On the nonlinear growth of anisotropic clustering

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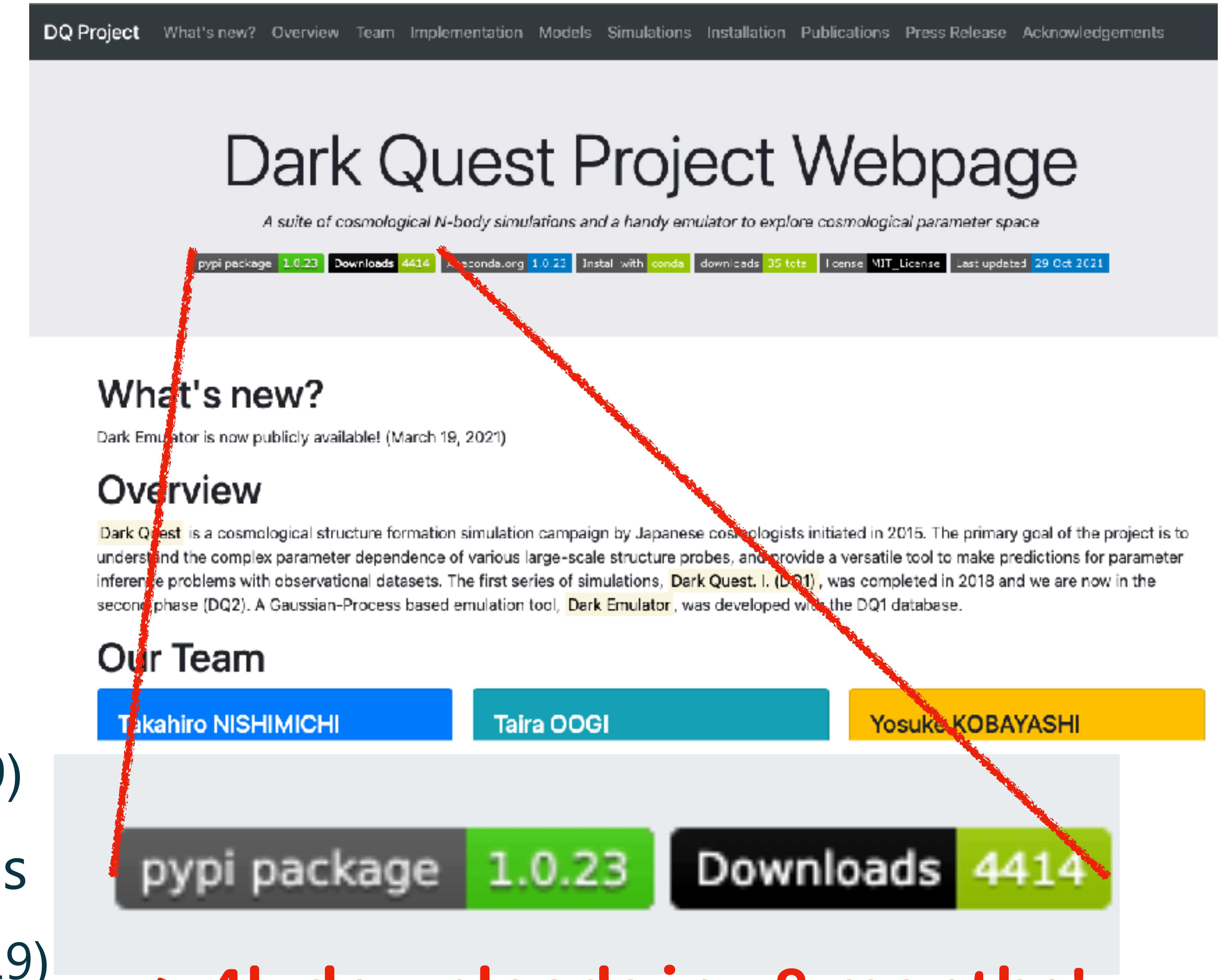
Disclaimer

- My main focus has been on large scales (BAO, RSD, PNG, neutrino masses, etc.) through *galaxy clustering* and *weak gravitational lensing*
- I participated in some collaborative studies with many of the participants of this WS to work on the IA signal, and I would like them to present those results
- What I am presenting from now on are very preliminary results, still NOT directly on the shape of galaxies nor halos, but would be closely related (I believe)

Dark Emulator now public

- 2021 3/19 public release
- Project webpage:

  https://darkquestcosmology.github.io/
- Maintained under github
- Documentation: readthedocs
- Application to observation
  - SDSS DR12 Full shape RSD analysis
    - Kobayashi, TN, Takada, Miyatake (arXiv:2110.06969)
  - HSCxSDSS lensing + clustering (2x2 pt) analysis
    - Miyatake, Sugiyama, Takada, TN+ (arXiv:2111.02419)



> 4k downloads in ~8 months!

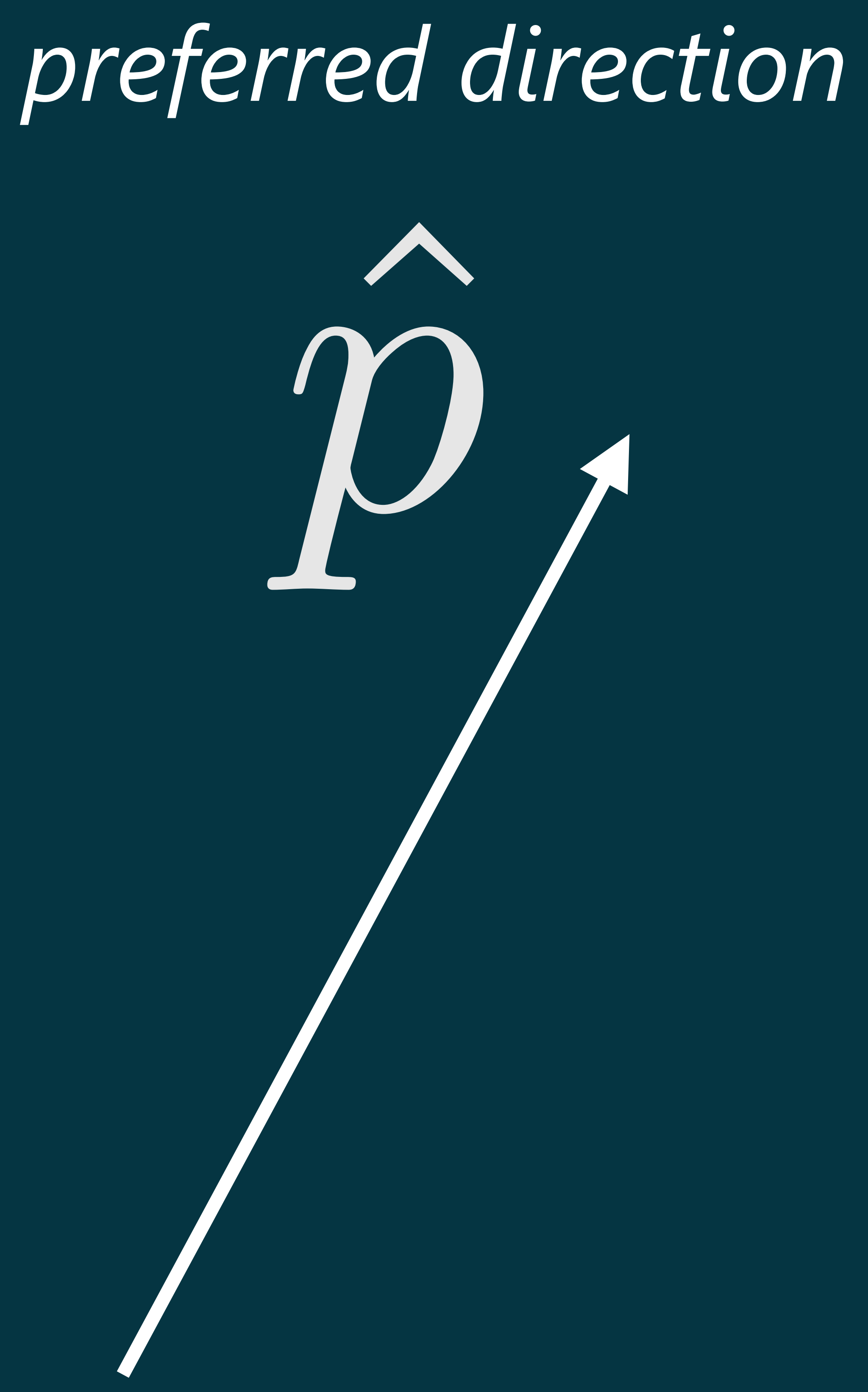
```
pip install dark_emulator
```

```
conda install -c nishimichi dark_emulator
```

- **Many papers on IA from our database (Teppei, Toshiki, …)**

Setup

- Consider a cosmological model where there exists a special direction due to some early universe physics
  - ex. vector inflation
- Assume that *background expansion* is **isotropic**
- Only *perturbations* are **anisotropic**
  - c.f., anisotropic separate Universe sims.
    - Masaki, TN, Takada arXiv:2003.10052, 2007.08727
    - See also Stücker+ 2003.06427 & Akitsu+ 2011.06584



The g_* stuff

- Assume adiabatic Gaussian initial condition
- Quadrupolar anisotropy in terms of the power spectrum

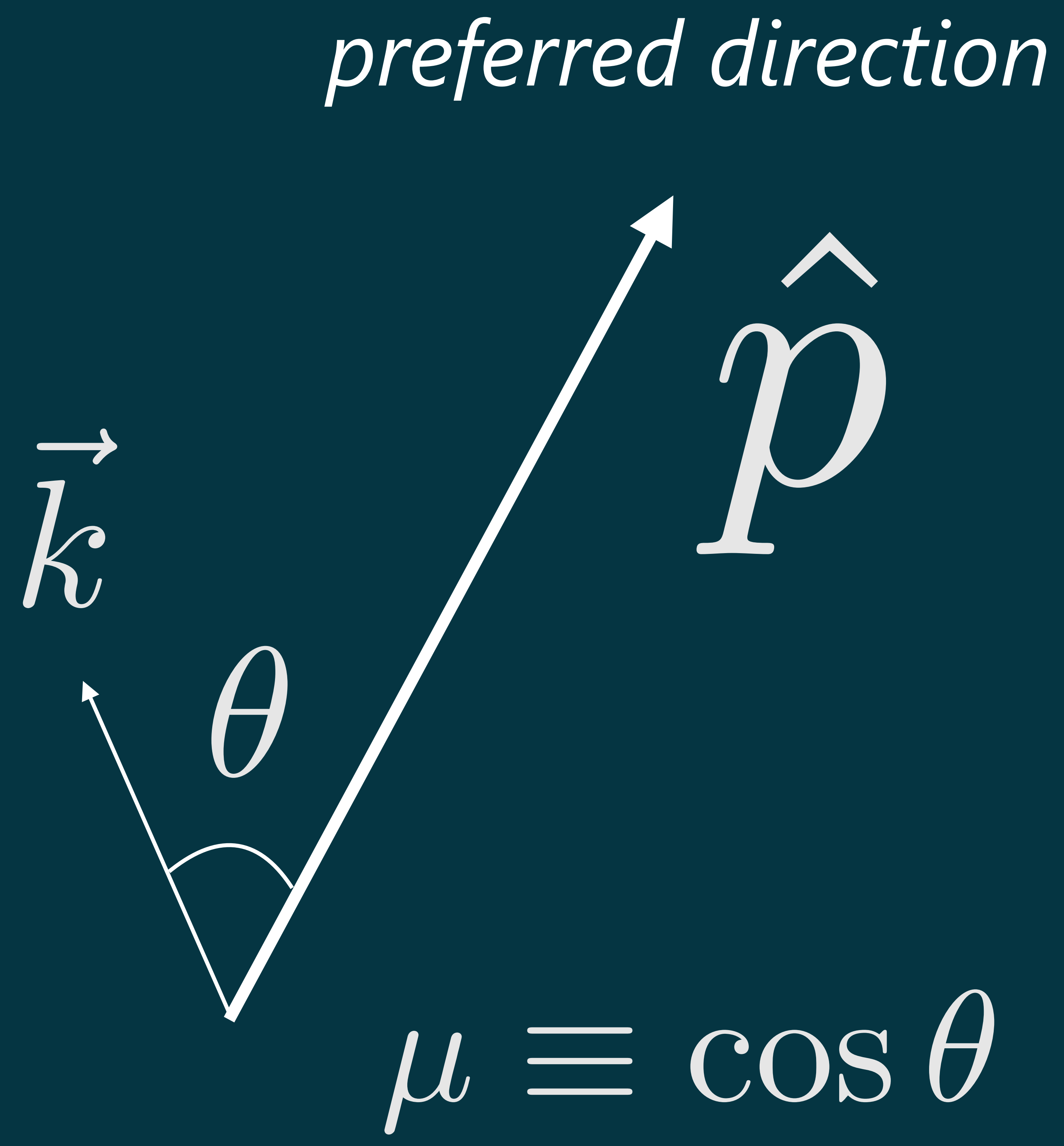
$$P_{\mathcal{R}}(\vec{k}) = P^0_{\mathcal{R}}(k)[1 + g\mathcal{L}_{\ell=2}(\mu)]$$

- Strength of anisotropy can depend on the scale (phenomenological parameterization)

$$g = g(k) = g_* \left(\frac{k}{k_*}\right)^q$$

Amplitude: g_*

Tilt: q (also denoted as n)



g_* from CMB

- Planck 2015 results. XX. Constraints on inflation

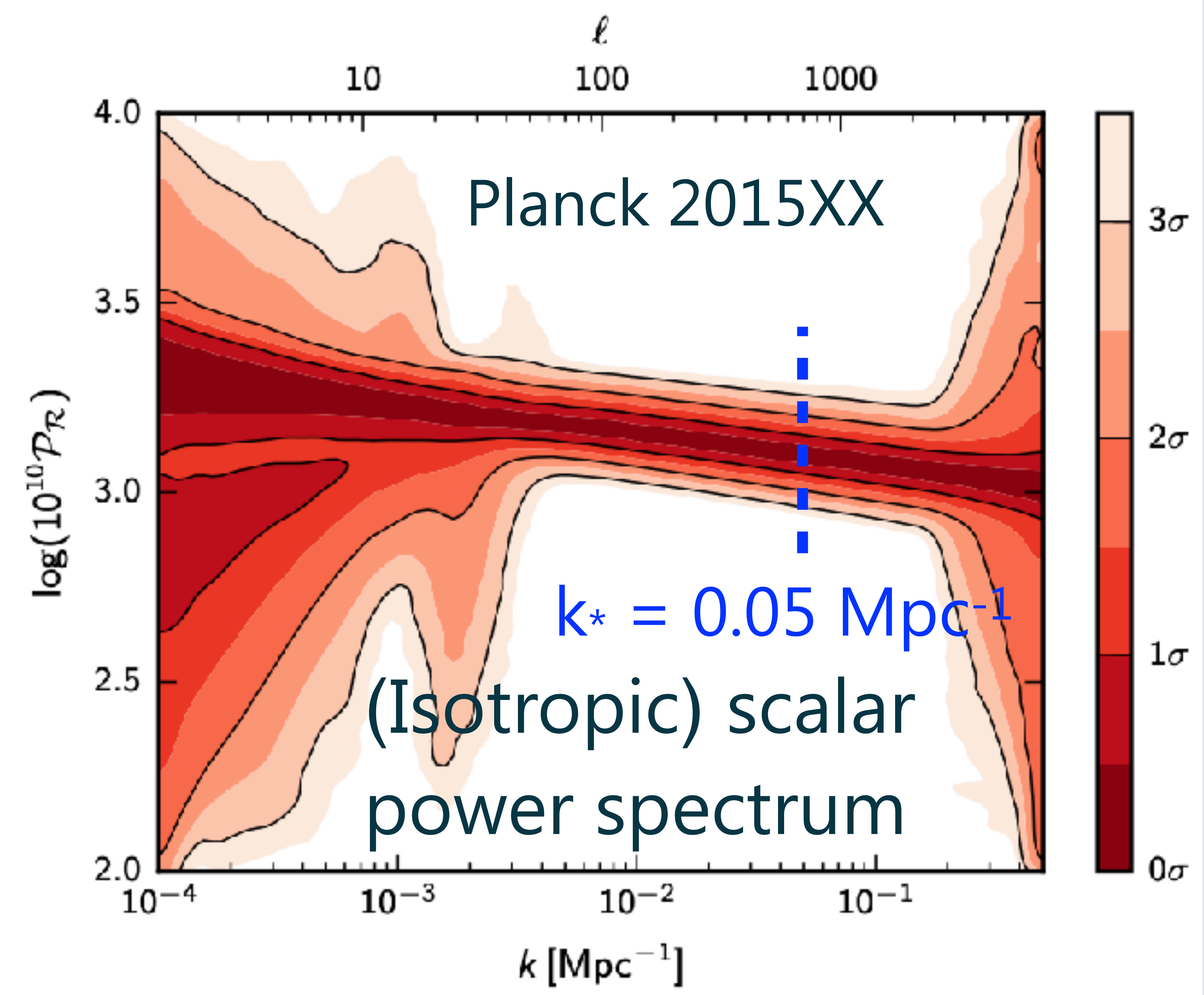
Table 17. Minimum-χ^2 g_* values for quadrupolar modulation, determined from the Commander, NILC, SEVEM, and SMICA foreground-cleaned maps. Also given are p-values, defined as the fraction of simulations with larger $|g_*|$ than the data. These results demonstrate that the data are consistent with cosmic variance in statistically isotropic skies.

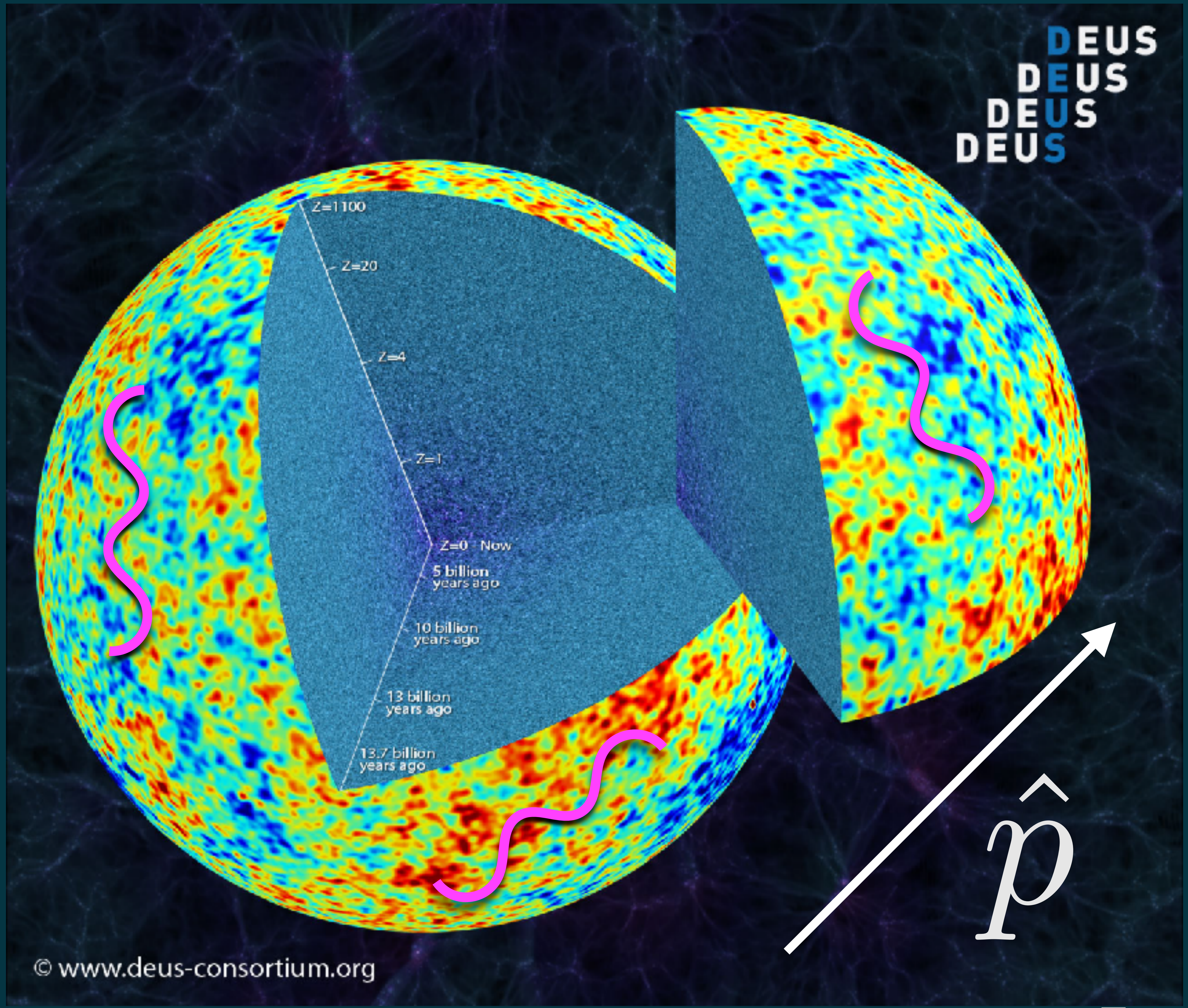
	Commander		NILC		SEVEM		SMICA	
q	g_*	p-value [%]	g_*	p-value [%]	g_*	p-value [%]	g_*	p-value [%]
−2 . . .	-7.39×10^{-5}	79.2	-7.66×10^{-5}	79.8	-7.43×10^{-5}	80.6	-7.52×10^{-5}	80.2
−1 . . .	5.99×10^{-3}	97.3	6.65×10^{-3}	95.8	6.27×10^{-3}	97.2	6.22×10^{-3}	96.9
0 . . .	-2.79×10^{-2}	12.5	-2.38×10^{-2}	26.9	-2.56×10^{-2}	20.7	-2.56×10^{-2}	20.0
1 . . .	-2.15×10^{-2}	8.2	-1.79×10^{-2}	23.7	-1.93×10^{-2}	17.8	-1.93×10^{-2}	16.7
2 . . .	-1.28×10^{-2}	9.7	-1.07×10^{-2}	23.7	-1.13×10^{-2}	20.4	-1.15×10^{-2}	18.1

※ $k_* = 0.05\ \mathrm{Mpc}^{-1}$

g_* from Large Scale Structure?

- # of accessible Fourier modes
  - 2D temperature vs 3D density
- Complementarity in terms of scales





Redshift space distortion

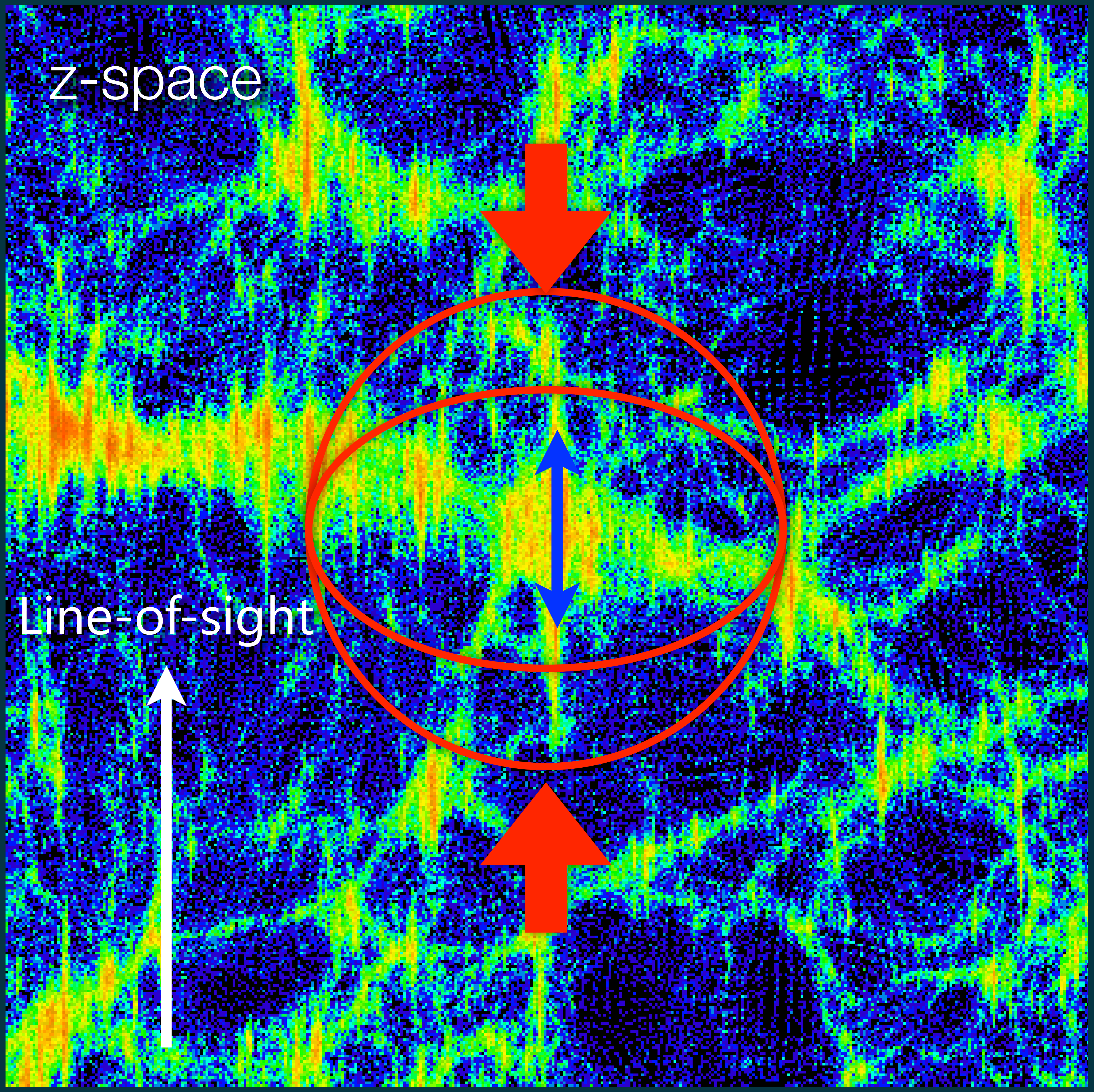
- (Unfortunately) LSS suffers from another source of anisotropy
  - which is quadrupolar at the leading order
- Alcock-Paczynski distortion can also enter

$$s = r + \frac{v_{\rm LoS}(r)}{aH(z)}\hat{e}_{\rm LoS}$$

line-of-sight displacement due to peculiar velocity

position in real space

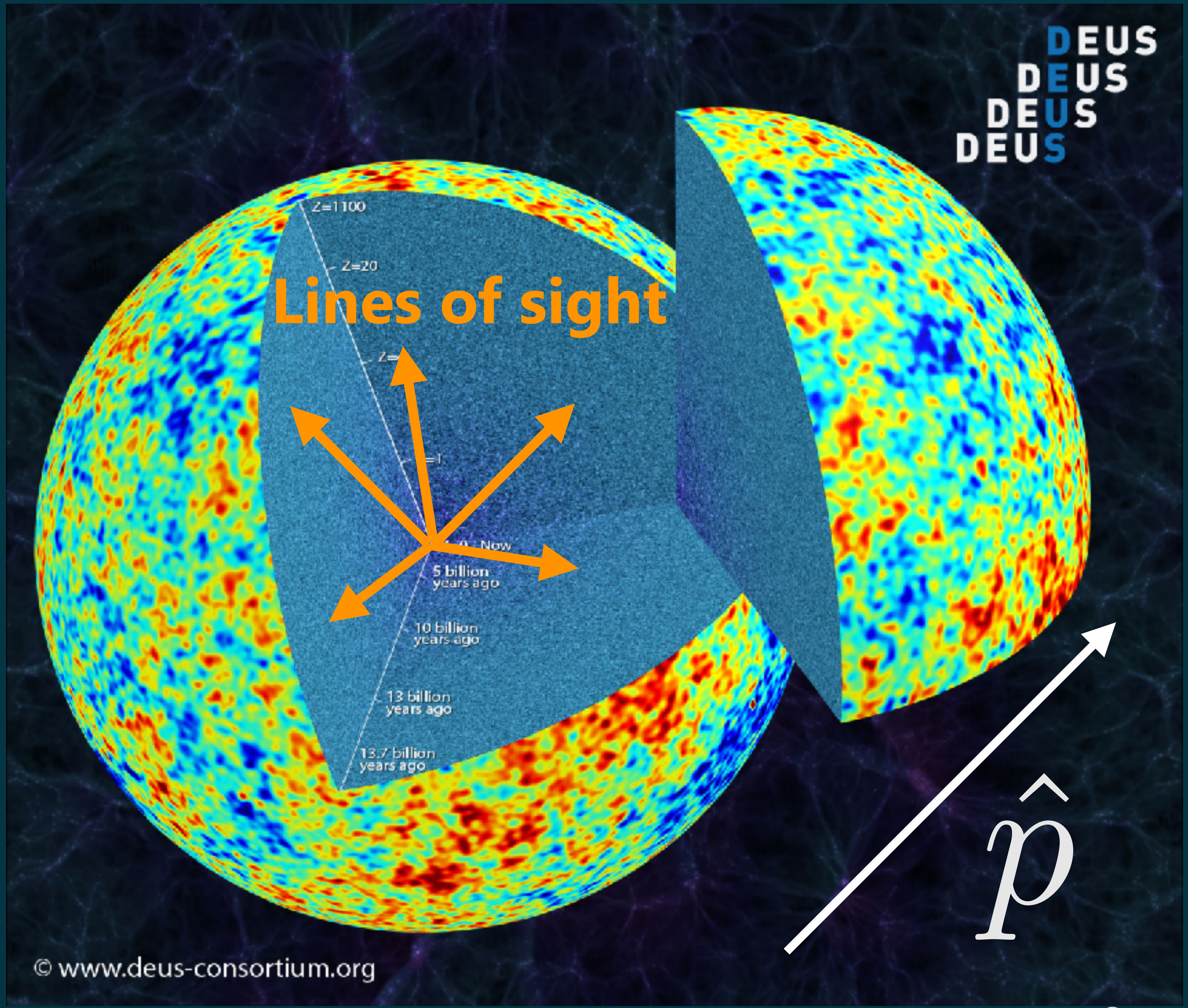
position in z-space



Redshift space distortion

- Since the line of sight direction is not the same over the sky, we can make distinction
  - The wider the better
  - If your survey is narrow and you are unlucky, you cannot
- Mathematical formulation is available in the literature for multiple directions
  - BipoSH expansion
  - TripoSH expansion

Varshalovich+`88
Hajian+`03,`04,`06
Shiraishi+`17



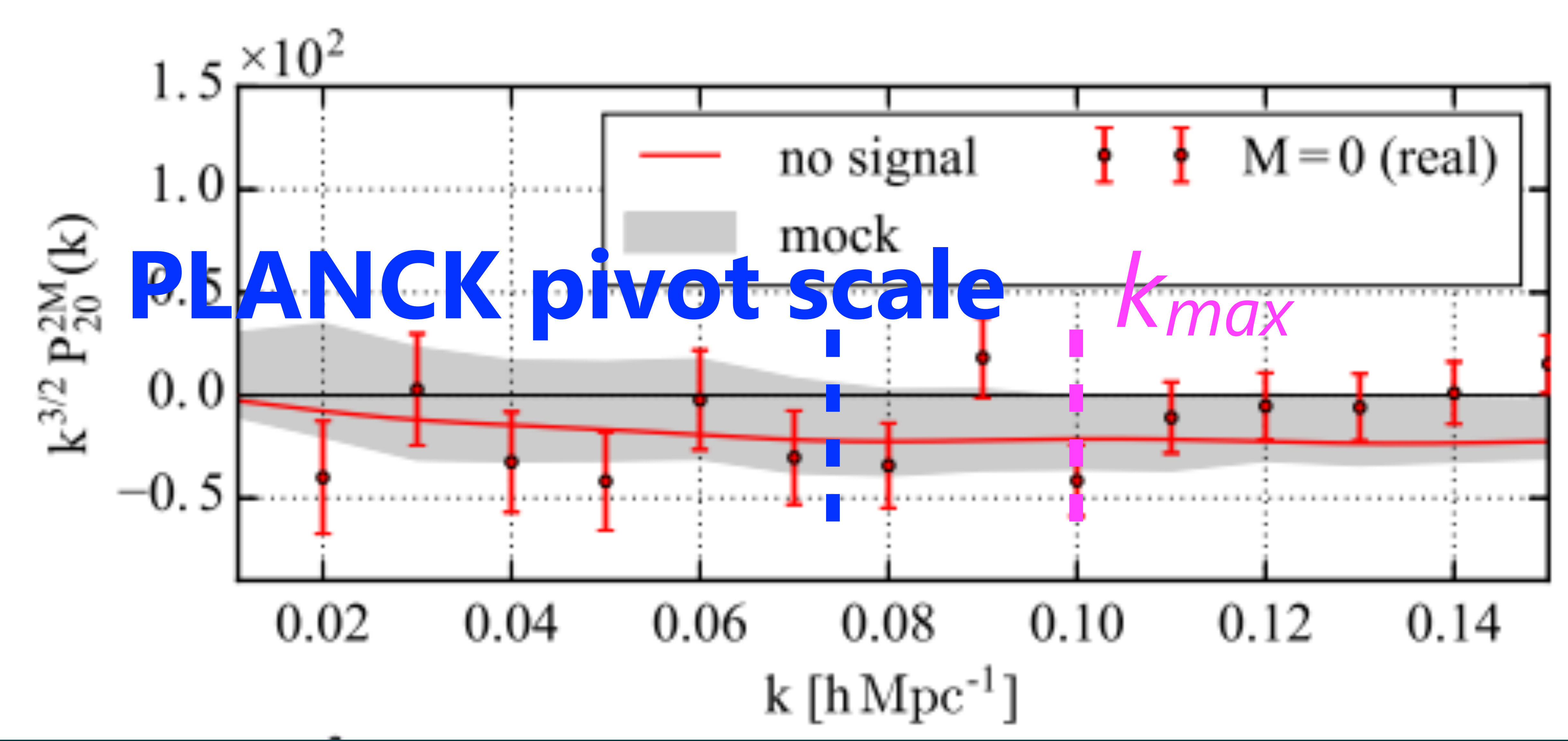
Recent example

- **Analysis using SDSS BOSS DR12**
  - Sugiyama, Shiraishi, Okumura '18

RSD — SA signal

$$P_g(\vec{k},\hat{n}) = P_K(\vec{k})\left[1+\sum_{L\geq 2}^{L=\text{even}}\sum_M g_{LM}\, f(k)\, Y_{LM}(\hat{k})\right]$$

$$P^{LM}_{\ell\ell'}(k) \equiv H^L_{\ell\ell'}\,\mathcal{P}^{LM}_{\ell\ell'}(k), \qquad H^L_{\ell\ell'} = \begin{pmatrix} \ell & \ell' & L \\ 0 & 0 & 0\end{pmatrix}$$



BipoSH expansion

$$P_g(\vec{k},\hat{n}) = \sum_{LM}\sum_{\ell\ell'}\mathcal{P}^{LM}_{\ell\ell'}(k)\, S^{LM}_{\ell\ell'}(\hat{k},\hat{n}).$$

$$S^{LM}_{\ell\ell'}(\hat{k},\hat{n}) \equiv (-1)^M \sum_{mm'}\begin{pmatrix} \ell & \ell' & L \\ m & m' & -M\end{pmatrix} y_{\ell m}(\hat{k})\, y_{\ell' m'}(\hat{n}),$$

$$P_g(\vec{k},\hat{n}) = \sum_\ell P_\ell(k)\,\mathcal{L}_\ell(\hat{k}\cdot\hat{n}) + \sum_{L\geq 1, M}\sum_{\ell\ell'}\mathcal{P}^{LM}_{\ell\ell'}(k)\, S^{LM}_{\ell\ell'}(\hat{k},\hat{n}),$$

RSD

Statistical anisotropy !!

$$P^{L\geq 2, M}_{\ell\ell'}(k) = \sqrt{\frac{2L+1}{4\pi}}\, g_{LM}\, f(k) \times (2\ell+1)\,(H^L_{\ell\ell'})^2\, P_{\ell'}(k),$$

$$g_{2M} = \frac{8\pi}{15}\, g_*\, Y^*_{2M}(\hat{p}),$$

Marginalize over $\hat{p}$ to get g_*

Recent example

- **Analysis using SDSS BOSS DR12**
  - Sugiyama, Shiraishi, Okumura '18

$$-0.09 < g_* < 0.08$$

for q=0

RSD

$$P_{\rm g}(\vec{k},\hat{n}) = \sum_\ell P_\ell(k)\,\mathcal{L}_\ell(\hat{k}\cdot\hat{n}) + \sum_{L\geq 1, M}\sum_{\ell\ell'} \mathcal{P}_{\ell\ell'}^{LM}(k)\,S_{\ell\ell'}^{LM}(\hat{k},\hat{n}),$$

Statistical anisotropy !!

Power spectrum					
Spectral index	CMASS NGC	CMASS SGC	LOWZ NGC	LOWZ SGC	All
$n = -2$	$-0.05 < g_* < 0.05$	$-0.52 < g_* < 0.14$	$-0.08 < g_* < 0.08$	$-0.13 < g_* < 0.16$	$-0.040 < g_* < 0.044$
$n = -1$	$-0.13 < g_* < 0.11$	$-0.14 < g_* < 0.21$	$-0.15 < g_* < 0.14$	$-0.21 < g_* < 0.25$	$-0.084 < g_* < 0.096$
$n = +1$	$-0.09 < g_* < 0.05$	$-0.66 < g_* < 0.10$	$-0.07 < g_* < 0.07$	$-0.11 < g_* < 0.12$	$-0.068 < g_* < 0.047$

What is still missing?

- Analysis limited on scales larger than k=0.1h/Mpc (applicable range of linear-theory template)
- In standard (isotropic) setting, no matter how strong nonlinearity grows, statistical anisotropy cannot be generated except one with the LoS as the special direction
  - SA at any scale is a smoking gun (even w/o a theoretical template)!
- How does primordial SA evolve in nonlinear regime?
  - Eventually go away? (e.g., relaxation inside halos)

Only one previous study

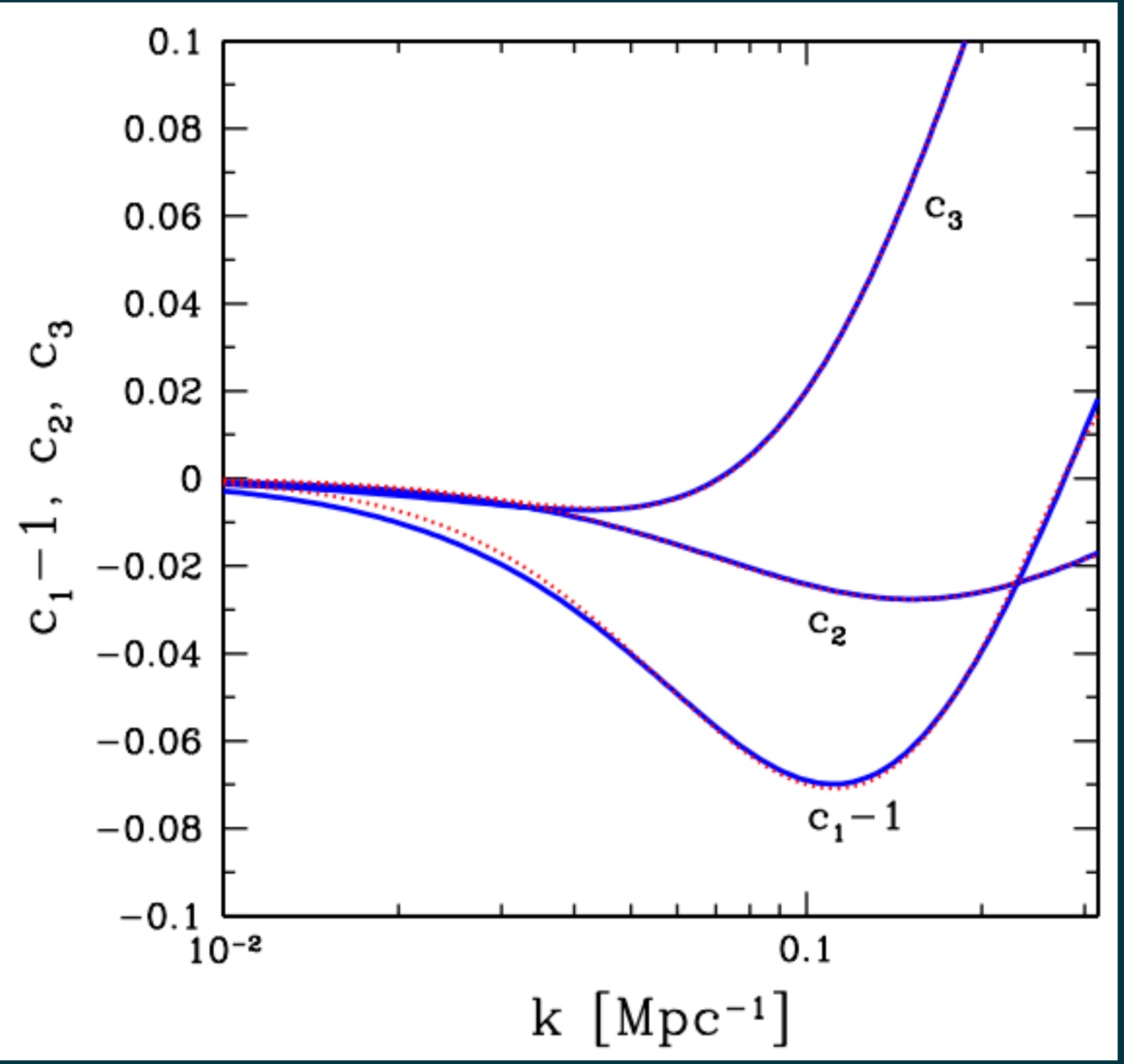
$$P^{(2)}(\boldsymbol{k}) = A^{(2)}(k)[1 + g_2^{(2)}(k)\mathcal{P}_2(\mu_k) + g_4^{(2)}(k)\mathcal{P}_4(\mu_k)],$$

$$A^{(2)}(k) = A_{\rm lin}(k) + B_{00}(k) + g_*^2 B_{20}(k),$$

$$g_2^{(2)}(k) = g_* c_1(k) + g_*^2 c_2(k),$$

$$g_4^{(2)}(k) = g_*^2 c_3(k),$$

- Follow the evolution of the quadrupolar SA using 1-loop PT for g initially constant in k
- Slight suppression on weakly nonlinear regime (~7% at z=0)
  - Turn to amplification at larger k?
    - Breakdown of PT?
  - Mixing of different multipoles?
    - Order g_*^2 effect



Ando & Kamionkowski (2009) PRL **100**, 071301

Simulations

- # of particles 1024^3
- Initial condition
  - 2nd order Lagrangian PT
  - Glass preinitial (vs Grid)
- Evolution
  - TreePM code based on FDPS

    (TN, Tanaka, Yoshikawa in prep)

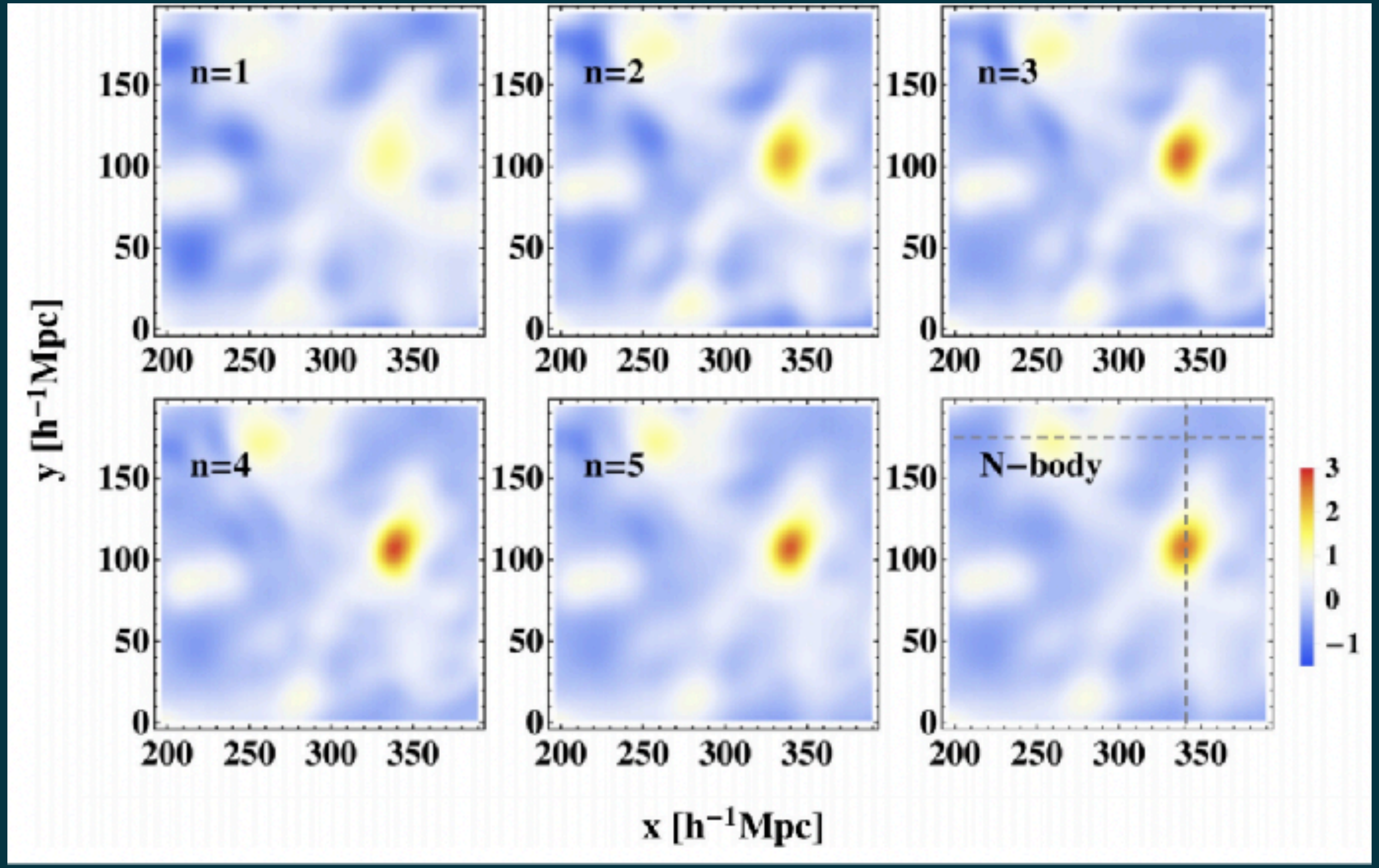
	Coming box size	# of realizations
Isotropic	1024 Mpc/h	4
Isotropic HR	512 Mpc/h	4
Isotropic Grid IC	1024 Mpc/h	1
$g_* = 0.1$, $q = 0$	1024 Mpc/h	4
$g_* = 0.1$, $q = 0$ HR	512 Mpc/h	4
$g_* = 0.1$, $q = 0$ Grid IC	1024 Mpc/h	1
$g_* = 0.2$, $q = 0$	1024 Mpc/h	4
$g_* = 0.4$, $q = 0$	1024 Mpc/h	4
$g_* = 0.005$, $q = -2$	1024 Mpc/h	4
$g_* = 0.05$, $q = -1$	1024 Mpc/h	4
$g_* = 0.01$, $q = 1$	1024 Mpc/h	4

GridSPT

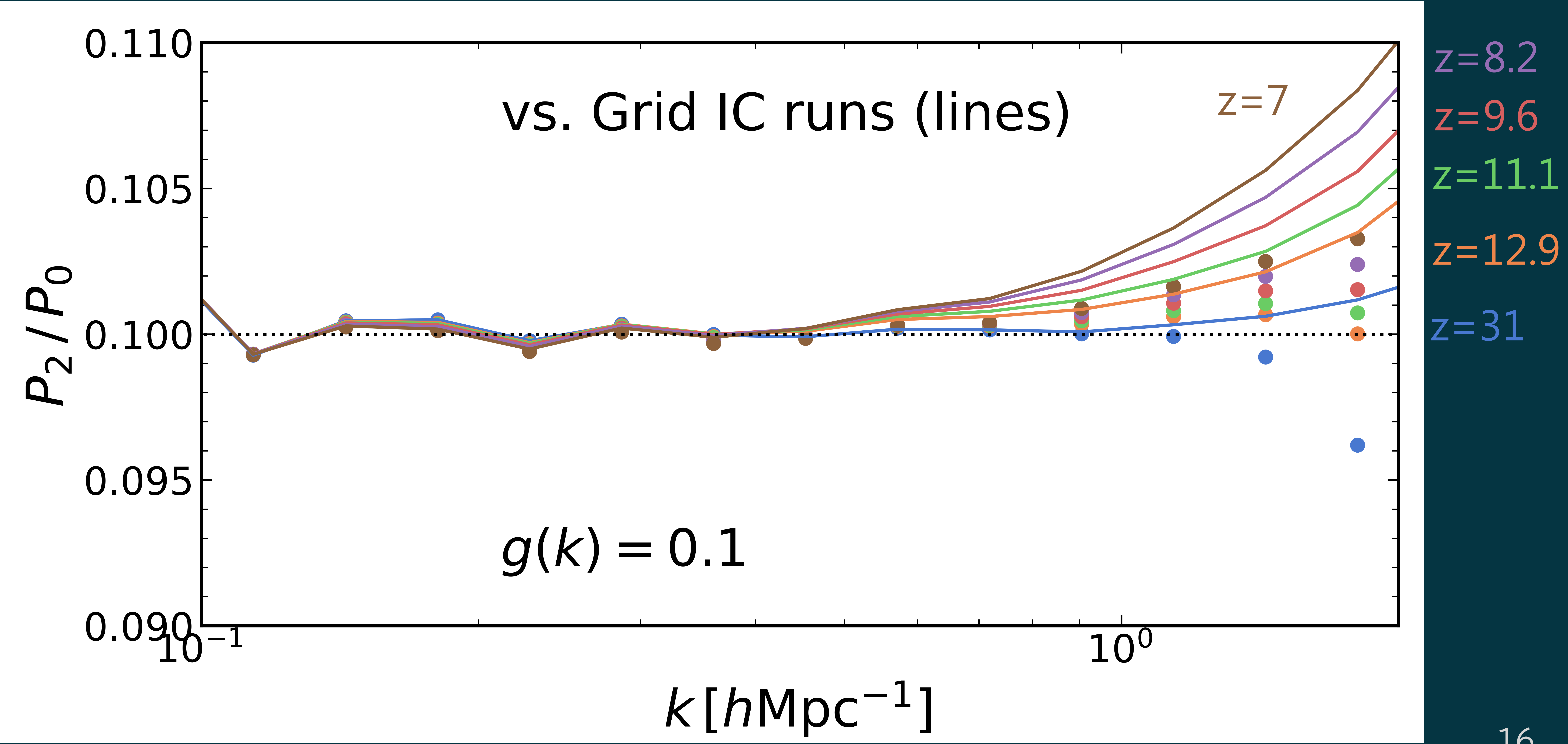
Taruya, TN & Jeong (2018) PRD **98**, 103532

- Numerical random realizations to systematically evaluate higher order terms
  - Can make an apple-to-apple comparison with simulations (with the identical sample variance)
  - Use as its for anisotropic linear fields
- A cutoff scale around the grid spacing scale (a bit larger scale for dealiasing)
  - 1024 Mpc/h, 1200^3 grid points
  - At 2-loop (1x5 order) , the 2/(n+1) rule gives
    - Resolution equivalent to 400^3 grid points (~2.5Mpc/h)

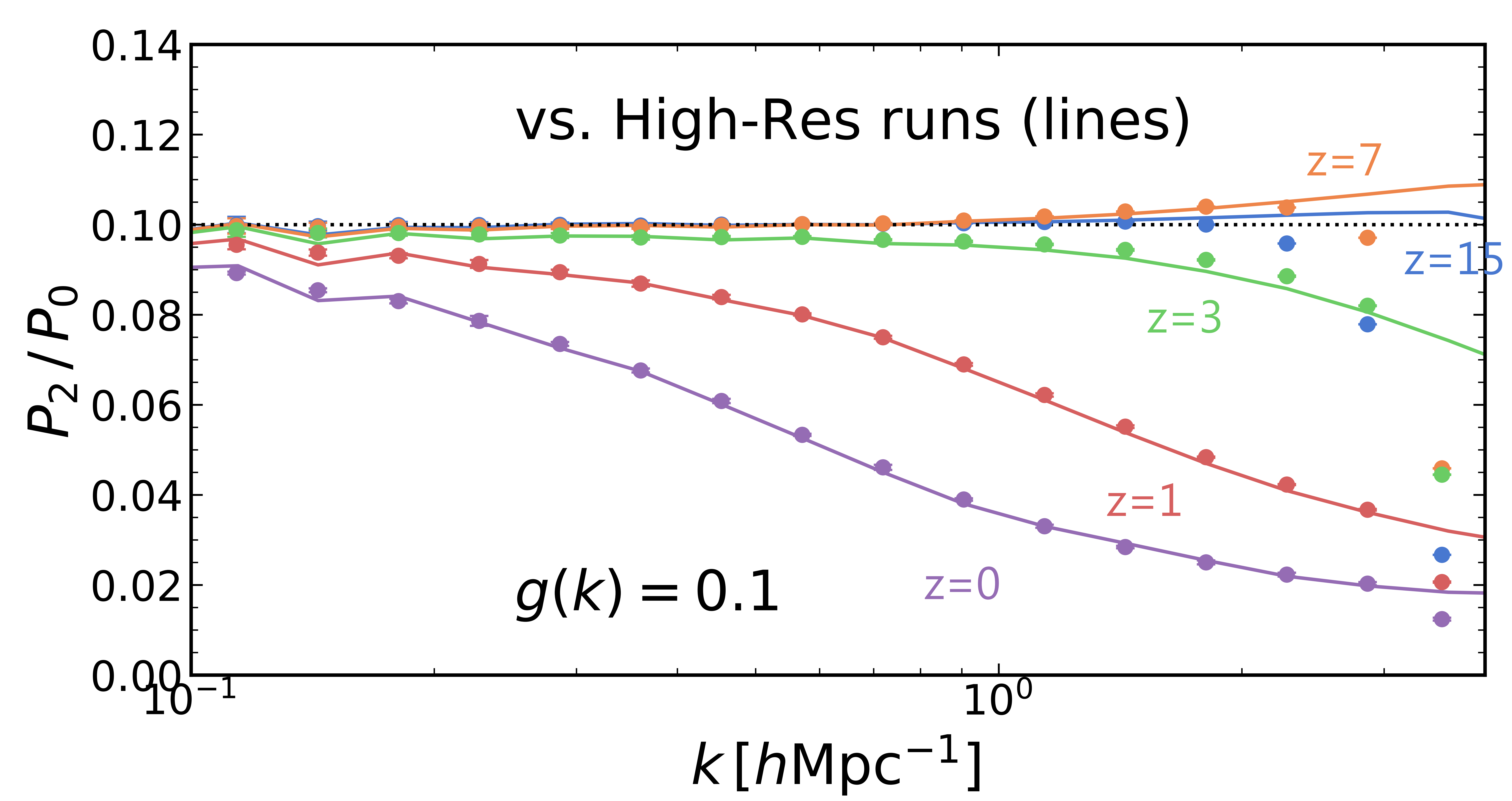
$$\begin{pmatrix} \delta_n(\boldsymbol{x}) \\ \theta_n(\boldsymbol{x}) \end{pmatrix} = \frac{2}{(2n+3)(n-1)} \begin{pmatrix} n+\frac{1}{2} & 1 \\ \frac{3}{2} & n \end{pmatrix} \sum_{m=1}^{n-1} \begin{pmatrix} (\nabla\delta_m)\cdot\boldsymbol{u}_{n-m} + \delta_m\,\theta_{n-m} \\ [\partial_j(\boldsymbol{u}_m)_k][\partial_k(\boldsymbol{u}_{n-m})_j] + \boldsymbol{u}_m\cdot(\nabla\theta_{n-m}) \end{pmatrix}$$



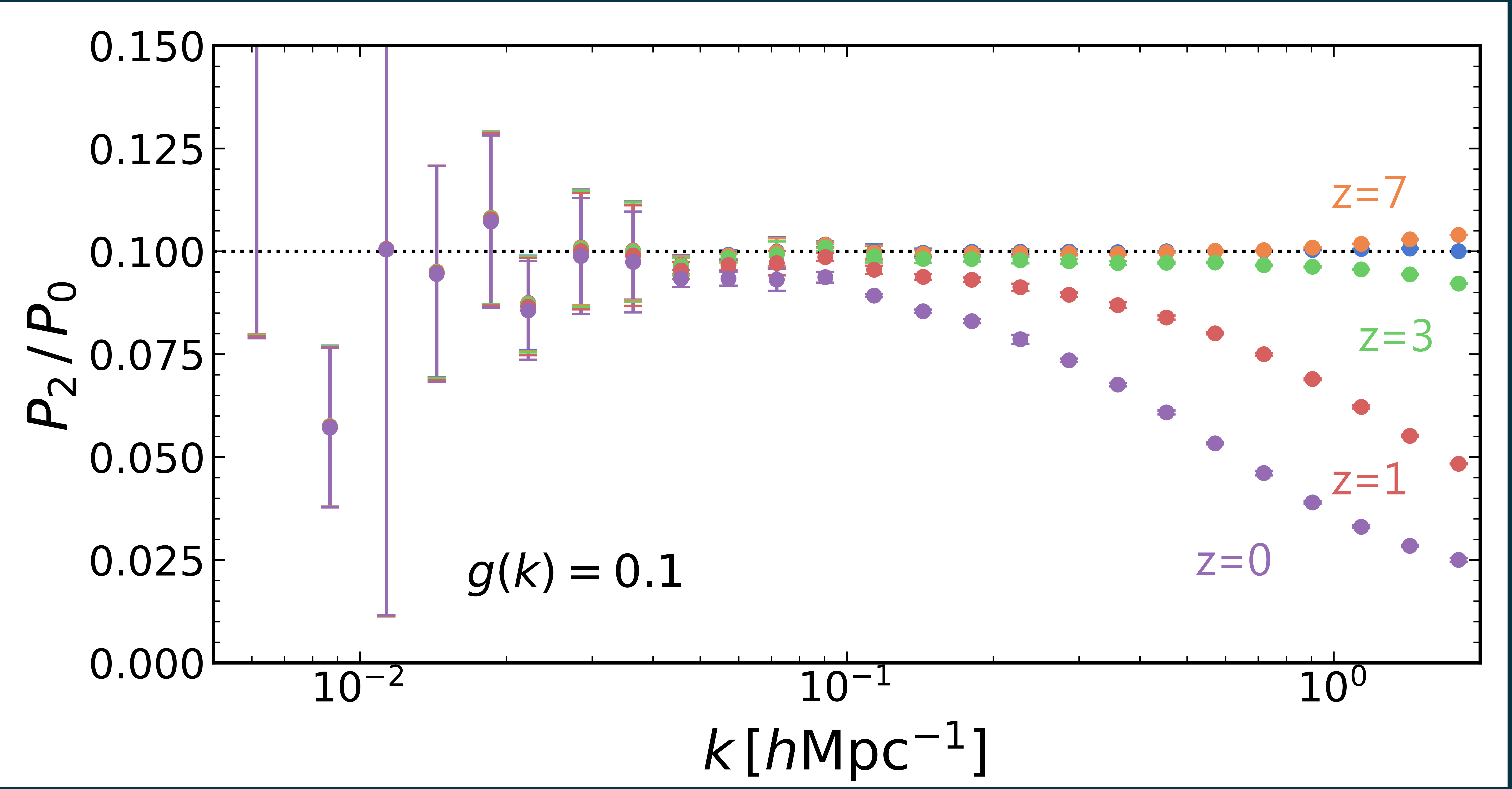
High z evolution (Grid vs Glass IC)



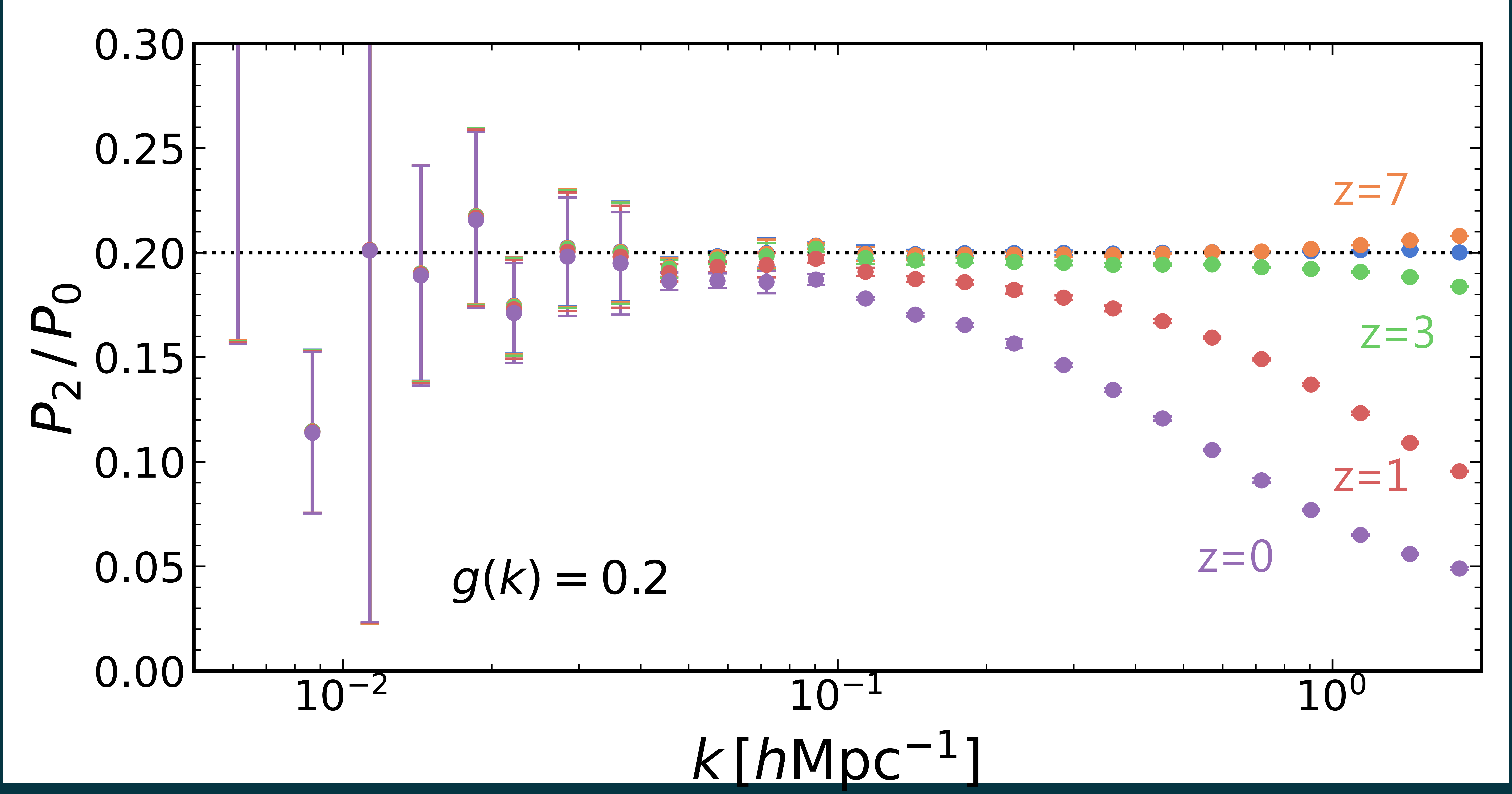
Resolution study



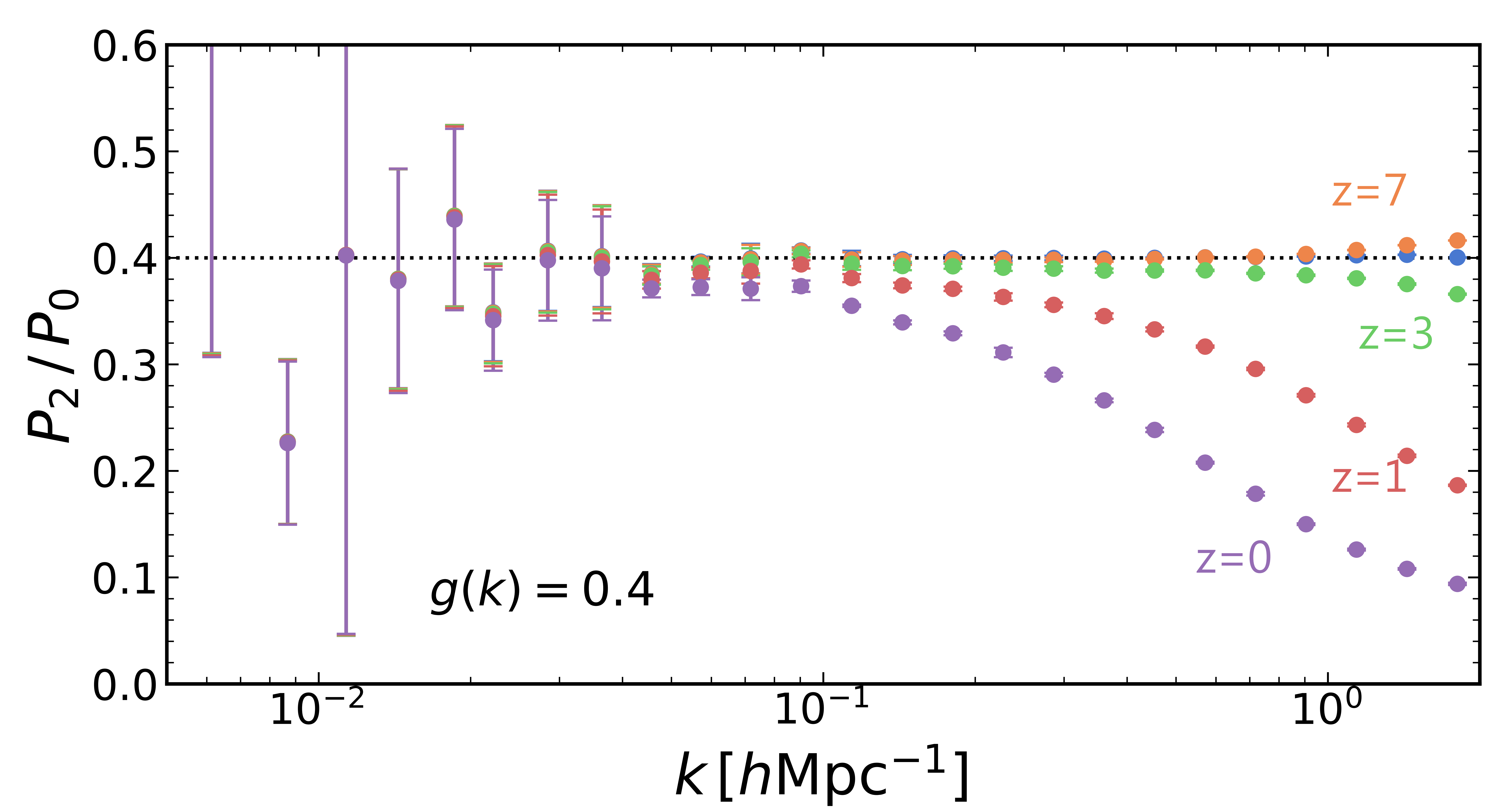
Results (quadrupole signal)



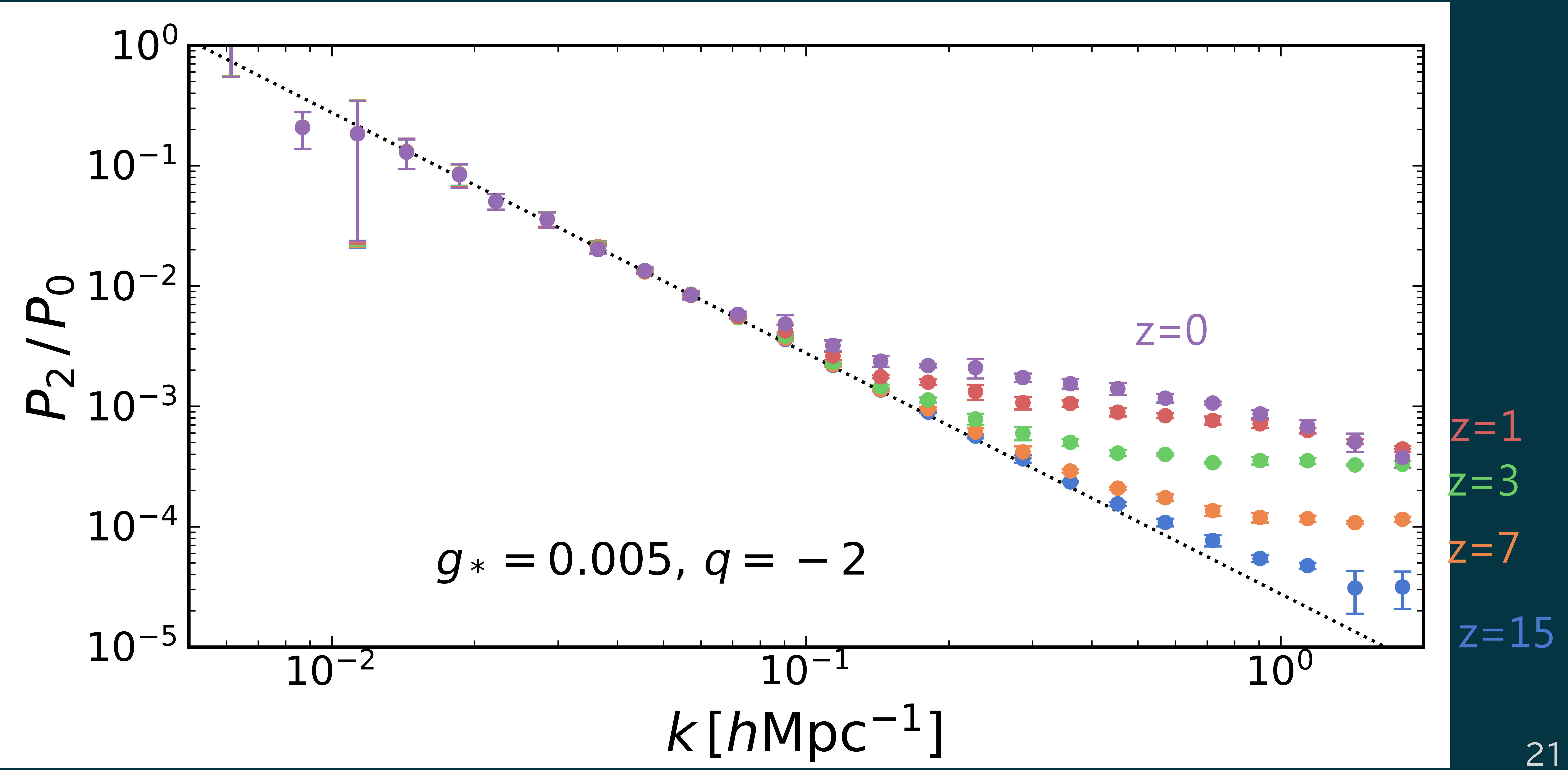
Results (quadrupole signal)



Results (quadrupole signal)

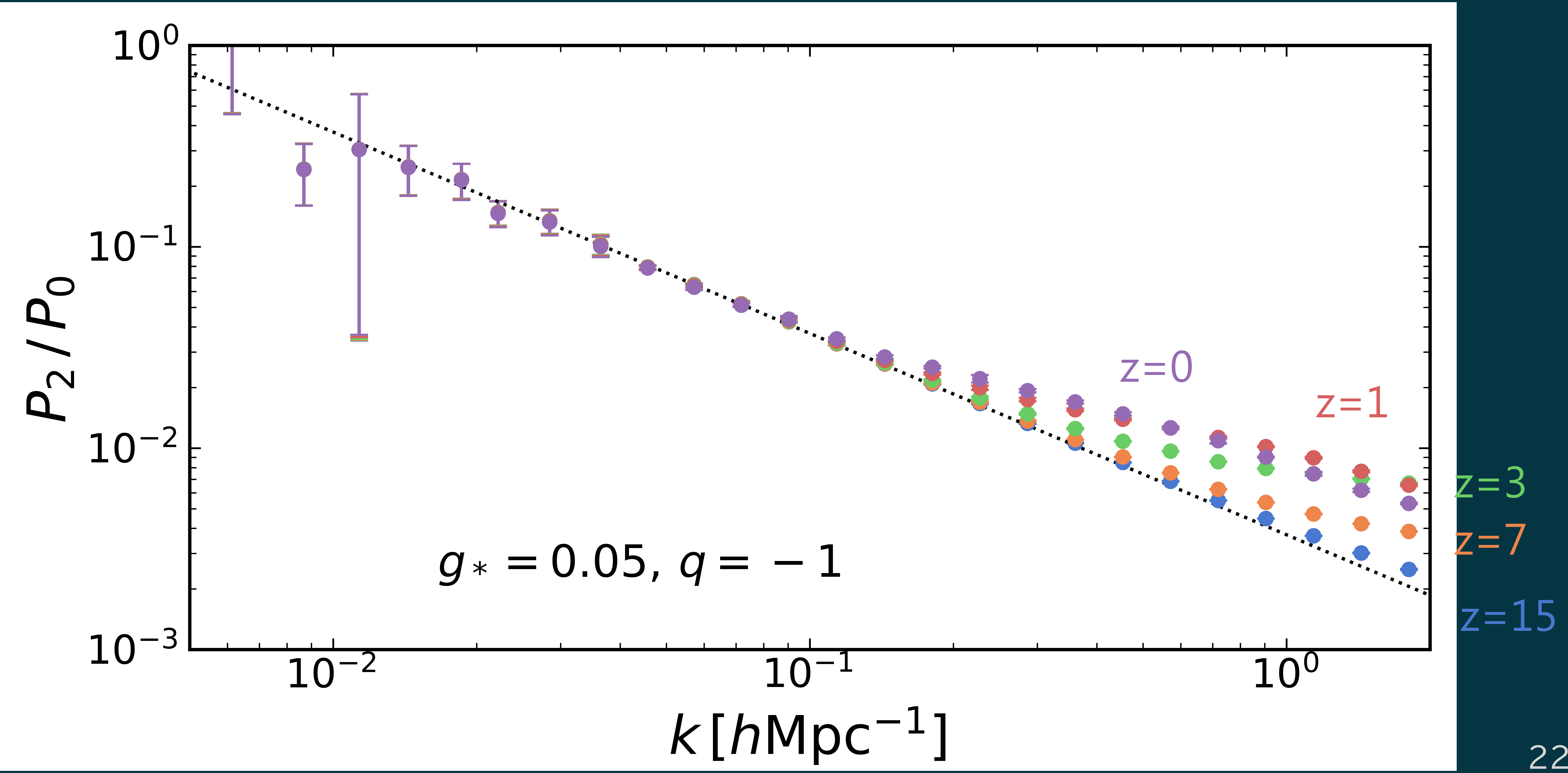


Results



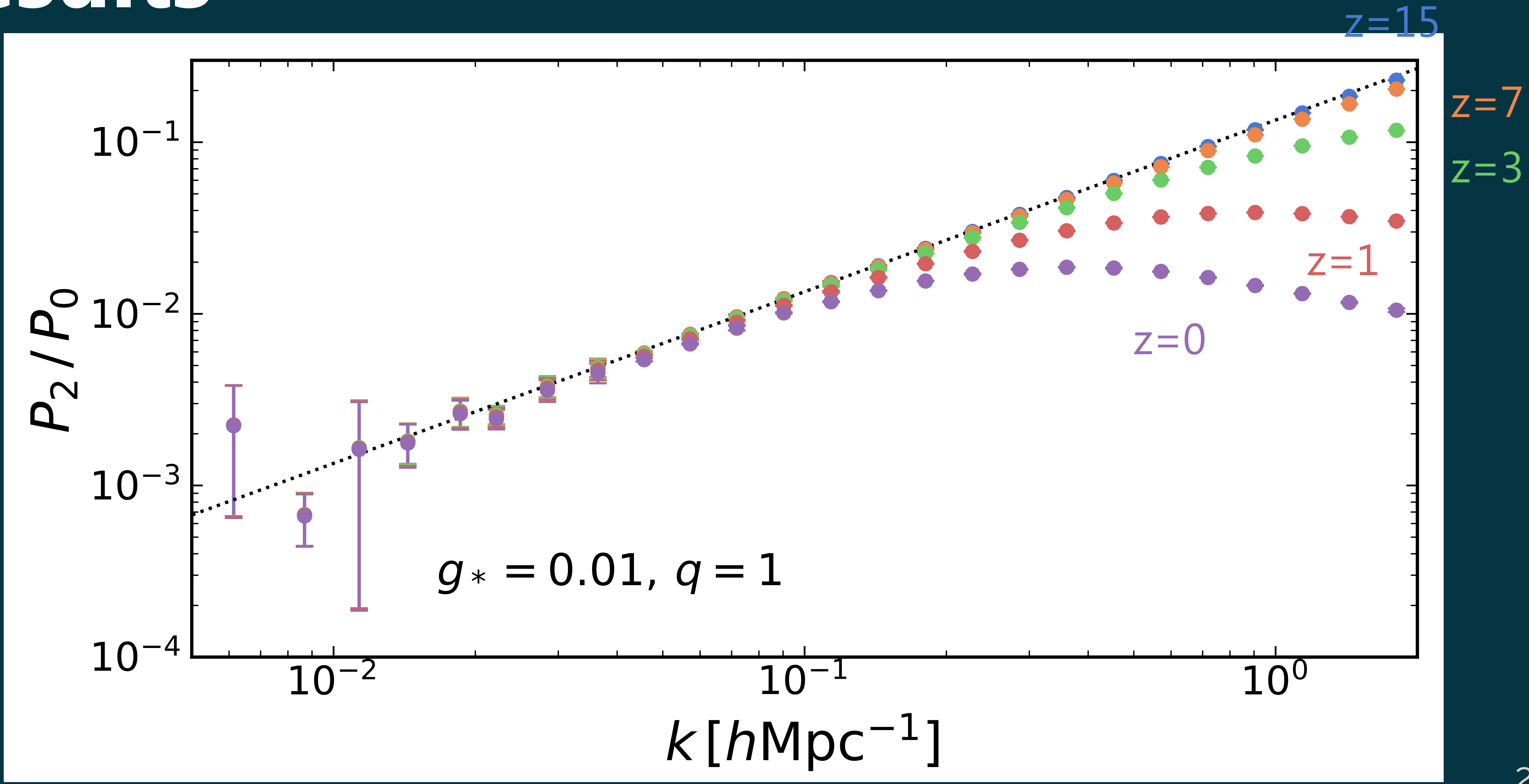
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
P_2 / P_0
10^{-2}
10^{-1}
10^{0}
$k\,[h\mathrm{Mpc}^{-1}]$
$g_* = 0.005,\ q = -2$
z=0
z=1
z=3
z=7
z=15

Results



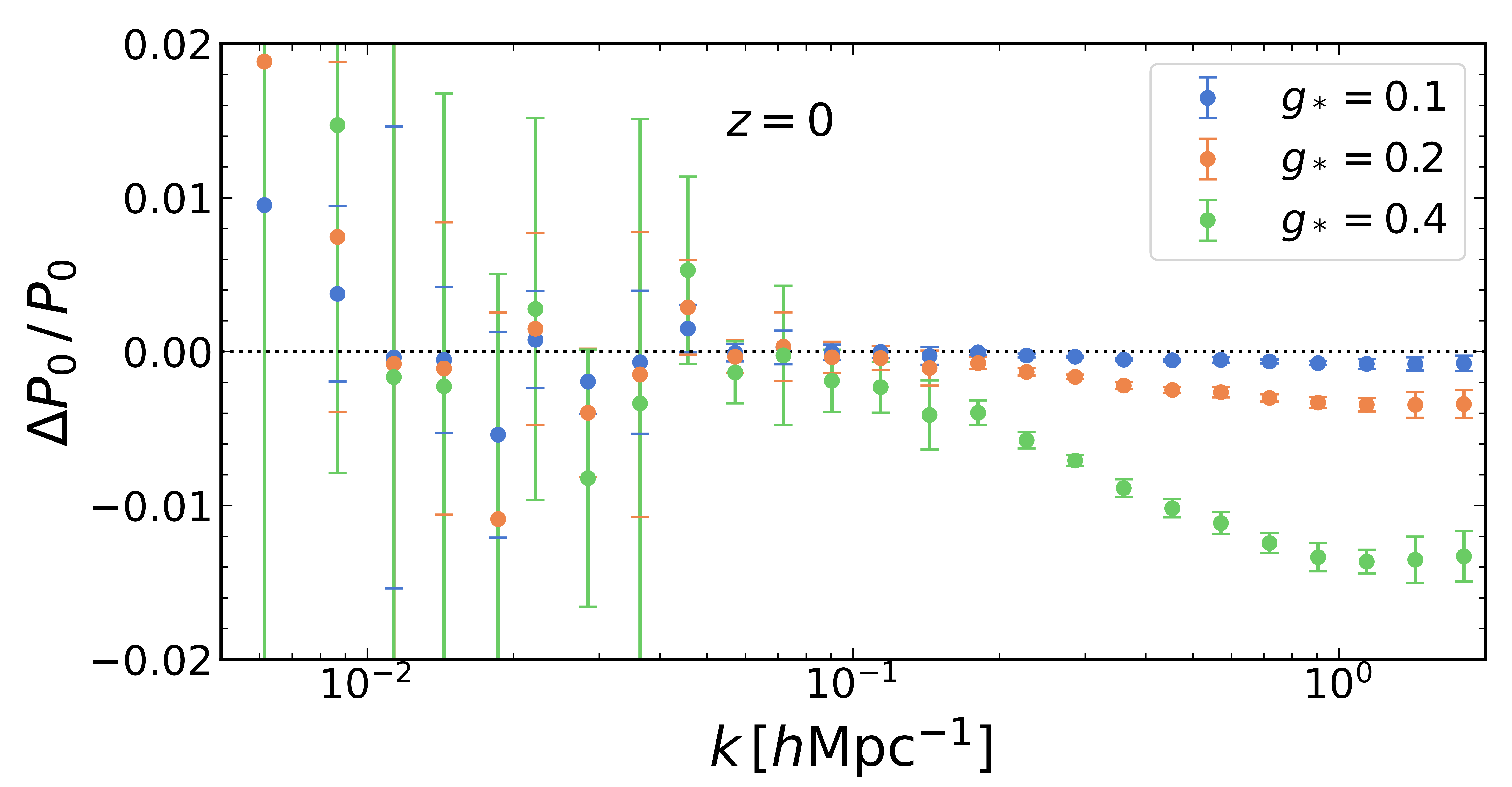
10^0
10^{-1}
10^{-2}
10^{-3}
P_2 / P_0
10^{-2}
10^{-1}
10^0
$k\,[h\mathrm{Mpc}^{-1}]$
$g_* = 0.05,\ q = -1$
z=0
z=1
z=3
z=7
z=15

Results

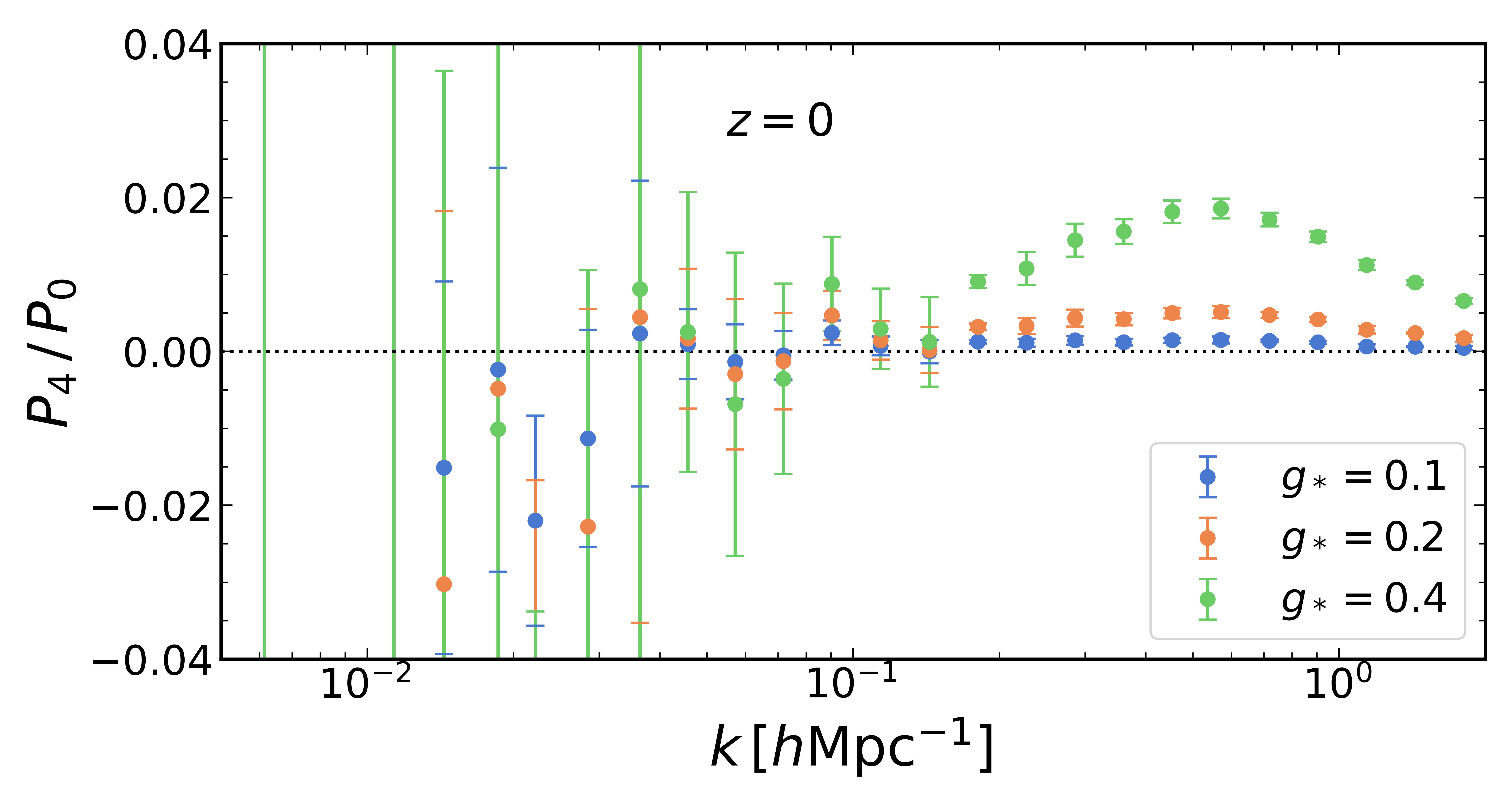


z=15
z=7
z=3
z=1
z=0
P_2 / P_0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$g_* = 0.01,\ q = 1$
10^{-2}
10^{-1}
10^{0}
$k\,[h\mathrm{Mpc}^{-1}]$

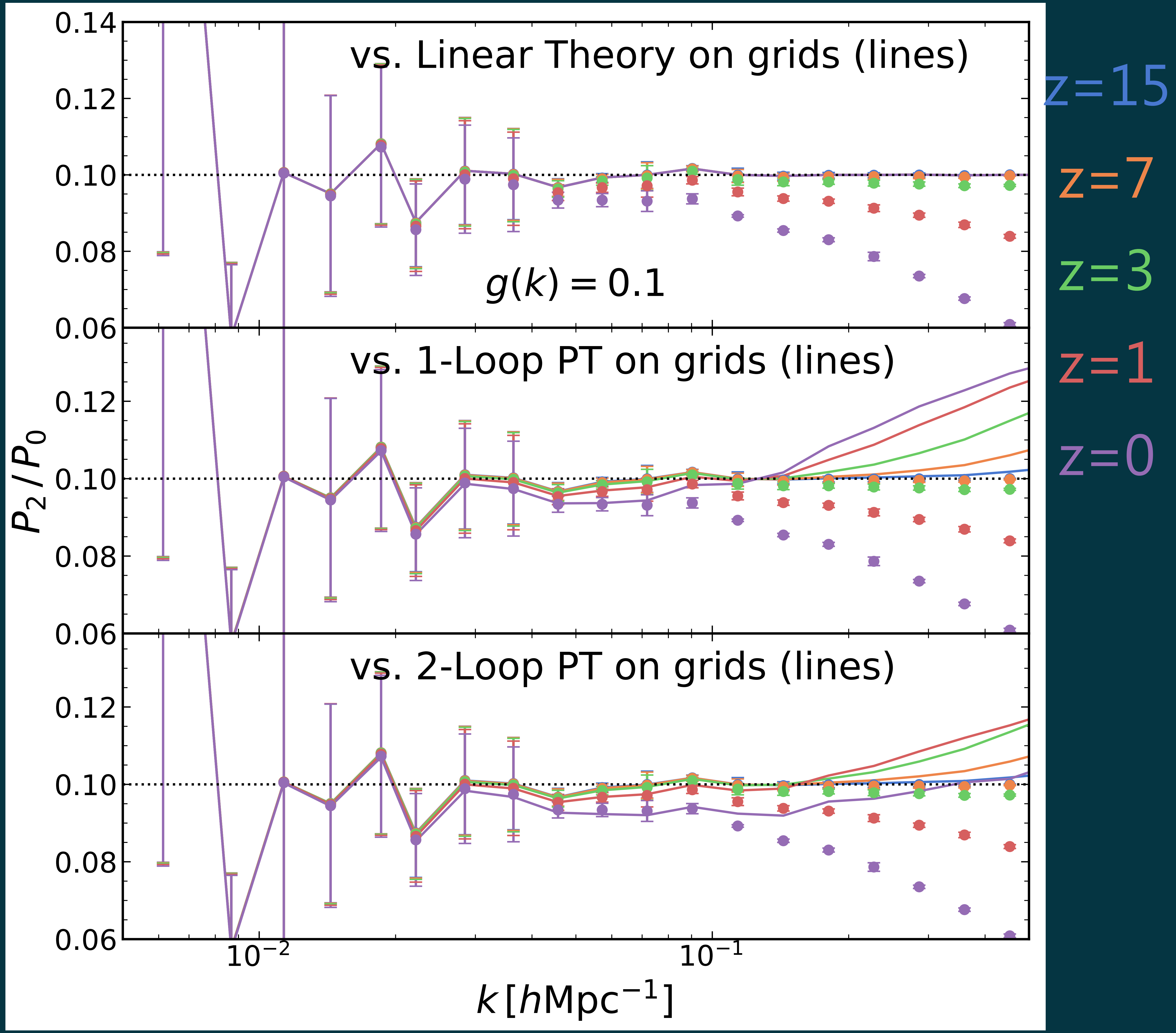
Leakage to other multipoles



Leakage to other multipoles



Comparison with GridSPT

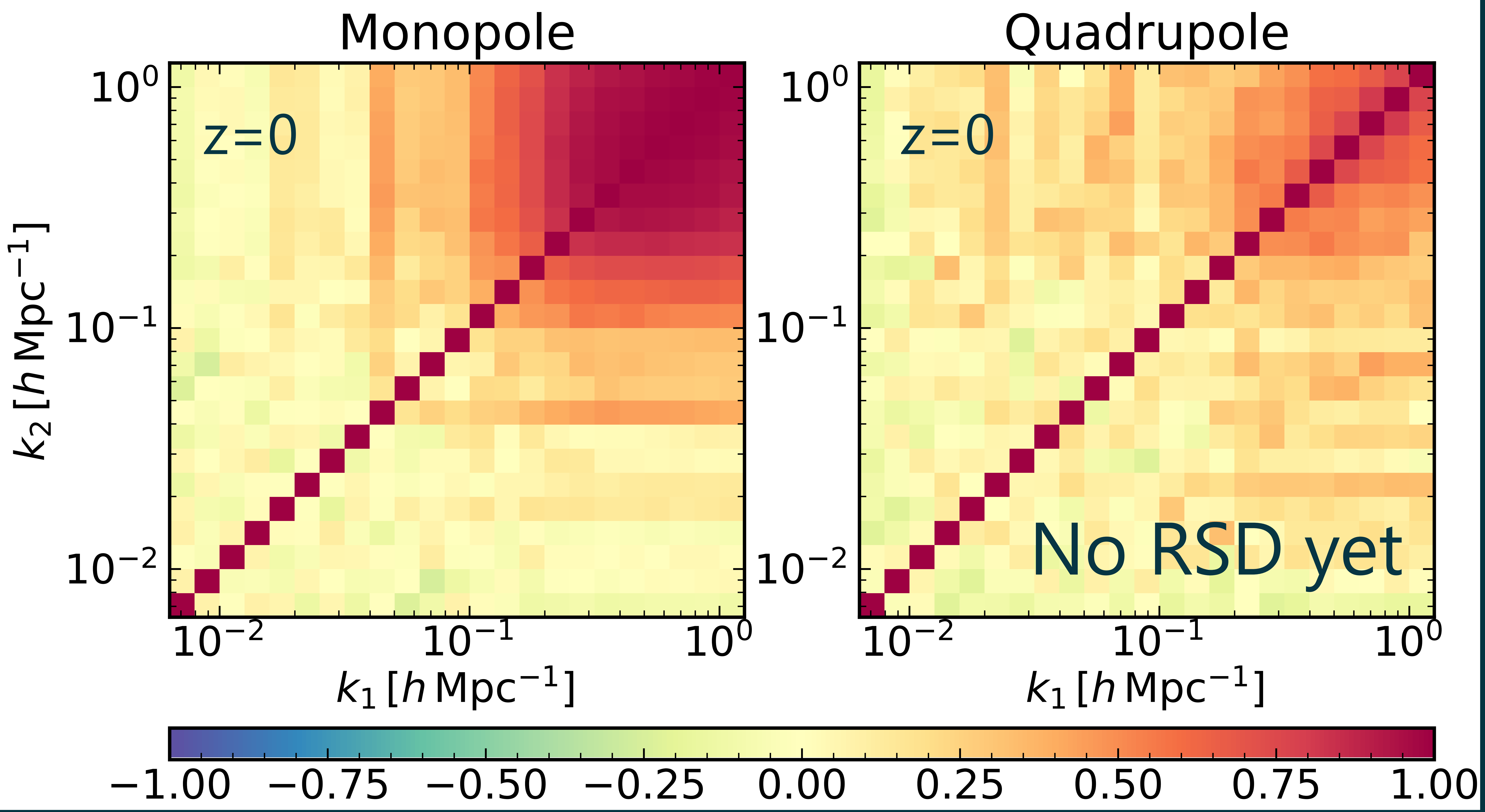


- ~7% dip predicted by 1-loop PT is consistent with simulations
- A slightly better agreement at 2 loop
- Nonperturbative soon kicks in to suppress the quadrupoles signal

Comparison with GridSPT

Information gain?

$$\mathrm{Cov}(k_1, k_2) = \langle [\hat{P}(k_1) - P(k_1)][\hat{P}(k_2) - P(k_2)] \rangle$$



$$r(k_1, k_2) = \frac{\mathrm{Cov}(k_1, k_2)}{\sqrt{\mathrm{Var}(k_1)\mathrm{Var}(k_2)}}$$

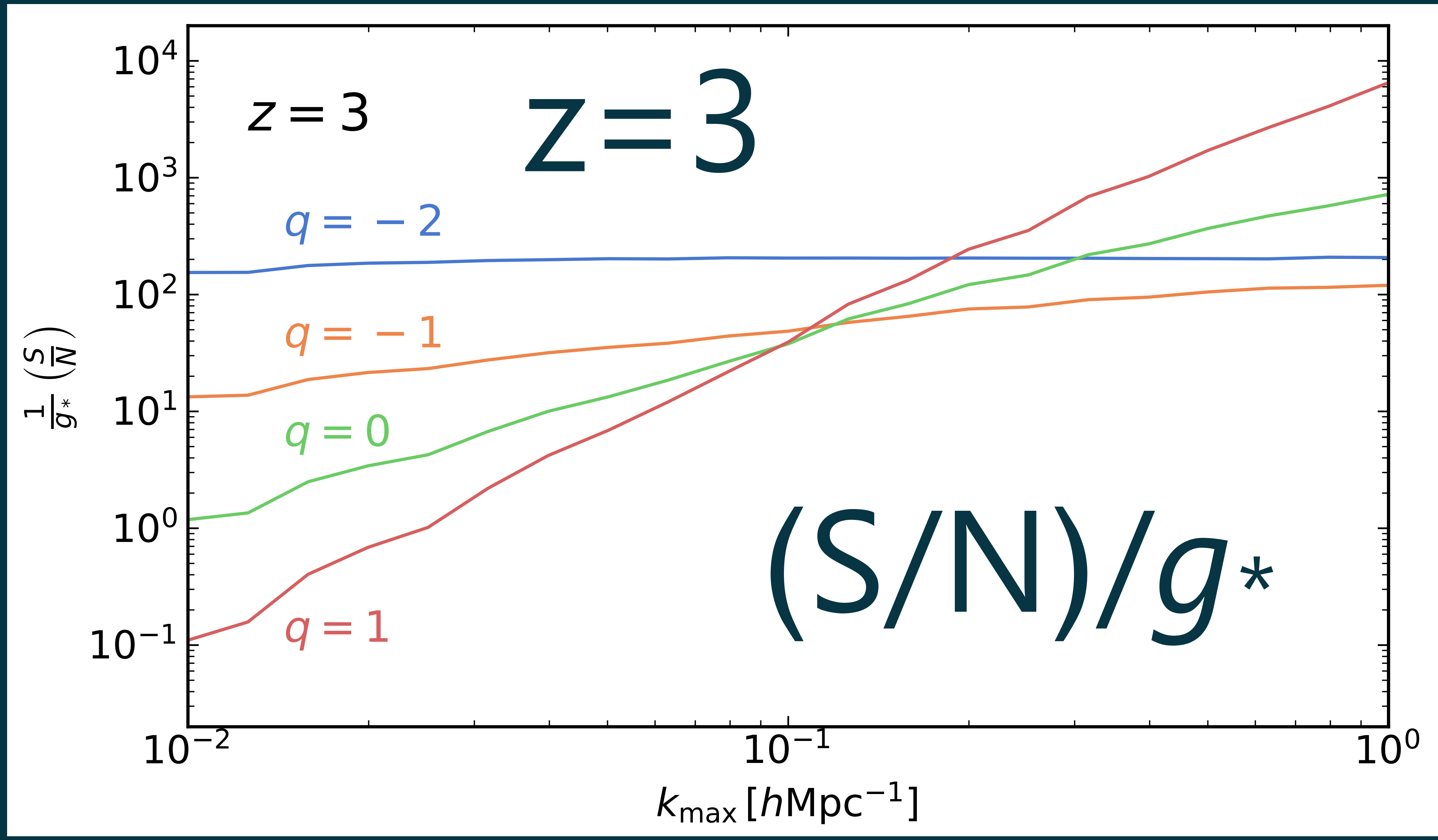
- 100 realizations of isotropic simulations
- monopole power at different k no longer independent on nonlinear regime
- Weaker off-diagonal covariance for quadrupole
- Can gain much by going beyond k~0.1h/Mpc

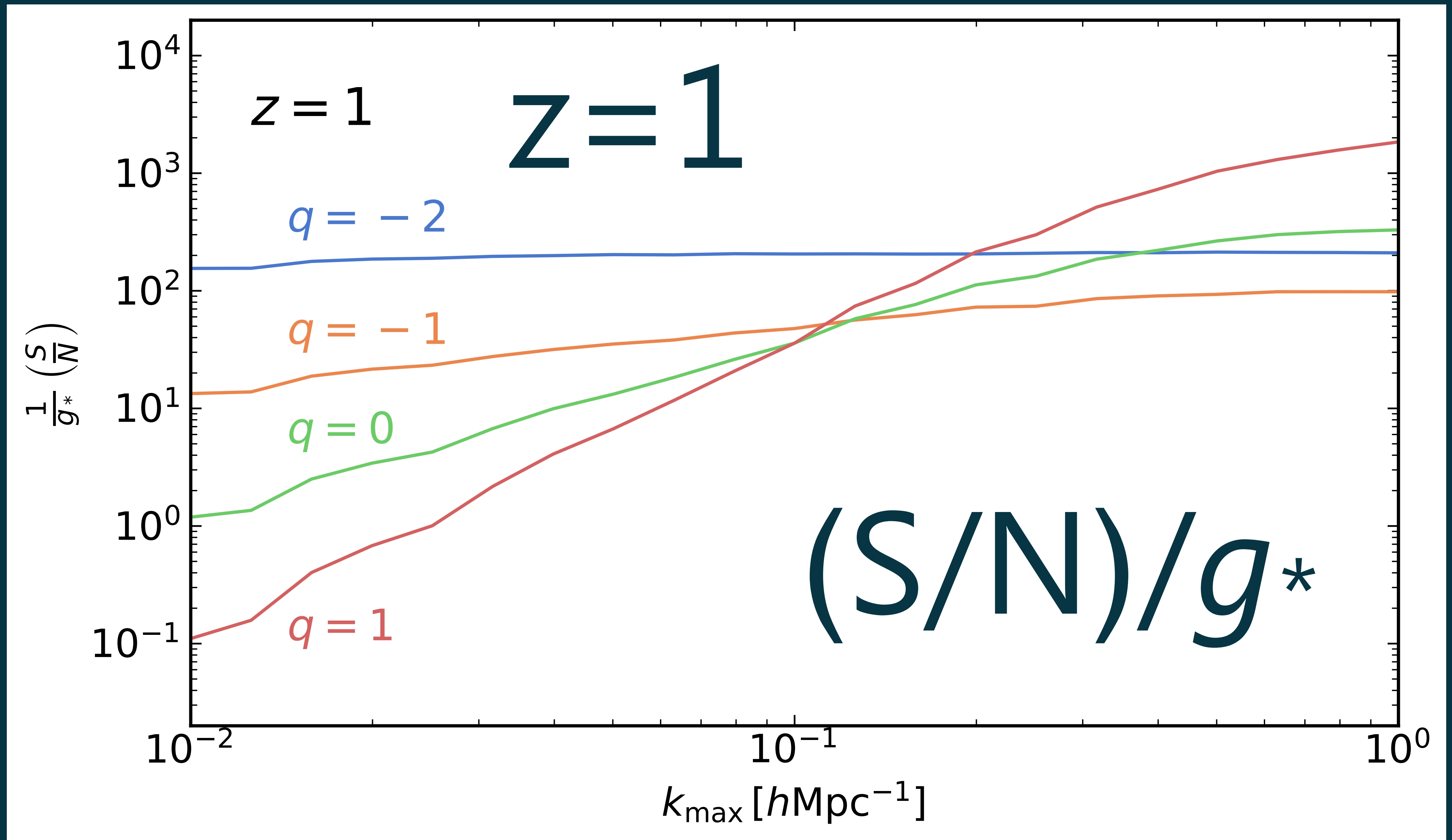
Signal to noise

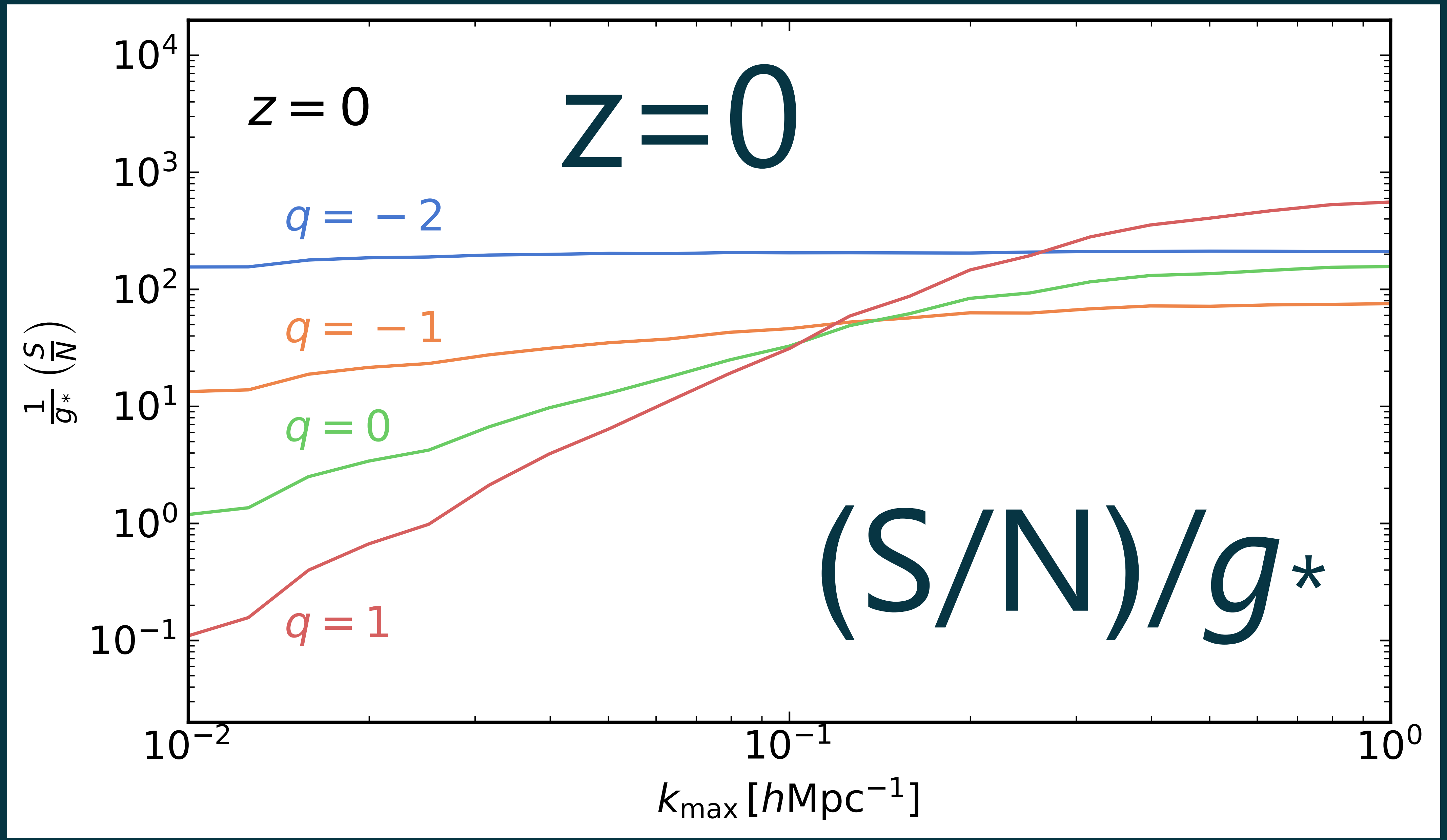
- Quantify the information in (1024Mpc/h)³

$$\left(\frac{S}{N}\right)^2 = \sum_{k_1,k_2<k_{\max}} P_2(k_1;g)\mathrm{Cov}^{-1}_{\mathrm{iso}}(k_1,k_2)P_2(k_2;g)$$

- For models with large q, potential factor >10 gain







Conclusion and outlook

- Follow the evolution of the quadrupoles SA signal using N-body simulations
  - Check with GridSPT (1 and 2 loop)
    - Good agreement on weakly nonlinear regime
  - On strongly nonlinear regime, there is a competition between mode transfer from large scale and isotropization
  - Off-diagonal covariance of quadrupole is smaller than monopole
    - Potential gain from nonlinear scales