

Universal properties in externally driven systems and active matter with spatial anisotropy

KA, Takasan & Kawaguchi, Phys Rev Research 4, 013194 (2022)

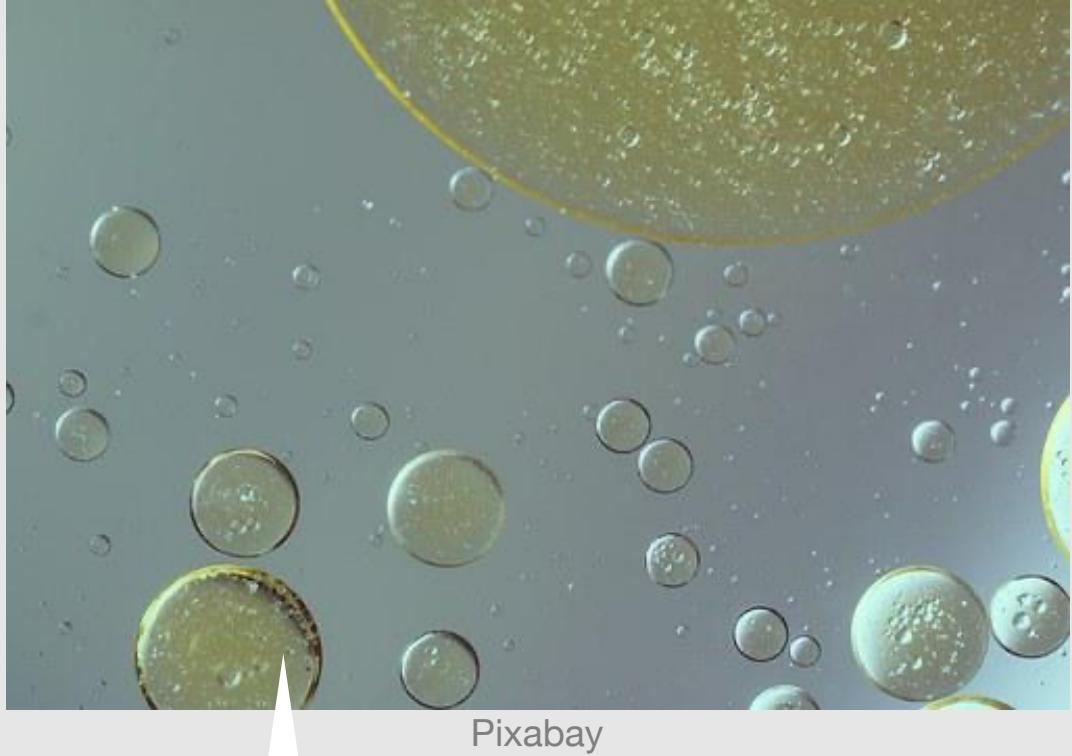
Kyosuke Adachi (RIKEN BDR/iTHEMS)

Collaborators: K. Takasan, H. Nakano, and K. Kawaguchi

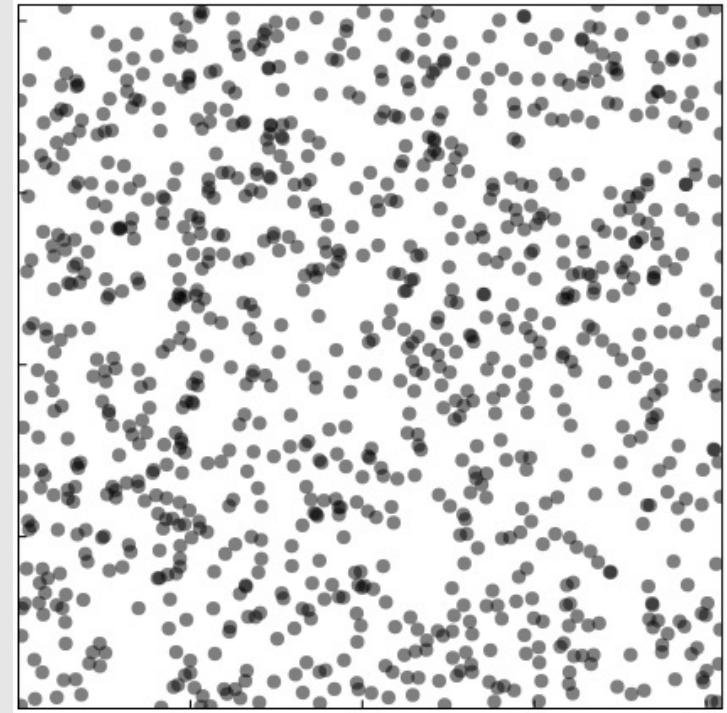
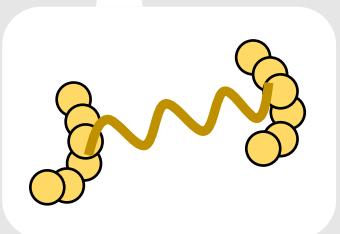
Introduction

Equilibrium phase separation by attractive force

1/15



Pixabay

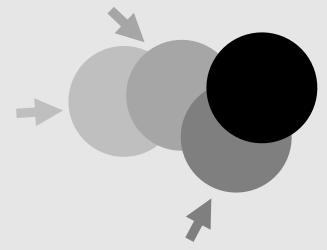


$$i = 1, 2, \dots, N$$

$$\frac{d\mathbf{r}_i}{dt} = \mu \sum_{j(\neq i)} \mathbf{F}_{ij} + \xi_i$$

Translational noise

Attractive force + Exclusion



Motility-induced phase separation with no attractive force

2/15

Active Brownian particles (2D)

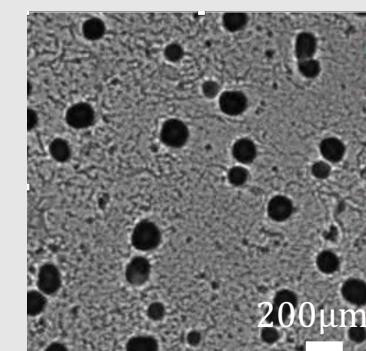
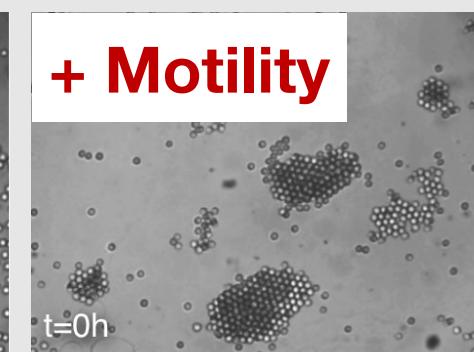
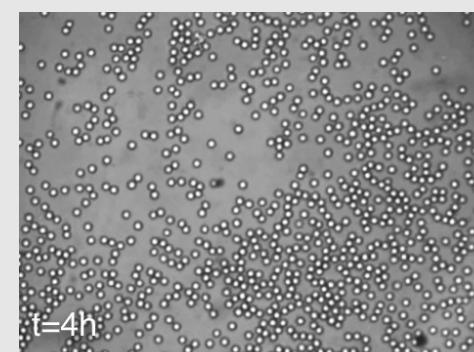
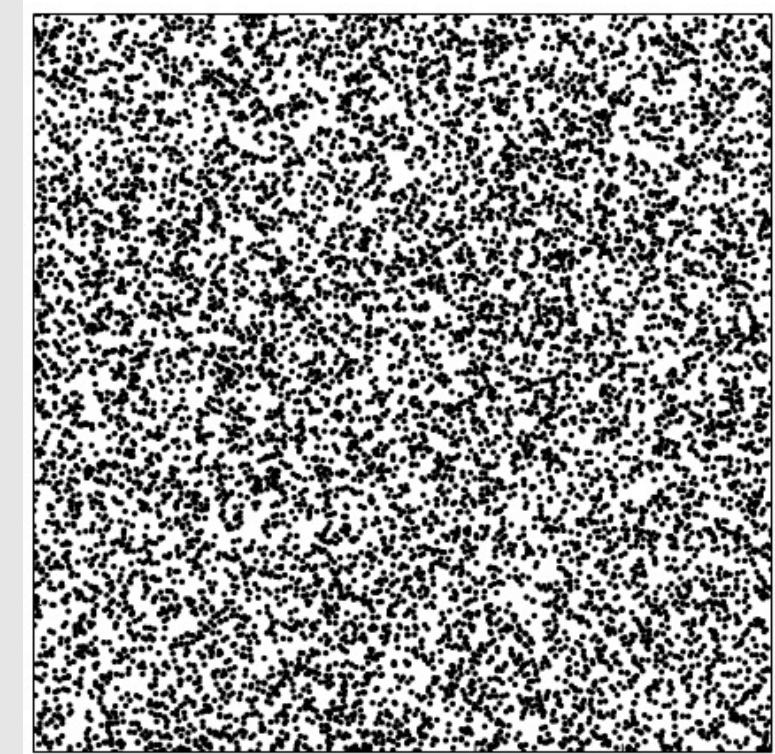
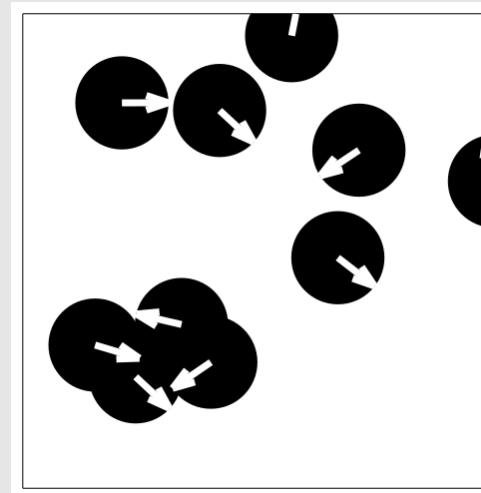
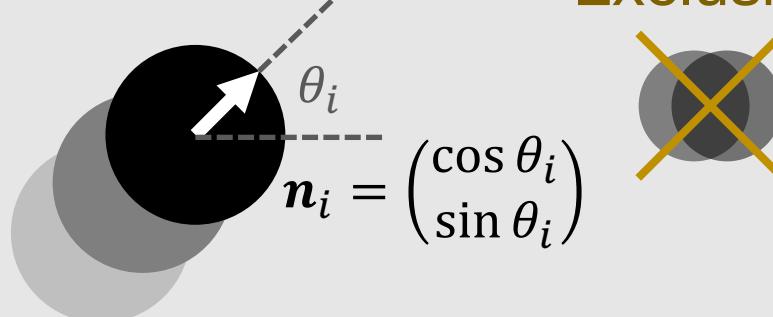
Fily & Marchetti, PRL (2012)

$$\frac{d\theta_i}{dt} = \xi_i^\theta \quad \text{Rotational noise}$$

$$\frac{dr_i}{dt} = v_0 n_i + \mu \sum_{j(\neq i)} F_{ij} + \xi_i \quad \text{Translational noise}$$

Motility

Exclusion



Buttinoni et al., PRL (2013)

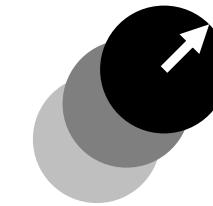
Liu et al., PRL (2019)

Comparison of motility-induced & equilibrium phase separation

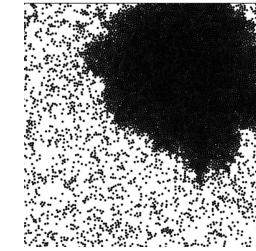
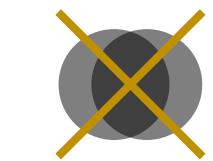
3/15

Motility-induced
phase separation

Motility



Exclusion

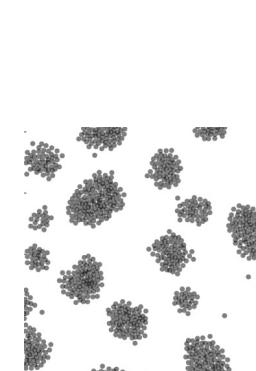
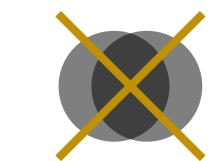


Equilibrium
phase separation

**Attractive
force**



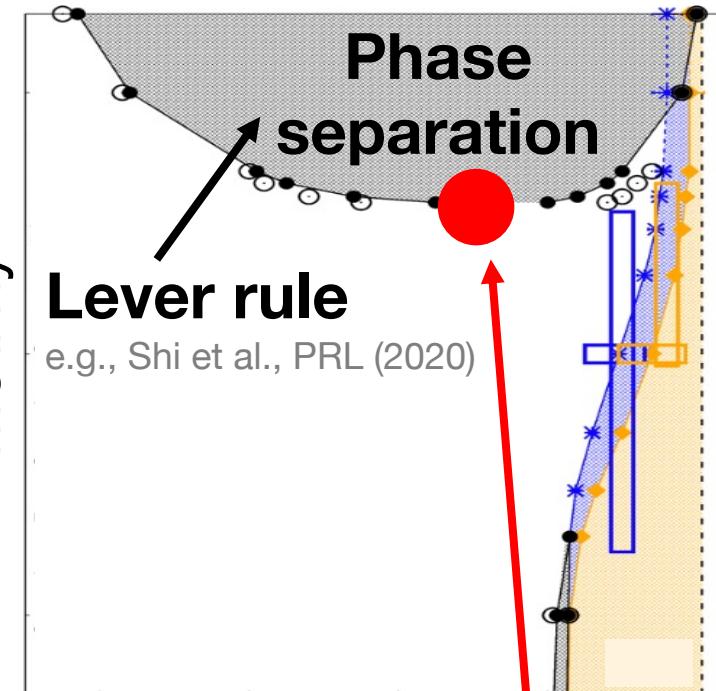
Exclusion



Typical phase diagram (2D)

Digregorio et al., PRL (2018)

Motility



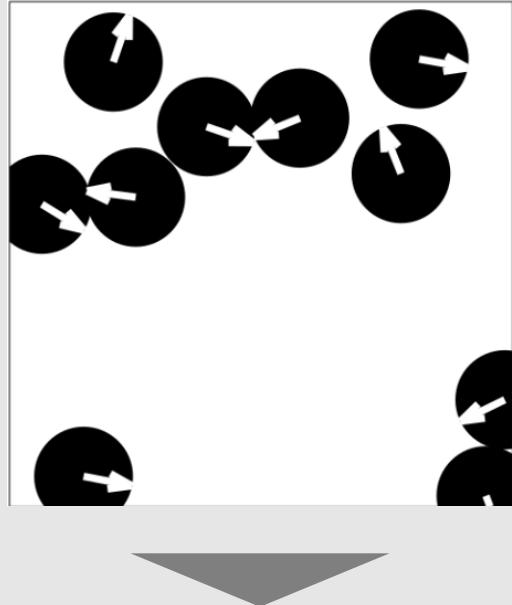
Critical phenomena

Siebert et al., PRE (2018); Partridge & Lee, PRL (2019)
Dittrich et al., EPJE (2021); Speck, PRE (2022)

Our focus: two classes of nonequilibrium phase separation

4/15

Motility-induced phase separation

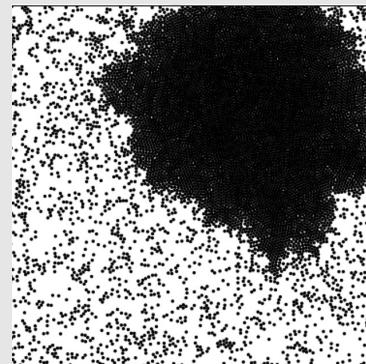


Motility

Exclusion



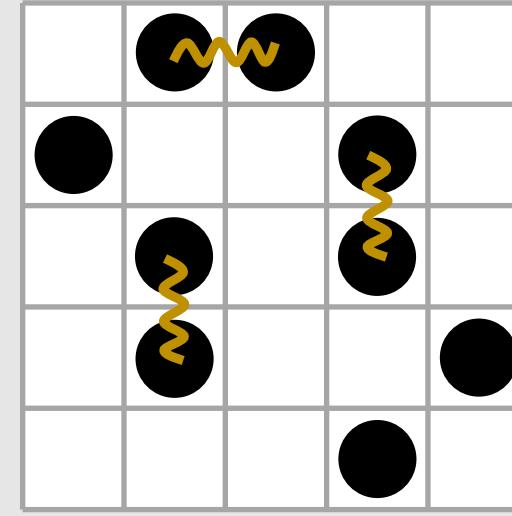
Any connection?



Spatial anisotropy
will be the key

Phase separation under external driving

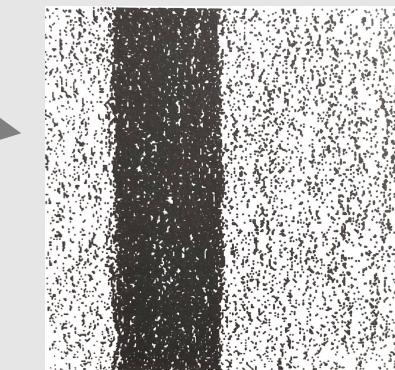
e.g., Schmittmann & Zia, "Statistical Mechanics of Driven Diffusive Systems"



Attractive
force

Exclusion

External driving



Marro & Dickman (1999)

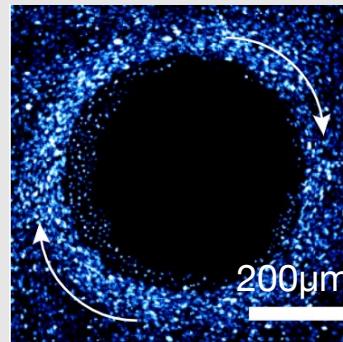
Question

5/15

Effect of inhomogeneity

Dish surface coated with laminin

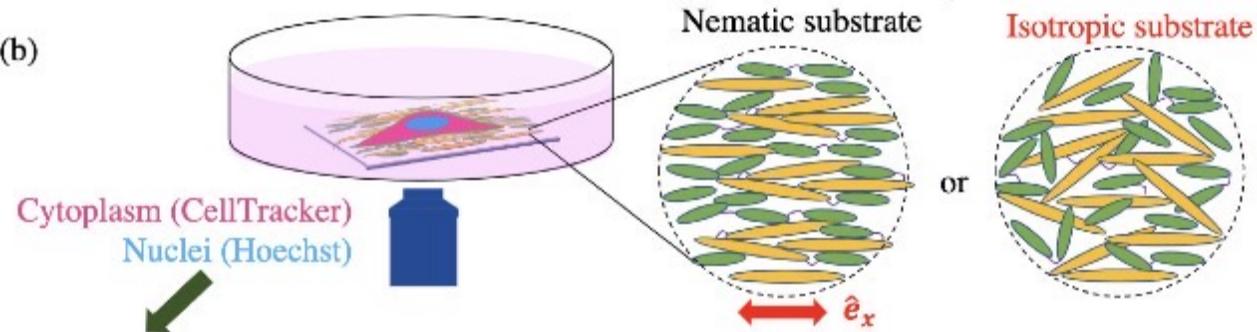
Area passivated
with poloxamer



Yamauchi et al., arXiv:2008.10852

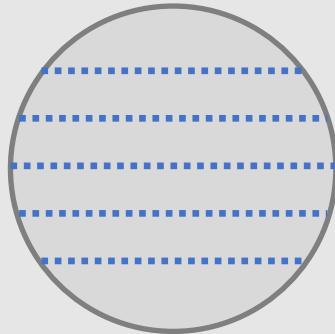
Effects of anisotropy

(b)

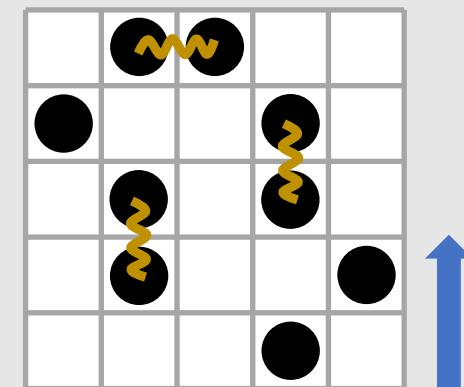
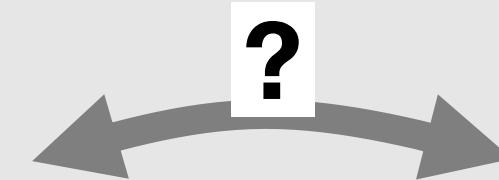
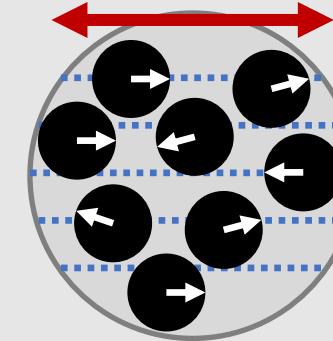


Luo et al., arXiv:2206.12574

Substrate orientation



Anisotropy of motility

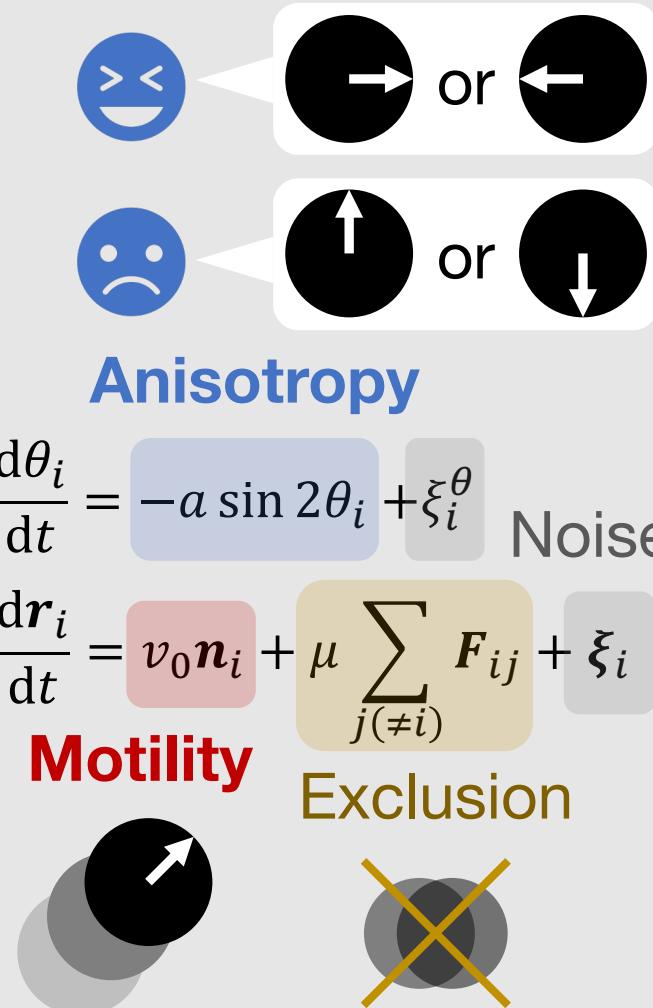


Can anisotropic motility in active matter provide a connection to externally driven systems?

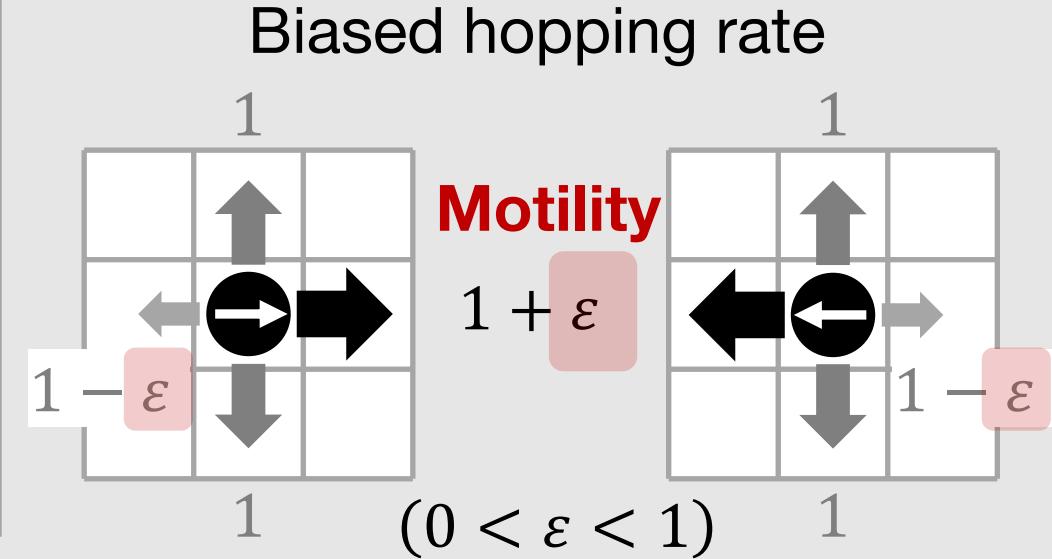
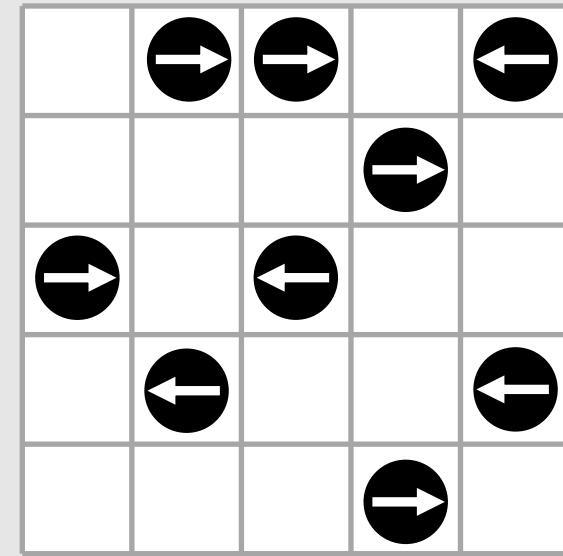
We consider two models with anisotropic motility

6/15

Active Brownian particles (2D)
Nakano & KA, in prep.



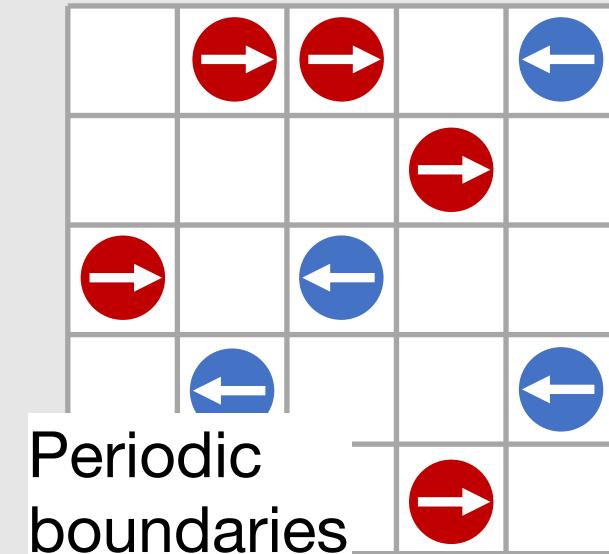
Active lattice gas (2D)
KA, Takasan & Kawaguchi, Phys Rev Research 4, 013194 (2022)
Thompson et al., J Stat Mech (2011); Kourbane-Houssene et al., PRL (2018)



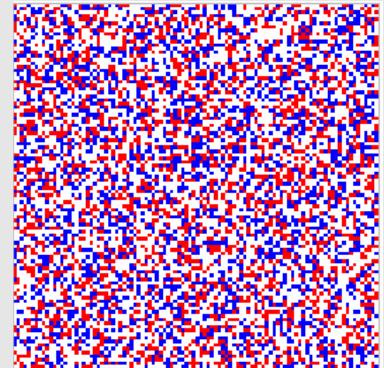
Connection to externally driven systems: phase transition property

Anisotropic phase separation reminiscent of driven systems

7/15

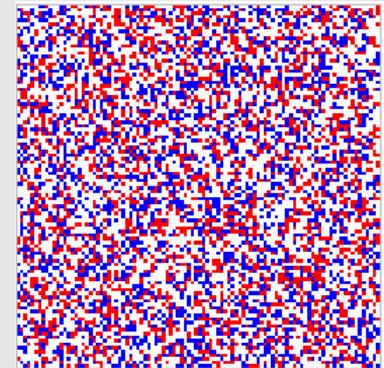


Homogeneous



$$\varepsilon = 0.1$$

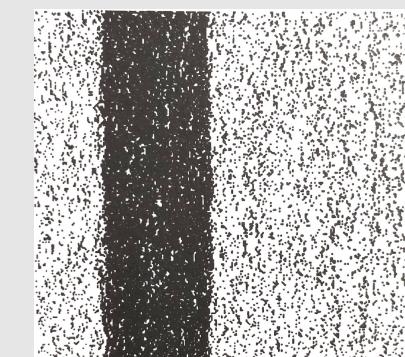
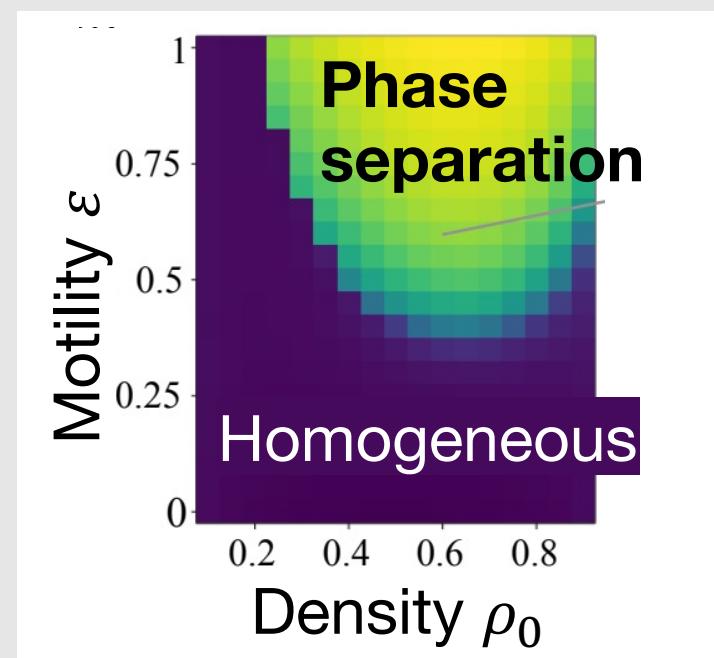
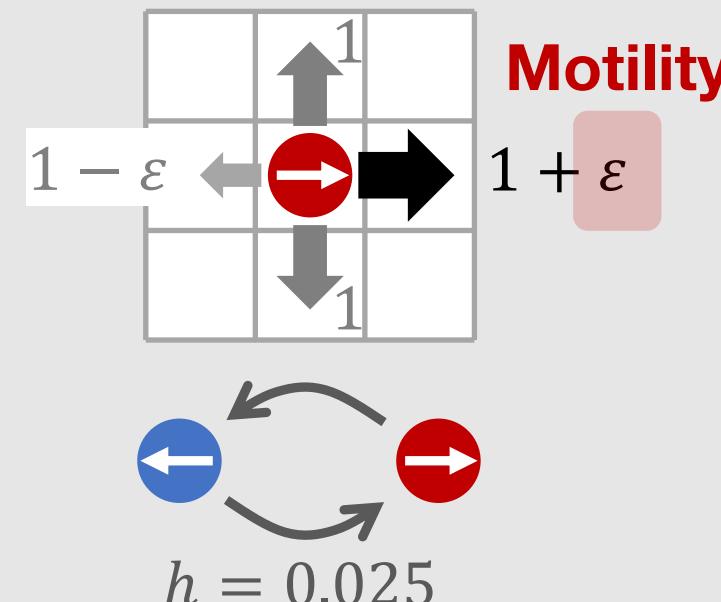
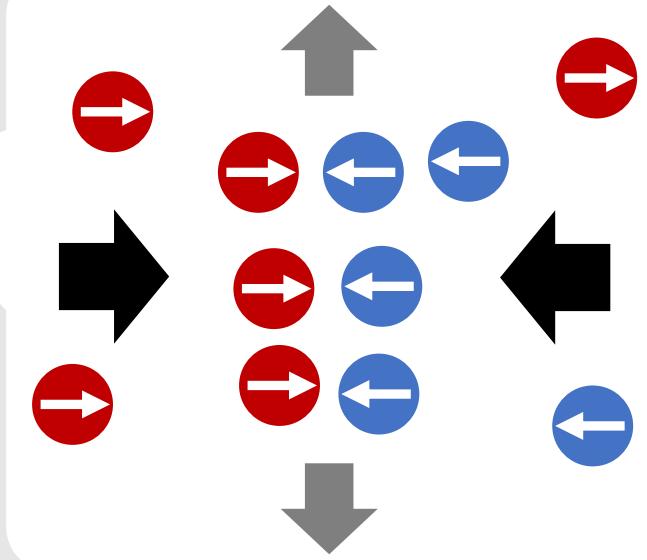
Phase separation



$$\rho_0 = 0.5$$

$$\varepsilon = 1$$

PRR 4, 013194 (2022)



Marro & Dickman (1999)

Attractive force



External driving

Approximate fluctuating hydrodynamic equation

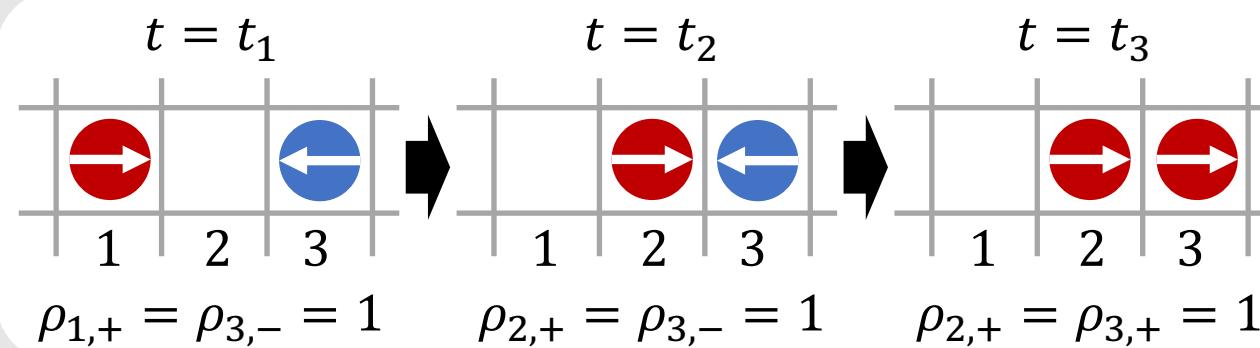
8/15

Andrianov et al., PRE (2006); Lefèvre & Biroli, J Stat Mech (2007); Partridge & Lee, PRL (2019)

PRR 4, 013194 (2022)

Trajectory

$$\{\rho_{i,s}(t)\}_{t=0}^T$$



Probability for trajectory

$$P[\{\rho_{i,s}(t)\}] = \int D\tilde{\rho} \exp(-S[\rho, \tilde{\rho}])$$

$$D\tilde{\rho} := \prod_{i,s,t} d\tilde{\rho}_{i,s}(t)$$

$$S[\rho, \tilde{\rho}] := -i \int_0^T dt \sum_{i,s} \tilde{\rho}_{i,s} \partial_t \rho_{i,s} - \int_0^T dt \sum_{i,s} \rho_{i,s} \left\{ \sum_{j(\sim i)} (1 - \rho_j) [e^{i(\tilde{\rho}_{i,s} - \tilde{\rho}_{j,s})} - 1] + h [e^{i(\tilde{\rho}_{i,s} - \tilde{\rho}_{i,-s})} - 1] \right\}$$

$$- \varepsilon \int_0^T dt \sum_{i,s} s \rho_{i,s} \left\{ (1 - \rho_{i+\hat{x}}) [e^{i(\tilde{\rho}_{i,s} - \tilde{\rho}_{i+\hat{x},s})} - 1] - (1 - \rho_{i-\hat{x}}) [e^{i(\tilde{\rho}_{i,s} - \tilde{\rho}_{i-\hat{x},s})} - 1] \right\}$$



Long-wavelength approximation with $\rho_{i,s}(t) \in \{0,1\} \rightarrow \rho_s(\mathbf{r}) \in [0,1]$

$$S[\rho, \tilde{\rho}] \rightarrow -i \int_0^T dt \int d^2\mathbf{r} \sum_s \tilde{\rho} \{ \partial_t \rho_s - (\Delta \rho_s - \rho_{-s} \Delta \rho_s + \rho_s \Delta \rho_{-s}) + 2\varepsilon s \partial_x [(1 - \rho) \rho_s] + hs(\rho_+ - \rho_-) \} + \frac{1}{2} \int_0^T dt \int d^2\mathbf{r} \left[\sum_s 2(1 - \rho) \rho_s (\nabla \tilde{\rho}_s)^2 + h \rho (\tilde{\rho}_+ - \tilde{\rho}_-)^2 \right]$$



MSRJD approach (functional integral \Leftrightarrow Langevin eq.)

e.g., Tauber, "Critical dynamics"

$$\partial_t \rho_s = \Delta \rho_s - \rho_{-s} \Delta \rho_s + \rho_s \Delta \rho_{-s} - 2s\varepsilon \partial_x [(1 - \rho) \rho_s] - h(\rho_s - \rho_{-s}) + \xi_s$$

$$\begin{cases} \langle \xi_s(\mathbf{r}, t) \rangle = 0 \\ \langle \xi_s(\mathbf{r}, t) \xi_{s'}(\mathbf{r}', t') \rangle = M_{ss'} \delta^{(2)}(\mathbf{r} - \mathbf{r}') \delta(t - t') \\ M_{ss'} := \delta_{ss'} [-2\nabla \cdot (1 - \rho) \rho_s \nabla] + (2\delta_{ss'} - 1) h \rho \end{cases}$$

Interpretation of hydrodynamic equation

9/15

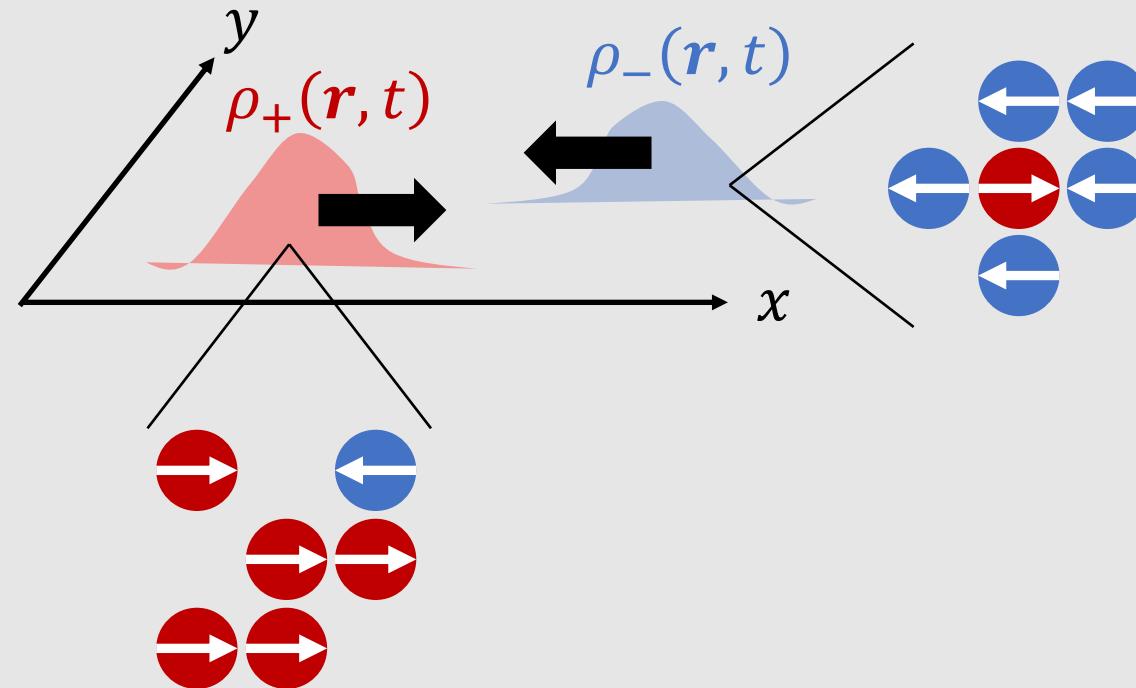
PRR 4, 013194 (2022)

$$\partial_t \rho_s = \Delta \rho_s - \rho_{-s} \Delta \rho_s + \rho_s \Delta \rho_{-s} - 2s\varepsilon \partial_x [(1-\rho) \rho_s] - h(\rho_s - \rho_{-s}) + \xi_s$$

Diffusion

Motility**Rotation** Noise

$$s \in \{+, -\}$$



Critical dynamics based on tree-level renormalization group

10/15

PRR 4, 013194 (2022)

$$\partial_t \rho_s = \Delta \rho_s - \rho_{-s} \Delta \rho_s + \rho_s \Delta \rho_{-s} - 2s\varepsilon \partial_x[(1-\rho)\rho_s] - h(\rho_s - \rho_{-s}) + \xi_s$$

↓
 $\rho := \rho_+ + \rho_-$
 $m := \rho_+ - \rho_-$

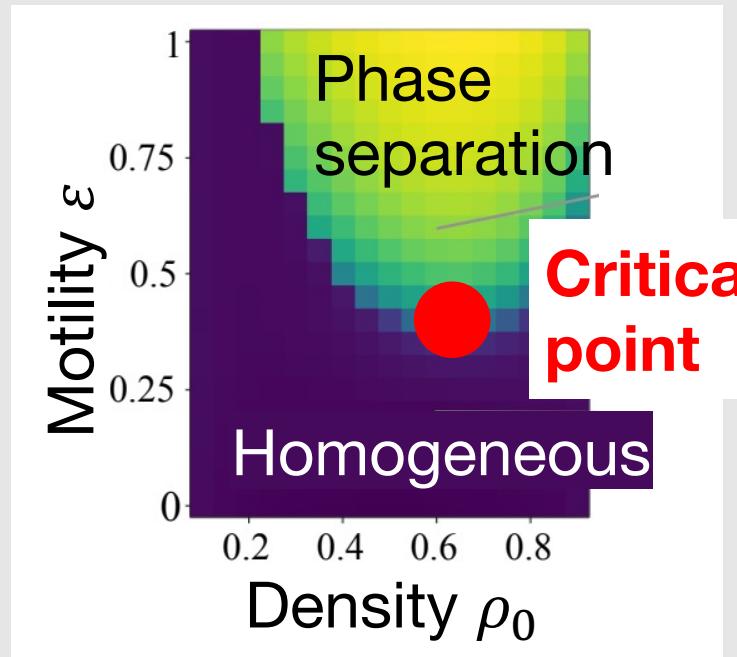
$$\begin{cases} \partial_t \rho = \Delta \rho - 2\varepsilon \partial_x[(1-\rho)m] + \xi_\rho \\ \partial_t m = (1-\rho)\Delta m + m\Delta\rho - 2\varepsilon \partial_x[(1-\rho)\rho] - 2hm + \xi_m \end{cases}$$

Exponential decay of m

- ↓ Adiabatic approximation $\partial_t m = 0$
↓ Neglect irrelevant terms for anisotropic phase separation

$$\partial_t \phi = \tau_x \partial_x^2 \phi + \tau_y \partial_y^2 \phi - a \partial_x^4 \phi + u \partial_x^2 \phi^3 + \xi$$

$$\phi(\mathbf{r}, t) := \rho(\mathbf{r}, t) - \rho_0$$



Connection to externally driven systems: transition property

11/15

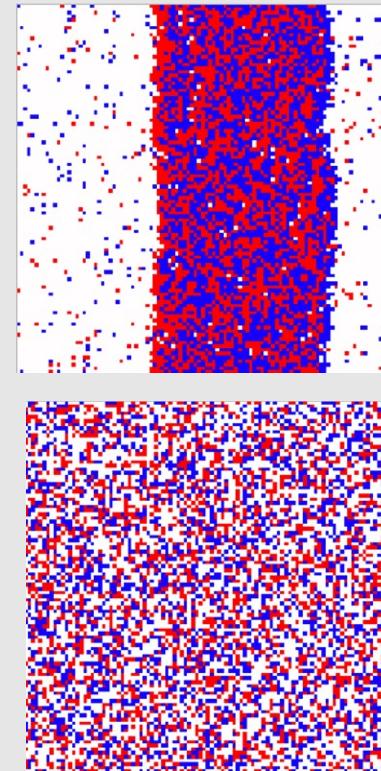
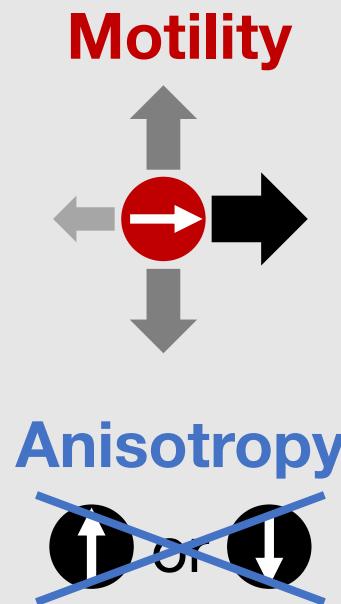
$$\partial_t \phi = \tau_x \partial_x^2 \phi + \tau_y \partial_y^2 \phi - a \partial_x^4 \phi + u \partial_x^2 \phi^3 + \xi$$

PRR 4, 013194 (2022)

Same as critical dynamics of randomly driven lattice gas

Præstgaard et al., Eur Phys J (2000)

Active lattice gas



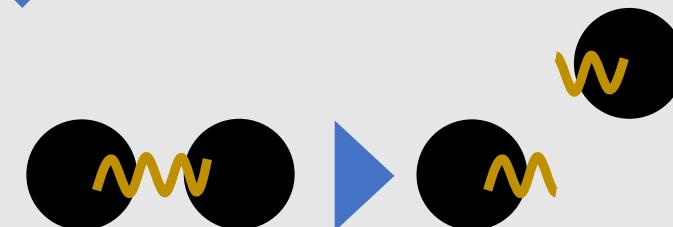
Randomly driven lattice gas

Præstgaard et al., Eur Phys J (2000)

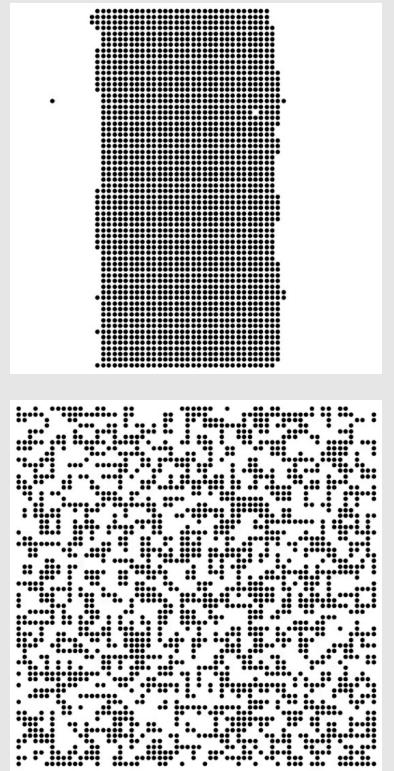
Attractive force



Temporally random external driving



Temperature ↘



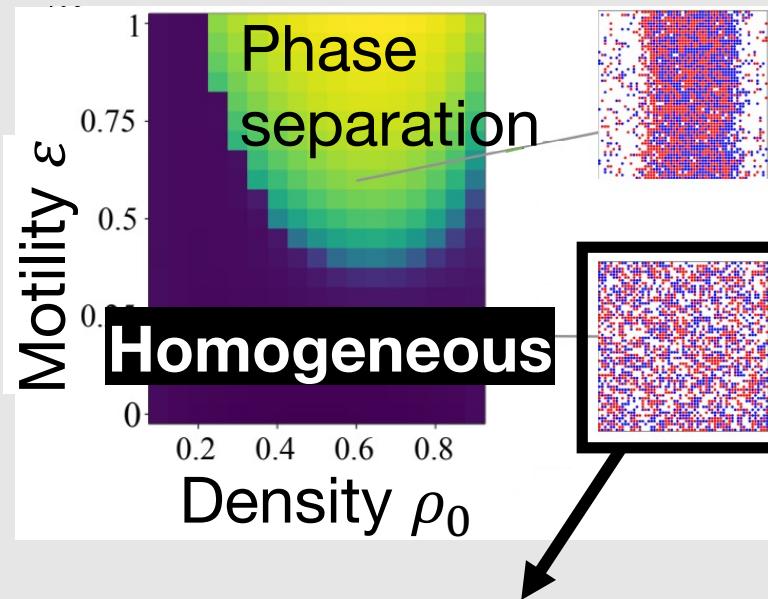
by H. Nakano

**Connection to externally driven systems:
homogeneous state property**

Singular structure factor & long-range correlation

12/15

PRR 4, 013194 (2022)

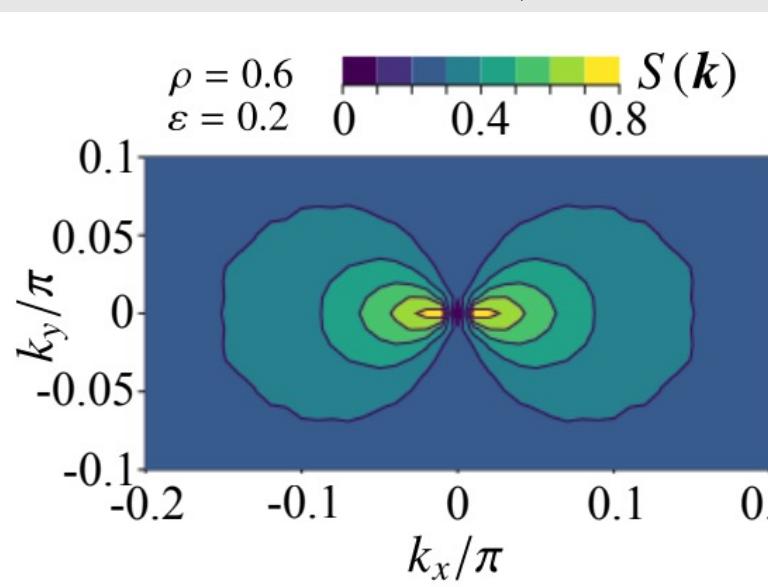
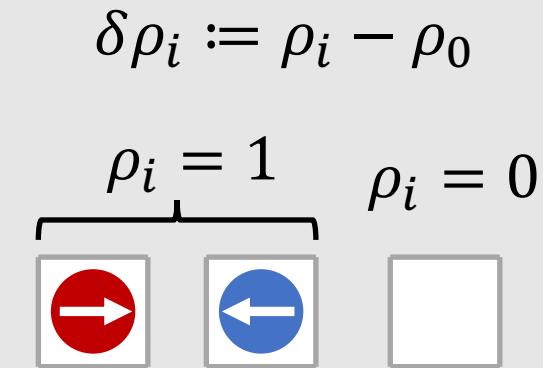


Density correlation function

$$C(\mathbf{r}_i - \mathbf{r}_j) := \langle \delta\rho_i \delta\rho_j \rangle$$

Fourier transformation

Structure factor $S(\mathbf{k})$



Singularity of $S(\mathbf{k})$ at $\mathbf{k} = 0$

$$S(k_x = 0, k_y \rightarrow 0) < S(k_x \rightarrow 0, k_y = 0)$$

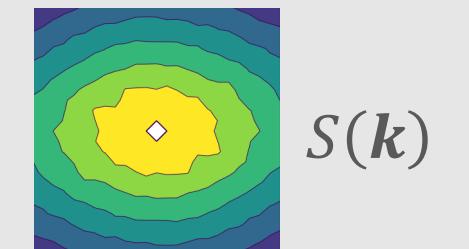
Inverse Fourier transformation

Schmittmann & Zia (1995)

Power-law decay of $C(\mathbf{r})$

$$C(\mathbf{r} \rightarrow \infty) \sim (-x^2 + y^2)/r^4$$

cf. Typical equilibrium models



Exponential decay of $C(\mathbf{r})$

Generic singularity of structure factor in homogeneous state

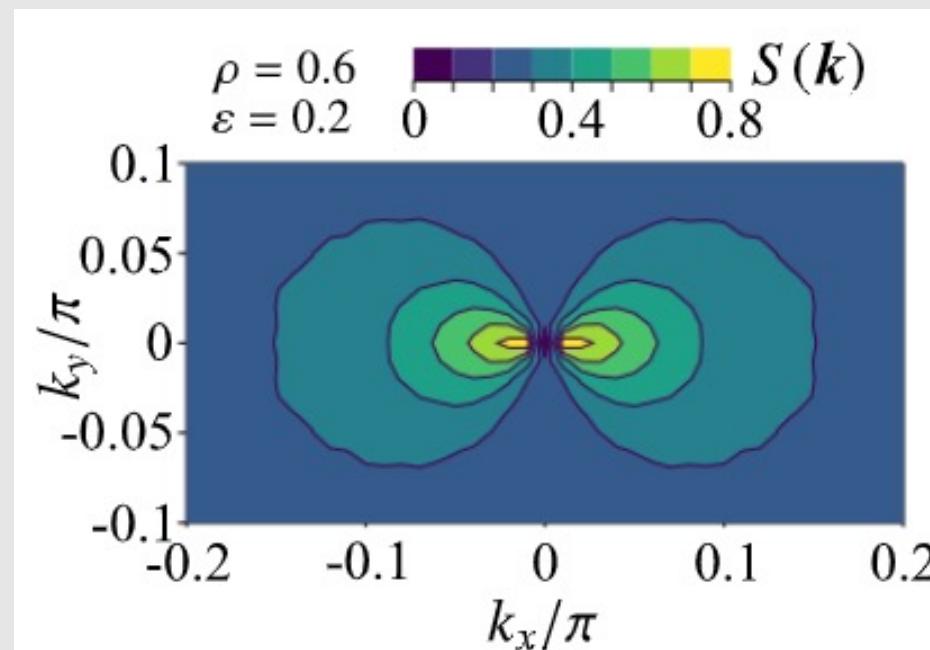
13/15

PRR 4, 013194 (2022)

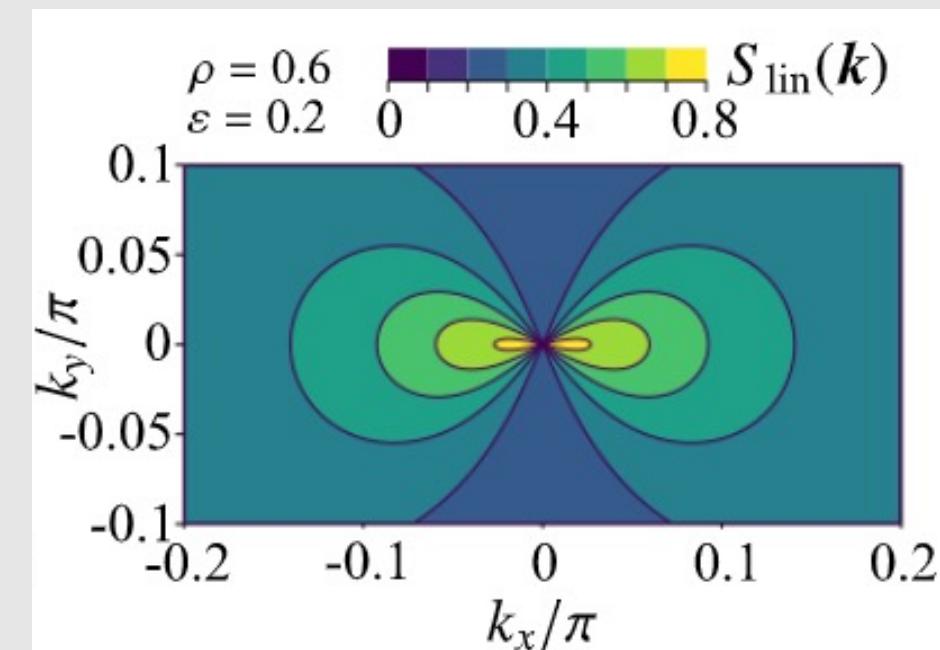
$$S_{\text{lin}}(\mathbf{k}) = (1 - \rho_0)\rho_0 \frac{[2h + (1 - \rho_0)\mathbf{k}^2]\mathbf{k}^2 + 4\varepsilon^2(1 - \rho_0)k_x^2}{[2h + (1 - \rho_0)\mathbf{k}^2]\mathbf{k}^2 - 4\varepsilon^2(1 - \rho_0)(2\rho_0 - 1)k_x^2}$$

Anisotropic motility ε  Singularity of $S(\mathbf{k})$ regardless of parameter details

Simulation



Plot of $S_{\text{lin}}(\mathbf{k})$



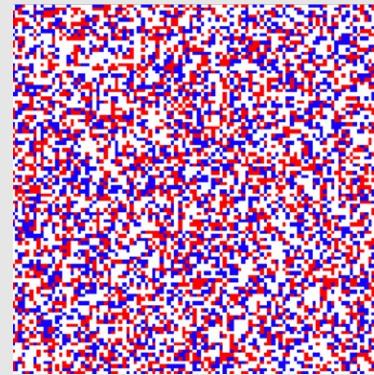
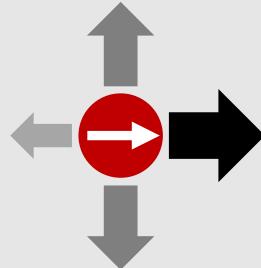
Connection to externally driven systems: homogeneous state property

14/15

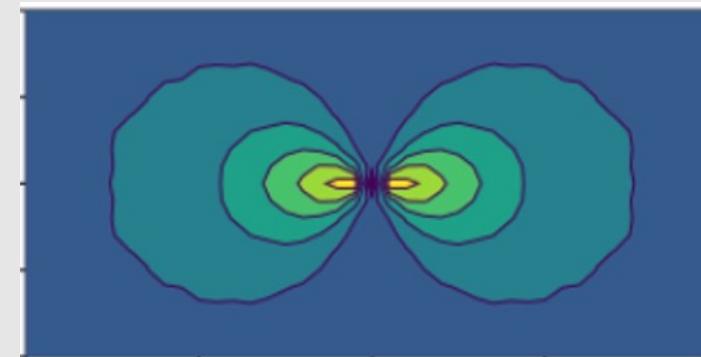
PRR 4, 013194 (2022)

Active lattice gas

Motility



Anisotropy

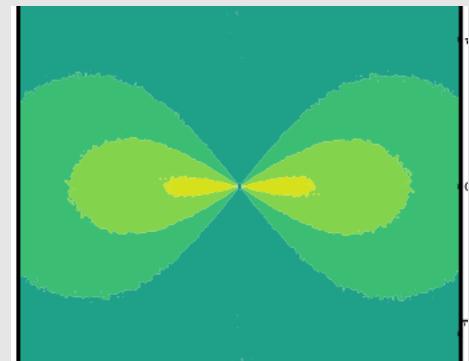
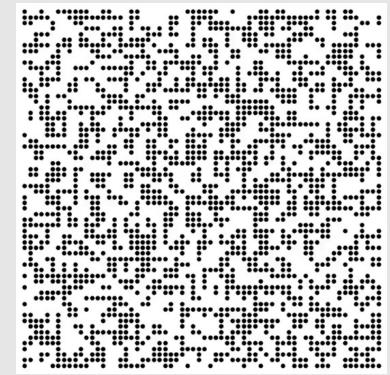
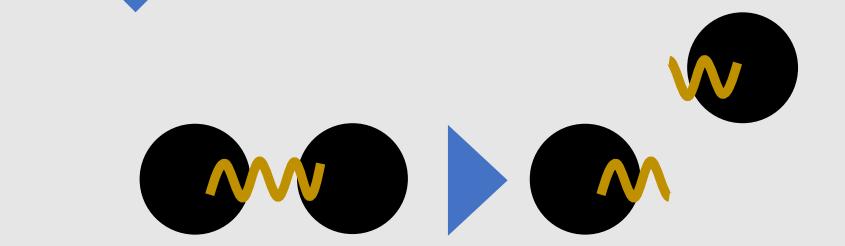


Randomly driven lattice gas

Attractive force



Temporally random external driving



by H. Nakano

Spatial anisotropy + violation of the detailed balance

e.g., Schmittmann & Zia, Phys Rep (1998)

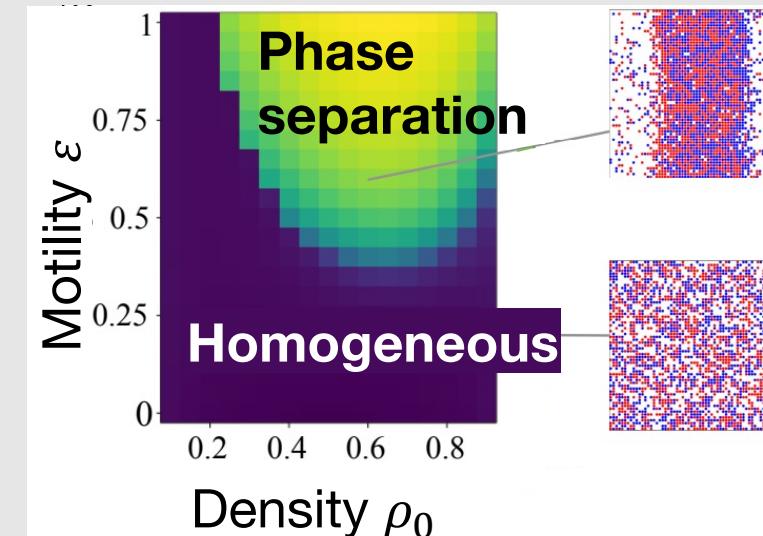
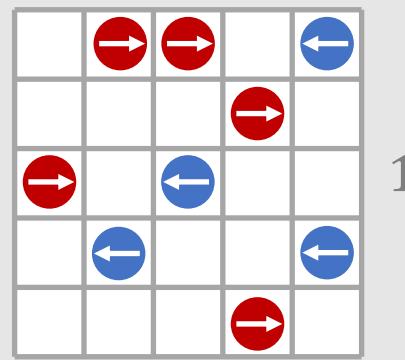
Universal properties of active & driven systems with anisotropy

15/15

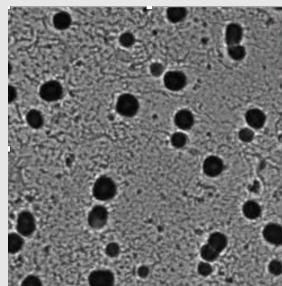
KA, Takasan & Kawaguchi, Phys Rev Research 4, 013194 (2022)

Nakano & KA, in prep.

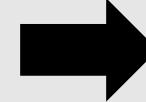
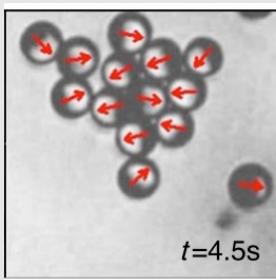
Active lattice gas



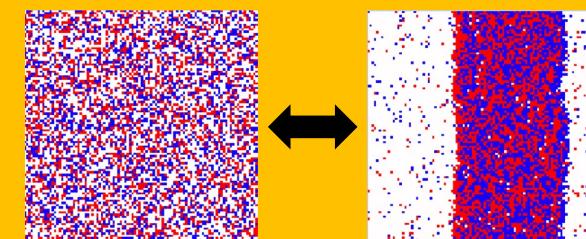
Active matter
with exclusion



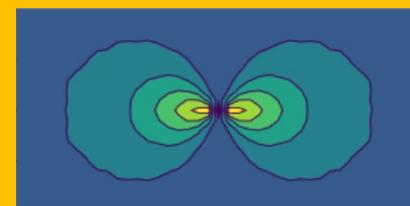
+ Spatial
anisotropy



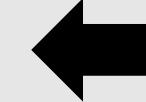
Anisotropic phase separation



Singular structure factor



+ External
driving



Particles with
attractive force

