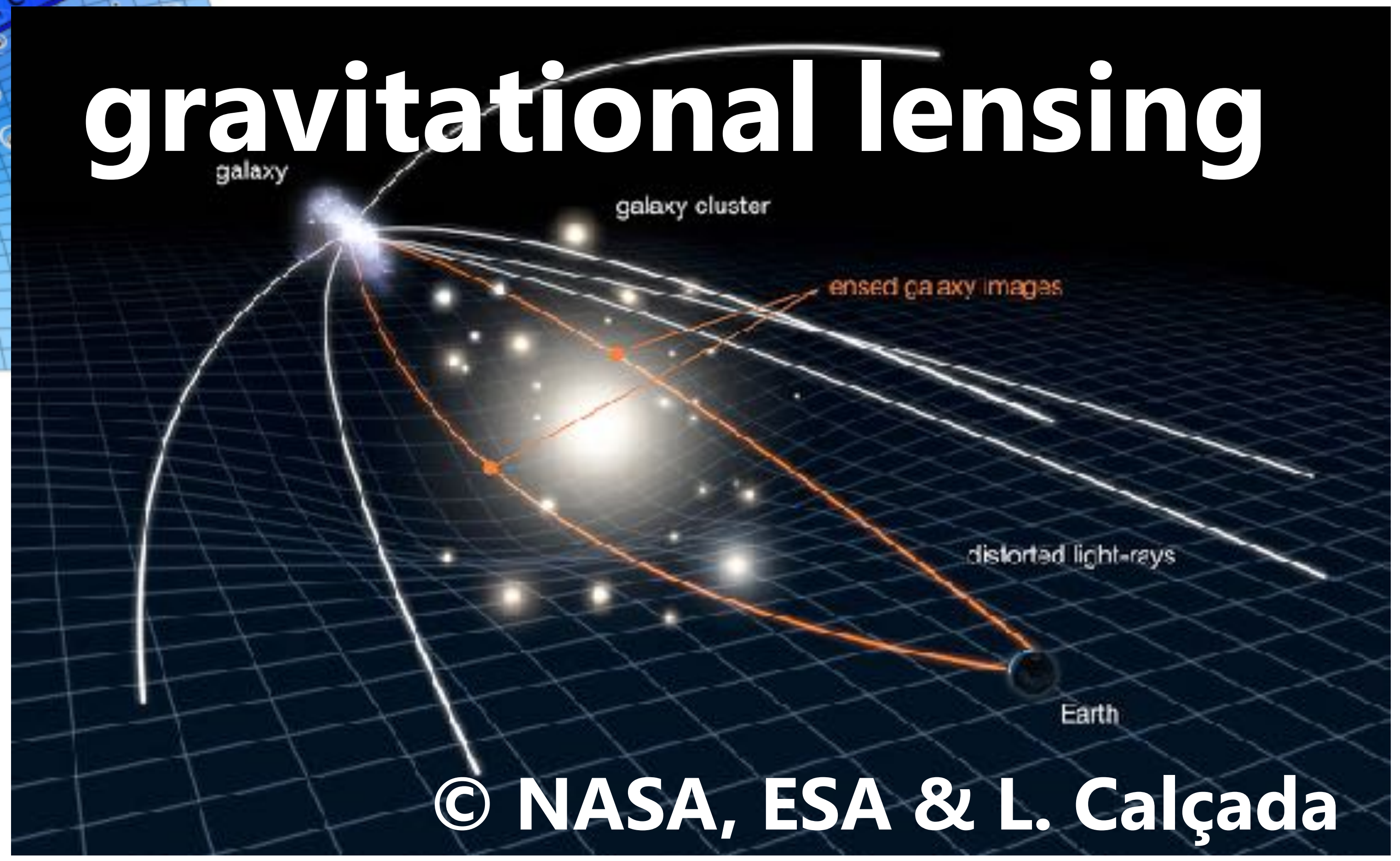
$$+\ \boldsymbol{z}_{\mathbf{grav}} + \boldsymbol{z}_{\mathbf{ISW}} + \boldsymbol{z}_{\mathbf{Shapiro}} + \dots$$

$$(\theta_{\mathrm{lens}}, \varphi_{\mathrm{lens}})$$



integrated Sachs-Wolfe





2. Redshift space distortions

T.Matsubara [ApJ 537 L77 (2000)], A.Challinor and A.Lewis [1105.5292], C.Bonvin and R.Durrer [1105.5280], J.Yoo [1409.3223], and many works

Perturbed FLRW metric: $\mathrm{d}s^2 = \left[-(1+2\Phi)\mathrm{d}t^2 + a^2(1-2\Psi)\mathrm{d}x^2\right]$

Observed redshift including all effects: $1+z = \dfrac{(k_\mu u^\mu)_{\mathrm{S}}}{(k_\mu u^\mu)_{\mathrm{O}}}$

Redshift space ↔ real space

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} + \frac{1+z}{H}\left(-\Phi + \frac{1}{2}v^2 - \int_t^{t_0}\left(\dot{\Phi}+\dot{\Psi}\right)\mathrm{d}t'\right)\hat{\boldsymbol{r}} - \int_0^{\chi}(\Psi+\Psi)\mathrm{d}\chi'\,\hat{\boldsymbol{r}} - \int_0^{\chi}(\chi-\chi')\nabla_\perp(\Phi+\Psi)\mathrm{d}\chi'$$

Doppler term other relativistic contributions

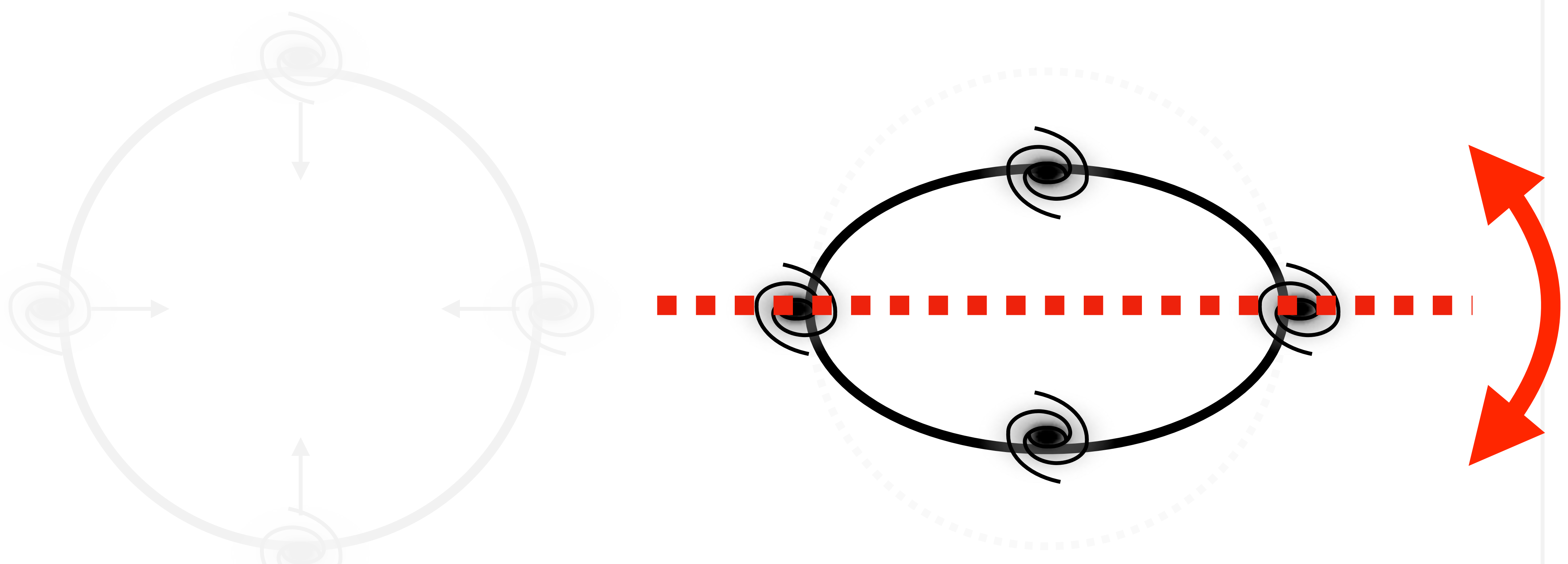
conservation law: $\left(1+\delta^{(\mathrm{s})}(\boldsymbol{s})\right)\mathrm{d}^3s = \left(1+\delta(\boldsymbol{r})\right)\mathrm{d}^3r$

Redshift-space density ↔ real-space density

$$\delta^{(\mathrm{s})} = b\delta - \frac{1}{\mathcal{H}}\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial\boldsymbol{r}}(\hat{\boldsymbol{r}}\cdot\boldsymbol{v}) - \left(\frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\hat{\boldsymbol{r}}\cdot\boldsymbol{v} + \frac{1}{\mathcal{H}}\left(\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial\boldsymbol{r}}\Psi + \mathcal{H}\hat{\boldsymbol{r}}\cdot\boldsymbol{v} + \hat{\boldsymbol{r}}\cdot\dot{\boldsymbol{v}}\right)$$

$$-2\Phi + \Psi + \frac{\dot{\Phi}}{\mathcal{H}} + \frac{1}{r}\int_0^r \mathrm{d}r'\left(2 - \frac{r-r'}{r'}\Delta_\Omega\right)(\Phi+\Psi) + \left(\frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\left(\Psi + \int_0^r \mathrm{d}r'\left(\dot{\Psi}+\dot{\Phi}\right)\right)$$

3. Recalling the Doppler effect



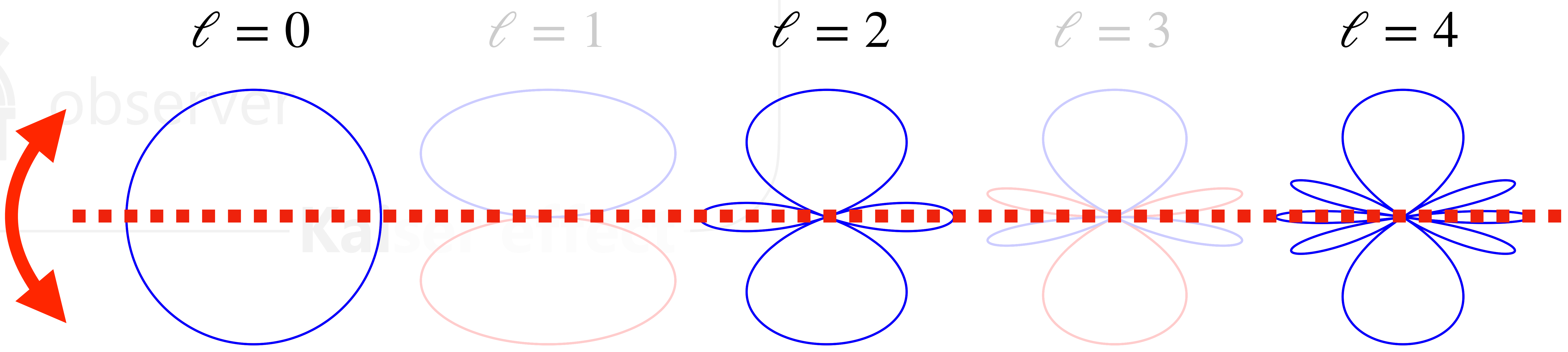
Kaiser formula
(constant line-of-sight vector)

$$\delta^{(\mathrm{s})}(\boldsymbol{k}) = \left(b + f\,(\hat{\boldsymbol{k}}\cdot\hat{z})^2\right)\delta_{\mathrm{L}}(\boldsymbol{k})$$

(line-of-sight vector)²

symmetric distortions along the line-of-sight direction

even multipole anisotropies in RSD



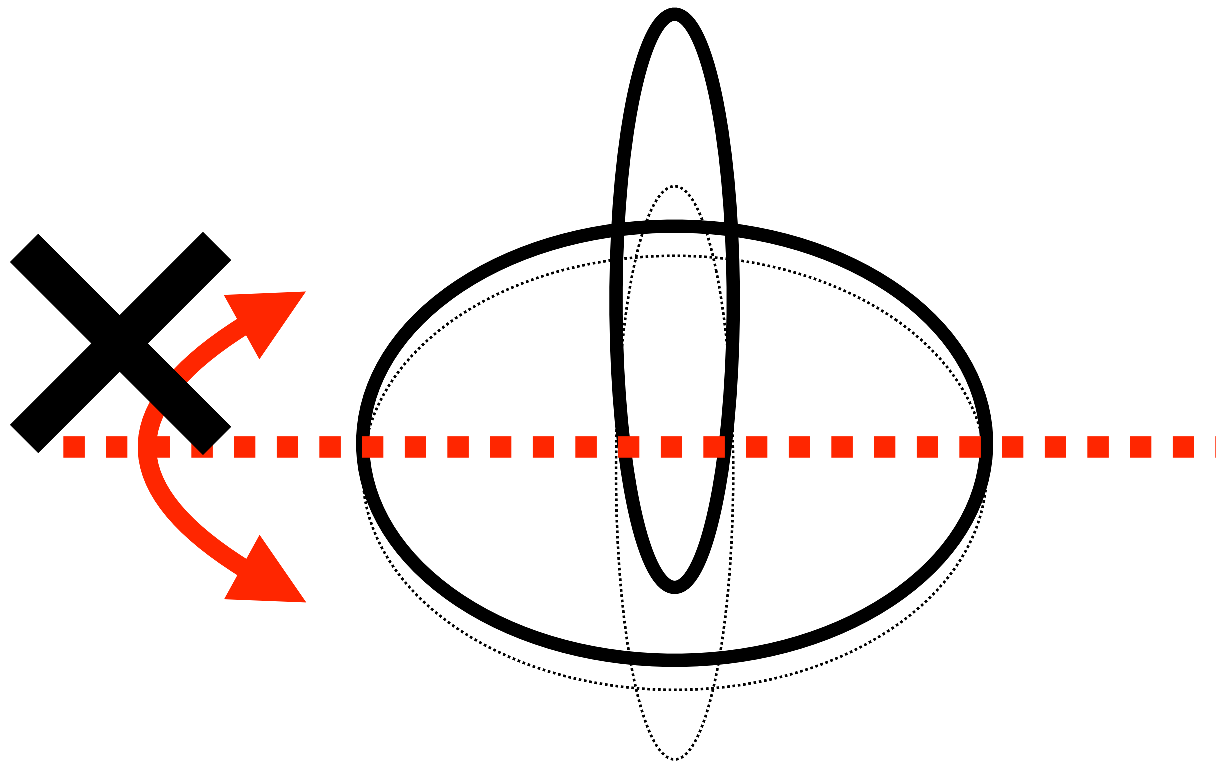
4. Relativistic contributions

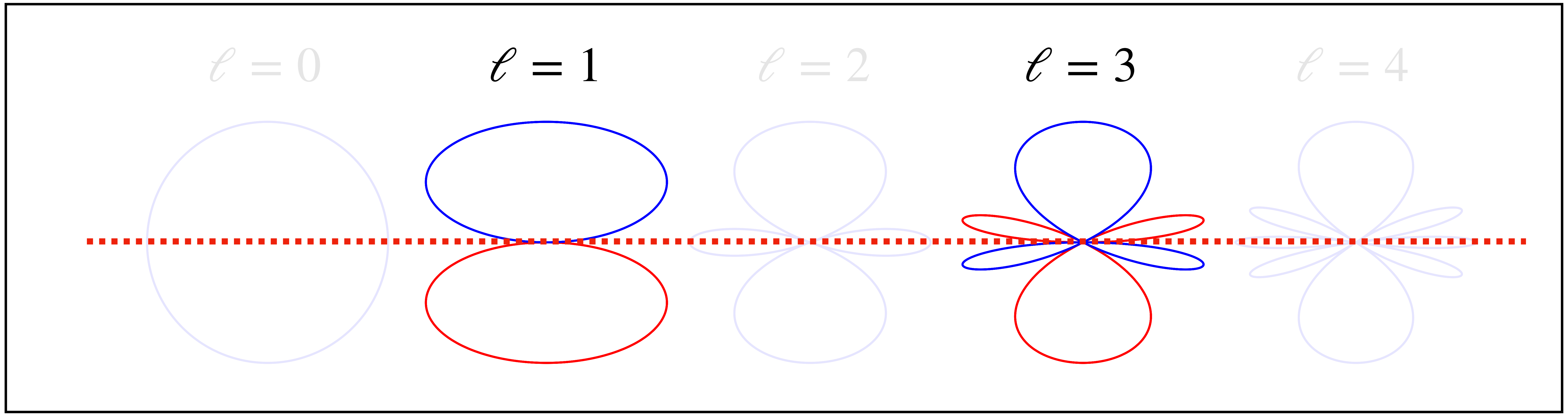
Redshift-space density ↔ real-space density

$$\delta^{(\mathrm{s})} = b\delta - \frac{1}{\mathcal{H}}\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial \boldsymbol{r}}(\hat{\boldsymbol{r}}\cdot\boldsymbol{v}) - \left(\frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\hat{\boldsymbol{r}}\cdot\boldsymbol{v} + \frac{1}{\mathcal{H}}\left(\hat{\boldsymbol{r}}\cdot\frac{\partial}{\partial \boldsymbol{r}}\Psi + \mathcal{H}\hat{\boldsymbol{r}}\cdot\boldsymbol{v} + \hat{\boldsymbol{r}}\cdot\dot{\boldsymbol{v}}\right)$$

$$- 2\Phi + \Psi + \frac{\dot{\Phi}}{\mathcal{H}} + \frac{1}{r}\int_0^r \mathrm{d}r' \left(2 - \frac{r-r'}{r'}\Delta_\Omega\right)(\Phi+\Psi) + \left(\frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2}\right)\left(\Psi + \int_0^r \mathrm{d}r' \left(\dot{\Psi} + \dot{\Phi}\right)\right)$$

(line-of-sight vector)$^{\text{odd}}$ in relativistic effects ⇒ odd multipoles





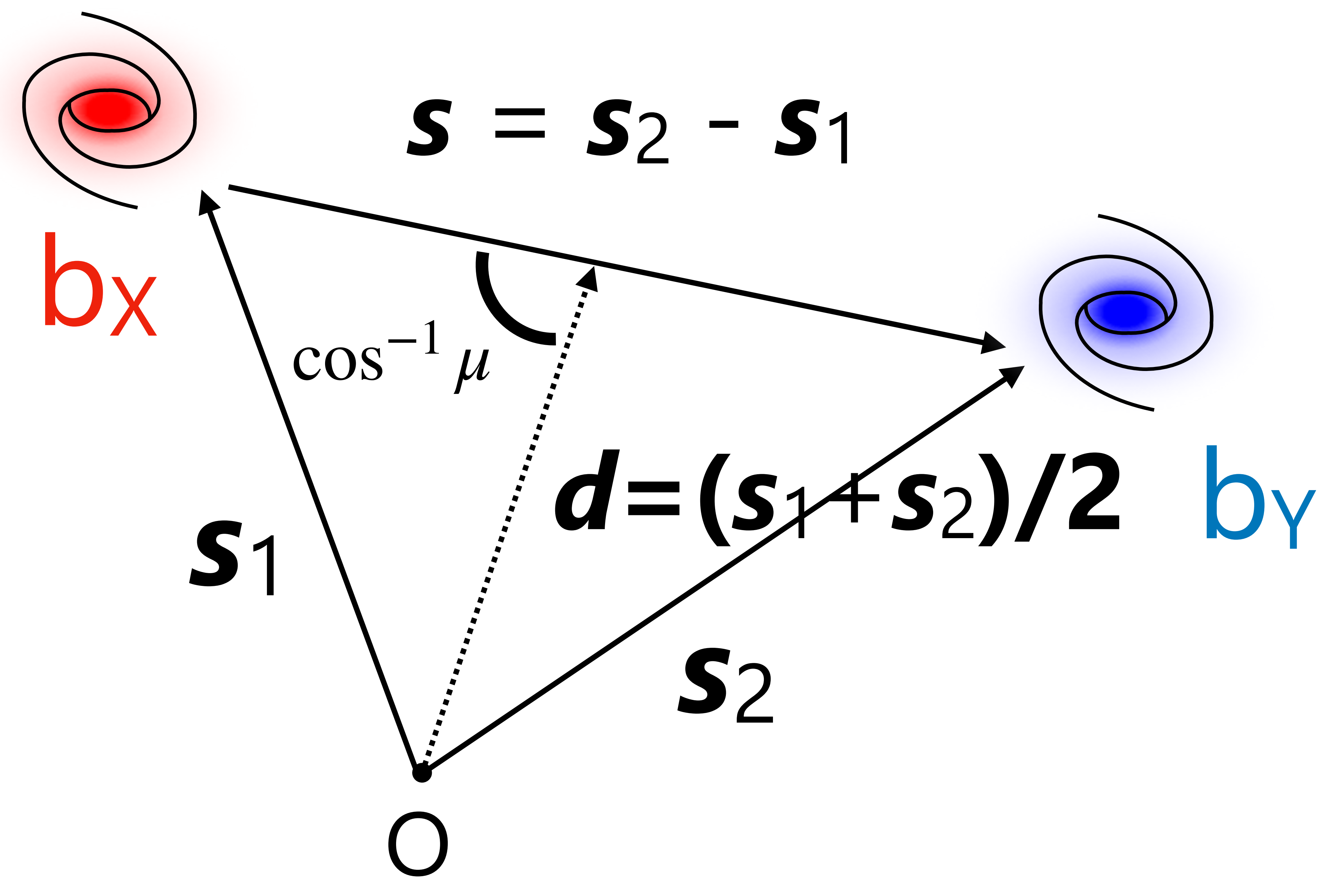
Relativistic effects induces the asymmetric distortions along the LOS direction

5. Lowest-order odd multipole: dipole (l = 1)

$$\langle \delta_X^{(S)}(\boldsymbol{s}_1)\delta_Y^{(S)}(\boldsymbol{s}_2)\rangle = \xi^{(S)}(s, d, \mu)$$

$$\xi_\ell(s,d) = \frac{2\ell+1}{2}\int_{-1}^{1} \xi^{(S)}(s,d,\mu)\,P_\ell(\mu)\,\mathrm{d}\mu$$

$$(P_1(\mu) = \mu = \hat{s}\cdot\hat{d})$$



From **linear theory**, the dipole moment is …

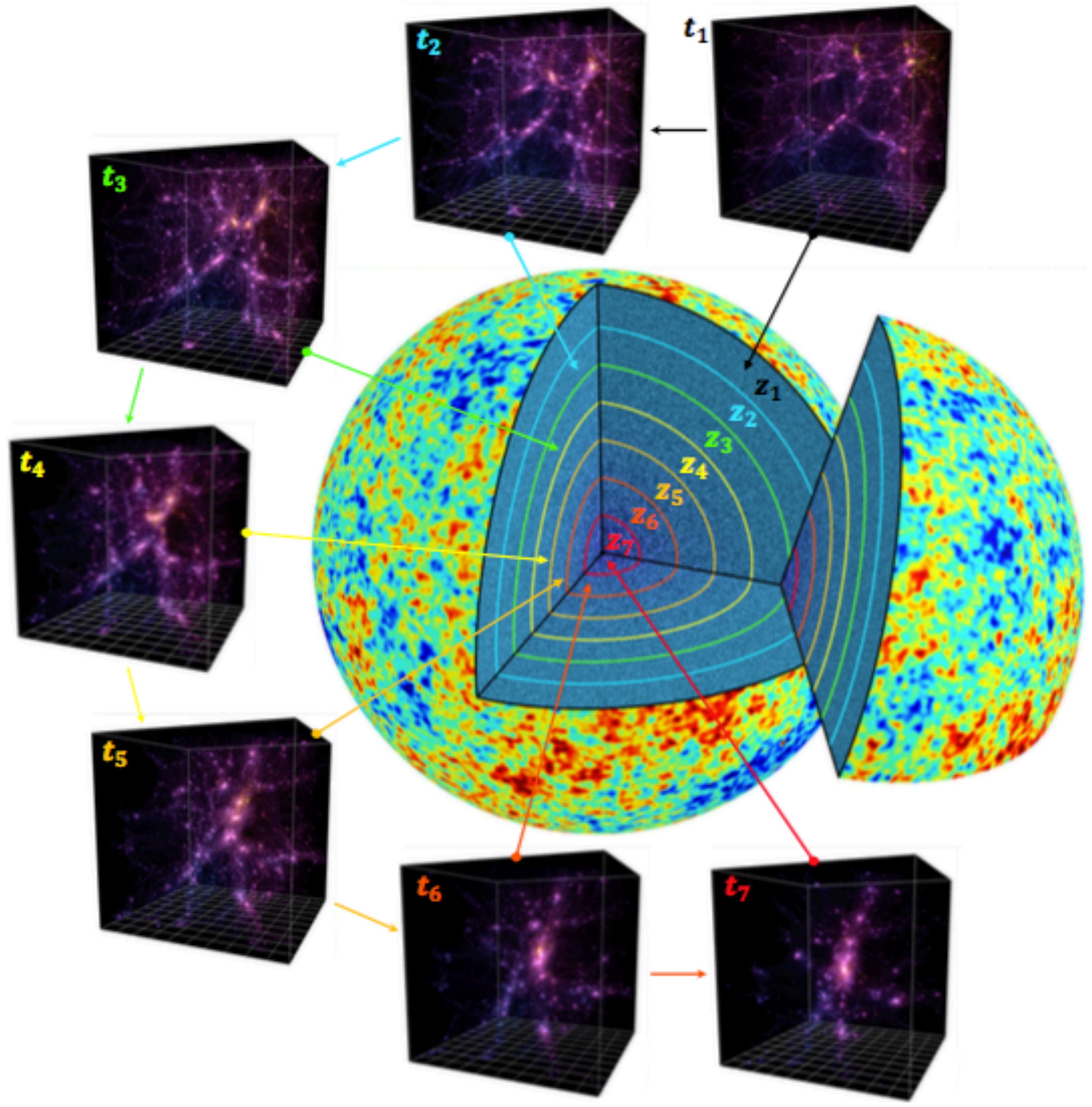
A.Challinor and A.Lewis [1105.5292], C.Bonvin and R.Durrer [1105.5280], J.Yoo [1409.3223], and many works

- observed only when cross-correlating different biased objects
- induced by the Doppler term beyond the plane-parallel limit (wide-angle correction $\xi_1 \sim O((s/d)^n)$)
- proportional to $$\xi_1(s) \propto \Delta b = (b_X - b_Y)$$

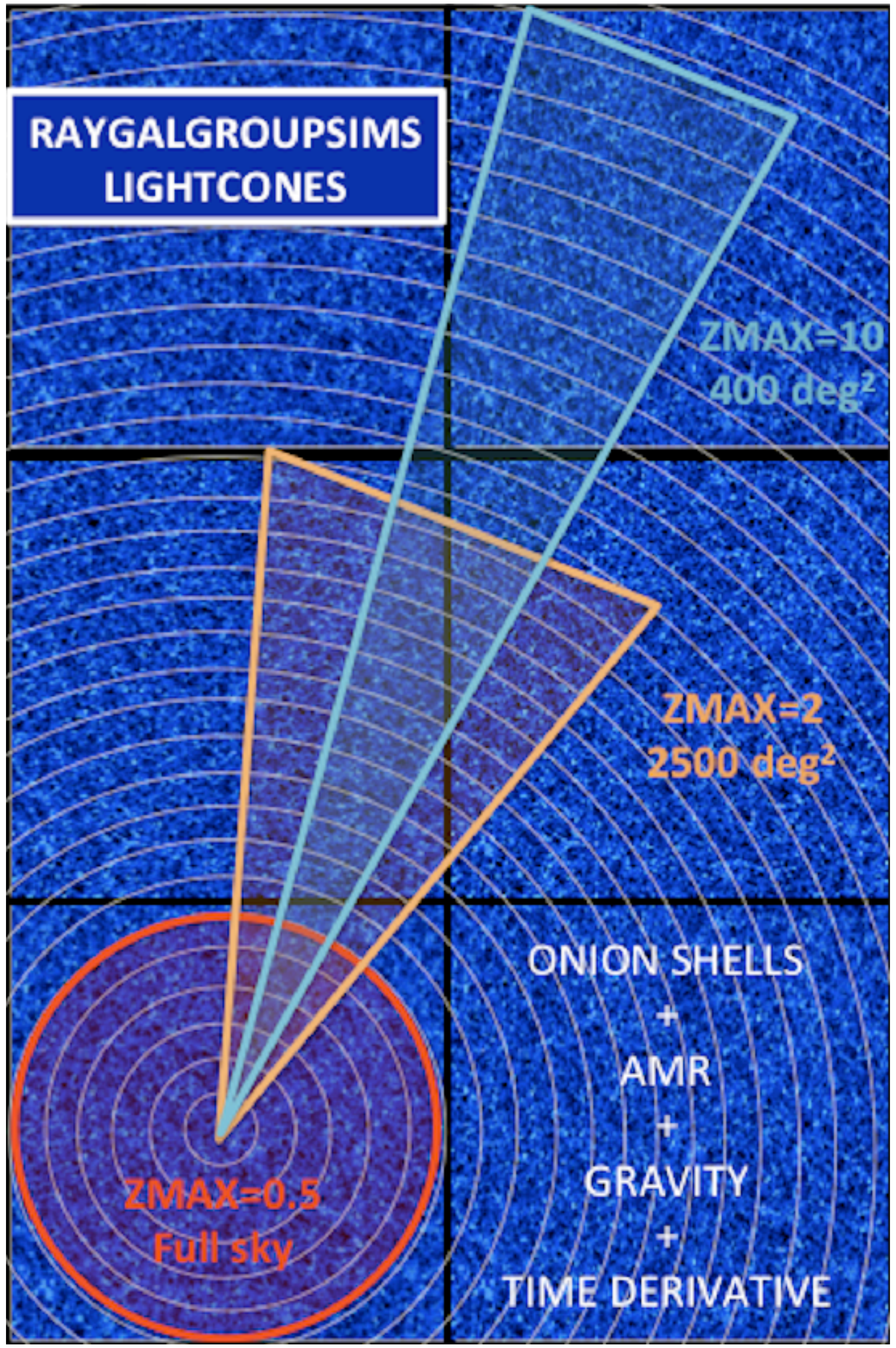
6. Beyond linear regime: RayGalGroupSims (RayGal)

M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, S.Saga [1803.04294]
Y.Rasera, M-A.Breton, ..., S.Saga, A.Taruya, ... [2111.08745]

- storing gravitational potential data on light cone
- tracing back the light ray to the source by direct integration of geodesic equation
- the observed (angular) position and redshift

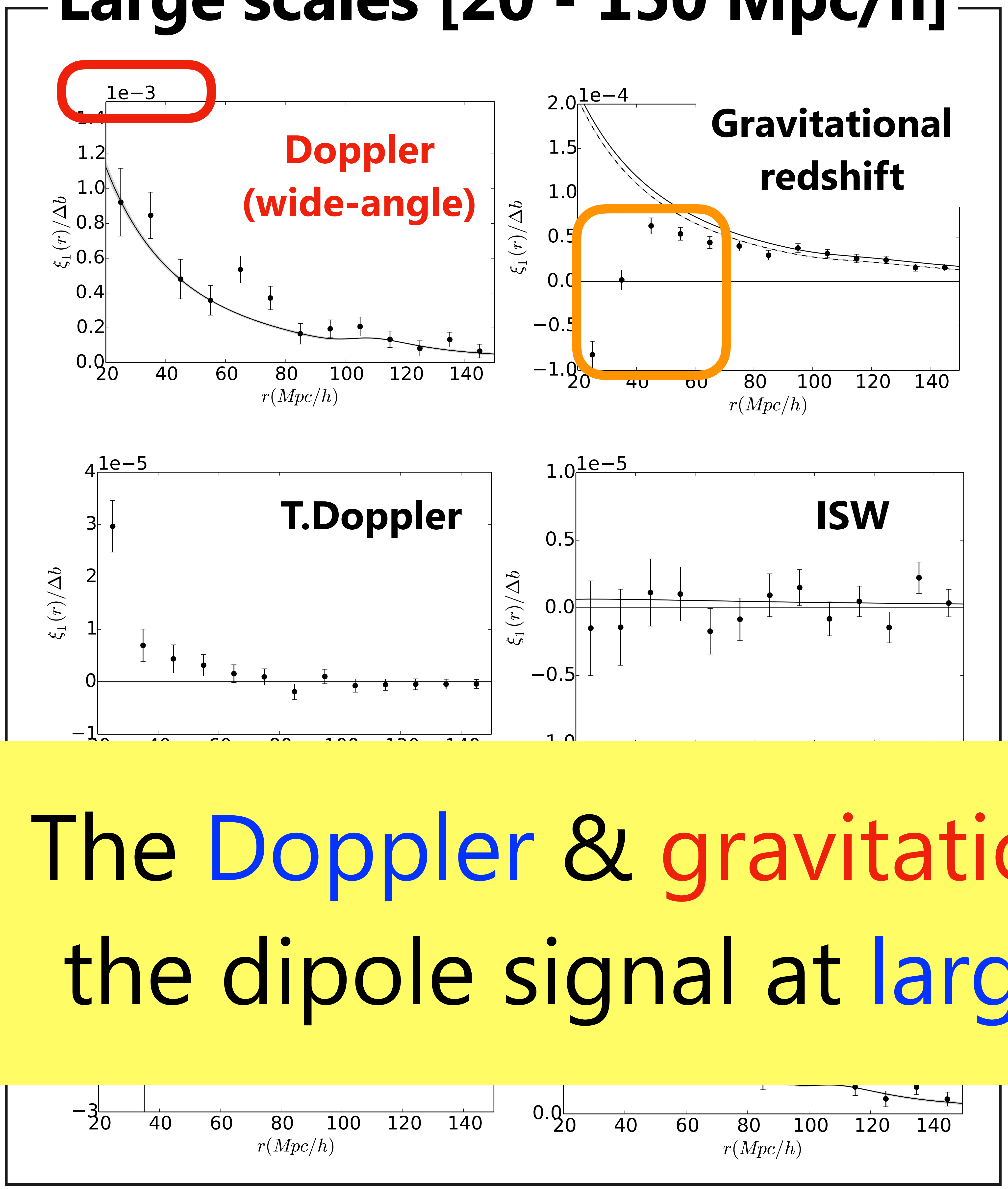


$$1 + z = \frac{\left(g_{\mu\nu}k^\mu k^\nu\right)_{\rm source}}{\left(g_{\mu\nu}k^\mu k^\nu\right)_{\rm observer}}$$

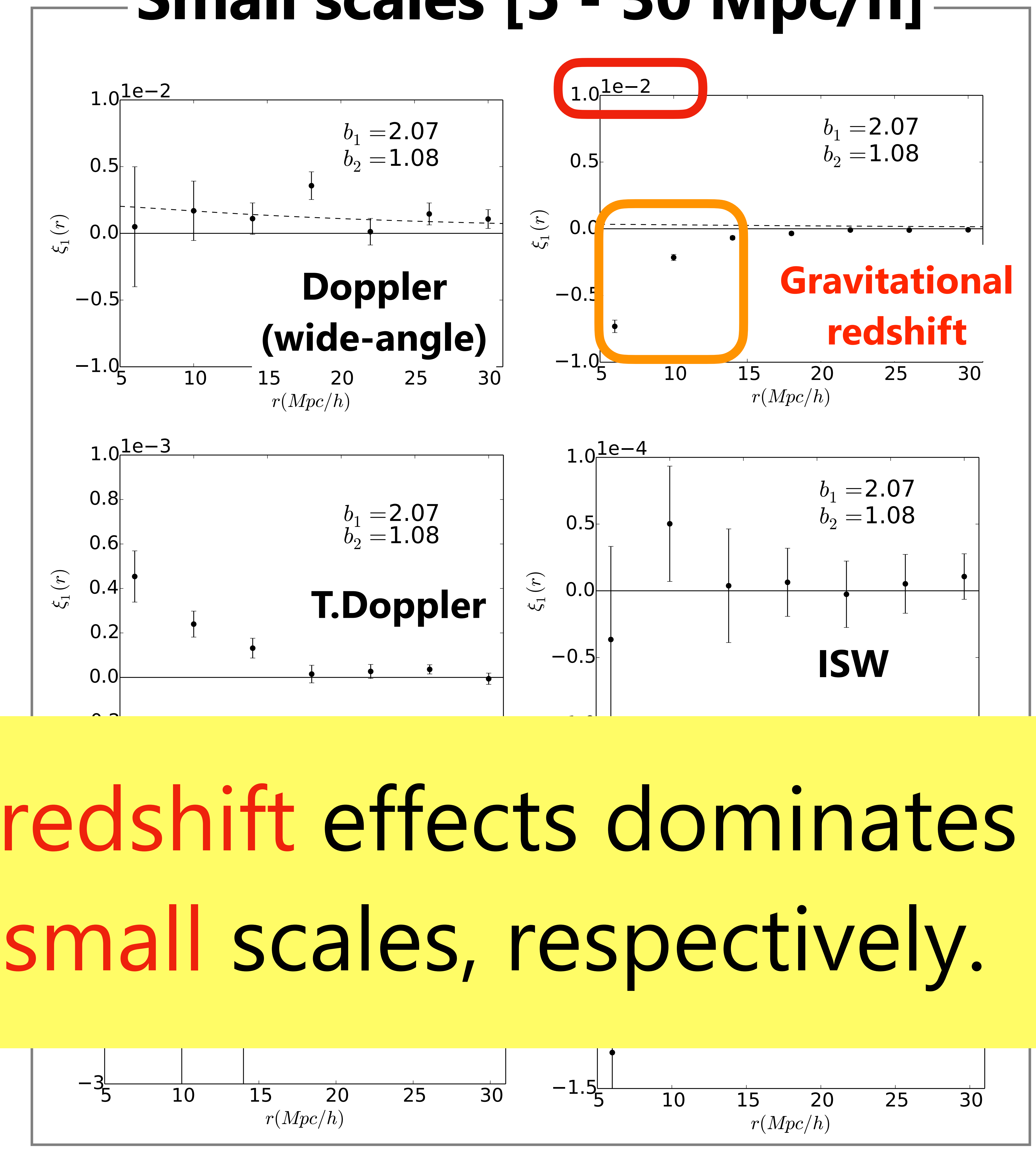


7. Measurements in RayGalGroupSims (z=0.34)

Large scales [20 - 150 Mpc/h]



Small scales [5 - 30 Mpc/h]



The Doppler & gravitational redshift effects dominates the dipole signal at large & small scales, respectively.

8. Our model

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]

Doppler & gravitational redshift effects dominates the dipole signal at large & small scales, respectively.

$$\boldsymbol{s} = \boldsymbol{r} + \frac{1+z}{H}(\boldsymbol{v}\cdot\hat{\boldsymbol{r}})\hat{\boldsymbol{r}} - \frac{1+z}{H}\Phi \quad \text{~~+ minor relativistic terms~~}$$

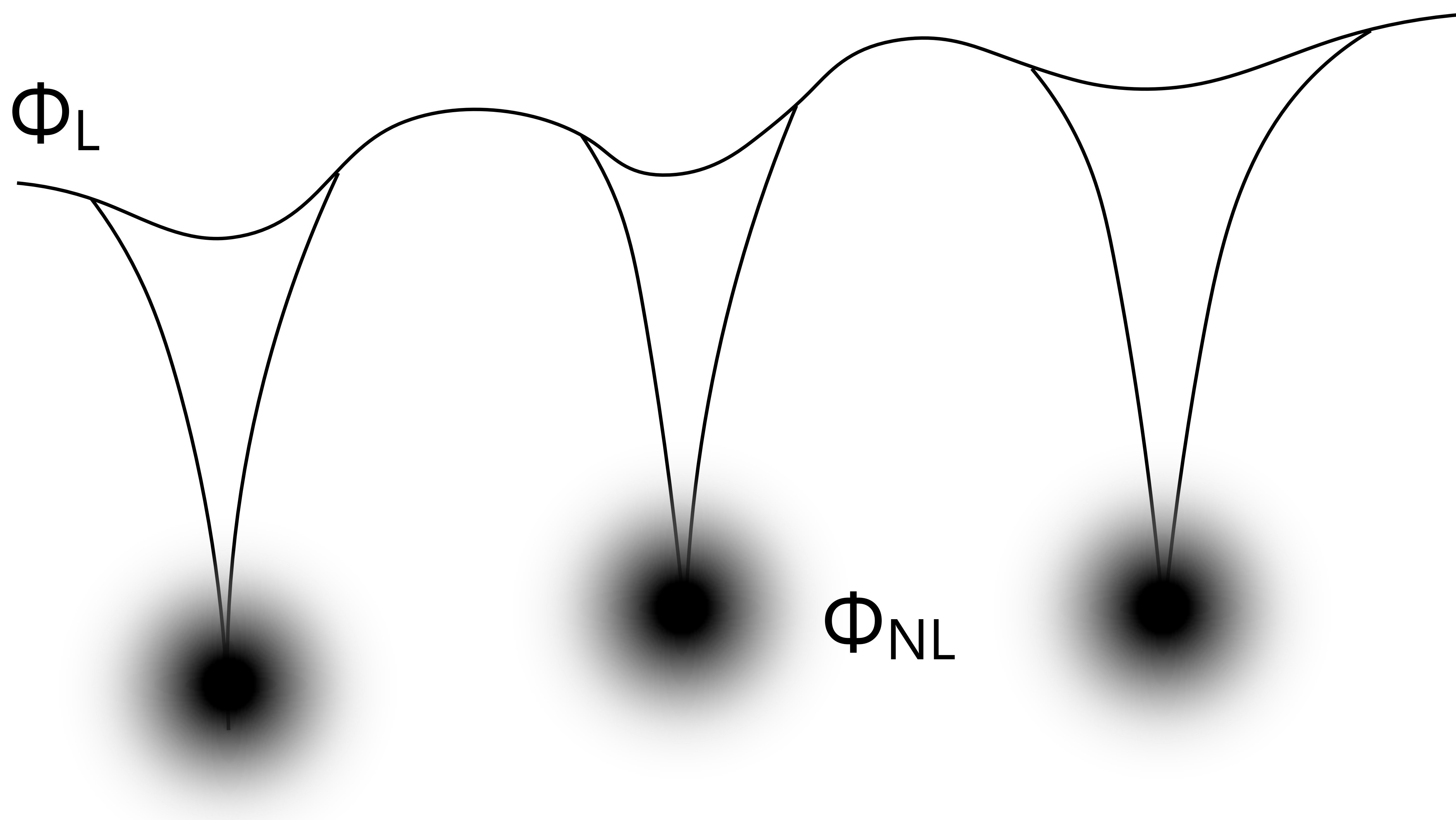
Φ must be modified by the gravitational potential of haloes: $\Phi = \Phi_{\rm L} + \Phi_{\rm halo}$

($\Phi_{\rm halo}(M, z)$ is estimated by using the NFW profile)

Our model for redshift-space density field

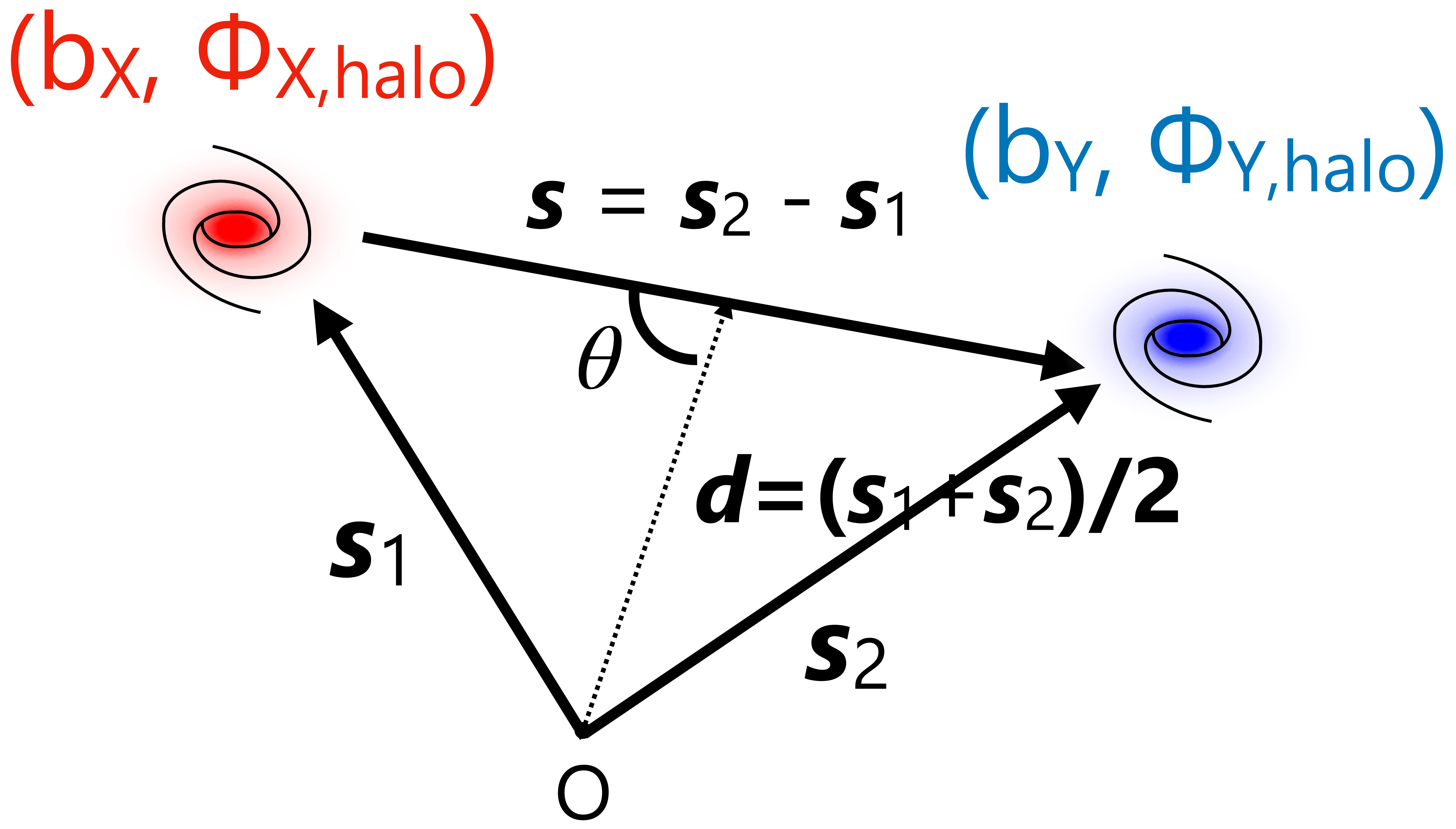
$$\delta = \delta^{(\rm real)} + \delta^{(\rm Doppler)} + \delta^{(\rm grav)} + \delta^{(\rm halo)}$$

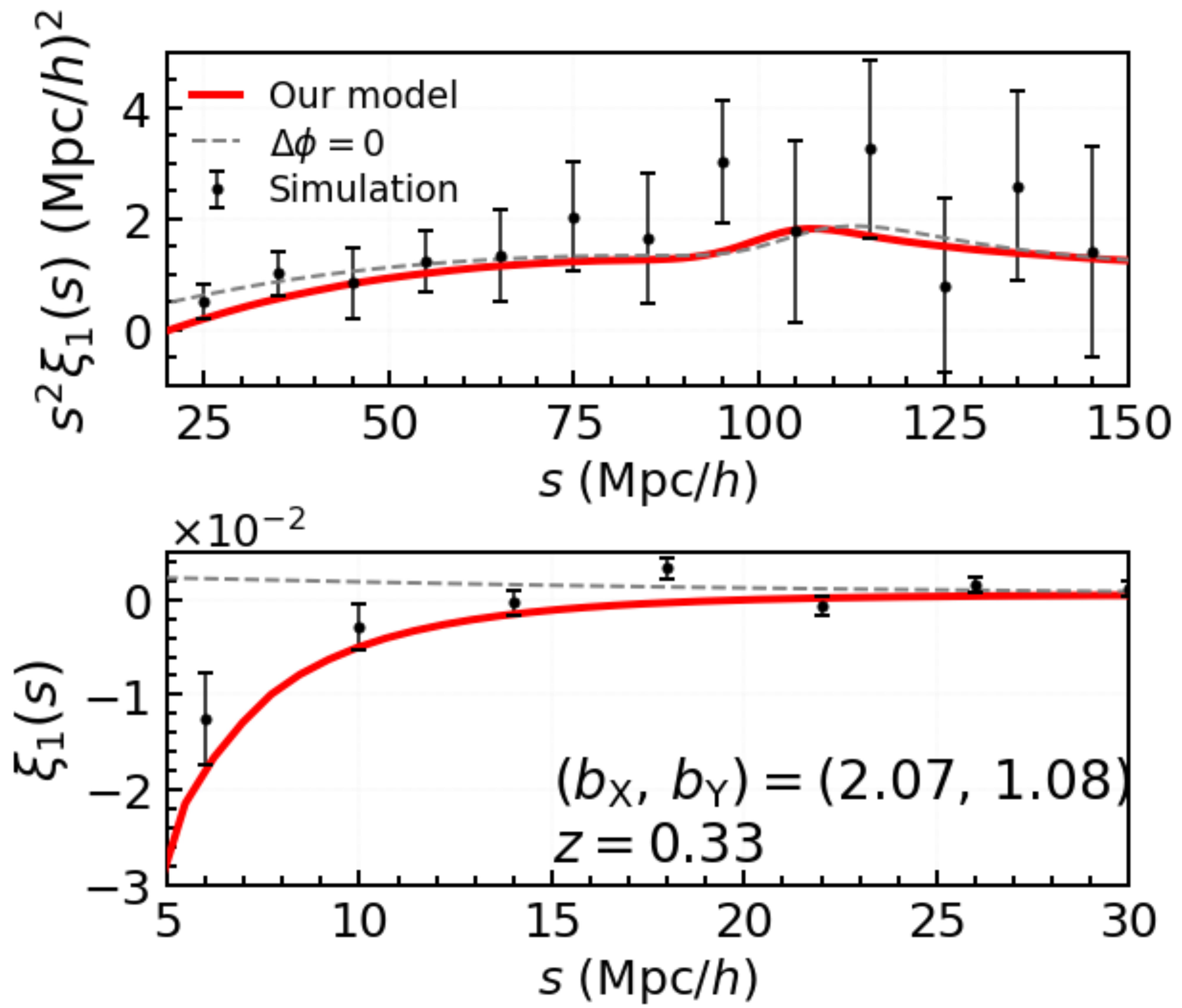
$\delta^{(\rm Doppler)} \leftrightarrow v$, $\delta^{(\rm grav)} \leftrightarrow \Phi_{\rm L}$, $\delta^{(\rm halo)} \leftrightarrow \Phi_{\rm halo}$



9. Our model

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]





Analytical results : the dipole amplitude is proportional to

- $\Delta b = (b_X - b_Y)$ for Doppler & linear potential contributions
- $\Delta\Phi = (\Phi_{X,halo} - \Phi_{Y,halo})$ for halo potential contribution

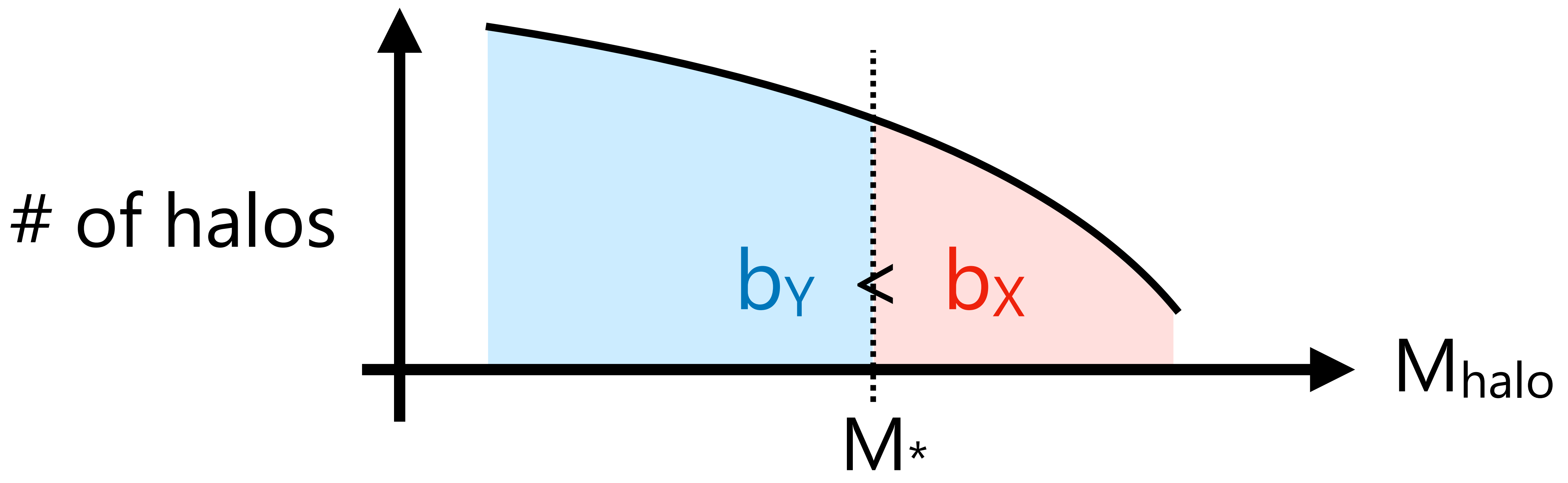
10. Future detectability

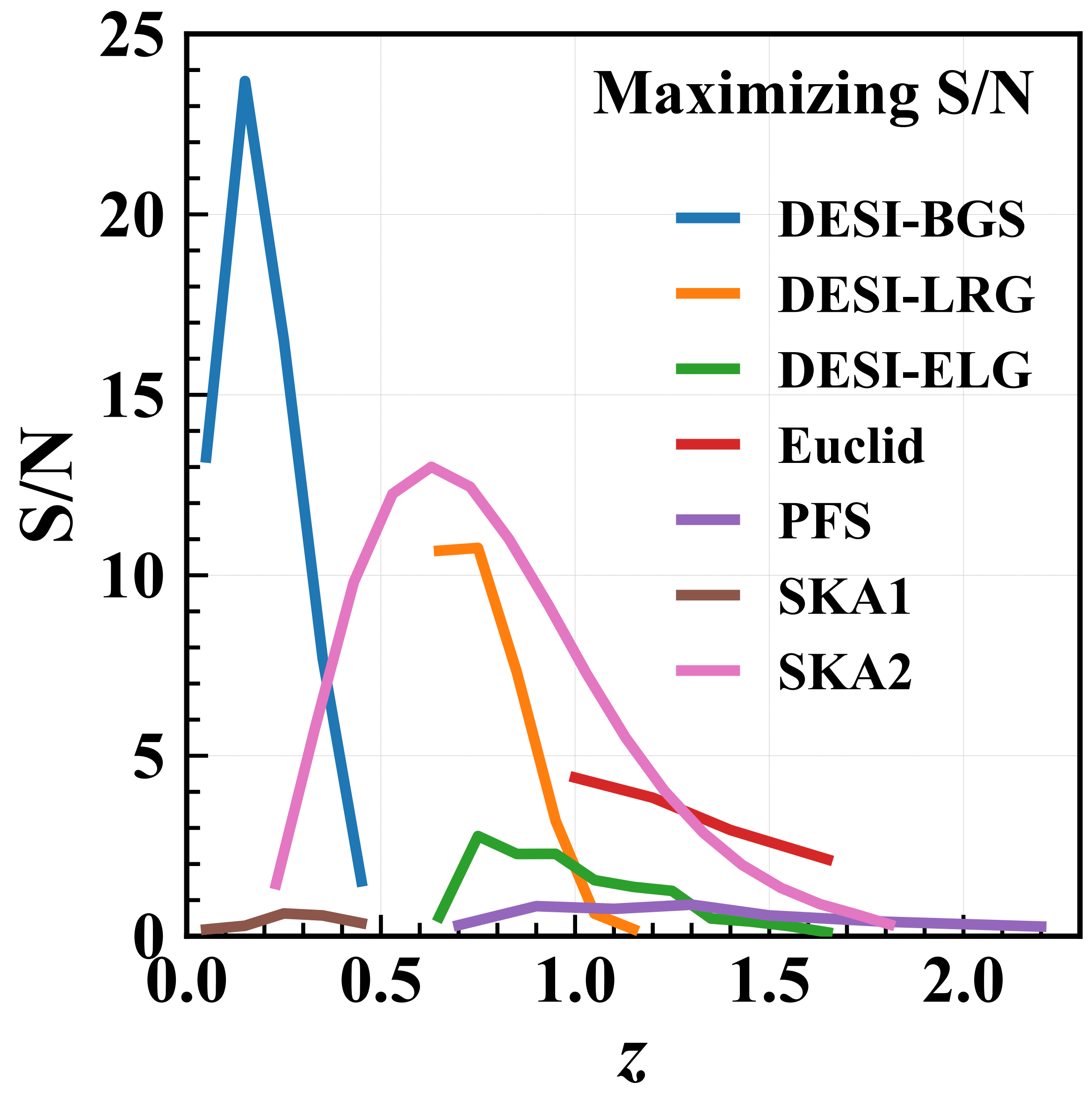
Cross-correlating two different targets ⇒ non-zero dipole

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2109.06012]

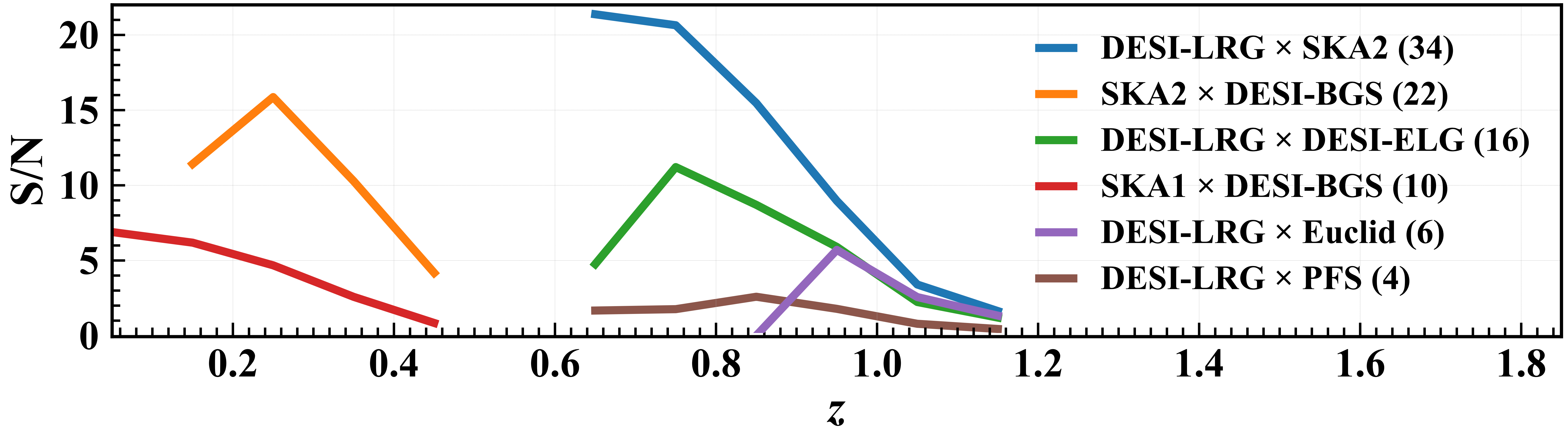
(1) split a sample into at least two subsamples

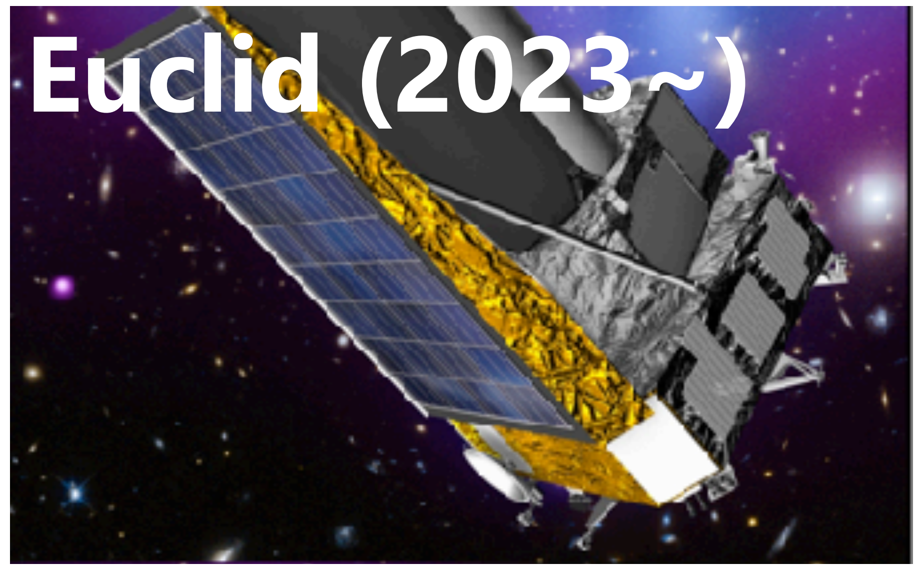
no unique way, for example, we assume galaxies follow the halo distribution

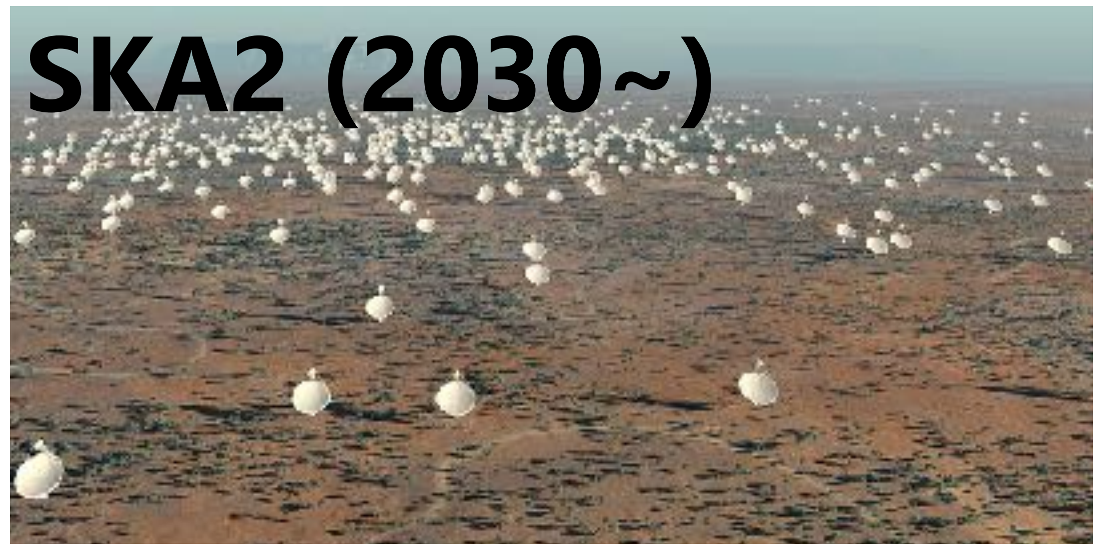




(2) combining two surveys







c.f. 2.8σ detection in BOSS galaxies

S. Alam et al. [1709.07855]

11. Dipole in galaxy-galaxy cross-correlation

Dipole signal can be a new probe of gravity theory

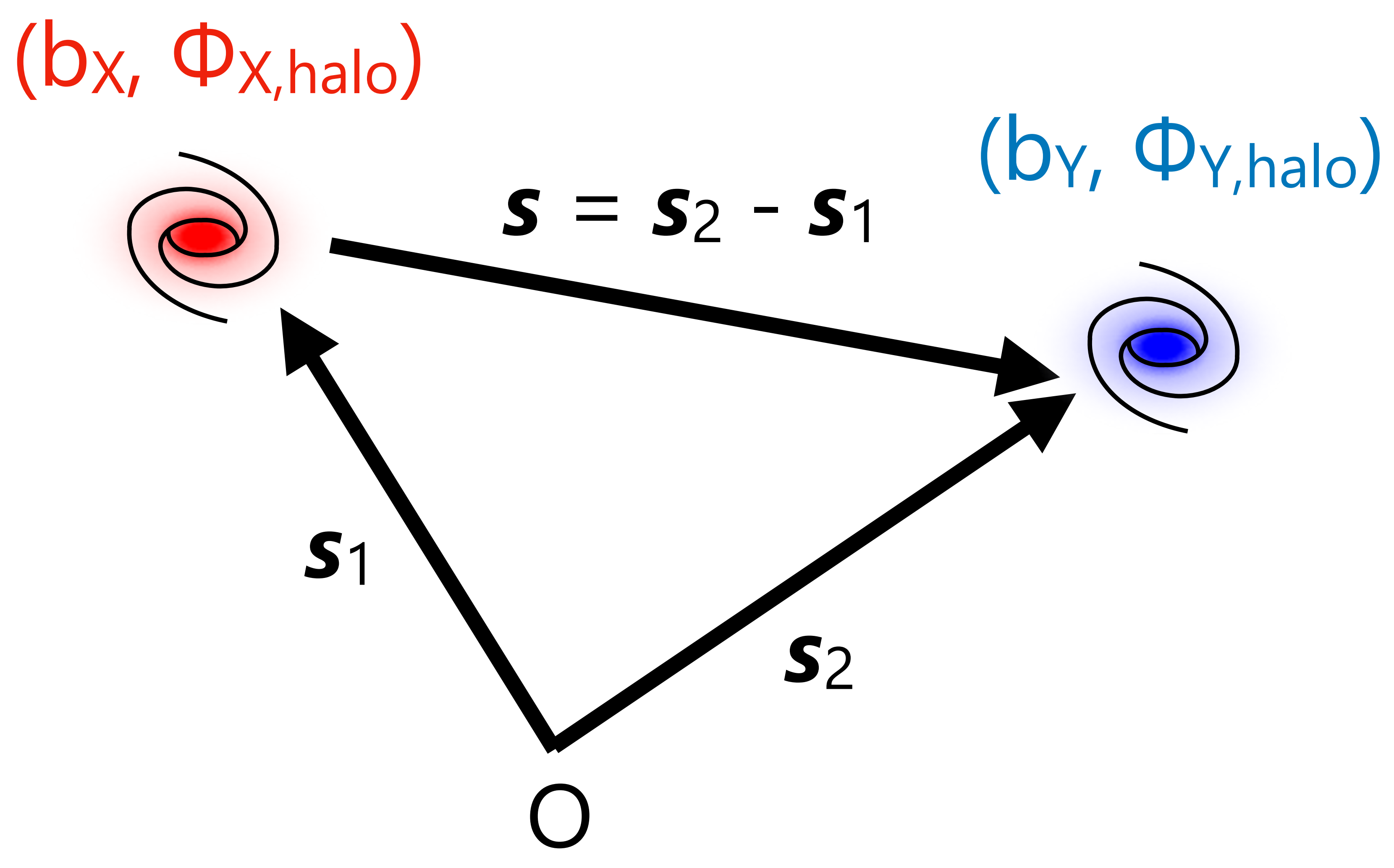
- S/N > 10−20
- test of equivalence principle
- modified gravity

C.Bonvin, F-.O. Franco, P.Fleury [2004.06457]
D. Sobral-Blanco, C.Bonvin [2102.05086]
S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2112.07727]
C.Bonvin, L.Pogosian [2209.03614]
and many works.

We need at least 2 samples to observe non-zero dipole

- no unique way to split, contamination, uncertainty

$$\xi_{XY,1} \sim (b_X - b_Y)(\cdots) + (\Phi_{X,\rm halo} - \Phi_{Y,\rm halo})(\cdots)$$



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1. Galaxy density-ellipticity correlations

$$\xi = \langle \delta(s_1)\gamma_{+/\times}(s_2) \rangle$$

P.Catelan et al. (2001)
C.M.Hirata & U.Seljak (2004)

Our model
(Doppler + gravitational redshift)

$$\delta = \delta^{(\mathrm{real})} + \delta^{(\mathrm{Doppler})} + \delta^{(\mathrm{grav})} + \delta^{(\mathrm{halo})}$$

$\delta^{(\mathrm{Doppler})} \leftrightarrow v$, $\delta^{(\mathrm{grav})} \leftrightarrow \Phi_L$, $\delta^{(\mathrm{halo})} \leftrightarrow \Phi_{\mathrm{halo}}$

Linear alignment model

Galaxy shape projection onto a plane perpendicular to the **non-fixed** LOS:

$$\gamma^{\mathrm{I}}_{ij}(\boldsymbol{x}) = b_{\mathrm{K}} \left[P_{ik}(\hat{\boldsymbol{x}}) P_{jl}(\hat{\boldsymbol{x}}) - \frac{1}{2} P_{ij}(\hat{\boldsymbol{x}}) P_{kl}(\hat{\boldsymbol{x}}) \right] \left(\frac{\partial_i \partial_j}{\partial^2} - \frac{1}{3}\delta_{ij} \right) \delta_{\mathrm{L}}(\boldsymbol{x})$$

$$P_{ij}(\hat{\boldsymbol{x}}) \equiv \delta_{ij} - \hat{x}_i \hat{x}_j$$

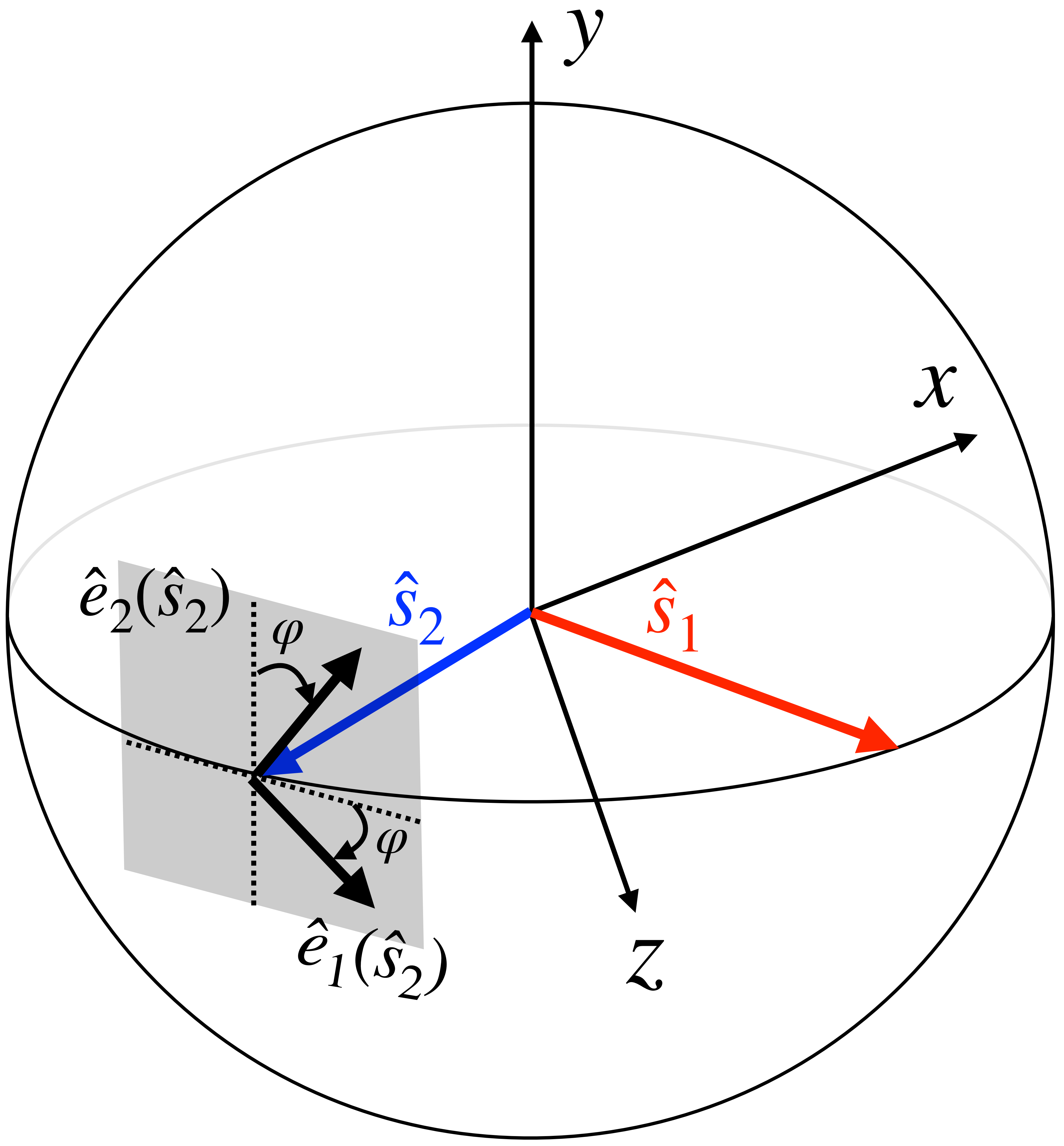
Two independent components:

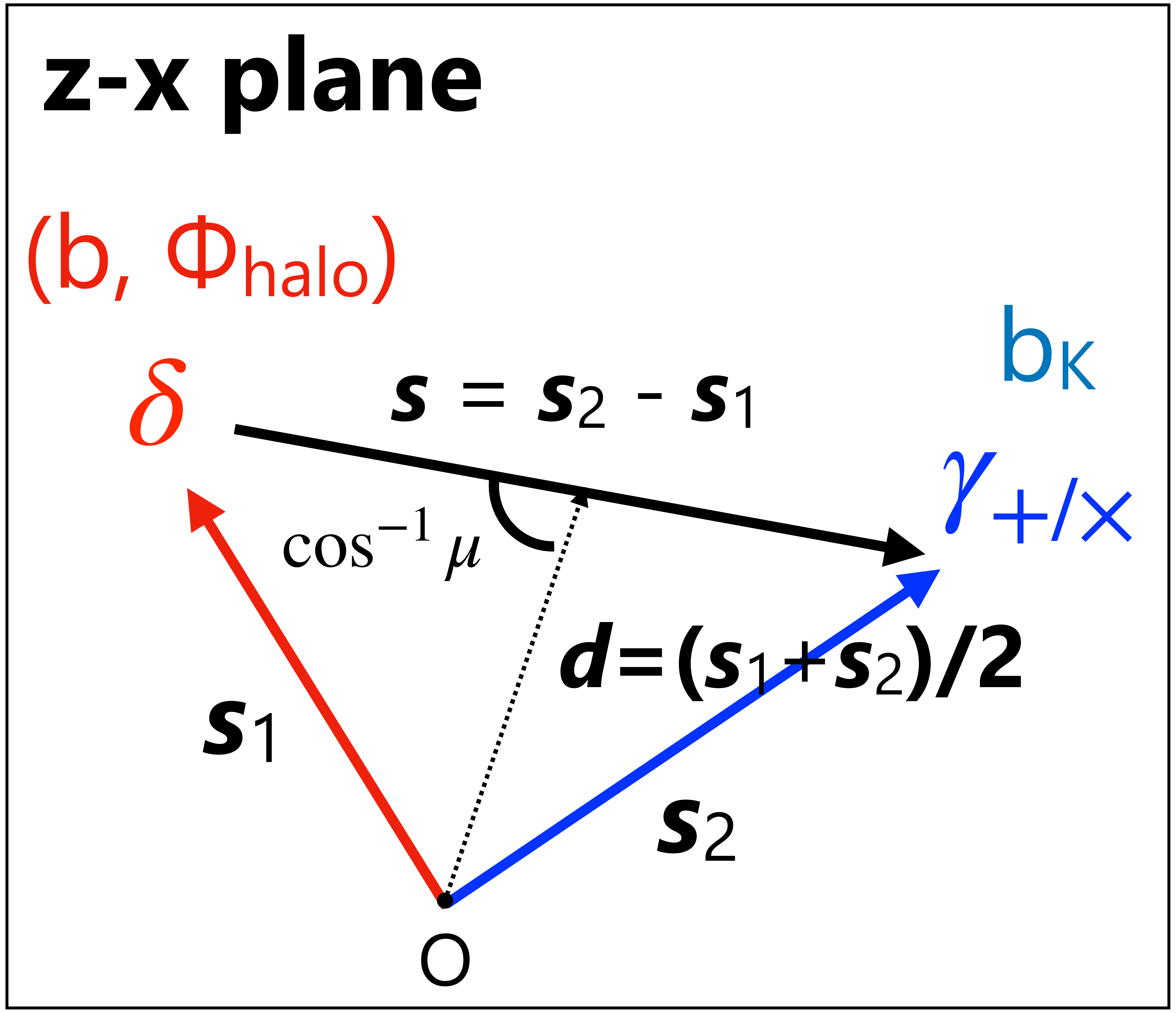
$$\begin{pmatrix} \gamma_+(\boldsymbol{x}) \\ \gamma_\times(\boldsymbol{x}) \end{pmatrix} = \begin{pmatrix} \hat{e}_{1i}(\hat{x})\hat{e}_{1j}(\hat{x}) - \hat{e}_{2i}(\hat{x})\hat{e}_{2j}(\hat{x}) \\ 2\hat{e}_{1i}(\hat{x})\hat{e}_{2j}(\hat{x}) \end{pmatrix} \gamma^{\mathrm{I}}_{ij}(\boldsymbol{x})$$

$$\xi = \langle \delta(s_1)\gamma_+(s_2) \rangle = \langle \delta^{(\mathrm{real})}(s_1)\gamma_+(s_2) \rangle + \langle \delta^{(\mathrm{Doppler})}(s_1)\gamma_+(s_2) \rangle + \langle \delta^{(\mathrm{grav})}(s_1)\gamma_+(s_2) \rangle + \langle \delta^{(\mathrm{halo})}(s_1)\gamma_+(s_2) \rangle$$

$$\equiv \xi^{(\mathrm{real})} + \xi^{(\mathrm{Doppler})} + \xi^{(\mathrm{grav})} + \xi^{(\mathrm{halo})}$$

2. Configurations





$$\xi(s, d, \mu, \varphi) = \langle \delta(s_1)\gamma_{+/\times}(s_2) \rangle \qquad \varphi = 0 \longrightarrow \langle \delta(s_1)\gamma_{\times}(s_2) \rangle = 0$$

3. Analytical results

S.Saga, T.Okumura, A.Taruya, T.Inoue [2207.03454]

We decomposed the contributions: $\xi \equiv \xi^{(\mathrm{real})} + \xi^{(\mathrm{Doppler})} + \xi^{(\mathrm{grav})} + \xi^{(\mathrm{halo})}$

Non-zero multipoles		even multipoles	odd multipoles
(real) (Doppler)	plane-parallel	$\xi_0^{(\mathrm{real})}$ $\xi_2^{(\mathrm{real})}$ $\xi_0^{(\mathrm{Doppler})}$ $\xi_2^{(\mathrm{Doppler})}$ $\xi_4^{(\mathrm{Doppler})}$ (T.Okumura & A.Taruya [1912.04118])	0
	wide-angle	0	$\xi_1^{(\mathrm{real})}$ $\xi_3^{(\mathrm{real})}$ $\xi_1^{(\mathrm{Doppler})}$ $\xi_3^{(\mathrm{Doppler})}$
(grav) (halo)	plane-paralle	0	$\xi_1^{(\mathrm{grav})}$ $\xi_3^{(\mathrm{grav})}$ $\xi_1^{(\mathrm{halo})}$ $\xi_3^{(\mathrm{halo})}$ $\xi_5^{(\mathrm{halo})}$
	wide-angle	$\xi_0^{(\mathrm{grav})}$ $\xi_2^{(\mathrm{grav})}$ $\xi_4^{(\mathrm{grav})}$ $\xi_0^{(\mathrm{halo})}$ $\xi_2^{(\mathrm{halo})}$ $\xi_4^{(\mathrm{halo})}$ $\xi_6^{(\mathrm{halo})}$	0

All terms are $\xi \propto (1-\mu^2)$ → Sum of their multipoles vanishes

3. Analytical results

S.Saga, T.Okumura, A.Taruya, T.Inoue [2207.03454]

We decomposed the contributions into four pieces: $\xi \equiv \xi^{(\mathrm{real})} + \xi^{(\mathrm{Doppler})} + \xi^{(\mathrm{grav})} + \xi^{(\mathrm{halo})}$

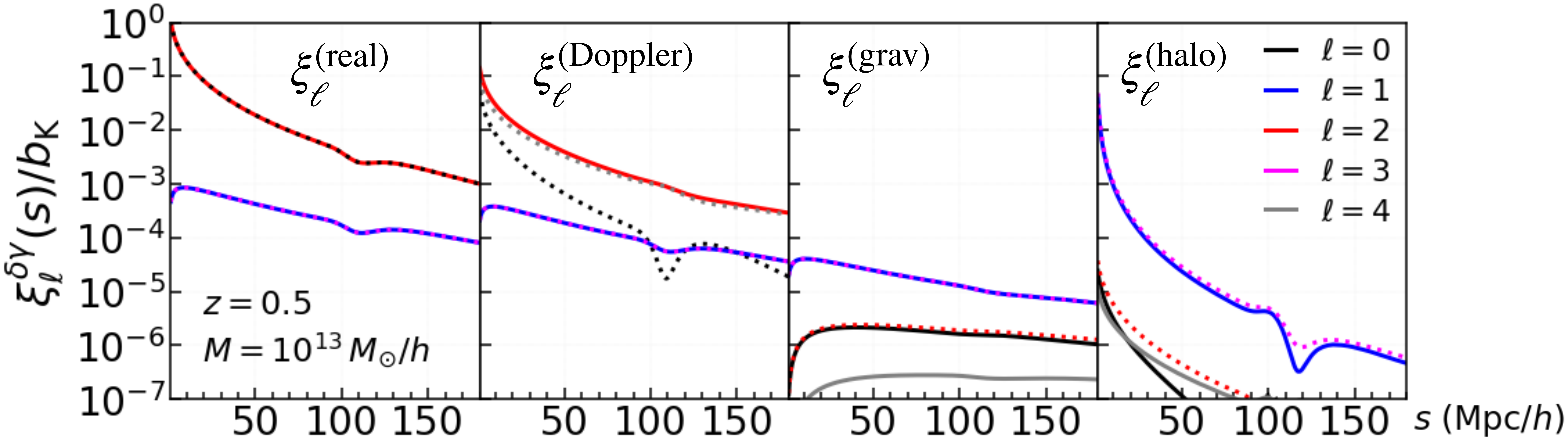
Non-zero multipoles		even multipoles	odd multipoles
(real) (Doppler)	plane-parallel	$\xi_0^{(\mathrm{real})} + \xi_2^{(\mathrm{real})} = 0$ $\xi_0^{(\mathrm{Doppler})} + \xi_2^{(\mathrm{Doppler})} + \xi_4^{(\mathrm{Doppler})} = 0$	0
			$\xi_1^{(\mathrm{real})} + \xi_3^{(\mathrm{real})} = 0$ $\xi_1^{(\mathrm{Doppler})} + \xi_3^{(\mathrm{Doppler})} = 0$
(grav) (halo)			$\xi_1^{(\mathrm{grav})} + \xi_3^{(\mathrm{grav})} = 0$ $\xi_1^{(\mathrm{halo})} + \xi_3^{(\mathrm{halo})} + \xi_5^{(\mathrm{halo})} = 0$
	wide-angle	$\xi_0^{(\mathrm{grav})} + \xi_2^{(\mathrm{grav})} + \xi_4^{(\mathrm{grav})} = 0$ $\xi_0^{(\mathrm{halo})} + \xi_2^{(\mathrm{halo})} + \xi_4^{(\mathrm{halo})} + \xi_6^{(\mathrm{halo})} = 0$	0

$$\sum_{\ell=\mathrm{all}} \xi_\ell^{(\mathrm{all})} = \sum_{\ell=\mathrm{even}} \xi_\ell^{(\mathrm{all})} = \sum_{\ell=\mathrm{odd}} \xi_\ell^{(\mathrm{all})} = 0$$

All terms are $\xi \propto (1-\mu^2)$ → Sum of their multipoles vanishes

4. Multipoles

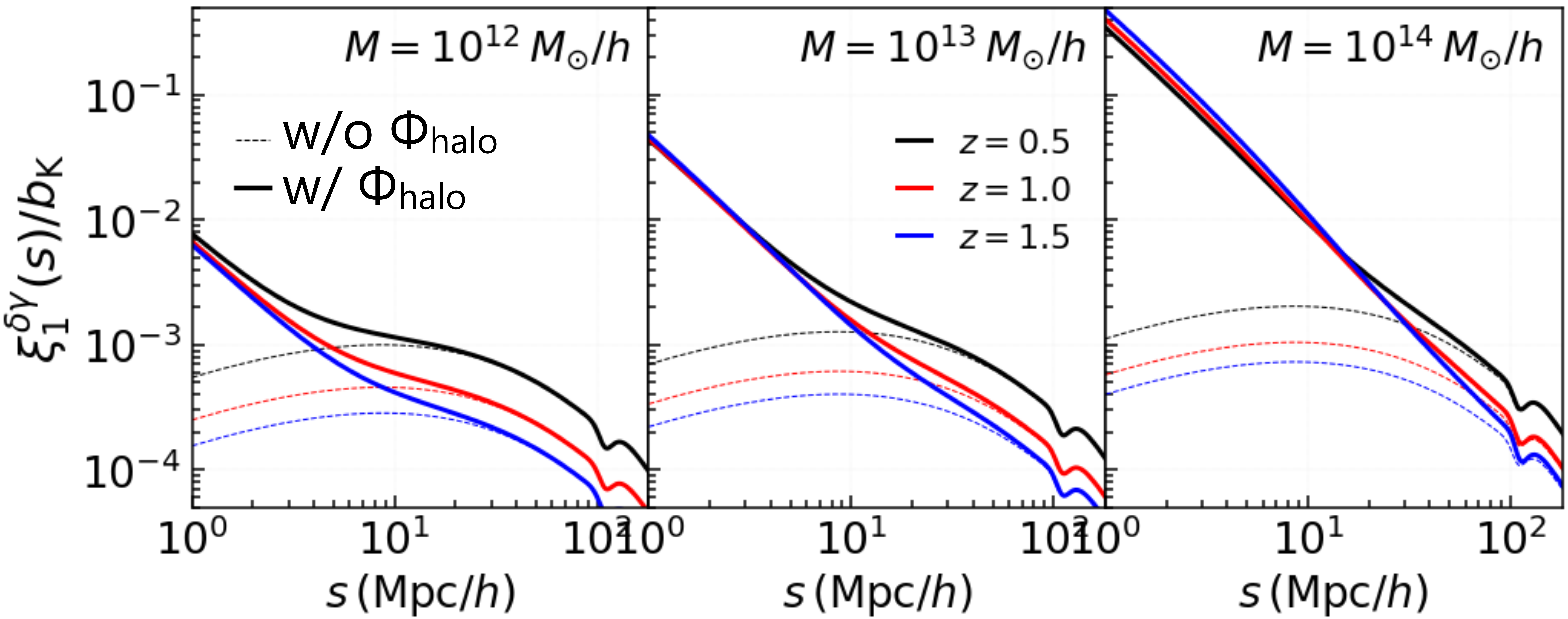
S.Saga, T.Okumura, A.Taruya, T.Inoue [2207.03454]



		even multipoles	odd multipoles
real **Doppler**	plane-parallel	dominant at all scales	0
	wide-angle	0	dominant at large scales
~~**grav**~~ **halo**	plane-parallel	0	dominant at small scales
	wide-angle	subdominant	0

5. Dipole moment

S.Saga, T.Okumura, A.Taruya, T.Inoue [2207.03454]



6. Dipole covariance

S.Saga, T.Okumura, A.Taruya, T.Inoue [2207.03454]

Schematically...

$$b_K = -0.0134\, A_{IA}\Omega_{m0}/D_+(z)$$

$$\mathrm{COV}_1(s_1, s_2) \sim \frac{1}{V}\sum_{\ell,\ell'}\left(\xi^{gg}_{\ell}\times\xi^{II}_{\ell'} + \xi^{gI}_{\ell}\times\xi^{gI}_{\ell'}\right)$$

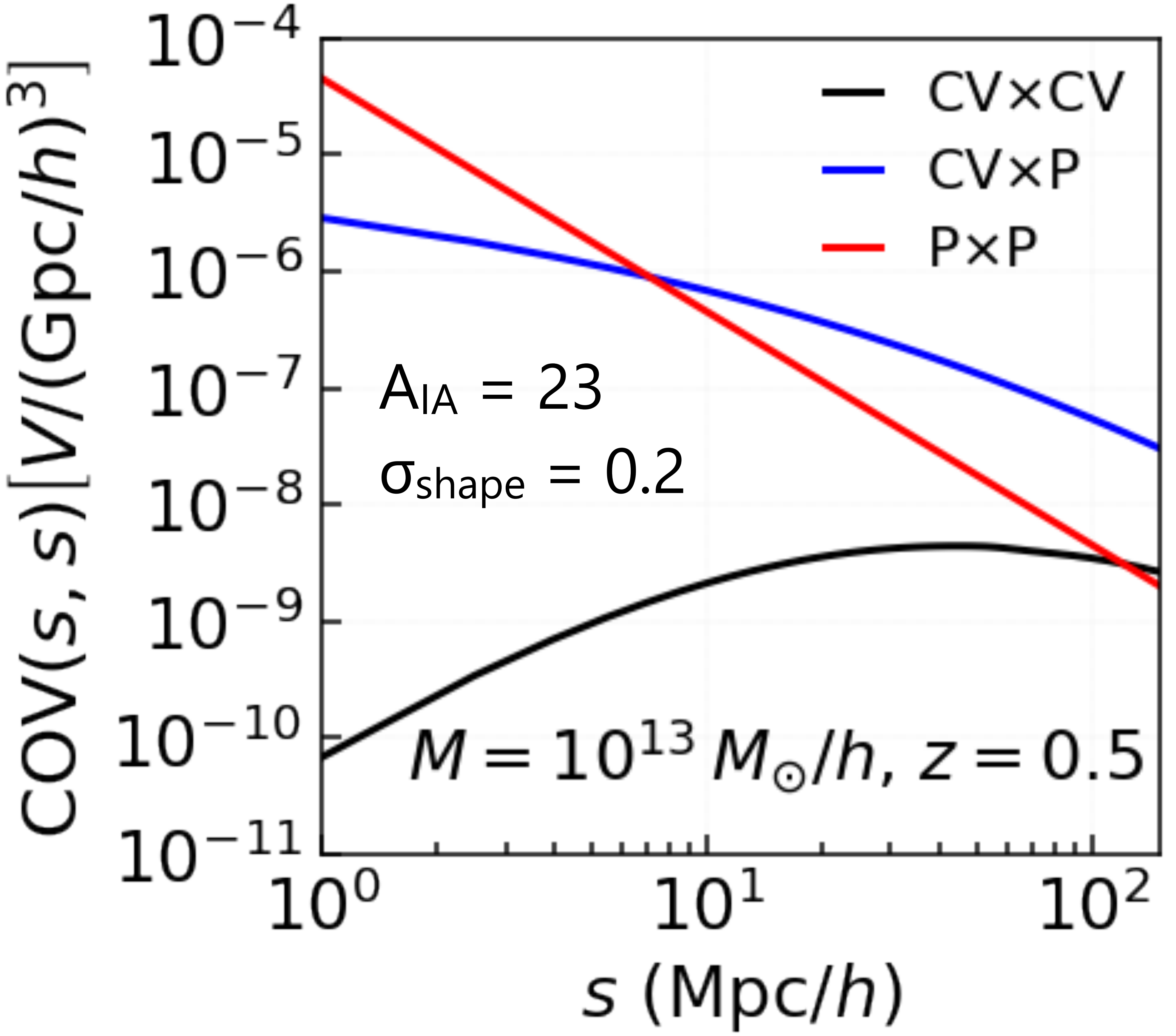
cosmic variance x cosmic variance (CVxCV)

$$+\frac{1}{V}\sum_{\ell'}\left(\frac{1}{n_g}\times\xi^{II}_{\ell'} + \frac{\sigma^2_{\rm shape}}{n_g}\times\xi^{gg}_{\ell'}\right)$$

cosmic variance × Poisson (CVxP)

$$+\frac{1}{V}\frac{1}{n_g}\frac{\sigma^2_{\rm shape}}{n_g}$$

Poisson x Poisson (PxP)

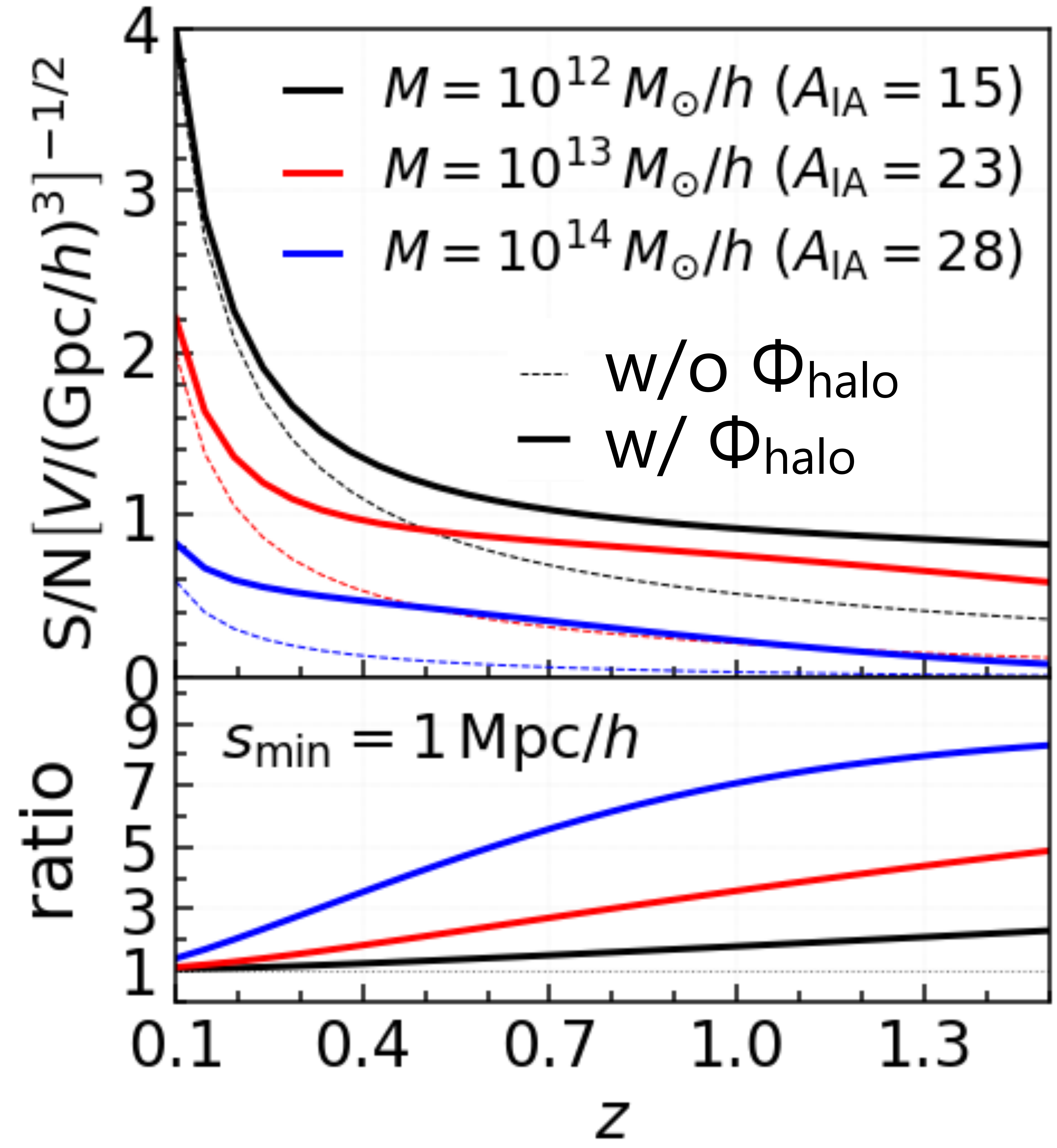


Note: we treat carefully the angular dependence by using $Y_{\ell,m}(\theta,\varphi)$ in computing the covariance matrix.

$$\xi = \langle\delta(s_1)\gamma_{+/\times}(s_2)\rangle \propto \sin 2\varphi \text{ or } \cos 2\varphi$$

7. Signal-to-noise ratio

S.Saga, T.Okumura, A.Taruya, T.Inoue [2207.03454]



$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)^2 = \sum_{s_1,s_2=s_{min}}^{s_{max}} \xi_1(s_1)(\mathrm{COV}_1(s_1,s_2))^{-1}\xi_1(s_2)$$

$(s_{min}, s_{max}) = (1, 150)$ Mpc/h

bias&number density: Sheth&Tormen (1999)

A_{IA} is chosen to match Kurita et al (2020)

$$b_K = -0.0134\, A_{IA}\Omega_{m0}/D_+(z)$$

SN reaches ~ 1–4 × [volume in $(\mathrm{Gpc/h})^3]^{1/2}$

Summary

Dipole anisotropy in galaxy-galaxy correlations

- we need two populations (SN reaches ~ 10−25)

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [2004.03772]
S.Saga, A.Taruya, Y.Rasera, M-A.Breton [2109.06012]

Dipole anisotropy in galaxy-IA correlations

- single galaxy populations + shape information
- rough estimates suggest SN ~ 1−4 × [volume in $(\mathrm{Gpc}/h)^3]^{1/2}$

S.Saga, T.Okumura, A.Taruya, T.Inoue [2207.03454]

Future prospects

- SN for specific surveys, systematic effects
- galaxy-galaxy cross-correlation + galaxy-IA cross-correlation
- test of gravity theory — T.Inoue et al. [in prep.]
- measurements in RayGalGroupSims
- dipole of another correlation?

$$\xi = \langle \delta(s_1) \square (s_2) \rangle$$