FFT-based estimators for line-of-sight dependent intrinsic alignment signals

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Intrinsic Alignments

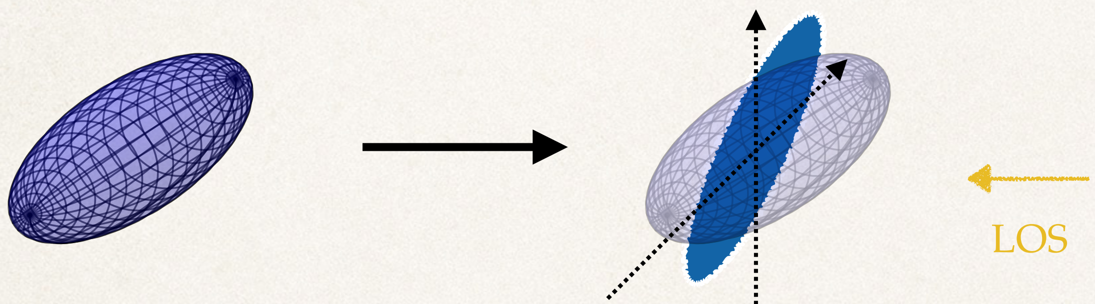
- Intrinsic Alignments (IA)

= Correlation between intrinsic galaxy / halo shapes and LSS

→ Extract cosmological information from IA signals

- Observables

= projected galaxy shapes (ellipticity γ_1, γ_2)

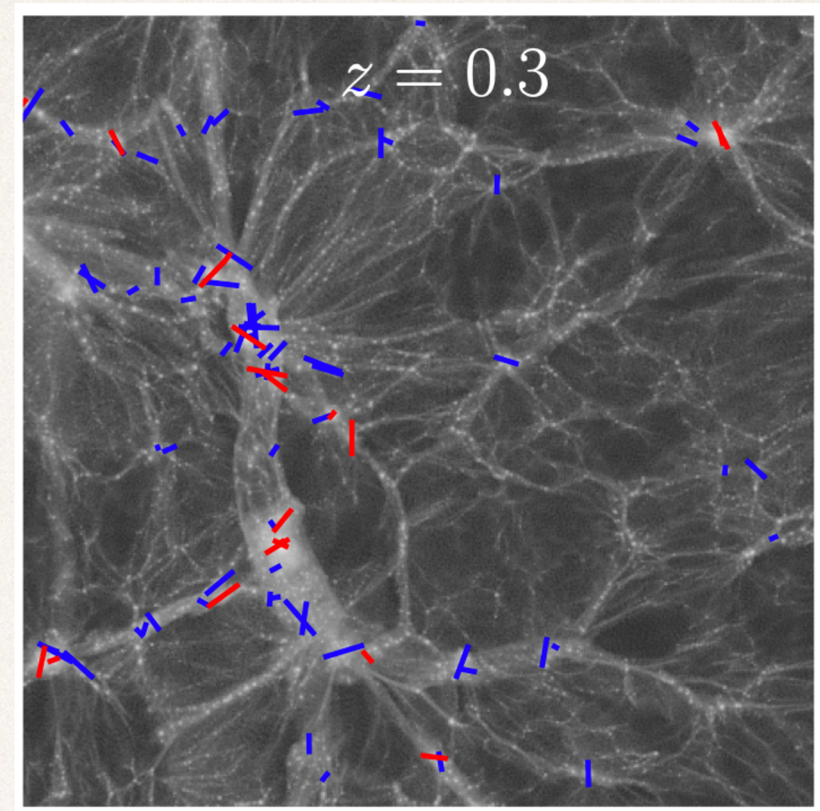


$$I_{ij}(\mathbf{x}) \qquad \underbrace{\gamma_{ij}(\mathbf{x})}_{\text{projected shape (observables)}} \equiv \underbrace{\mathcal{P}_{ijkl}(\hat{\mathbf{x}})}_{\text{projection tensor}} I_{kl}(\mathbf{x})$$

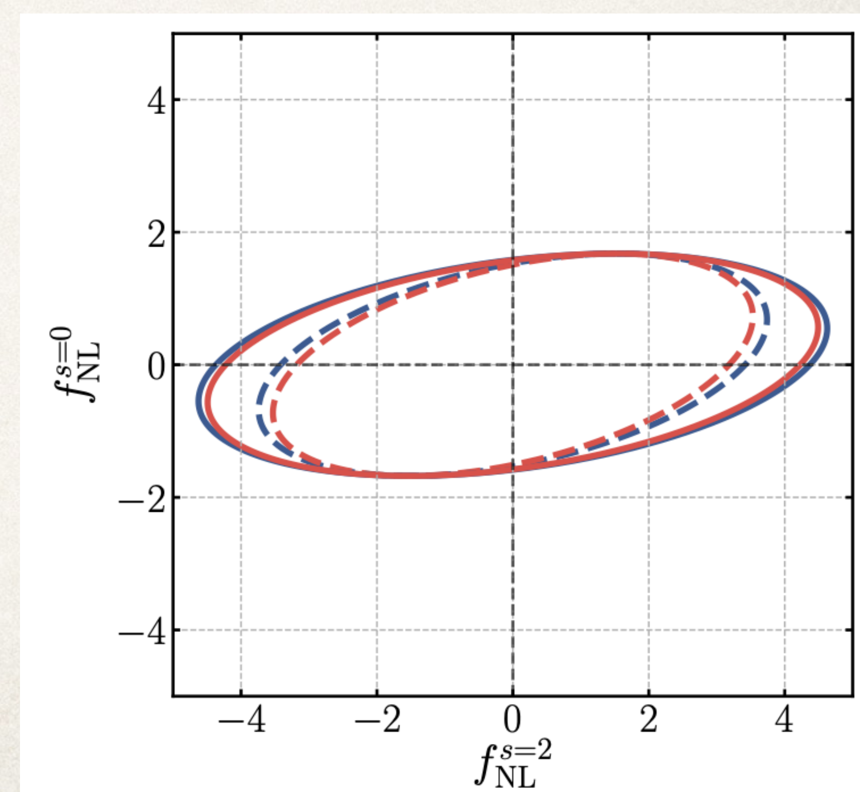
projected shape
(observables)

projection tensor

- Theoretical Forecasts
  - Standard cosmological parameters
    (Taruya&Okumura20, Okumura&Taruya21)
  - Anisotropic primordial non-Gaussianity
    (Schmidt+15, Kogai+18, +21, Akitsu+21)

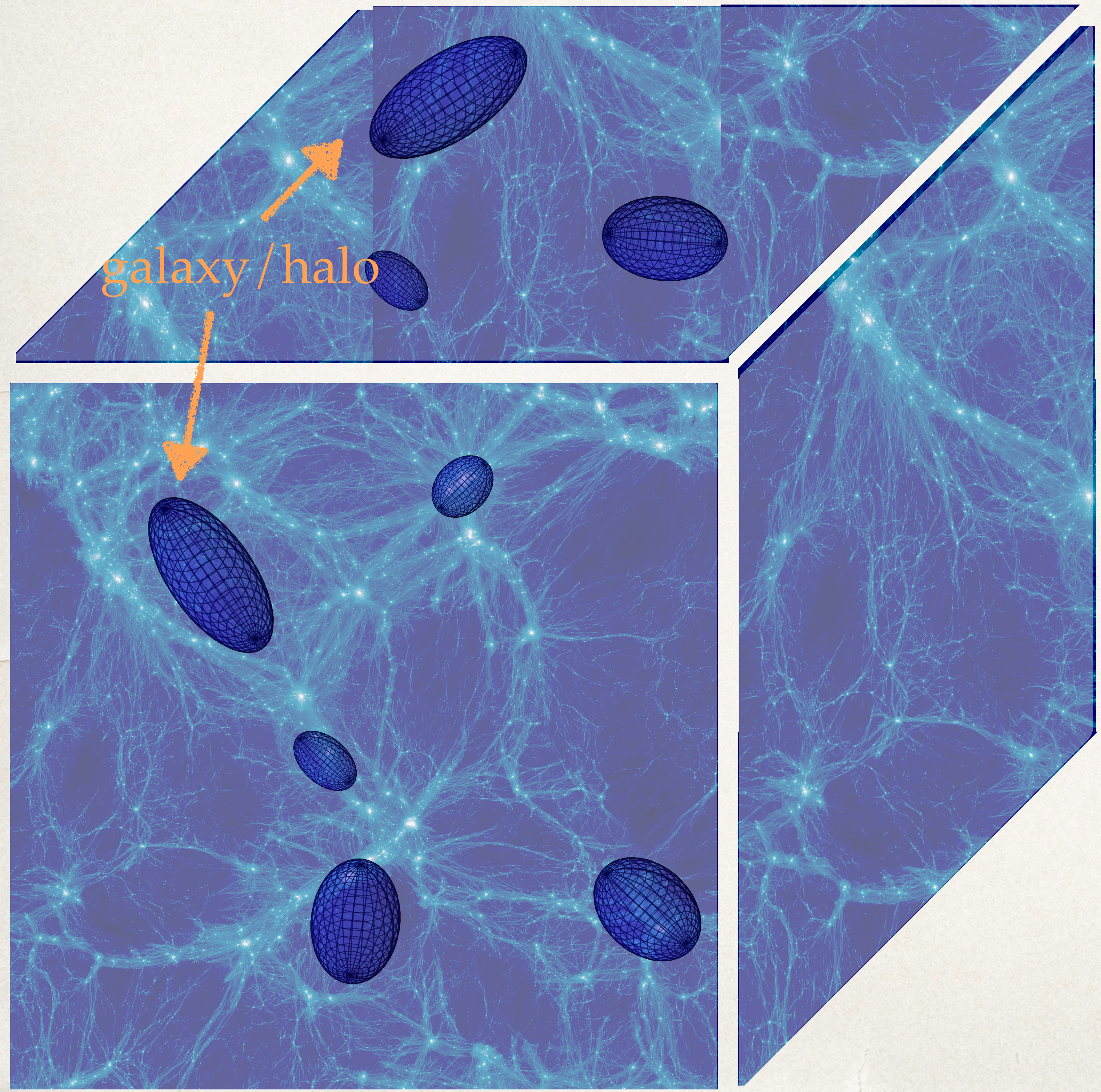


Shi+21



Akitsu+21

Measurements of IA PS (previous works)



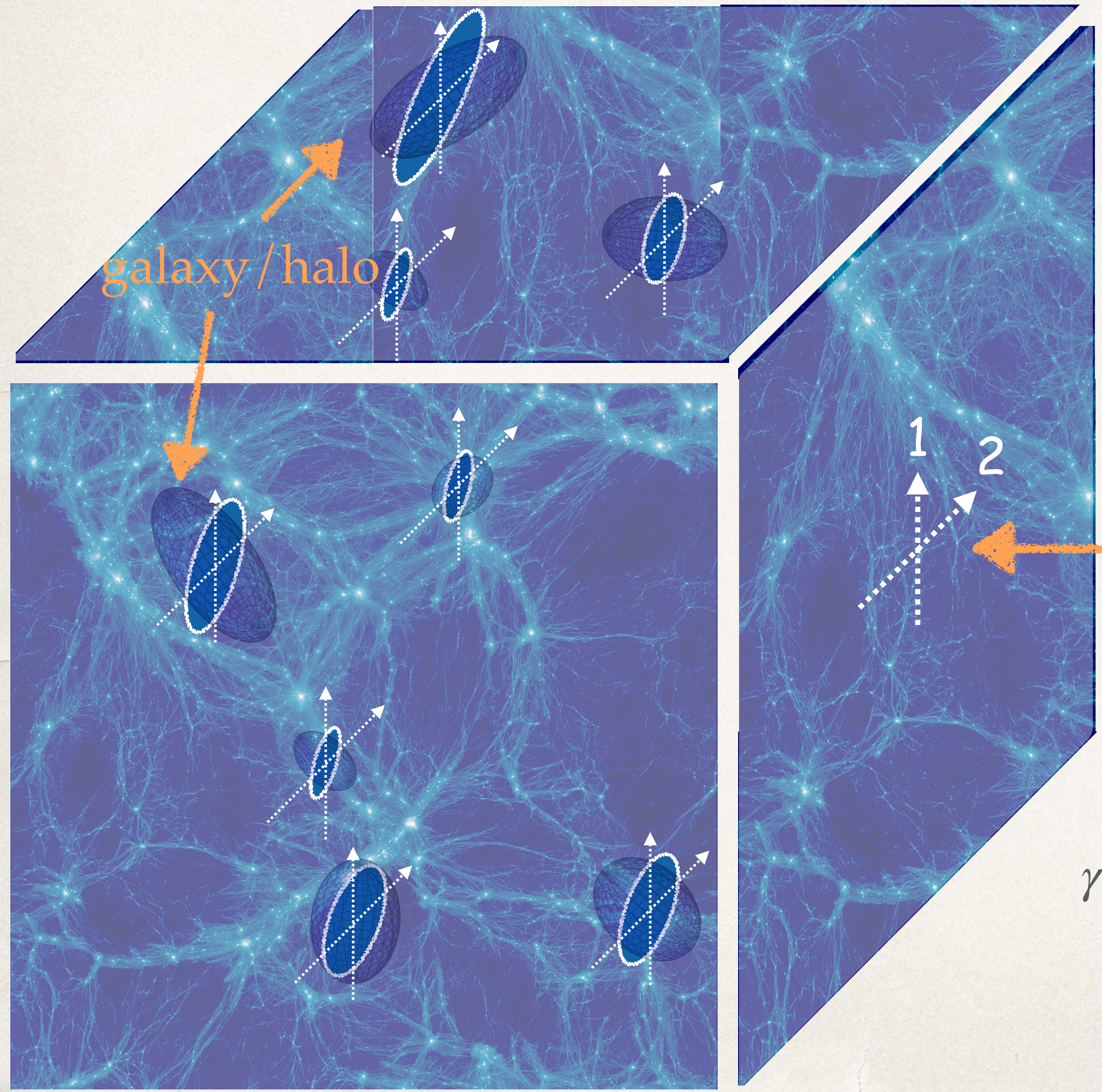
N-body halos: Kurita+20
IllustrisTNG galaxy: Shi+21a, +21b

<u>3d shape measurement</u>

- inertia tensor I_ij

$$I_{ij} = \sum_{p \in \text{member}} w(r_p) x_p^i x_p^j$$

Measurements of IA PS (previous works)



N-body halos: Kurita+20
IllustrisTNG galaxy: Shi+21a, +21b

Observable = 2d shape

$$I_{ij} = \begin{pmatrix} I_{11} & I_{12} & I_{13} \\ \cdot & I_{22} & I_{23} \\ \cdot & \cdot & I_{33} \end{pmatrix}$$

LOS = 3-axis

<u>Projected ellipticity</u>

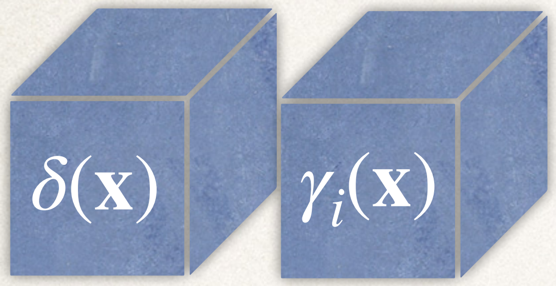
(two components)

$$\gamma_1 = \frac{I_{11} - I_{22}}{I_{11} + I_{22}} \qquad \gamma_2 = \frac{2I_{12}}{I_{11} + I_{22}}$$

Measurements of IA PS (previous works)

N-body halos: Kurita+20
IllustrisTNG galaxy: Shi+21a, +21b

- density & shape fields



$$\{\delta_m(\mathbf{x}), \delta_h(\mathbf{x}), \gamma_1(\mathbf{x}), \gamma_2(\mathbf{x})\}$$

matter halo shapes

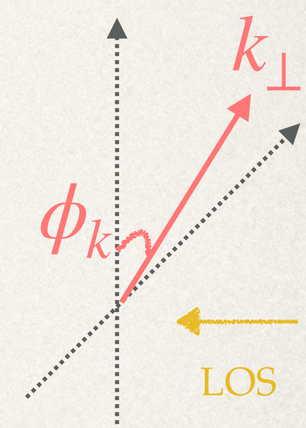
FFT

$$\{\delta_m(\mathbf{k}), \delta_h(\mathbf{k}), \gamma_1(\mathbf{k}), \gamma_2(\mathbf{k})\}$$

E/B decomposition

$$E(\mathbf{k}) \equiv \gamma_1(\mathbf{k})\cos 2\phi_k + \gamma_2(\mathbf{k})\sin 2\phi_k$$

$$B(\mathbf{k}) \equiv \gamma_1(\mathbf{k})\sin 2\phi_k - \gamma_2(\mathbf{k})\cos 2\phi_k$$



- IA Power Spectra

e.g.

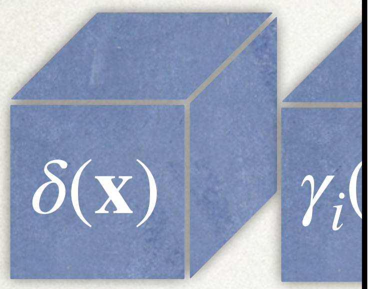
$$P_{E\delta}^{(\ell)}(k) = (2\ell+1)\int d\Omega_{\hat{\mathbf{k}}} E(\mathbf{k})\delta^*(\mathbf{k})\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}})$$

global LOS

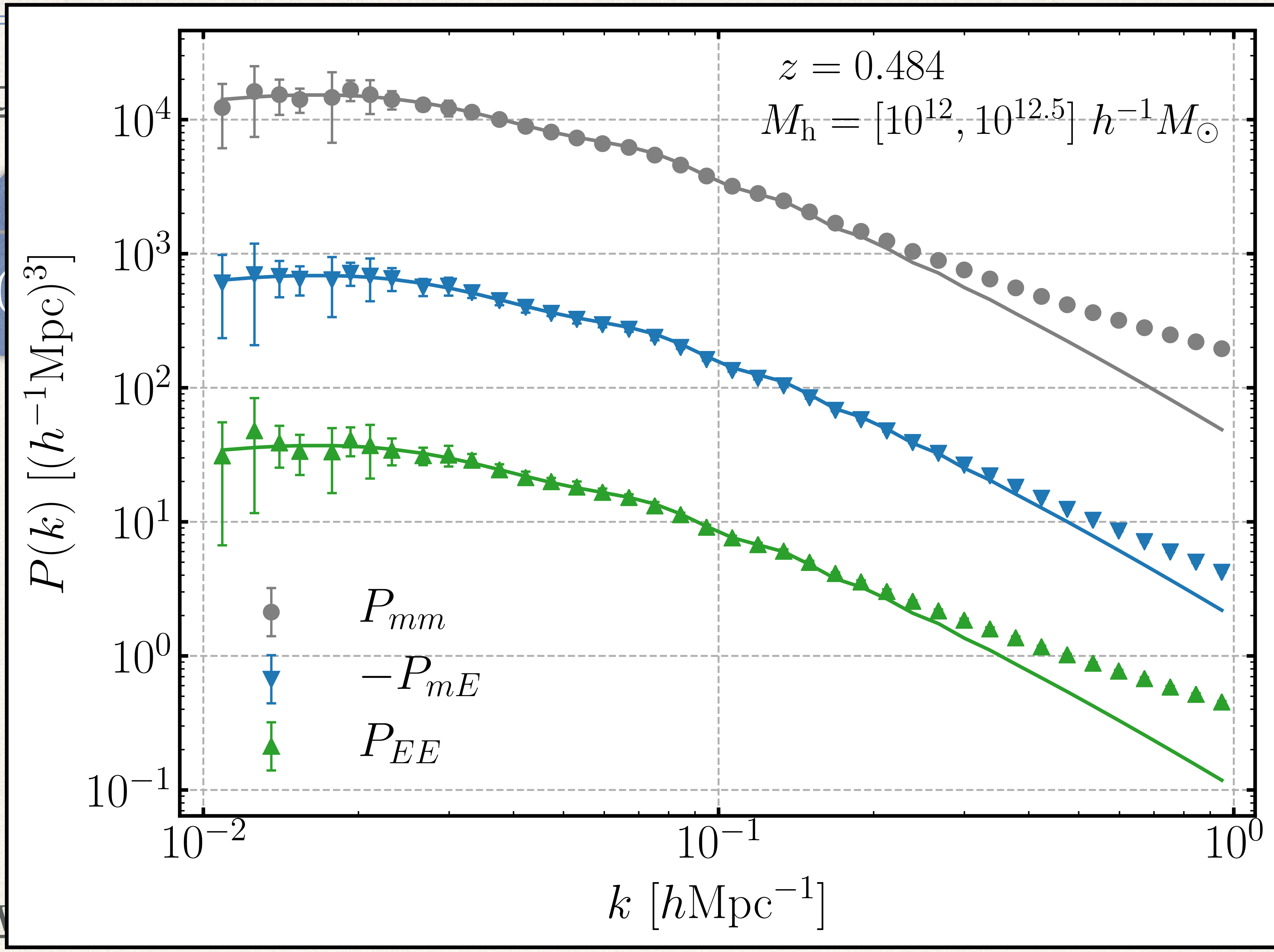
We call this type of estimators as Global plane-parallel (GPP) estimators.

Measurements of IA PS (previous works)

- densit



urita+20
axy: Shi+21a, +21b



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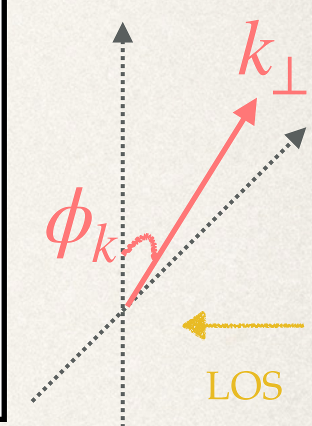


- IA Pow

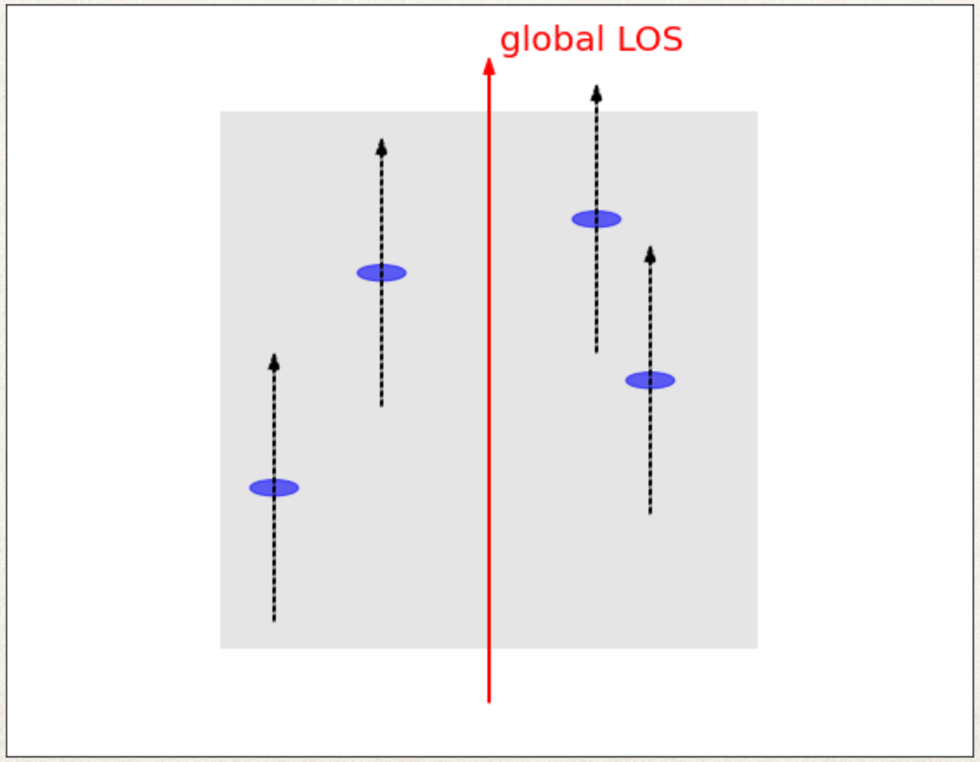
e.g.

$$P^{(\ell)}_{E\delta}(k) = (2\ell + 1)\int d\Omega_{\hat{\mathbf{k}}} E(\mathbf{k})\delta^*(\mathbf{k})\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\underline{\hat{\mathbf{n}}})$$

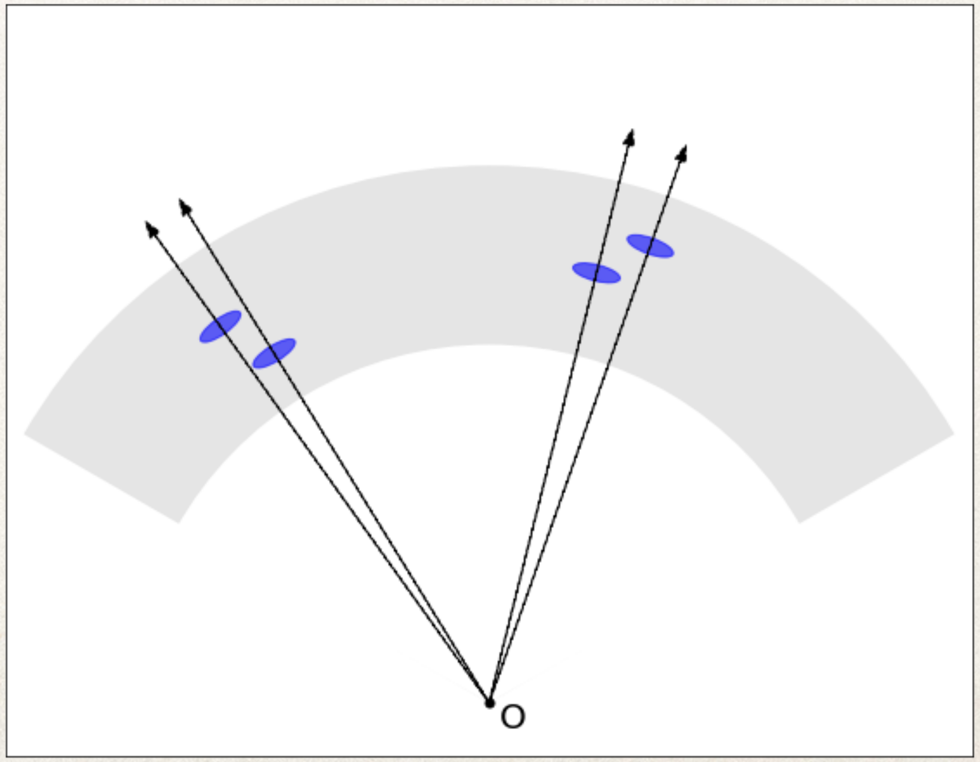
global LOS

Measurements of IA PS (previous works)

- Previous works (summary):
  - Measurements from simulation data (N-body halos: Kurita+20, IllustrisTNG galaxy: Shi+21a, +21b)
  - Global plane-parallel (GPP) approximation
    - To obtain $\gamma(\mathbf{x})$, we performed the projection with global line-of sight $\hat{\mathbf{n}}$ (=const.)
    - After the projection, we measured IA PS using GPP estimator
- Problems:
  - Different LOSs are need to be considered in a wide-angle survey.
  - → GPP estimator might not work.



Previous simulation-based studies

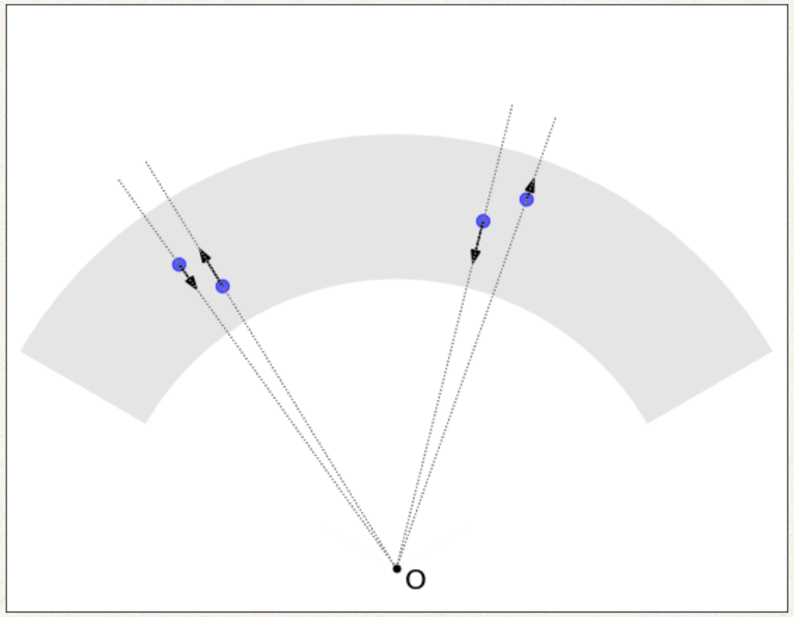


Realistic observations

Measurements from realistic surveys

- Purpose of this study:
  - develop IA estimator available for realistic wide-angle surveys.

- Hints:
  - The same problem occurs in the case of clustering power spectrum.
  - due to the Redshift-Space Distortions (RSD)



  - Yamamoto estimator (Yamamoto+06) resolves the problem, and has been used in the (also latest) BOSS clustering analysis.

  - In this work, we extend Yamamoto estimator to the case of Intrinsic Alignment signals.

Clustering estimator (review)

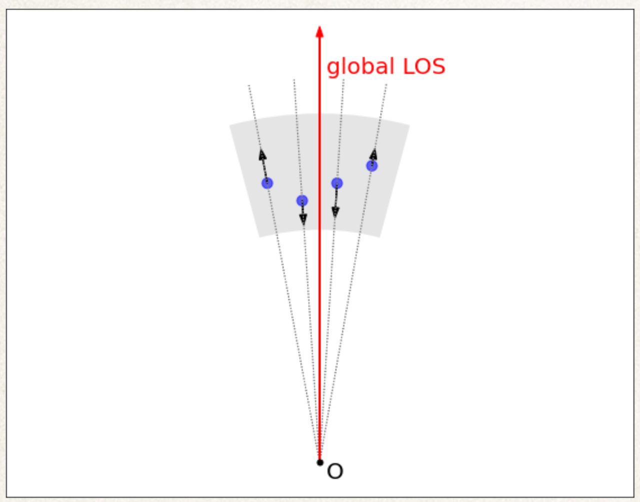
- Global plane-parallel (GPP) estimator:

$$P^{(\ell)}(k) = (2\ell+1)\int d\Omega_{\hat{\mathbf{k}}}\,\delta(\mathbf{k})\delta^*(\mathbf{k})\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}})$$

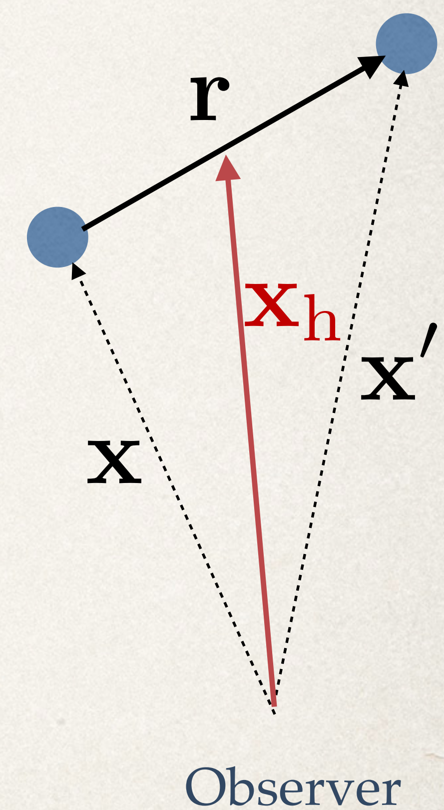
$$= (2\ell+1)\int d\Omega_{\hat{\mathbf{k}}}\int d\mathbf{x}\int d\mathbf{x}'\,\underline{\delta(\mathbf{x})\delta(\mathbf{x}')}\,e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{n}})$$

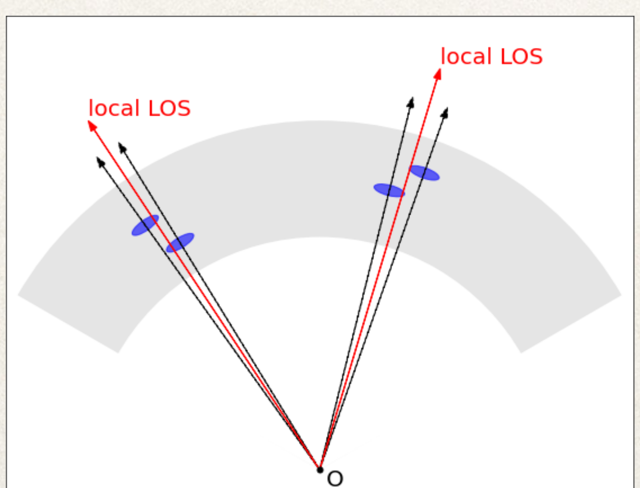
$$\xi(\mathbf{x},\mathbf{x}') = \xi(r, \hat{\mathbf{r}}\cdot\underline{\hat{\mathbf{n}}})$$

global LOS



- GPP estimator is valid for a small-angle surveys.





- GPP estimator is NOT valid for a wide-angle surveys like SDSS.

$$\xi(\mathbf{x},\mathbf{x}') = \xi(r, \hat{\mathbf{r}}\cdot\underline{\hat{\mathbf{x}}_\mathrm{h}})$$

local LOS

- => We need a Local PP estimator.

Clustering estimator (review)

- Yamamoto estimator (LPP estimator): Yamamoto+06

$$P^{(\ell)}(k) = (2\ell+1)\int d\Omega_{\hat{\mathbf{k}}}\int d\mathbf{x}\int d\mathbf{x}'\delta(\mathbf{x})\delta(\mathbf{x}')e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\underline{\hat{\mathbf{x}}_{\rm h}})$$

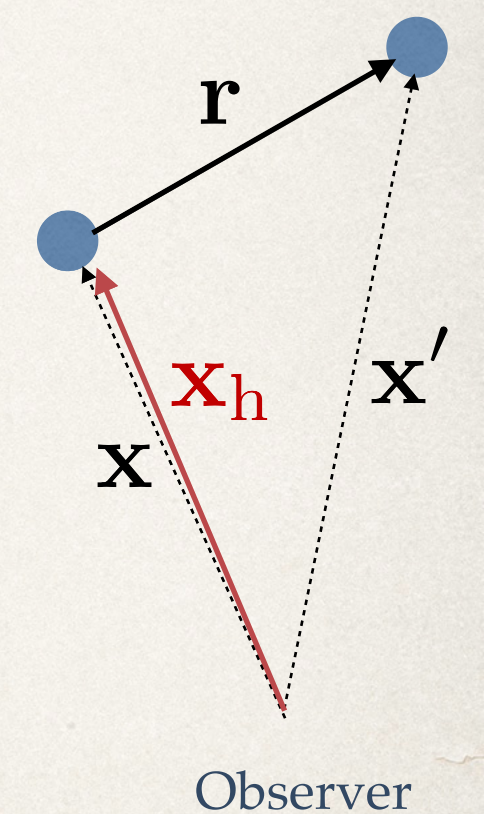
$$\simeq (2\ell+1)\int d\Omega_{\hat{\mathbf{k}}}\int d\mathbf{x}\int d\mathbf{x}'\delta(\mathbf{x})\delta(\mathbf{x}')e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\underline{\hat{\mathbf{x}}})$$

$$= (2\ell+1)\int d\Omega_{\hat{\mathbf{k}}}\left[\int d\mathbf{x}\delta(\mathbf{x})e^{-i\mathbf{k}\cdot\mathbf{x}}\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})\right]\left[\int d\mathbf{x}'\delta(\mathbf{x}')e^{+i\mathbf{k}\cdot\mathbf{x}'}\right]$$

→FFT →FFT

Replace $\hat{\mathbf{x}}_h$ with $\hat{\mathbf{x}}$

Bianchi+15
Scoccimarro 15
Hand+18



e.g.

$$\mathcal{L}_2(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}}) = \frac{3}{2}(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})^2 - \frac{1}{2} = \frac{3}{2}\hat{x}_i\hat{x}_j\hat{k}_i\hat{k}_j - \frac{1}{2}$$

$$\int d\mathbf{x}\delta(\mathbf{x})\mathcal{L}_2(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})e^{-i\mathbf{k}\cdot\mathbf{x}} \sim \left[\int d\mathbf{x}\delta(\mathbf{x})\hat{x}_i\hat{x}_j e^{-i\mathbf{k}\cdot\mathbf{x}}\right]\hat{k}_i\hat{k}_j$$

→FFT

- PS measurements for a wide-angle survey can be done with FFT-based numerical implementations.

2pt Statistics of Shapes

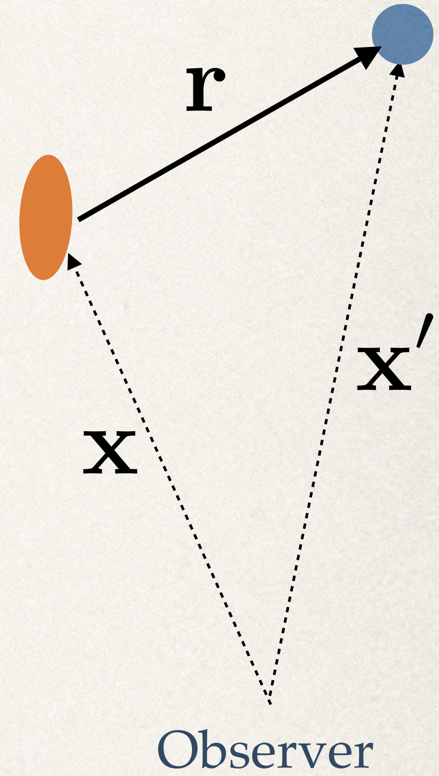
- Density-Shape Cross-Statistics
  - Complex expression:

$$\gamma(\mathbf{x}) \equiv \gamma_1(\mathbf{x}) + i\gamma_2(\mathbf{x})$$

  - Coordinate-independent 2pt CF:

$$\xi_{\gamma\delta}(\mathbf{r}) \equiv \langle \gamma_{\rm rot}(\mathbf{x};\mathbf{x}')\delta(\mathbf{x}')\rangle = \langle \gamma(\mathbf{x})\delta(\mathbf{x}')\rangle \underline{e^{-2i\phi_{\hat{\mathbf{r}},\hat{\mathbf{n}}}}}$$

(2D rotation)



  - Independent modes in Fourier space:

$$E(\mathbf{k}) + iB(\mathbf{k}) \equiv \gamma(\mathbf{k})e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{n}}}}$$

  - Coordinate-independent IA PS:

$$(2\pi)^3\delta_D^3(\mathbf{k}-\mathbf{k}')P_{\gamma\delta}(\mathbf{k}) \equiv \langle \gamma(\mathbf{k})\delta^*(\mathbf{k})\rangle e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{n}}}}$$

$P_{E\delta} + iP_{B\delta}$

  - Relation between PS and CF:

$$P_{\gamma\delta}(\mathbf{k}) = \int d\mathbf{r}\ \xi_{\gamma\delta}(\mathbf{r})e^{2i\phi_{\hat{\mathbf{r}},\hat{\mathbf{n}}}-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{n}}}}e^{-i\mathbf{k}\cdot\mathbf{r}}$$

$$= \int d\mathbf{x} \int d\mathbf{x}'\ \gamma(\mathbf{x})\delta(\mathbf{x})e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{n}}}}e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}$$

LPP estimator for IA (this work)

$$P_{\gamma\delta}(\mathbf{k}) = \int d\mathbf{x} \int d\mathbf{x}'\ \gamma(\mathbf{x})\delta(\mathbf{x}) e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{n}}}} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}$$

- Initial guess:

Define multipoles & Replace $\hat{\mathbf{n}}$ with $\hat{\mathbf{x}}_\mathrm{h}$

$$P^{(\ell)}_{\gamma\delta}(k) = (2\ell+1)\int d\Omega_{\hat{\mathbf{k}}} \int d\mathbf{x} \int d\mathbf{x}'\ \gamma(\mathbf{x})\delta(\mathbf{x}') e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{x}}_\mathrm{h}}} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}}_\mathrm{h})$$

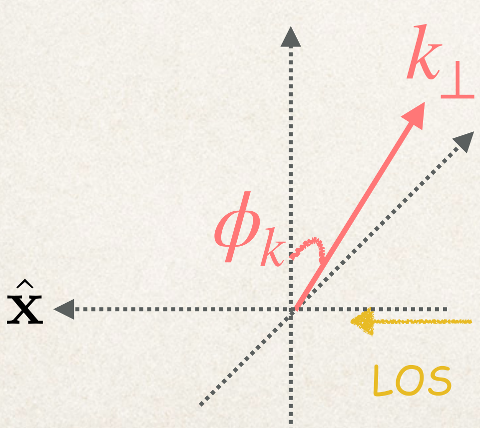
- Replace $\hat{\mathbf{x}}_h$ with $\hat{\mathbf{x}}$ to obtain a FFT-based impl.

$$P^{(\ell)}_{\gamma\delta}(k) \simeq (2\ell+1)\int d\Omega_{\hat{\mathbf{k}}} \int d\mathbf{x} \int d\mathbf{x}'\ \gamma(\mathbf{x})\delta(\mathbf{x}') e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{x}}}} e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')} \mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})$$

$$= (2\ell+1)\int d\Omega_{\hat{\mathbf{k}}} \left[\int d\mathbf{x}\gamma(\mathbf{x}) e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{x}}}} e^{-i\mathbf{k}\cdot\mathbf{x}} \mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})\right]\left[\int d\mathbf{x}'\delta(\mathbf{x}') e^{+i\mathbf{k}\cdot\mathbf{x}'}\right]$$

polarization tensor

$$e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{x}}}} = \frac{e^{(-2)}_{ij}(\hat{\mathbf{x}})\hat{k}_i\hat{k}_j}{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})^2}$$



$$\left[\int d\mathbf{x}\gamma(\mathbf{x}) e^{(-2)}_{ij}(\hat{\mathbf{x}}) \frac{\mathcal{L}_\ell(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})}{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})^2} e^{-i\mathbf{k}\cdot\mathbf{x}}\right]\hat{k}_i\hat{k}_j$$

non-separable due to the denominator

LPP estimator for IA (this work)

- Replace (usual) Legendre polynomial with associated Legendre polynomial (m=2):

$$\mathcal{L}_\ell(\mu) \to \mathcal{L}_\ell^{m=2}(\mu)$$

e.g.

$$\mathcal{L}_2^{m=2}(\mu) = 3(1-\mu^2)$$

$$\mathcal{L}_4^{m=2}(\mu) = \frac{15}{2}(1-\mu^2)(7\mu^2-1)$$

- Finally, our LPP estimator:

$$P_{\gamma\delta}^{(\ell)}(k) = (2\ell+1)\frac{(\ell-2)!}{(\ell+2)!}\int d\Omega_{\hat{\mathbf{k}}}\int d\mathbf{x}\int d\mathbf{x}'\ \gamma(\mathbf{x})\delta(\mathbf{x}')e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{x}}_\mathrm{h}}}e^{-i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\mathcal{L}_\ell^{m=2}(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}}_\mathrm{h})$$

$$\simeq (2\ell+1)\frac{(\ell-2)!}{(\ell+2)!}\int d\Omega_{\hat{\mathbf{k}}}\left[\int d\mathbf{x}\gamma(\mathbf{x})e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{x}}}}e^{-i\mathbf{k}\cdot\mathbf{x}}\mathcal{L}_\ell^{m=2}(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})\right]\left[\int d\mathbf{x}'\delta(\mathbf{x}')e^{+i\mathbf{k}\cdot\mathbf{x}'}\right]$$

→FFT

$$= \left[\int d\mathbf{x}\gamma(\mathbf{x})e_{ij}^{(-2)}(\hat{\mathbf{x}})\frac{\mathcal{L}_\ell^{m=2}(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})}{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})^2}e^{-i\mathbf{k}\cdot\mathbf{x}}\right]\hat{k}_i\hat{k}_j$$

→FFT

$$e^{-2i\phi_{\hat{\mathbf{k}},\hat{\mathbf{x}}}} = \frac{e_{ij}^{(-2)}(\hat{\mathbf{x}})\hat{k}_i\hat{k}_j}{1-(\hat{\mathbf{k}}\cdot\hat{\mathbf{x}})^2}$$

We obtain an estimator to measure $P_{\gamma\delta}^{(\ell)}(k)$ given observable fields, $\gamma(\mathbf{x})$, $\delta(\mathbf{x})$.

Validation Tests I: methods

- Measure IA power spectrum in hypothetical observations using simulation box.

$$\delta_{\mathrm{m}}(\mathbf{k}) \to T_{ij}(\mathbf{k}) \equiv (\hat{k}_i\hat{k}_j - \delta^{\mathrm{K}}_{ij}/3)\delta_{\mathrm{m}}(\mathbf{k}) \xrightarrow{\mathrm{FFT}} T_{ij}(\mathbf{x})$$

In this work, we assume $T_{ij}(\mathbf{x})$ as an original galaxy shape field for simplicity.

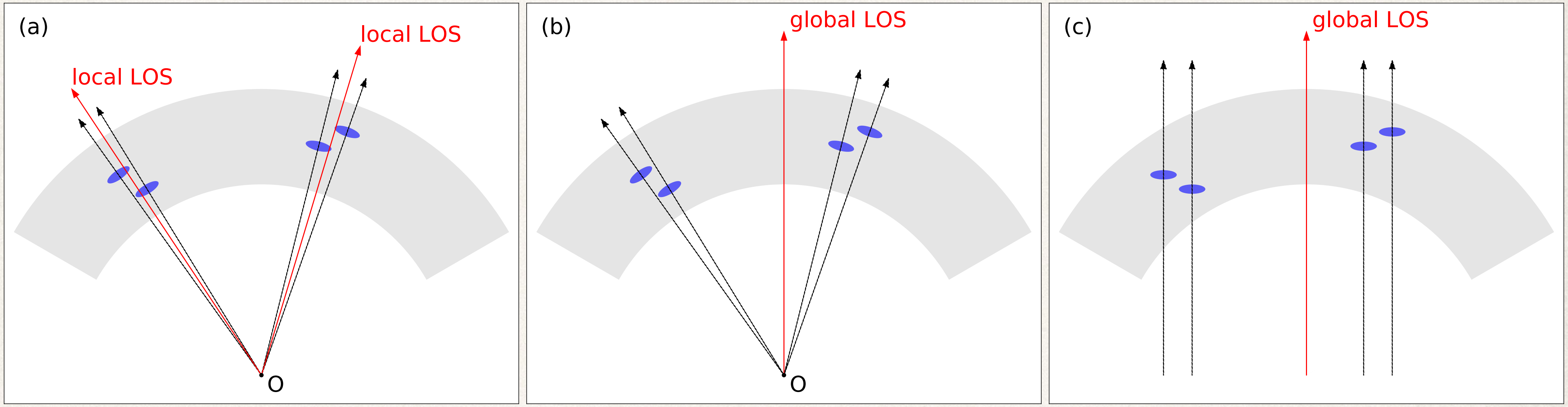
$$\gamma_{ij}(\mathbf{x}) \equiv \mathcal{P}_{ijkl}(\hat{\mathbf{x}})T_{kl}(\mathbf{x})$$

projected shape (observables) — projection

—> observables: $\delta_m(\mathbf{x}), \gamma(\mathbf{x})$

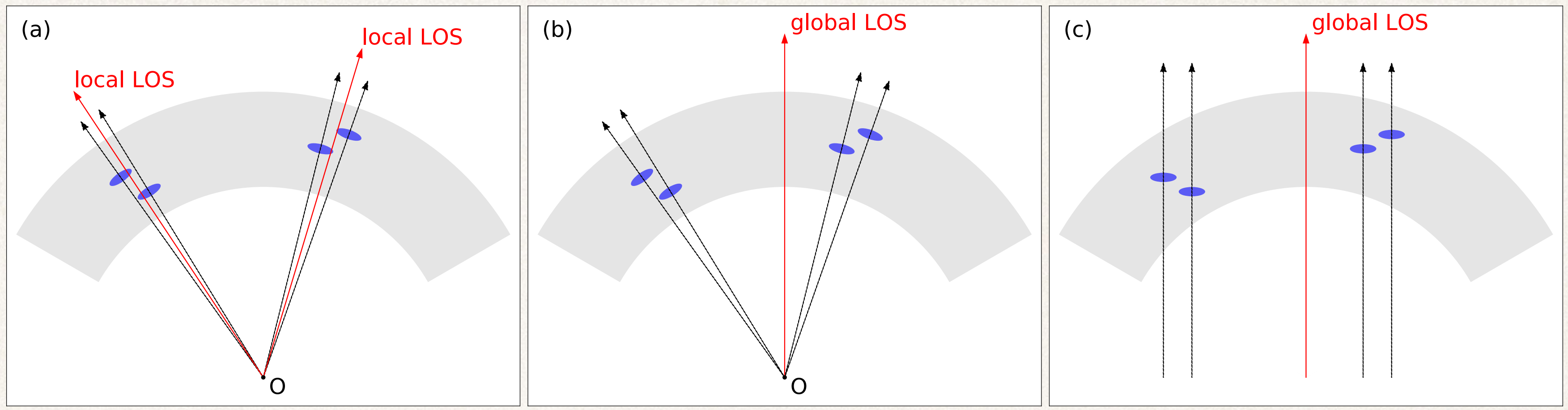
Validation Tests I: methods

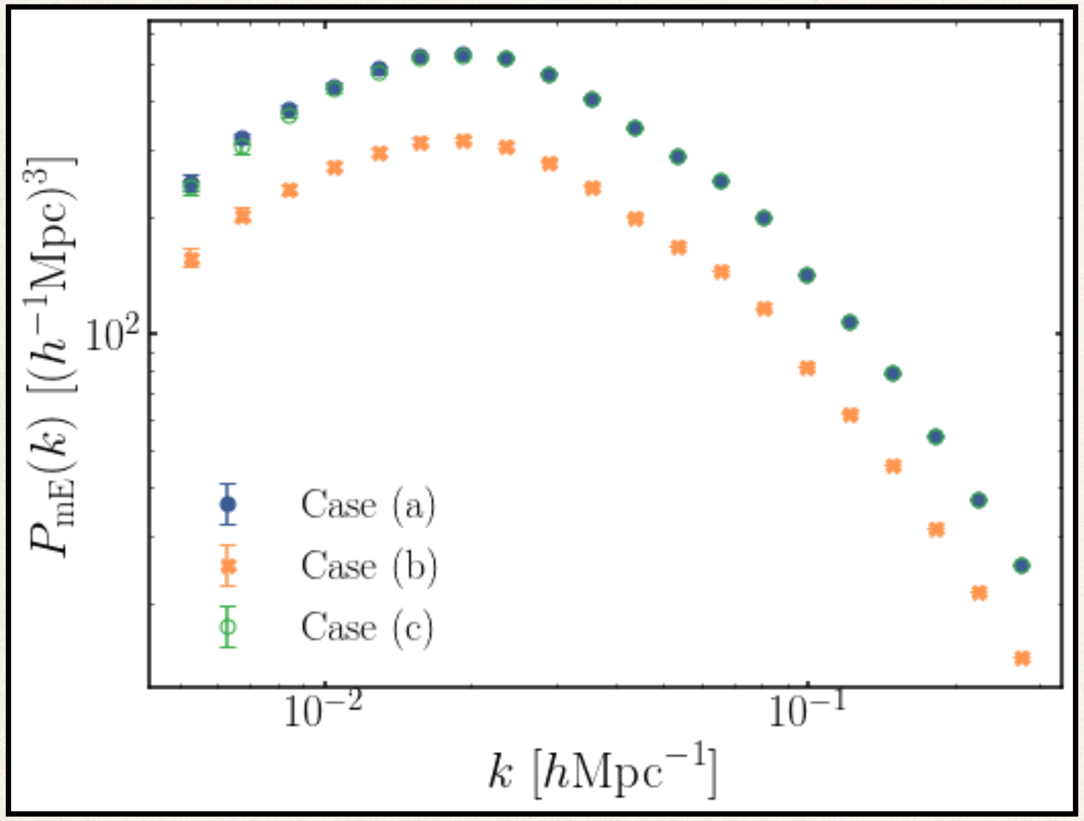
- We validate our estimator using hypothetical observations with three different configurations.
  - BOSS-like geometry (gray regions)

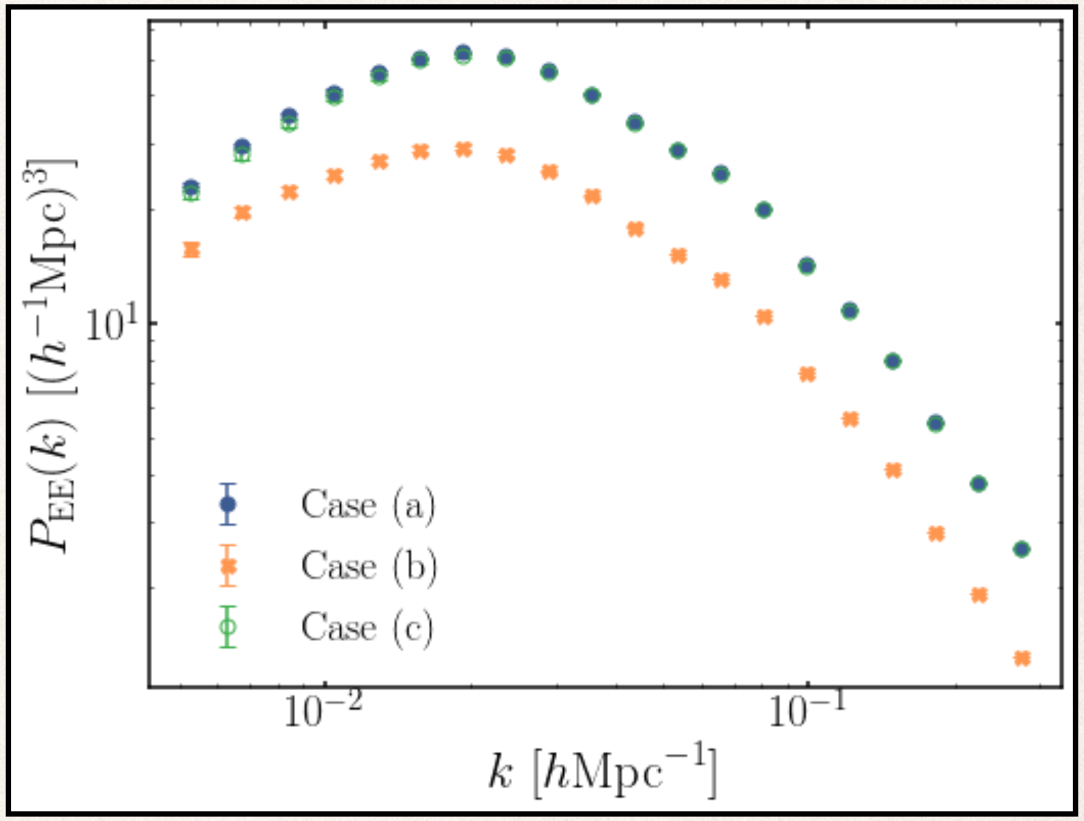


Projection	Observation-like	Observation-like	Distant obs. approx. $\left(P_{ijkl}(\hat{\mathbf{x}}) \rightarrow P_{ijkl}(\hat{\mathbf{n}})\right)$
Estimator	Local PP (this work)	Global PP	Global PP

Validation Tests I: results





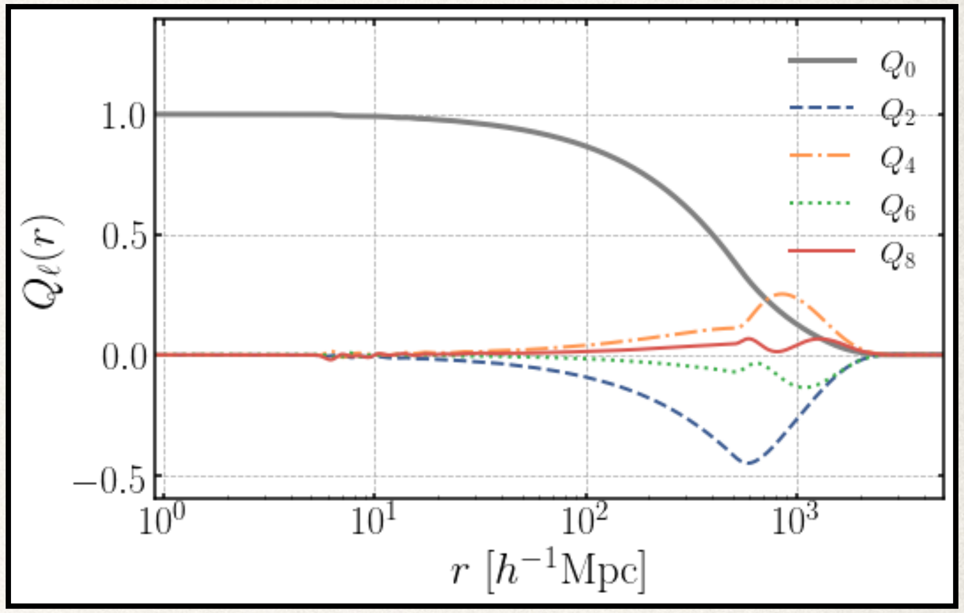


- We can obtain desirable signals in Case (a) using LPP estimator.
- On the other hand, GPP estimator causes bias in measured signals as in Case (b).

Validation Tests II: window convolution

- Comparing the measurements with the window-convolved theory.
  - Direct calculation of window convolution is very massive.
  - In this work, we also derived a window convolution scheme for IA using the pair counting method (Wilson+2015) introduced for galaxy clustering analysis.

$$P_{\gamma\delta} \xrightarrow{\text{hankel}} \xi_{\gamma\delta} \xrightarrow{\text{convolution}} \xi_{\gamma\delta} \otimes Q \xrightarrow[\text{back}]{\text{hankel}} P'_{\gamma\delta}$$



For matter auto-power:

$$P^{\rm win}_{\rm mm}(k) = 4\pi \int r^2 dr\ j_0(kr)\xi(r)Q_0(r)$$

For matter-tidal cross:

$$P^{\rm win}_{\rm Em}(k) = 4\pi \int r^2 dr\ j_2(kr)\xi(r)\{Q_0(r) - \frac{2}{7}Q_2(r) + \frac{1}{21}Q_4(r)\}$$

newly derived in this work

Validation Tests II: window convolution

- Compa
  - Dire
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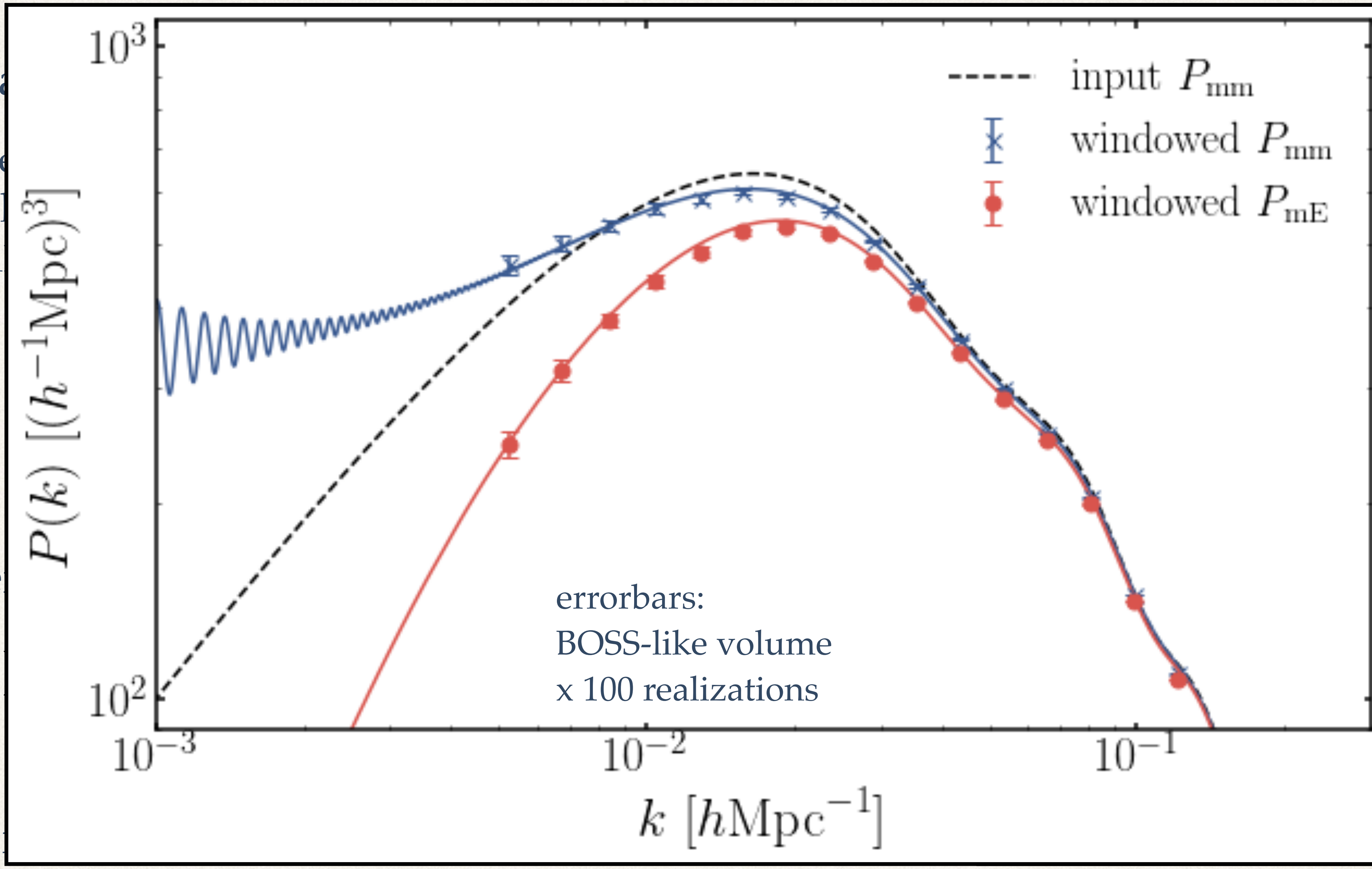
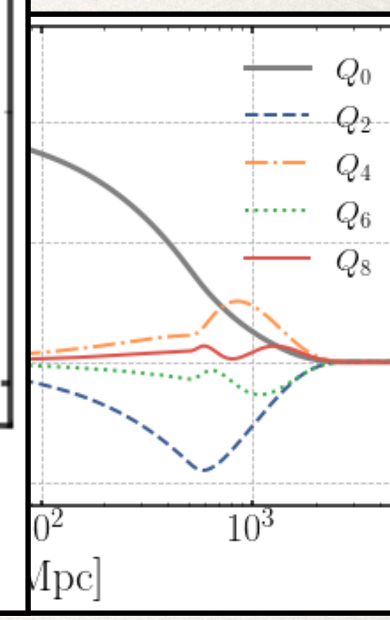
For matte

$P^{\rm win}_{\rm mm}(k$

For matte





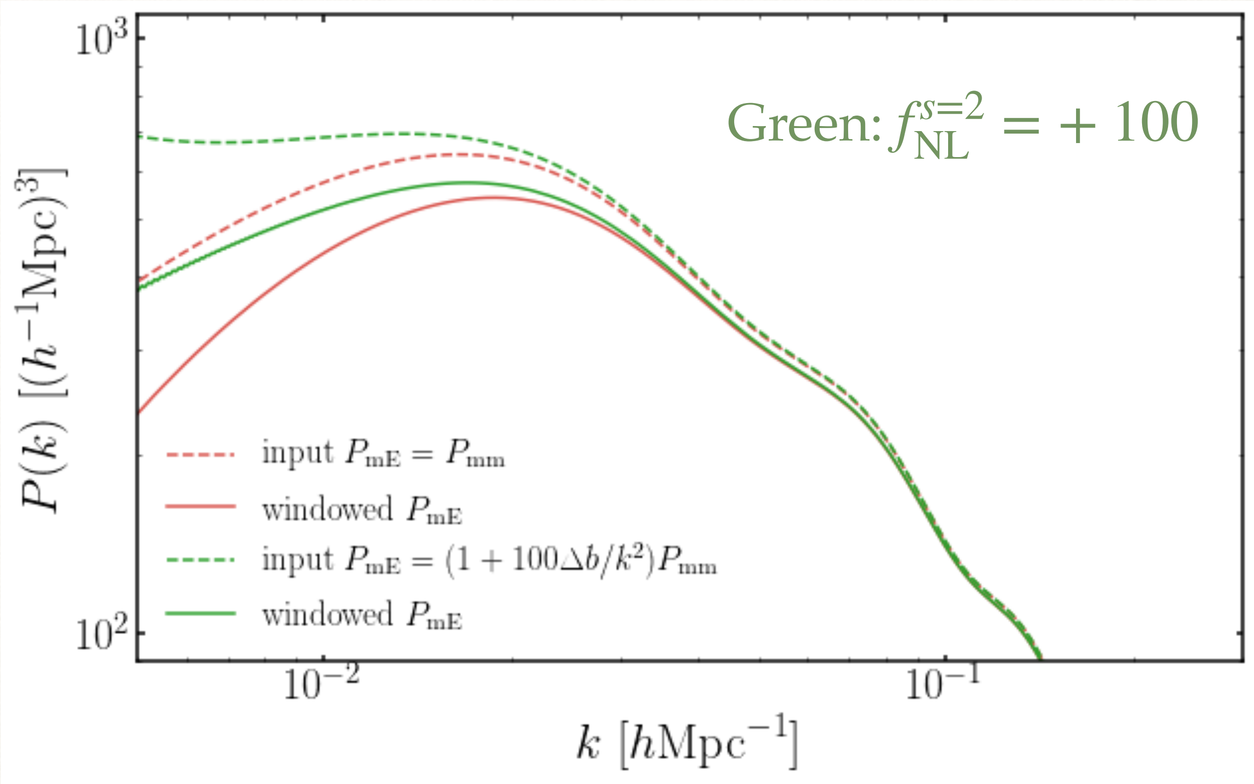
$$P^{\rm win}_{\rm Em}(k) = 4\pi \int r^2 dr\ j_2(kr)\xi(r)\{Q_0(r) - \frac{2}{7}Q_2(r) + \frac{1}{21}Q_4(r)\}$$

newly derived in this work

Summary

- We developed a Local PP estimator for IA power spectrum.
  - Available for wide-angle surveys
  - FFT-based implementation (similar to clustering estimator)

- We validated the estimator with hypothetical observations.
  - No measurement bias unlike in the case of (conventional) Global PP estimator
  - Measurements are consistent with theoretical predictions after appropriate window convolutions.



- We can also naturally compute the windowed IA power with spin-2 f_nl IC.