Drag law of three dimensional granular fluids

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Introduction

Drag law in fluid

Drag of a tracer in a flow is characterized by Reynolds number

- **Slow-speed region**
  \[ F = 6\pi \mu a U \] (3D sphere) (Stokes’ law)


- **High-speed region** \( F \propto U^2 \) (Newton’s law)
  impulsive force due to collisions
Previous studies (1)

Drag law in a granular media

The Cell is pulled by $V$.

Experimental setup

The Disk is fixed by a wire.

The drag force $f$ is measured.

Time average

Drag force $F$

Previous studies (2)

Drag law in a granular media

\[ F = F_0(\phi) + \alpha(\phi)V^2 \] is a good fitting function.

Yield force
Previous studies (3)

The origin of the term proportional to $V^2$

Dimensional analysis

- Force $\propto [\text{time}]^{-2}$
- Stiffness $k$ and pulling speed $V$ have the dimension of time.

$\Rightarrow$ drag force $\propto V^2$

The origin of the yield force

Another quantity having the dimension of time...
- Gravity acceleration $g$?
- Dry friction between the grains and the bottom plate?
Previous studies on 3D drag

- **3D drag simulation under gravity with friction**
  
  J. E. Hilton & A. Tordesillas, PRE, 88, 062203 (2013)

  \[ F = F_0 + \alpha V \]

- **Drag experiment of rod**
  
  K.A. Reddy, Y. Forterre, and O. Pouliquen, PRL, 106, 10

  \[ V = \exp \left( \frac{F - F_c}{F_0} \right) \]

  \[ \iff F = a + b \log(V) \]
Perfect fluidity in granular jet (1)
Chicago group suggested the perfect fluidity in granular jet problem.

\[ \nabla \cdot \mathbf{u} = 0, \]
\[ \rho (\partial_t + \mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \cdot \mathbf{\sigma} \]
\[ \mathbf{\sigma} = -p \mathbf{I} + \mu p \dot{\gamma} / |\dot{\gamma}| \]

Coulombic friction

Ellowitz et al. PRL 111, 168001 (2013)
Our previous study: Drag law in 2D granular media


- Active microrheology
- Frictionless system

When the system is moderately dense, the drag law is explained by:
- the perfect fluid +
- vacancy.

\[ F_{\text{drag}} = \left( \frac{3}{2} + \frac{2}{3} \sin^2 \theta_0 \right) \sin \theta_0 D \rho V^2 \]

Recently, Dr. Tanabe found our mistake!
Our previous study: Drag law in 2D granular media

Introduction of the **dry friction** of the bottom plate
⇒ Existence of the yield force
⇒ Friction is the **origin of the constant force.**
Our previous study & Motivation

Motivation

What determines the velocity dependence?

⇒ We clarify the relationship between $F_{\text{drag}}$ and $V$ based on the 3D DEM simulation.

Various $V$ dependence
- Linear regime (creep)
- Plateau regime (activation)

log-log plot

$F^*_{\text{drag}} = F_0 + \alpha V^2$

$F^*_{\text{drag}} = \frac{F_{\text{drag}}}{(\phi_c - \phi)^{-\beta}}$

$V^*$, $\phi$

$F_{\text{drag}} = F_0 + \alpha V^2$

$\phi_{c}$
Setup

3D DEM simulation for frictionless systems

- **Particles**
  - mass $m$, diameter $d$: monodispersity
  - Number of grains $\sim 10^4$
  - Restitution coefficient: $e = 0.8$

- **Intruder**
  - Diameter $D$
  - Pulling with $F = F_{ex}$ in $x$-direction

- **System**
  - Cylinder $L_x = 60d, R = 7.5d$
  - Boundary condition:
    - $x$-direction $\Rightarrow$ periodic boundary
    - $y, z$-direction $\Rightarrow$ flat physical boundary (curvature $R$)
Movie
Drag law \((D = d)\)

\[ F_{\text{drag}} = (\text{Yield force}) + (\text{term } \propto V) \]

Various \(V\) dependence

- \(V\) regimes
- \(V^2\) regime (perfect fluidity)
- Plateau regime

Similar to PRE, \(88, 062203\) (2013)
Drag law

$V^*$

$F_{\text{drag}}^*$

Logarithmic dependence

$\phi$

$0.635$

$\Rightarrow$ Why?
Log regime

Activated processes occur?

Contact force $F_c$

Activation

Average contact force $\langle F \rangle_c$

~ External force $F_{ex}$?

$$t^* = t \sqrt{\kappa / m}$$

Time evolution of the position and velocity of the tracer

$$x^* = x / d$$

$$p / (x / w) \sqrt{x_a} = x_a$$
Activated process

Time-averaged contact force $\bar{F}_c$ from the simulation

$$\bar{F}_c = \sum_{\text{contact}} \kappa \delta \cos \theta$$

$\kappa$: spring constant

**Good agreement $\Rightarrow$ Activated processes**

*Logarithmic dependence!*

Drag law \((D > d)\)

When the intruder is much larger than the surrounding particles, the plateau regime vanishes.⇒ This fact seems to validate our conjecture (activation process).
We have performed the three-dimensional drag simulation. This system is force-controlled (active microrheology). Many characteristic regimes

- Quadratic regime $\Rightarrow$ **Newtonian**
  (Similar to 2D system)

- Log regime $\Rightarrow$ **Activated process**
  This regime vanishes for larger $D$.

**Next question:**
What is the proper drag law?

Especially, the drag law for frictional grains under gravity.
$\Rightarrow$ We numerically study this problem.
Setup

- 3D DEM simulation by LAMMPS
- With gravity and friction
- Polydisperse particles (0.9d~1.1d: uniformly distributed)
- We control $V$, and measure $F_{\text{drag}}$.
- We introduce the Froude number (dimensionless speed)

$$Fr = \frac{V}{\sqrt{gD}}$$

$Fr = \frac{V}{\sqrt{gD}}$ = ratio of two characteristic time scales:

$t_1 = D/V$: forward motion in x-direction
$t_2 = \sqrt{D/g}$: falling in z-direction

Interaction model

- Hertzian model + dashpot (proportional to the relative velocity)

$$F_n = \sqrt{R_{\text{eff}}\delta}(K_n\delta - m_{\text{eff}}\gamma_n v_n)$$

$$F_t = -\min \left( \mu F_n, \sqrt{R_{\text{eff}}\delta}(K_t\Delta s_t + m_{\text{eff}}\gamma_t v_t) \right)$$

$K_n = 2 \times 10^8 \rho d g, K_t = 2.45 \times 10^8 \rho d g$

※All quantities are nondimensionalized in terms of $\rho, d, g$. 

In collaboration with S. Kumar and K. A. Reddy (IITC, India)

System I
40$d \times 40$d $\times 38$d
70,001 particles

System II
80$d \times 40$d $\times 80$d
300,001 particles
Result

- movie
Typical time evolution of the drag force

Typical velocity field
Drag force vs. Froude number (constant depth)

- $F_{\text{drag}}$ increases as the diameter $D$ increases. $F_{\text{drag}} = F_{\text{drag}}(\text{Fr})$.
- Especially, for $0 \leq \text{Fr} \leq 0.5$, $F_{\text{drag}} \approx F_Y \equiv \text{const}$.

**Question:** How can we scale these drag laws depending on the intruder diameter $D$ or the depth $h$?

$\Rightarrow$ We focus on the static (constant) regime ($0 < \text{Fr} \leq 0.5$).
Diameter $D$ dependence in the static regime

- $F_Y$ increases as $\mu$ increases.
- $F_Y$ depends the power of $(D^* + 1)$.
  ⇒ We define the exponent $\phi_\mu$.

Yield force is scaled by $D^* + 1$ with

$$\frac{F_Y^*}{f(\mu)(D^* + 1)^{\phi_\mu}} = \text{const.}$$

※$f(\mu)$ depends only on $\mu$.

Next, how is the depth dependence?
Depth $h$ dependence (constant diameter $D$)

- As well as $(D^* + 1)$, $F_Y$ also depends on the power of $h$.
  $\Rightarrow$ We define the exponent $\alpha_\mu$.
  $\alpha_\mu$ is the increasing function of $\mu$.

- Yield force is also scaled by $h$ with
  \[
  \frac{F_Y^*}{g(\mu) h^* \alpha_\mu} = \text{const.}
  \]
Scaling law

From the above discussions, the yield force can be scaled as

\[ F_Y \propto \rho (D^* + 1)^{\phi_{\mu}} h^{\alpha_{\mu}} g \]

And this scaling can be applied for the whole range of \( Fr \) except for the shallow region.

\[ \frac{F_{\text{drag}}}{F_Y} = F_{\text{dynamic}}^*(Fr) \]

Sum rule

Two exponents \( \phi_{\mu} \) and \( \alpha_{\mu} \) satisfy an approximate sum rule

**Sum rule**

\[ \phi_{\mu} + \alpha_{\mu} \approx 3 \]
Discussion

• Why is the sum rule $\phi_{\mu} + \alpha_{\mu} \approx 3$ satisfied?
  From the dimensional analysis, $F_{\text{drag}} = [\text{length}]^3$
  Which do other quantities have the dimension of length?
  $\Rightarrow D, h \Rightarrow (D + d)^{\phi_{\mu} h^{\alpha_{\mu}}}, \phi_{\mu} + \alpha_{\mu} = 3$

• Why $\phi_{\mu=0} \approx 2$?
  Collision cross section is given by $\pi (D + d)^2/4. \Rightarrow \phi_{\mu=0} \approx 2$

• Why does $\phi_{\mu} (\alpha_{\mu})$ decrease (increase)?
  Or really power dependence?
  We do not still have any answer.
  $\Rightarrow$ Larger (in height) simulation should be done.
Short summary of Part II

• We have performed DEM simulation to study the drag law in 3D granular media.
• There exist two regimes depending on the Froude number (static and dynamic parts).
• The drag law for the whole Fr regime can be scaled in terms of $D + d$ and $h$.
• There exists an approximately sum rule $(\phi_\mu + \alpha_\mu \approx 3)$ between two exponents.

Future work
• Larger simulations are needed.
• Force chain network
Question

• Can we use periodic boundary condition in the pulling direction?
  ⇒ It may affect the results  
    (especially for high velocity regime).

• When we watch the movie (in Part I), the surrounding particles seem to have a finite temperature?
  ⇒ What happens when the particles have the finite temperature?

We study passive microrheology.
Setup

Velocity control simulation
Soft core simulation (e=1)
N=20,000, monodisperse
Initial packing fraction: $\phi = 0.4$

- Intruder
  - Intruder is fixed.
  - Diameter $D = 5d$
  - Mass $M = \infty$

- Surrounding particles
  - Monodisperse
  - At $t = 0$, the velocity $V$ is added.
  - No overlaps at first.
3D simulation \((T = 0)\)

\[
\phi = 0.4 \\
e = 1 \\
V^* = 10^{-0.5}
\]
Force chains only exist near the intruder.
Density profile

White lines… stream lines

Large vacant region behind the intruder.
**Drag law \((T = 0)\)**

\[
F_{\text{drag}} = \left( \frac{3 + 2e}{8} - \frac{9}{64} \sin^2 \theta_0 \right) \sin^2 \theta_0 \rho D^2 V^2
\]

- The drag law is insensitive to the restitution coefficient.
- The drag force is proportional to \(V^2\).
- The drag force is two times smaller than that obtained by the perfect fluid + vacancy model.
Comparison with active microrheology

Difference between active (force control) and passive (velocity control) microrheology?

Setup:
2D system
Particle number: 10000
Restitution coefficient: e=0.9
Bidisperse: 1:1.4

Tendency is opposite!
Why? We still have no idea.
We consider the case that the particles have a finite temperature $T(>0)$.

**Introduction of dimensionless parameters**

- Dimensionless drag force
  \[ C_D = \frac{F}{\frac{1}{2}\rho V^2 S} \]
  (We measure $F$ at $V^* = 0.1$.)
- We also define the dimensionless velocity as
  \[ R = \frac{V}{v_T} = V\sqrt{\frac{m}{2T}}. \]
Movie \((R = 1)\)

\[ N = 20,000 \]
\[ \phi = 0.4 \]
\[ e = 1 \]
\[ V^* = 0.1 \]
Movie \((R = 0.1)\)

\[ N = 20,000 \]
\[ \phi = 0.4 \]
\[ e = 1 \]
\[ V^* = 0.1 \]
Density field \((R = 0.1)\)

Vacant regime becomes smaller. (Particles can go around the intruder.)
Drag law between $C_D$ vs. $R$

- Hard-core limit is realized for soft-core simulations.
- $R$ dependence
  
  $R$: large $\Rightarrow C_D = \text{const.} \Rightarrow F \propto V^2$ (Newtonian)
  
  $R$: small $\Rightarrow C_D \propto 1/V \Rightarrow F \propto V$ (Stokes’ law)

  Consistent with Stokesian drag $F = 4\pi\eta \frac{D}{2} V$ (slip surface)

  ($\eta$: viscosity for the surrounding particles $\leftarrow$ Garzo & Dufty, PRE (1999))
Discussion

Fluid

Our system

Reynold’s number

• Transient regime between $C_D \propto \frac{1}{R}$ and $C_D \propto \text{const}$.

Fluid: ○ ⇔ Our system: ✗

• High $R$ limit

Fluid: turbulence (Karman vortex and separation vortex)

⇔ Our system: converge to constant

What causes these?

→ roughness of the surface, thermal wall?

Summary of Part III

- We have performed simulations.
- Velocity control system.
- \( T = 0 \)
  
  Drag force \( \propto V^2 \) (Newtonian)

- \( T > 0 \)
  
  \( R \gg 1 \): Newtonian
  
  \( R \ll 1 \): Stokesian

  \( \rightarrow \) consistent with Stokes drag \( F = 4\pi\eta\frac{D}{2}V \)

Future work

- Transient behavior
- Introduction of surface roughness or thermal wall