



Imprints of relativistic effects on the dipole anisotropy of the density-intrinsic alignment cross-correlation

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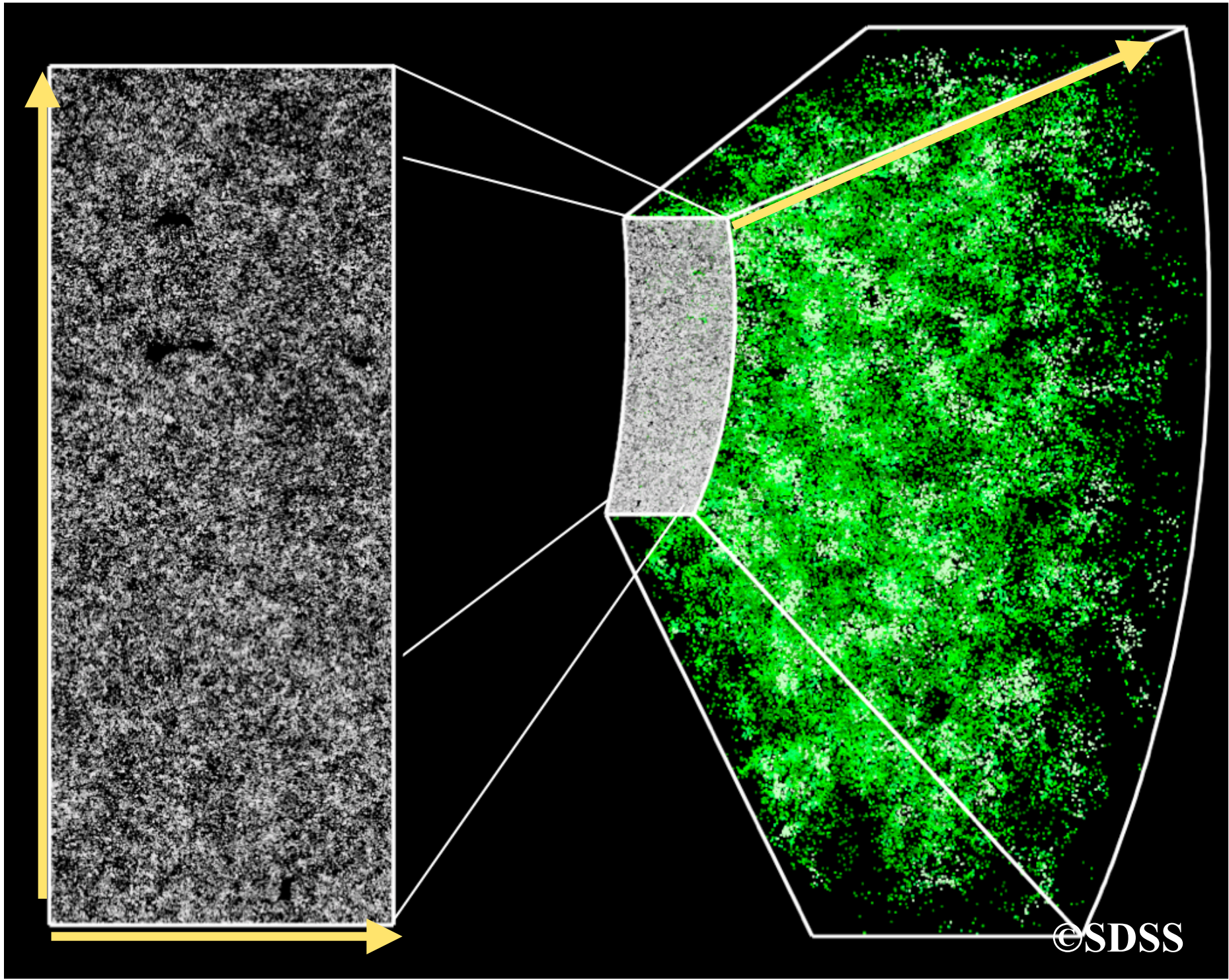
Collaborators:

T.Okumura (ASIAA), A.Taruya(YITP), T.Inoue (ASIAA)

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- 2. Dipole anisotropy in galaxy-galaxy correlations**
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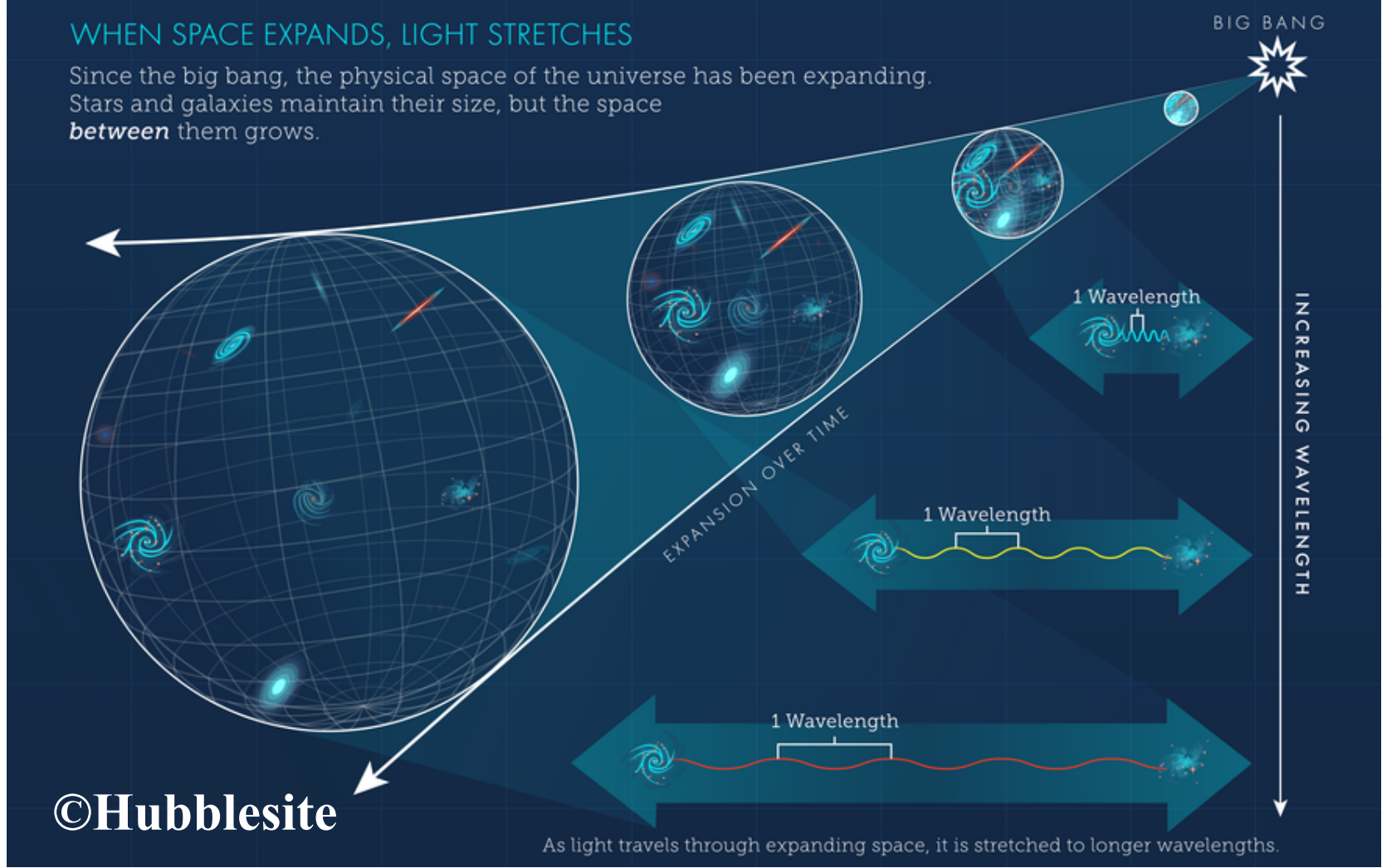
1.1. Galaxy redshift survey



3D map of our universe by measuring

redshift: $z = \frac{\lambda_{\text{obs}} - \lambda_{\text{em}}}{\lambda_{\text{em}}}$

angular position: (θ, ϕ)



Observed redshift

Cosmological redshift (Hubble flow) + **Doppler effect (peculiar velocity)**

Observed position (inferred from redshift) ≠ Actual position

1.2. Redshift space distortions (RSD)

Observed galaxy distribution appears distorted



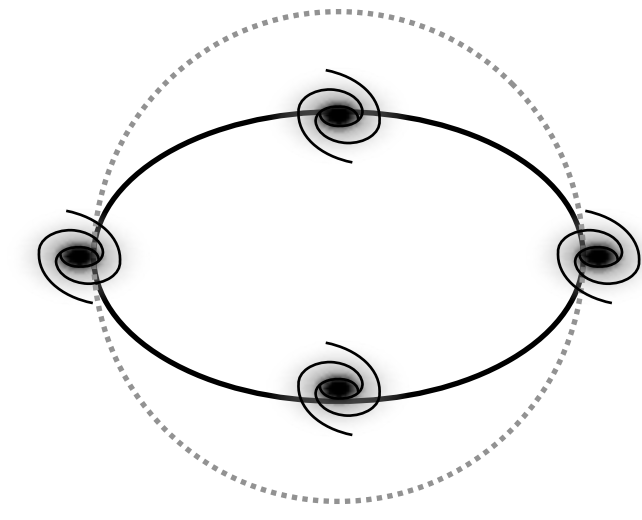
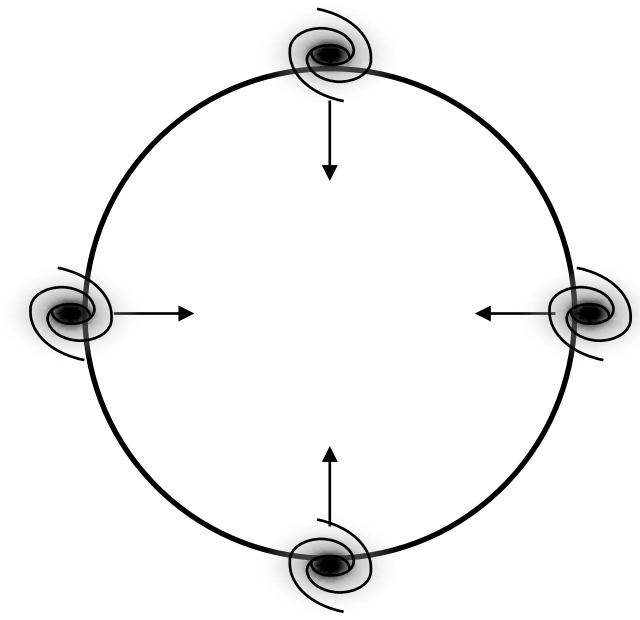
Redshift space distortions (RSD)

Primary source: Doppler effect induced by peculiar velocity

Large scale: coherent infall

Real space
(Actual position)

Redshift space
(Apparent position)



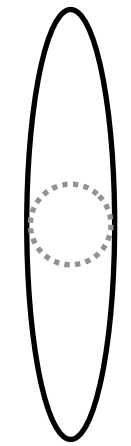
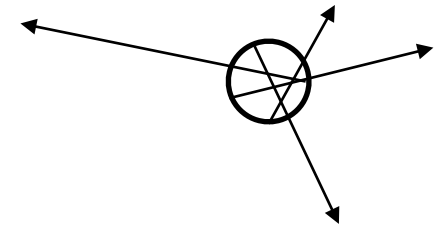
↑ line-of-sight direction
Observer

Kaiser effect

Small scale: virial motion

Real space
(Actual position)

Redshift space
(Apparent position)



↑ line-of-sight direction
Observer

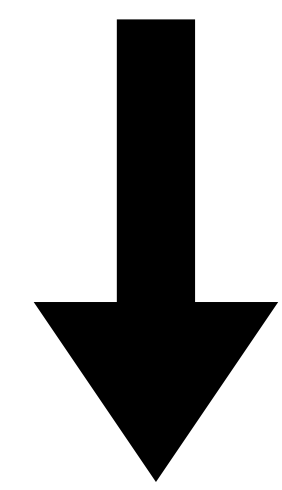
Finger-of-God effect

1.3 Cosmology - RSD

Redshift space ↔ Real space

N. Kaiser (1987)

redshift space $\mathbf{s} = \mathbf{r} + \frac{1+z}{H(z)} (\mathbf{v} \cdot \hat{\mathbf{z}}) \hat{\mathbf{z}}$ special relativity, $v \ll 1$
 real space $\hat{\mathbf{z}}$: constant line-of-sight vector



continuity equation (linear): $\dot{\delta}_L + \frac{1}{a} \nabla \cdot \mathbf{v} \simeq 0$

conservation law: $(1 + \delta^{(s)}(\mathbf{s})) d^3s = (1 + \delta(\mathbf{r})) d^3r$

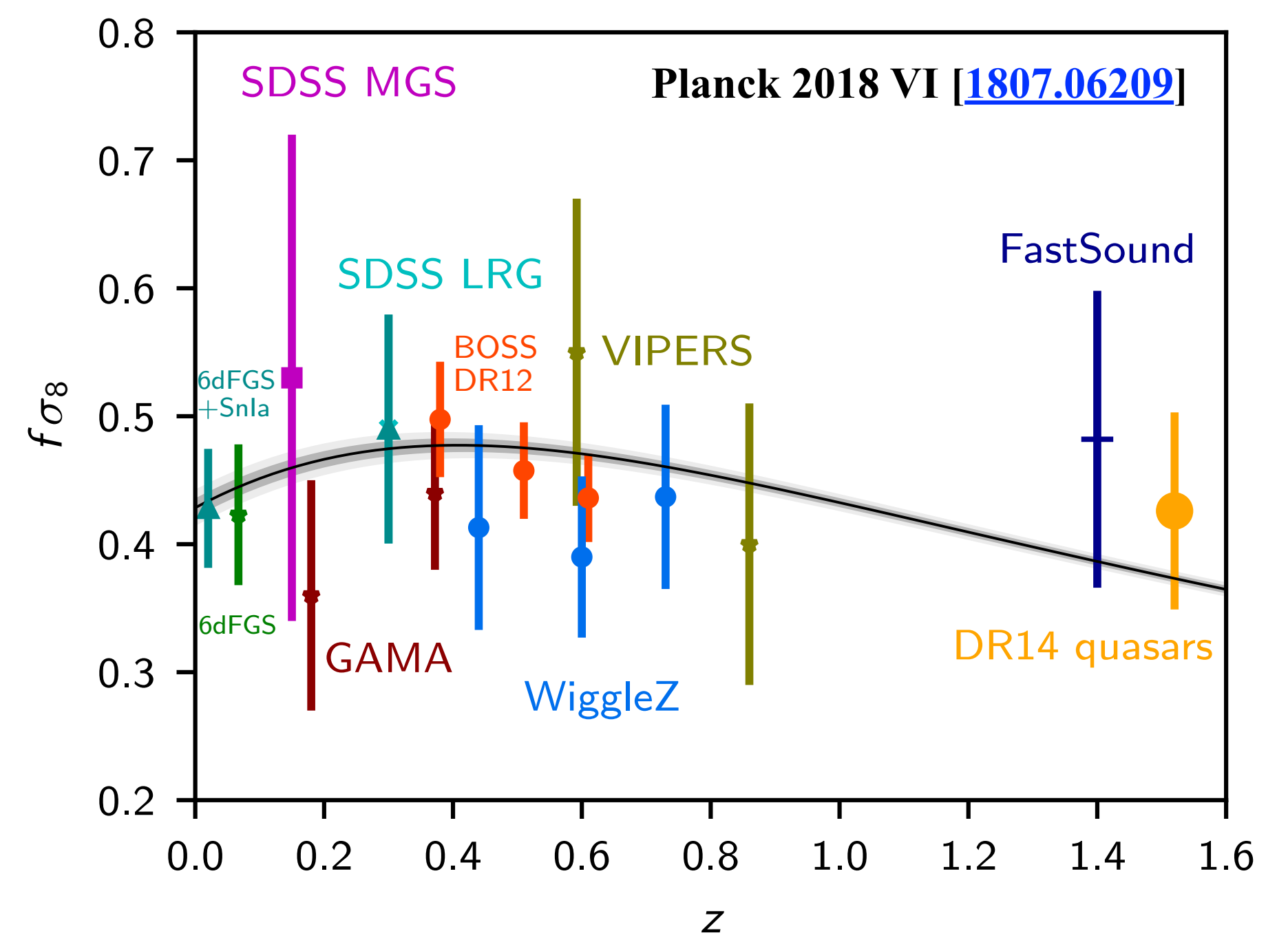
Kaiser formula

$$\delta^{(s)}(\mathbf{k}) = \left(b + f (\hat{\mathbf{k}} \cdot \hat{\mathbf{z}})^2 \right) \delta_L(\mathbf{k})$$

$f \equiv \frac{d \ln \delta_L}{d \ln a}$: linear growth rate

Linear growth rate depends on the gravity theory
 → a **probe of gravity** on cosmological scales

For Λ CDM: $f \approx (\Omega_m(a))^{0.55}$



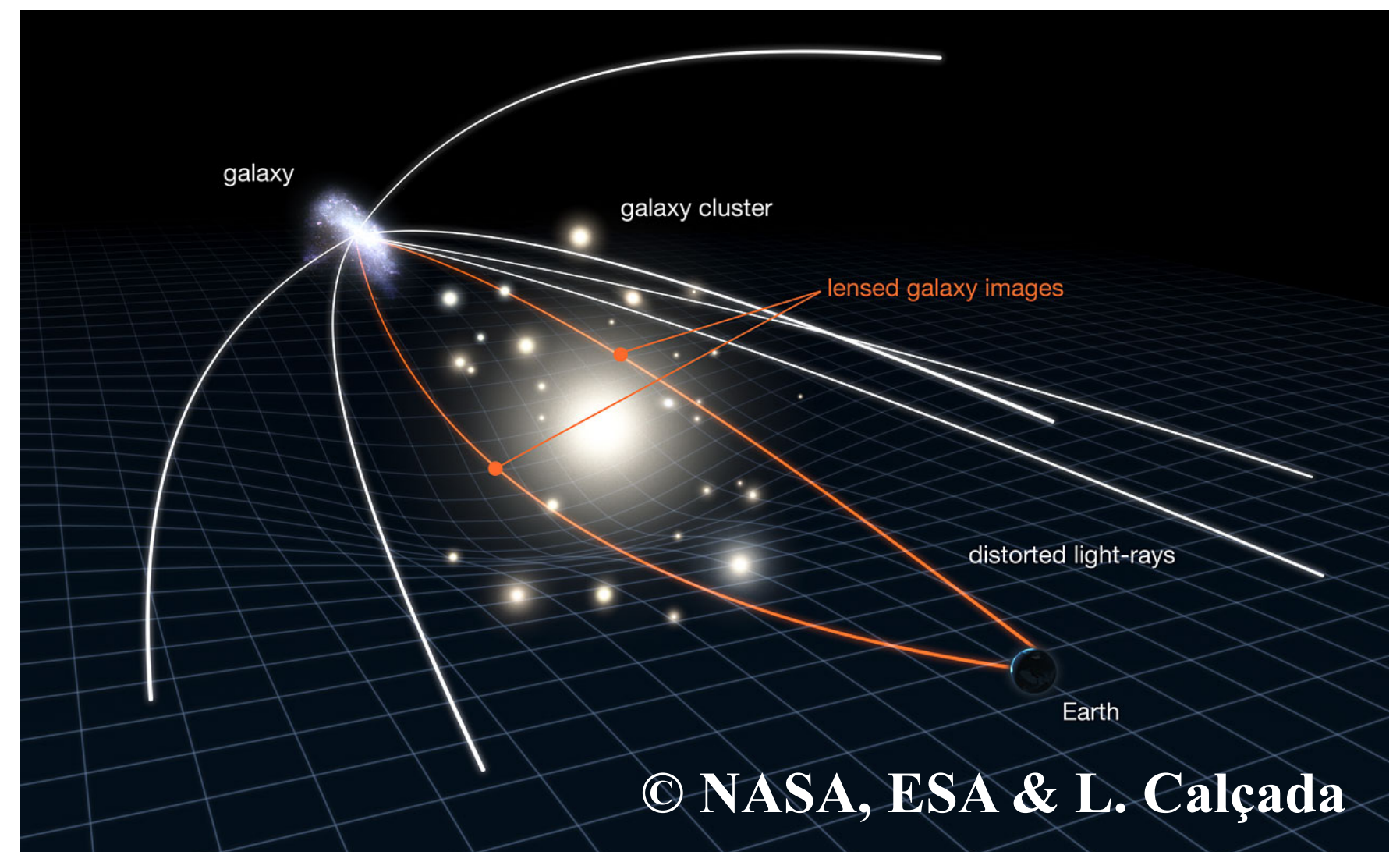
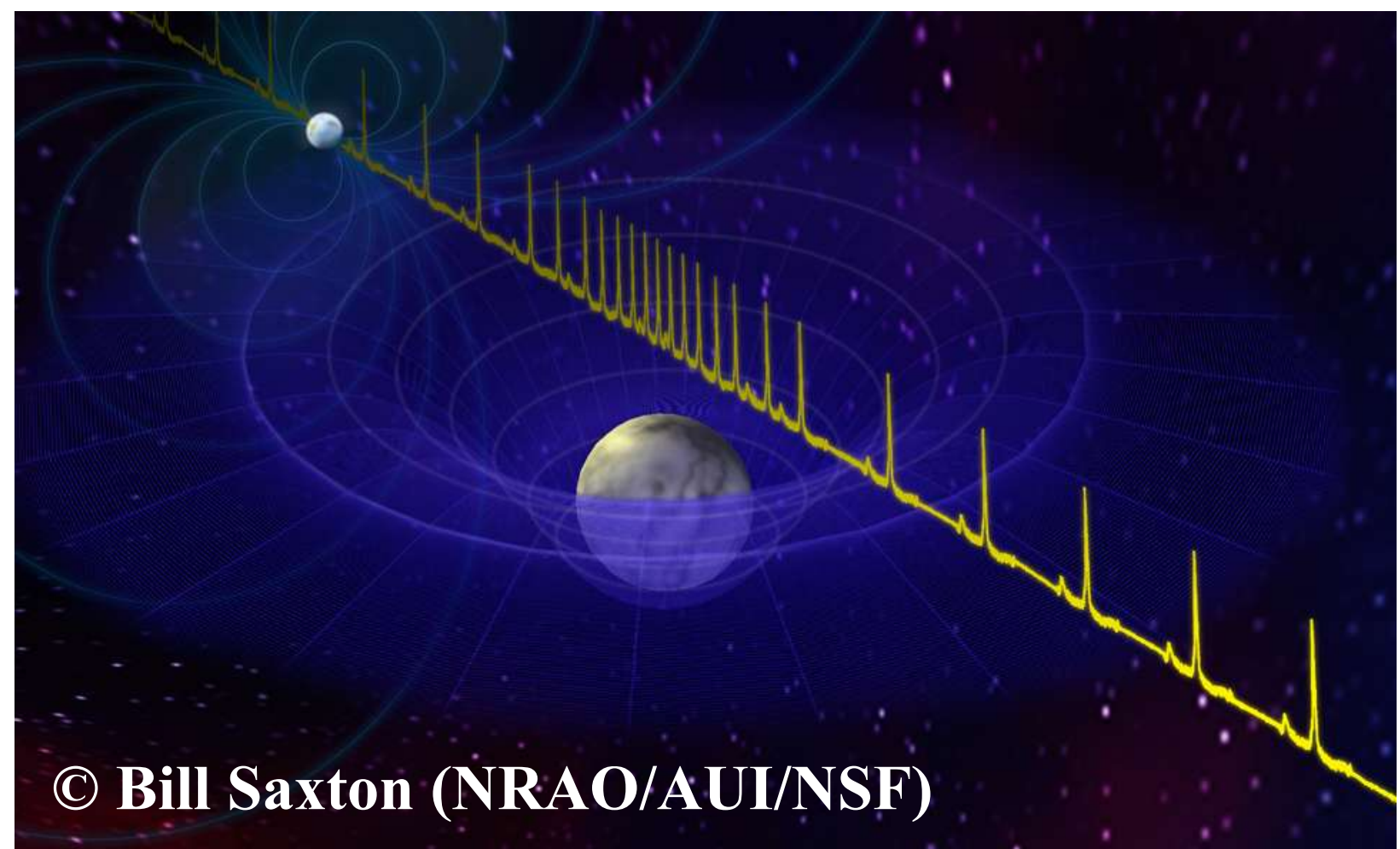
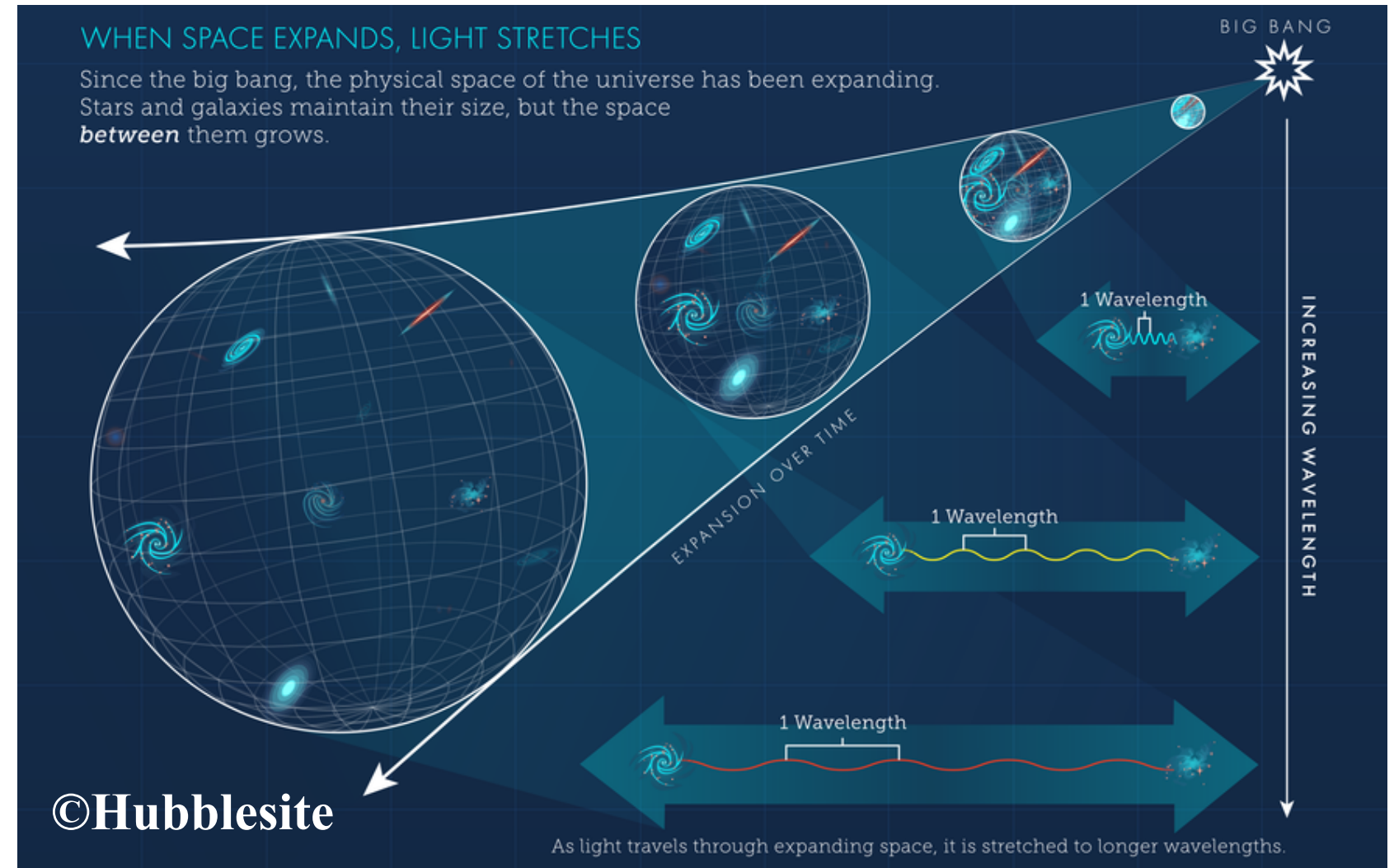
1.4. Other relativistic effects

Observed redshift

- Cosmological redshift (Hubble flow)
- + Doppler effect (peculiar velocity)

Other relativistic effects

- + Gravitational redshift
- + Integrated Sachs-Wolfe
- + Shapiro Time delay
- + Gravitational lensing
- + ...



1.5. How is the signal of relativistic effects observed?

A.Challinor and A.Lewis [[1105.5292](#)]
 C.Bonvin and R.Durrer [[1105.5280](#)]
 J.Yoo [[1409.3223](#)],
 and many works

$$\left\{ \begin{array}{l} \text{Perturbed FLRW: } ds^2 = [-(1 + 2\Phi)dt^2 + a^2(1 - 2\Psi)dx^2] \\ \text{Solve the geodesic eq.: } \frac{dk^\mu}{d\lambda} + \Gamma^\mu_{\alpha\beta} k^\alpha k^\beta = 0 \\ \text{Define observed redshift including all effects: } 1 + z = \frac{(k_\mu u^\mu)_S}{(k_\mu u^\mu)_O} \end{array} \right.$$

Redshift space including possible relativistic effects

$$\mathbf{s} = \mathbf{r} + \frac{1+z}{H} (\mathbf{v} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}$$

$$+ \frac{1+z}{H} \left(-\Phi + \frac{1}{2}v^2 - \int_t^{t_0} (\dot{\Phi} + \dot{\Psi}) dt' \right) \hat{\mathbf{r}} - \int_0^\chi (\Psi + \dot{\Psi}) d\chi' \hat{\mathbf{r}} - \int_0^\chi (\chi - \chi') \nabla_\perp (\Phi + \Psi) d\chi'$$

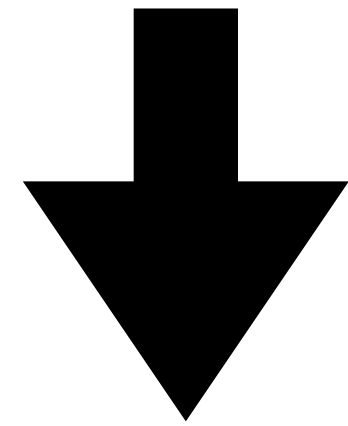
- gravitational redshift
- Transverse Doppler
- Shapiro time delay
- integrated Sachs-Wolfe
- gravitational lensing

1.6. How is the signal of relativistic effects observed?

Observed redshift including possible relativistic effects

$$\mathbf{s} = \mathbf{r} + \frac{1+z}{H} (\mathbf{v} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} + \frac{1+z}{H} \left(-\Phi + \frac{1}{2} v^2 - \int_t^{t_0} (\dot{\Phi} + \dot{\Psi}) dt' \right) \hat{\mathbf{r}} - \int_0^\chi (\Psi + \Upsilon) d\chi' \hat{\mathbf{r}} - \int_0^\chi (\chi - \chi') \nabla_\perp (\Phi + \Psi) d\chi'$$

c.f. Kaiser formula



conservation law: $(1 + \delta^{(S)}(s)) d^3s = (1 + \delta(r)) d^3r$
(linear approximation)

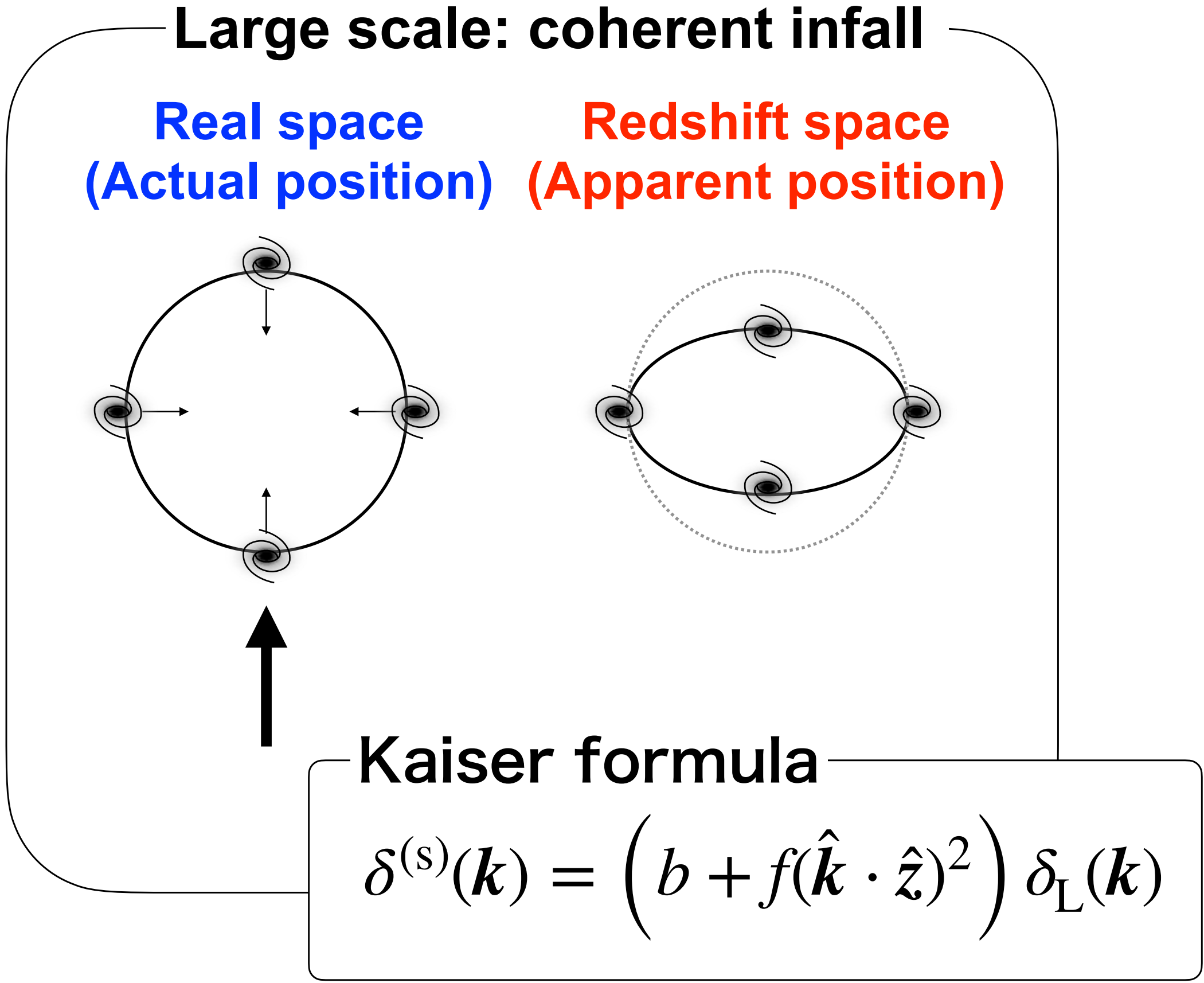
Linear density field with relativistic effects

$$\delta^{(s)} = b\delta - \frac{1}{\mathcal{H}} \hat{\mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{r}} (\hat{\mathbf{r}} \cdot \mathbf{v}) - \left(\frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) \hat{\mathbf{r}} \cdot \mathbf{v} + \frac{1}{\mathcal{H}} \left(\hat{\mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{r}} \Psi + \mathcal{H} \hat{\mathbf{r}} \cdot \mathbf{v} + \hat{\mathbf{r}} \cdot \dot{\mathbf{v}} \right) - 2\Phi + \Psi + \frac{\dot{\Phi}}{\mathcal{H}} + \frac{1}{r} \int_0^r dr' \left(2 - \frac{r-r'}{r'} \Delta_\Omega \right) (\Phi + \Psi) + \left(\frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) \left(\Psi + \int_0^r dr' (\dot{\Psi} + \dot{\Phi}) \right)$$

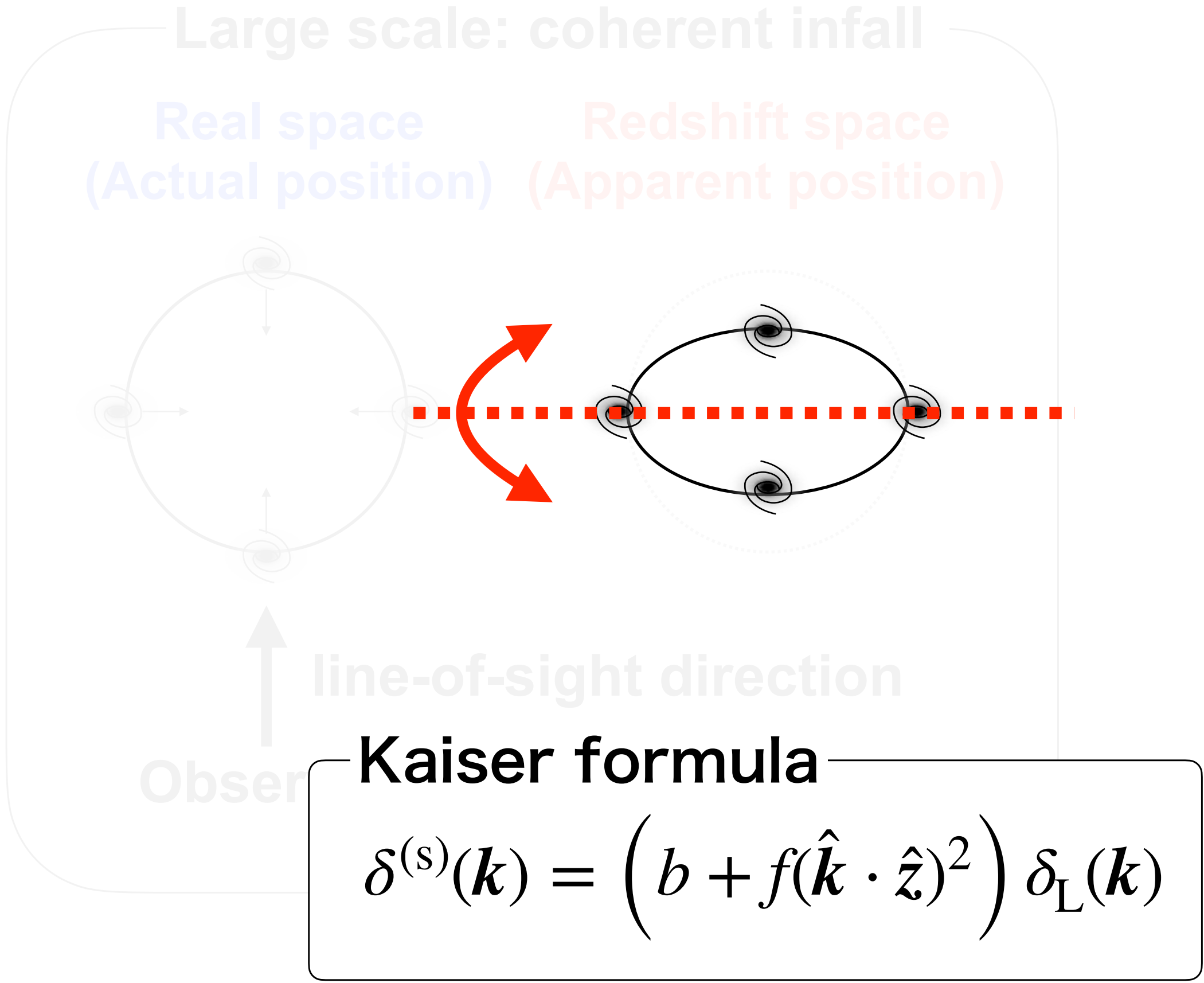
(line-of-sight vectors are highlighted in red)

Using this expression, we can intuitively understand how we will observe

1.7. Recalling Kaiser effect

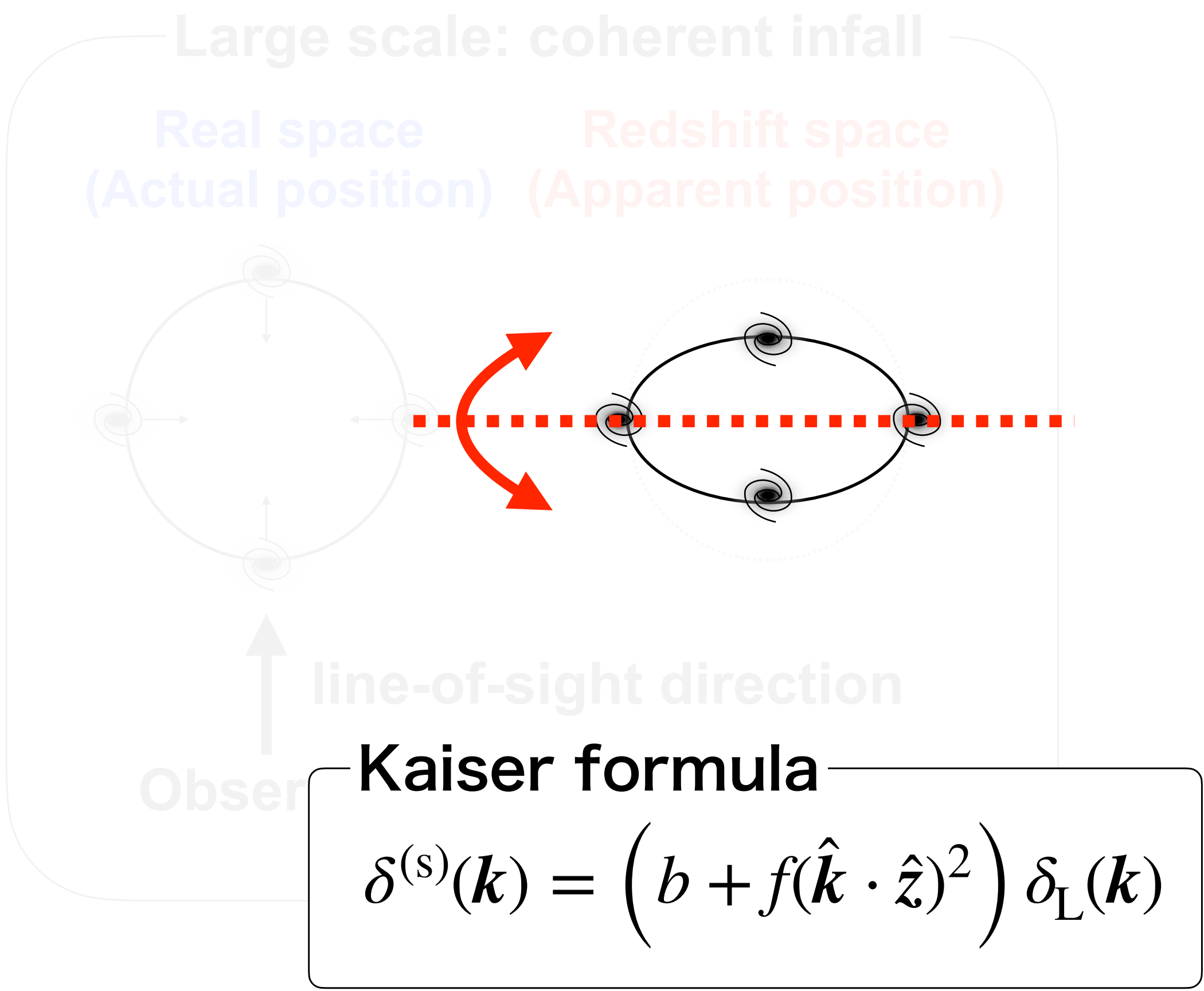


1.7. Recalling Kaiser effect



(line-of-sight vector)²
→ **even multipole anisotropies**

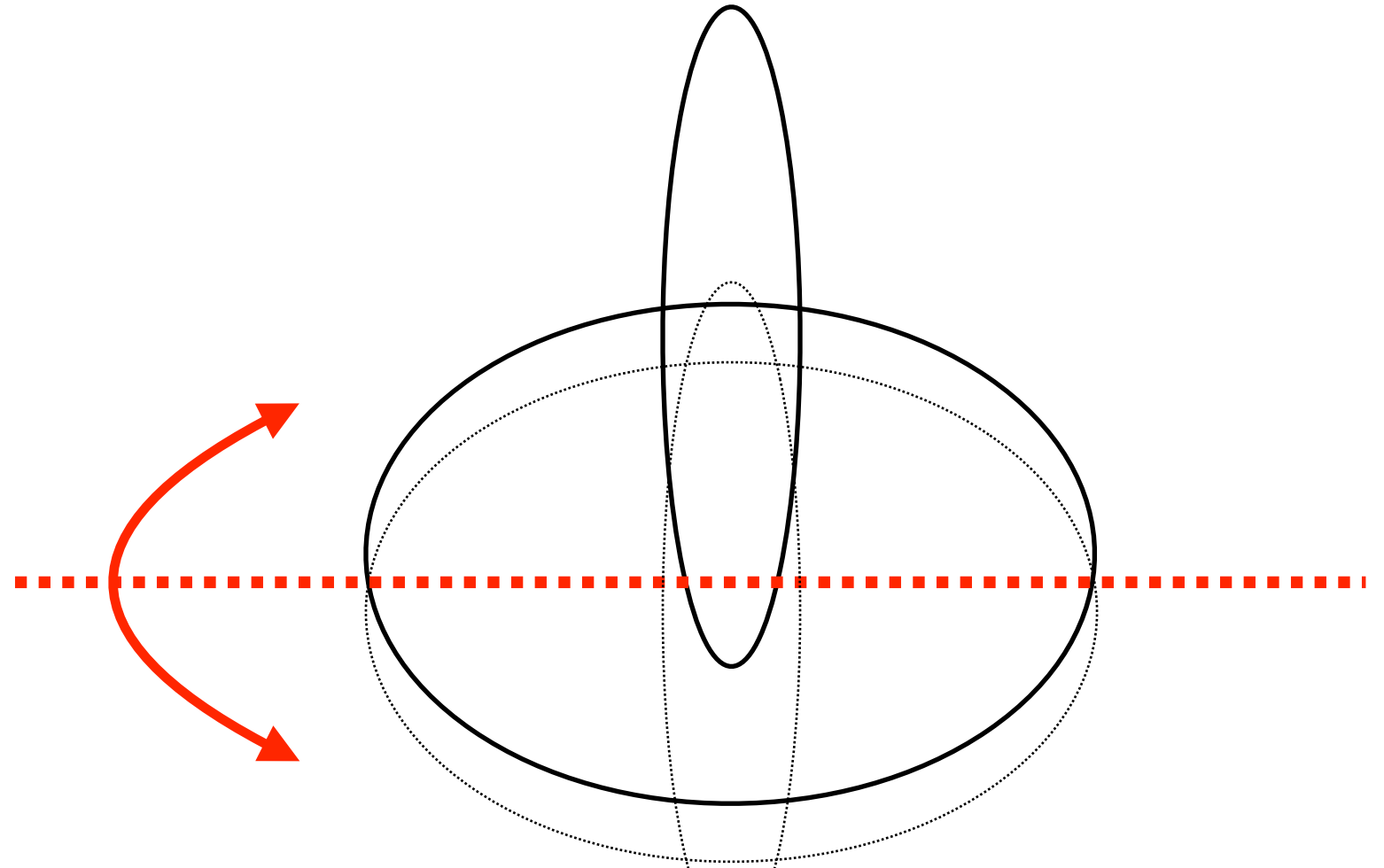
1.7. Recalling Kaiser effect



(line-of-sight vector)²
→ even multipole anisotropies

With relativistic effects

$$\delta^{(s)} = b\delta - \frac{1}{\mathcal{H}} \hat{\mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{r}} (\hat{\mathbf{r}} \cdot \mathbf{v}) - \left(\frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) \hat{\mathbf{r}} \cdot \mathbf{v} + \frac{1}{\mathcal{H}} \left(\hat{\mathbf{r}} \cdot \frac{\partial}{\partial \mathbf{r}} \Psi + \mathcal{H} \hat{\mathbf{r}} \cdot \mathbf{v} + \hat{\mathbf{r}} \cdot \dot{\mathbf{v}} \right) - 2\Phi + \Psi + \frac{\dot{\Phi}}{\mathcal{H}} + \frac{1}{r} \int_0^r dr' \left(2 - \frac{r-r'}{r'} \Delta_{\Omega} \right) (\Phi + \Psi) + \left(\frac{2}{r\mathcal{H}} + \frac{\dot{\mathcal{H}}}{\mathcal{H}^2} \right) \left(\mathbf{v} \cdot \hat{\mathbf{r}} \right)$$



(line-of-sight vector)^{odd}
→ odd multipole anisotropies

Dipole anisotropy

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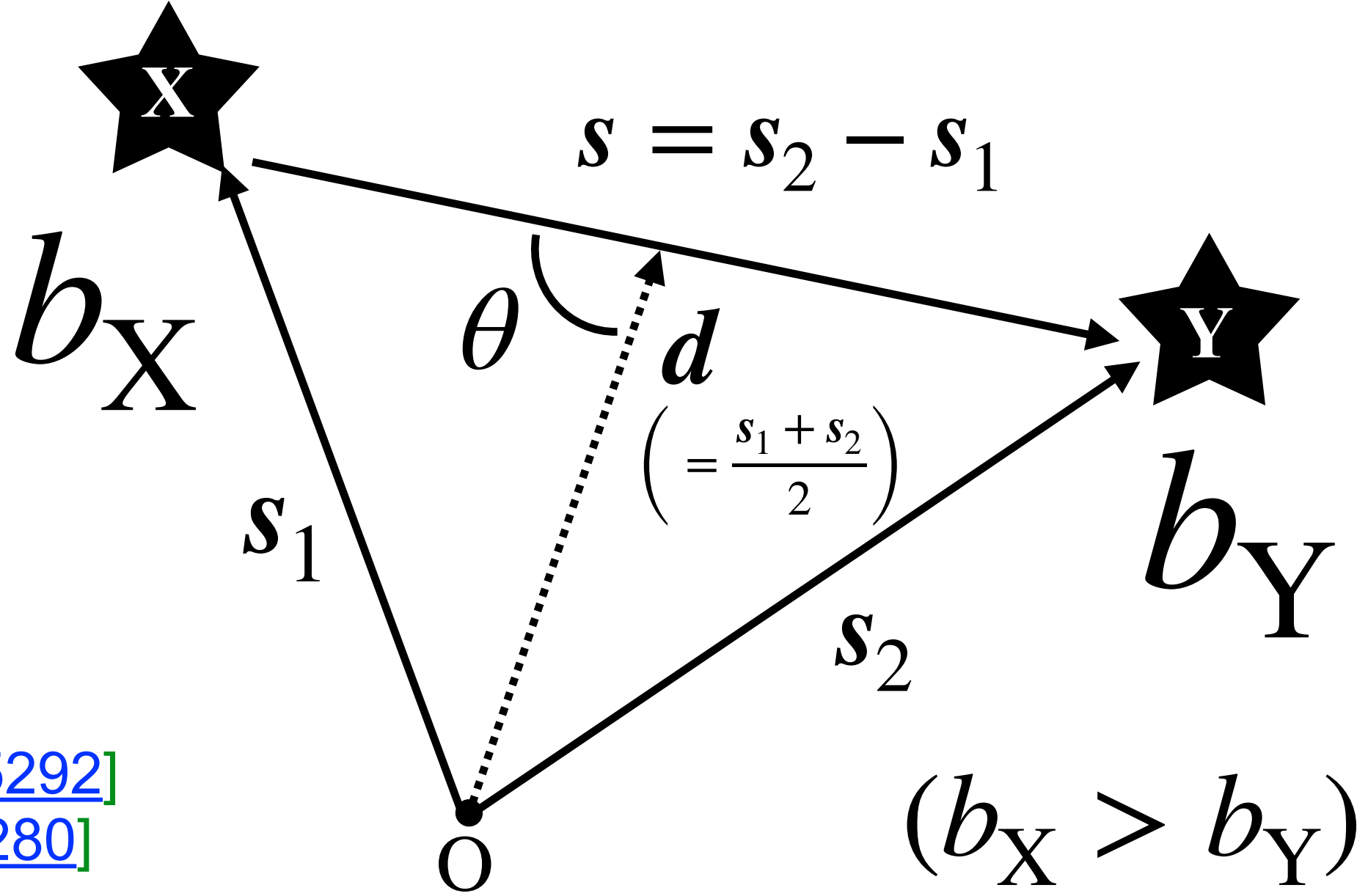
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2.1. Dipole anisotropy

$$\xi_1 = \frac{3}{2} \int_{-1}^1 (\xi^{(S)}(s_1, s_2) \cos \theta) d \cos \theta$$

Linear theory: $\xi_1 \propto (b_X - b_Y)$

A.Challinor and A.Lewis [1105.5292]
 C.Bonvin and R.Durrer [1105.5280]
 J.Yoo [1409.3223], ...



Note: wide-angle effect

Beyond the distant-observer limit, the Doppler effect induces non-zero dipole:

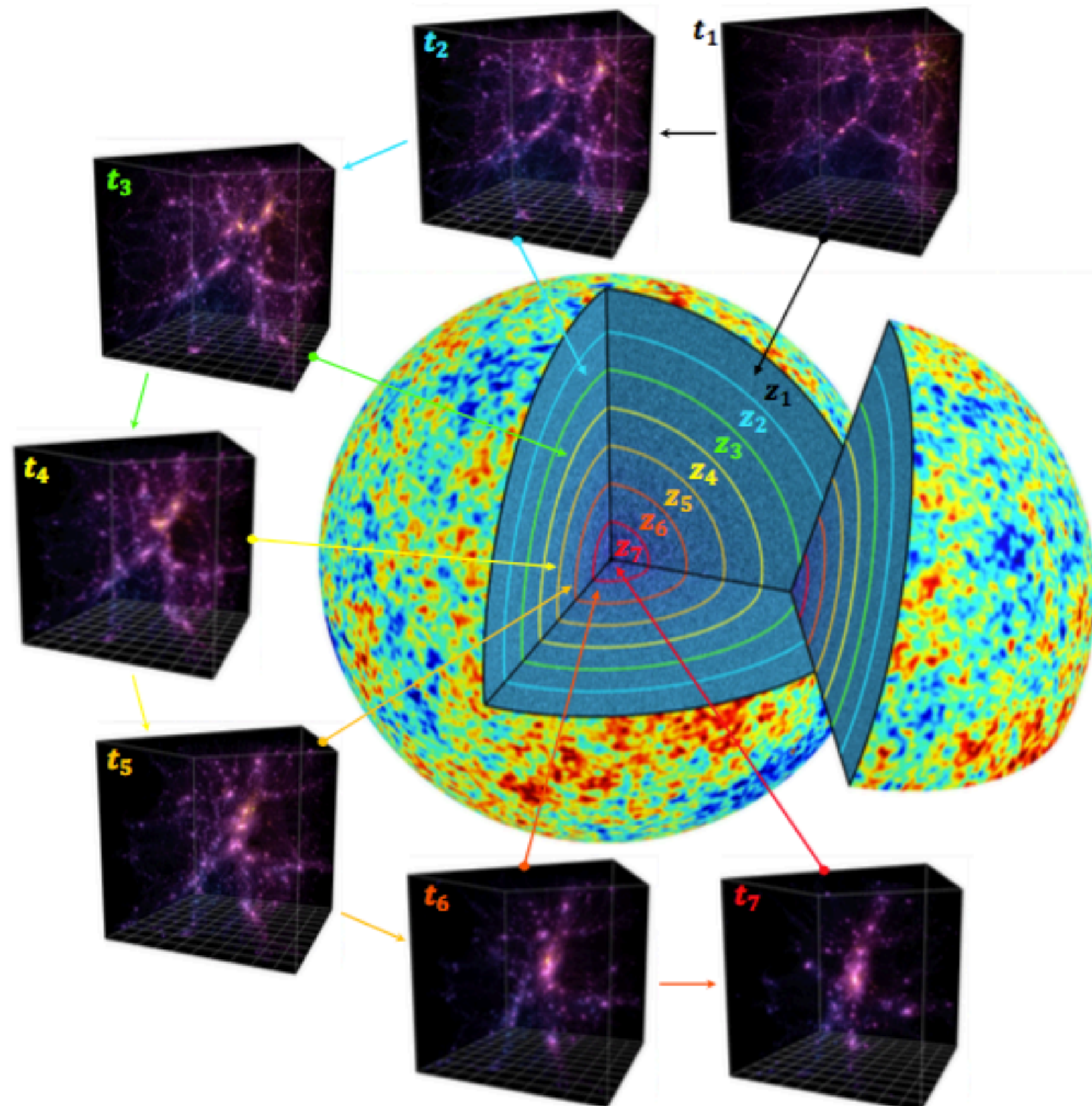
$$s = r + \frac{1+z}{H(z)} (v \cdot \hat{z}) \hat{z} \quad \hat{z}: \text{constant line-of-sight vector} \rightarrow \xi_1 \propto (b_X - b_Y)$$

Cross-correlating different biased objects is essential

2.2. Dipole in simulations

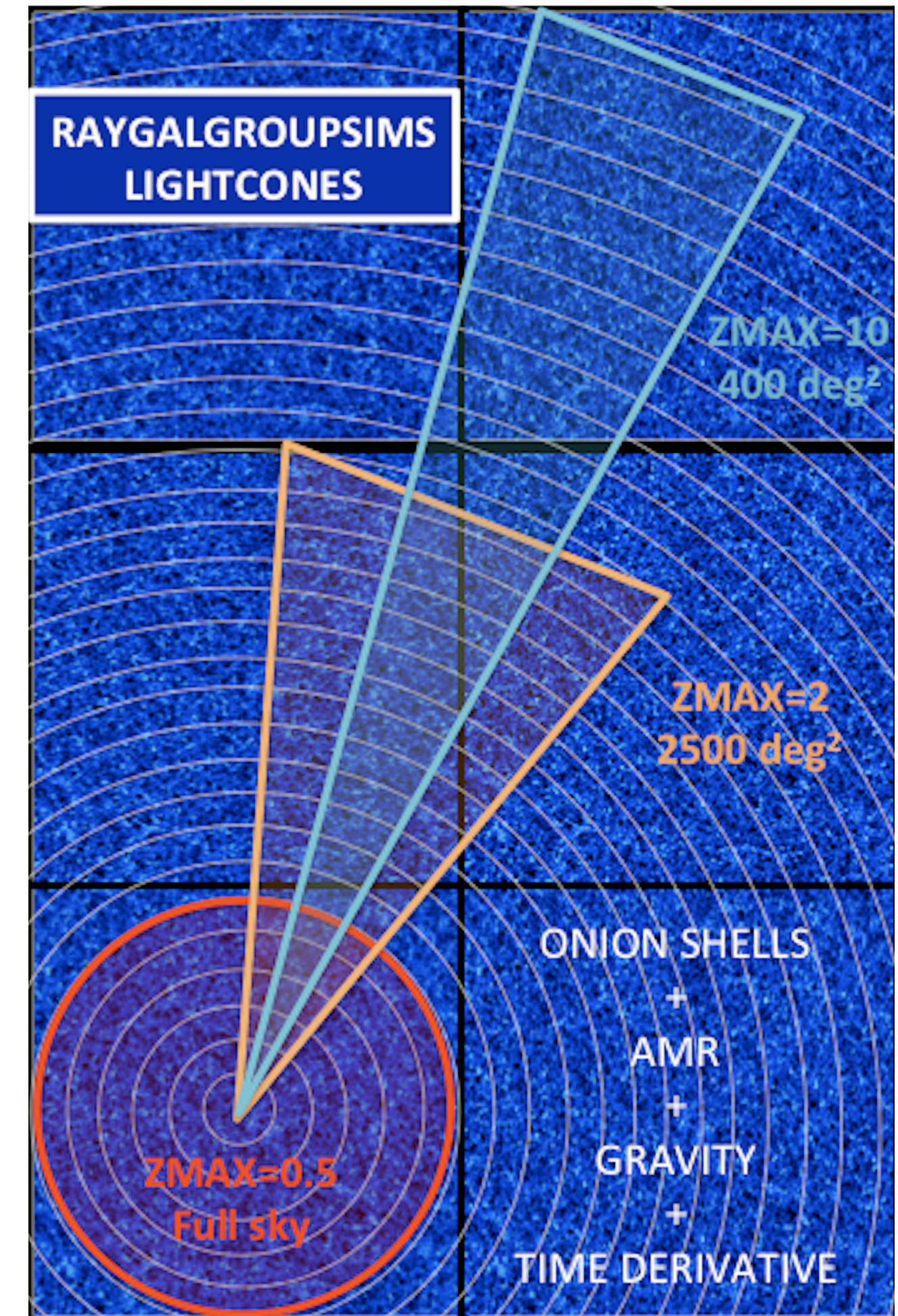
M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, S.Saga [[1803.04294](#)]

- Using cosmological N-body code RAMSES.
- Storing gravitational potential data on light cone
- Tracing back the light ray to the source by direct integration of geodesic equation
- We obtain "Observed" position and redshift



$$1 + z = \frac{\left(g_{\mu\nu} k^\mu k^\nu \right)_{\text{source}}}{\left(g_{\mu\nu} k^\mu k^\nu \right)_{\text{observer}}}$$

$$g_{\mu\nu} k^\mu k^\nu = -ak^0 \left(1 + \phi + \mathbf{v} \cdot \hat{\mathbf{n}} + \frac{1}{2}v^2 \right)$$



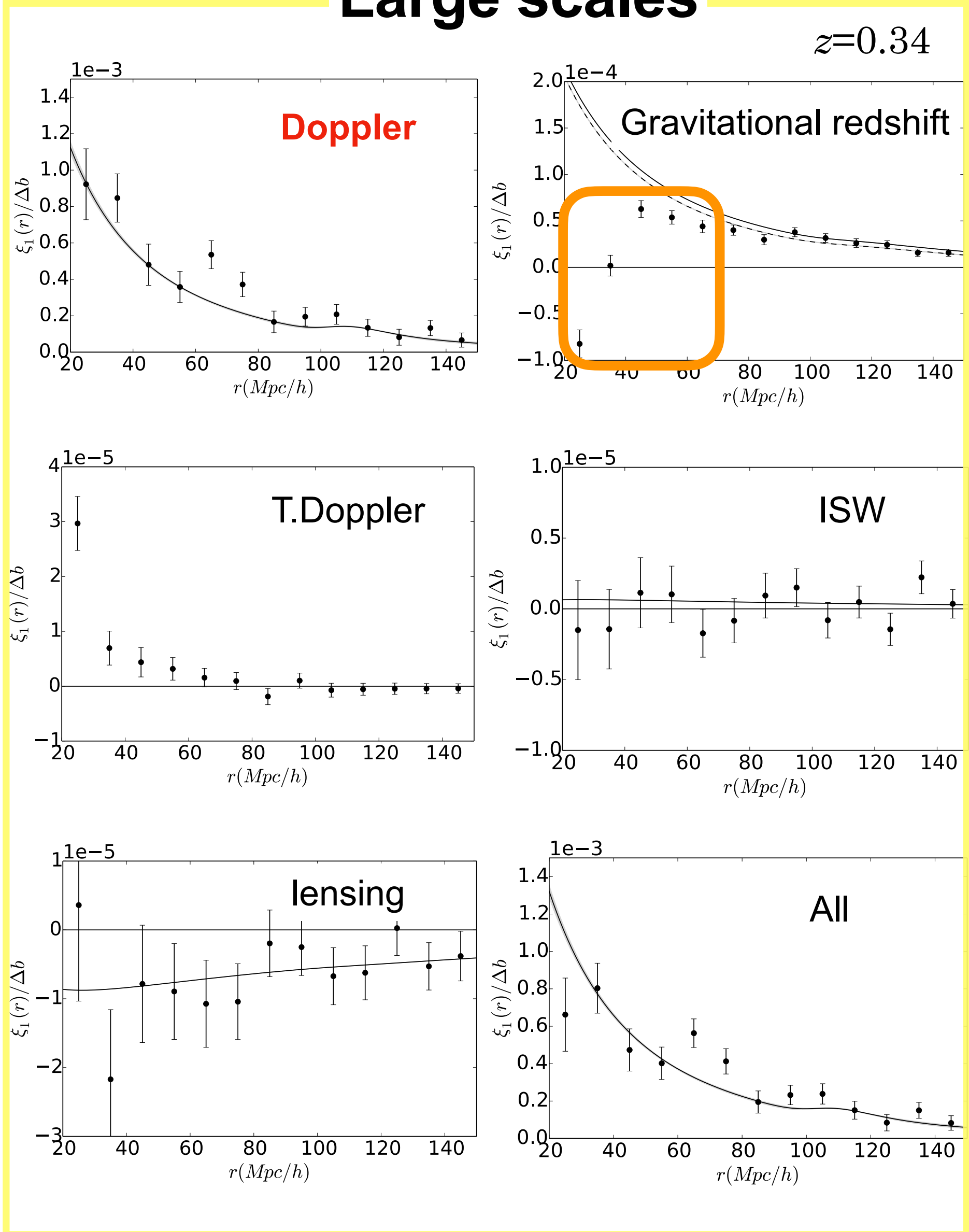
([RayGalGroupSims](#) by M-A.Breton and Y.Rasera)

Light-cone catalogue with all relativistic effects

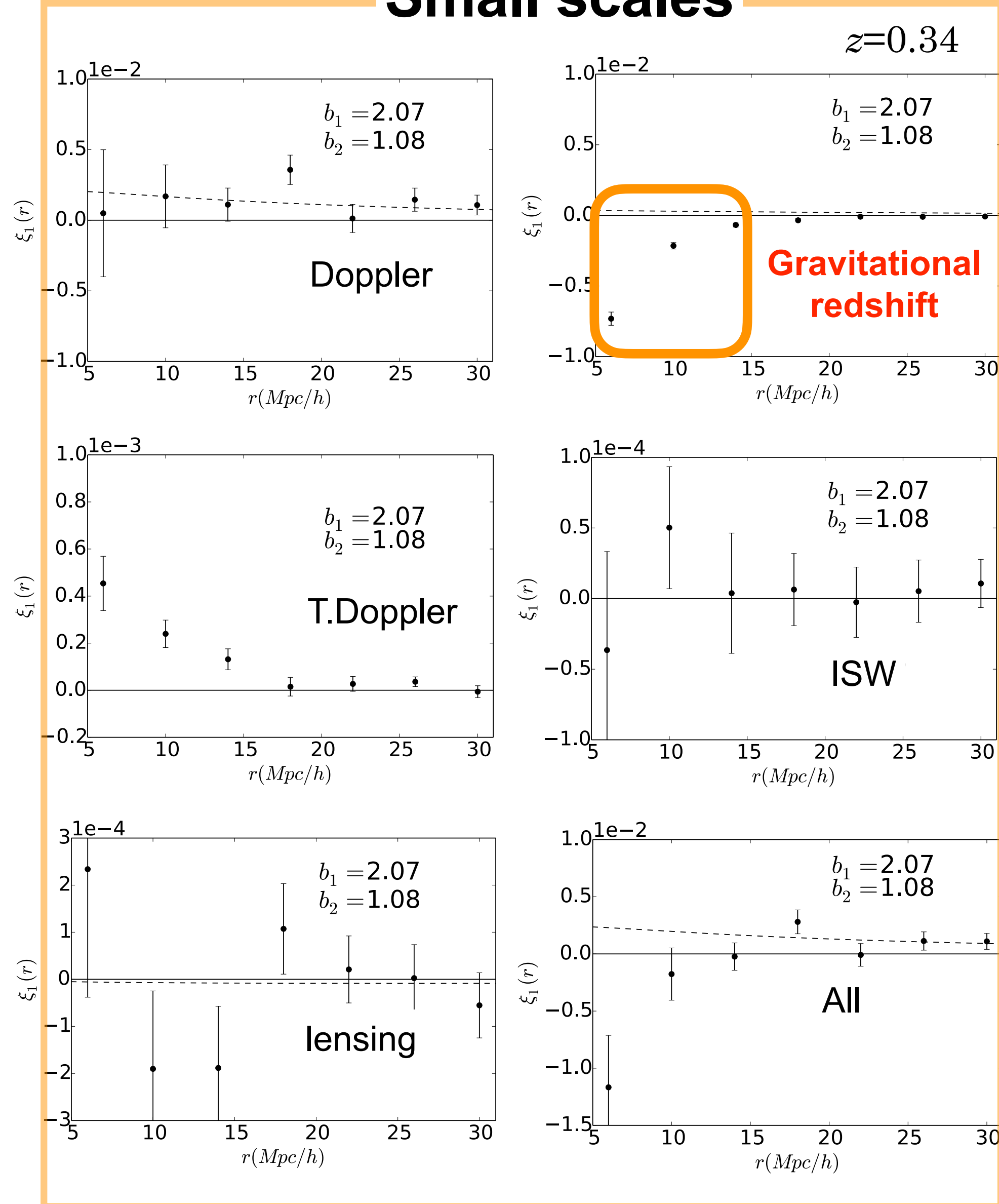
2.3. Measurements in simulations

M-A.Breton, Y.Rasera, A.Taruya, O.Lacombe, S.Saga [1803.04294]

Large scales



Small scales



2.4. Our model

S.Saga, A.Taruya, M-A.Breton, Y.Rasera [[2004.03772](#)]

Picking up dominant contributions:

$$\mathbf{s} = \mathbf{r} + \frac{1+z}{H} (\mathbf{v} \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}} + \frac{1+z}{H} \left(-\Phi + \frac{1}{2} v^2 - \int_t^{t_0} (\dot{\Phi} + \dot{\Psi}) dt' \right) \hat{\mathbf{r}} - \int_0^\chi (\Psi + \Psi) d\chi' \hat{\mathbf{r}} - \int_0^\chi (\chi - \chi') \nabla_\perp (\Phi + \Psi) d\chi'$$

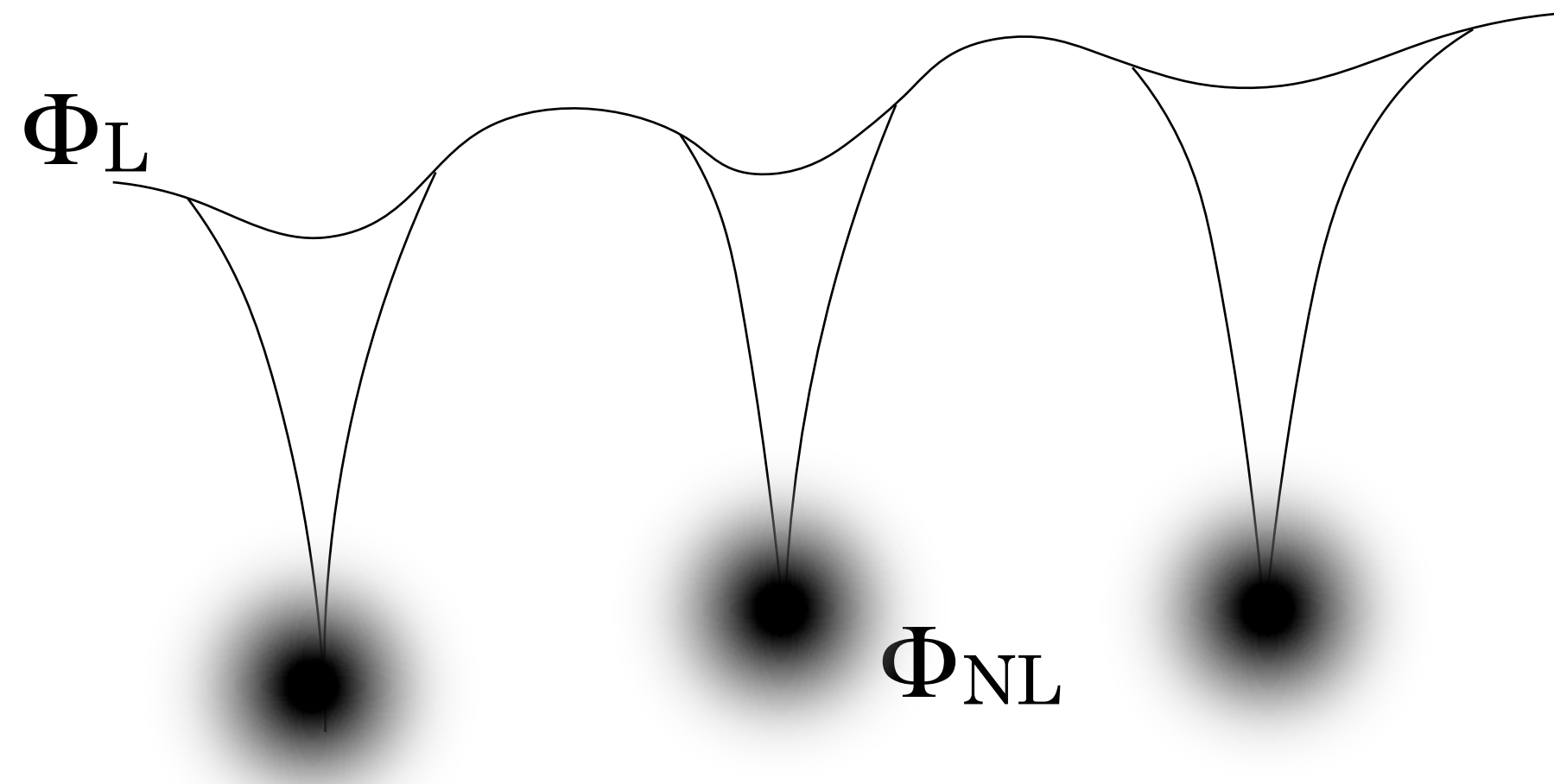
Modelling the potential

$$\Phi = \Phi_L + \Phi_{NL}$$

Linear potential

Non-linear halo potential

Φ_{NL} is estimated by NFW profile



$$\delta(\text{std}) = \text{Real space} + \text{Doppler effect} \equiv \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{s}} \left[b + f\mu_k^2 - if \frac{2}{ks} \mu_k \right] \delta_L(\mathbf{k}), \quad \mathcal{M} \equiv -3\Omega_{m0} H_0^2 / (2a^2 H)$$

$$\delta(\text{pot}) = \text{Linear potential} \equiv \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{s}} \left[(iks\mu_k + 2) \frac{\mathcal{M}}{sk^2} \right] \delta_L(\mathbf{k}),$$

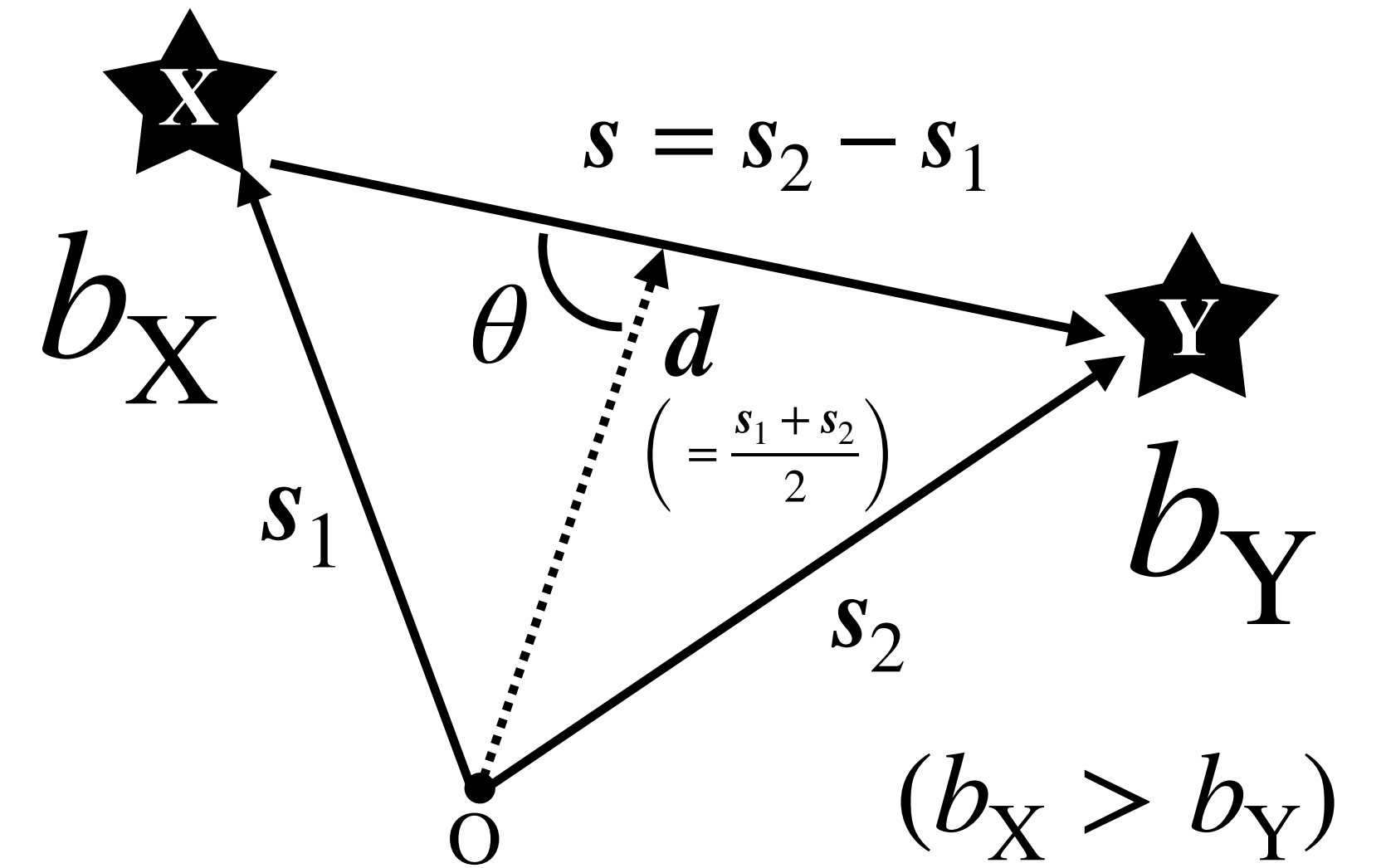
$$\delta(\text{NL}) = \text{NL potential} \equiv -\frac{\Phi_{NL}}{aHs} \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{s}} \left[-1 + (1 - 2f)\mu_k^2 - i(1 + f) \frac{2}{ks} \mu_k - ibks\mu_k - ifks\mu_k^3 \right] \delta_L(\mathbf{k}),$$

2.5. Dipole in the galaxy-galaxy correlations

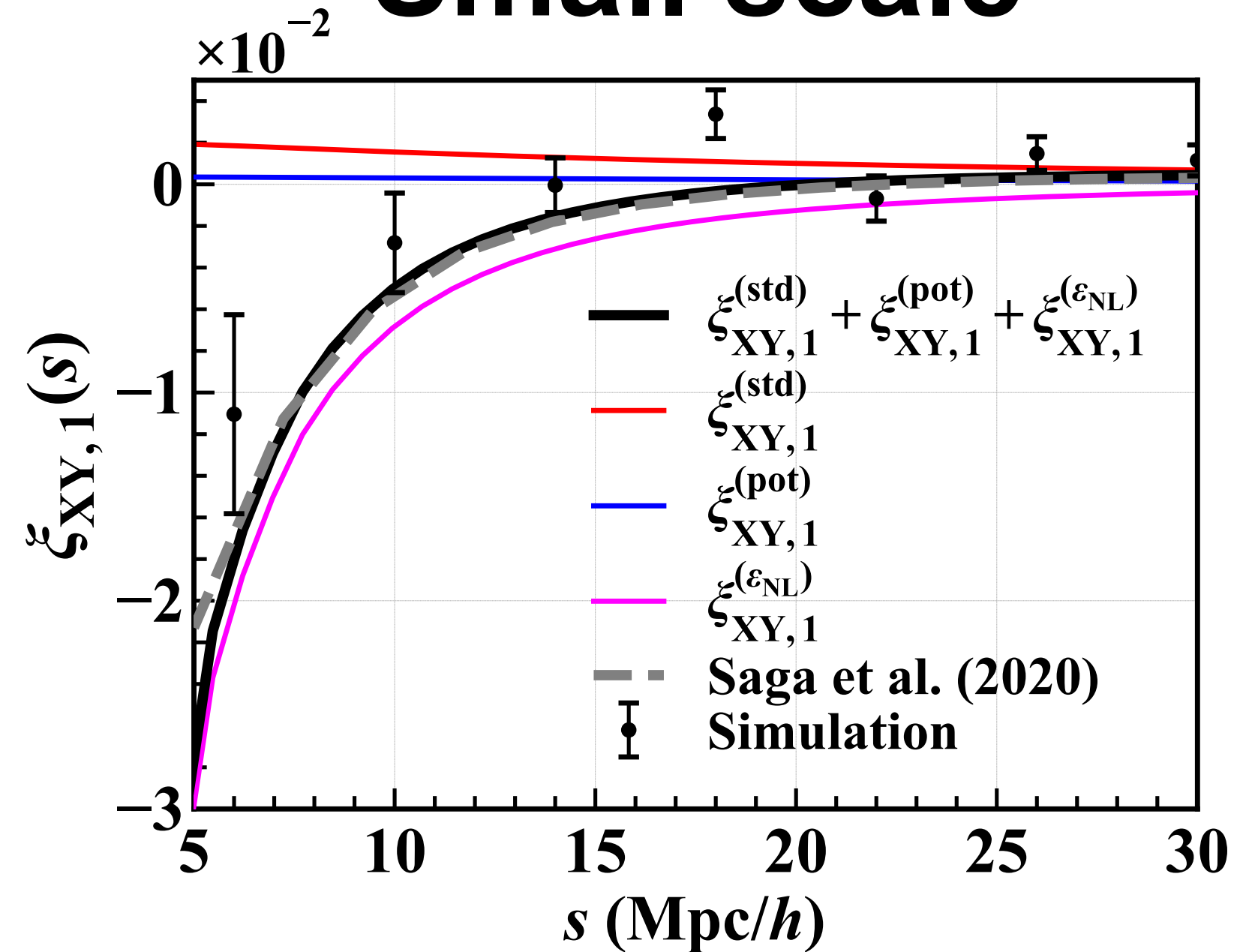
$$\xi_{XY}^{(\text{std})} = \langle \delta_X^{(\text{std})}(s_1) \delta_Y^{(\text{std})}(s_2) \rangle$$

$$\xi_{XY}^{(\text{pot})} = \langle \delta_X^{(\text{std})}(s_1) \delta_Y^{(\text{pot})}(s_2) \rangle + X \leftrightarrow Y$$

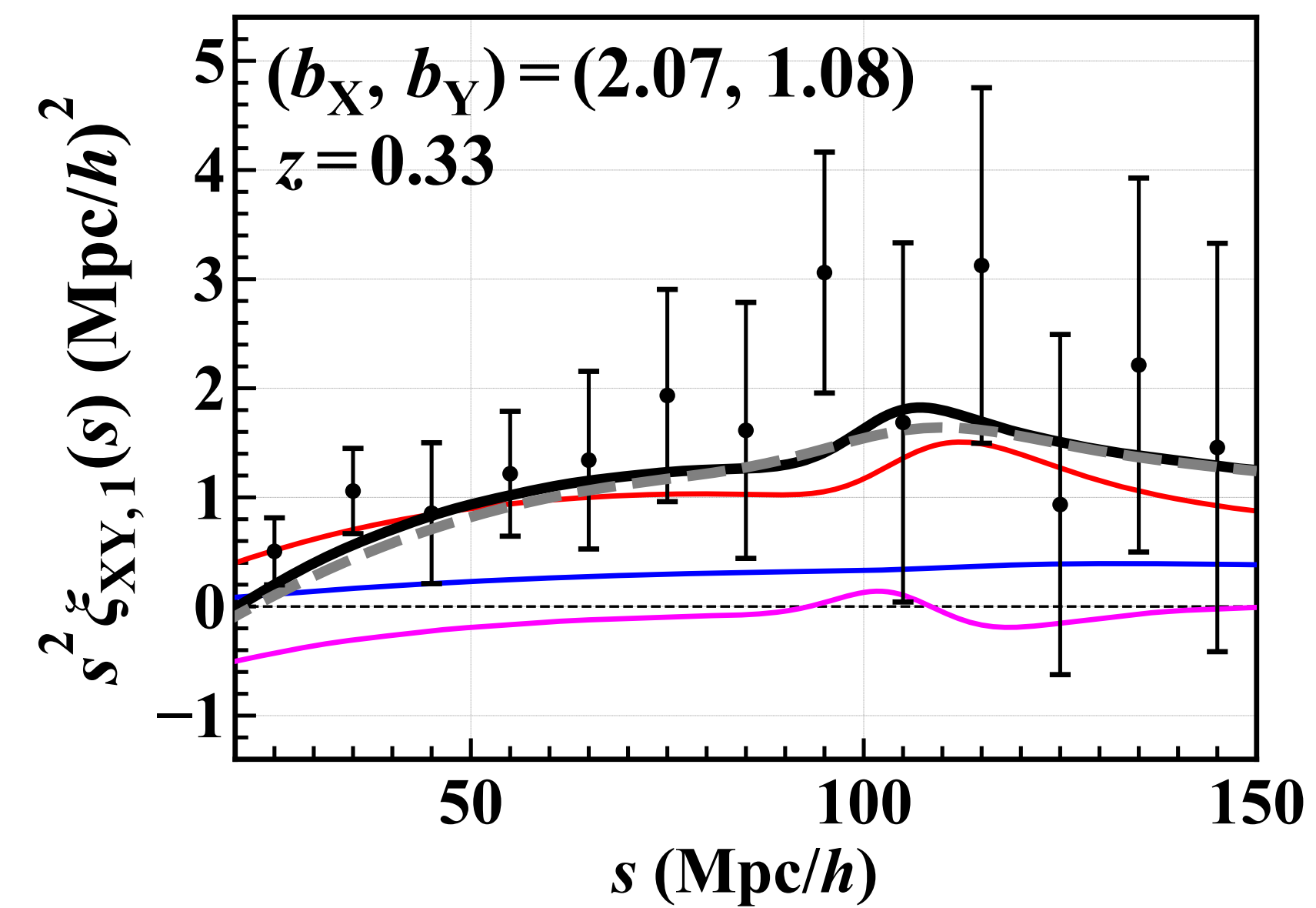
$$\xi_{XY}^{(\text{NL})} = \langle \delta_X^{(\text{std})}(s_1) \delta_Y^{(\text{NL})}(s_2) \rangle + X \leftrightarrow Y$$



Small scale



Large scale

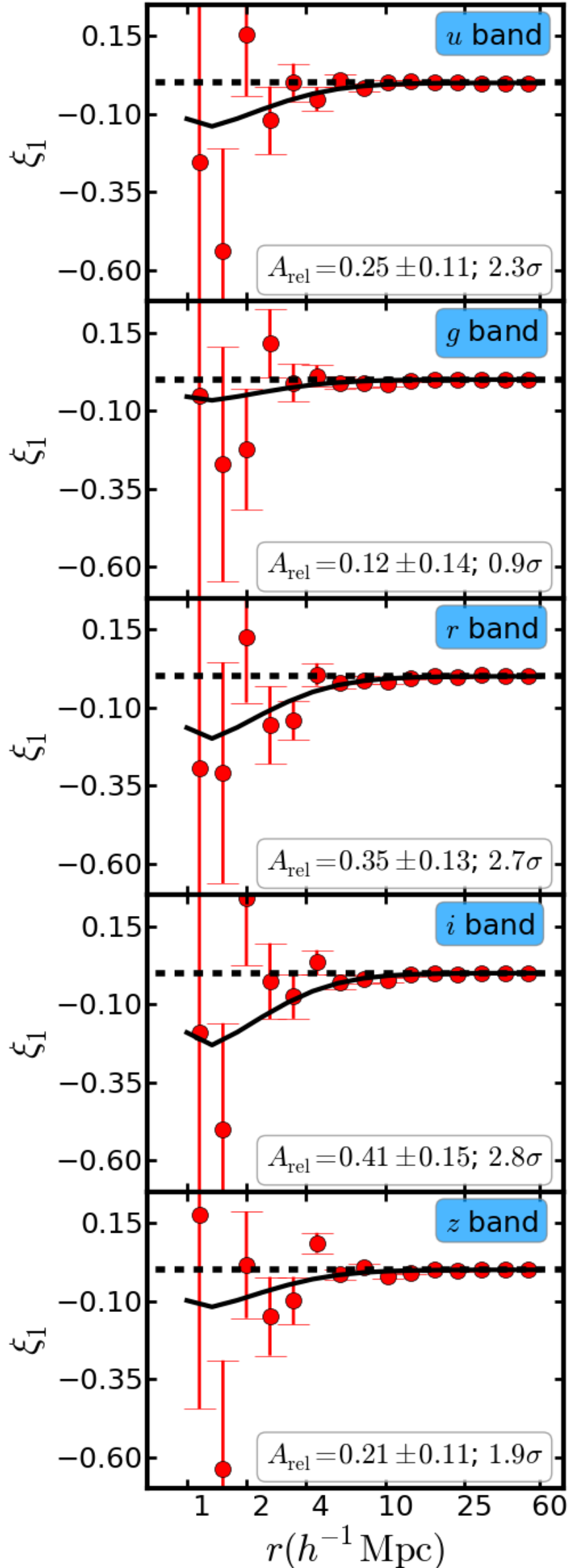
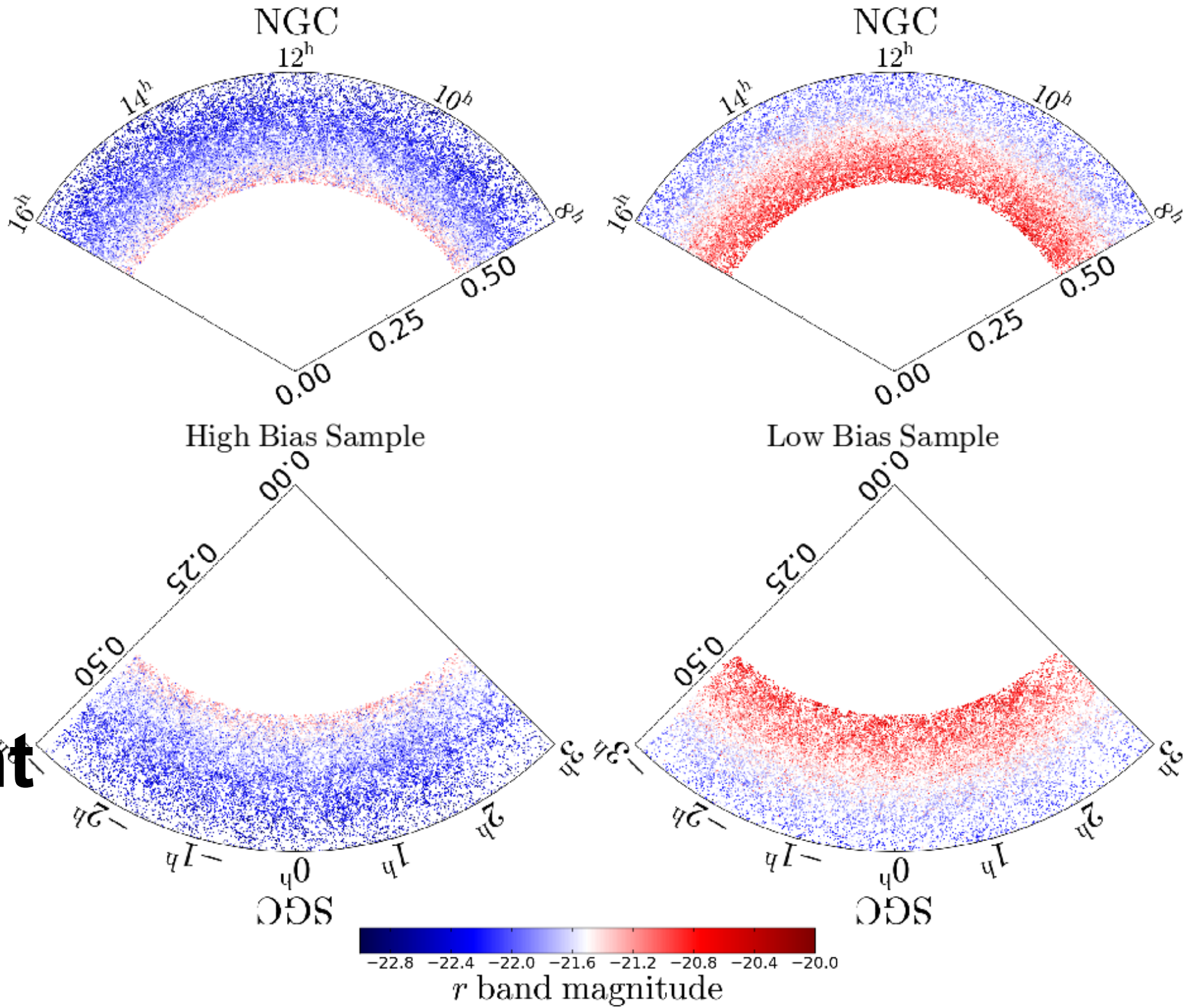


2.6. Recent detection of the dipole

S. Alam et al. [[1709.07855](#)]

2.8 σ detection of relativistic effects in SDSS DR12 CMASS galaxy sample

- 765,433 LRGs
- $0.44 < z < 0.70$
- using the absolute magnitude of galaxies to separate into different biased samples



Deep & Wide future surveys



New probe of gravity!

2.7. Future detectability

S.Saga, A.Taruya, Y.Rasera, M-A.Breton ([2109.06012](#))



Cross-correlation between different biased objects

Strategy 1: Split a single sample into two subsamples

Assume galaxies follow the halo distribution with $M_{\text{halo}} > M_{\text{min}}$

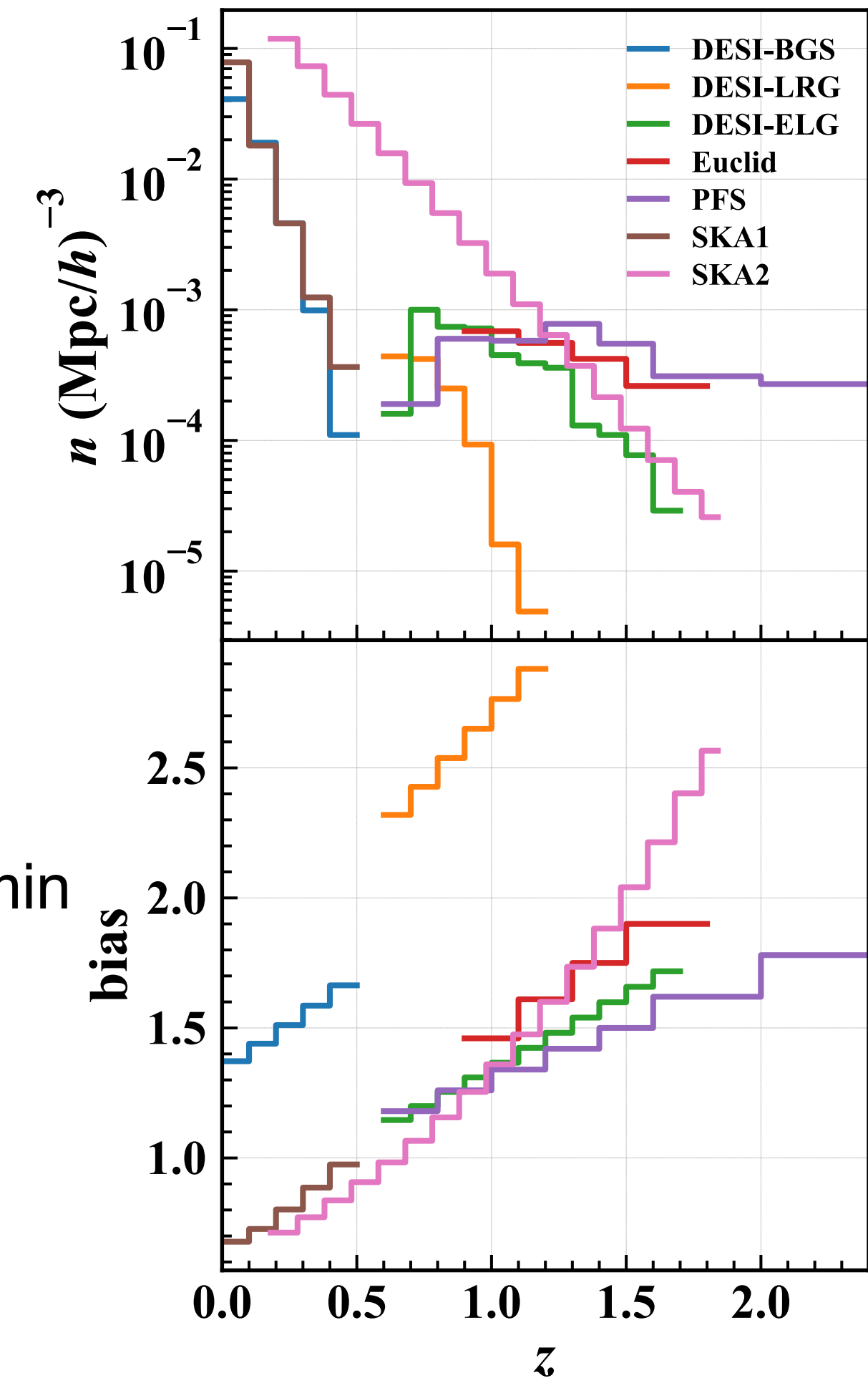
subsample with small bias: $[M_{\text{min}}, M^*]$

subsample with large bias: $[M^*, \infty]$

The uncertainty of how M^* is determined remains.

Strategy 2: Combining several types of samples from different surveys

Assume survey regions are fully overlapped



2.8. Results

S.Saga, A.Taruya, Y.Rasera, M-A.Breton ([2109.06012](#))

Strategy 1: Split a single sample into two subsamples

Assuming galaxies follow the halo distribution with $M_{\text{halo}} > M_{\text{min}}$

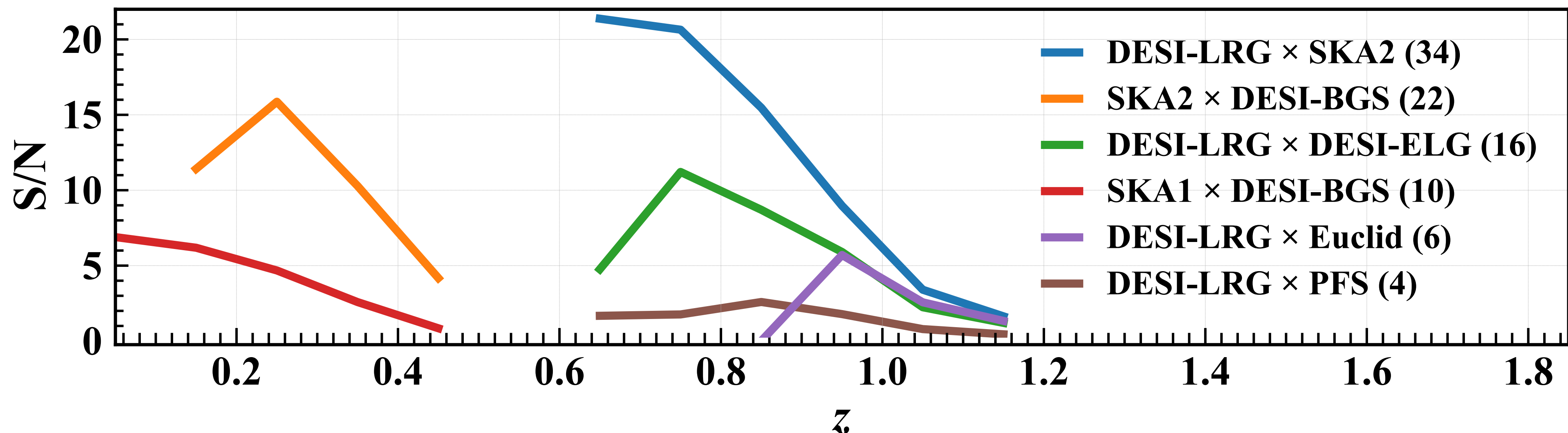
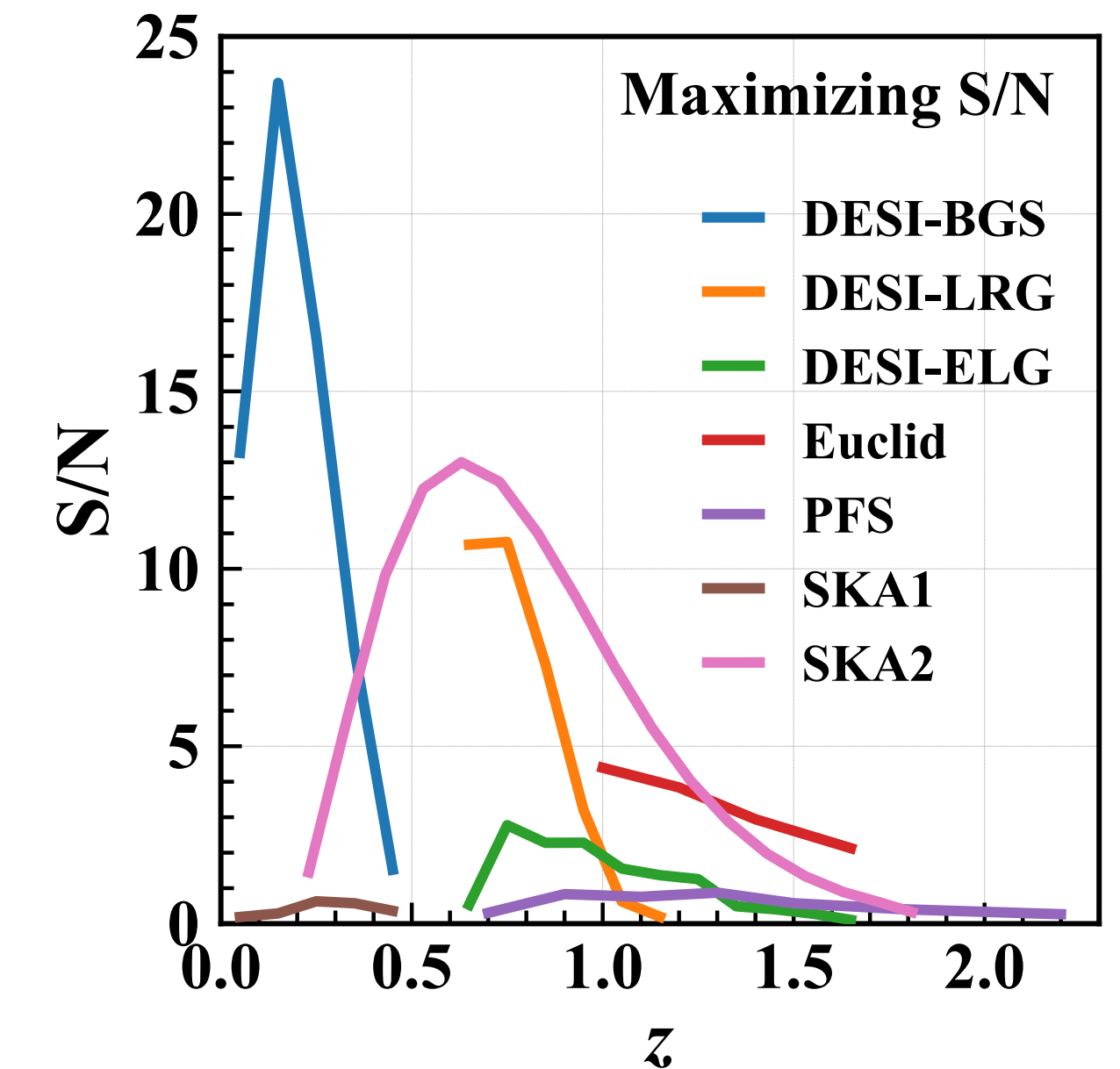
subsample with small bias: $[M_{\text{min}}, M^*]$

subsample with large bias: $[M^*, \infty]$

The uncertainty of how M^* is determined remains.

Strategy 2: Combining several types of samples from different surveys

Assuming survey regions are fully overlapped



2.9. Short summary

Gravitational redshift effects(halo potential)



Dipole in the galaxy-galaxy cross-correlation with **different biased objects**



Future surveys can detect them with large SN~10–20!

🤔 However two or more different biased samples are needed
uncertainty: how we get (sub)samples...?

**cross-correlation between...
one type of biased samples & its shape information**

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3.1. Galaxy-Intrinsic alignment correlations

$$\xi = \langle \delta(s_1) \gamma_{+/\times}(s_2) \rangle$$

Our model

Linear alignment model

P.Catelan et al. (2001)
C.M.Hirata & U.Seljak (2004)

Projection onto a plane perpendicular to the **(not-fixed)** line-of-sight direction:

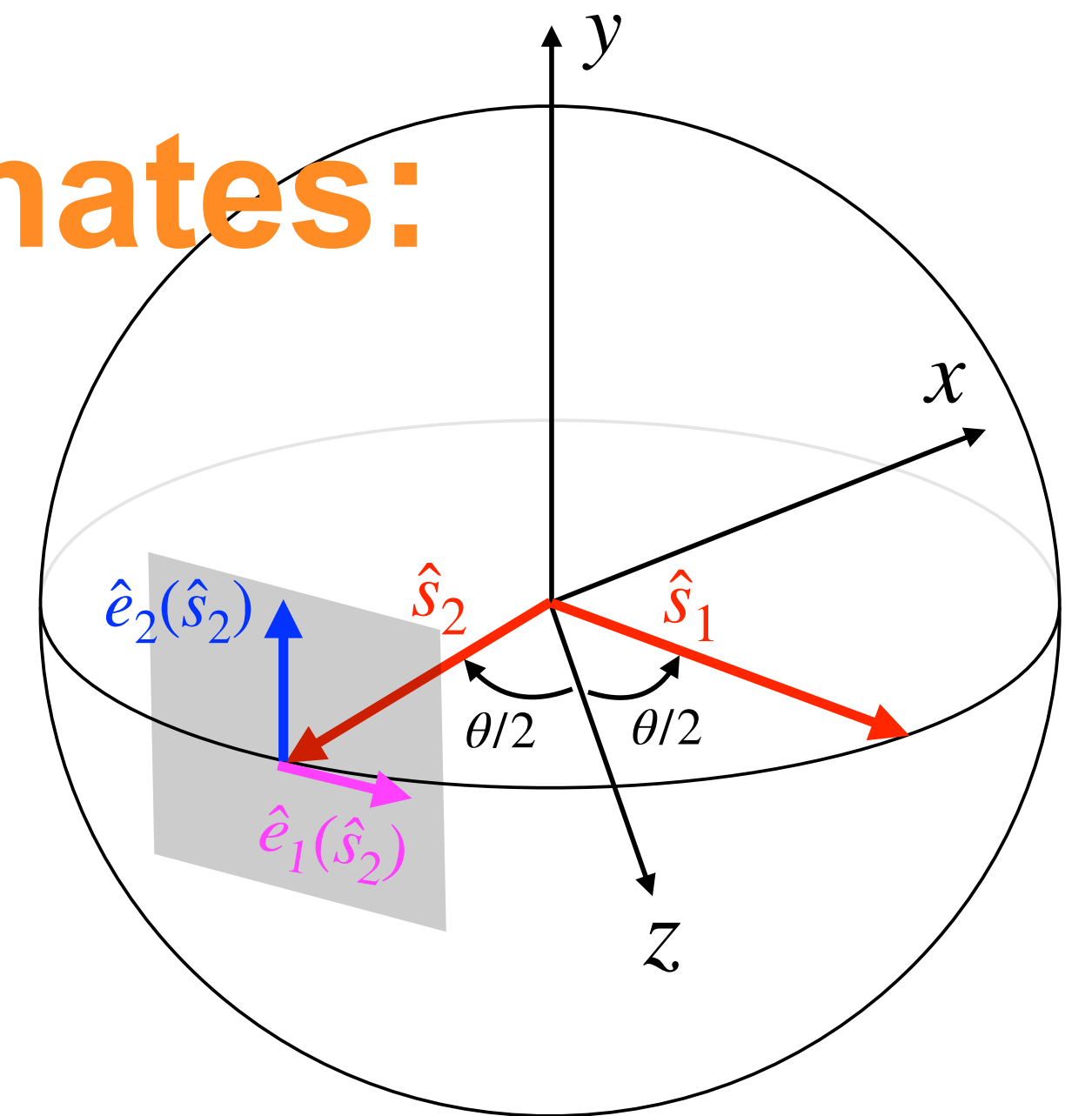
$$\gamma_{ij}^I(\mathbf{x}) = b_K \left[\mathcal{P}_{ik}(\hat{\mathbf{x}}) \mathcal{P}_{jl}(\hat{\mathbf{x}}) - \frac{1}{2} \mathcal{P}_{ij}(\hat{\mathbf{x}}) \mathcal{P}_{kl}(\hat{\mathbf{x}}) \right] \left(\frac{\partial_i \partial_j}{\partial^2} - \frac{1}{3} \delta_{ij} \right) \delta_L(\mathbf{x})$$

$$\mathcal{P}_{ij}(\hat{\mathbf{x}}) \equiv \delta_{ij} - \hat{x}_i \hat{x}_j$$

Two independent components:

$$\begin{pmatrix} \gamma_+(\mathbf{x}) \\ \gamma_\times(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \hat{e}_{1i}(\hat{\mathbf{x}}) \hat{e}_{1j}(\hat{\mathbf{x}}) - \hat{e}_{2i}(\hat{\mathbf{x}}) \hat{e}_{2j}(\hat{\mathbf{x}}) \\ 2\hat{e}_{1i}(\hat{\mathbf{x}}) \hat{e}_{2j}(\hat{\mathbf{x}}) \end{pmatrix} \gamma_{ij}^I(\mathbf{x})$$

Coordinates:



$$\langle \delta(s_1) \gamma_\times(s_2) \rangle = 0$$

3.3. Demonstration: multipole moments

$$\xi = \langle \delta(s_1) \gamma_+(s_2) \rangle = \langle \delta^{(\text{std})}(s_1) \gamma_+(s_2) \rangle + \langle \delta^{(\text{pot})}(s_1) \gamma_+(s_2) \rangle + \langle \delta^{(\text{NL})}(s_1) \gamma_+(s_2) \rangle$$

$$\equiv \xi^{(\text{std})} + \xi^{(\text{pot})} + \xi^{(\text{NL})}$$

$$\xi_\ell = \frac{2\ell + 1}{2} \int_{-1}^1 (\xi^{(S)}(s_1, s_2) P_\ell(\cos \theta)) d \cos \theta$$

Our model

$\delta^{(\text{std})}$ = Real space + Doppler effect

$$\equiv \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{s}} \left[b + f\mu_k^2 - if \frac{2}{ks} \mu_k \right] \delta_L(\mathbf{k}),$$

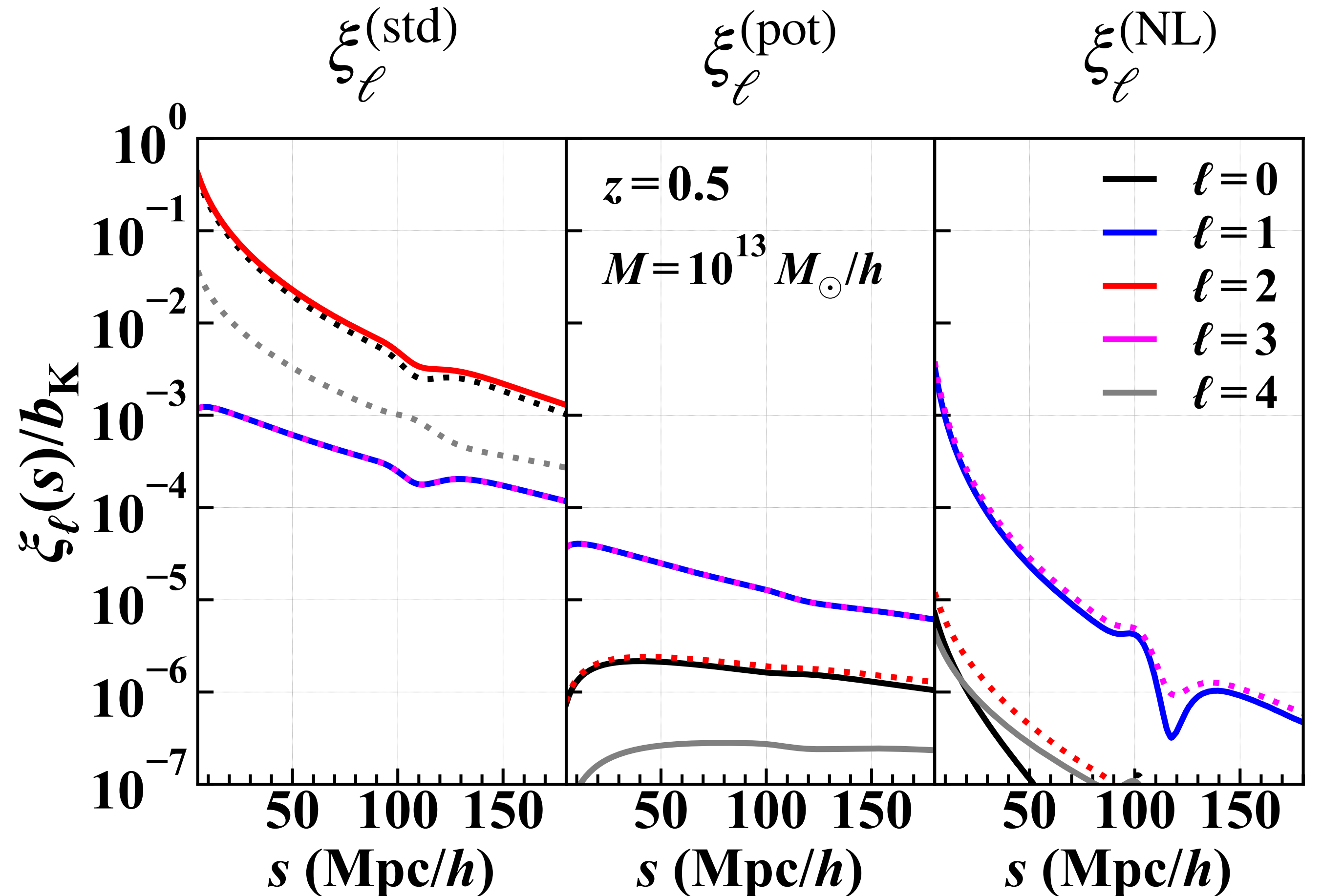
$\delta^{(\text{pot})}$ = Linear potential

$$\equiv \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{s}} \left[(iks\mu_k + 2) \frac{\mathcal{M}}{sk^2} \right] \delta_L(\mathbf{k}),$$

$$\mathcal{M} \equiv -3\Omega_{m0} H_0^2 / (2a^2 H)$$

$\delta^{(\text{NL})}$ = NL potential

$$\equiv -\frac{\Phi_{\text{NL}}}{aHs} \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{s}} \left[-1 + (1 - 2f)\mu_k^2 - i(1 + f) \frac{2}{ks} \mu_k - ibks\mu_k - ifks\mu_k^3 \right] \delta_L(\mathbf{k}),$$



3.3. Demonstration: multipole moments

$$\xi = \langle \delta(s_1) \gamma_+(s_2) \rangle = \langle \delta^{(\text{std})}(s_1) \gamma_+(s_2) \rangle + \langle \delta^{(\text{pot})}(s_1) \gamma_+(s_2) \rangle + \langle \delta^{(\text{NL})}(s_1) \gamma_+(s_2) \rangle$$

$$\equiv \xi^{(\text{std})} + \xi^{(\text{pot})} + \xi^{(\text{NL})}$$

$$\xi_\ell = \frac{2\ell + 1}{2} \int_{-1}^1 (\xi^{(S)}(s_1, s_2) P_\ell(\cos \theta)) d \cos \theta$$

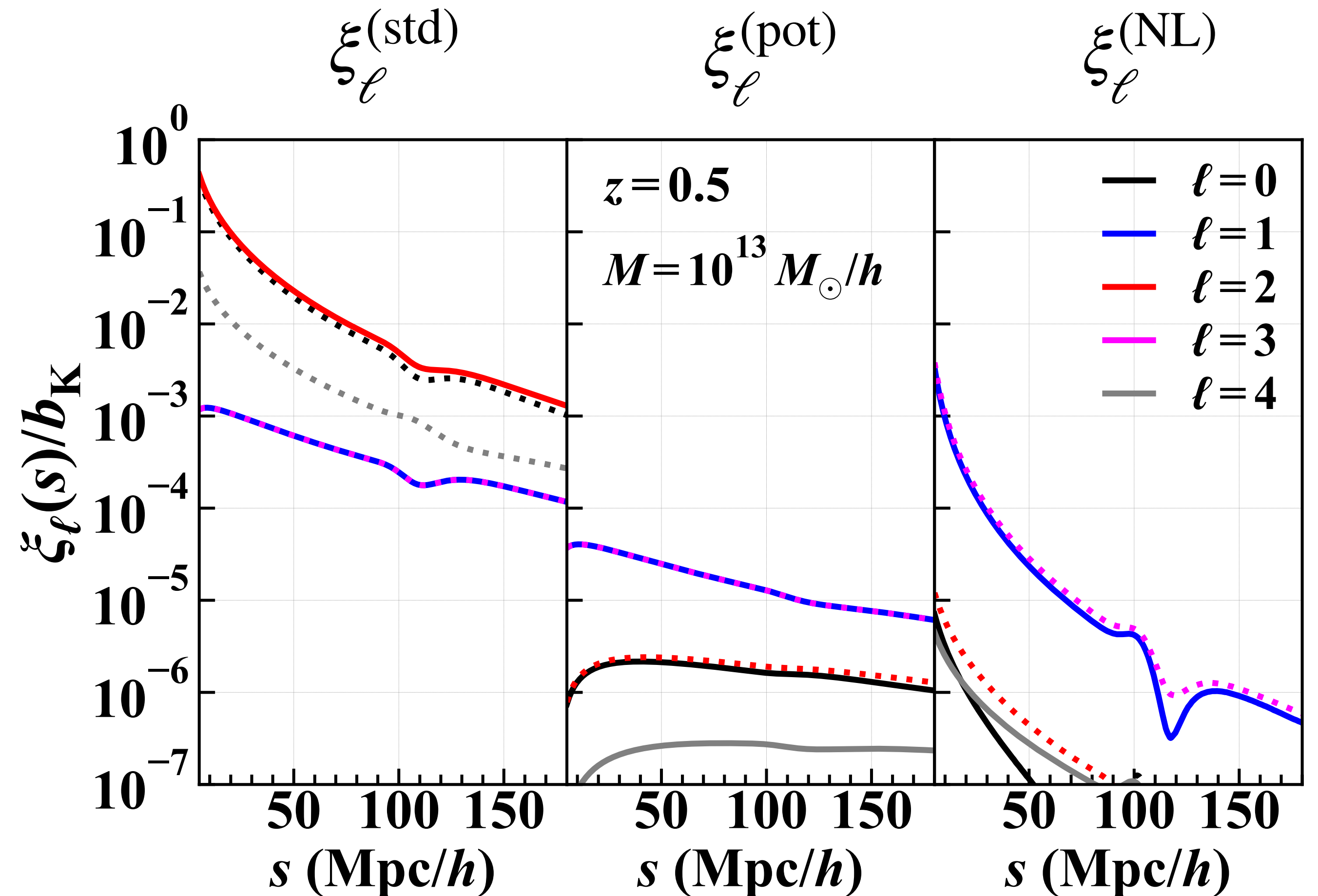
Similarities with GG correlations:

- even multipoles are dominated by "std"
- wide-angle effects in "std" produce odd multipoles
- "NL" dominates small-scale odd multipoles

Difference:

- std & NL dipole are the same sign (no sign flip at small scales)

➔ **GI dipole**



3.4. Demonstration: dipole signals

New simple formulae including **wide-angle effects & gravitational redshift effect**

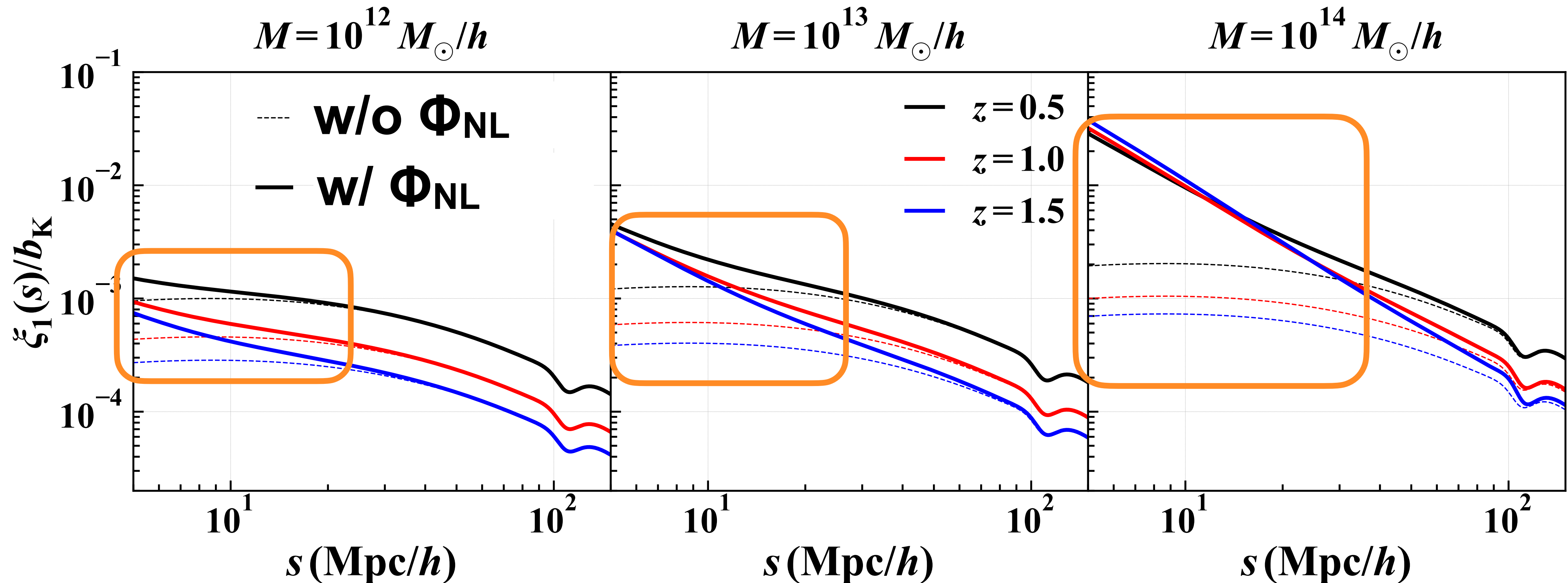
$$\xi_1^{(\text{std})} = \frac{2}{5}(b+f)b_K \Xi_2^{(0)}$$

$$\xi_1^{(\text{pot})} = -\frac{2}{5} \mathcal{M} s b_K \Xi_3^{(1)}$$

$$\xi_1^{(\text{NL})} = -\frac{2}{5} \frac{\Phi_{\text{NL}}}{aHs} b_K b \Xi_3^{(-1)} - \frac{2}{105} f \frac{\Phi_{\text{NL}}}{aHs} b_K \left(7\Xi_3^{(-1)} - 2\Xi_5^{(-1)} \right)$$

$$\mathcal{M} \equiv -3\Omega_{\text{m}0} H_0^2 / (2a^2 H)$$

$$\Xi_\ell^{(n)}(s) = \int \frac{k^2 dk}{2\pi^2} \frac{j_\ell(ks)}{(ks)^n} P_L(k)$$



3.5. Demonstration: dipole covariance

$$A_{IA} = 23$$

Schematically, ...

$$\text{COV}_1(s_1, s_2) \sim \frac{1}{V} \sum_{\ell, \ell'} \left(\xi_{\ell}^{\text{gg}} \times \xi_{\ell'}^{\text{II}} + \xi_{\ell}^{\text{gl}} \times \xi_{\ell'}^{\text{gl}} \right)$$

CV×CV term

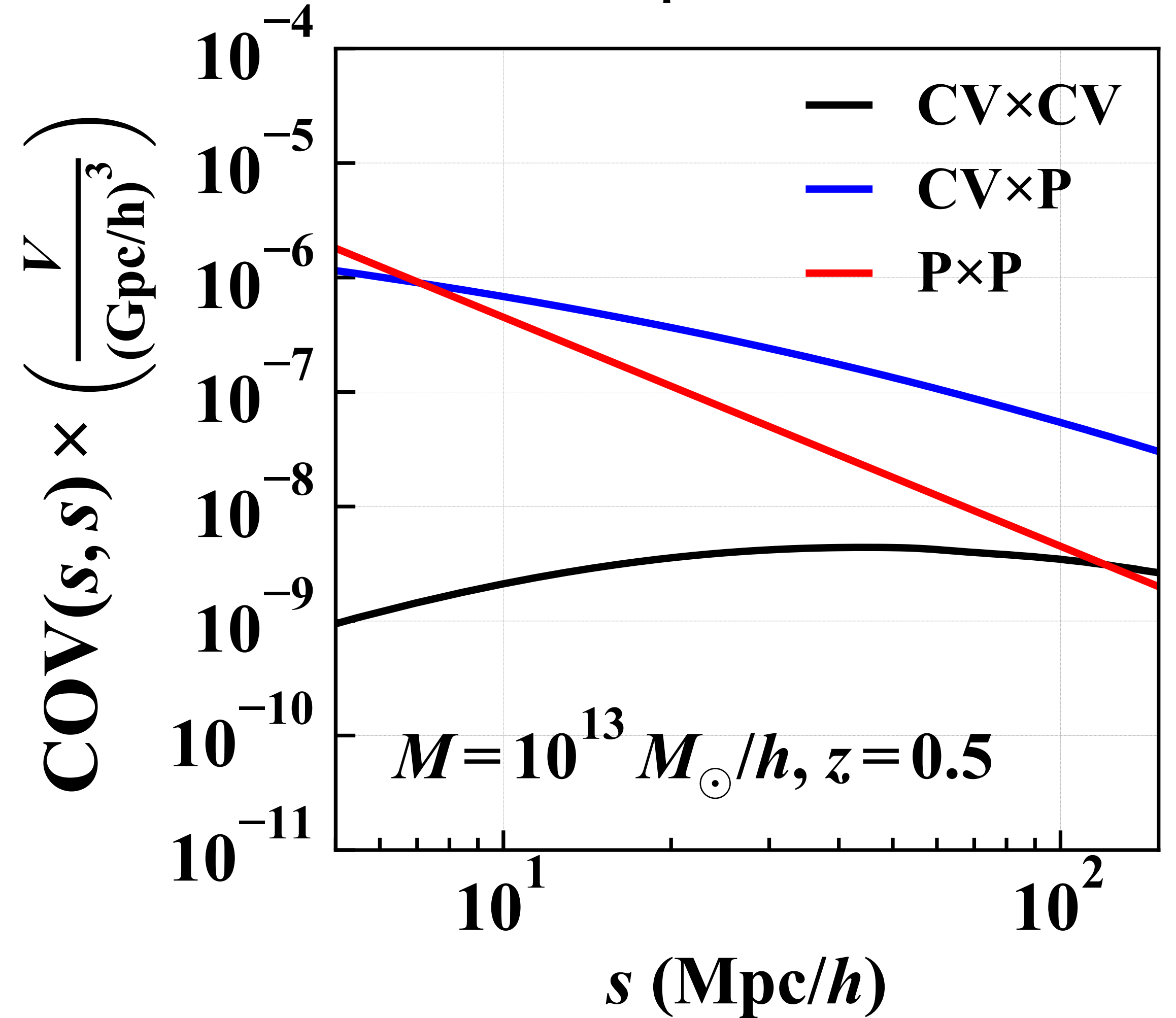
$$\frac{1}{V} \sum_{\ell'} \left(\frac{1}{n_g} \times \xi_{\ell'}^{\text{II}} + \frac{\sigma_{\text{shape}}^2}{n_g} \times \xi_{\ell'}^{\text{gg}} \right)$$

CV×P term

$$\frac{1}{V} \frac{1}{n_g} \frac{\sigma_{\text{shape}}^2}{n_g}$$

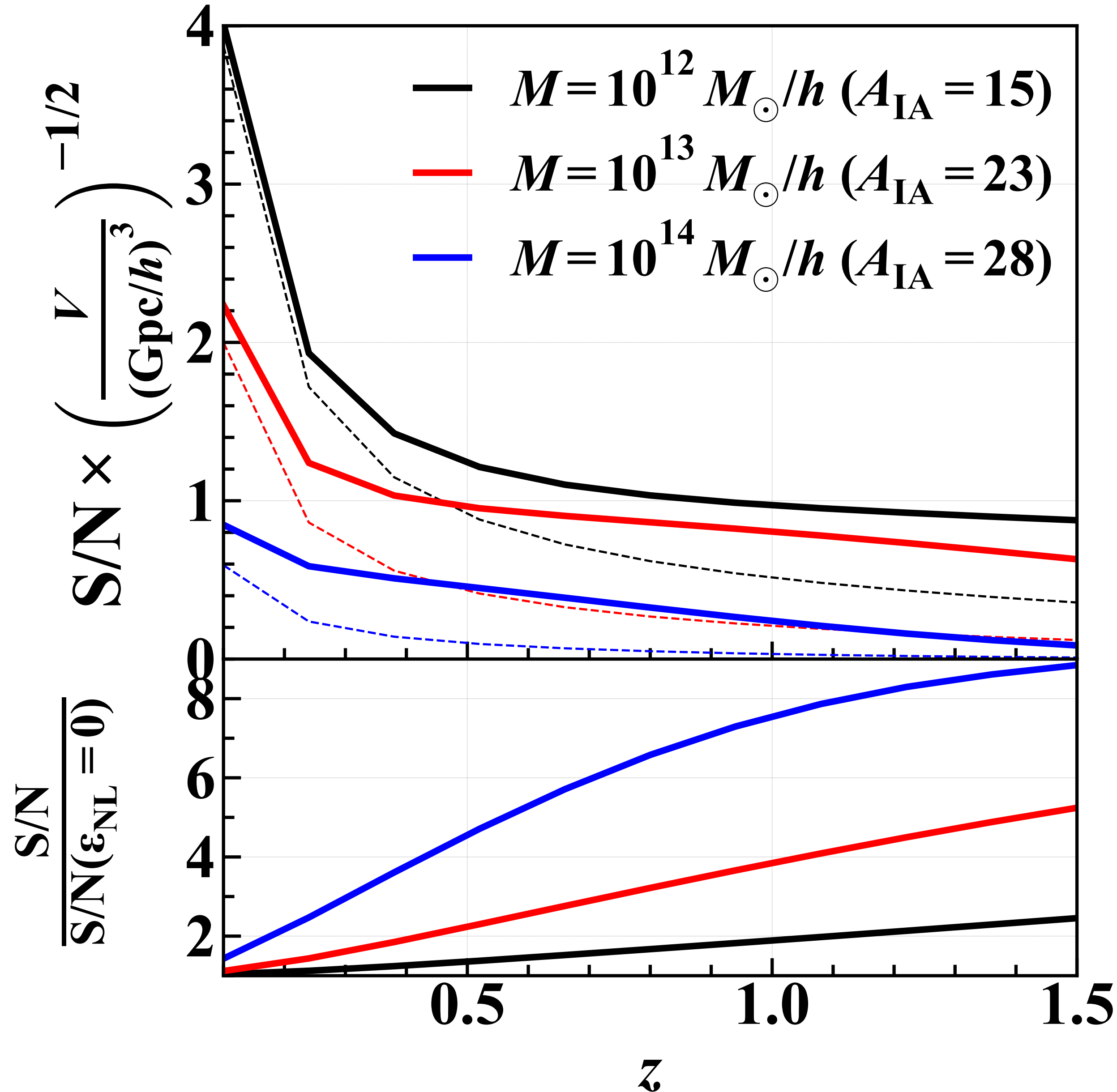
P×P term

NOTE: instead using Legendre polynomial $P_{\ell}(\cos \theta)$, we need to carefully treat the angular dependence by using $Y_{\ell, m}(\theta, \phi)$ in computing covariance matrix.



$$\sigma_{\text{shape}} = 0.2$$

3.6. Demonstration: signal-to-noise ratio



$$\left(\frac{S}{N} \right)^2 = \sum_{s_1, s_2 = s_{\min}}^{s_{\max}} \xi_1(s_1) (\text{COV}_1(s_1, s_2))^{-1} \xi_1(s_2)$$

$(s_{\min}, s_{\max}) = (1, 150) \text{ Mpc}/h$

bias&number density: Sheth&Tormen(1999)

A_{IA} is chosen to match Kurita et al.(2020)

- low-z SN is dominated by wide-angle effect
- measurements at high-z have more chances

4. Summary

The galaxy-Intrinsic alignment cross-correlation can be a new probe of gravitational redshift effects

Dipole anisotropy in galaxy-galaxy correlations

- Two populations are needed S.Saga, A.Taruya, M-A.Breton, Y.Rasera [[2004.03772](#)]
- SN reaches $\sim 10-25$ S.Saga, A.Taruya, Y.Rasera, M-A.Breton [[2109.06012](#)]

Dipole anisotropy in galaxy-IA correlations:

- Single galaxy populations + shape information
- Rough estimation implies SN reaches $\lesssim 1-4 \times [\text{volume in (Gpc/h)}^3]^{1/2}$

SS et al, in prep.

Future prospects:

- SN for specific surveys + systematic effects
- Test of gravity theory
- Measurements in RayGalGroupSims