EFT of galaxy shapes

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with:

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Discussion outline:

Agenda:

non-EFT/PT part

- Galaxy shapes, tensors and IA in 3D
- The role of symmetries
- Projections onto 2D planes

EFT/PT part

- EFT/bias expansion for galaxy shapes
- One-loop power spectrum and tree-level bispectrum

Ellipsoids, 2-tensors, galaxy shapes

How can we describe the field of ellipsoids?

Ellipsoid – 3 parameters;

$$T^0_{ij} = \begin{pmatrix} 1/a^2 & 0 & 0 \\ 0 & 1/b^2 & 0 \\ 0 & 0 & 1/c^2 \end{pmatrix}$$

Rotation matrix – 3 Euler angles;

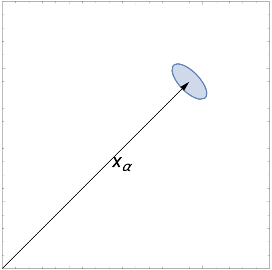
$$\mathcal{R}_{ij}(\psi, \theta, \phi) \implies \boldsymbol{T} = \boldsymbol{\mathcal{R}} \boldsymbol{T}^0 \boldsymbol{\mathcal{R}}^T$$

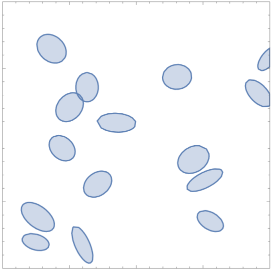
Ellipsoid equation;

$$(\boldsymbol{x} - \boldsymbol{x}_\alpha) \cdot \boldsymbol{T}^{(\alpha)} \cdot (\boldsymbol{x} - \boldsymbol{x}_\alpha) = 1$$

Tensor field:

$$T_{ij}(\boldsymbol{x}) = \sum_\alpha T^{(\alpha)}_{ij}(\boldsymbol{x}_\alpha) \delta^{\mathrm{D}}(\boldsymbol{x} - \boldsymbol{x}_\alpha)$$





Ellipsoids, 2-tensors, galaxy shapes

Intrinsic galaxy shape field:

$$I_{ij}(\boldsymbol{x}) = \sum_{\alpha} I_{ij}(\boldsymbol{x}_\alpha)\delta^{\mathrm{D}}(\boldsymbol{x} - \boldsymbol{x}_\alpha)$$

Number-wighted galaxy size

$$\mathrm{tr}[I_{ij}(\boldsymbol{x})] = \overline{s^2}\left(1 + \delta_{\mathrm{s}}(\boldsymbol{x})\right),$$

where $\langle I_{ij}\rangle = \overline{s^2}/3\ \delta^{\mathrm{K}}_{ij}$.

Shape fluctuation field

$$S_{ij}(\boldsymbol{x}) = \frac{I_{ij}(\boldsymbol{x}) - \langle I_{ij}\rangle}{\mathrm{tr}\langle I_{ij}\rangle} = g_{ij}(\boldsymbol{x}) + \frac{1}{3}\delta_{\mathrm{s}}(\boldsymbol{x})\delta^{\mathrm{K}}_{ij}$$

Trace-free galaxy shape perturbations: $g_{ij}(\boldsymbol{x}) \equiv \mathrm{TF}[S_{ij}(\boldsymbol{x})]$.

Alternative shape field definition:

$$I_{ij}(\boldsymbol{x}) = \sum_{\alpha} \frac{I_{ij}(\boldsymbol{x}_\alpha)}{\mathrm{tr}\langle I_{\alpha,ij}\rangle}\delta^{\mathrm{D}}(\boldsymbol{x} - \boldsymbol{x}_\alpha) \quad \Longrightarrow \quad \mathrm{tr}[I_{ij}(\boldsymbol{x})] = \langle n_{\mathrm{g}}\rangle\left(1 + \delta_{\mathrm{n}}(\boldsymbol{x})\right)$$

Symmetries and Spherical Tensors

$SO(3)$ representations:

$$\text{Spherical tensors :}\quad \boldsymbol{Y}^{(\ell)m}(\hat{k}') = \sum_{q=-\ell}^{\ell} \left(\mathcal{D}^{(\ell)}\right)^{m}{}_{q} \boldsymbol{Y}^{(\ell)q}(\hat{k})$$

Rank $0, 1, 2$ form the any orthogonal basis constructed as:

$$\begin{aligned}
&\text{scalar :} \quad && \boldsymbol{Y}^{(0)} = 1, \\
&\text{vector :} \quad && \boldsymbol{Y}_i^{(0)} = \hat{k}_i, \quad \boldsymbol{Y}_i^{(\pm 1)} = e_i^{\pm}, \\
&\text{tensor :} \quad && \boldsymbol{Y}_{ij}^{(0)} = \hat{k}_i\hat{k}_j - \tfrac{1}{3}\delta_{ij}^K, \ \boldsymbol{Y}_{ij}^{(\pm 1)} = \hat{k}_j e_i^{\pm} + \hat{k}_i e_j^{\pm}, \ \boldsymbol{Y}_{ij}^{(\pm 2)} = e_i^{\pm} e_j^{\pm},
\end{aligned}$$

This gives the expansion

$$T_{ij}(\boldsymbol{k}) = \frac{1}{3} T_0^{(0)}(\boldsymbol{k}) \delta_{ij}^K + \sum_{m=-2}^{2} T_2^{(m)}(\boldsymbol{k}) \boldsymbol{Y}_{ij}^{(m)}(\hat{k})$$

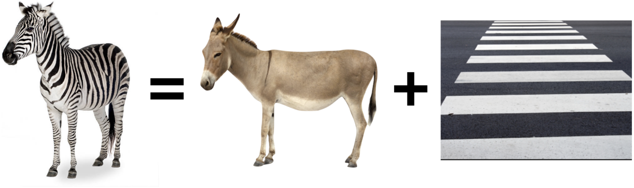
This equivalent to the usual cosmological SVT decomposition.

Decomposition of tensor correlators

We are interested in statistical N-point functions:

$$\langle S_{ij}(\boldsymbol{k}_1) S_{lm}(\boldsymbol{k}_2)\rangle' = P_{ij,lm}(k_1),$$
$$\langle S_{ij}(\boldsymbol{k}_1) S_{lm}(\boldsymbol{k}_2) S_{rs}(\boldsymbol{k}_3)\rangle' = B_{ij,lm,rs}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3)$$

Given that we can decomposed the S_{ij} tensor:



separation into "dynamics" + "symmetries"

$$\langle S_1 S_2 \rangle \quad = \quad \langle \delta_1 \delta_2 \rangle \quad + \quad \{\boldsymbol{Y}^{(\ell)m}\}$$

Symmetries and spherical tensors

Statistics isotropy and homogeneity and parity invariance:

$$\left\langle S_{\ell}^{(m)}(\boldsymbol{k})\; S_{\ell'}^{(m')}(\boldsymbol{k}')\right\rangle = (2\pi)^3 \delta^K_{mm'} \delta^D_{\boldsymbol{k}+\boldsymbol{k}'} P_{\ell\ell'}^{(|m|)}(k).$$

All contributions given by the five scalar functions $P_{\ell\ell'}^{(m)}$

$$\begin{aligned}
\langle \delta(\boldsymbol{k})\delta(\boldsymbol{k}')\rangle &= P_{00}^{(0)}(k) \\
\langle \delta(\boldsymbol{k}) g_{ij}(\boldsymbol{k}')\rangle &= \boldsymbol{Y}_{ij}^{(0)} P_{02}^{(0)}(k) \\
\langle g_{ij}(\boldsymbol{k}) g_{lm}(\boldsymbol{k}')\rangle &= \boldsymbol{Y}_{ij}^{(0)} \boldsymbol{Y}_{lm}^{(0)} P_{22}^{(0)}(k) + \sum_{q=1,2} \boldsymbol{Y}_{ij}^{\{(q)} \boldsymbol{Y}_{lm}^{(-q)\}} P_{22}^{(q)}(k)
\end{aligned}$$

Bispectrum:

$$\langle \delta(\boldsymbol{k}_1) g_{ij}(\boldsymbol{k}_2) g_{lm}(\boldsymbol{k}_3)\rangle = \boldsymbol{Y}_{ij}^{(0)}\left(\hat{k}_2\right) \boldsymbol{Y}_{lm}^{(0)}\left(\hat{k}_3\right) B_{022}^{(0)}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3) + \ldots$$

Reminder:

$$S_{ij}(\boldsymbol{k}) = g_{ij}(\boldsymbol{k}) + \frac{1}{3}\delta_{\mathrm{s}}(\boldsymbol{k})\delta^{\mathrm{K}}_{ij}$$

Projections: flat-sky

3D shape of galaxies get projected onto the onto the sky:

$$\gamma_{I,ij}(\boldsymbol{r}) = \mathrm{TF}\left[\mathcal{P}_{ik}(\hat{r})\mathcal{P}_{jl}(\hat{r})\right] g_{kl}(\boldsymbol{r})$$

where $\mathcal{P}_{ij}(\hat{r}) \equiv \delta^K_{ij} - \hat{r}_i\hat{r}_j$.
Integrating along the line of sight for photometric survey

$$\hat{\gamma}_{I,ij}(\boldsymbol{\theta}) = \int d\chi\, W(\chi)\gamma_{I,ij}\big(\chi\hat{n}, \chi\boldsymbol{\theta}\big),$$

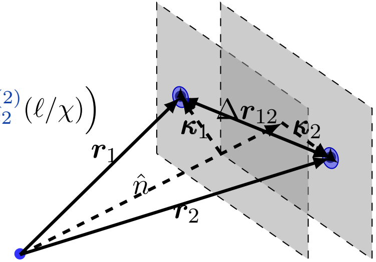
These rotation of the basis leads to the following spectra

$$C_{\delta E}(\ell) = \int d\chi \frac{W^2(\chi)}{\chi^2} P^{(0)}_{02}(\ell/\chi),$$

$$C_{EE}(\ell) = \int d\chi \frac{W^2(\chi)}{\chi^2} \left(2P^{(0)}_{22}(\ell/\chi) + P^{(2)}_{22}(\ell/\chi)\right)$$

$$C_{BB}(\ell) = \int d\chi \frac{W^2(\chi)}{\chi^2} P^{(1)}_{22}(\ell/\chi)$$

$$C_{\delta B}(\ell) = C_{EB}(\ell) = 0.$$



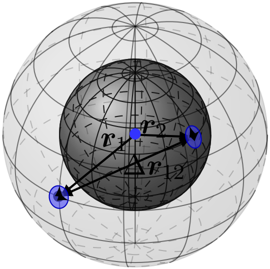
Projections: full-sky

Note that configuration space basis vectors are eigenfunctions of the projection operator $\mathcal{P}.m^{\pm} = m^{\pm} \implies$ projections are simple!

$$\hat{\gamma}_{\pm 2}(\hat{r}) = \boldsymbol{M}_{ij}^{(\pm 2)\dagger} \hat{\gamma}_{I,ij}(\hat{r}) = \int d\chi \; W(\chi) g_{\pm 2}(\chi \hat{r})$$

Spin weighted spherical harmonics:

$$\hat{\gamma}_{\pm 2} = \sum_{\ell m} {}_{\pm}\hat{\gamma}_{\ell m \pm 2} Y_{\ell}^{(m)}$$



Full-angle power spectrum:

$$\left\langle X_{\ell m}^* X'_{\ell m} \right\rangle = \delta^{\mathrm{K}}_{\ell \ell'} \delta^{\mathrm{K}}_{m m'} C_{\ell}^{XX'}$$

Leads to familiar E and B mode full sky form:

$$\langle {}_s\hat{\gamma}^*_{\ell m} | {}_{s'}\hat{\gamma}_{\ell' m'} \rangle' = \sum_{q=0}^{2} \int_k P_{22}^{(q)}(k) \left[\int_{\chi_1} W_1 \mathcal{J}_{\ell,2}^{s,\{q}(\chi_1 k) \right] \left[\int_{\chi_2} W_2 \mathcal{J}_{\ell,2}^{s',-q\}}(\chi_2 k) \right]$$

[Shiraishi++:21]

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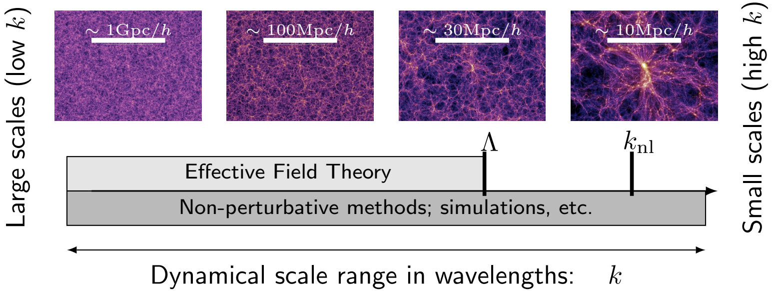
[Blazek,ZV+:15, Schmitz++:18, Blazek++:19]

Strategy and model development



Describe the matter density on large-scales (small fluctuations).

EFT methods:

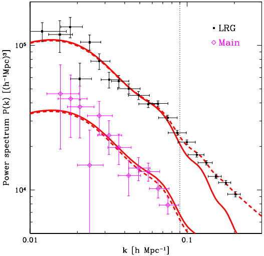
a) UV physics unknown, and we have scale separation (inflation, baryonic fluids, dielectrics)

b) UV physics known, but long-wavelengths are of interest (phonons, QCD (CPT))

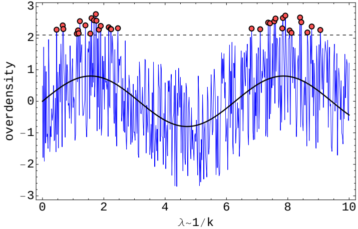
Bias coefficients incorporate complicated galaxy formation physics: halo formation, merger history, feedback (SN, AGN), . . .

Galaxies and biasing of dark matter halos

- cosmological theory (sims) give dark matter distribution, but not galaxy distribution.
- what we observe from survey are galaxies, not dark matter.
- Bias: How does galaxy distribution related to the matter?



[Tegmark et al, 2006]



- galaxies form at high peaks: $\Longrightarrow$ exhibit higher clustering
- Tracer detriments the amplitude: $P_g(k) = b^2 P_m(k) + \ldots$

Canonical approaches to galaxy biasing

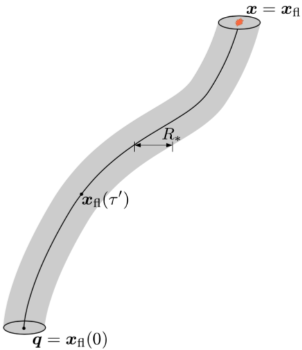
Local biasing model: relation to dark matter

$$\delta_{\mathsf{h}} = c_\delta \delta + c_{\delta^2}\delta^2 + c_{\delta^3}\delta^3 + \dots$$ [Fry+:93]

Quasi-local (in space): [McDonald+:09]

$$\delta_{\mathsf{h}}(\boldsymbol{x}) = c_\delta \delta(\boldsymbol{x}) + c_{\delta^2}\delta^2(\boldsymbol{x}) + c_{\delta^3}\delta^3(\boldsymbol{x}) + c_{s^2}s^2(\boldsymbol{x}) + c_{\delta s^2}\delta(\boldsymbol{x})s^2(\boldsymbol{x}) + c_\epsilon \epsilon + \dots,$$

with effective (bias) coefficients c_l and operators:



[from Desjacques++:18]

$$s_{ij}(\boldsymbol{x}) = \partial_i \partial_j \phi(\boldsymbol{x}) - \frac{1}{3}\delta^{\rm K}_{ij}\delta(\boldsymbol{x}), \quad \dots$$

where ϕ is the gravitational potential, and white noise (stochasticity) ϵ.

Complete set set of operators including non-locality in time effects!

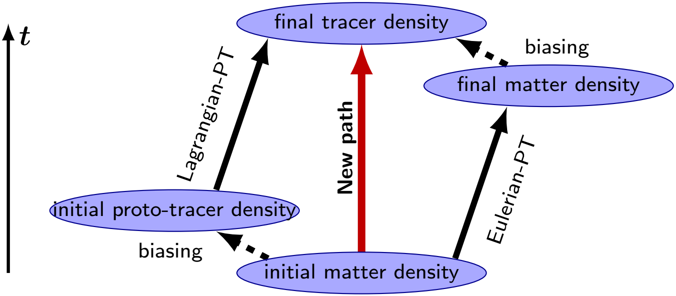
[Angulo, ZV++:15, Fujita, ZV++:16, Desjacques++:18 Fujita&ZV:20]

A new look at bias expansion: "Monkey bias"

A new idea: [Fujita, ZV:20]

(I.) construct a bias from linear density - as a Monkey would,

(II.) impose physical constraints - consistency relations in LSS



A new look at bias expansion

A new idea: [Fujita, ZV:20]

(I.) construct a bias from linear density - as a Monkey would,

(II.) impose physical constraints - consistency relations in LSS

How do we describe the system for a tracer?

Balance equations:

$$\partial_\tau \delta_\alpha(\boldsymbol{x}) + \boldsymbol{\nabla} \cdot \left([1+\delta_\alpha]\, \boldsymbol{u}_\alpha \right)(\boldsymbol{x}) = S_\delta[\delta](\boldsymbol{x}),$$

$$\partial_\tau \boldsymbol{u}_\alpha(\boldsymbol{x}) + \mathcal{H}\boldsymbol{u}_\alpha(\boldsymbol{x},\tau) + \boldsymbol{u}_\alpha(\boldsymbol{x},\tau)\cdot\boldsymbol{\nabla}\boldsymbol{u}_h(\boldsymbol{x},\tau) = -\boldsymbol{\nabla}\phi(\boldsymbol{x},\tau) + S_u[\delta](\boldsymbol{x}),$$

The lhs. terms are:

$$\nabla^2\phi(\boldsymbol{x}) \propto \delta_m(\boldsymbol{x}),$$

and small scale sources $S_\delta(\boldsymbol{x})$, $S_u(\boldsymbol{x})$, suppressed by some scale k_*.

The key notion is the separation of scales in the system, i.e. gravity dominates on large scales.

I. Specifying the non-linear terms

This is the "Monkey part":

$$\text{Continuity eq.}: \quad \partial_\tau \delta + (\text{linear terms}) = -\delta\theta - \partial_i \delta \frac{\partial_i}{\partial^2}\theta,$$

$$\text{Euler eq.}: \quad \partial_\tau \theta + (\text{linear terms}) = -\frac{\partial_i \partial_j}{\partial^2}\theta \frac{\partial_i \partial_j}{\partial^2}\theta,$$

where δ is the density and θ is the velocity divergence.
Solution is constructed by the iterative "Monkey" process

$$\left\{ XY, \quad \partial_i X \frac{\partial_i}{\partial^2} Y, \quad \frac{\partial_i \partial_j}{\partial^2} X \frac{\partial_i \partial_j}{\partial^2} Y \right\},$$

where X and Y are drown from the list of the lower order operators.

New bias basis:

$$\delta_\alpha = a_1 \delta_L + b_1 \delta_L^2 + b_2 \partial_i \delta_L \frac{\partial_i}{\partial^2} \delta_L + b_3 \frac{\partial_i \partial_j}{\partial^2} \delta_L \frac{\partial_i \partial_j}{\partial^2} \delta_L + \dots$$

In the paper we keep terms up to the third order terms in PT.

II. Constraining the coefficients

Consistency relations of LSS are direct consequence of the equivalence principle and adiabatic initial conditions:

$$\langle \delta^m_{\boldsymbol{k}}(\tau)\delta^{\rm g}_{\boldsymbol{q}_1}(\tau_1)\ldots\delta^{\rm g}_{\boldsymbol{q}_n}(\tau_n)\rangle' \sim -P_{\rm g}(\boldsymbol{k},\tau)\sum_\alpha \frac{D(\eta_\alpha)}{D(\eta)}\frac{\boldsymbol{k}\cdot\boldsymbol{q}_\alpha}{k^2}\langle \delta^{\rm g}_{\boldsymbol{q}_1}(\eta_1)\ldots\delta^{\rm g}_{\boldsymbol{q}_n}(\eta_n)\rangle',\ k\to 0.$$

[Peloso+:13, Kehagias+:13, Creminelli++:13...]

Tree-level statistics is the simplest way to impose the constraints:

$$\lim_{k\to 0}\langle \delta_{\boldsymbol{k}}\delta^\alpha_{\boldsymbol{q}_1}\delta^\beta_{\boldsymbol{q}_2}\rangle' = \left({a_1}^{(\alpha)}{b_2}^{(\beta)} - {a_1}^{(\beta)}{b_2}^{(\alpha)}\right)\frac{\boldsymbol{k}\cdot\boldsymbol{q}_1}{2k^2}P_\ell(k)P_\ell(q_1) + \mathcal{O}(k^0),$$

By requiring the IR-divergent term to vanish we get:

$$\frac{{b_2}^{(\alpha)}}{{a_1}^{(\alpha)}} = \frac{{b_2}^{(\beta)}}{{a_1}^{(\beta)}} = \mathcal{C}(\tau).$$

The $\mathcal{C}(\tau)$ is universal, tracers independent, function of time.

Fixing the dynamical degrees of freedom

New bias expansion:

$$\delta_g = a_1\left[\delta_L + \mathcal{C}\,\partial_i\delta_L\frac{\partial_i}{\partial^2}\delta_L\right] + b_1\delta_L^2 + b_3\left(\frac{\partial_i\partial_j}{\partial^2}\delta_L\frac{\partial_i\partial_j}{\partial^2}\delta_L\right) + (\text{3rd order})$$

How to determine the universal coefficients $\mathcal{C}(\tau)$?

Easy way is to fix it to dark matter: $\mathcal{C} = 1$.

These coefficients reflect dynamics and modifications of GR!

Example: clustering quintessence [Sefusatti&Vernizzi:11, Fasiello&ZV:17]

$$\mathcal{C} = 1 - \epsilon(\tau),$$

where ϵ depends on the quintessence field and τ.

This motivates the construction of optimal estimators for $\mathcal{C}$.

Scalar field biasing: effective approach

Alternative systematisation in terms of derivatives of potential ϕ :

$$\Pi^{[1]}_{ij} = \frac{2}{3\Omega_m \mathcal{H}^2} k_i k_j \phi,$$

with higher operators O_h:

$$\begin{aligned}
&(1) \qquad \mathrm{tr}\left[\Pi^{[1]}\right],\\
&(2) \qquad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^2\right], \quad \left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^2,\\
&(3) \qquad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^3\right], \quad \mathrm{tr}\left[\left(\Pi^{[1]}\right)^2\right]\mathrm{tr}\left[\Pi^{[1]}\right], \quad \left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^3, \quad \mathrm{tr}\left[\Pi^{[1]}\Pi^{[2]}\right],
\end{aligned}$$

and additional derivative operators $R_*^2 \nabla^2 \mathrm{tr}\left[\Pi^{[1]}\right], \ldots$.

– series allows one to estimate the higher order (theory) errors

– coefficients - physics from the R_* scale (some degeneracies)

Tracer field is then given

$$\delta_{\mathrm{s}}(\boldsymbol{x}) = \sum_O b_O^{(s)} \mathrm{tr}[O_{ij}](\boldsymbol{x}),$$

Biasing of shapes in 3D: effective approach

Expansion of the field of galaxy shapes:

$$g_{ij}(\boldsymbol{x}) = \sum_O b_O^{(g)} \mathrm{TF}[O_{ij}](\boldsymbol{x}).$$

where the list of operators (up to higher derivatives and stochastic contributions) is

$$\begin{aligned}
&(1) \qquad \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}, && [\mathrm{Hirata\&Seljak:04}] \\
&(2) \qquad \mathrm{TF}\left[\Pi^{[2]}\right]_{ij},\ \ \mathrm{TF}\left[\left(\Pi^{[1]}\right)^2\right]_{ij},\ \ \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right], \\
&(3) \qquad \mathrm{TF}\left[\Pi^{[3]}\right]_{ij},\ \ \mathrm{TF}\left[\Pi^{[1]}\Pi^{[2]}\right]_{ij},\ \ \mathrm{TF}\left[\Pi^{[2]}\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right], \\
&\qquad\quad \mathrm{TF}\left[\left(\Pi^{[1]}\right)^3\right]_{ij},\ \ \mathrm{TF}\left[\left(\Pi^{[1]}\right)^2\right]_{ij}\mathrm{tr}\left[\Pi^{[1]}\right],\ \ \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}\left(\mathrm{tr}\left[\Pi^{[1]}\right]\right)^2 \ldots
\end{aligned}$$

Derivative operators relevant for leading power spectrum corrections

$$R_*^2 \nabla^2 \mathrm{TF}\left[\Pi^{[1]}\right]_{ij}.$$

Density weighting of IA?

Galaxy number weighting of the shape fluctuation field

$$S_{ij}(\boldsymbol{x}) = (1+\delta_{\rm n}(\boldsymbol{x}))\left(\tilde{g}_{ij}(\boldsymbol{x}) + \frac{1}{3}\tilde{\delta}_{\rm s}(\boldsymbol{x})\delta^{\rm K}_{ij}\right)$$

The connection to the earlier definition:

$$g_{ij}(\boldsymbol{x}) = (1+\delta_{\rm n}(\boldsymbol{x}))\,\tilde{g}_{ij}(\boldsymbol{x}), \qquad \delta_{\rm s}(\boldsymbol{x}) = (1+\delta_{\rm n}(\boldsymbol{x}))\,\tilde{\delta}_{\rm s}(\boldsymbol{x}).$$

linear order: $b_1^{\rm g} = \tilde{b}_1^{\rm g}$ and $b_1^{\rm s} = \tilde{b}_1^{\rm s}$
second order: $b_1^{\rm n}\tilde{b}_1^{\rm g}$ and $b_1^{\rm n}\tilde{b}_1^{\rm s}$.

However, there is an indep. op. $\mathrm{tr}[\Pi^{[1]}]\mathrm{TF}[\Pi^{[1]}]$ in g_{ij} and $\tilde{g}_{ij}$,

There is full degeneracy of these operators in the EFT.

Higher derivatives and stochasticity

Higher derivatives:

Taylor expansion $\implies$ suppression in R_* (extend of operators)
Leading operator:

$$R_*^2 \nabla^2 \mathrm{TF}[\Pi^{[1]}]_{ij} \, .$$

At higher order:

$$R_*^2 \nabla^2 \mathrm{TF}[(\Pi^{[1]})^2]_{ij} \, , \; R_*^2 \mathrm{TF}[\partial_k \Pi^{[1]} \partial^k \Pi^{[1]}]_{ij} \, ,$$

and many others... rapidly increase at higher order.

Stochasticity:

Fields ϵ_{ij}, ϵ_O are uncorrelated with the O_{ij}.

$$\langle \epsilon_O(\boldsymbol{k}) \epsilon_{O'}(\boldsymbol{k}') \rangle' , \quad \langle \epsilon_{ij}(\boldsymbol{k}) \epsilon_{kl}(\boldsymbol{k}') \rangle' = \left(\delta^{\mathrm{K}}_{ik} \delta^{\mathrm{K}}_{jl} + \delta^{\mathrm{K}}_{il} \delta^{\mathrm{K}}_{jk} - \frac{2}{3} \delta^{\mathrm{K}}_{ij} \delta^{\mathrm{K}}_{kl} \right) P^{\mathrm{g}}_{\epsilon}$$

Beyond leading order:

$$\begin{array}{ll} 1^{\mathrm{st}} & \epsilon_{ij} \\ 2^{\mathrm{nd}} & \epsilon^{\delta}_{ij} \mathrm{tr}[\Pi^{[1]}], \quad \epsilon_{\Pi^{[1]}} \mathrm{TF}[\Pi^{[1]}]_{ij} \, , \end{array}$$

One-loop results

Perturbative form of the shear tensor field

$$S_{ij}(\boldsymbol{k}) = \sum_{n=1}^{\infty} (2\pi)^3 \delta^D_{\boldsymbol{k}-\boldsymbol{q}_{1n}} \mathcal{K}^{(n)}_{ij,\mathrm{bias}}(\boldsymbol{q}_1 \ldots, \boldsymbol{q}_n) \delta_L(\boldsymbol{q}_1) \ldots \delta_L(\boldsymbol{q}_n),$$

where $\mathcal{K}^{(n)}_{\mathrm{bias}}$ are bias kernels (up to third order for one-loop).
PT results up to one-loop power spectrum

$$P^{\mathrm{one-loop}}_{ijlm} = P^{ab,\mathrm{lin}}_{ijlm} + P^{(22)}_{ijlm} + P^{(13)}_{ijlm} + P^{(31)}_{ijlm},$$

Linear, and loop (22), (13) contributions

$$P^{\mathrm{lin}}_{ijlm}(\boldsymbol{k}) = \frac{k_i k_j k_l k_m}{k^4} c^2_{\Pi^{[1]}} P_{\mathrm{lin}}(k),$$
$$P^{(22)}_{ijlm}(\boldsymbol{k}) = 2\ \mathcal{K}^{(2)}_{ij}(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) \mathcal{K}^{(2)}_{lm}(\boldsymbol{q}, \boldsymbol{k}-\boldsymbol{q}) P_{\mathrm{lin}}(q) P_{\mathrm{lin}}(|\boldsymbol{k}-\boldsymbol{q}|),$$
$$P^{(13)}_{ijlm}(\boldsymbol{k}) = 3 c_{\Pi^{[1]}} \frac{k_i k_j}{k^2} P_{\mathrm{lin}}(k)\ \mathcal{K}^{(3)}_{lm,b}(\boldsymbol{k}, \boldsymbol{q}, -\boldsymbol{q}) P_{\mathrm{lin}}(q).$$

Similar, but more cumbersome, for bispectrum...

One-loop results

Bias parameters:

$$
\begin{aligned}
a, \in \{\mathrm{n},\, \mathrm{s}\} : \quad & \overbrace{\left\{b_1^a\right\}}^{P_{11}} \bigcup \overbrace{\left\{b_{2,1}^a, b_{2,2}^a\right\}}^{P_{22}} \bigcup \overbrace{\left\{b_{3,1}^a\right\}}^{P_{13}} \bigcup \left\{b_{R_*}^a\right\} \bigcup \left\{\text{stoch.}\right\}, \\
\mathrm{g} : \quad & \underbrace{\left\{b_1^{\mathrm{g}}\right\}}_{P_{11}} \bigcup \underbrace{\left\{b_{2,1}^{\mathrm{g}}, b_{2,2}^{\mathrm{g}}, b_{2,3}^{\mathrm{g}}\right\}}_{P_{22}} \bigcup \underbrace{\left\{b_{3,1}^{\mathrm{g}}, b_{3,2}^{\mathrm{g}}\right\}}_{P_{13}} \bigcup \left\{b_{R_*}^{\mathrm{g}}\right\} \bigcup \left\{\text{stoch.}\right\}.
\end{aligned}
$$

Shot noise:

$$
P_{02}^{(0)}(k) = 0, \quad P_{22}^{(0)}(k) = P_{22}^{(1)}(k) = P_{22}^{(2)}(k) = 2P_\epsilon
$$

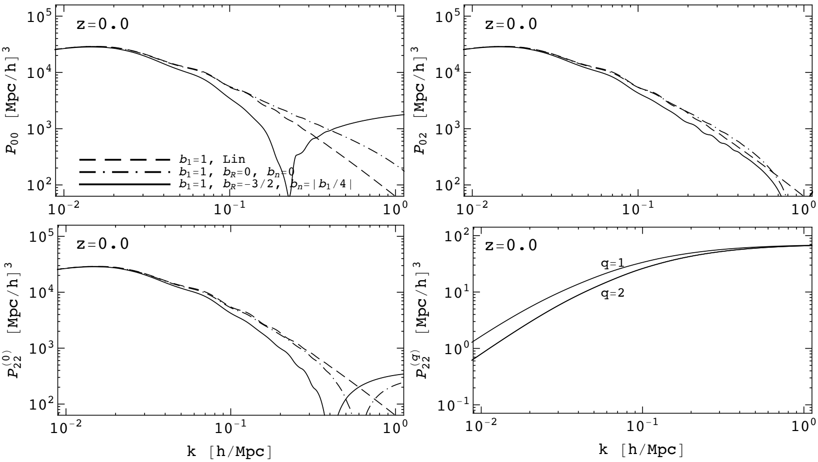
which gives

$$
C_{nE}(\ell) = 0, \quad C_{EE}(\ell) = C_{BB}(\ell) = \mathcal{W} P_\epsilon
$$

i.e., $C_{EE}(\ell) - C_{BB}(\ell)$ is shot noise free.

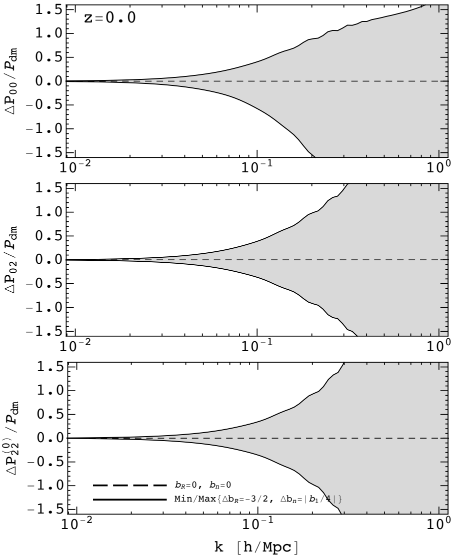
3D correlators

$P_{00}^{(0)}$, $P_{02}^{(0)}$, $P_{22}^{(0)}$, $P_{22}^{(1)}$ and $P_{22}^{(2)}$.



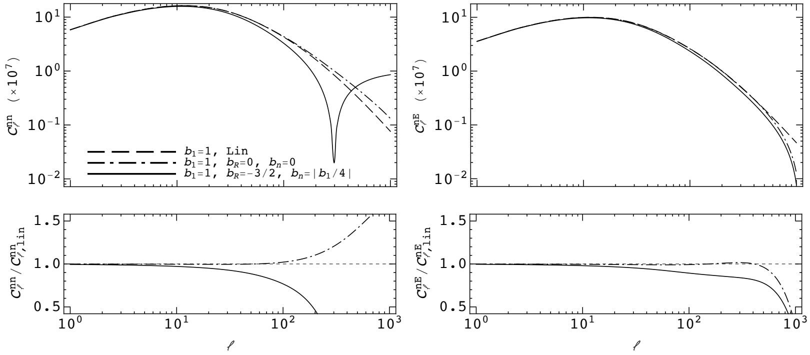
3D correlators

$P_{00}^{(0)}$, $P_{02}^{(0)}$, $P_{22}^{(0)}$



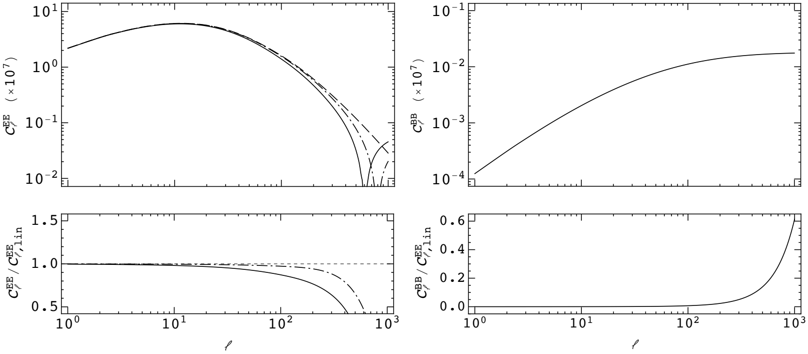
Projected correlators

$C_\ell^{\rm nn}$ and $C_\ell^{\rm nE}$



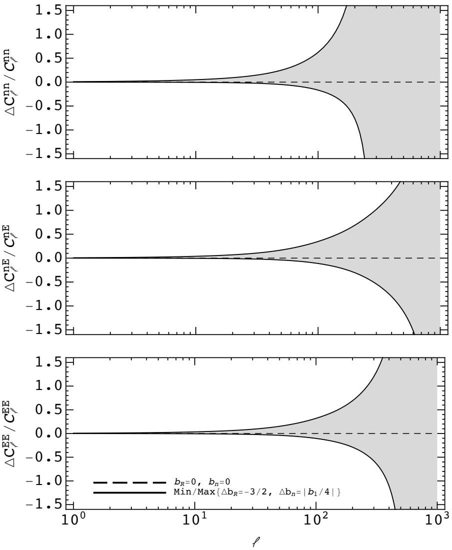
Projected correlators

C_ℓ^{EE} and C_ℓ^{BB}

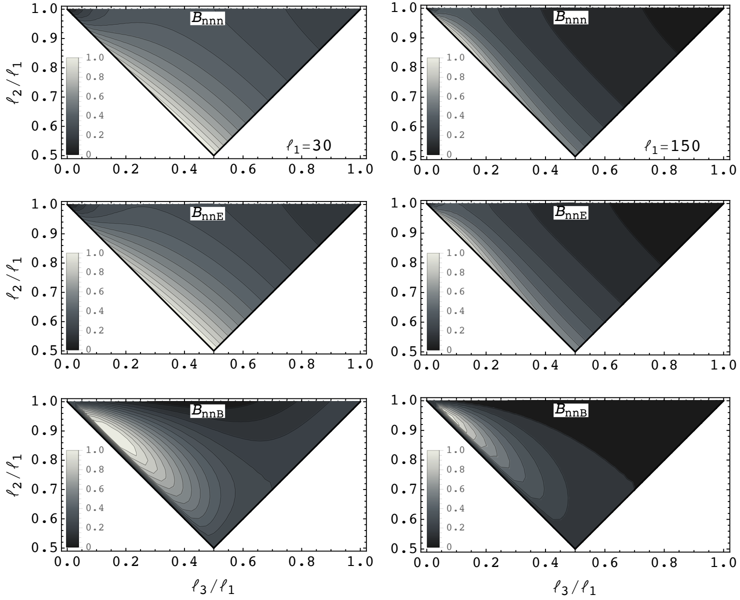


Projected correlators

$C_\ell^{\rm nn}$, $C_\ell^{\rm nE}$ and $C_\ell^{\rm EE}$

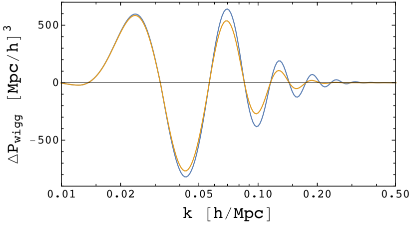


Bispectrum



IR resummation

Why do we usually care? Because of the BAO!



Long displacements can be resummed - without affecting the UV

At leading order:

$$\left[P^{\rm IR}\right]^{(q)}_{\ell\ell'} = \left[P^{\rm nw}_L\right]^{(q)}_{\ell\ell'} + e^{-\frac{1}{2}\Sigma^2 k^2}\left[P^{\rm w}_L\right]^{(q)}_{\ell\ell'}, \quad P^{\rm w}_L = P_L - P^{\rm nw}_L$$

with long-displacement dispersion $\Sigma^2 = \int_0^\Lambda \frac{dk}{6\pi}\left[1 - j_0(kR_*)\right]P_L(k)$,

Straightforward prescription for higher loops!

[Simonović++;15,ZV++;16, Ivanov++;16]

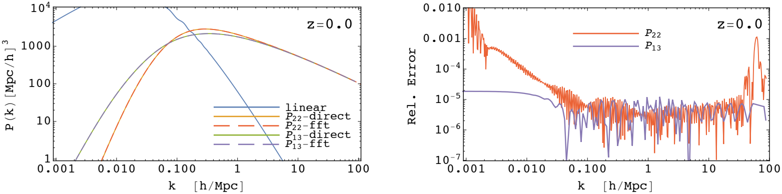
Efficient Evolution of Loops

Problem: we get cosmo. parameters - MCMC runs - slow!

$$P_{1-\text{loop}} = P_{\text{lin}} + P_{22} + 2P_{13} + P_{\text{c.t.}}$$

$$P_{22} \sim \int_q f(q)g(k-q)P_q^{\text{lin}}P_{k-q}^{\text{lin}} = \int_0^\infty r^2 j_0(rk)\left[\xi^{\text{lin}}(r)\right]^2$$

Solution: Mellin transform used to reduce the problem to Hankel/Bessel!



Very fast to evaluate - useful is FFTLog [Hamilton:00]

Works for EPT & LPT [Schmittfull&ZV:16x2, McEwen++:16, Simonović++:17]

What we were talking about:

Summary:

- description of IA as biased tensor field on large scales:
  "symmetry + dynamics(eft)"
- use of spherical tensors to disentangle the symmetry structure: allows the full sky treatment
- EFT framework allows us to determine the scale dependence on large scales, while the small scale effects are condensed into the bias parameters
- One-loop power spectrum results and tree-level bispectrum results are available