# Current status of absorbing-state transitions & connections to reversible-irreversible transitions

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@YITP focus meeting on
Rheology of disordered particles - suspensions, glassy and granular materials

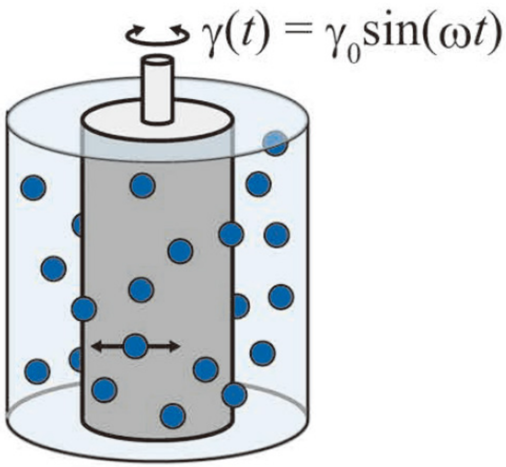
A review talk including:

- KaT, M. Kuroda, H. Chaté & M. Sano,
  Phys. Rev. Lett. 99, 234503 (2007); Phys. Rev. E 80, 051116 (2009).
- M. Takahashi, M. Kobayashi & KaT, arXiv:1609.01561

# Reversible-Irreversible Transitions (RIT) & Absorbing-State Transitions (AST)

## RIT in suspensions

discovered by Pine et al. (Nature 2005) in a Couette cell experiment




small $\gamma_0$: particles motion reversible

large $\gamma_0$: particles motion irreversible

concerns rheological properties

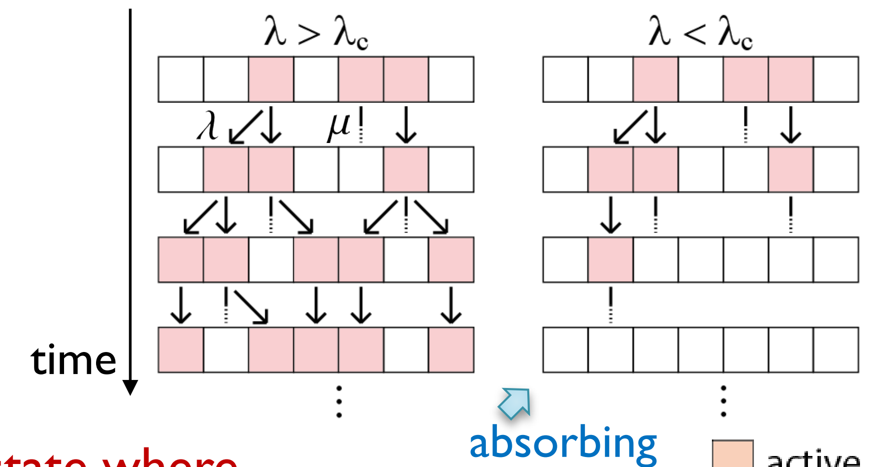
## AST in stat phys

- Non-equilibrium phase transitions into "absorbing states"
- A few universality classes established. "test bed of physics of noneq critical phenomena" [Hinrichsen, Adv. Phys. 49, 815 (2000); Henkel et al., *Noneq Phase Transitions* (2009)]
- Established based on toy models, but relevance in real systems has been recognized recently.
- RIT is a type of AST.

# Absorbing-State Transitions [Hinrichsen, Adv. Phys. 49, 815 (2000)]

Prototypical model: contact process

Active sites can stochastically
(1) activate a neighbor (at rate $\lambda$)
(2) become inactive (at rate $\mu$)




| |
|---|
| $\lambda > \lambda_c$ : active sites persist |
| $\lambda < \lambda_c$ : active sites die out |

absorbing state = a global state where no further state change is allowed.

Order parameter: density of active sites

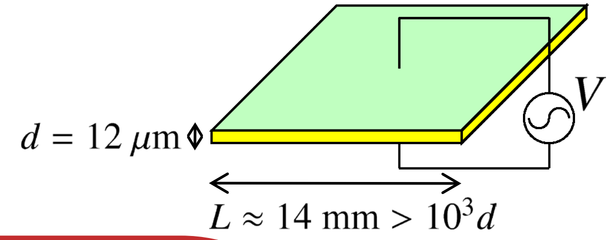
$$\rho \sim \begin{cases} (\lambda - \lambda_c)^\beta & (\lambda \geq \lambda_c) \\ 0 & (\lambda \leq \lambda_c) \end{cases}$$

Critical exponents are universal.
⇒ universality class

Directed percolation (DP) universality class: most fundamental case

"DP conjecture": [Janssen 1981, Grassberger 1982]
AST are usually DP, in the absence of symmetry, conservation law, long-range interactions, quenched disorder.

# Real Example



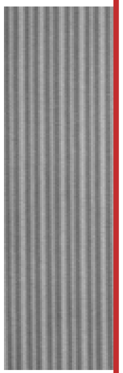

Electro-convection of nematic liquid crystal

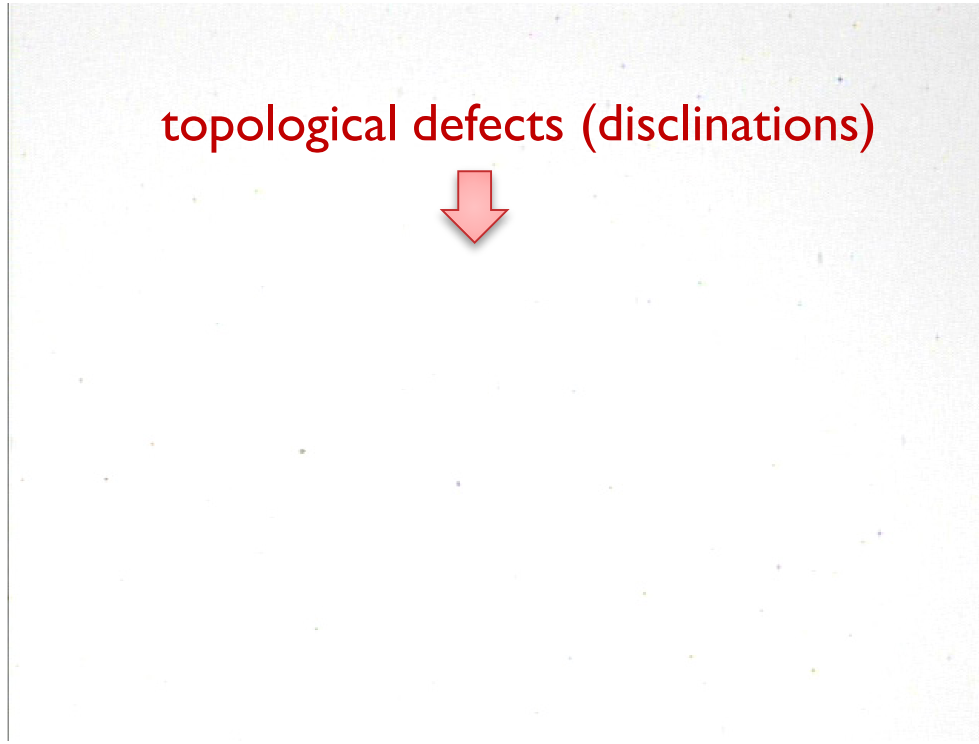
Anisotropic m…

Convecti… …e transition?

transmit'd light images

100 μm

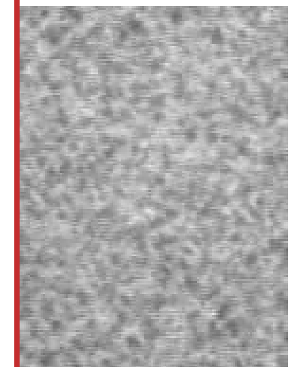




topological defects (disclinations)

72V, 200Hz, speed x3

voltage will be turned off

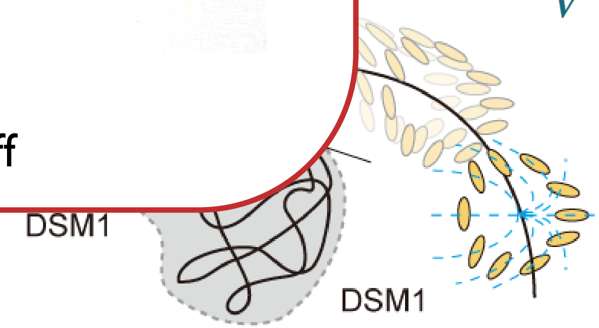


no convection

r…

…d turbulence (…SM2)

0 ≈ 8 V $V$

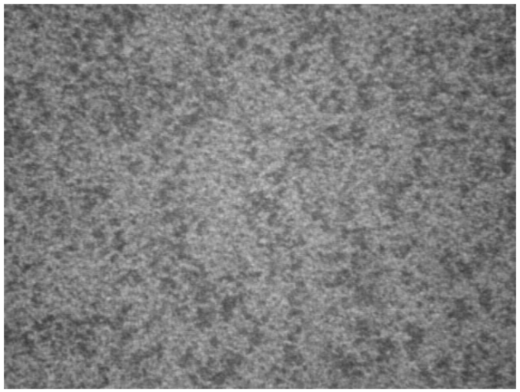


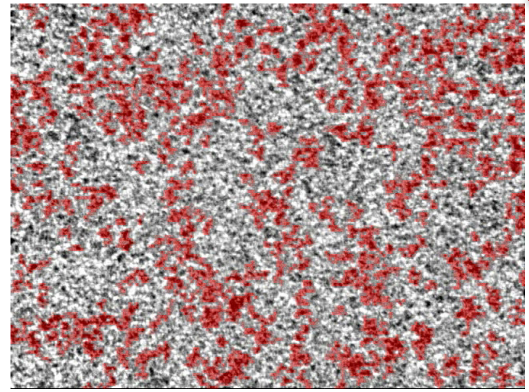
[KaT *et al.* PRL 99, 234503 (2007);
PRE 80, 051116 (2009)]

# Near the Transition

DSM1 & DSM2 coexist (DSM2 patches amid DSM1)

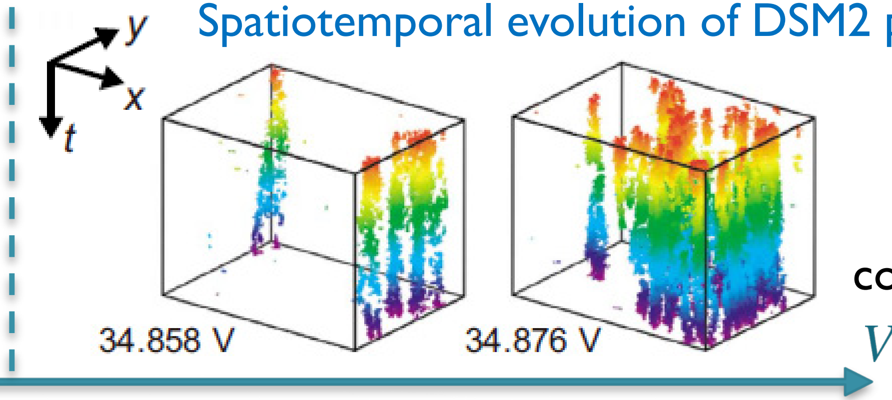
DSM2 painted in red





Spatiotemporal evolution of DSM2 patches

DSM2 patches die out




color = time

$V_c$ ← Transition for DSM2 patches to survive.
Absorbing-state transition (DSM2 nucleation is very rare)

Order parameter $\rho$ = DSM2 area fraction

# Critical Phenomena

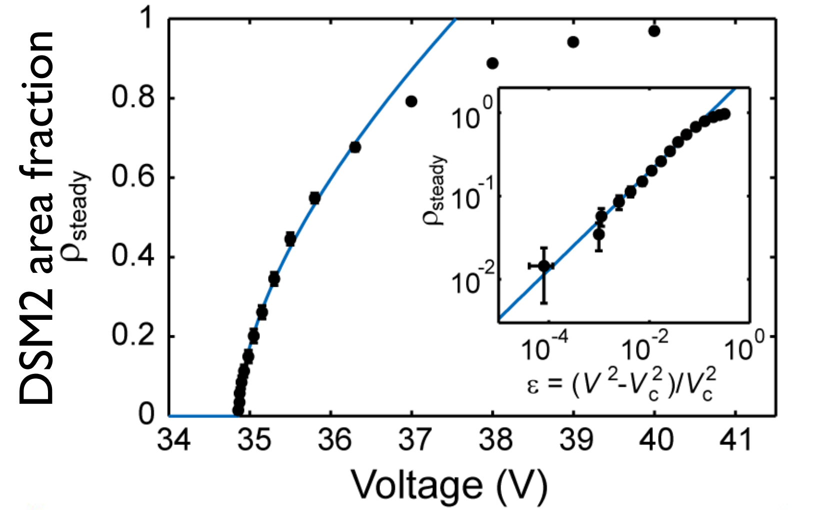
[KaT *et al.* PRL 99, 234503 (2007); PRE 80, 051116 (2009)]

- Steady state

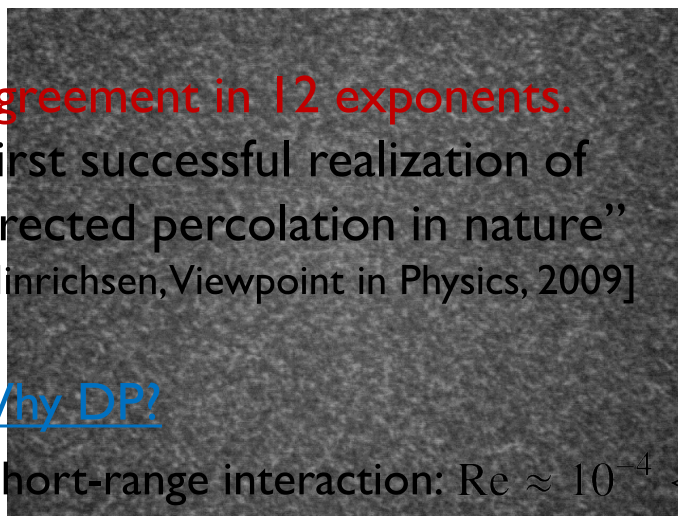
$$\rho_{\text{steady}} \sim (V^2 - V_c^2)^\beta$$

$\beta = 0.59(4)$ $\beta^{\text{DP}} \approx 0.583$

agreement with (2+1)d DP class




- Relaxation from fully active state



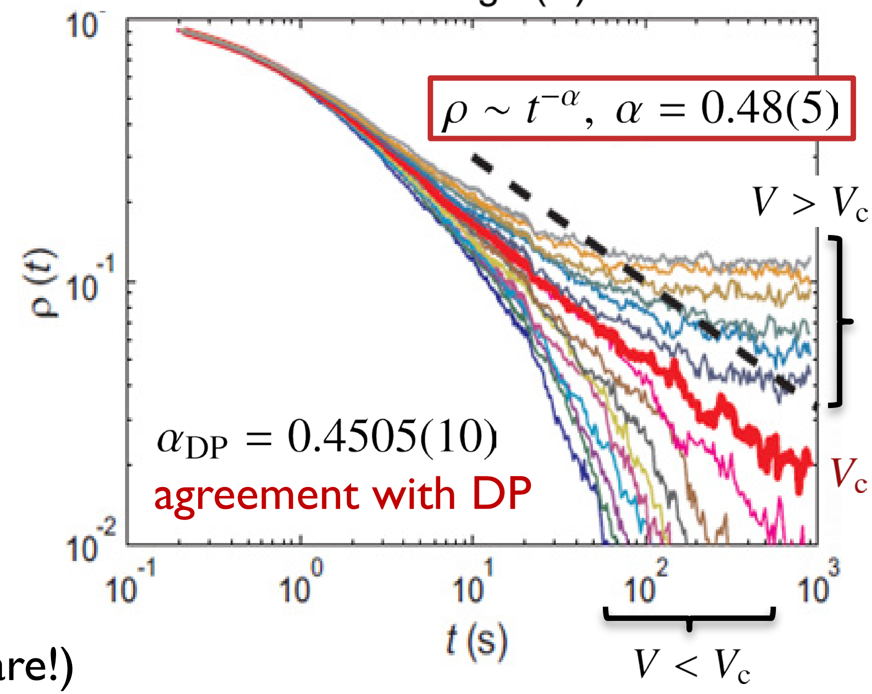
Agreement in 12 exponents.

"first successful realization of directed percolation in nature"

[Hinrichsen, Viewpoint in Physics, 2009]

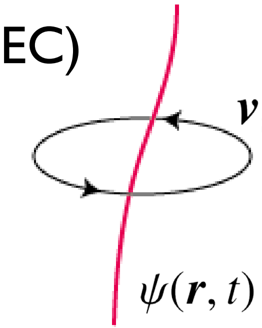
Why DP?

- short-range interaction: $\text{Re} \approx 10^{-4} \ll 1$
- (almost perfectly) abs.-state transition.
  (∵generation of topological defects is rare!)



# Quantum Turbulence: Another Topological-Defect Turbulence

- In quantum fluids (e.g., superfluid He, cold atom BEC) vortices are quantized (= topological defects).



$$\boldsymbol{v}_s = \frac{\hbar}{m}\nabla\theta$$

$$\psi(\boldsymbol{r},t) = |\psi(\boldsymbol{r},t)|e^{i\theta(\boldsymbol{r},t)}$$

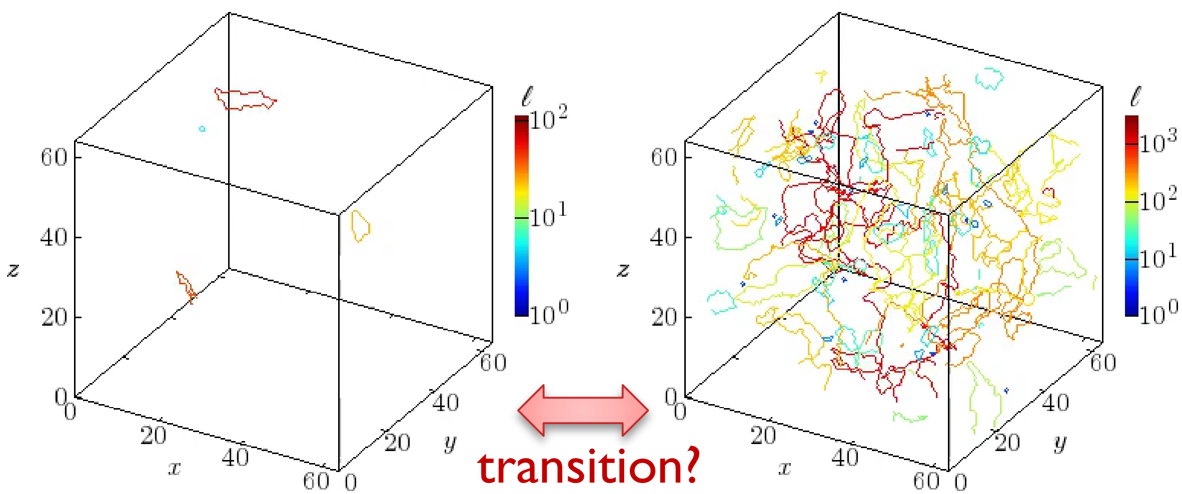
- Quantum turbulence
  = turbulence made of quantum vortices

- *In silico* example: Gross-Pitaevskii eq. with dissipation + random potential

$$(i\hbar - \gamma)\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) = -\frac{\nabla^2}{2m}\psi + [V(\boldsymbol{r},t) - \mu]\psi + g|\psi|^2\psi$$

(potential amplitude = $V$)

low $V$
($V$ = 10)




high $V$
($V$ = 15)

sustained vortices

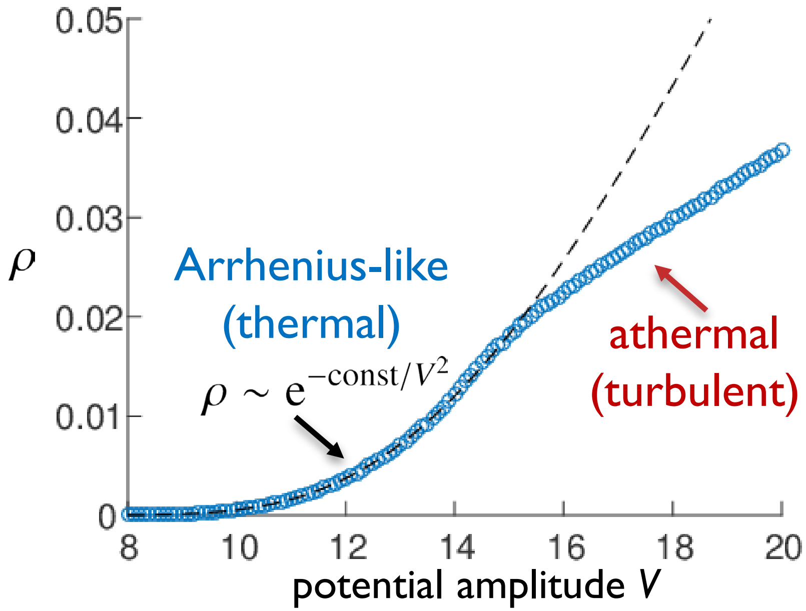
[Takahashi, Kobayashi & KaT, arXiv:1609.01561]

# Transition to Quantum Turbulence

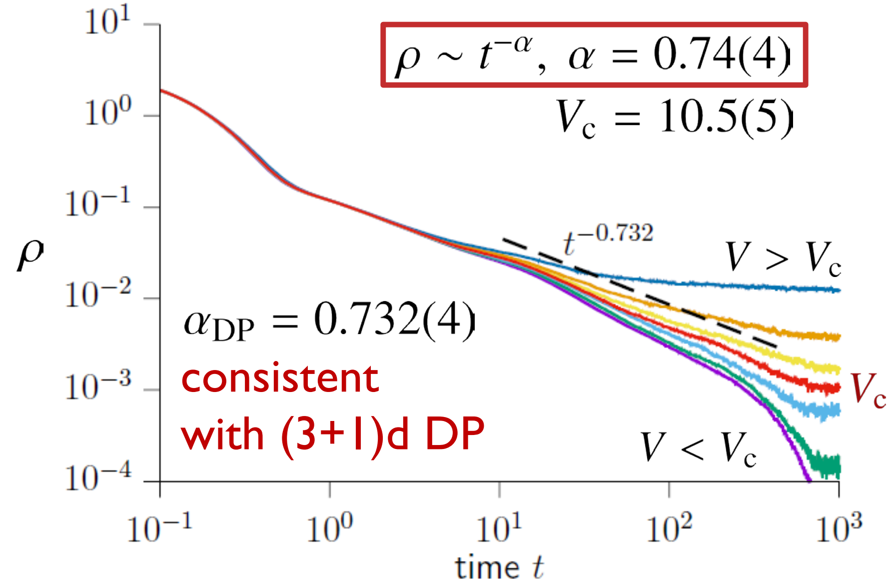
[Takahashi, Kobayashi & KaT, arXiv:1609.01561]

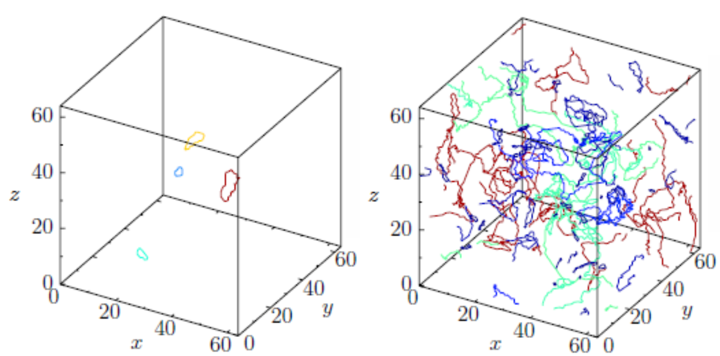
Order parameter $\rho$ = density of quantum vortices

steady state



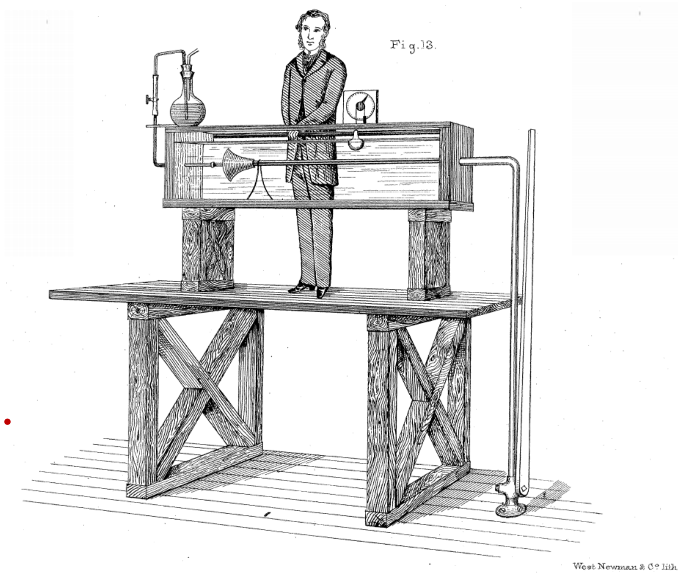

relaxation from
developed turbulence




- Data suggest (3+1)d DP class.
- "Absorbing state" underlying, thanks to energy barrier for defect generation, but smeared by thermal excitation.

# Newtonian Fluids




[Reynolds 1883]

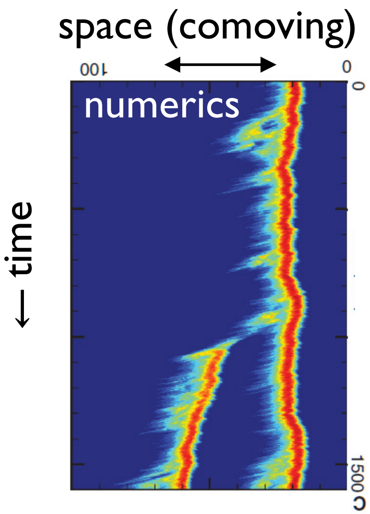
Transition to turbulence in pipe flow

- Experimentally, transition near $\mathrm{Re} \approx 2000$.
- Laminar flow is linearly stable up to $\mathrm{Re} = \infty$.
  (nonlinear effect is crucial)

Question:
Turbulence generated by a nonlinear perturbation can persist or decay?
(at a given Reynolds number)

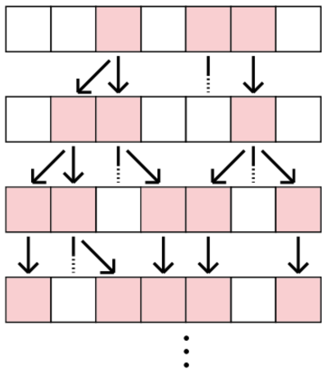
# Near the Transition to Turbulence in Pipe

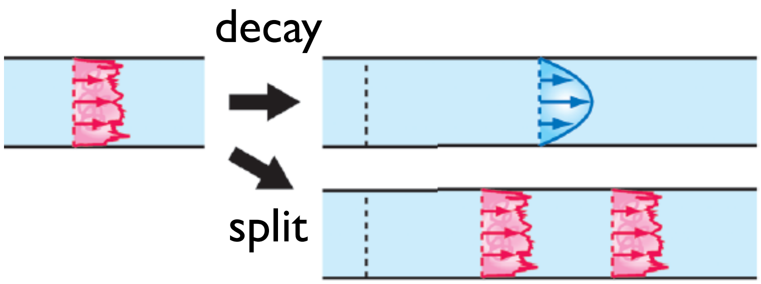
Turbulence localized: "turbulent puff" [see; Hof group, Science 2011 & refs therein]

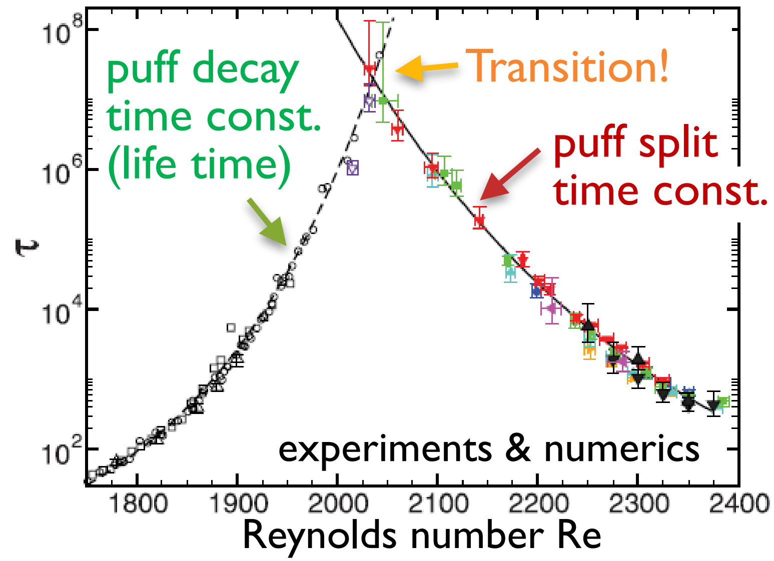



Puffs decay / split stochastically.

⇧ contact process? DP class?






For pipe,
direct test of DP is unrealistic...

However,
similar transition in channel flow/Taylor-Couette ⇨ DP-class exponents!
[Sano & Tamai, Nat. Phys. 12, 249 (2016); Lemoult et al., Nat. Phys. 12, 254 (2016)]

# Current Status of DP-class Transitions

- 4 real / realistic examples, where all independent exponents were checked & agreed. (at least barely)

| Systems | why absorbing? |
|---|---|
| liquid-crystal turbulence (exp) | topological defect |
| quantum turbulence (num) | topological defect |
| Newtonian turbulence (exp & num) | laminar stability |
| active matter turbulence (num) | ? (numerically checked) |

← [Yeomans group, Nat. Comm. 8, 15326 (2017)]

but further studies needed to answer why DP in those systems.

- So what?
  - Unified description near the transition

    DP continuum equation

    $$\frac{\partial \rho}{\partial t} = a\rho - b\rho^2 + D\nabla^2\rho + \sqrt{\rho}(\text{noise})$$

  - Not only the dependence on the control parameter, but how it ages, how it reacts against perturbations, etc., are known.
  - Theory & analysis & techniques developed for AST may be employed.

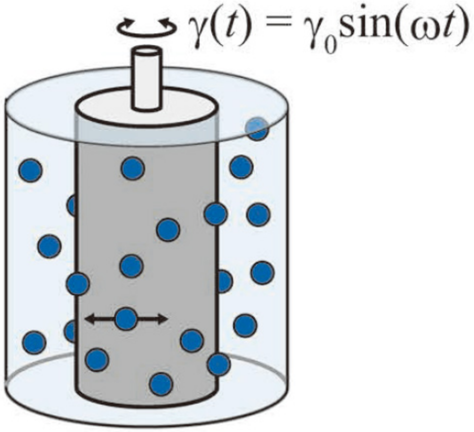
- Other classes:

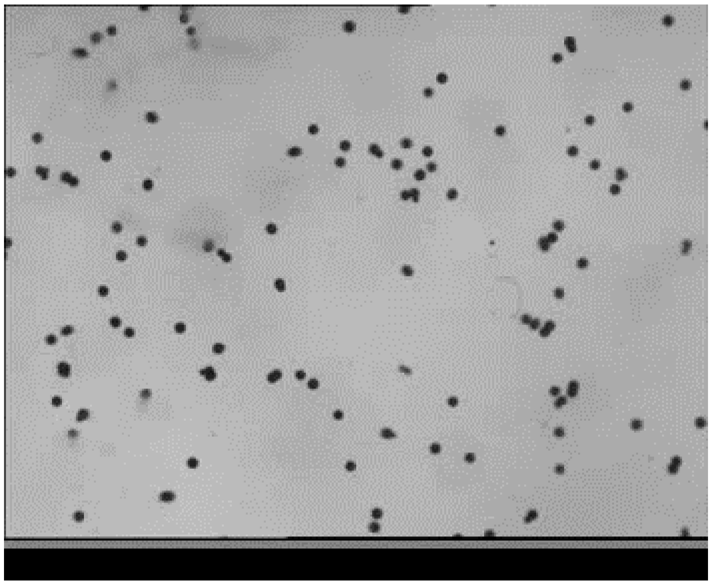
  voter (with $Z_2$ symmetry), C-DP (with conservation law), etc.

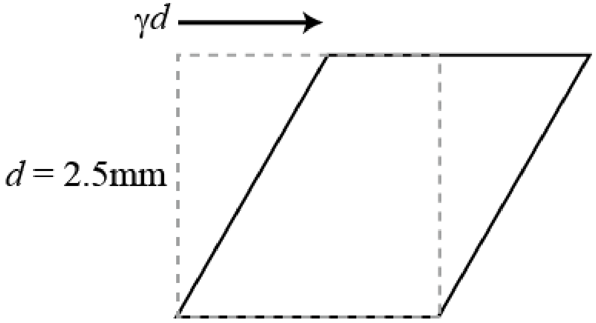
# So... Reversible-Irreversible Transition (RIT)

Couette cell experiment by Pine et al. [Nature 438, 997 (2005)]

- diameter 230μm ⇨ Brownian motion negligible
- oscillatory shear $\gamma(t) = \gamma_0 \sin \omega t$
- Re = $10^{-3}$ ⇨ Stokes flow
  $-\nabla p + \nabla^2 v = 0$ & boundary reversible
- volume fraction $\phi$ = 0.1-0.4 ⇨ no jamming
- density & index matching, some particles are dyed.



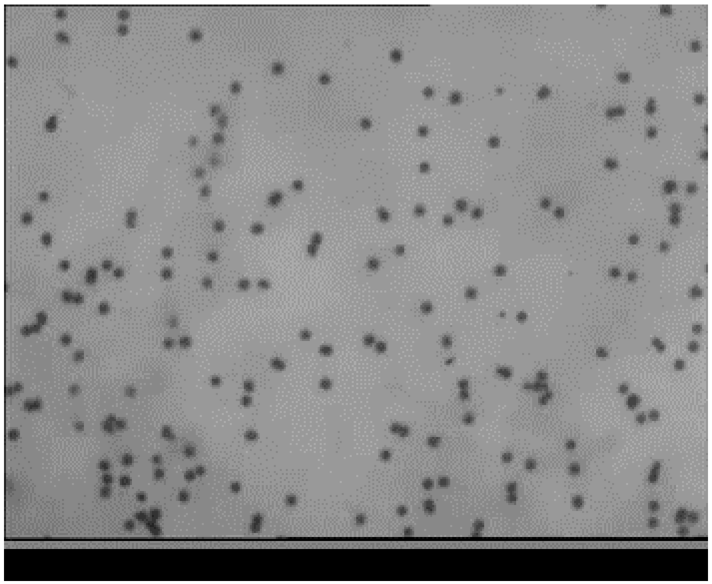





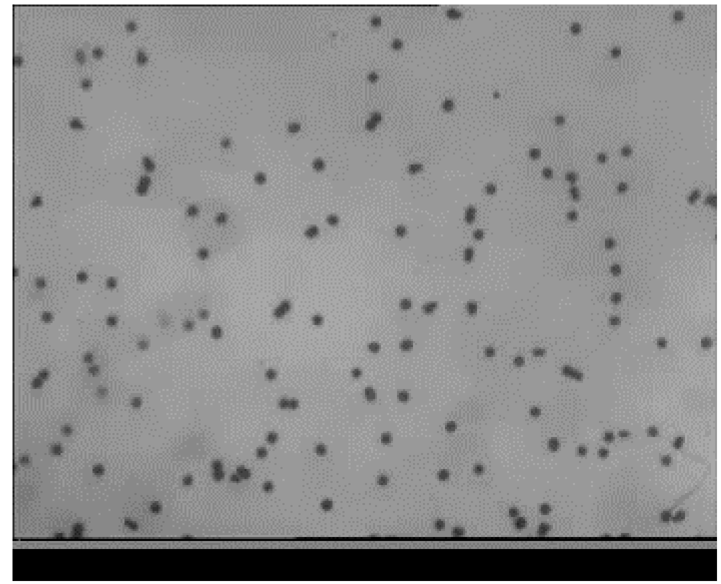
# Stroboscopic Imaging

[Pine et al., Nature 438, 997 (2005)]

$\gamma_0 = 1.0$ $(\phi = 0.3)$ $\gamma_0 = 2.5$



reversible motion



irreversible motion

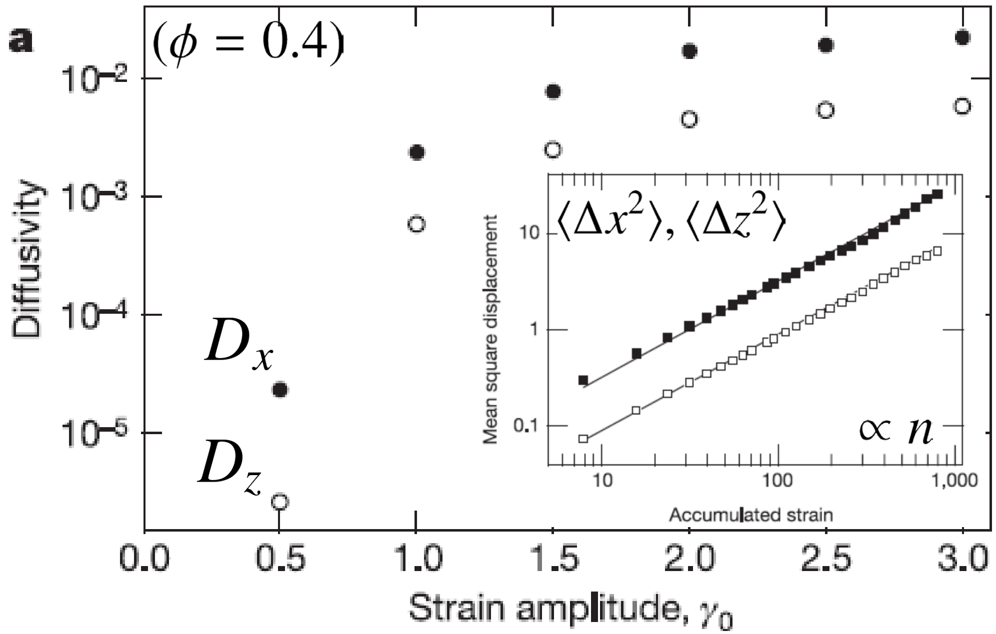
"reversible-irreversible transition"

# An Order Parameter

[Pine et al., Nature 438, 997 (2005)]

diffusion coefficient

(in stroboscopic imaging)




$$\langle \Delta x^2 \rangle = 2D_x n$$
$$\langle \Delta z^2 \rangle = 2D_z n$$

x: flow direction

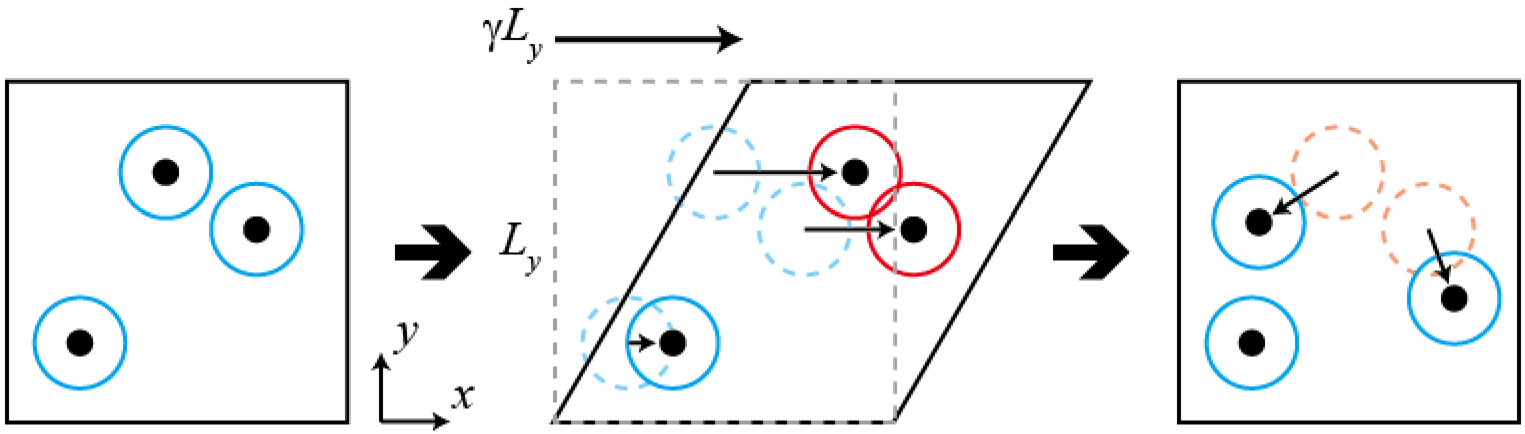
z: axial direction

$n$ = cycle number

Suggested the existence of a well-defined transition point.

# Possible Mechanism

Model by Corté et al. [Nat. Phys. 4, 420 (2008)]



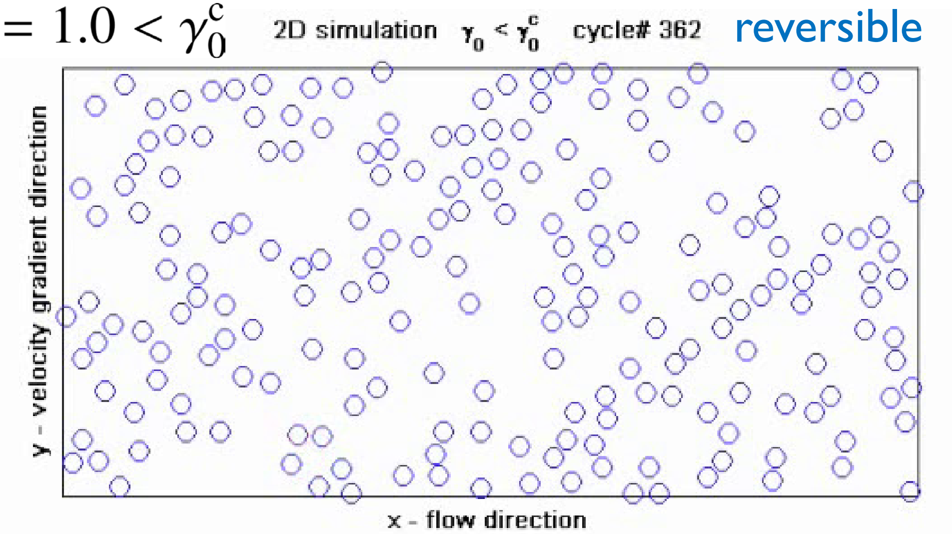

- Finite range of interaction.
- Oscillatory shear: $\gamma(t) = \gamma_0 \sin \omega t$
- When particles collide ⇨ random displacements
- Model for the dilute case.

# RIT in Corté et al.'s Model

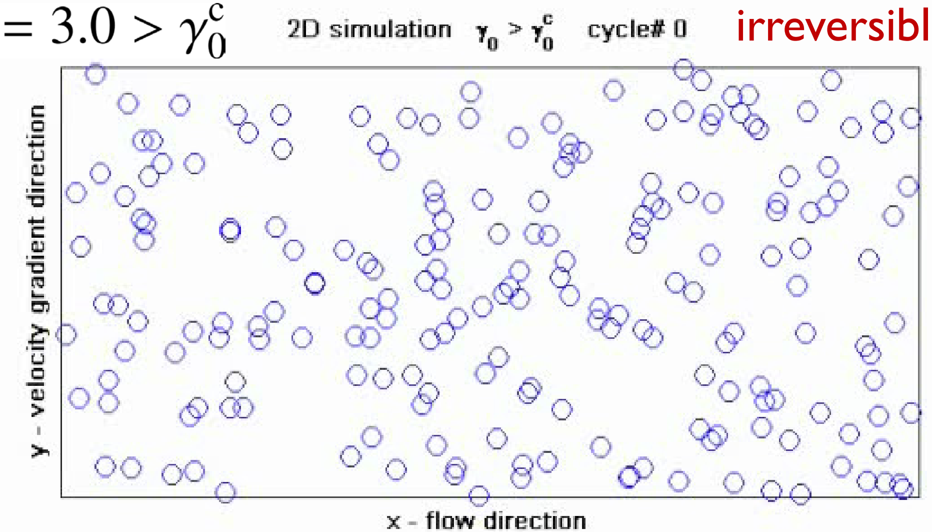
[Corté et al., Nat. Phys. 4, 420 (2008)]

Stroboscopic sampling

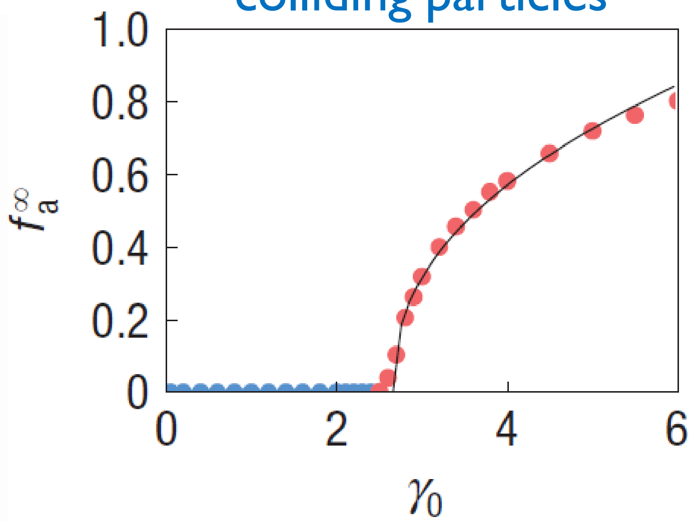
$\gamma_0 = 1.0 < \gamma_0^c$ reversible




$\gamma_0 = 3.0 > \gamma_0^c$ irreversible




fraction of colliding particles




**RIT is an AST!**

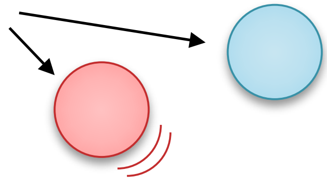
"Random organization"

Total particle number is conserved.

⇨ C-DP class?

# C-DP Class

active (A)
or inactive (B)



(# of active (colliding) particles) + (# of inactive particles) = const.

A (diffusive) B (non-diffusive)

| | | |
|---|---|---|
| A | → | B |
| A + B | → | 2A |

$\rho(x, t) = [A]$

$\phi(x, t) = [A] + [B]$

C-DP continuum equation

$$\begin{cases} \dfrac{\partial \rho}{\partial t} = a\rho - b\rho^2 + D\nabla^2\rho + \sqrt{\rho}(\text{noise}) + c\rho\phi \\ \dfrac{\partial \phi}{\partial t} = D_\phi \nabla^2 \rho \end{cases}$$

- Infinitely many absorbing states (any $\phi(x,t)$ with $\rho(x,t) = 0$ is absorbing)
- Critical exponents are different from DP, but unfortunately close...

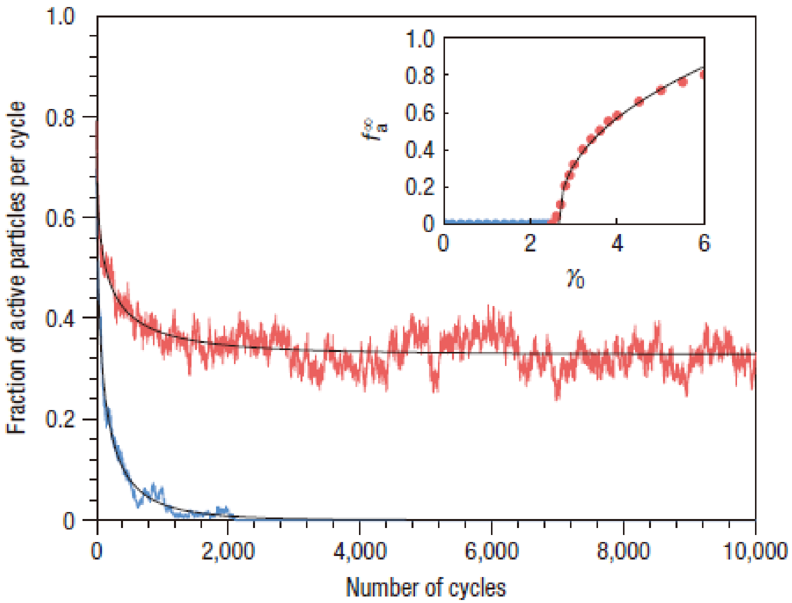
| | (2+1)d DP | (2+1)d C-DP | (3+1)d DP | (3+1)d C-DP |
|---|---|---|---|---|
| $\beta$ | 0.584(3) | 0.624(29) | 0.813(11) | 0.840(12) |
| $\nu_\perp$ | 0.733(3) | 0.799(14) | 0.584(6) | 0.593(13) |
| $\nu_\parallel$ | 1.295(6) | 1.225(29) | 1.11(1) | 1.081(27) |

surface critical behavior is useful to distinguish them [Bonachela & Munoz 2007]

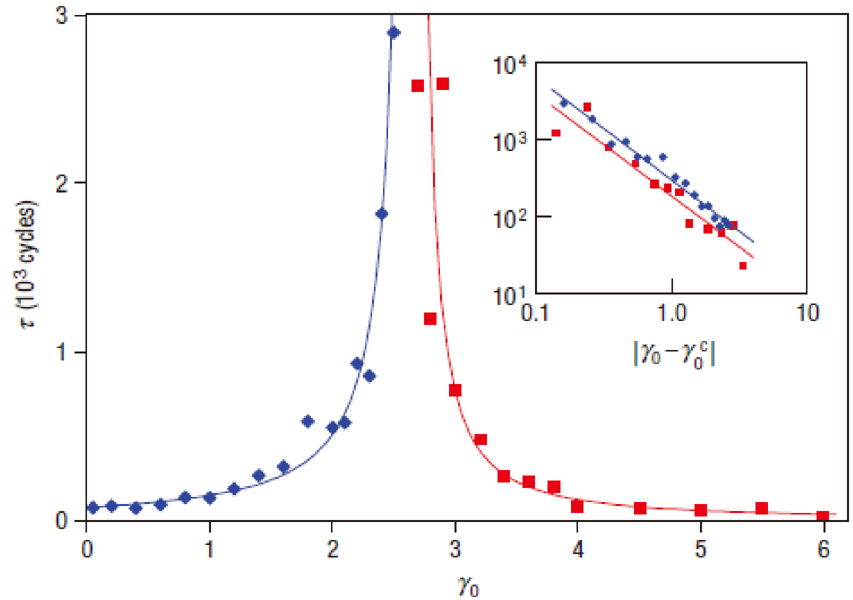
# Critical Behavior in Corté et al.'s Model

[Corté et al., Nat. Phys. 4, 420 (2008)]

order parameter
(fraction of colliding particles)




relaxation time




➩ $\beta = 0.45(2)$

$\beta^{\rm CDP} = 0.639(9), \beta^{\rm DP} = 0.584(3)$

➩ $\nu_{||} = 1.33(2)$

$\nu_{||}^{\rm CDP} = 1.225(29), \nu_{||}^{\rm DP} = 1.295(6)$

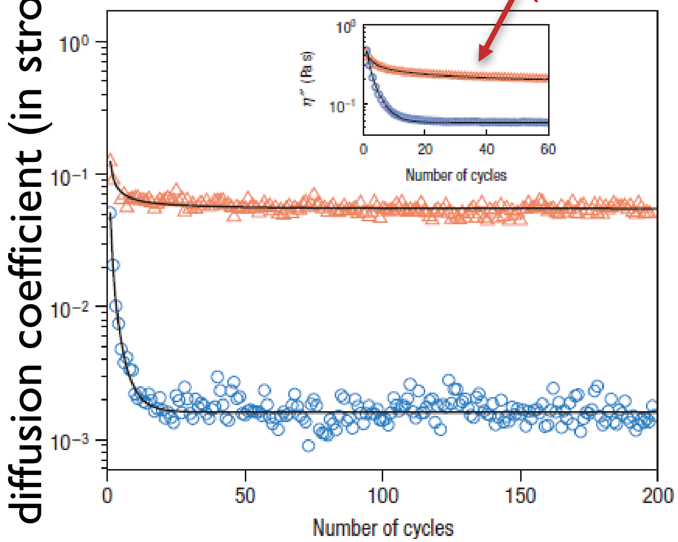
*not* in quantitative agreement with C-DP or DP...

# Critical Behavior in Experiment

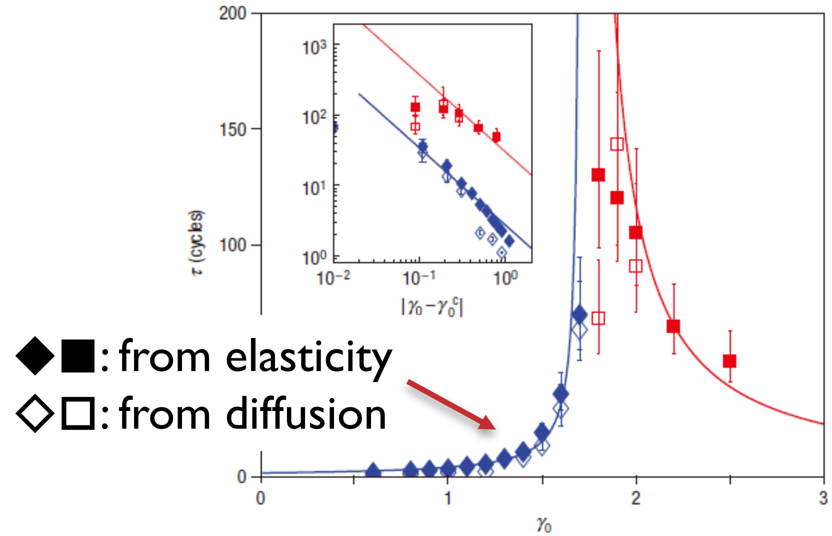
[Corté et al., Nat. Phys. 4, 420 (2008)]

order parameter

imaginary part of complex viscosity (elastic component)




relaxation time




⇨ $\beta = 0.45(10)$

$\beta^{\mathrm{CDP}}_{(2+1)\mathrm{d}} = 0.639(9), \beta^{\mathrm{DP}}_{(2+1)\mathrm{d}} = 0.584(3)$

$\beta^{\mathrm{CDP}}_{(3+1)\mathrm{d}} = 0.840(12), \beta^{\mathrm{DP}}_{(3+1)\mathrm{d}} = 0.813(11)$

⇨ $\nu_{\|} = 1.1(3)$

$\nu^{\mathrm{CDP}}_{\|,(2+1)\mathrm{d}} = 1.225(29), \nu^{\mathrm{DP}}_{\|,(2+1)\mathrm{d}} = 1.295(6)$

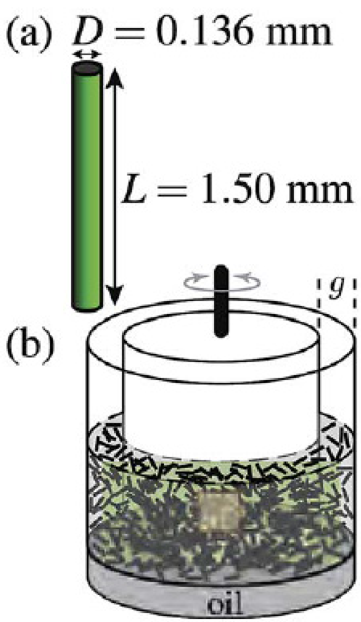
$\nu^{\mathrm{CDP}}_{\|,(3+1)\mathrm{d}} = 1.081(27), \nu^{\mathrm{DP}}_{\|,(3+1)\mathrm{d}} = 1.11(1)$

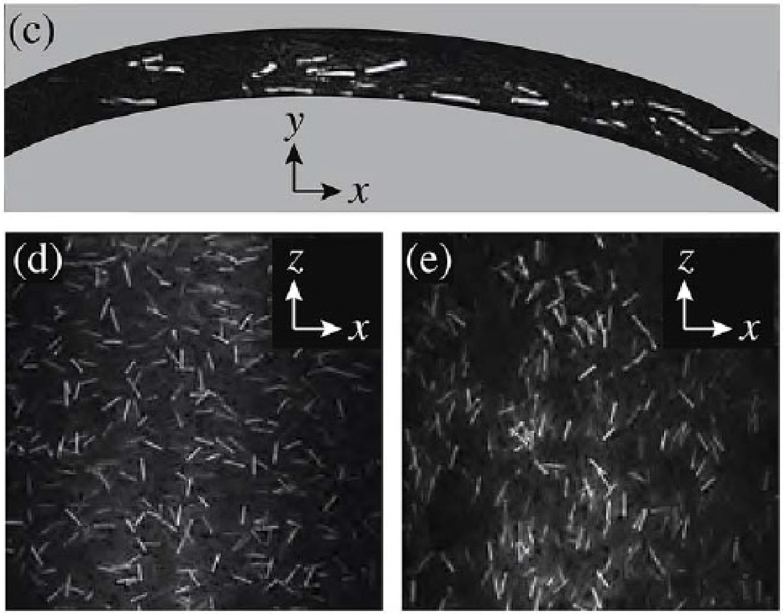
- *Not* in agreement with C-DP/DP. Hydrodynamic long-range interactions?
- Elastic component behaves like order parameter. Rheological consequence!

purely viscous (reversible) ⇨ viscoelastic (irreversible)

# With Rods...

[Franceschini et al., PRL 107, 250603 (2011);
Soft Matter 10, 6722 (2014)]



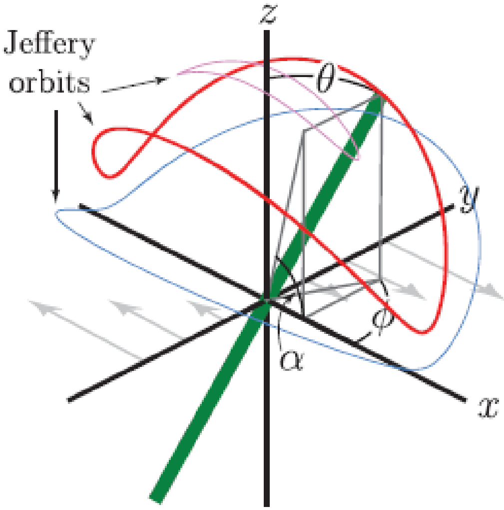




before

after

motion of a rod

Jeffery orbit




affects
effective volume fraction

Rods get aligned through interactions.

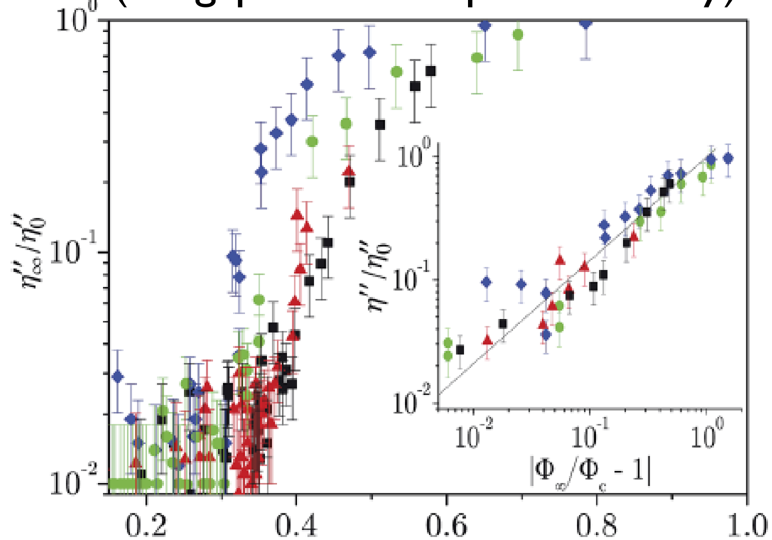
⇨ Tilting of Jeffery orbit changes in time, so does the effective volume fraction.

# With Rods...

[Franceschini et al., PRL 107, 250603 (2011);
Soft Matter 10, 6722 (2014)]

**steady-state order parameter**

(imag. part of complex viscosity)



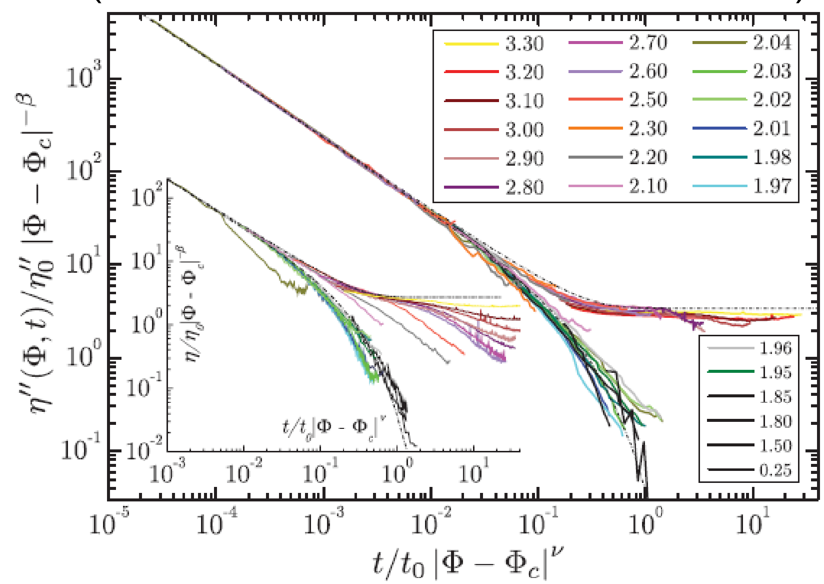

effective volume fraction $\Phi_\infty$

➪ $\beta = 0.84(4)$

$\beta^{\mathrm{CDP}}_{(3+1)\mathrm{d}} = 0.840(12), \beta^{\mathrm{DP}}_{(3+1)\mathrm{d}} = 0.81(1)$

**relaxation process**

(inset: with constant volume fraction)




➪ $\beta = 0.86(7), \nu_{||} = 1.09(5)$

$\beta^{\mathrm{CDP}}_{(3+1)\mathrm{d}} = 0.840(12), \beta^{\mathrm{DP}}_{(3+1)\mathrm{d}} = 0.81(1)$

$\nu^{\mathrm{CDP}}_{||,(3+1)\mathrm{d}} = 1.081(27), \nu^{\mathrm{DP}}_{||,(3+1)\mathrm{d}} = 1.105(5)$

Agreement with C-DP class! (also with DP class)

Why different from spheres? Hydrodynamic long-range effect?

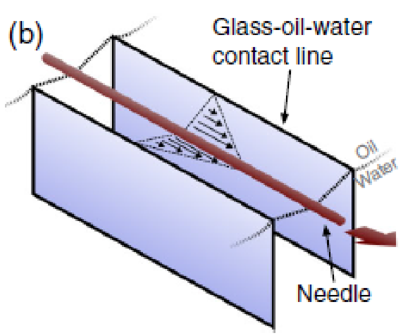
# Dense Case

high volume fraction, particles jammed ⇨ all particles interact & cage effect

- Less well understood, different observations from different systems, but reversible-irreversible transition seems to exist as well.
- Three regimes [Keim & Arratia, PRL 112, 028302 (2014); see also Regev et al., PRE 2013]

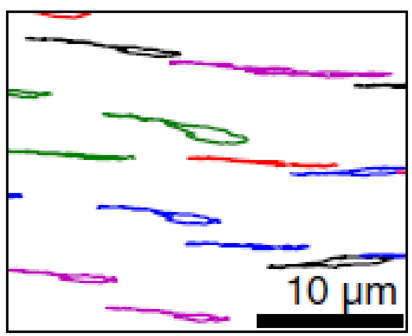
> ➢ $\gamma_0 \ll \gamma_c$ : reversible (back & forth), nearly affine deformation, elastic
> ➢ $\gamma_0 \lesssim \gamma_c$ : reversible (loop), non-affine with T1 events, viscosity emerges
> ➢ $\gamma_0 \gtrsim \gamma_c$ : irreversible, plastic deformation, related to yielding?

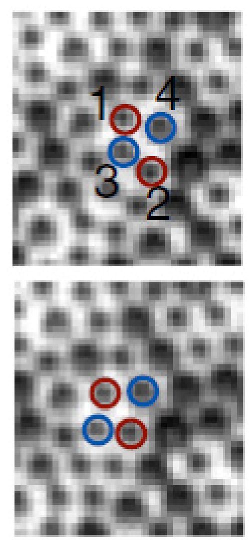
(Keim & Arratia, PRL 2014)

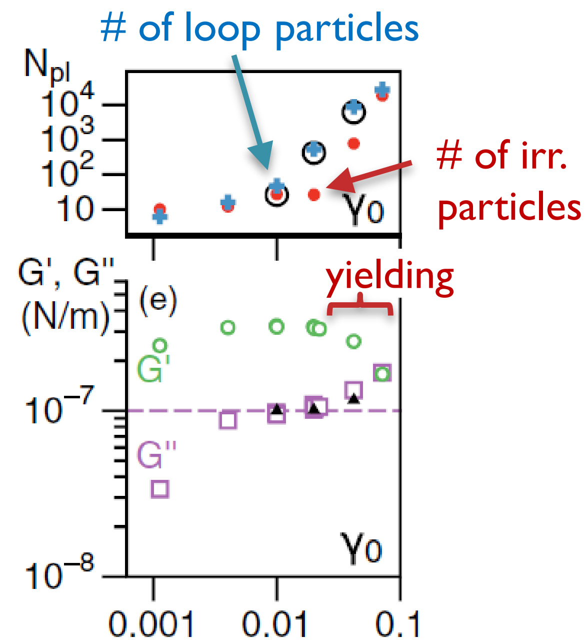



bidisperse PS particles, electrostatically jammed

loop-reversibility regime



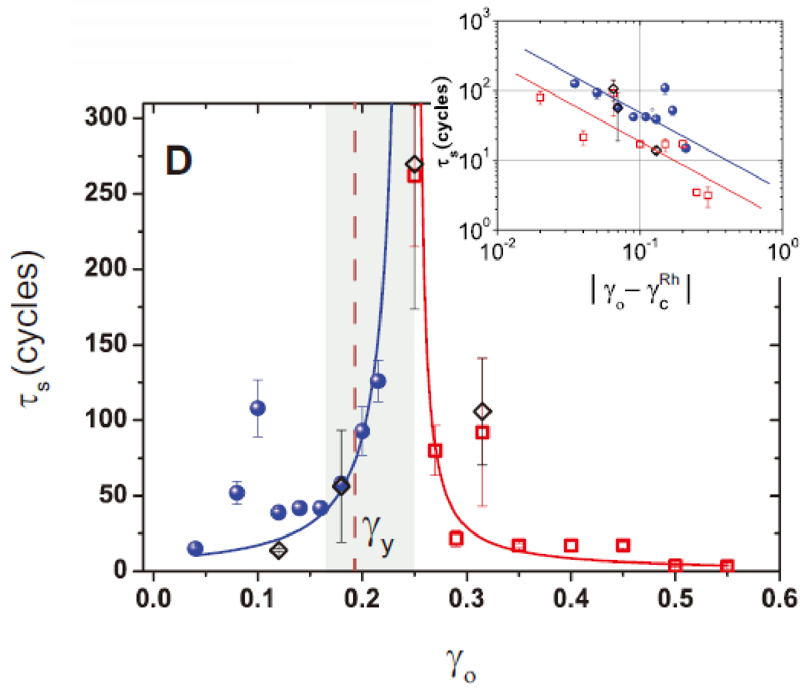



# Question 1: Continuous vs Discontinuous

## confocal rheometer experiment

bidisperse PNIPAM particles, $\phi \approx 0.67$

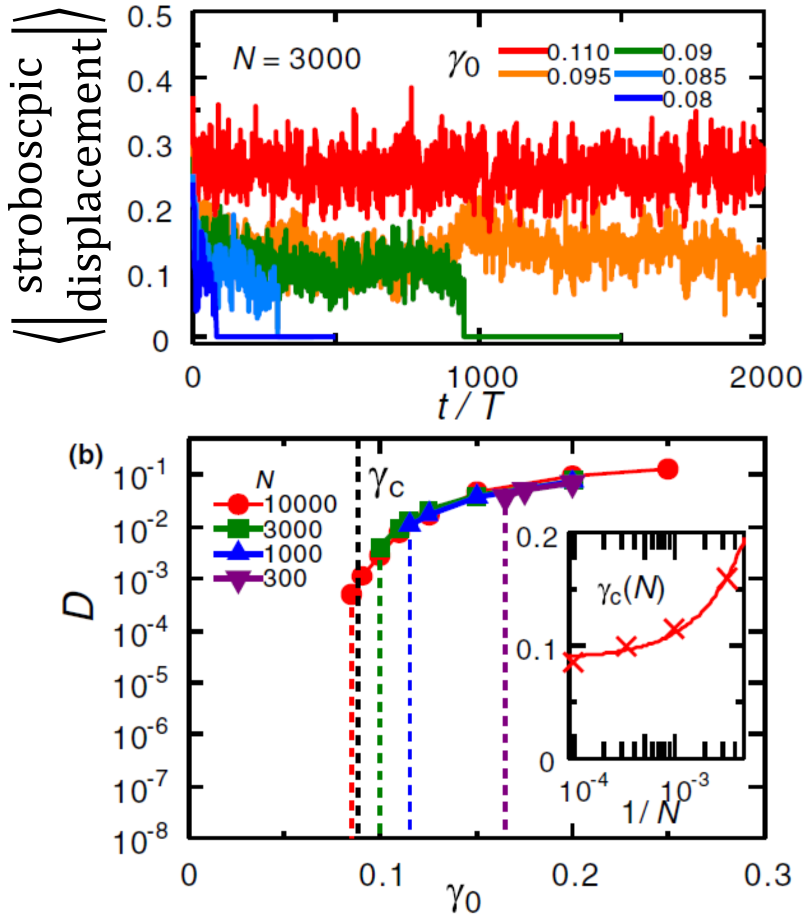
[Hima Nagamanasa et al., PRE 2014]




## simulations

soft repulsive particles, $\phi \approx 0.80$

[Kawasaki & Berthier, PRE 2016]

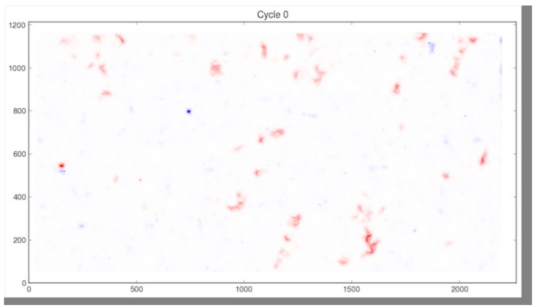


# Question 2: Homogenous vs Inhomogeneous

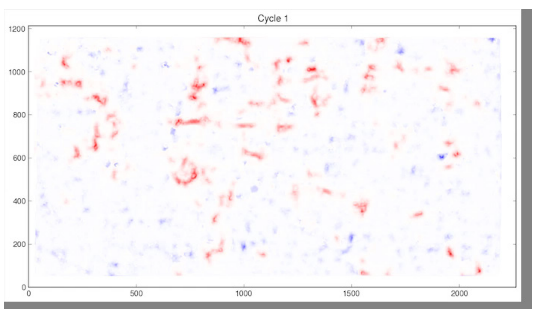
### interfacial rheometer experiment

bidisperse PS particles, electrostatically jammed
[Keim & Arratia, PRL 2014]

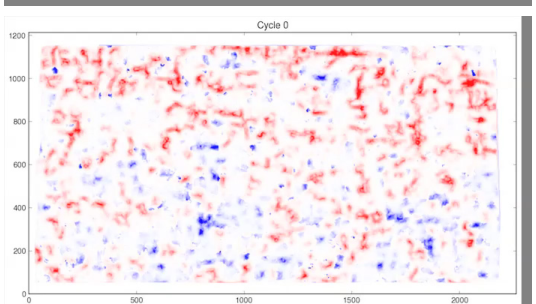
color = non-affine deformation events




$\gamma_0 = 0.01$
reversible
(back & forth)



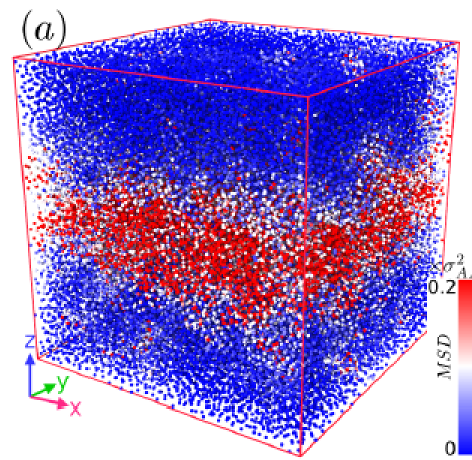

$\gamma_0 = 0.02$
reversible
(loop)



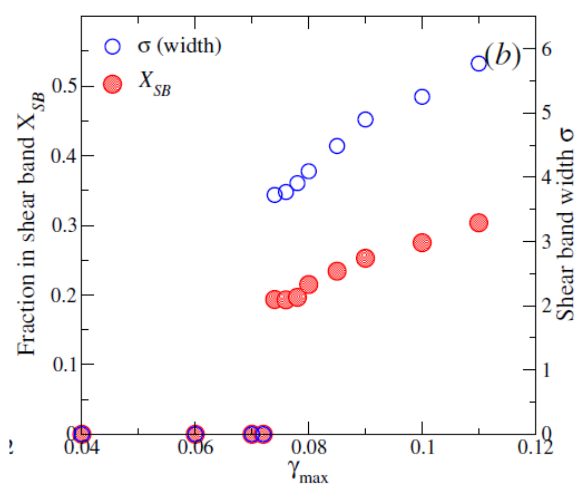

$\gamma_0 = 0.04$
irreversible

### simulations

bidisperse Lennard-Jones w/ cut-off
[Parmar, Kumar, Sastry, arXiv:1806.02464]




shear banding
for $\gamma_0 \gtrsim \gamma_{\text{yield}}$



# Question 3: Connection to the Dilute Limit

Tendencies (there may be counter-examples!)

| | | dilute ($\phi \ll \phi_J$) | dense ($\phi \gtrsim \phi_J$) |
|---|---|---|---|
| Particle motion (as $\gamma_0 \nearrow$) | | reversible<br>→ irreversible | reversible (back & forth)<br>[→ reversible (loop)]<br>→ irreversible |
| Rheology | | purely viscous<br>→ elasticity emerges | purely elastic<br>→ viscosity emerges<br>→ yielding |
| RIT | continuity | continuous | discontinuous? continuous? |
| | homogeneity | homogeneous | shear banding?<br>homogeneously disordered? |
| | as an absorbing-state transition | ?? (spheres)<br>C-DP class (rods) | ?? (even if continuous & homogeneous) |

How can those be connected?
[cf. phase diagram in
Schreck et al., PRE 2013]

AST with disorder?
any possibility of
"activated scaling" ??
[see Vojta, PRE 2012]

# Summary

**Current status of absorbing-state transitions**

- Transitions into an absorbing state (no further state change allowed)
- Most fundamental = directed percolation (DP) class.
  - Established based on toy models (mostly decades ago)
  - Now relevant in real experiments & realistic models: turbulence in liquid crystal, quantum fluid, Newtonian flow, active matter
  - Practical criteria for being in the DP class?

**Connections to reversible-irreversible transitions**

- Transition between reversible & irreversible particle motion in suspensions under oscillatory shear.
- A kind of AST (almost by definition), rheological consequences.
- Dilute case: relation to C-DP class (DP with conserved field), but confirmed with rods only.
- Dense case: largely unsettled, intriguing features (loop reversibility, relation to yielding, ...) and many open problems!