Multi species asymmetric simple exclusion process with impurity activated flips

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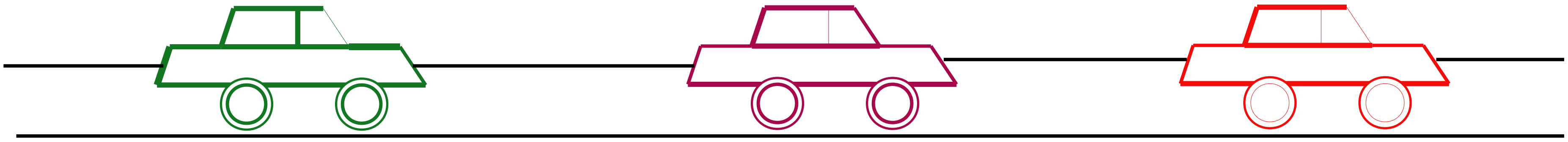
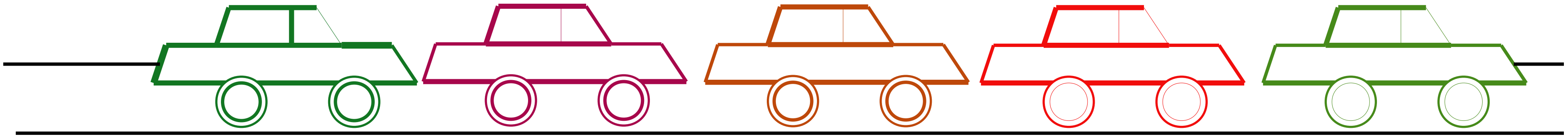
[References: SciPost Physics 14, 016 (2023); arXiv:2208.03297 (2022)]

Non-equilibrium systems

Transport phenomena [net non-zero current]



Phase transitions (e.g. traffic jam)



One general aim

understanding

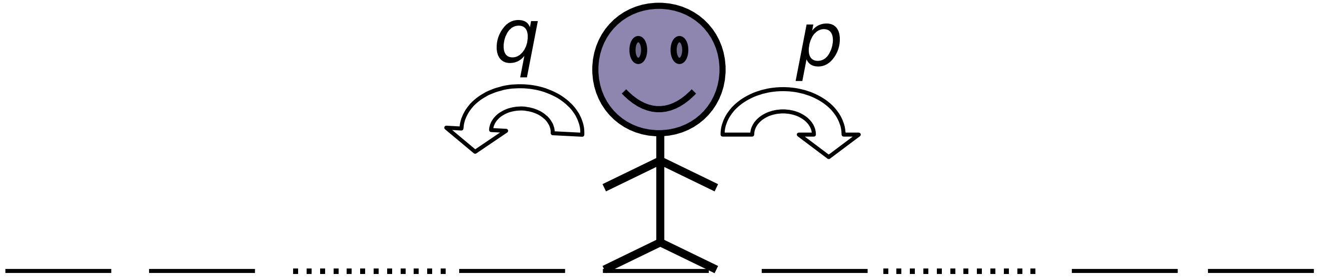
complex non-equilibrium phenomena

using

simple exactly solvable toy models

→ non-equilibrium steady state probability distribution

A random walker



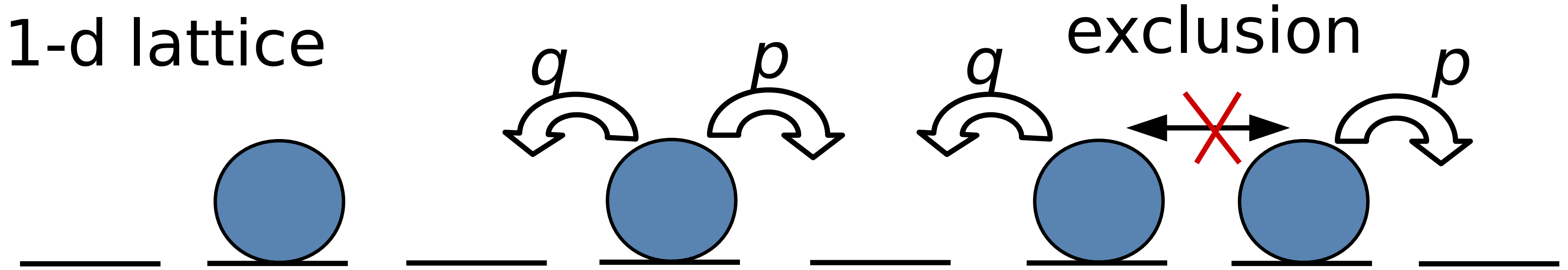
periodic lattice

Steady state: all equally likely configurations

Generalization: Many *interacting* random walkers

Model: Asymmetric Simple Exclusion Process (ASEP)

Asymmetric Simple Exclusion Process



Exactly solvable model

Boundary induced phase transitions

Applications: protein transport, *traffic flow*

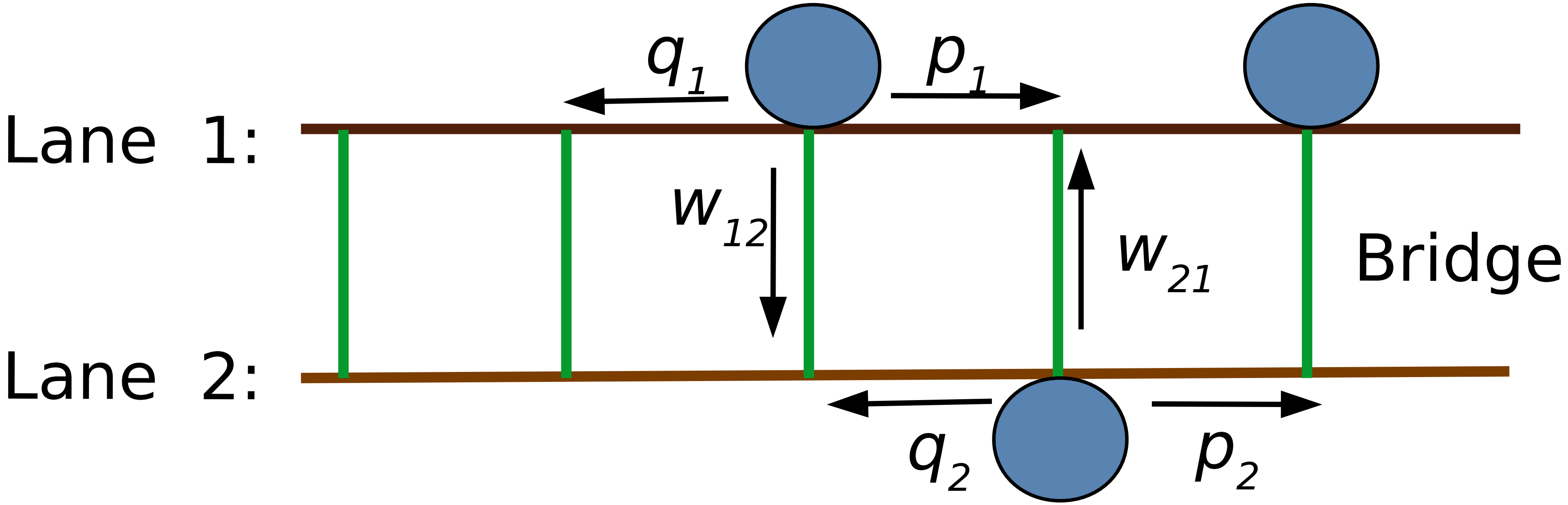
Multi lane ASEP

more realistic traffic flow

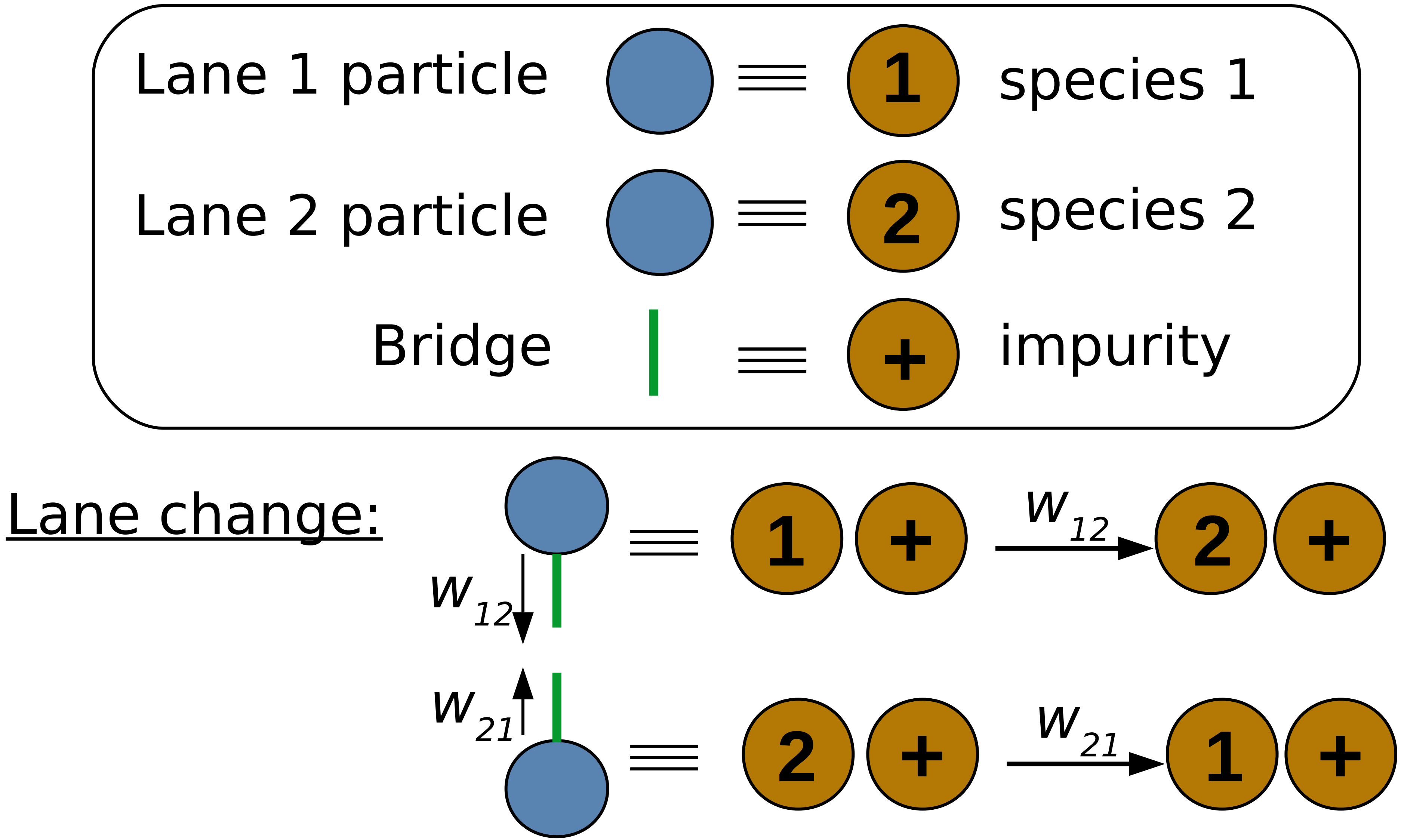
no exact solution

Question: approximate mapping of multi lane ASEP
to
1-d model that allows exact solution ?

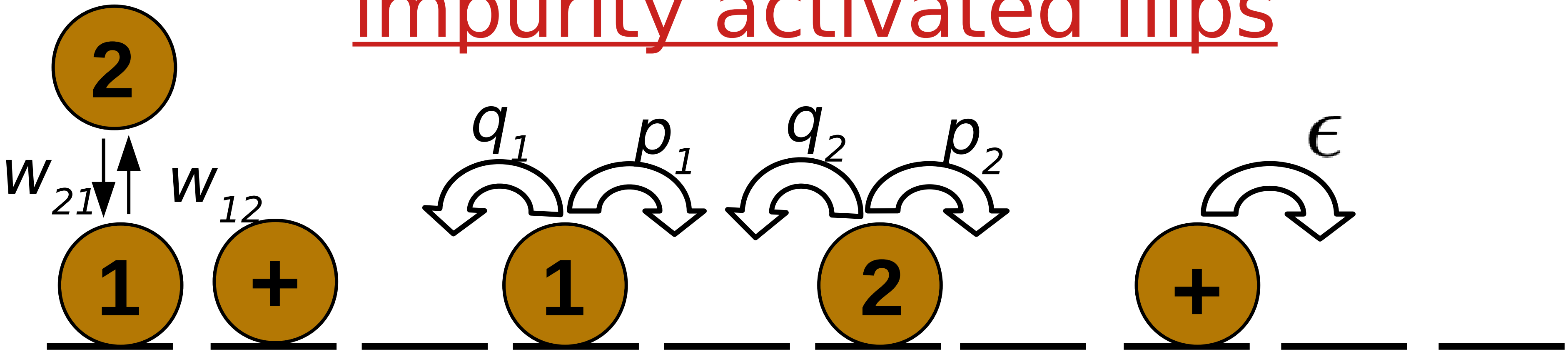
Motivation: Two lane (multi lane) ASEP to 1-d ASEP with two (multiple) species



Question: equivalent 1-d model ?


Lane 1 particle
species 1
Lane 2 particle
species 2
Bridge
impurity
Lane change:
w_{12}
w_{21}

Multi species ASEP with impurity activated flips



(i) disordered

(ii) non-conserved

(iii) non-ergodic

→

exact steady state
and
observables ?

Steady state: Matrix Product Ansatz

components represented by matrices

species "K" $\longrightarrow D_K$ impurity $\longrightarrow A$ vacancy $\longrightarrow E$

$$\ldots 012+\ldots \equiv \ldots E D_1 D_2 A \ldots$$

Probability of any configuration:

$$P(\{s_i\}) \propto \mathrm{Tr}\left[\prod_{i=1}^{L} X_i\right],$$

$$X_i = E\,\delta_{s_i,0} + A\,\delta_{s_i,+} + \sum_{K=1}^{\mu} D_K\,\delta_{s_i,K}.$$

Tasks: (i) matrix algebra
(ii) matrix representations

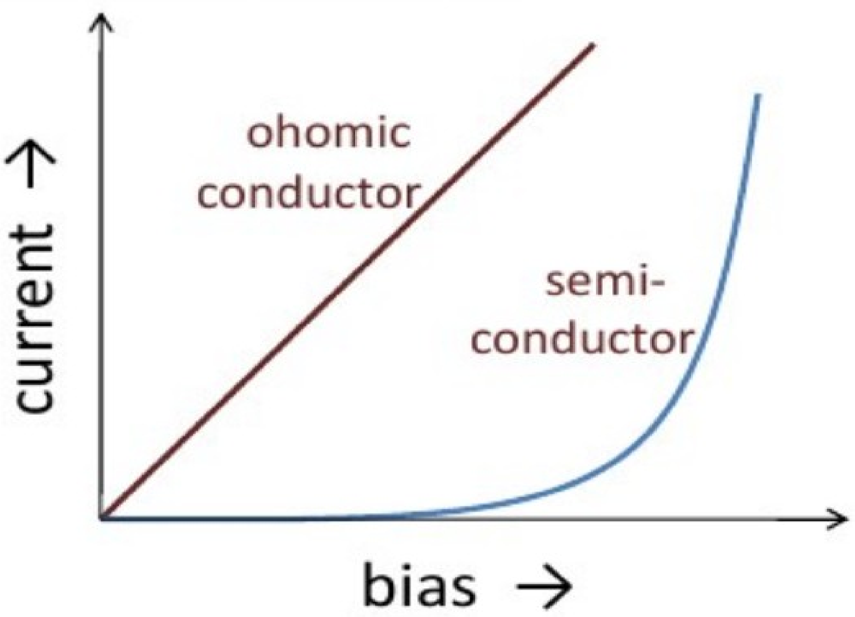
RESULTS:

totally asymmetric motion $(q_1{=}0{=}q_2)$	⇒	*finite dimensional* matrices
partially asymmetric motion $(q_1{\neq}0,\, q_2{\neq}0)$	⇒	*infinite dimensional* matrices

[SciPost Physics 14, 016 (2023)]

Negative differential mobility

general notion:



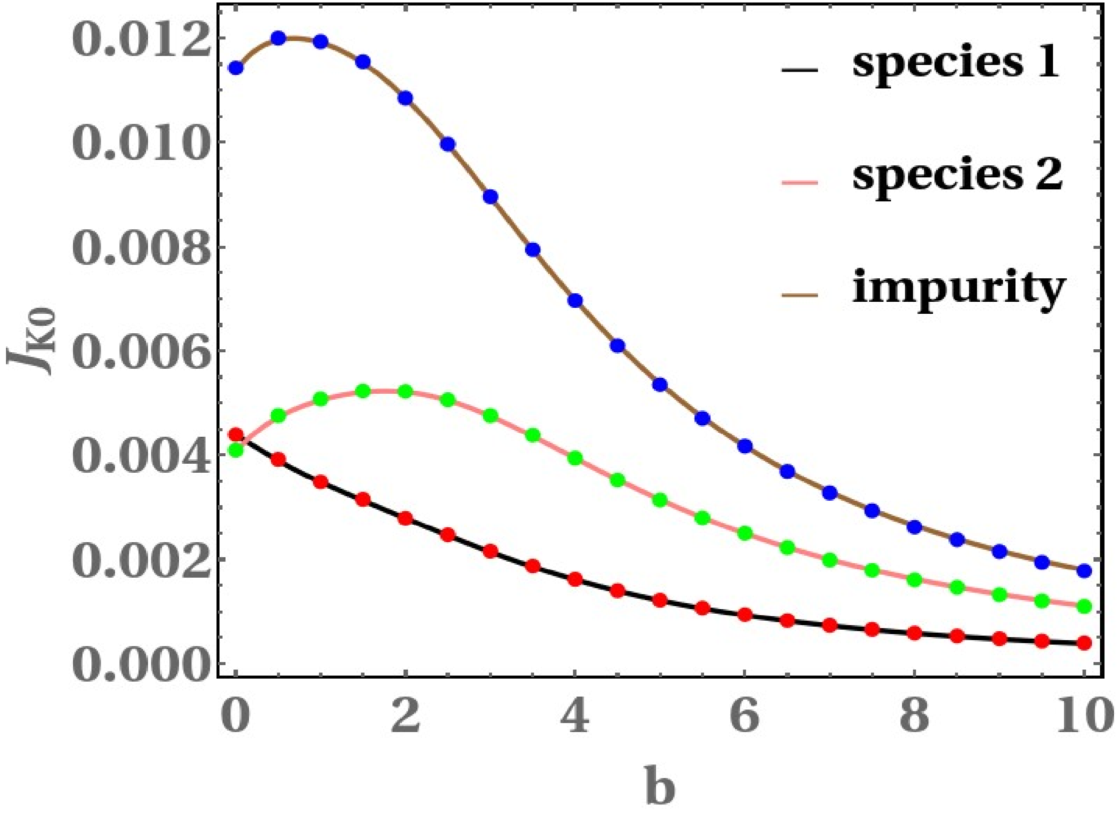
current increases with increasing bias

our observation:

decreasing current with increasing bias

Drift current of species "K": J_{K0}

Bias: $\ln(p_1/q_1) = b$ $\quad p_1 = 1, q_1 = e^{-b}$ $\quad p_2 = \frac{1}{1+b^2} = q_2$



decreasing current with increasing bias

Clustering

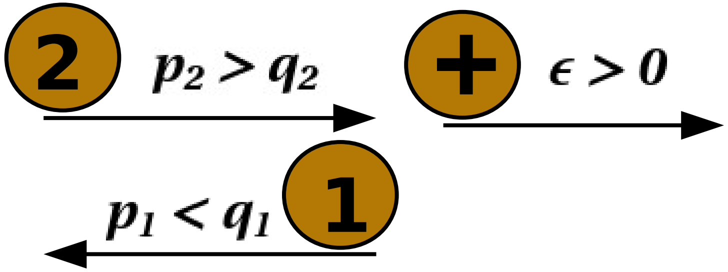
clustering from counter-flow

- pedestrians moving in opposite directions in a narrow lane

  ⇓

  traffic jam at high density

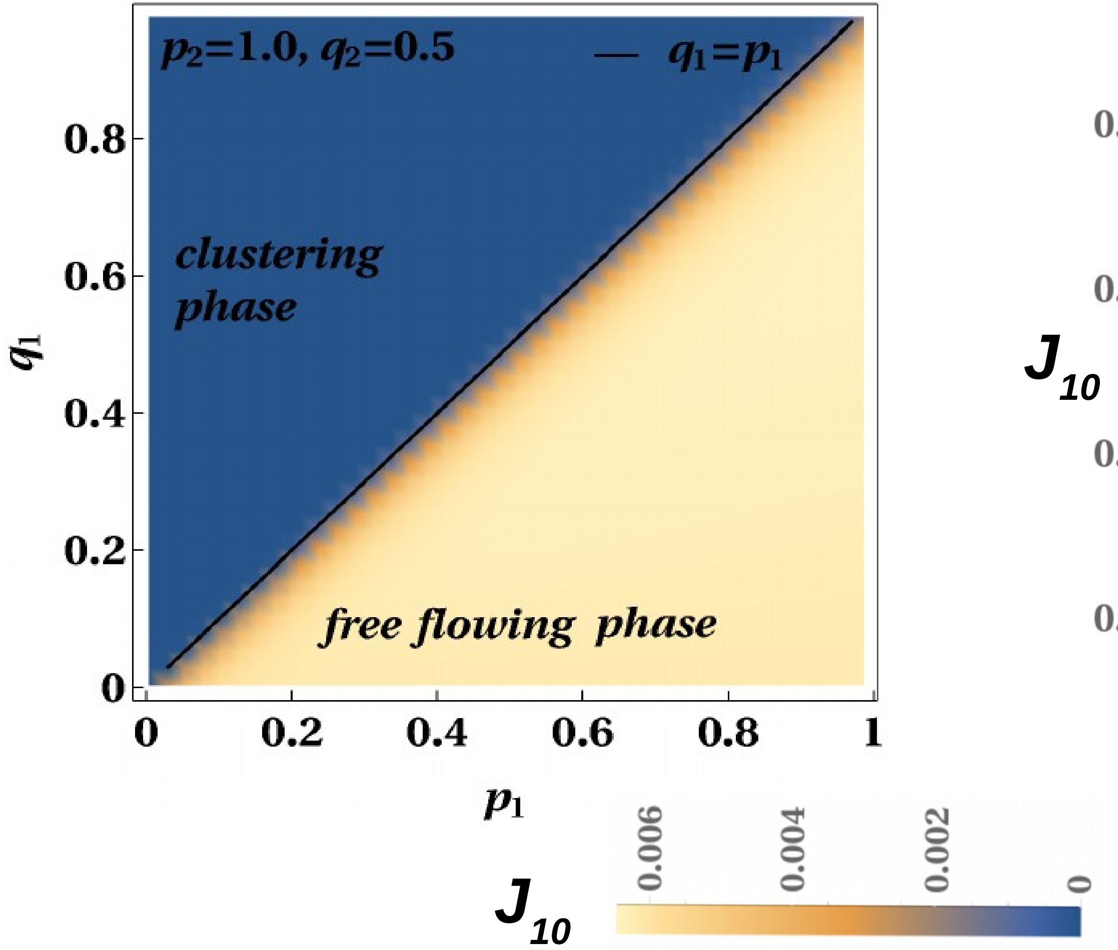
counter-flow in our model

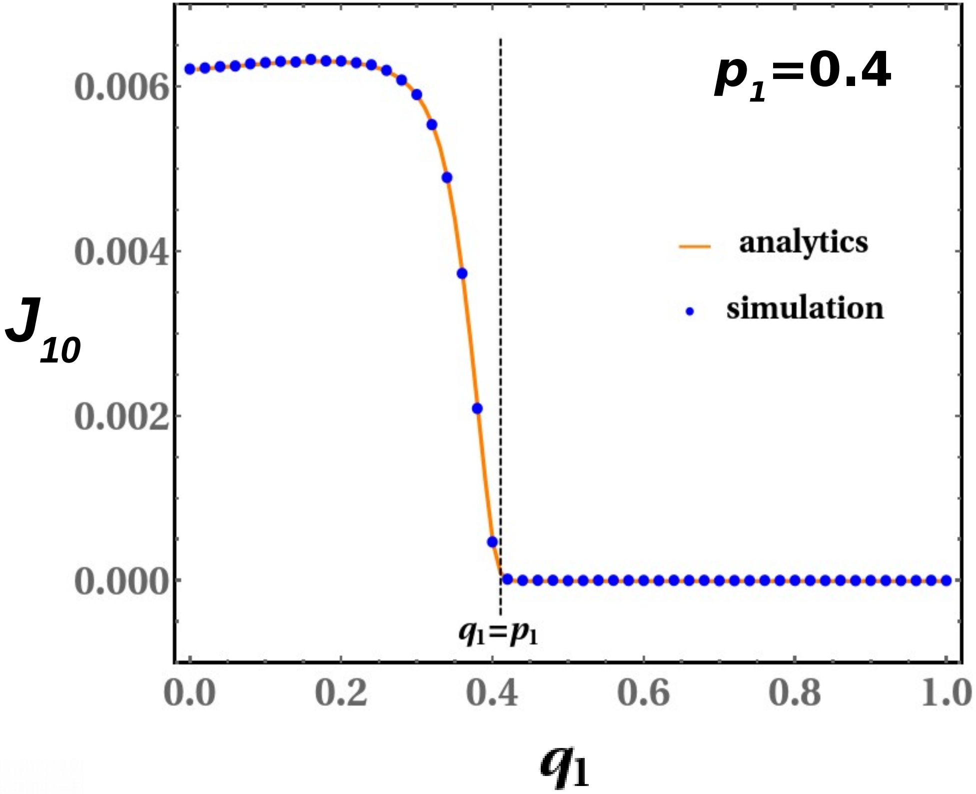


⇒ cluster formation ?

Two different phases

current: $J_{10} = p_1\langle 10\rangle - q_1\langle 01\rangle$





Summary

- multi lane traffic flow

  ⇩ 1-d approximation

  multi species ASEP with impurity activated flips

- exact matrix product steady state [SciPost Physics 14, 016 (2023)]
- negative differential mobility [SciPost Physics 14, 016 (2023)]
- cluster formation [arXiv:2208.03297 (2022)]

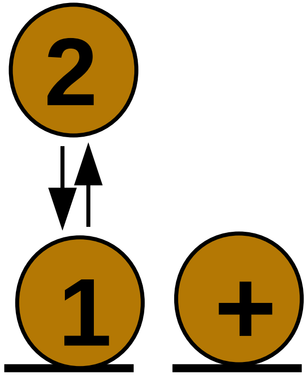
connection to run-and-tumble particles

two possible “run” directions in one dimension:

: right running RTP

: left running RTP

[arXiv:2208.03297 (2022)]



: tumbling of RTP

[impurity causes tumbling or chemotaxis]

THANK YOU

Extra slide:1

$0+201\mathbf{1}+2 \longrightarrow 0+201\mathbf{2}+2$ *(accessible)*

$0+20\mathbf{1}1+2 \not\longrightarrow 0+20\mathbf{2}1+2$ *(not accessible)*

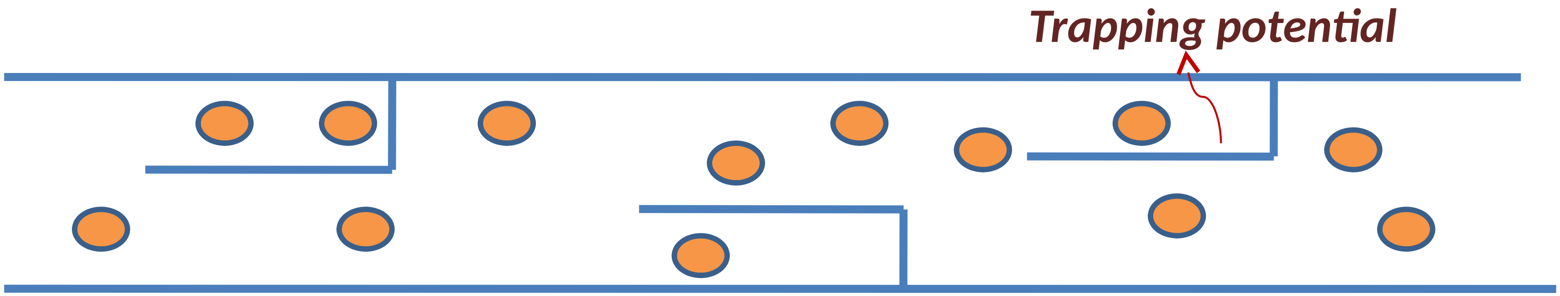
only a subspace of the whole configuration space is accessible, for a given initial configuration

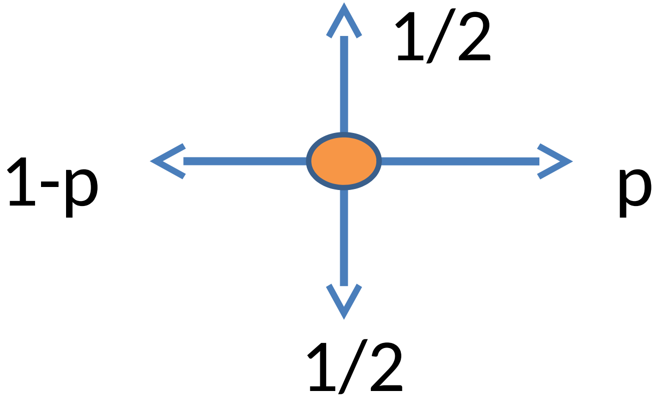
$\Downarrow$

NON-ERGODIC

Extra slide:2

- *non-interacting particles*





(a) p=1/2 *J=0*

(b) p=0.51 , only few particles get trapped ,
positive response

(c) p=0.95 , most of the particles get trapped,
negative response

[Baerts *et.al.* , Phys. Rev. E 88, 052109 (2013)]

Mechanism: some kind of trapping that decreases dynamical activity

Extra slide:3

Matrix algebra:

$$
\begin{aligned}
p_K D_K E - q_K E D_K &= D_K \\
\epsilon A E &= A \\
\sum_{\substack{I=1\\ I\neq K}}^{\mu} w_{IK} D_I A &= D_K A \sum_{\substack{I=1\\ I\neq K}}^{\mu} w_{KI}
\end{aligned}
$$

Representation:

(3 species, q_k=0)

$$
D_1 = d_1 \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\quad
D_2 = d_2 \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
\quad
D_3 = d_3 \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

$$
E = \begin{pmatrix} \frac{1}{\epsilon} & 0 & 0 & 0 \\ \frac{1}{p_1} - \frac{1}{\epsilon} & \frac{1}{p_1} & 0 & 0 \\ \frac{1}{p_2} - \frac{1}{\epsilon} & 0 & \frac{1}{p_2} & 0 \\ \frac{1}{p_3} - \frac{1}{\epsilon} & 0 & 0 & \frac{1}{p_3} \end{pmatrix}
\quad
A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

$$
\begin{aligned}
d_1 &= w_{21}w_{31} + w_{23}w_{31} + w_{32}w_{21} \\
d_2 &= w_{12}w_{32} + w_{13}w_{32} + w_{31}w_{12} \\
d_3 &= w_{13}w_{23} + w_{12}w_{23} + w_{21}w_{13}
\end{aligned}
$$

Extra slide:4

Bacteria push the limits of chemotactic precision to navigate dynamic chemical gradients

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Motile bacteria often survive by consuming ephemeral sources of dissolved organic matter (DOM) produced, for example, in the ocean by phytoplankton lysis and exudation or sloppy feeding and excretion by larger organisms (1–4). The microscale interactions between nutrient sources and bacteria underpin ocean biogeochemistry and are strongly influenced by the ability of bacteria to actively navigate toward favorable conditions. Past experiments on chemotaxis using *Escherichia coli* and other model bacteria have generally focused on stable gradients of intermediate to high nutrient concentrations, where bacteria can readily detect chemical gradients (5–7). However, the environments that wild bacteria navigate are often characterized by short-lived, microscale chemical gradients where background conditions are highly dilute (8, 9). In such ephemeral chemical fields, bacteria experience a gradient in DOM concentration as a noisy, dynamic signal, rather than as a steady concentration ramp (10).

Vibrio ordalii

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