Interaction and drag in non-equilibrium environments: from the study of granular materials

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collaboration with Takahiro Tanabe (Meiji Univ.)

Rheology of disordered particles-suspensions, glassy and granular particles-
Talk on June 21st, 2018

Outline

- Introduction: What do we know about drag in granular media?
- Previous study for pure 2D drag in granular media
- Previous studies on the interaction between intruders
- Simulations of two intruders in 2D granular media
  - Two intruders in a steady motion
  - Two intruders under an oscillation
- Phenomenological theory for the drag and the interactions for intruders
- Discussion & conclusions

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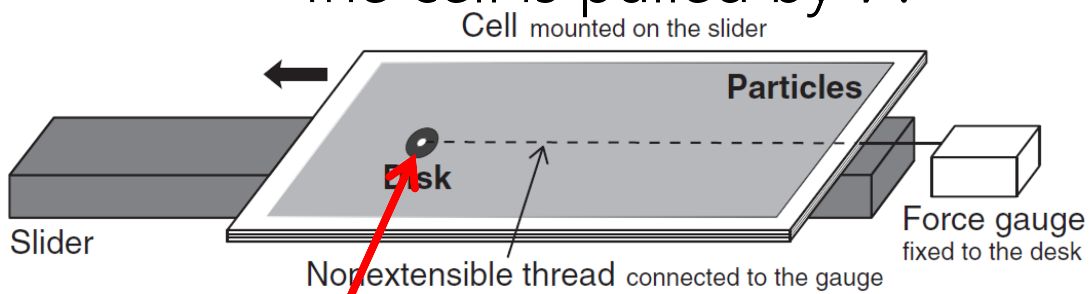
Drag law in a granular media

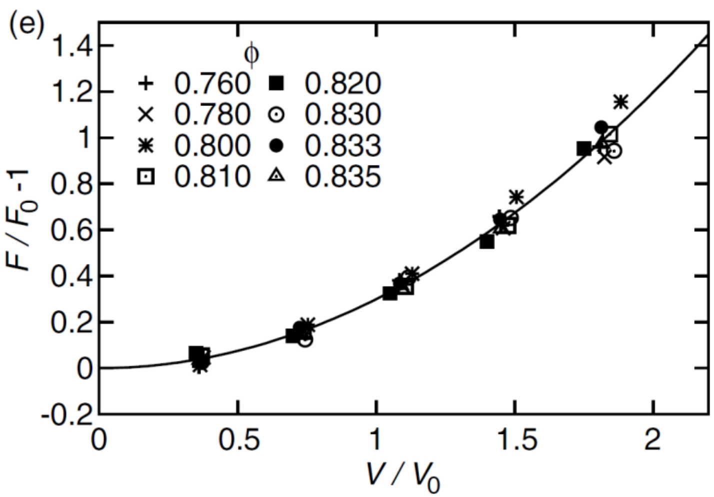
Y. Takehara & K. Okumura, PRL, **112**, 148001 (2014).

Experimental setup

The cell is pulled by V.



The disk is fixed by a wire.

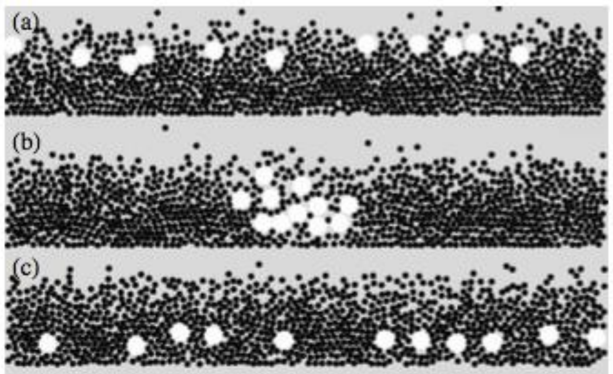


See S. Takada's talk yesterday.

Interactions of intruders in granular assemblies

Brazil Nuts Effect (ED)

D. A. Sanders, et al, PRL, **93**, 20, (2004)



$\rho_L < \rho_S$

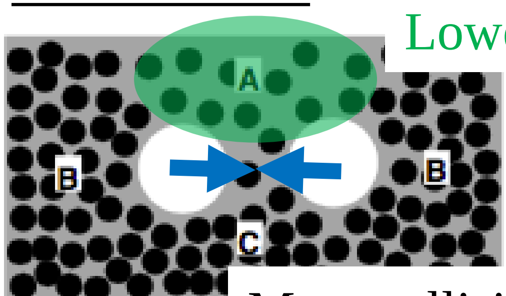
$\rho_L = \rho_S$

$\rho_L > \rho_S$

$$\boldsymbol{\rho_L/\rho_S \approx 1.0 \Longrightarrow}$$ **Attraction**

ρ_L, ρ_S: Large/Small granular density

Mechanism

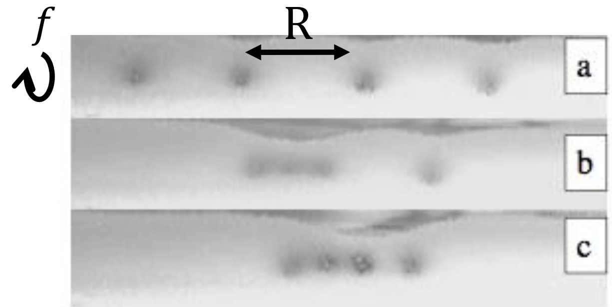


Lower density region

More collisions at region B than region A cause attraction (going to lower density)

Rolling Cylinder (Exp.)

I. Zuriguel, et al, PRL, **95**, 258002, (2005)



(a) $f > f_{th}$

$\Longrightarrow$ Separation

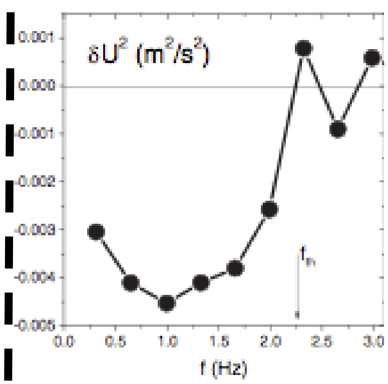
(b),(c) $f < f_{th}$

$\Longrightarrow$ Attraction

$$\boldsymbol{f < f_{th} \Longrightarrow}$$ **Attraction**

$$\boldsymbol{f > f_{th} \Longrightarrow}$$ **Separation**

Mechanism



$f < f_{th} \Longrightarrow \delta U^2 < 0$

(large fluctuation outside)

$\Longrightarrow \delta P = \rho\, \delta U^2/2 < 0$

Pressure difference makes attraction.

$$\delta U^2 = -[U_{rms}(out)^2 + V_{rms}(out)^2 - (U_{rms}(in)^2 + V_{rms}(in)^2)]$$

Purpose of this talk

- This talk is dedicated to the theory for the drag force and the interaction of intruders mainly in granular media
- For this purpose, we review previous works.
- Then, we introduce recent numerical simulations by Tanabe.
- Finally, I will explain some phenomenological theories to understand the results of simulations and experiments.

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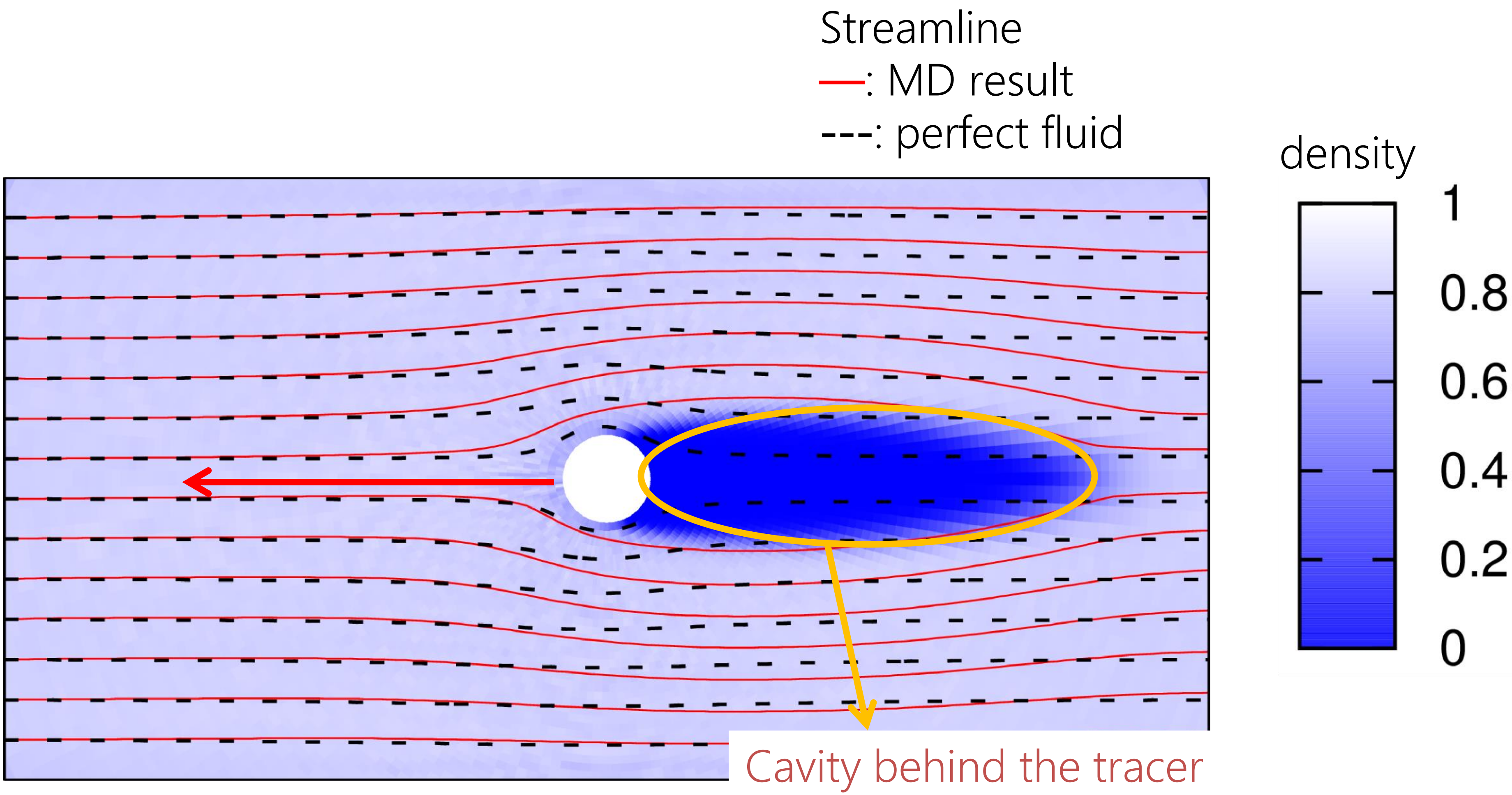
Density profile & Streamline

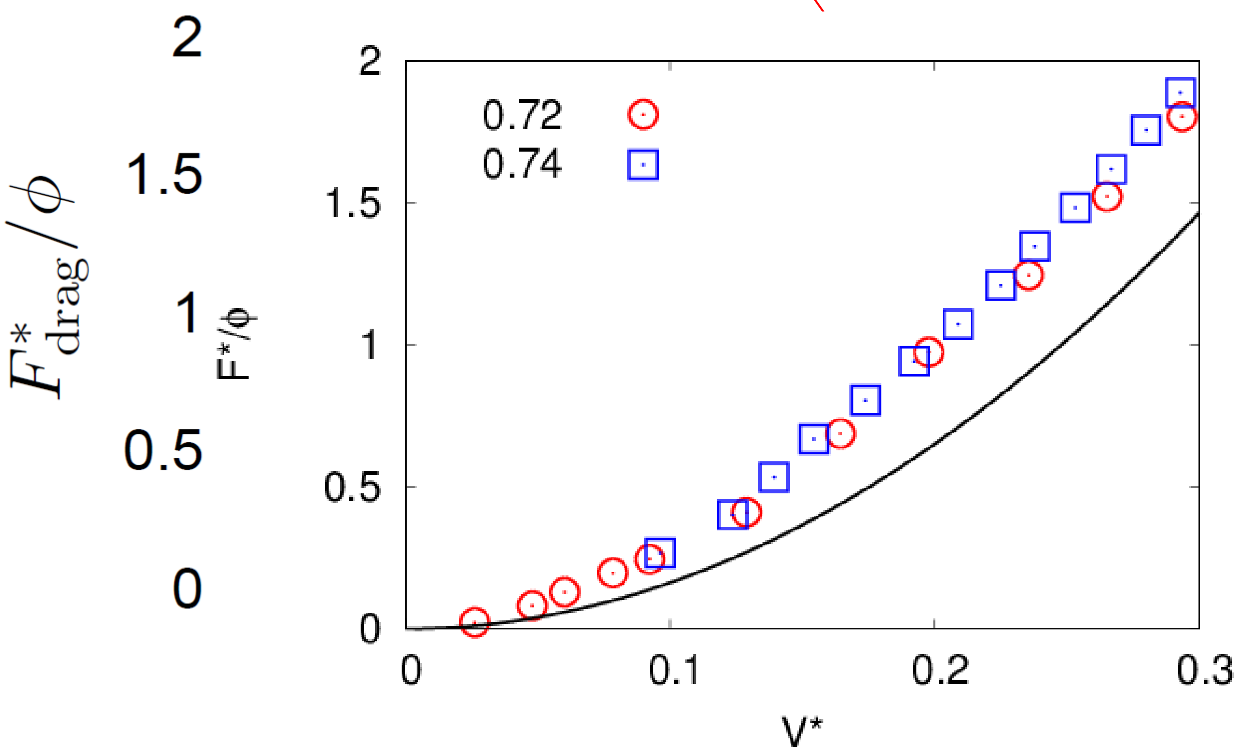


Cavity behind the tracer
(Dead water model can be used.)

The streamline obtained from MD
is well fitted by that of the perfect fluid.

Comparison with simulation result

$$F_{\mathrm{drag}} = \left(\frac{3+2e}{2} - \frac{2}{3}\sin^2\theta_0\right)\sin\theta_0 D\rho V^2$$



Simulation results are well fitted **without any fitting parameters** !
This is consistent with Chicago group for granular jet.

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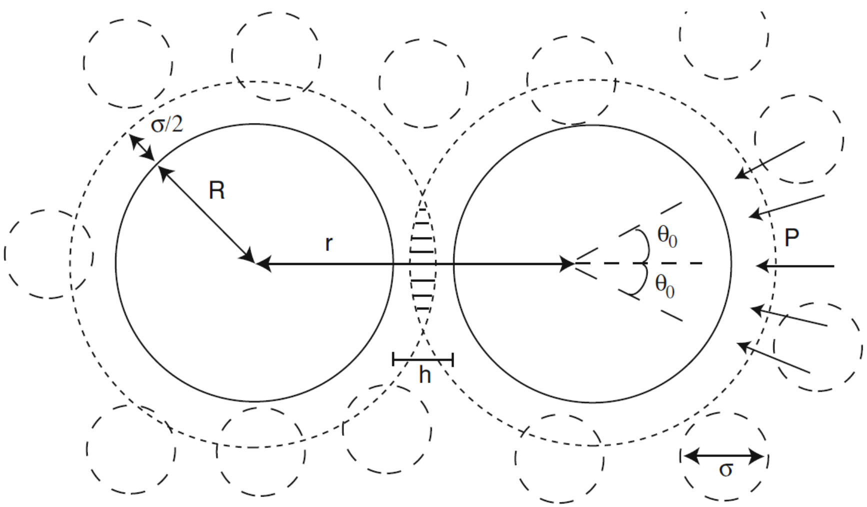
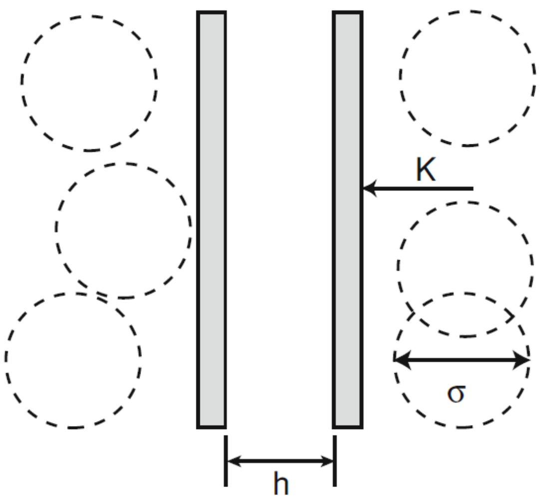
Depletion force & adiabatic piston

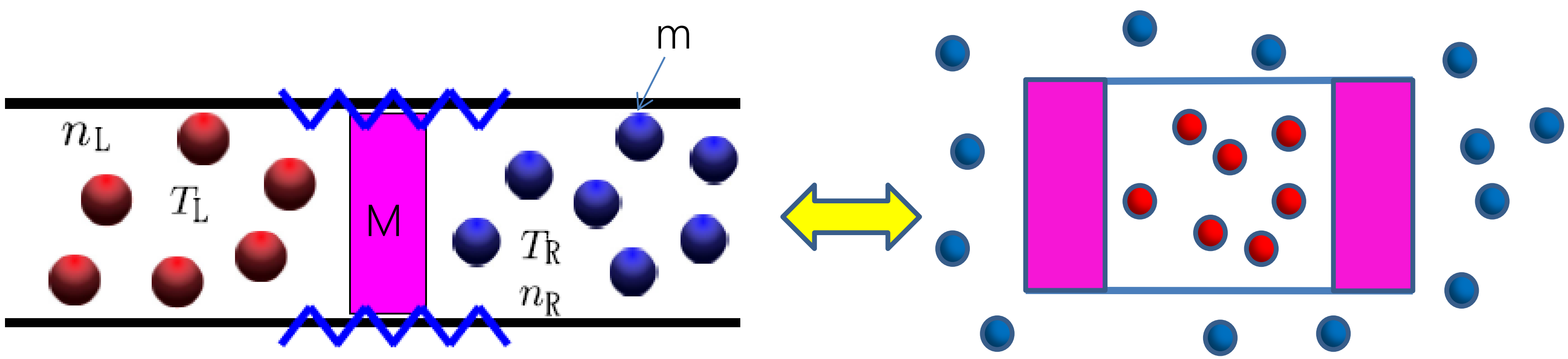
- * The problem is related to the depletion force (in a constriction region).
- * Note that the conventional depletion force is the result of the violation of the pressure balance=> acceleration.
- * However, we are interested in the adiabatic motion under the pressure balance.=> Casimir effect or adiabatic piston





The adiabatic piston

- * The adiabatic piston problem is equivalent to fluctuating 1d depletion force problem.
- * If the density is lower in the depletion region, the attractive interaction between pistons exist under the pressure balance as a result of non-Gaussian correrction.



$$V_{\mathrm{ad}} \equiv \epsilon \frac{\sqrt{\pi}}{4} v_{T_e}^2 \left(\frac{1}{v_{T_\mathrm{L}}} - \frac{1}{v_{T_\mathrm{R}}} \right) \qquad v_T \equiv \sqrt{2k_B T/M} \qquad \epsilon \equiv \sqrt{m/M}$$

Interaction of non-Gaussian systems

We begin with the generalized Fokker-Planck equation:

$$\partial_t P = \sum^N \{\gamma^{-1}\nabla_k(P\nabla_k U) + \mathcal{L}_k P\}$$

$$\mathcal{L}_k = \lambda \int \sum_{n=1}^{\infty} \frac{(-\boldsymbol{x}\cdot\nabla_k)^n}{n!} \mathcal{W}(\boldsymbol{x}) d\boldsymbol{x} = \lambda \int (e^{-\boldsymbol{x}\cdot\nabla_k} - 1)\mathcal{W}(\boldsymbol{x}) d\boldsymbol{x}.$$

$$\mathcal{L}_k = \lambda[\tilde{\mathcal{W}}(i\nabla_k) - 1]. \qquad \tilde{\mathcal{W}}(\boldsymbol{q}) := \int d\boldsymbol{x} e^{-i\boldsymbol{q}\cdot\boldsymbol{x}} \mathcal{W}(\boldsymbol{x})$$

If the hopping is nonlocal, we need higher order derivative in Fokker-Planck equation.

Effective interaction in steady state

$$0 = \gamma^{-1} P_{\rm ss} \nabla_k U + D b_{d\nu} \nabla_k (1 - a^2 c_{d\nu} \nabla_k^2) P_{\rm ss}$$

⇕

$$P_{\rm ss} = \frac{e^{-\beta U}}{Z} \left[1 - a^2 \beta W + O(a^4)\right]$$

$$\beta = 1/(\gamma D b_{d\nu})$$

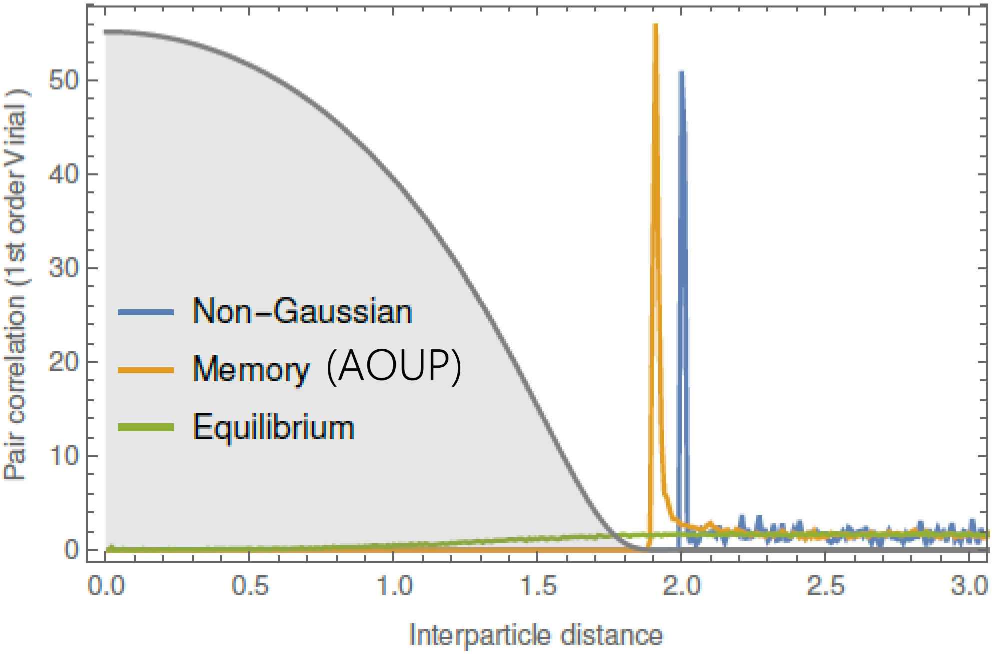
$$W = c_{d\nu} \left\{ \beta \sum_{k=1}^{N} \nabla_k^2 U - \frac{3\beta^2}{2} \sum_{\{k,\ell\}=1}^{N} \nabla_k U \cdot \nabla_\ell U + \beta^3 \int (\nabla_k U)^3 d\boldsymbol{r} \right\}$$

Non-Gaussian noise and AOUP

- Both models have effective attractive interactions.
- Nevertheless, the AOUP exhibits strong phase separation but the non-Gaussian model does not.

AOUP=Active Ornstein Uhrenbeck process

Fodor et al., PRL **117**, 038103 (2016).



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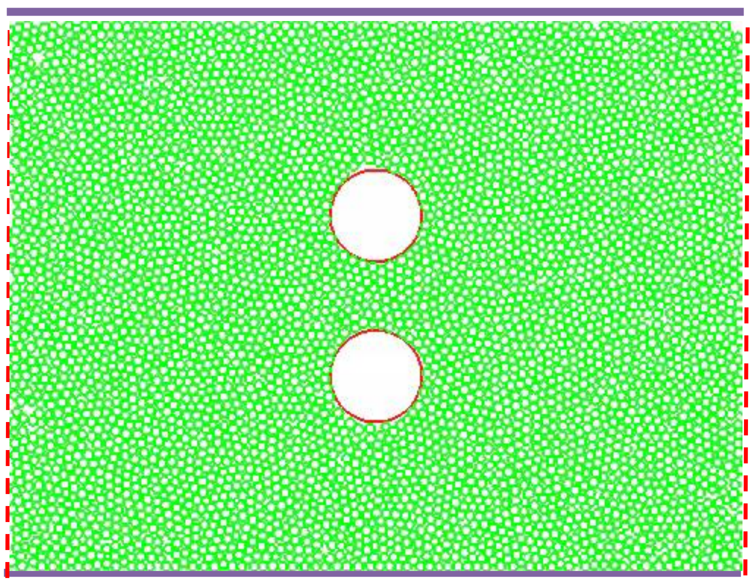
Simulation for two intruders

$$m_i \ddot{r}_i = \sum_j F_c^{i,j} + F_{ex}$$

i : Disk index
$\boldsymbol{r}_i$: Position m_i : Mass

$$v_{ex}^x = V^* \left(D\sqrt{k_n/M_S}\right)$$

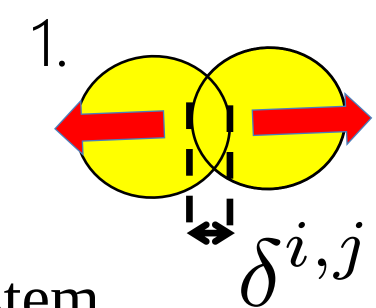


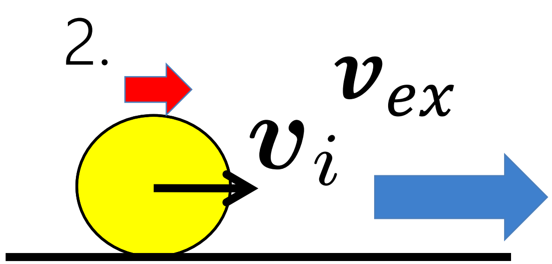
1. Repulsive force (only normal force)

$$F_c^{i,j} = k_n \delta^{i,j} n^{i,j} - \gamma_n \boldsymbol{v}_n^{i,j}$$

2. Driving force (follow tray oscillation)

$$F_{ex} = -\mu(\boldsymbol{v}_i - \boldsymbol{v}_{ex})$$





- Fixed flat wall $L_y = 60D_S$
- Periodic boundary $L_x = 80D_S$
- Two intruders are pinned in system

Explanation for simulations

$v_{ex}^{x} = V^{*}\ (D\sqrt{k_n/M_S})$

- Diameter of disks (density: const.)

$D_S = D(1 \pm 0.1r)$: Small disks $\quad D_L = 10D$: Intruder

(poly-dispersity: Uniform distr.)

- Intruders are fixed on system with distance L_0 for y-direction

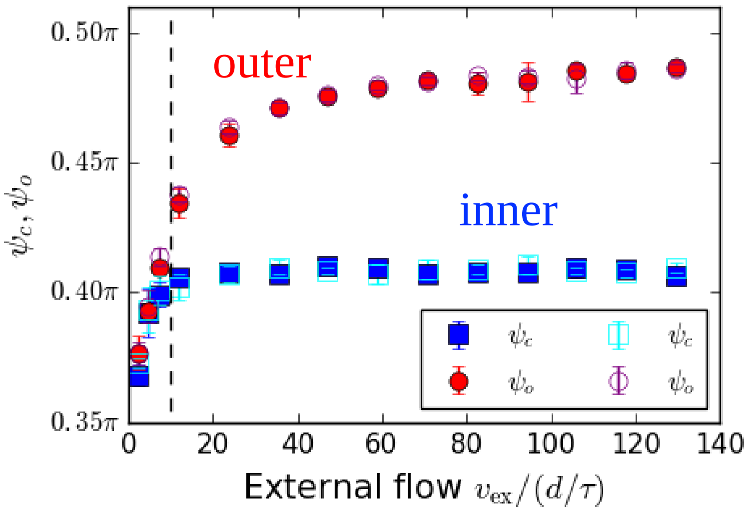
i : Disk index	
$\boldsymbol{r}_i$: Position	m_i : Mass

- Control parameter: non-dimensional speed V^*

$$m_i \ddot{r}_i = \sum_j F_c^{i,j} + F_{ex}$$

- Fixed flat wall $L_y = 60D_S$
- Periodic boundary $L_x = 80D_S$
- Two intruders are pinned or mobile in system

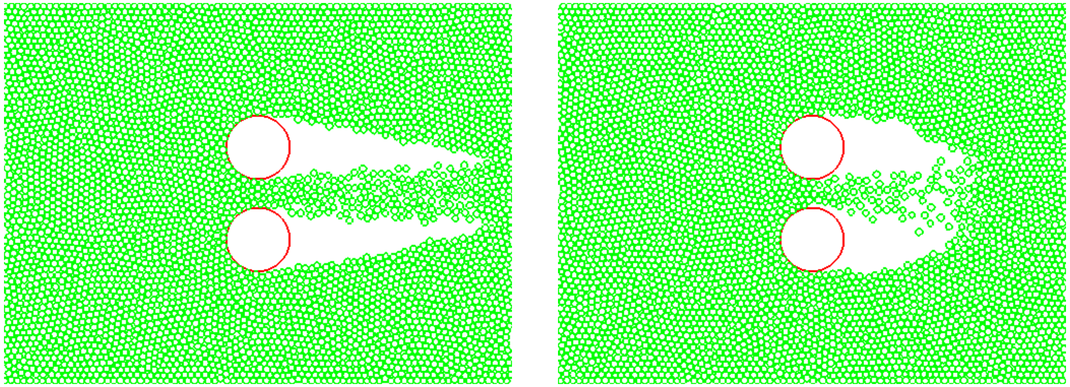
Separation angle: Low V differs from high V.



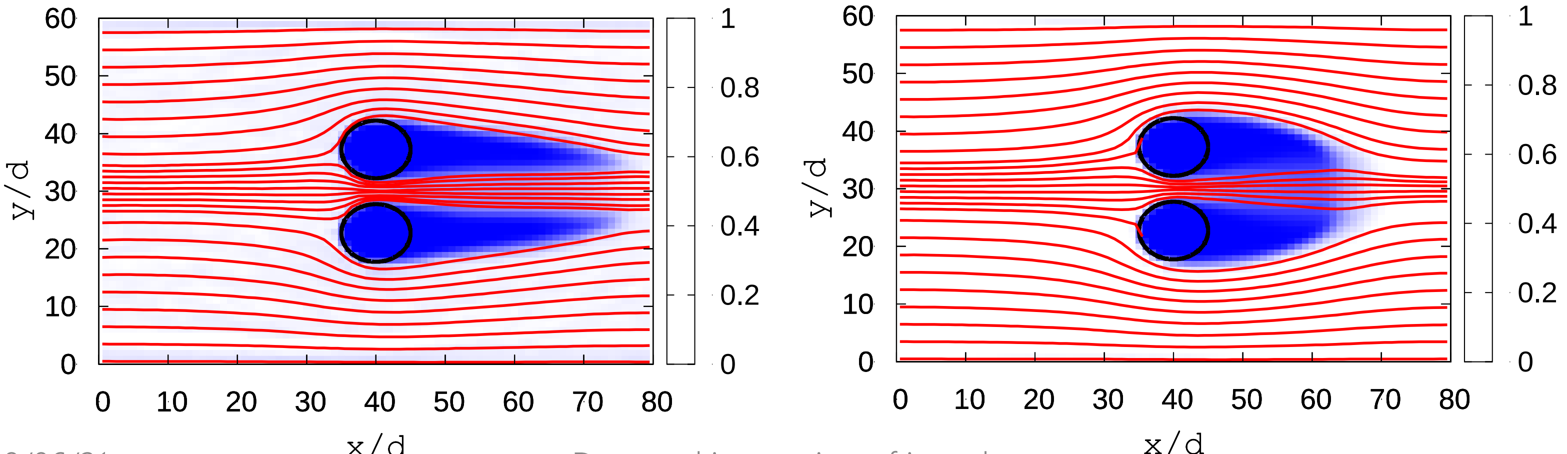
Flow time>relaxation time

$D/v_{ex} > \tau \Rightarrow v_{ex}\tau/d < 10$: Flow is different.

$v_{ex}\tau/d$=4.71 $v_{ex}\tau/d$=23.57

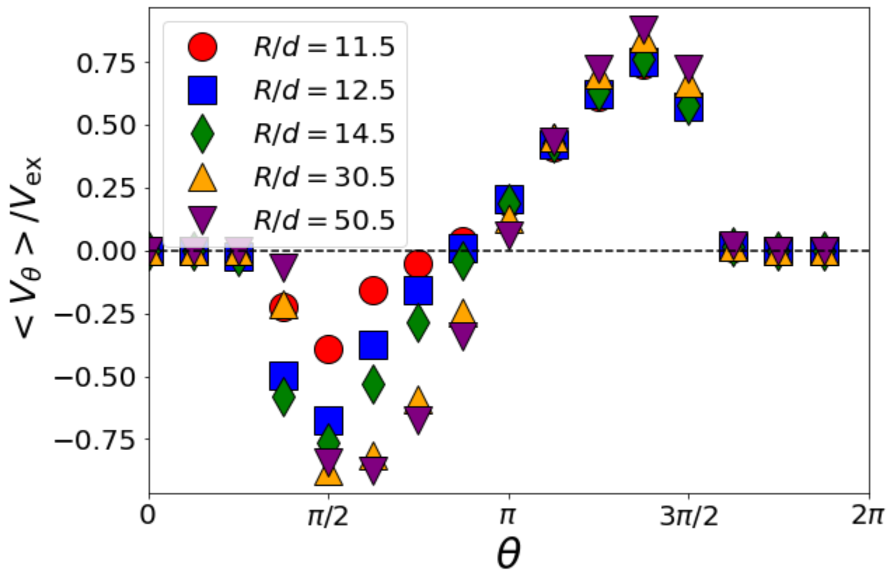


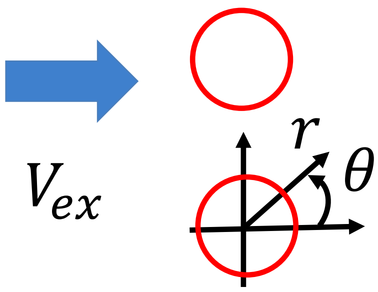
$v_{ex}\tau/d$=4.71 No coalescence of wakes $v_{ex}\tau/d$=23.57



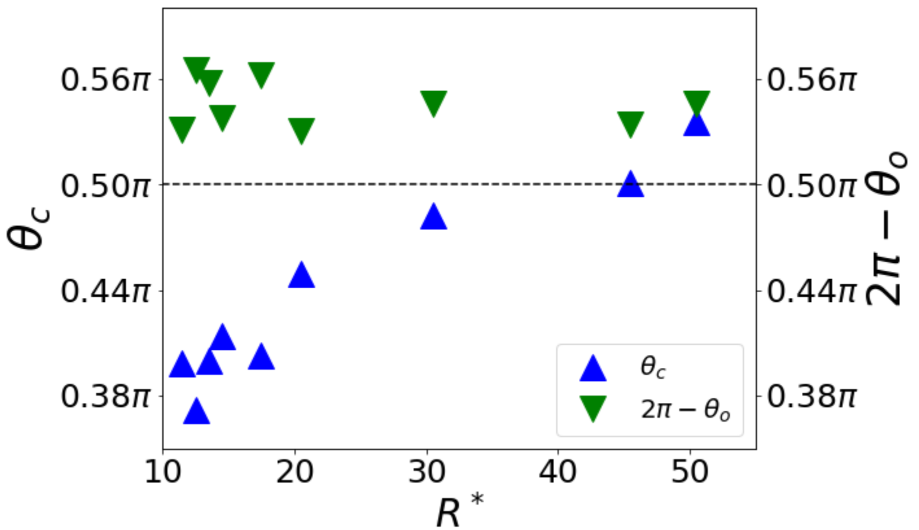
Granular flow : Velocity & Separation angle

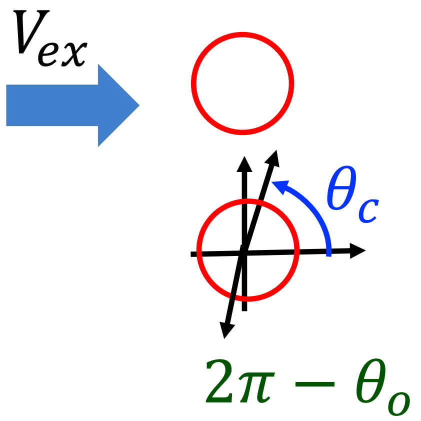
Velocity field



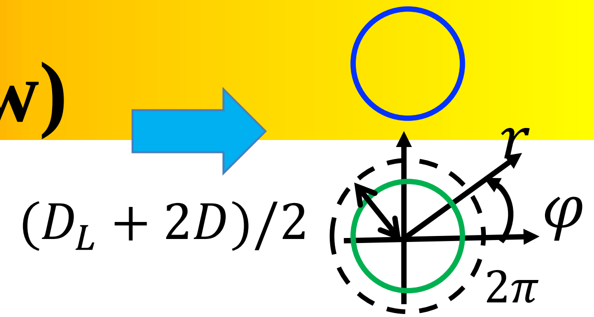


Separation angle



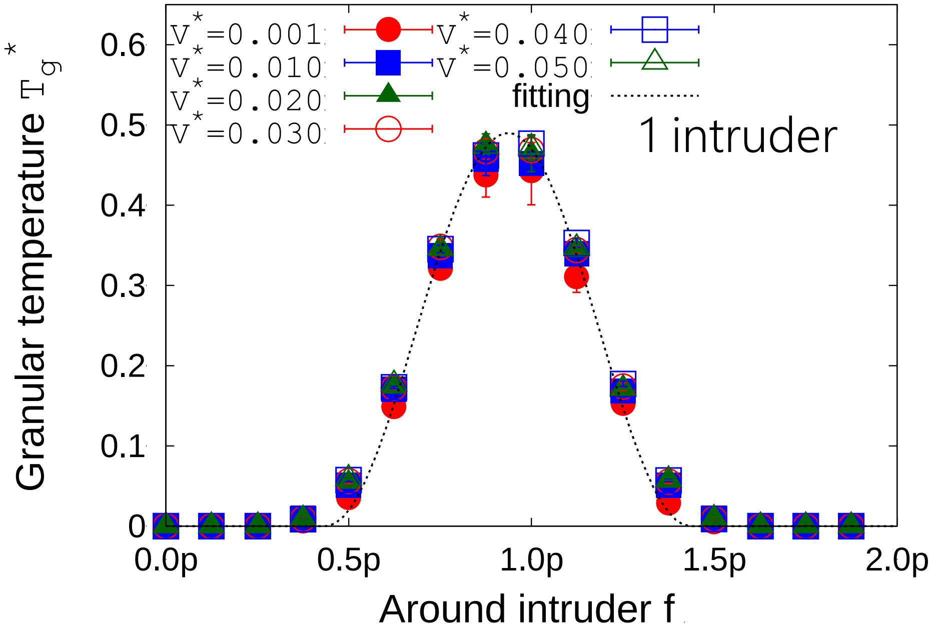


Granular temperature (steady flow)

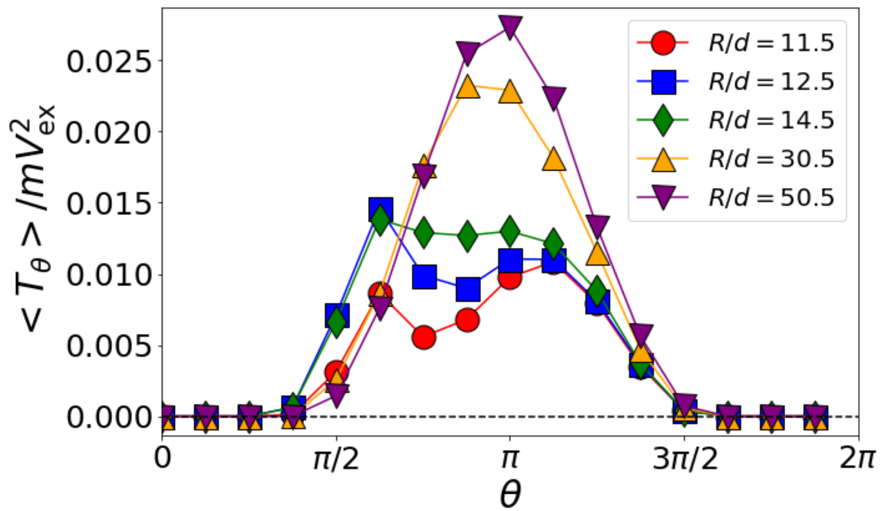


Fitting with $\cos^2\theta$

One intruder

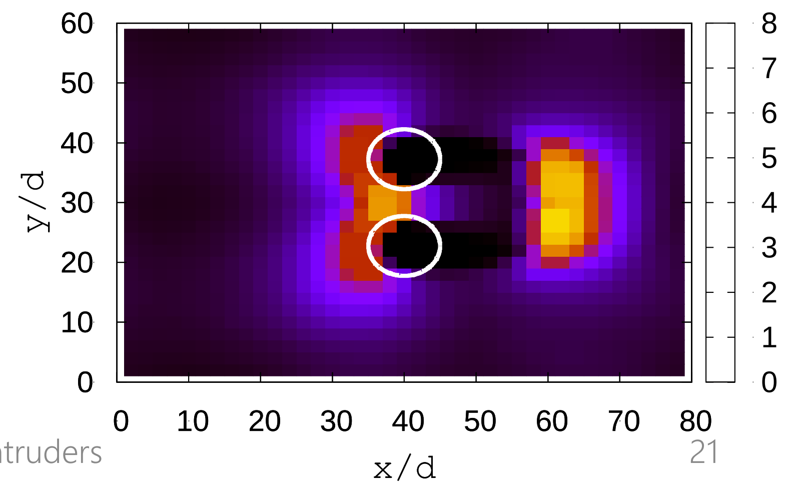


2 intruders



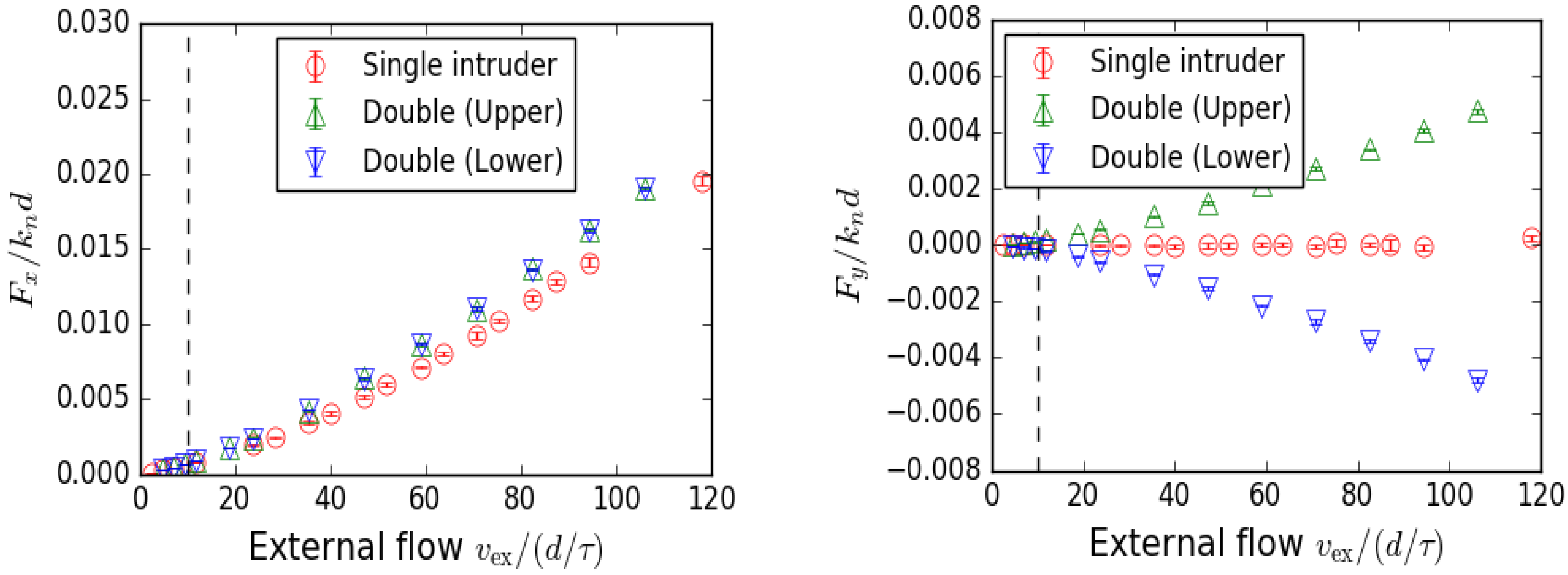
$$T_G(\varphi) = \begin{cases} 0, & |\varphi + \varDelta| < \varphi_S \\ \gamma(\cos((\varphi+\varDelta)-\pi))^2, & otherwise \end{cases}$$

$(\gamma = 0.47, \varDelta = 0.203, \varphi_S = 1.5691)$
(φ_S: separation angle)

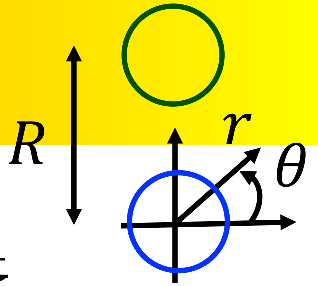


Repulsive force between two intruders

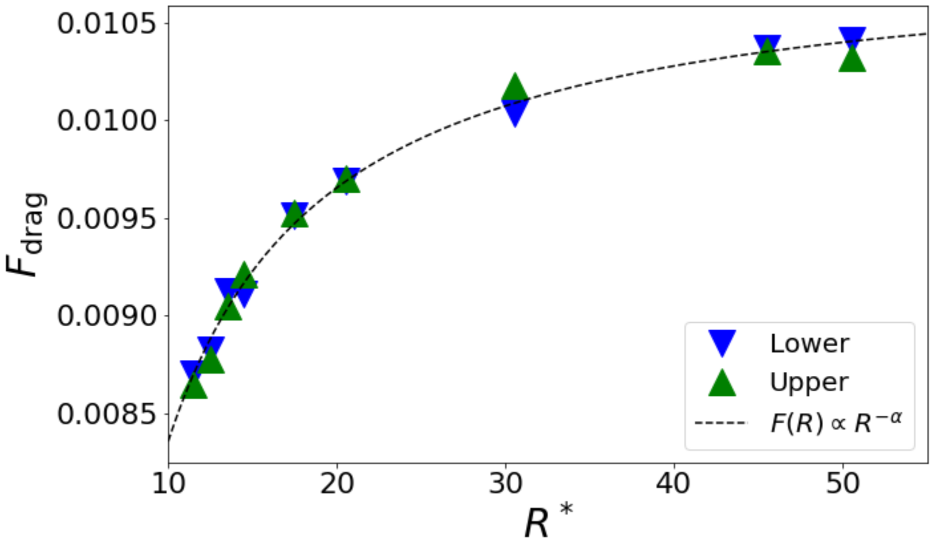
- The effective repulsive interaction exists between two.
- => This is contrast to perfect fluidity (Bernoulli).



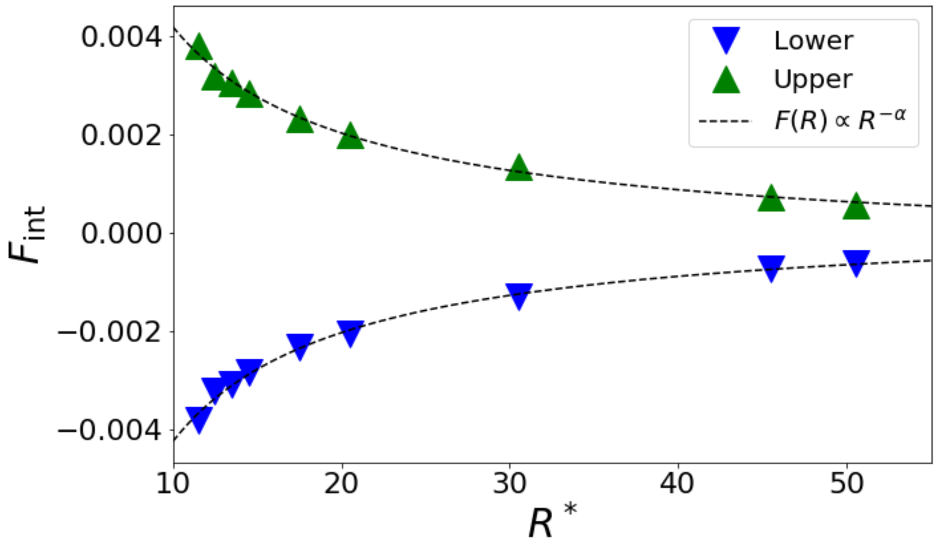
Drag & interaction force for R



Drag force F_{drag}



Interaction force F_{int}



Both $F_{\mathrm{drag}}(R)$ & $F_{\mathrm{int}}(R)$ can be fit as $F(R) = a + bR^{-\alpha}$ $(\alpha \approx 1)$.

F_{drag} is suppressed when R is small because the flow speed decreases.

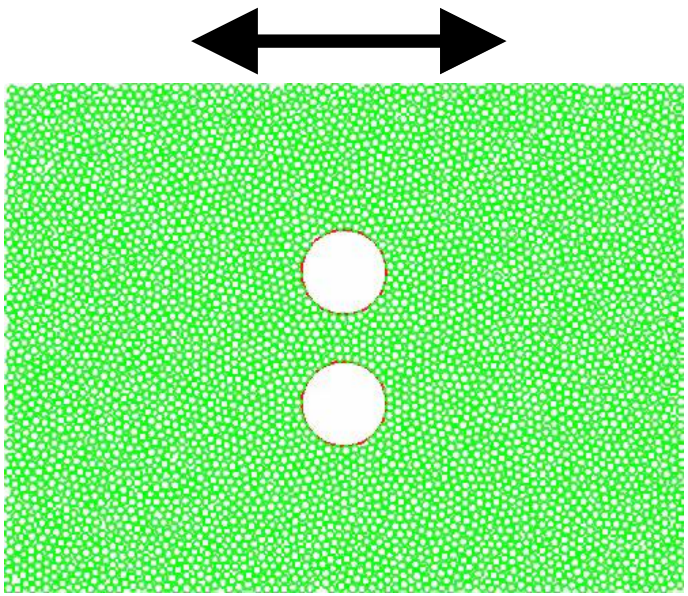
F_{int} approaches 0 as increasing R.

Oscillatory flow

$$m_i \ddot{r}_i = \sum_j F_c^{i,j} + F_{ex}$$

i : Disk index
$\boldsymbol{r}_i$: Position m_i : Mass

$$v_{ex}^x(t) = 2\pi A\nu \ \sin(2\pi\nu t)$$



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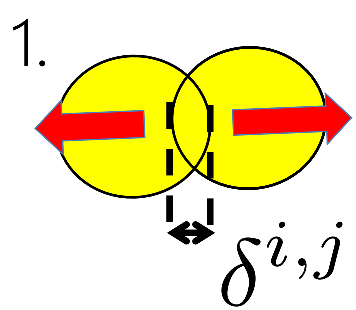
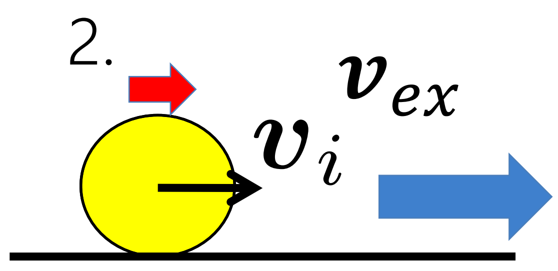
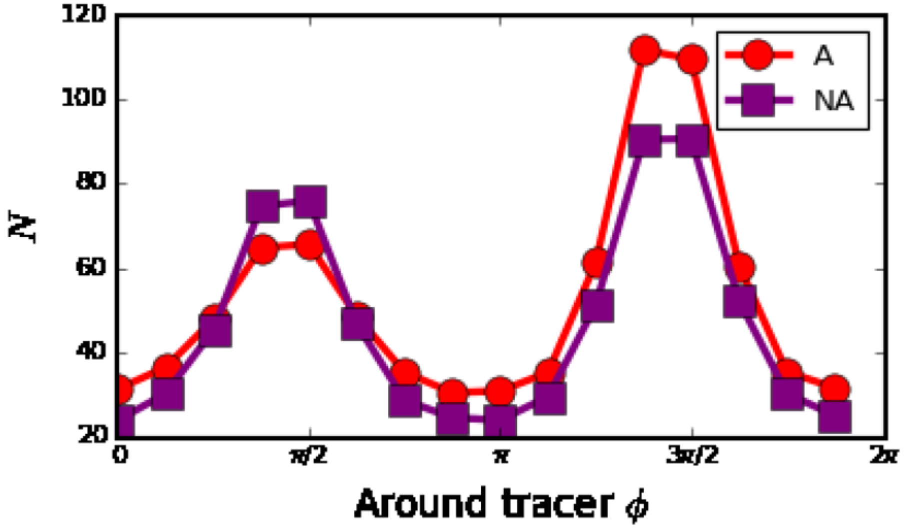
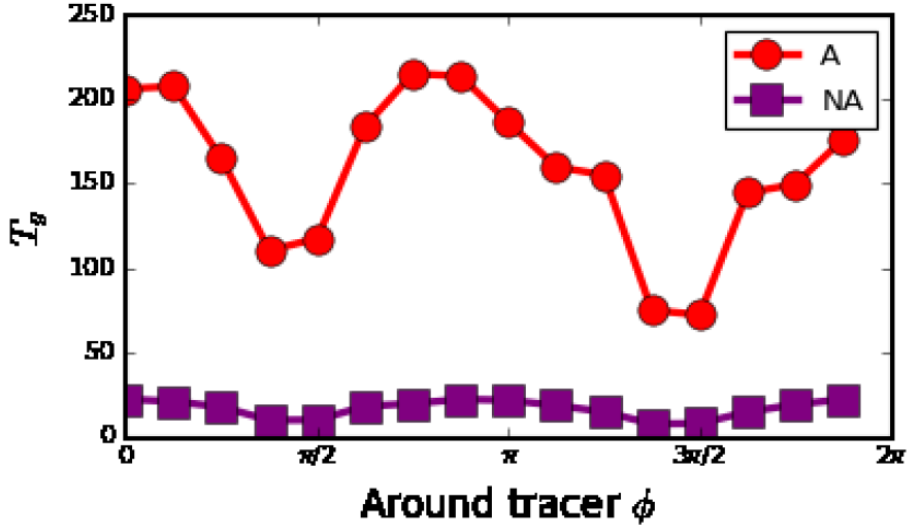




Origin of attractive force

- There are wakes in front and back of intruders.
- Grains in the channel are squeezed out.
- Density in the channel is much lower than that outside.
- Pressure difference exists.





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Outline of the theory

- We assume that Enskog theory for binary systems can be used.
- Then, the environment is assumed to be Maxwellian.
- Then, we adopt Kramers-Moyal expansion.
- We can determine the drag force if we know the temperature.
- The temperature is determined by a phenomenological simple model.
- The interaction is also guessed from a simple crude theory.

Enskog theory for mixtures

The distribution function for $i-$species $f_i(\boldsymbol{V},t)$ for the velocity

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\boldsymbol{\nabla}\right)f_i(\boldsymbol{V},t)=\sum_{j=1}^{2}J_{ij}^E(\boldsymbol{V}|f_i,f_j),$$

$$J_{ij}^E(\boldsymbol{V}_1|f_i,f_j)=d_{ij}g_{ij}\int d\boldsymbol{V}_2\int d\hat{\boldsymbol{\sigma}}\Theta(\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{V}_{12})(\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{V}_{12})$$
$$\times\left[\frac{1}{e_{ij}^2}f_i(\boldsymbol{r},\boldsymbol{V}_i'';t)f_j(\boldsymbol{r}+\boldsymbol{\sigma},\boldsymbol{V}_2'';t)-f_i(\boldsymbol{r},\boldsymbol{V}_i;t)f_j(\boldsymbol{r}-\boldsymbol{\sigma},\boldsymbol{V}_2;t)\right]$$

with the radial distribution at contact g_{ij} between i and j,

the precollisional velocities $\boldsymbol{V}_j''$, $\boldsymbol{V}_{12}=\boldsymbol{V}_1-\boldsymbol{V}_2$

restitution coefficient e_{ij} between i and j

Model

intruder (mass M and the diameter D)

in another species (consisting of N identical disks of the mass m

a small parameter $\epsilon \equiv \sqrt{m/M}$

the geometric factor

$$\chi = \frac{1}{2} n(d+D) g_{dD}(\varphi)$$

$$g_{dD}(\varphi) = \frac{1}{1-\varphi} + \frac{9\varphi}{16(1-\varphi)^2} z \qquad z = 2dD\langle\sigma\rangle/((d+D)\langle\sigma^2\rangle)$$

Boltzmann-Enskog equation

- Boltzmann-Enskog equations for the intruder and grains are given by

$$\frac{\partial P(\boldsymbol{V},t)}{\partial t} = \int d\boldsymbol{V}' \left[W_{\rm tr}(\boldsymbol{V}|\boldsymbol{V}')P(\boldsymbol{V}',t) - W_{\rm tr}(\boldsymbol{V}'|\boldsymbol{V})P(\boldsymbol{V},t)\right] + \mathscr{B}_{\rm tr}P(\boldsymbol{V},t)$$

$$\frac{\partial p(\boldsymbol{v},t)}{\partial t} = \int d\boldsymbol{v}' \left[W_{\rm g}(\boldsymbol{v}|\boldsymbol{v}')p(\boldsymbol{V}',t) - W_{\rm g}(\boldsymbol{v}'|\boldsymbol{v})p(\boldsymbol{v},t)\right] + \mathscr{B}_{\rm g}p(\boldsymbol{v},t) + \chi J_E[\boldsymbol{v}|p,p],$$

Transition rate

- Transition rate is given by

$$W_{\rm tr}(\boldsymbol{V}|\boldsymbol{V}') = \chi \int d\boldsymbol{v}' \int d\hat{\boldsymbol{\sigma}} p(\boldsymbol{v}',t)\Theta(-(\boldsymbol{V}'-\boldsymbol{v}')\cdot\hat{\boldsymbol{\sigma}})(\boldsymbol{V}'-\boldsymbol{v}')\cdot\hat{\boldsymbol{\sigma}}\delta\left(\boldsymbol{V}-\boldsymbol{V}'+\frac{\epsilon^2}{1+\epsilon^2}(1+e)[(\boldsymbol{V}'-\boldsymbol{v}')\cdot\hat{\boldsymbol{\sigma}}]\hat{\boldsymbol{\sigma}}\right)$$

and

$$W_{\rm g}(\boldsymbol{v}|\boldsymbol{v}') = \frac{\chi}{N}\int d\boldsymbol{v}' \int d\hat{\boldsymbol{\sigma}} P(\boldsymbol{V}',t)\Theta(-(\boldsymbol{V}'-\boldsymbol{v}')\cdot\hat{\boldsymbol{\sigma}})(\boldsymbol{V}'-\boldsymbol{v}')\cdot\hat{\boldsymbol{\sigma}}\delta\left(\boldsymbol{v}-\boldsymbol{v}'+\frac{1}{1+\epsilon^2}(1+e)[(\boldsymbol{v}'-\boldsymbol{V}')\cdot\hat{\boldsymbol{\sigma}}]\hat{\boldsymbol{\sigma}}\right),$$

$$\boldsymbol{V}' = \boldsymbol{V} - \frac{(1+e)}{e}\frac{M}{M+m}\hat{\boldsymbol{\sigma}}\hat{\boldsymbol{\sigma}}\cdot(\boldsymbol{V}-\boldsymbol{v})] = \boldsymbol{V} + (1+e)\frac{1}{1+\epsilon^2}\hat{\boldsymbol{\sigma}}(\hat{\boldsymbol{\sigma}}\cdot(\boldsymbol{V}'-\boldsymbol{v}')).$$

$\hat{\boldsymbol{\sigma}}$ is the unit normal at contact and $\Theta(x)$ is Heaviside's step function

If we assume $p(\boldsymbol{v}) = (m/2\pi T)^{d/2}\exp[-mv^2/(2T)]$, one can rewrite

$$W_{\rm tr}(\boldsymbol{v}_1'|\boldsymbol{v}_1) = \frac{\chi|\Delta v|^{2-d}}{k(\epsilon,e)^2}\sqrt{\frac{m}{2\pi T}}\exp[-mv_{2\sigma}^2/2T].$$

If we are interested in the case of $d=2$,

$$W_{\rm tr}(\boldsymbol{V}'|\boldsymbol{V}) = \frac{\chi}{k(\epsilon,e)^2}\sqrt{\frac{m}{2\pi T}}\exp\left[-\frac{m[V_\sigma' - V_\sigma + k(\epsilon,e)V_\sigma]^2}{2Tk(\epsilon,e)^2}\right],$$

Kramers-Moyal expansion

$$\frac{\partial P(\boldsymbol{V},t)}{\partial t} = \mathcal{L}_{\text{gas}} P(\boldsymbol{V},t) + \mathscr{B}_{\text{tr}} P(\boldsymbol{V},t),$$

Introducing $W(\boldsymbol{V};\boldsymbol{v}) \equiv W_{\text{tr}}(\boldsymbol{V}'|\boldsymbol{V})$

$$\begin{aligned}
\mathcal{L}_{\text{gas}} P(\boldsymbol{V},t) &= -\int d\boldsymbol{v} W(\boldsymbol{V};\boldsymbol{v}) P(\boldsymbol{V},t) + \int d\boldsymbol{v} W(\boldsymbol{V}-\boldsymbol{v};\boldsymbol{v}) P(\boldsymbol{V}-\boldsymbol{v},t) \\
&= -\int d\boldsymbol{v} W(\boldsymbol{V};\boldsymbol{v}) d\boldsymbol{v} P(\boldsymbol{V},t) + \sum_{n=0}^{\infty} \frac{(-)^n}{n!} \left(\frac{\partial}{\partial \boldsymbol{V}}\right)^n \cdot \int d\boldsymbol{v} \boldsymbol{v}^n W(\boldsymbol{V};\boldsymbol{v}) P(\boldsymbol{V},t) \\
&= \sum_{n=1}^{\infty} \frac{(-)^n}{n!} \left(\frac{\partial}{\partial \boldsymbol{V}}\right)^n \cdot \int d\boldsymbol{v} \boldsymbol{v}^n W(\boldsymbol{V};\boldsymbol{v}) P(\boldsymbol{V},t),
\end{aligned}$$

where we have used the formal relation

$$f(\boldsymbol{V}-\boldsymbol{v}) = \sum_{n=0}^{\infty} \frac{(-\boldsymbol{v})^2}{n!} \cdot \left(\frac{\partial}{\partial \boldsymbol{V}}\right)^n f(\boldsymbol{V}) = \exp[-\boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{V}}] f(\boldsymbol{V})$$

Summary of Kramers-Moyal expansion

$$\mathcal{L}_{\rm gas} P(\boldsymbol{V},t) = \sum_{n=1}^{\infty} \frac{(-1)^n \partial^n}{\partial V_{\alpha_1} \cdots \partial V_{\alpha_n}} D^{(n)}_{\alpha_1 \cdots \alpha_n}(\boldsymbol{V}) P(\boldsymbol{V},t)$$

$$D^{(n)}_{\alpha_1 \cdots \alpha_n}(\boldsymbol{V}) = \frac{1}{n!} \int d\boldsymbol{V}' (V'_{\alpha_1} - V_{\alpha_1}) \cdots (V'_{\alpha_n} - V_{\alpha_n}) W_{\rm tr}(\boldsymbol{V}'|\boldsymbol{V}).$$

Thermal system

- Drag law

$$F_{\rm ex} = -MD_x^{(1)}(\boldsymbol{V}).$$

- If we assume that the thermal speed is larger than pulling speed, we obtain

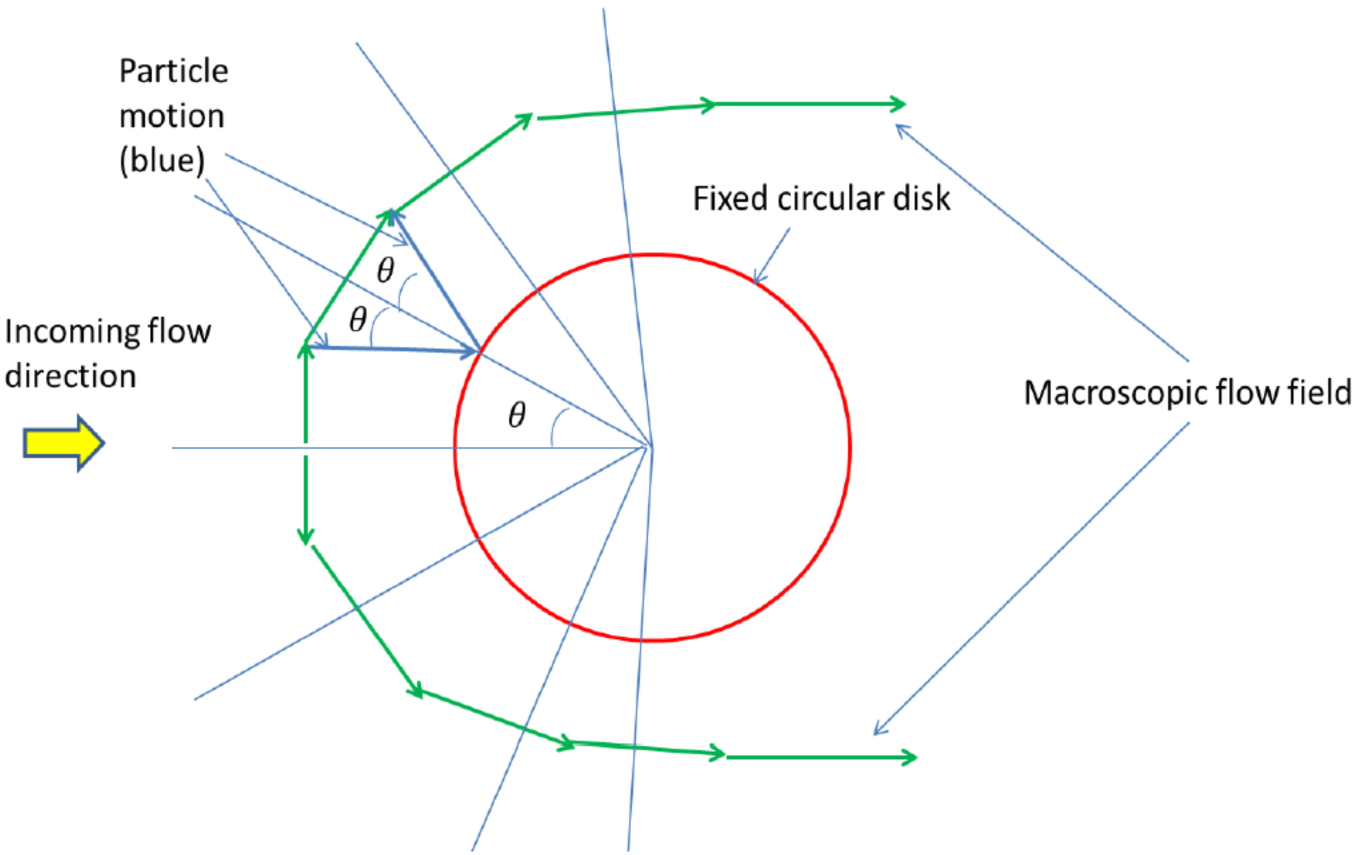
$$D_x^{(1)} \approx -2(1+e)\epsilon^2/(3(1+\epsilon^2))\chi\sqrt{2\pi T/m}V_x \text{ for } V_x \ll \sqrt{T/m},$$

- Then, we reach

$$F_{\rm ex} \approx \frac{\chi(1+e)}{3(1+\epsilon^2)}\sqrt{2\pi m T}V_x$$

Granular or athermal system

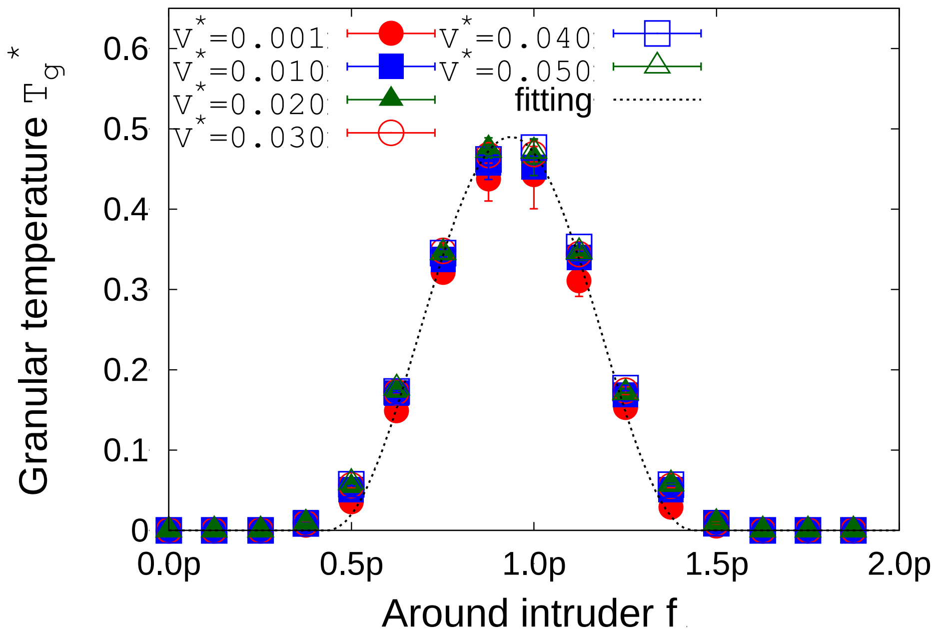
- We need to determine the temperature for athermal systems.
- For this, we adopt the simple picture:



Mean flow and the temperature

- The mean flow is the tangential projection of particle velocity.
- The fluctuation is the normal velocity:

$$\delta v_n = \pm v \cos\theta \qquad T(\theta) \approx m\langle(\Delta v)_n^2\rangle = mv^2 \cos^2\theta.$$



Drag law for granular systems

Using the average temperature

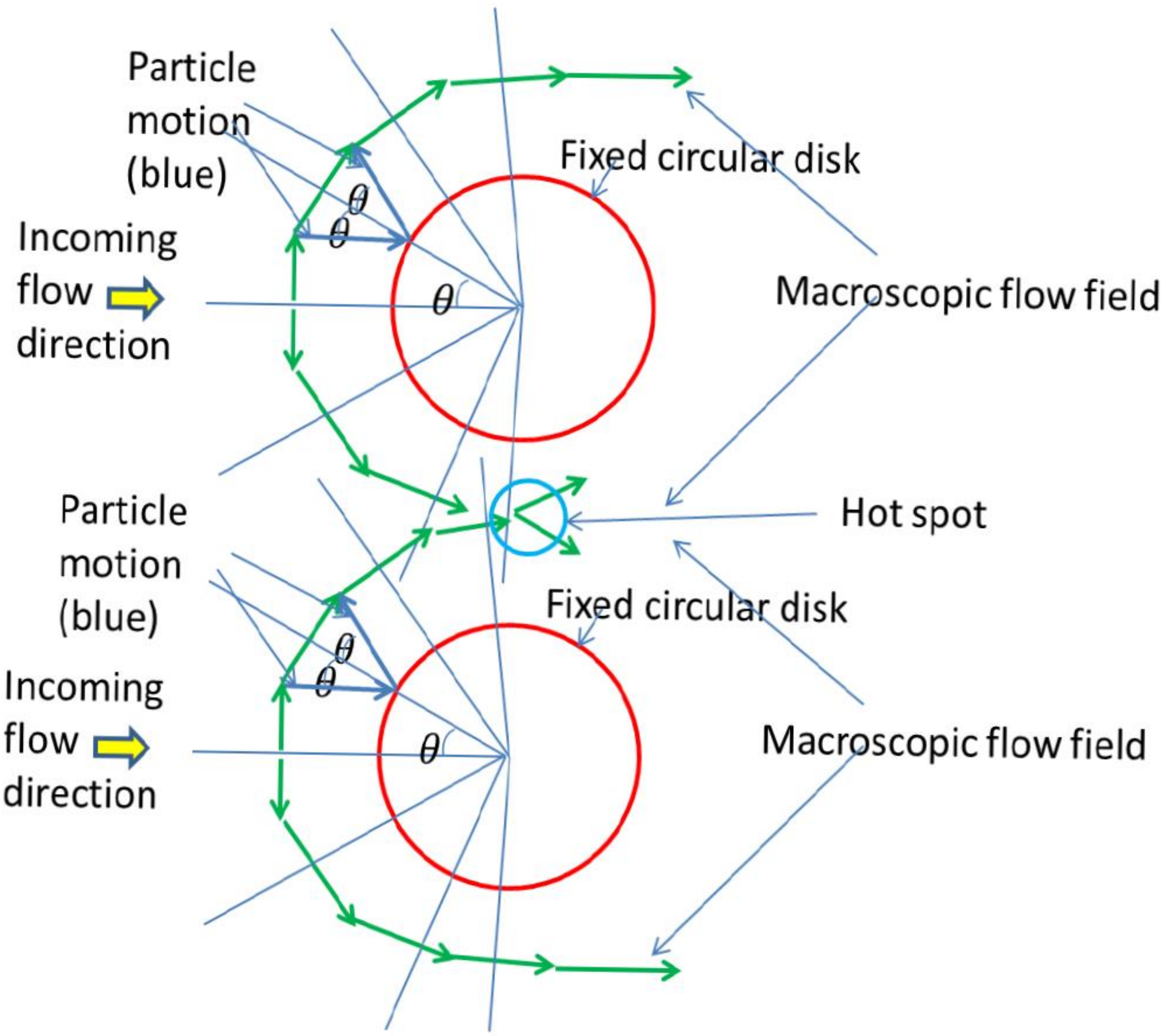
$$T \approx \overline{T(\theta)} \equiv \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} d\theta T(\theta) = m\frac{V^2}{\pi n_0}\int_{-\pi/2}^{\pi/2} d\theta \cos^2\theta = mV^2/2$$

We obtain

$$F_{\rm drag} = \frac{\sqrt{\pi(1+e)\epsilon^2}}{6(1+\epsilon^2)} n(d+D) g_{dD}(\varphi) e^{-1/2}(5I_0(1/2)+3I_1(1/2))V^2,$$

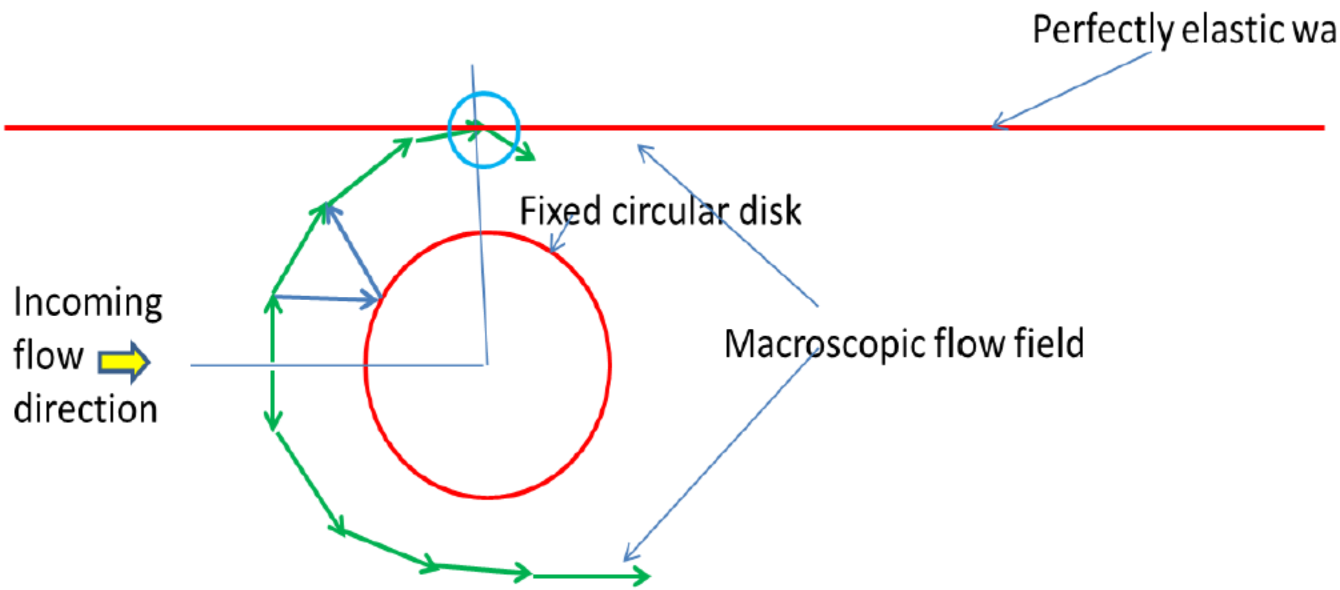
Interaction between two intruders

- Let us consider a steady flow:



A simplified model

- Two-body problem can be reduced to one-body problem:



The perpendicular component to the wall is $u \sin(\pi/2 - \theta_0) = u \cos\theta_0 = (V_0/2)\sin 2\theta_0$.

$$\Delta T \approx m(V^2/4)\sin^2(2\theta_0),$$
$$\Delta T(\theta) \approx (1/4)mV^2 \sin^2(2\theta_0)\exp[-2(\theta - \theta_0)^2]$$

Two-body interaction

- Induced pressure

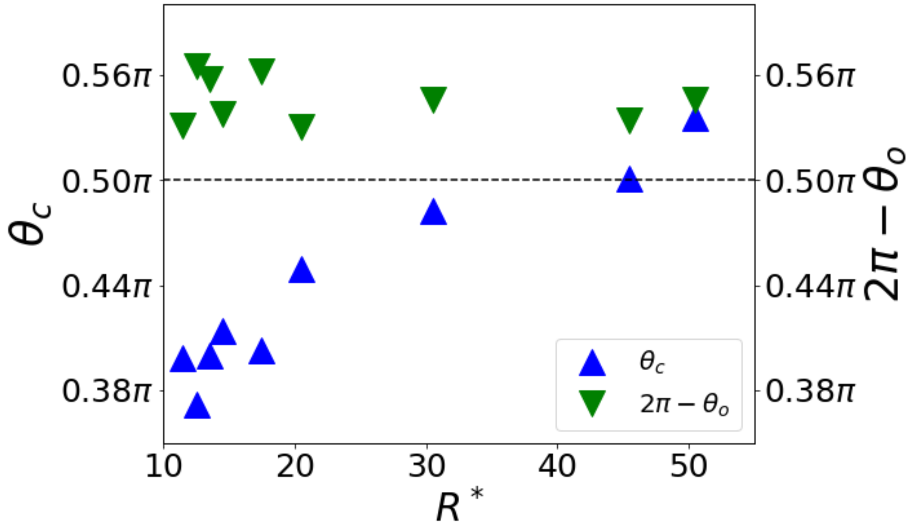
$$\Delta P(\theta) \approx n g_{dd}(\varphi) \Delta T(\theta).$$

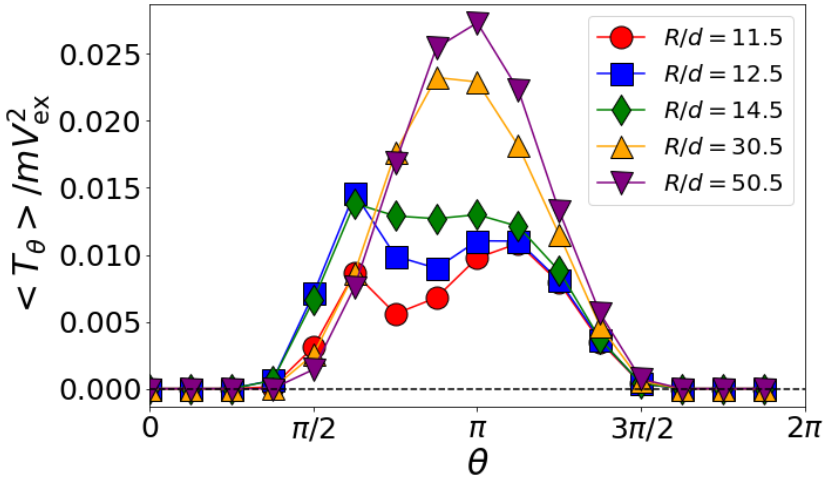
- Total induced force is repulsive:

$$F_{\text{tot}} \approx (1/4) n g_{dD}(\varphi) m V^2 \sin^2(2\theta_0) \int_{-\infty}^{\infty} d\psi e^{-2\psi^2} = (1/4)\sqrt{\pi/2}\, n g_{dD}(\varphi) m V^2 \sin^2(2\theta_0).$$

the separation angle θ_0 $\quad \theta_0 \approx 2\pi/5$ in the simulation.

$F_{\text{tot}} = 0$ if $\theta_0 = \pi/2$





Separation angle?

- Although the perfect fluid model cannot
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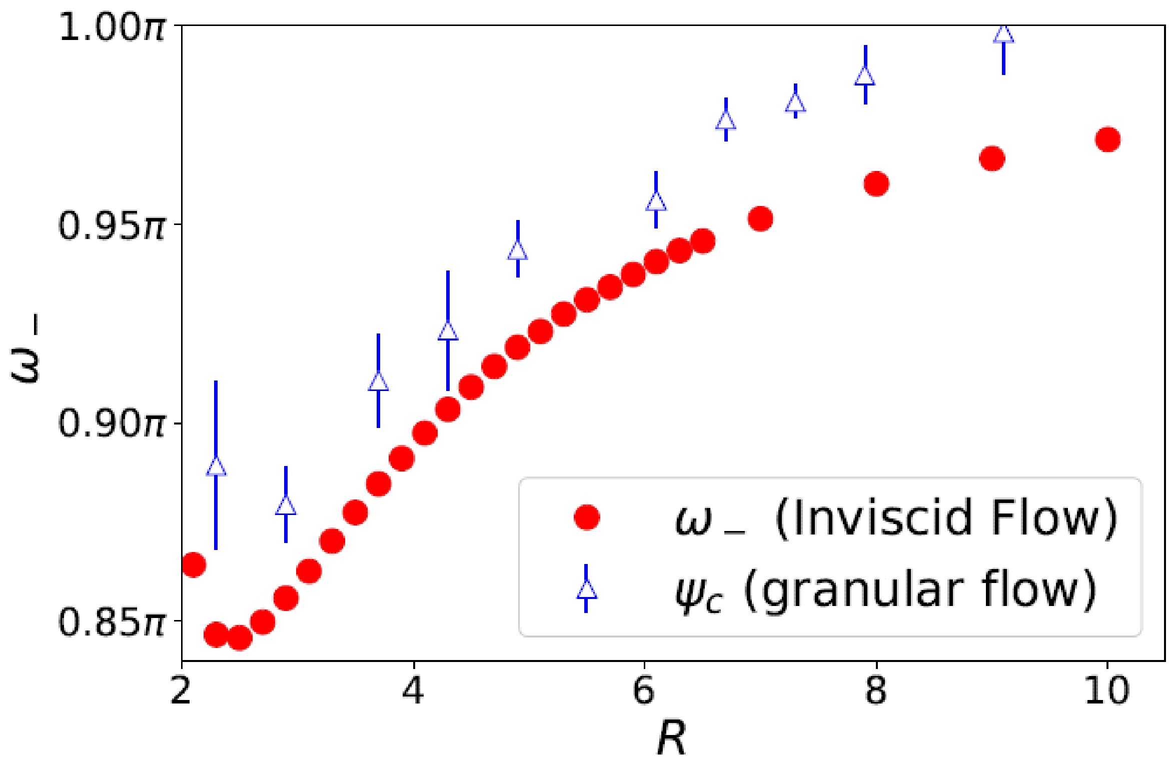
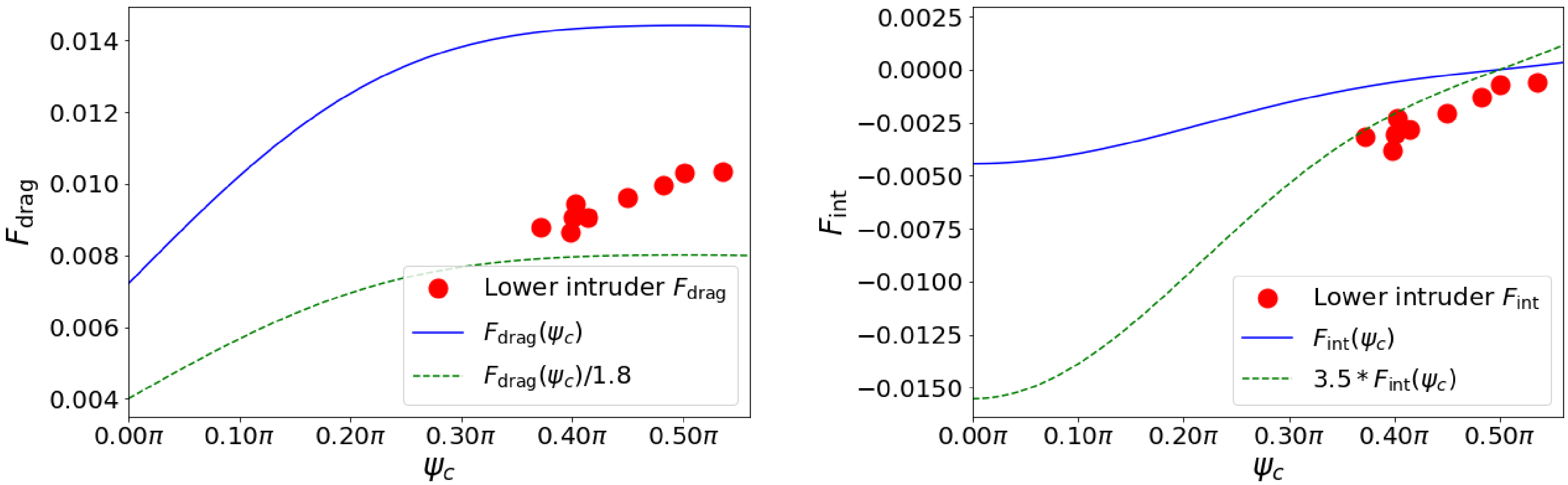


Application of perfect fluid model to two single bodies

$$F_{drag} = \frac{D}{2}\rho V^2 \left[\left(\left(e+\frac{3}{2}\right)-\frac{2}{3}\sin^2\psi_c\right)\sin\psi_c + \left(e+\frac{5}{6}\right)\right]$$

$$F_{int} = \frac{D}{2}\rho V^2 \left(-\left(e-\frac{1}{2}\right)-\frac{2}{3}\cos^2\psi_c\right)\cos\psi_c$$

<u>Drag and resistance anaginst **separation angle** ψ_c</u> where $\psi_o = 3\pi/2$



Note that pure two body calculation gives attractive interaction even if there exists the separation.

Outline

- Introduction: What do we know about drag in granular media?
- Previous study for pure 2D drag in granular media
- Previous studies on the interaction between intruders
- Simulations of two intruders in 2D granular media
  - Two intruders in a steady motion
  - Two intruders under an oscillation
- Phenomenological theory for the drag and the interactions for intruders
- Discussion & conclusions

Attractive interaction in oscillation

- The origin of the attractive interaction in oscillatory systems can be understood qualitatively:
- There are wakes (cavities) in front and back of the intruders.
- Grains in the channel can be squeezed out because the wake regions do not have pressure.
- The density in the channel is much lower than the outside density.
- The pressure also has the same tendency.
- So the intruders approach with each other.

Conclusions

- Perfect fluidity seems to work for the drag force in pure 2D.
- However, the interaction between two intruders cannot be expressed by the perfect fluid model.
- Instead, a simple model can be used for the drag force and the interaction in a steady flow.
- The qualitative picture for the interaction in an oscillatory flow exists, but there is no quantitative description.

THANK YOU