Statistical properties of galaxy shapes with angular dependent Primordial Non-Gaussianities

- Intrinsic galaxy alignment from angular dependent primordial non-Gaussianity (18) (1804.06284) K.Kogai, T. Matsubara, A.J. Nishizawa and Y. Urakawa
- Galaxy imaging surveys as spin-sensitive detector for cosmological colliders (21) (2009.05517) K. Kogai, K. Akitsu, F. Schmidt and Y. Urakawa

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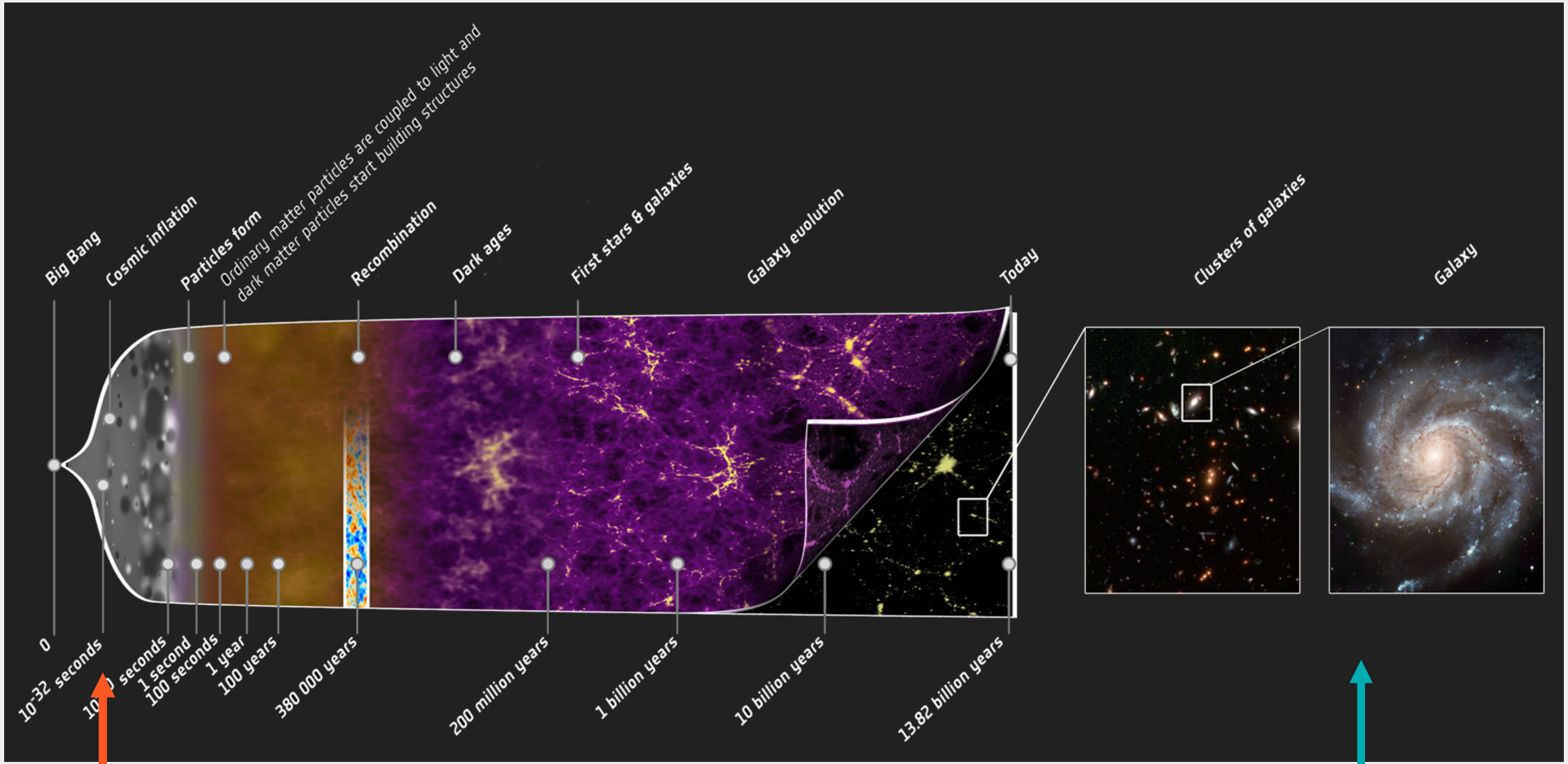
Image:NASA



Generation of (initial) fluctuation during inflation

Time evolution

Structure formation

Primordial Non-Gaussianity (PNG)

Primoradial perturbation characterizes physics during inflation

If Primordial perturbation is Gaussian, the statistic is only the power spectrum.

$$\langle \Phi(\boldsymbol{k})\Phi(\boldsymbol{k}')\rangle = (2\pi)^3\delta_{\mathrm{D}}(\boldsymbol{k}+\boldsymbol{k}')P_{\Phi}(k)$$

If non-Gaussian, the bispectrum characterizes the leading effect of the deviation from Gaussian

$$\langle \Phi(\boldsymbol{k}_1)\Phi(\boldsymbol{k}_2)\Phi(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta_{\mathrm{D}}(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_{\Phi}(k_1,k_2,k_3)$$

The bispectrum is a good indicator of the features of physics during inflation.

Local type PNGs

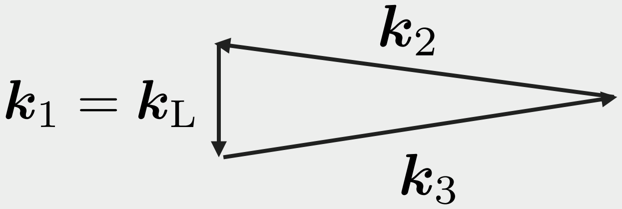
Simple case $\Phi(\boldsymbol{x}) = \Phi_{\rm G}(\boldsymbol{x}) + f_{\rm NL}^{\rm loc}(\Phi_{\rm G}(\boldsymbol{x})^2 - \langle \Phi_{\rm G}^2 \rangle)$

Bispectrum $B_\Phi(k_1, k_2, k_3) = 2 f_{\rm NL}^{\rm loc}[P(k_1)P(k_2) + \text{perms.}]$

$$\boldsymbol{k}_{\rm L} = \boldsymbol{k}_1$$
$$\boldsymbol{k}_{\rm S} = \boldsymbol{k}_2 - \boldsymbol{k}_{\rm L}/2$$
$$k_{\rm L} \ll k_2 \sim k_3$$

The local type bispectrum has the peak in squeezed limit

$$B_\Phi(k_1, k_2, k_3) \simeq 4 f_{\rm NL}^{\rm loc} P(k_{\rm L}) P(k_{\rm S})$$



Indicator to distinguish the inflation models

Single-field inflation $f_{\rm NL}^{\rm loc} \approx 0$ Maldacena(03), ...

Multi-field inflation $|f_{\rm NL}^{\rm loc}| > 0$ Lyth+(03), ...

Planck result $f_{\rm NL}^{\rm loc} = -0.9 \pm 5.1$

Property of Local type PNGs

We can understand by splitting the perturbation into short and long wavelength modes.

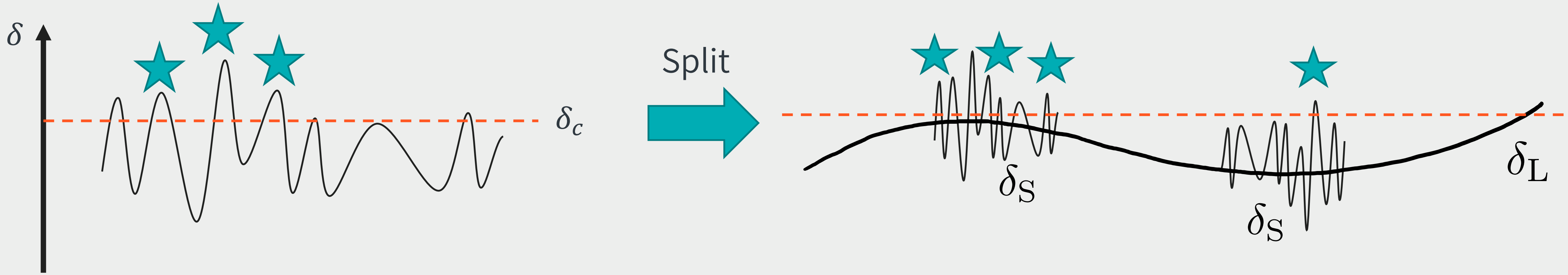
Gaussian case $\Phi = \Phi_{\mathrm{G,S}} + \Phi_{\mathrm{G,L}}$

Local type non-Gaussian $\Phi_{\mathrm{S}} = \Phi_{\mathrm{G,S}} + f_{\mathrm{NL}}^{\mathrm{loc}}(\Phi_{\mathrm{G,S}}^2 - \langle \Phi_{\mathrm{G,S}}^2 \rangle) + 2 f_{\mathrm{NL}}^{\mathrm{loc}} \Phi_{\mathrm{G,L}} \Phi_{\mathrm{G,S}}$

The short-mode power spectrum depends on the position $P_\Phi(\boldsymbol{k}_{\mathrm{S}}; \boldsymbol{x}) = [1 + 4 f_{\mathrm{NL}}^{\mathrm{loc}} \Phi_{\mathrm{G,L}}(\boldsymbol{x})] P_\Phi(k_s)$

Galaxy (halo) Bias

Halo forms when δ is larger than the collapse threshold δ_c

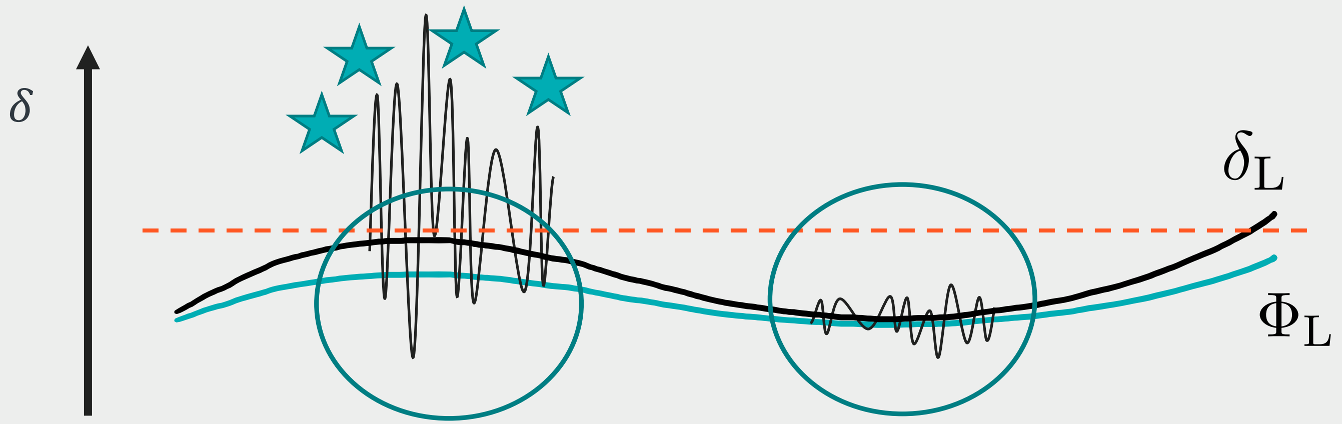


Halo is easy to forms on the top of the long-wavelength density perturbation

$$\delta_{\rm n}(\boldsymbol{x}) = b_\delta \delta_{\rm L}(\boldsymbol{x}) + \mathcal{O}(\delta^2) \qquad b_\delta^{(0)} = \frac{d\delta_{\rm n}}{d\delta_{\rm L}}$$

Scale-dependent bias

Dalal+(07)



The short-wavelength mode power spectrum also depends on the long-wavelength mode $\Phi_{\rm L}$

$$P_\delta(\boldsymbol{k}_{\rm S};\boldsymbol{x}) = [1+4f_{\rm NL}^{\rm loc}\Phi_{\rm G,L}(\boldsymbol{x})]P_\delta(k_s)$$

Number density contrast

$$\delta_{\rm n}(\boldsymbol{k}) = b_\delta\delta_{\rm L}(\boldsymbol{k}) + 4b_\Phi f_{\rm NL}^{\rm loc}\Phi_{\rm L}(\boldsymbol{k}) + \mathcal{O}(\delta^2)$$
$$= [b_\delta + 4b_\Phi f_{\rm NL}^{\rm loc}\mathcal{M}^{-1}(k)]\delta_{\rm L}(\boldsymbol{k}) + \mathcal{O}(\delta^2)$$

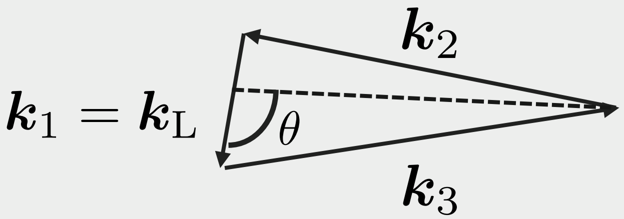
$$\mathcal{M}^{-1} \propto k^{-2}(k\to 0)$$

Poisson Eq.

$$\delta_{\rm m}(\boldsymbol{k}) = \mathcal{M}(k)\Phi(\boldsymbol{k})$$

$$\mathcal{M}(k) \propto k^2\ (k\to 0)$$

Angular dependent PNGs



Arkani-Hamed&Maldacena(15)

Template of primordial bispectrum in the squeezed limit (soft limit)

$$B_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \simeq \sum_{\ell=0,2,\cdots} A_\ell \mathcal{P}_\ell(\hat{\boldsymbol{k}}_{\rm S} \cdot \hat{\boldsymbol{k}}_{\rm L}) \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{\Delta_\ell} P_\phi(k_{\rm s}) P_\phi(k_{\rm L})$$

$$\boldsymbol{k}_{\rm L} = \boldsymbol{k}_1$$
$$\boldsymbol{k}_{\rm S} = \boldsymbol{k}_2 - \boldsymbol{k}_{\rm L}/2$$
$$k_{\rm L} \ll k_2 \sim k_3$$
$$\cos\theta \equiv \hat{\boldsymbol{k}}_{\rm S} \cdot \hat{\boldsymbol{k}}_{\rm L}$$

$$\mathcal{P}_\ell(\hat{\boldsymbol{k}}_{\rm S} \cdot \hat{\boldsymbol{k}}_{\rm L})$$

If spin-ℓ particles exists during inflation, the ℓ-th Legendre polynomial appears

If massive particles, the scale dependence and oscillation appears

$$\left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{3/2 \pm i\nu_\ell} \quad \rightarrow \quad \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{3/2} \cos\left(\nu_\ell \ln(k_{\rm L}/k_{\rm S}) + \psi_\ell\right)$$

ν_ℓ : parameter depending on the mass and spin

Shape bias

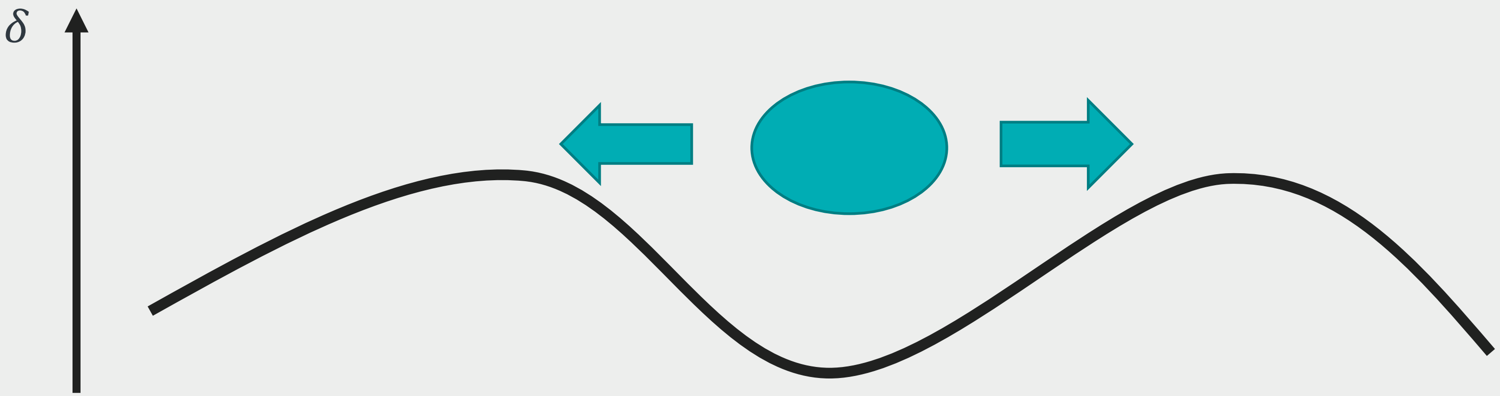
In a scheme similar to the galaxy number density,
Galaxy shape may respond to the tidal field of large scale structure.

Catelan+(00)

$$g_{ij}(\boldsymbol{x}) = b_K K_{ij}(\boldsymbol{x})$$

$$K_{ij}(\boldsymbol{x}) = \left(\frac{\partial_i \partial_j}{\nabla^2} - \frac{1}{3}\delta_{ij}\right)\delta(\boldsymbol{x})$$

The galaxy shape is a bias tracer of the tidal field.



Galaxy shape and Angular dependent PNGs

Schmidt+(15)

Primordial bispectrum in the squeezed limit

$$B_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = A_2 \mathcal{P}_2(\hat{\boldsymbol{k}}_\mathrm{S} \cdot \hat{\boldsymbol{k}}_\mathrm{L}) P_\phi(k_\mathrm{s}) P_\phi(k_\mathrm{L})$$

→ The short-mode power spectrum has the dependency on position.

$$P_\mathrm{m}(\boldsymbol{k}_\mathrm{S}; \boldsymbol{x}) = [1 + A_2 \alpha_{ij}(\boldsymbol{x}) \hat{k}^i_\mathrm{S} \hat{k}^j_\mathrm{S}] P_\mathrm{m}(k_s)$$

$$\alpha_{ij}(\boldsymbol{x}) = \frac{3}{2} \int \frac{d^3 \boldsymbol{k}_\mathrm{L}}{(2\pi)^3} [\hat{k}_\mathrm{L,i} \hat{k}_\mathrm{L,j}]^\mathrm{TL} \Phi_\mathrm{L,G}(\boldsymbol{k}_\mathrm{L}) e^{i \boldsymbol{k}_\mathrm{L} \cdot \boldsymbol{x}}$$

Galaxy shape function also depends on the primordial long-wavelength perturbation.

$$g_{ij}(\boldsymbol{k}) = b_K K_{ij}(\boldsymbol{k}) + b_\alpha A_2 \alpha_{ij}(\boldsymbol{k})$$
$$= [b_K + \frac{3}{2} b_\alpha A_2 \mathcal{M}^{-1}(k)] K_{ij}(\boldsymbol{k})$$

$\propto k^{-2}$ on large scale

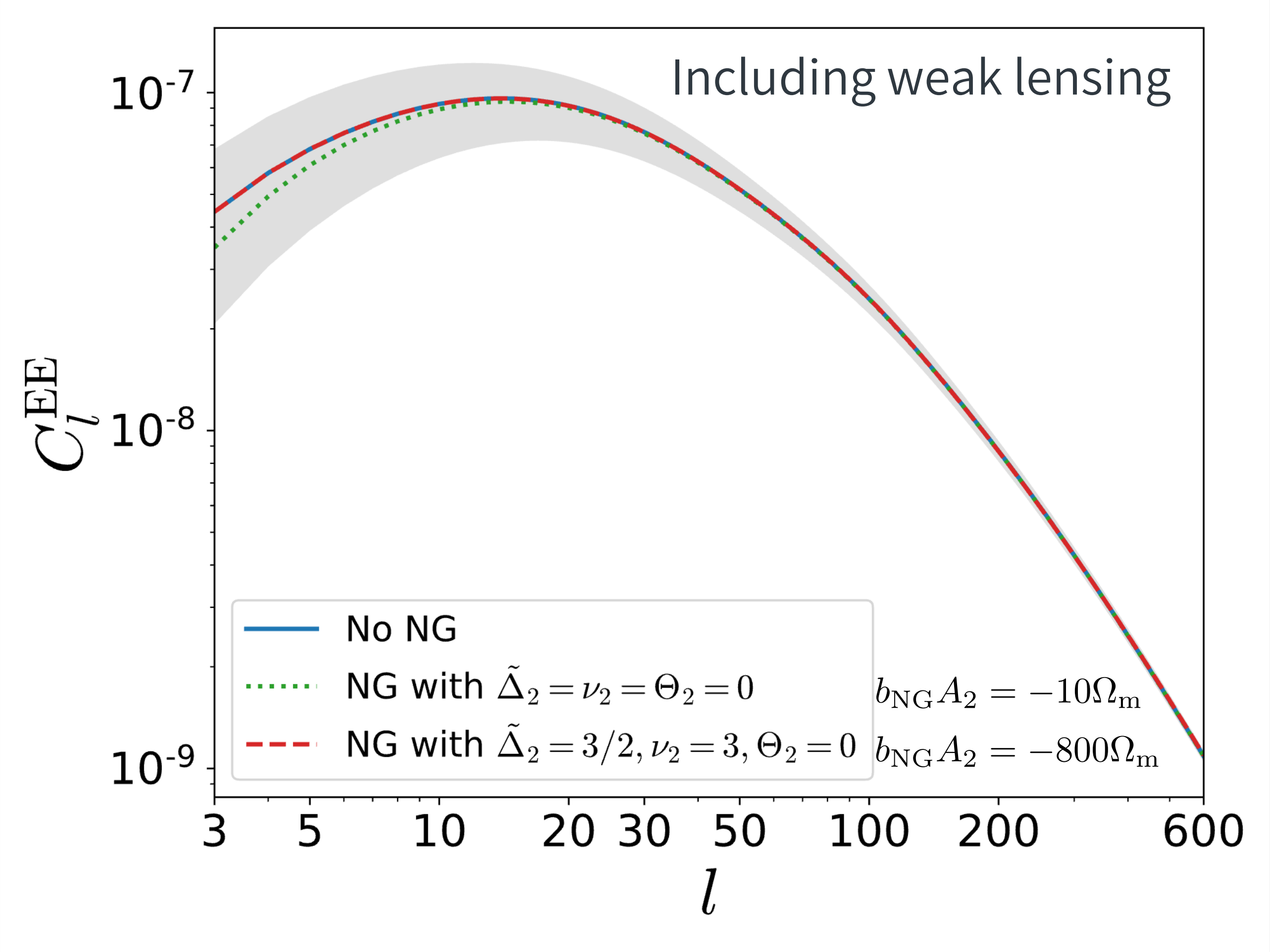
Poisson Eq.

$$\delta_\mathrm{m}(\boldsymbol{k}) = \mathcal{M}(k) \Phi(\boldsymbol{k})$$

$$\mathcal{M}(k) \propto k^2 \; (k \to 0)$$

Angular power spectrum

Schmidt+(15)
KK+(18)



Angular power spectrum

$$C_l = \frac{1}{2\pi}\frac{(l+2)!}{(l-2)!}\int dk k^2 P_{\rm m0}(k)[F(k)]^2$$

Blue (Gaussian)

$$F(k) = \int dz \frac{dN}{dz} D(z)\left[\frac{j_l(k\chi)}{(k\chi)^2}\right][b_K]$$

Green (spin-2 PNG)

$$F(k) = \int dz \frac{dN}{dz} D(z)\left[\frac{j_l(k\chi)}{(k\chi)^2}\right][b_K + 3b_{\rm NG}A_2\mathcal{M}^{-1}(k)]$$

Red (massive spin-2 PNG)

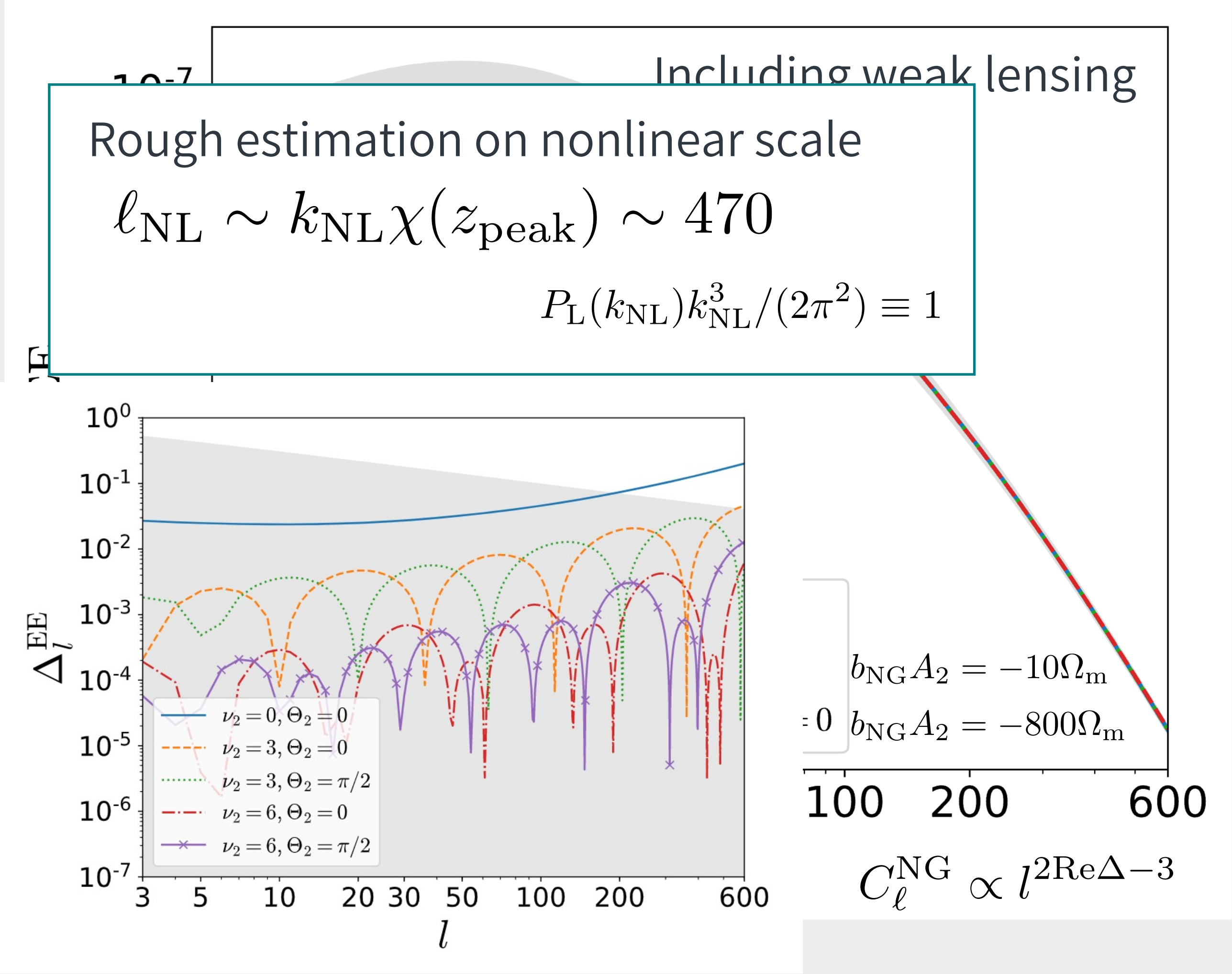
$$F(k) = \int dz \frac{dN}{dz} D(z)\left[\frac{j_l(k\chi)}{(k\chi)^2}\right]\left[b_K + 3b_{\rm NG}A_2\mathcal{M}^{-1}(k)\left(\frac{k}{k_*}\right)^{\frac{3}{2}}\cos(\nu_2\ln(\frac{k}{k_*}) + \Theta_2)\right]$$

Assumptions

$$\frac{dN}{dz} \propto \left(\frac{z}{0.51}\right)^{1.24}\exp\left[-\left(\frac{z}{0.51}\right)^{1.01}\right] \qquad b_K = -0.1\Omega_{\rm m}\left(\frac{D(0)}{D(z)}\right)$$

Angular power spectrum

Schmidt+(15)
KK+(18)



Rough estimation on nonlinear scale

$$\ell_{\rm NL} \sim k_{\rm NL}\chi(z_{\rm peak}) \sim 470$$

$$P_{\rm L}(k_{\rm NL})k_{\rm NL}^3/(2\pi^2) \equiv 1$$

Angular power spectrum

$$C_l = \frac{1}{2\pi}\frac{(l+2)!}{(l-2)!}\int dk k^2 P_{\rm m0}(k)[F(k)]^2$$

Blue (Gaussian)

$$F(k) = \int dz \frac{dN}{dz} D(z) \left[\frac{j_l(k\chi)}{(k\chi)^2}\right][b_K]$$

Green (spin-2 PNG)

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Assumptions

$$\frac{dN}{dz} \propto \left(\frac{z}{0.51}\right)^{1.24}\exp\left[-\left(\frac{z}{0.51}\right)^{1.01}\right] \qquad b_K = -0.1\Omega_{\rm m}\left(\frac{D(0)}{D(z)}\right)$$

Simulation Result

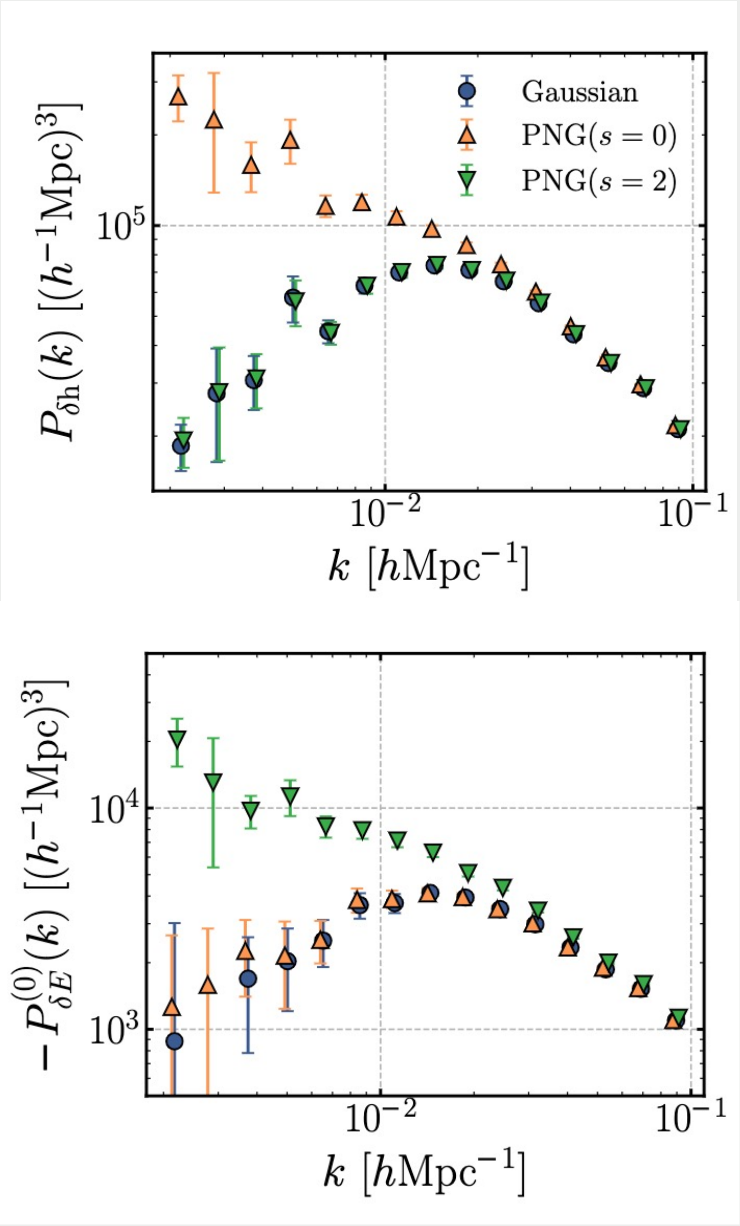
Akitsu+(20)

Matter-halo correlation

$\langle \delta \delta_{\rm n} \rangle$

Matter-shape correlation

$\langle \delta g_{ij} \rangle$



The primordial bispectrum in squeezed limit

Blue circular

$$B_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) = 0$$

Orange triangle

$$B_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \sim f_{\rm NL}^{\rm loc} P_\phi(k_{\rm s}) P_\phi(k_{\rm L})$$

Green Inverse trigonometric

$$B_\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) \sim A_2 \mathcal{P}_2(\hat{\boldsymbol{k}}_{\rm S} \cdot \hat{\boldsymbol{k}}_{\rm L}) P_\phi(k_{\rm s}) P_\phi(k_{\rm L})$$

Angular independent PNG influences on only $\delta_{\rm n}$

Angular dependent PNG influences on only g_{ij}

$\mathcal{P}_2(\hat{\boldsymbol{k}}_{\rm S} \cdot \hat{\boldsymbol{k}}_{\rm L})$

Can we detect the higher multipoles PNGs by using galaxy?

$\mathcal{P}_\ell(\hat{\boldsymbol{k}}_{\rm L} \cdot \hat{\boldsymbol{k}}_{\rm S})\ (\ell = 4, 6, 8, \cdots)$

Shape definition

The surface brightness in 2D polar coordinate

$$I(\boldsymbol{\theta}) = I_0(r) + \sum_{n=2}^{\infty} [c_n(r)\cos(n\phi) + s_n(r)\sin(n\phi)]$$

(Usual) Shear (2nd moment / quadrupole) $n = 2$

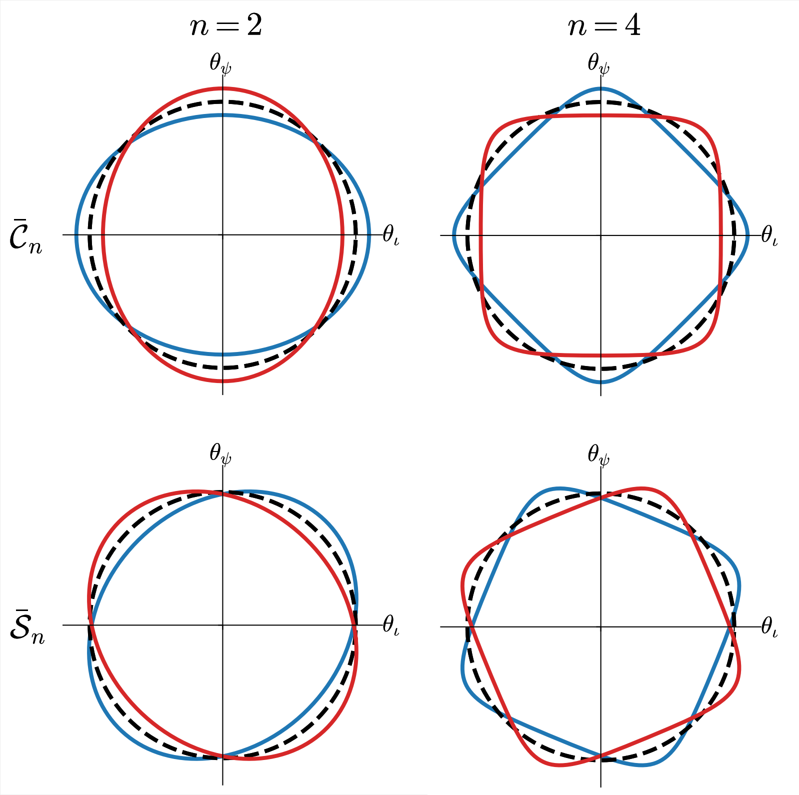
$$\gamma^{(2)}_{ab} \propto \int d^2\boldsymbol{\theta}\,[\theta_a\theta_b]^{\mathrm{TL}} I(\boldsymbol{\theta})$$

4th moment / hexadecapole $n = 4$

$$\gamma^{(4)}_{abcd} \propto \int d^2\boldsymbol{\theta}\,[\theta_a\theta_b\theta_c\theta_d]^{\mathrm{TL}} I(\boldsymbol{\theta})$$

General definition of shape moment decomposition

$$\gamma^{(n)}_{a_1 a_2 \cdots a_n} \propto \int d^2\boldsymbol{\theta}[\theta_{a_1} \cdots \theta_{a_n}]^{\mathrm{TL}} I(\boldsymbol{\theta})$$



(Scale-dependent) Bias for shapes

KK+(21)

$$\gamma_{a_1\cdots a_n} = [\text{projection operator}] g_{i_1\cdots i_n}$$

Shape bias

$$g_{i_1\cdots i_n}(\boldsymbol{x}) = \mathcal{G}_{i_1\cdots i_n}[\delta_{\rm L}(\boldsymbol{x}), K_{{\rm L}\,ij}(\boldsymbol{x}); P_\delta(\boldsymbol{k}_{\rm s};\boldsymbol{x})]$$

$$n = 4$$

$$g_{ijkm}(\boldsymbol{x}) = b_K^{(4)}[K_{ij}(\boldsymbol{x})K_{km}(\boldsymbol{x})]^{\rm TL} + \mathcal{O}(\delta^3)$$

Leading term : non-linear

PNG $B_\Phi \simeq A_4 \mathcal{P}_4(\hat{\boldsymbol{k}}_{\rm S}\cdot\hat{\boldsymbol{k}}_{\rm L})P_\phi(k_{\rm s})P_\phi(k_{\rm L})$

Power spectrum in local region is modulated

$$P_\Phi(\boldsymbol{k}_{\rm s};\boldsymbol{x}) = [1 + A_4 \hat{k}^i_{\rm S}\hat{k}^j_{\rm S}\hat{k}^k_{\rm S}\hat{k}^l_{\rm S}\alpha_{{\rm L}ijkl}(\boldsymbol{x})]P_\phi(\boldsymbol{k}_{\rm s})$$

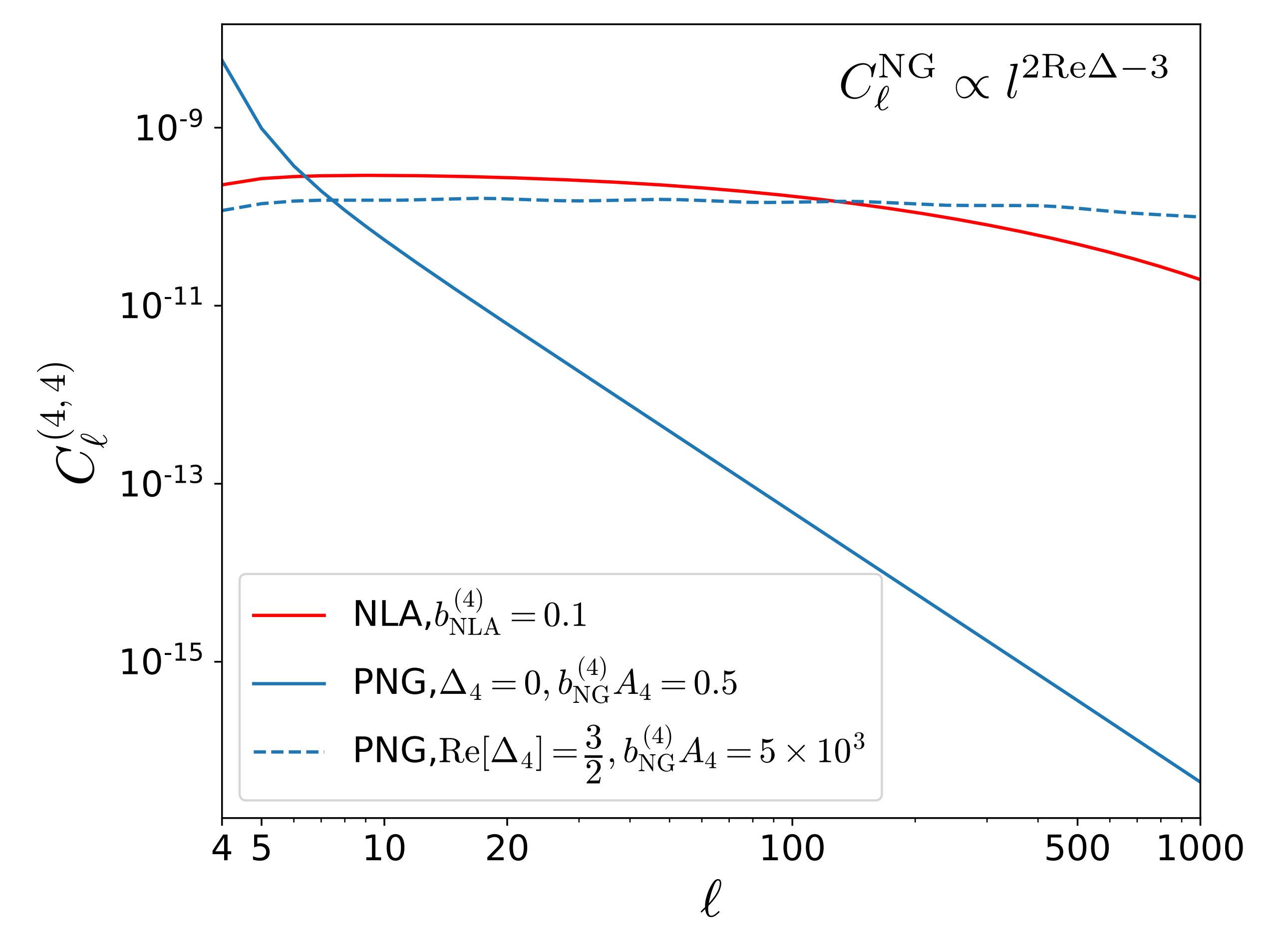
$$\alpha_{{\rm L}ijkl}(\boldsymbol{x}) = \frac{35}{8}\int\frac{d^3k_{\rm L}}{(2\pi^3)}[\hat{k}_{{\rm L}i}\hat{k}_{{\rm L}j}\hat{k}_{{\rm L}k}\hat{k}_{{\rm L}l}]^{\rm TL}\Phi_{\rm G,L}(\boldsymbol{k}_{\rm L})e^{i\boldsymbol{k}_{\rm L}\cdot\boldsymbol{x}}$$

Shape function with PNG

$$g_{ijkm}(\boldsymbol{k}) = b_{\rm NG}^{(4)}A_4\alpha_{ijkl}(\boldsymbol{k})$$
$$= \frac{35}{8}b_{\rm NG}^{(4)}A_4[\hat{k}_i\hat{k}_j\hat{k}_k\hat{k}_m]^{\rm TL}\mathcal{M}^{-1}(k)\delta(\boldsymbol{k})$$

Applying a similar scheme,
each moment of galaxy shape may respond to each mode of angular dependent PNGs.

Angular Power spectrum



Auto-correlation of 4th moment shape function

$$\langle (\gamma^{(4)})^2 \rangle \sim b^{(4)}_{\rm NG} A_4 \langle \alpha^2_{ijkl} \rangle + b^{(4)}_{\rm NLA} \langle K^2 K^2 \rangle$$

$$\langle K^2 K^2 \rangle \sim \int d^3\boldsymbol{p} P_{\rm m}(|\boldsymbol{k} - \boldsymbol{p}|) P_{\rm m}(\boldsymbol{p})$$

Note: we show only the scale dependence since some parameters are unknown.

Summary

- Galaxy shapes may become promising detectors for angular dependent PNGs (e.g., from higher spin particles).

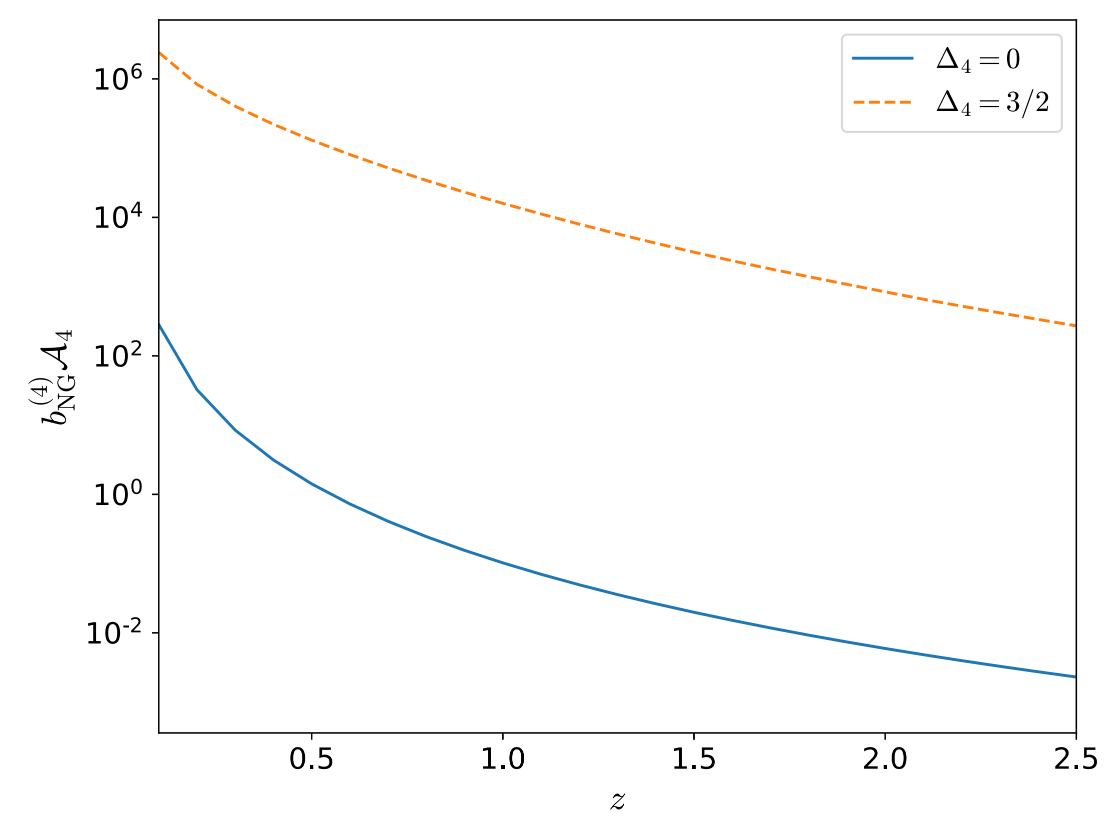
  - Each moment of galaxy shape respond to each mode of multipoles in PNGs.

$$g_{i_1 i_2 \cdots i_n} \Longleftrightarrow B_\Phi \sim \mathcal{P}_n(\boldsymbol{k}_{\rm L} \cdot \boldsymbol{k}_{\rm S}) P(k_{\rm s}) P(k_{\rm L})$$

- For future work, we need to investigate shape noise for each moment shape, how responsive to PNGs, especially for scale-dependent PNGs. (e.g., by using simulation)

Forecast

$$[\mathrm{S/N}]^2(z) \sim f_{\mathrm{sky}} \sum_{l=4}^{l_{\max}} (2l+1) \left[\frac{C^{(4,4)\mathrm{PNG}}(l,z)}{C^{(4,4)\mathrm{PNG}}(l,z) + C^{(4,4)\mathrm{NLA}}(l,z)} \right]^2$$



S/N=5

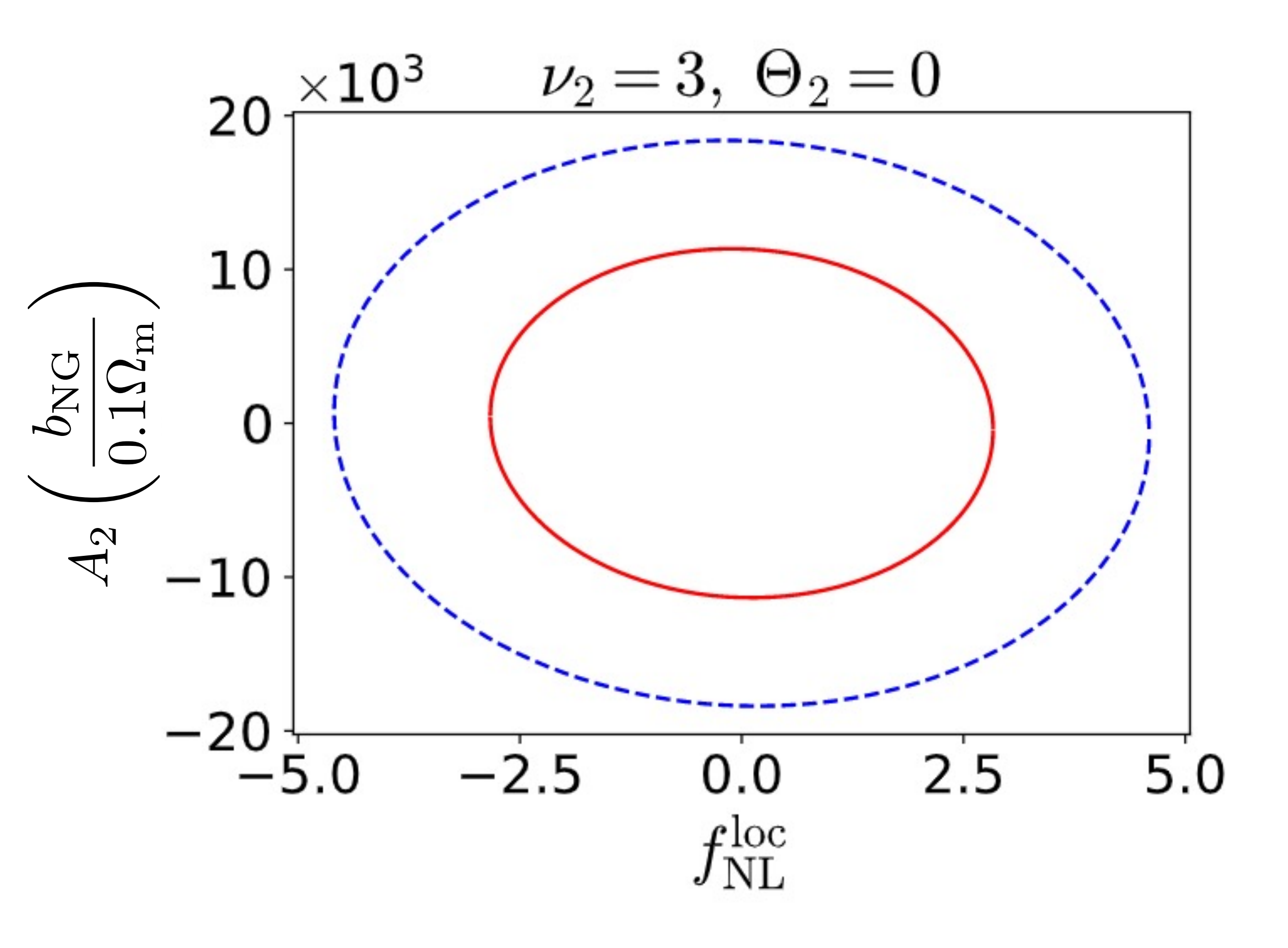
$$\frac{dN}{dz} \propto \left(\frac{z}{0.51}\right)^{1.24} \exp\left[-\left(\frac{z}{0.51}\right)^{1.01}\right]$$

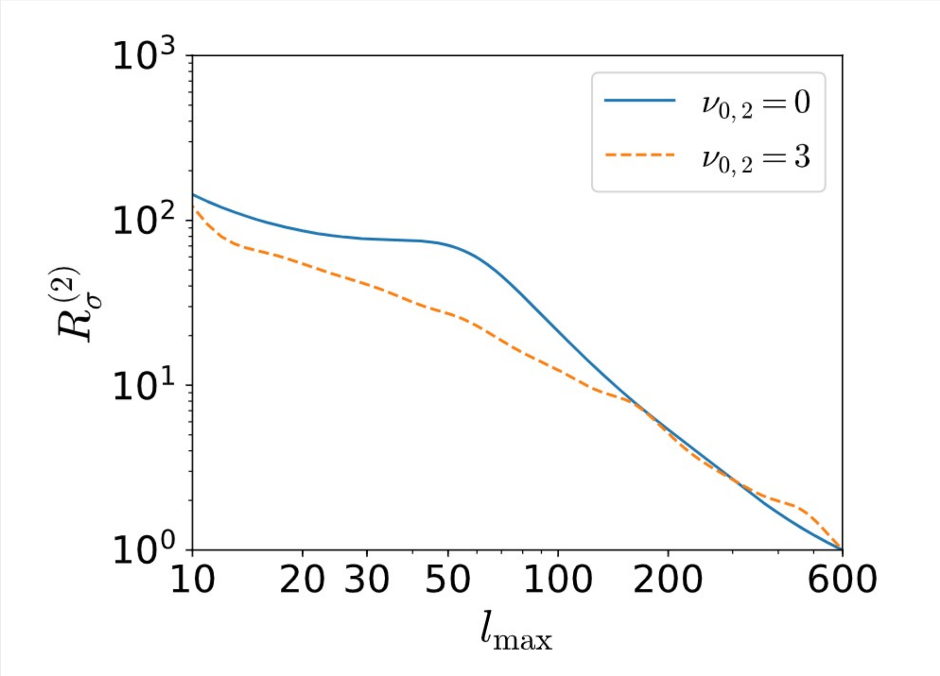
$$\ell_{\rm NL} \sim k_{\rm NL}\chi(z_{\rm peak})$$

$$\chi(z_{\rm peak}) \sim 2{\rm Gpc}$$

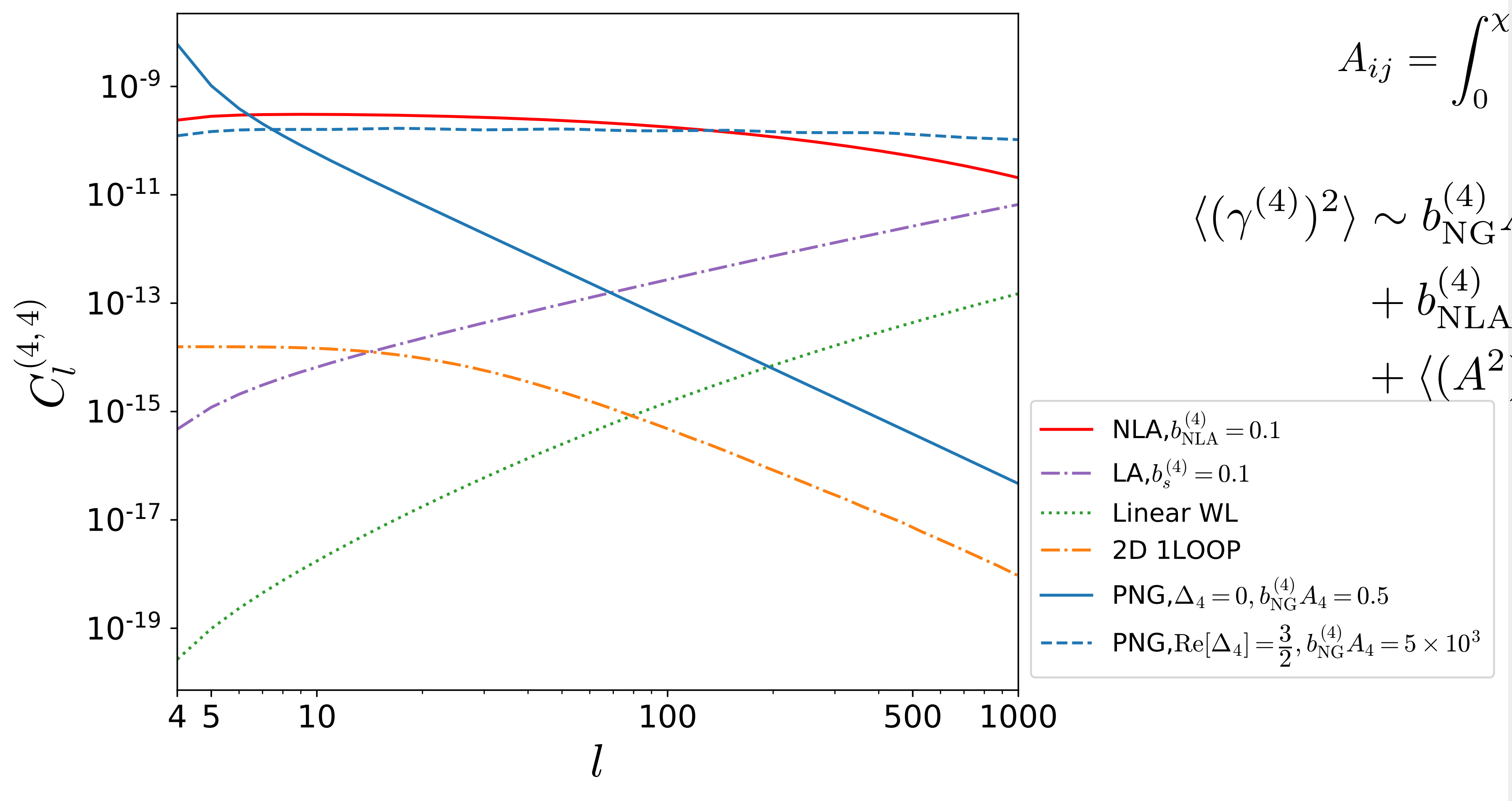
$$k_{\rm NL} \sim D(z =_{\rm peak})k_{\rm NL}(z=0)$$

$$\ell_{\rm NL} \sim 470 \qquad P_{\rm L}(k_{\rm NL})k_{\rm NL}^3/(2\pi^2) \equiv 1$$





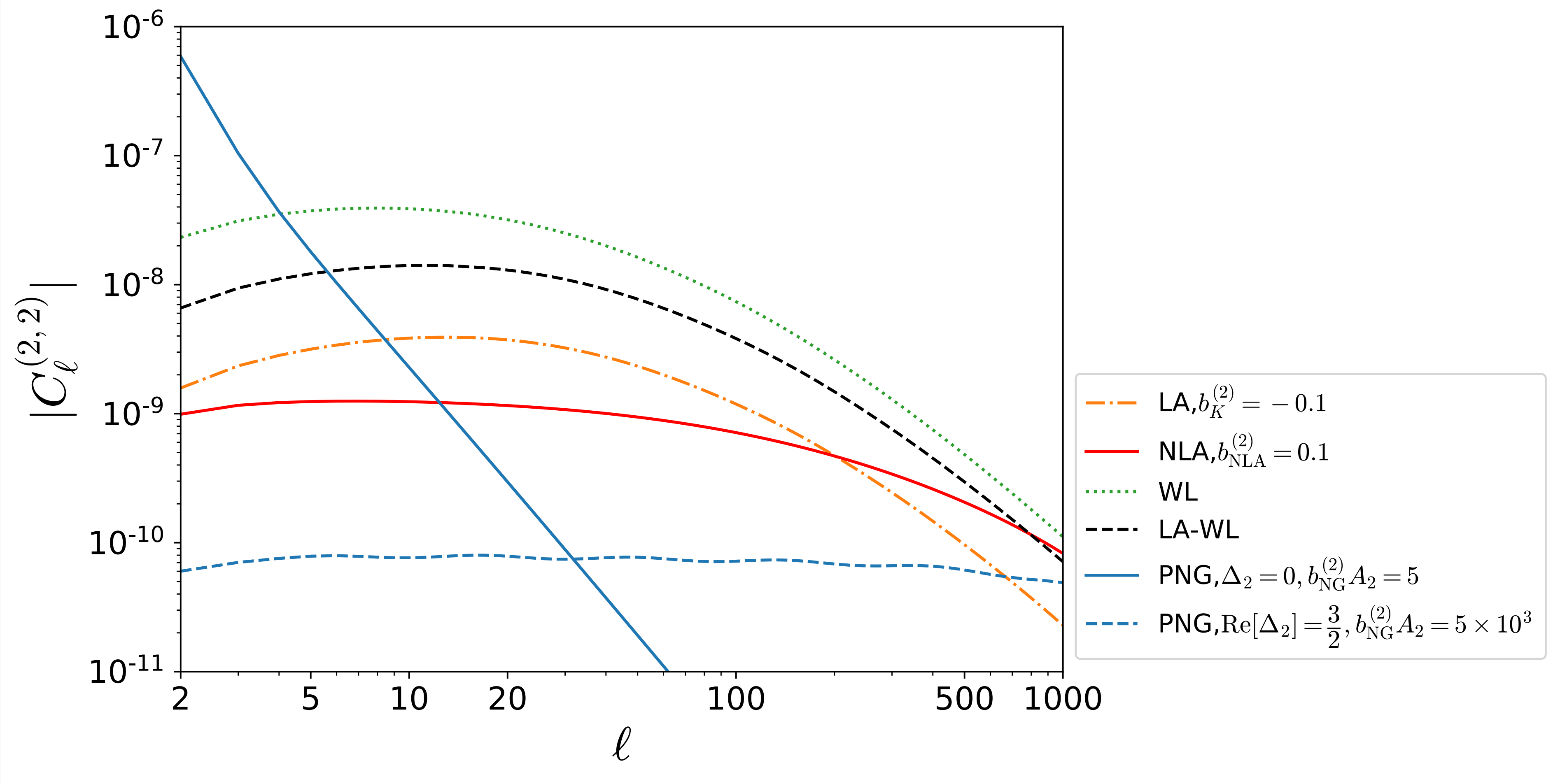
Weak Lensing



The deformation matrix of weak lensing

$$A_{ij}=\int_0^\chi d\chi' \frac{\chi-\chi'}{\chi}\partial_i\partial_j(\Phi+\Psi)$$

$$\begin{aligned}\langle(\gamma^{(4)})^2\rangle \sim\ & b^{(4)}_{\rm NG}A_4\langle\alpha^2_{ijkl}\rangle \\ &+ b^{(4)}_{\rm NLA}\langle K^2K^2\rangle + b^{(4)}_s\langle(\partial^2K)^2\rangle \\ &+ \langle(A^2)(A^2)\rangle + \langle(\partial^2A)^2\rangle\end{aligned}$$



$|C_\ell^{(2,2)}|$
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
2
5
10
20
100
500
1000
ℓ
LA,$b_K^{(2)} = -0.1$
NLA,$b_{\rm NLA}^{(2)} = 0.1$
WL
LA-WL
PNG,$\Delta_2 = 0, b_{\rm NG}^{(2)} A_2 = 5$
PNG,$\mathrm{Re}[\Delta_2] = \frac{3}{2}, b_{\rm NG}^{(2)} A_2 = 5 \times 10^3$

Power spectra

$$P_{\rm NLA}(k) \sim b_{\rm NLA} \int d^3p P(p) P(|\boldsymbol{k}-\boldsymbol{p}|)$$

$$B_\Phi \simeq A_4 \mathcal{P}_4(\hat{\boldsymbol{k}}_{\rm S}\cdot\hat{\boldsymbol{k}}_{\rm L}) \left(\frac{k_{\rm L}}{k_{\rm S}}\right)^{\Delta_4} P_\phi(k_{\rm s}) P_\phi(k_{\rm L})$$