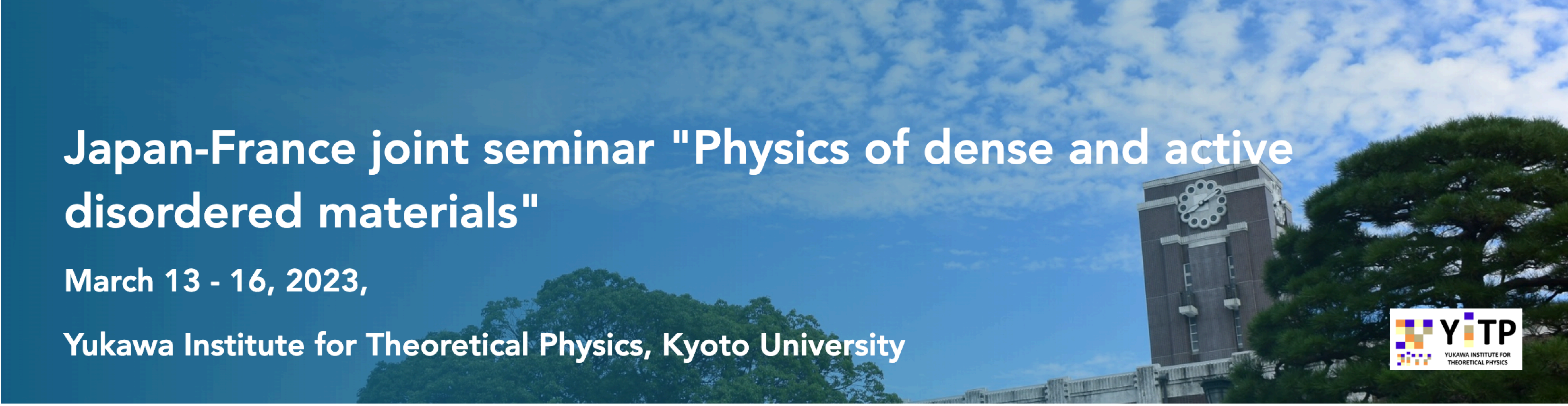

Control parameter dependence of fluctuation near jamming

Harukuni Ikeda, Gakushuin University

H. Ikeda, PRL **125** (3), 038001 (2020)
H. Ikeda, JCP **158**, 056101 (2023)

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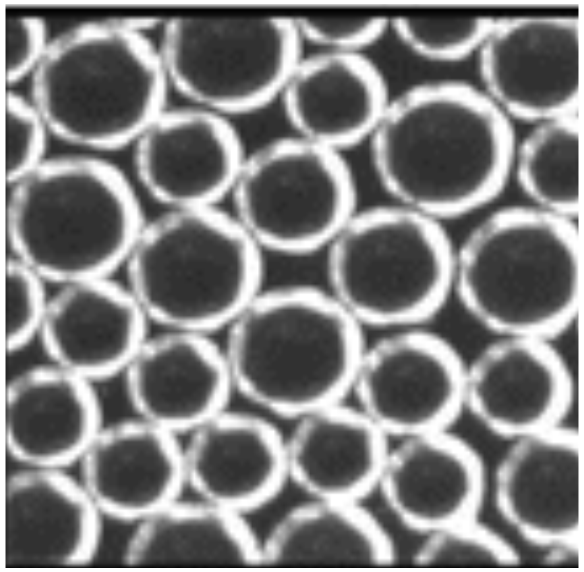
Introduction: What is granular material?

Consisting of large enough particles (0.1mm)
so that thermal fluctuations are negligible

M&M Candies

Grains

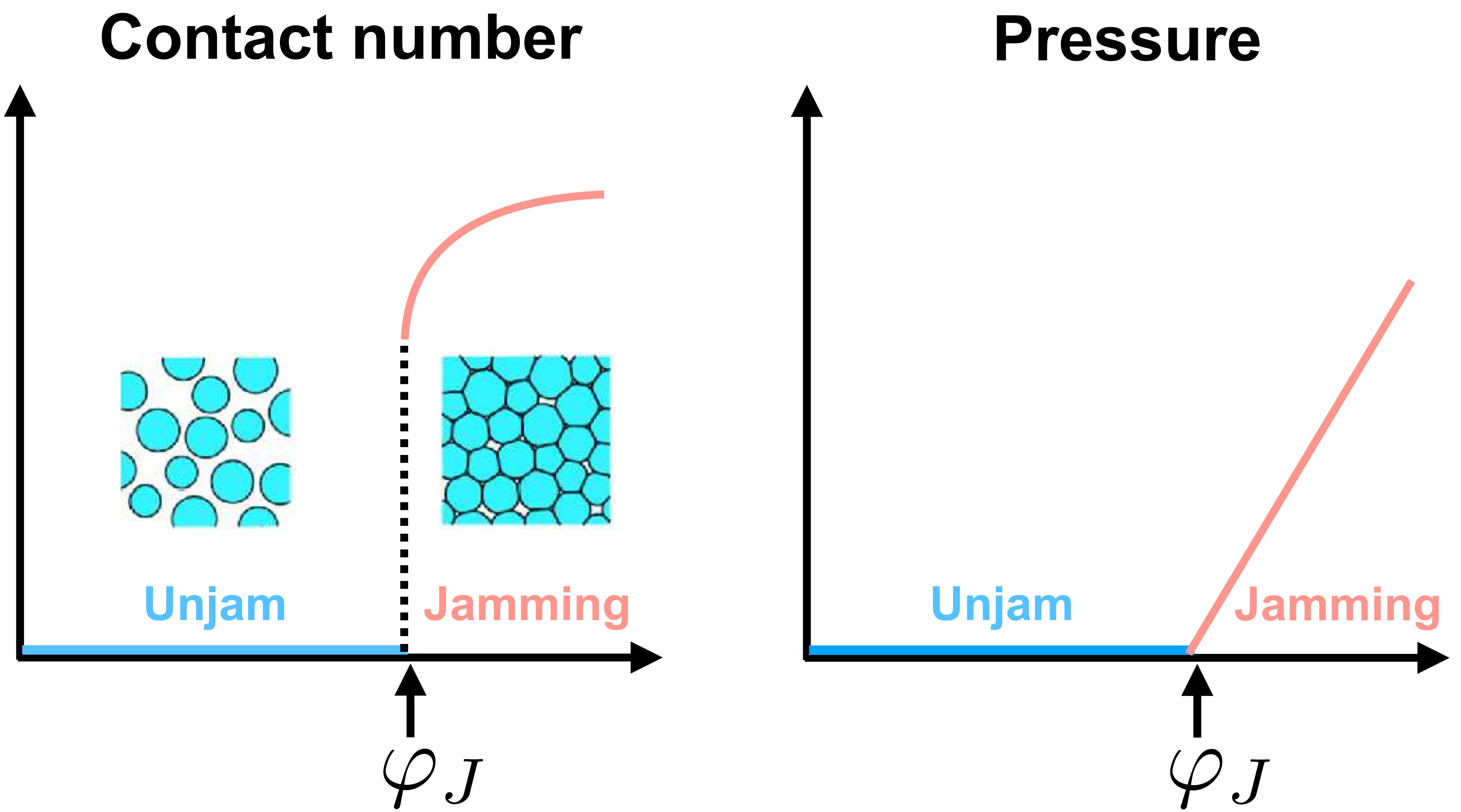


Forms

Sand & rock

Snow powders

Introduction: What is the jamming transition?



The jamming transition is a phase transition from fluid to solid at zero temperature

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- **Introduction**
- Dimensional dependence of the jamming transition
- Sample to sample fluctuation

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- Introduction
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- Introduction
- **Dimensional dependence of the jamming transition**
    - Review for known results in d=2 and d=3
    - Results in quasi 1d.
- Sample to sample fluctuation

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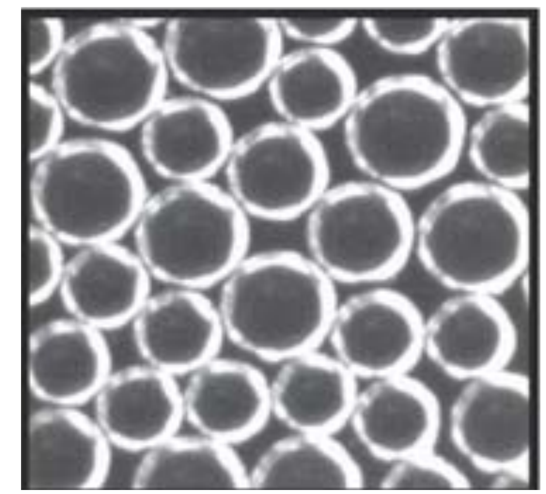
Jamming in *2d* and *3d*

Frictionless spherical particles

Harmonic Spheres

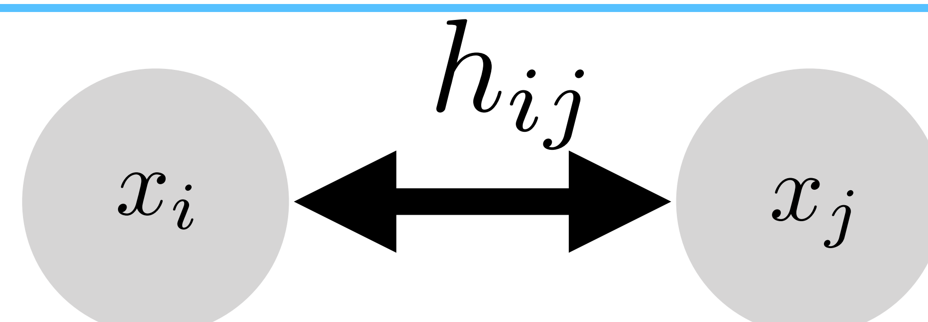
$$E = \sum_{i<j} v(h_{ij}) = \frac{\varepsilon}{2} \sum_{i<j} h_{ij}^2 \theta(-h_{ij})$$



Wet foams

Gap function

$$h_{ij} = |x_i - x_j| - \sigma_{ij}$$



Stability argument by Maxwell

J. C. Maxwell (1864)

# of constraints > # of degrees of freedom

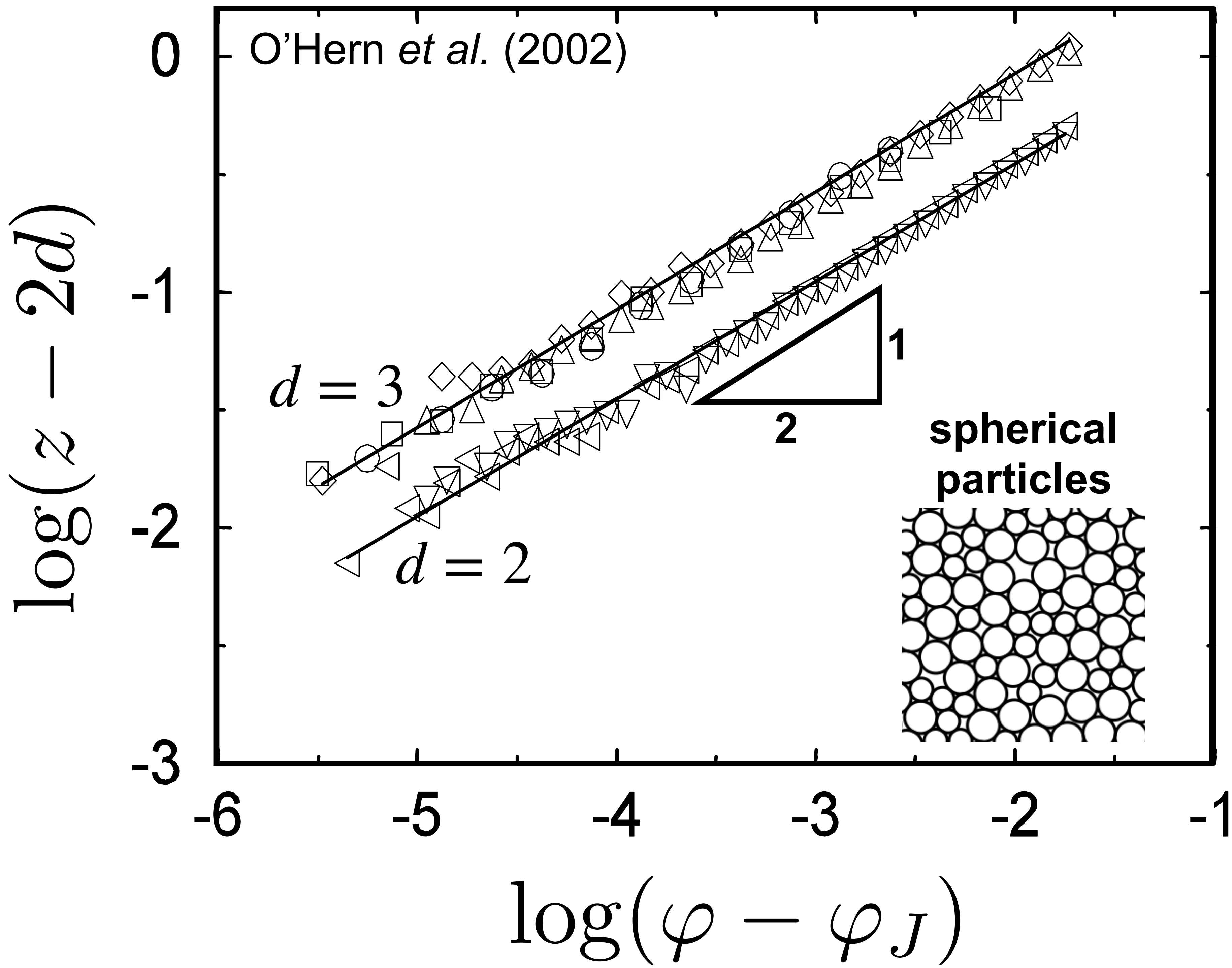
# of constraints = # of contacts = Nz/2

# of degrees of freedom = Nd

$$z \geq 2d$$

$$z_J = 2d \text{ (isostatic)}$$

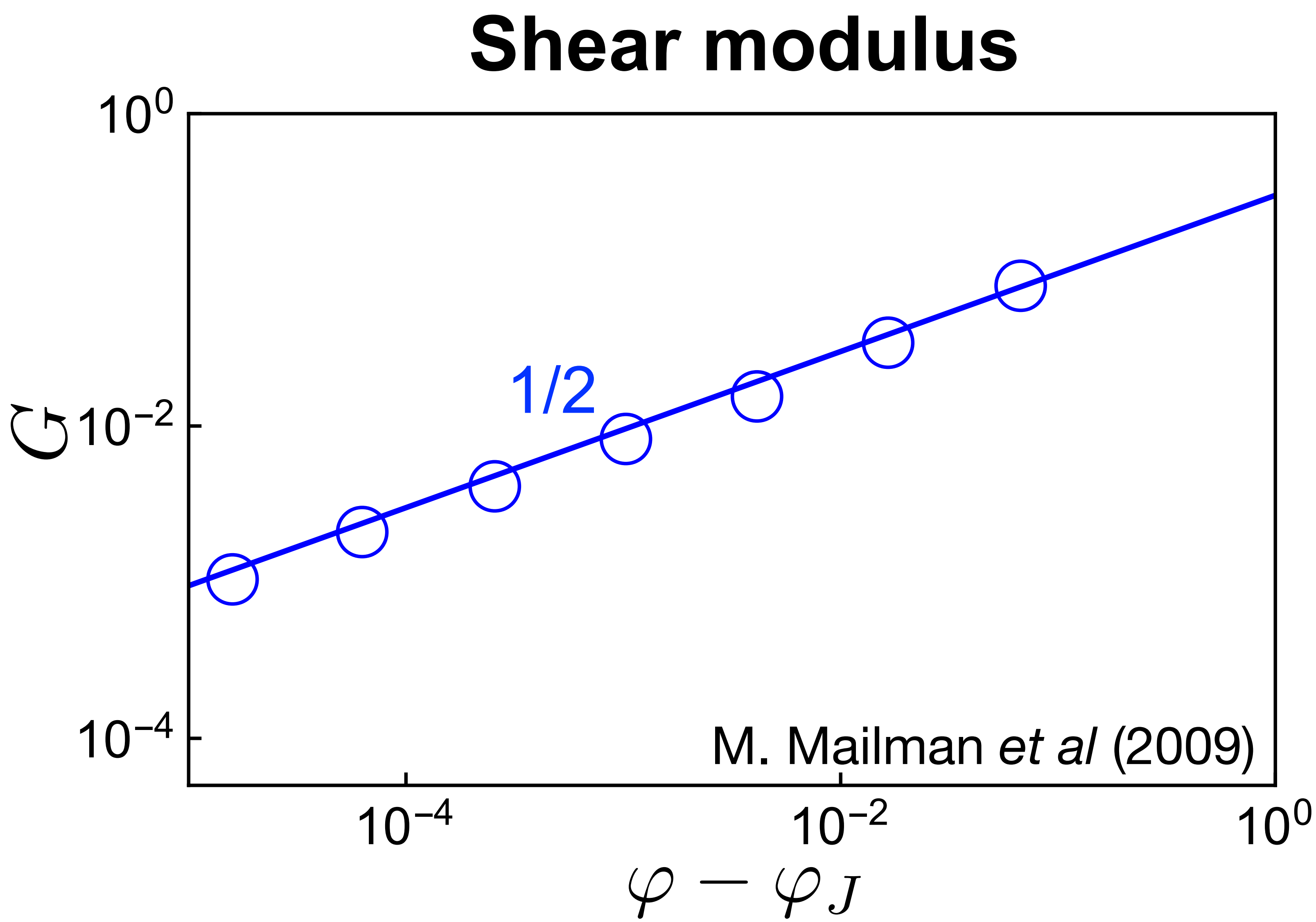
Jamming in *2d* and *3d*



Scaling relation $z - 2d \propto (\varphi - \varphi_J)^{0.5}$

The critical exponent does not depend on the spatial dimensions!

Jamming in *2d* and *3d*

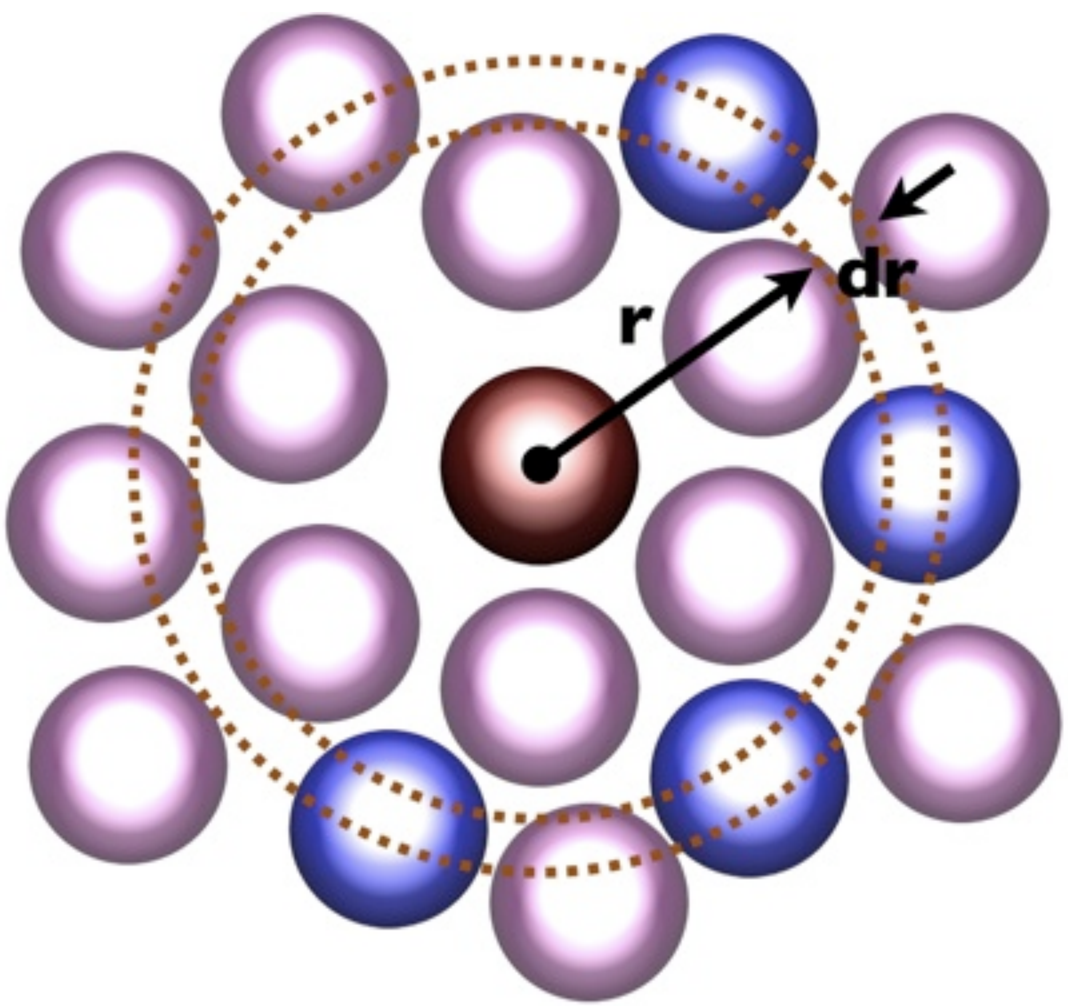


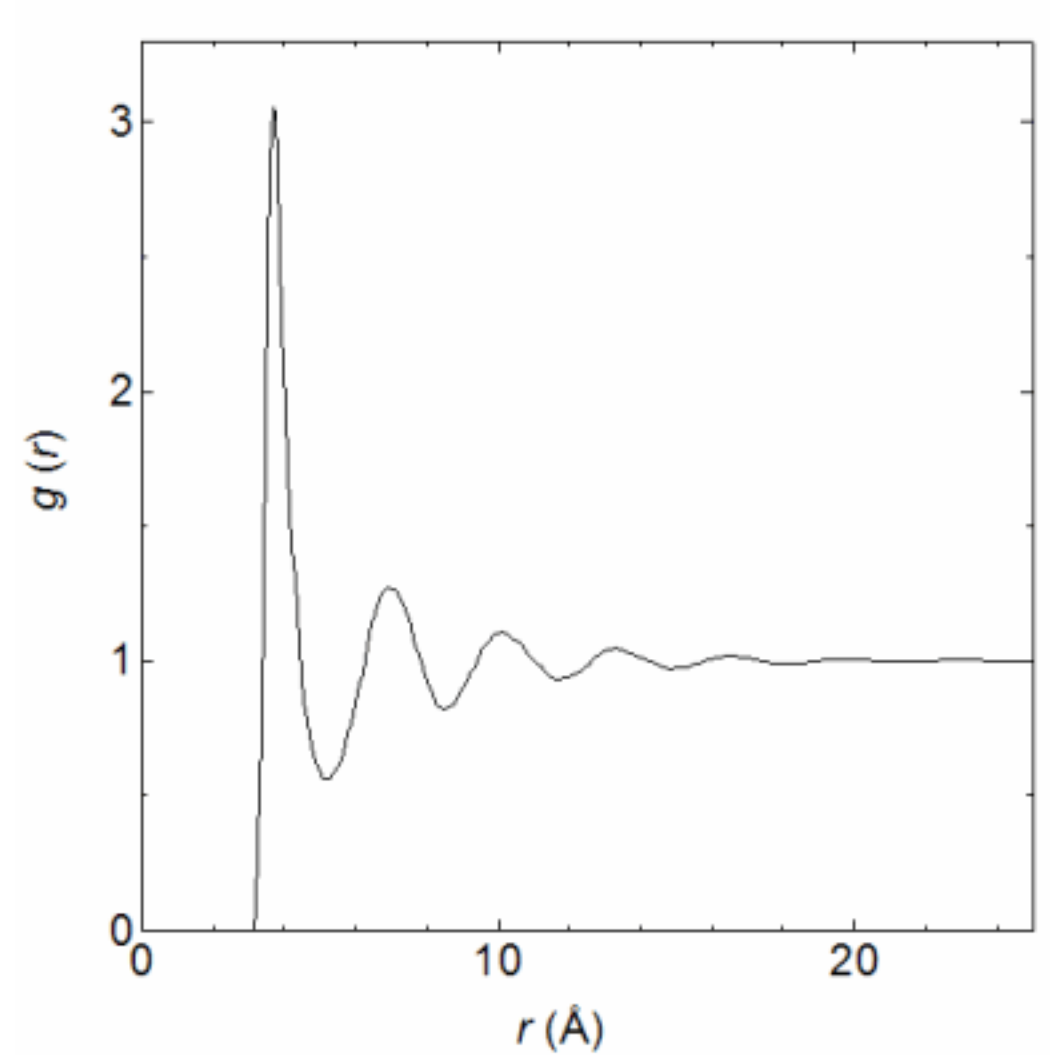
Scaling relation $G \sim (\varphi - \varphi_J)^{1/2}$

The critical exponent does not depend on the spatial dimensions!

Radial distribution function

$$g(r) = \frac{1}{N} \sum_{i \neq j} \delta(r - |\vec{x}_i - \vec{x}_j|)$$

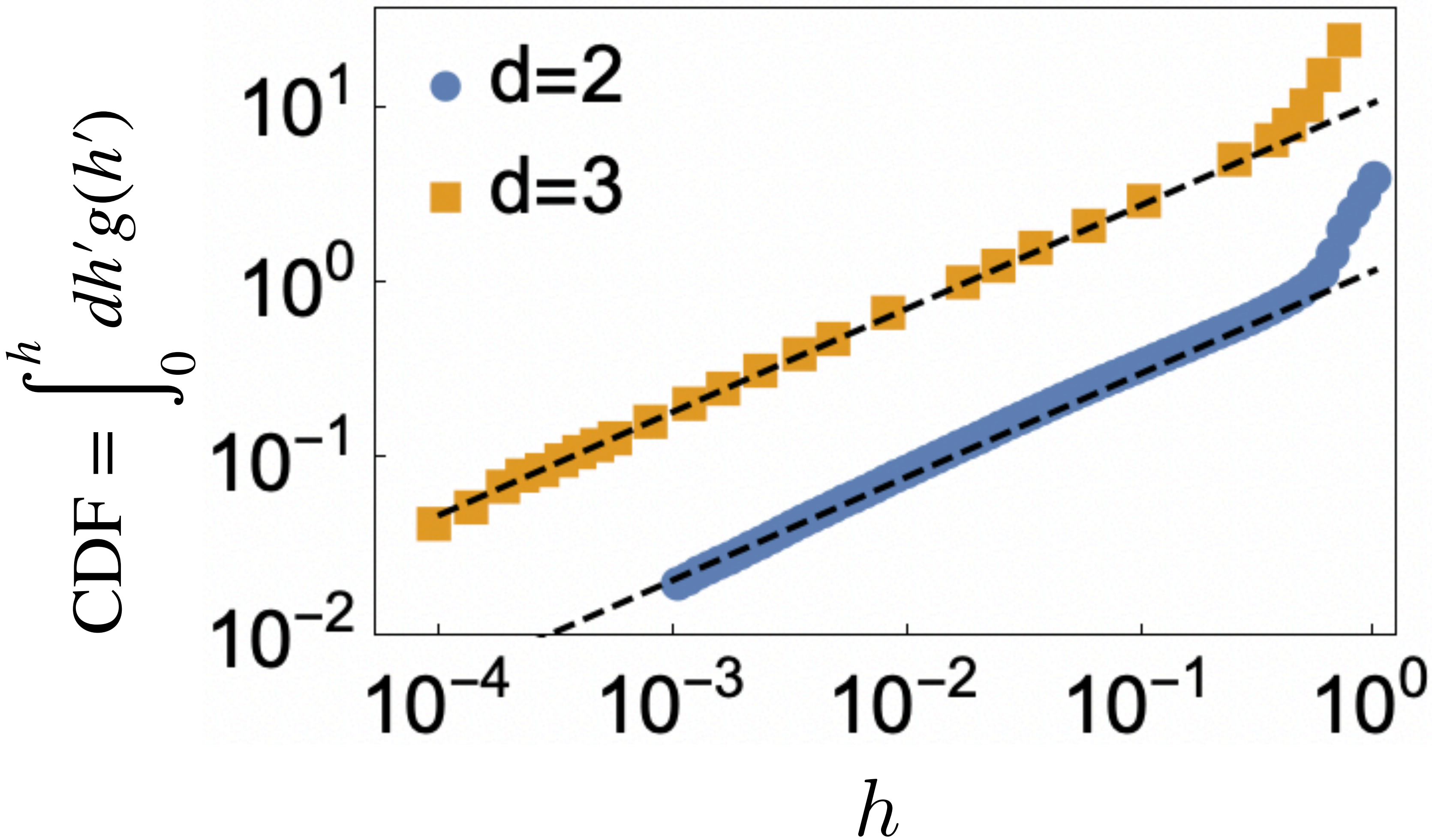




Jamming in *2d* and *3d*

Radial distribution function

P. Charbonneau, J. Kurchan, G. Parisi, P. Urbani, and F. Zamponi *et al.* (2014),
P. Charbonneau, E. Corwin, R. Dennis, R. Rojas, **H.I**, G. Parisi, and F. Ricci-Tersenghi (2021)



Scaling relation $g(h) \sim h^{-\gamma}, \gamma = 0.41...$

The critical exponent does not depend on the spatial dimensions!

Jamming in *2d* and *3d*

Excess contact number

$$z - 2d \propto (\varphi - \varphi_J)^{0.5}$$

Radial distribution function

$$g(h) \sim h^{-\gamma}, \gamma = 0.41$$

The critical exponent does not depend on the spatial dimensions.
Furthermore, the exponents agree with the mean-field prediction.

P. Charbonneau *et al.* (2014)

The upper critical dimension <= 2.

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- Introduction
- **Dimensional dependence of the jamming transition**
  - **Review for known results in d=2 and d=3**
  - Results in quasi 1d.
- Sample to sample fluctuation

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- Introduction
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    - Review for known results in d=2 and d=3
    - **Results in quasi 1d.**
- Sample to sample fluctuation

Jamming of quasi 1d system

Motivation

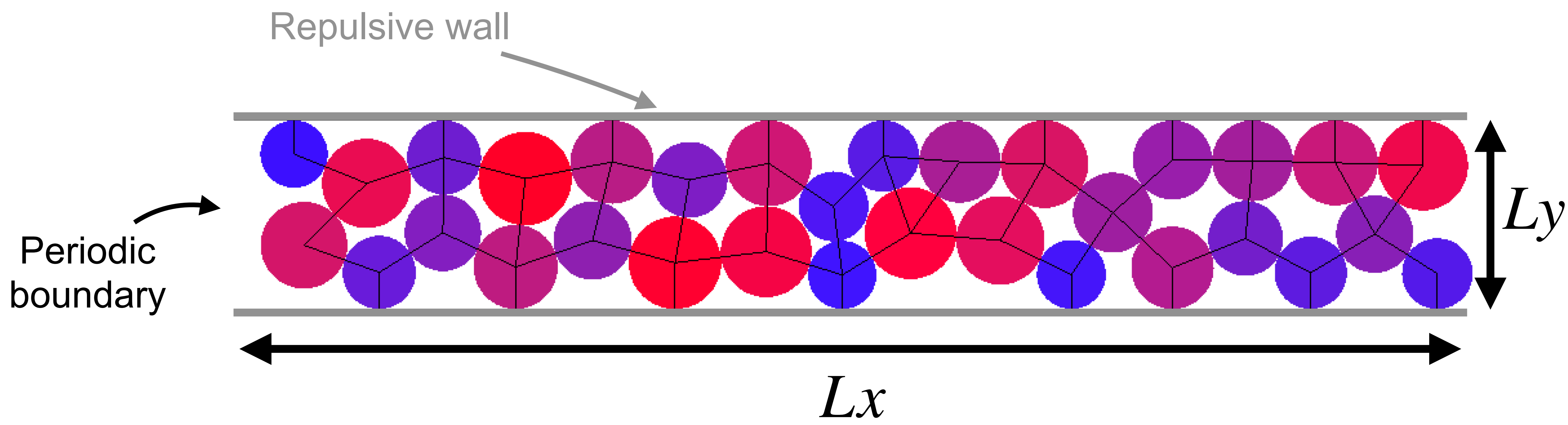
The upper critical dimension <=2

What will happen below the upper critical dimension?

Let we consider the quasi-one-dimensional system

Jamming of quasi 1d system

Setting



Interaction potential

Interaction between particles

$$V_N = \sum_{i<j}^{1,N} v(h_{ij}) + \sum_{i=1}^{N} v(h_i^{\mathrm{b}}) + \sum_{i=1}^{N} v(h_i^{\mathrm{t}})$$

Interaction between walls and particles

$$v(h) = k\frac{h^2}{2}\theta(-h)$$

Jamming of quasi 1d system

Algorithm

We want to determine the jamming transition point, where particles begin to interact and has a finite interaction potential.

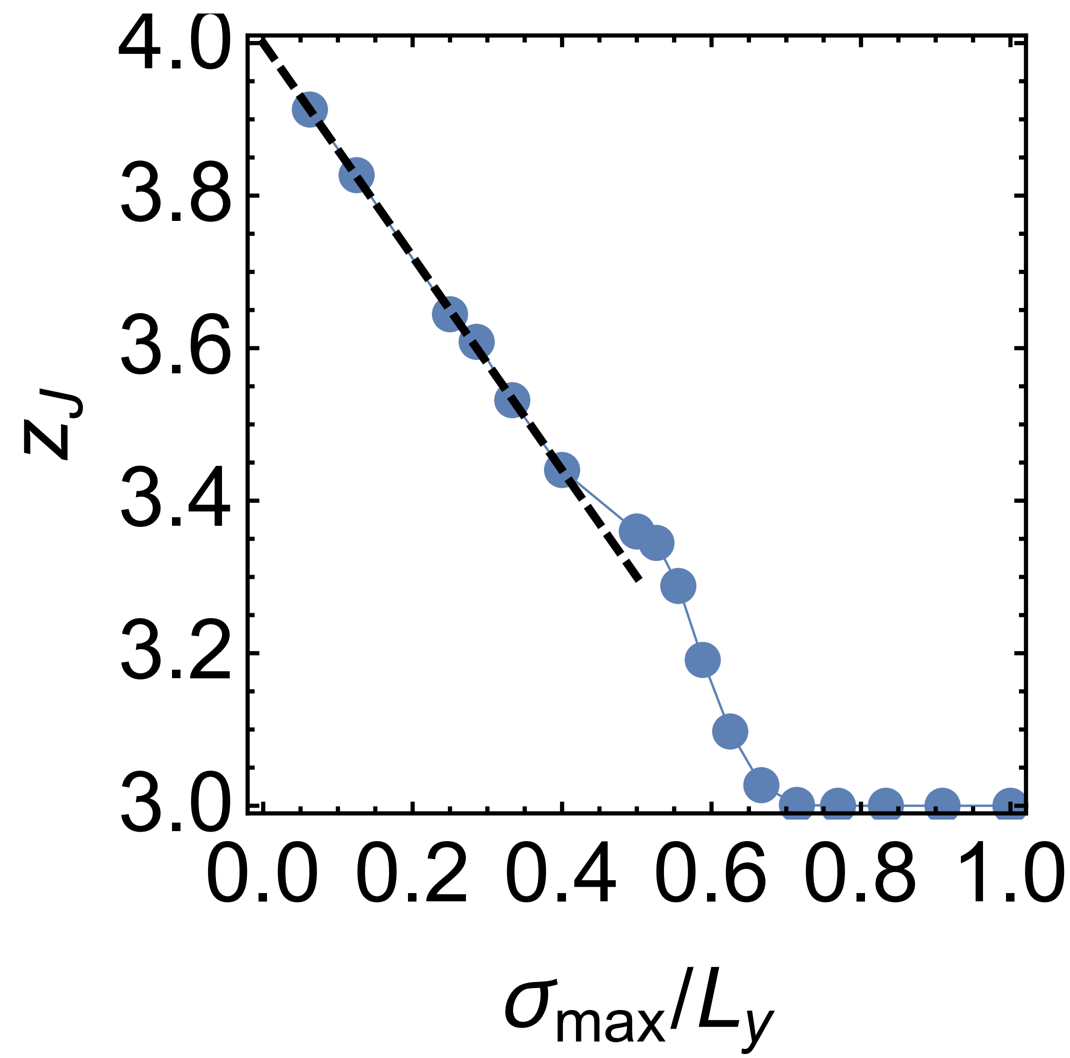
1. Start from a random configuration.
2. Increase the packing fraction $\varphi \rightarrow \varphi + \delta\varphi$.
3. Remove contact by the energy minimization.
4. Repeat 2 and 3. $\delta\varphi \rightarrow -\delta\varphi/2$ each time the transition point is crossed.

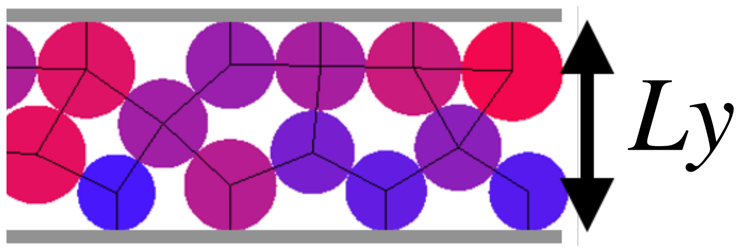
Animation

Ly is fixed during the compression.

Jamming of quasi 1d system

Results at the jamming transition point





Does Maxwell's isostatic condition hold in quasi 1d?

Unlike the bulk two dimensional system,
the contact number is not 4(=$2d$)!

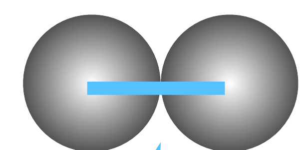
Jamming of quasi 1d system

Mxwell’s isostatic condition revisited

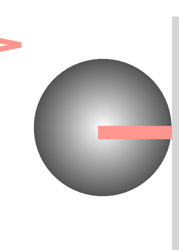
of constraints = # of degrees of freedom

# of degrees of freedom = Nd = 2N

Inter particle



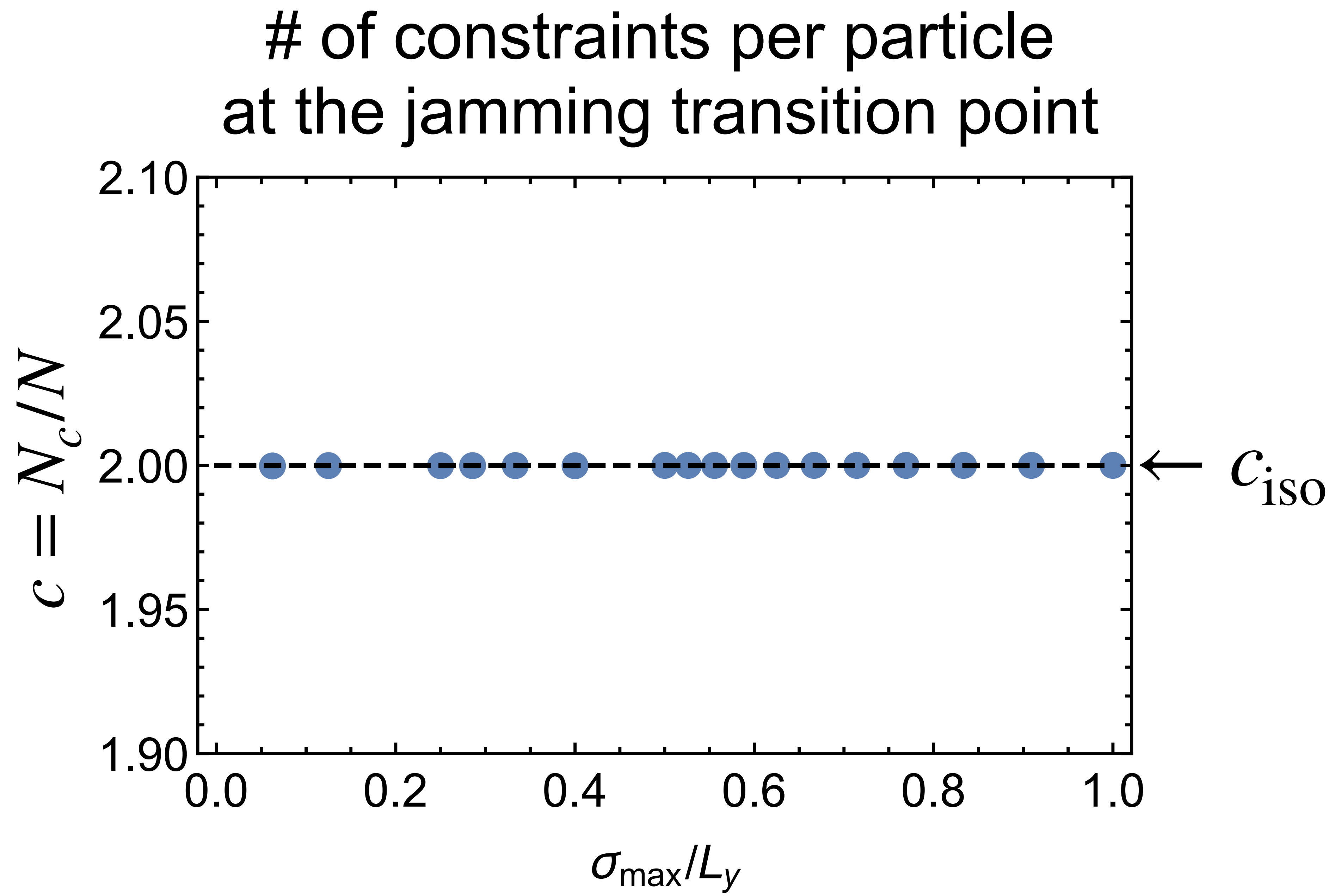
Wall



# of constraints = $N_c = \frac{Nz - N_w}{2} + N_w$

Isostatic number: $c_{\mathrm{iso}} \equiv N_c/N = 2$

Maxwells condition revisited



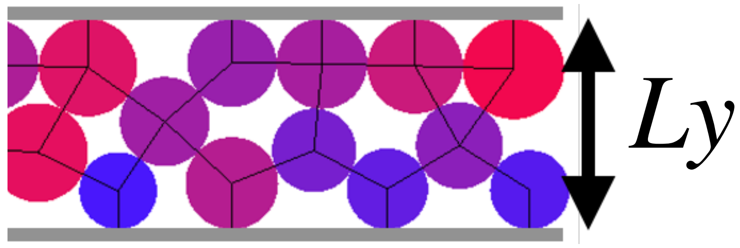
The system is always isostatic!

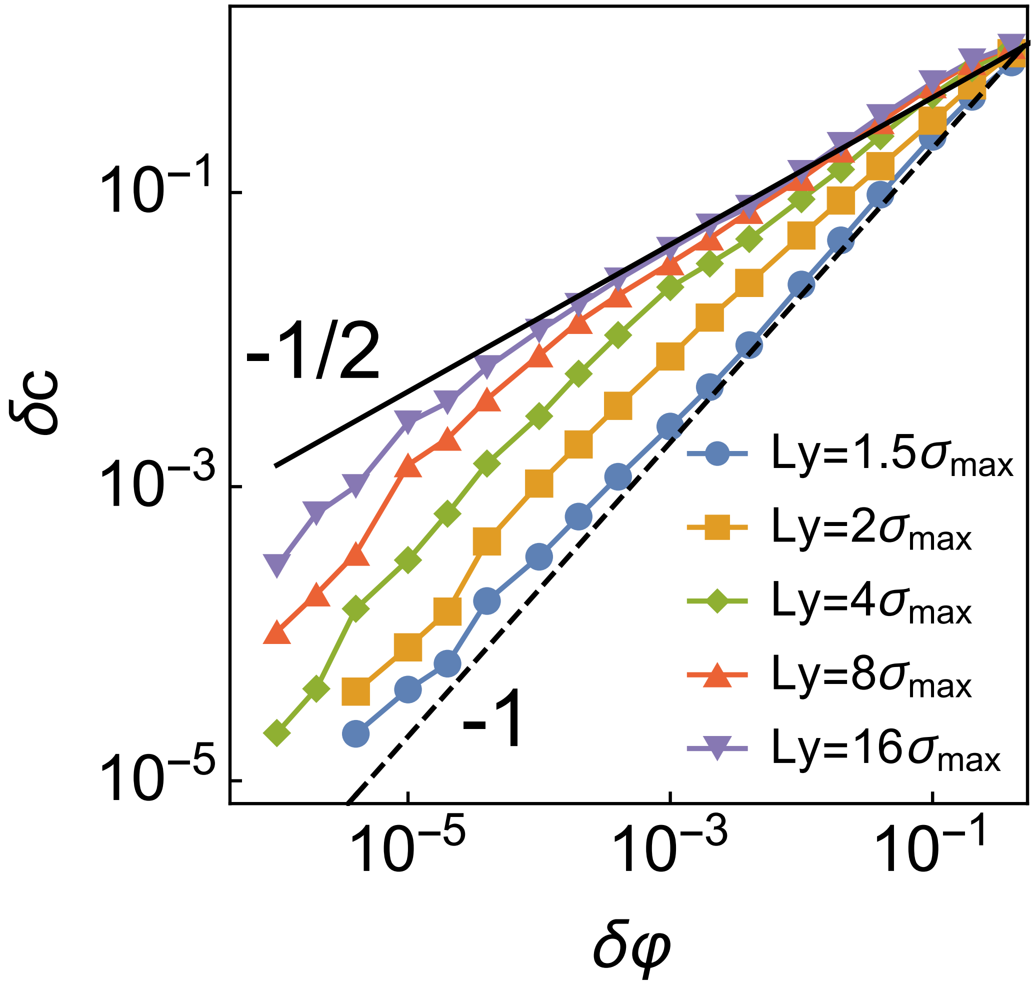
Jamming of quasi 1d system

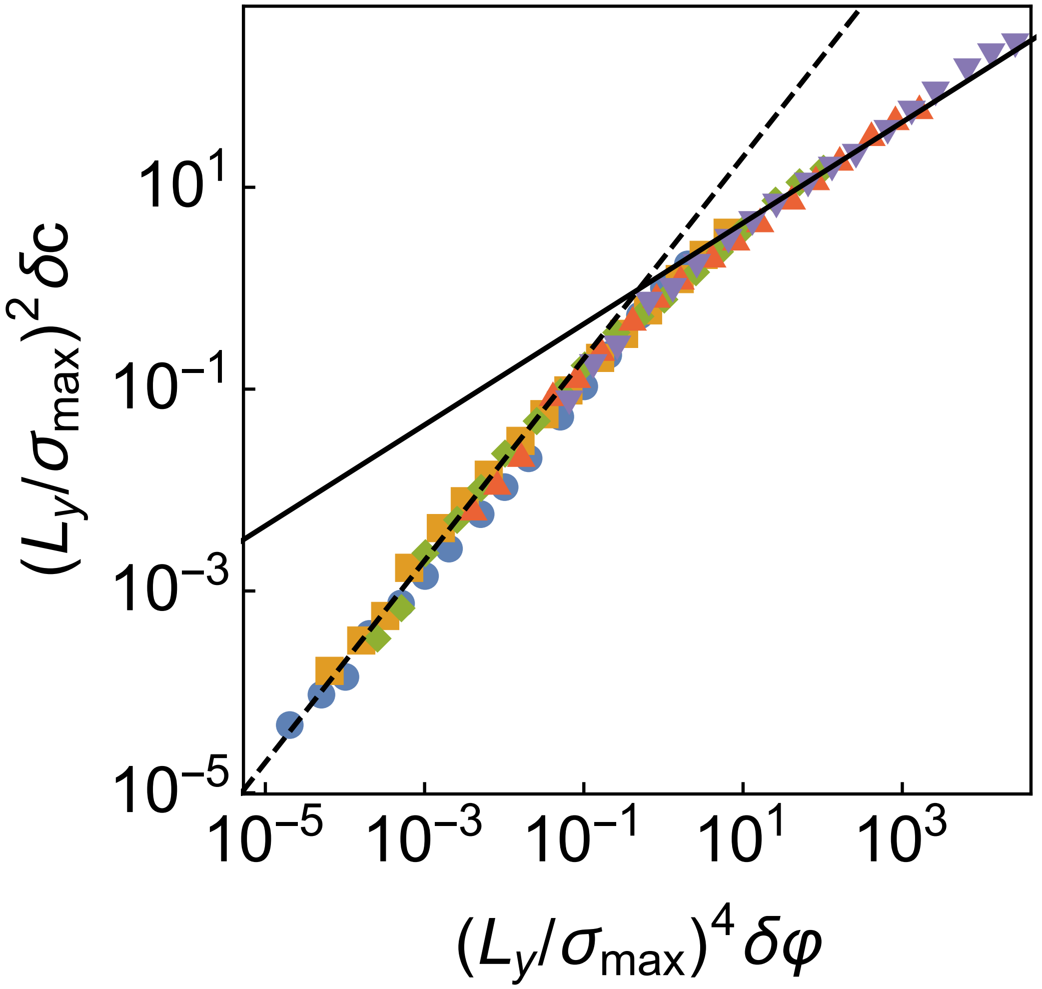
Scaling of the excess constraints

Excess constraints

$$\delta c = \frac{c - c_{\text{iso}}}{N} \ (\propto \delta z)$$





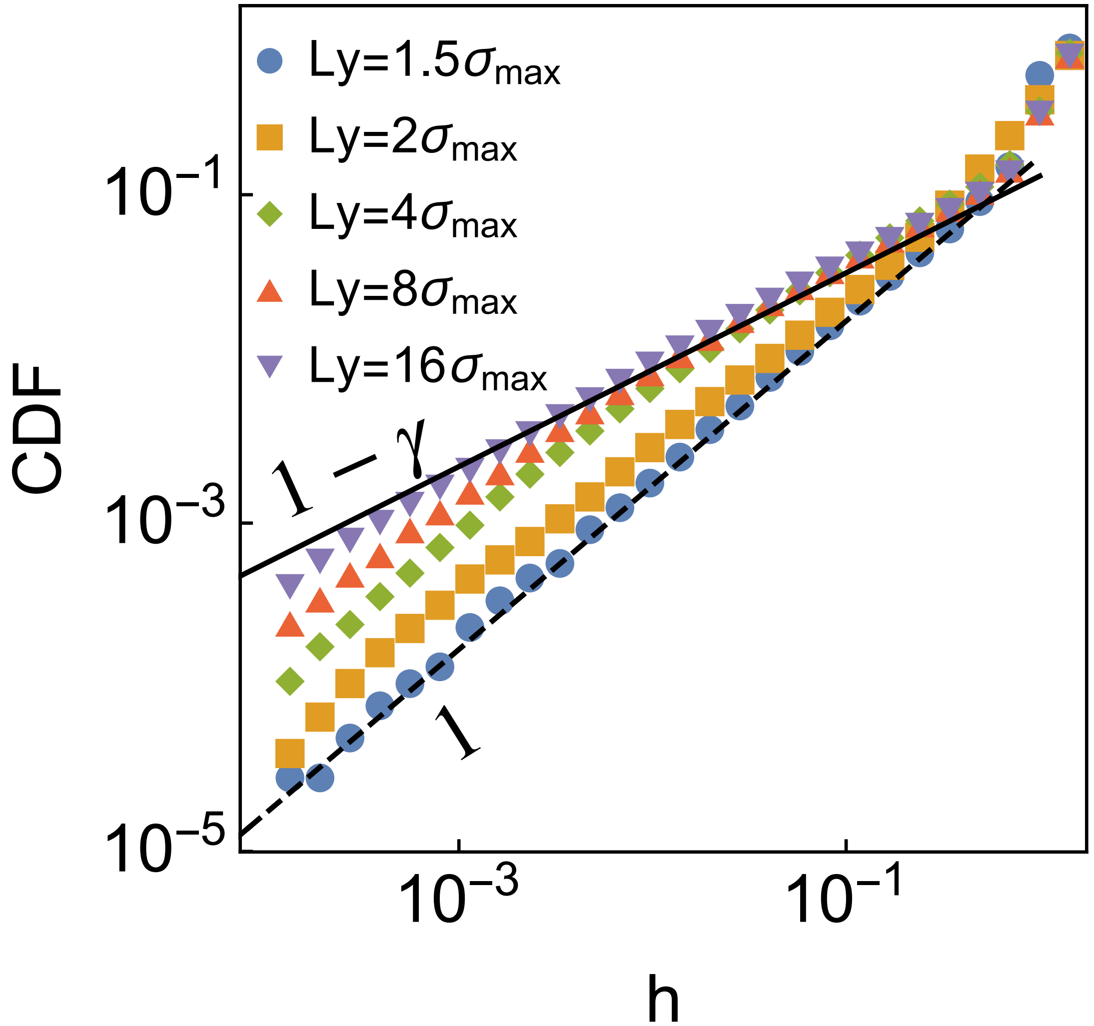


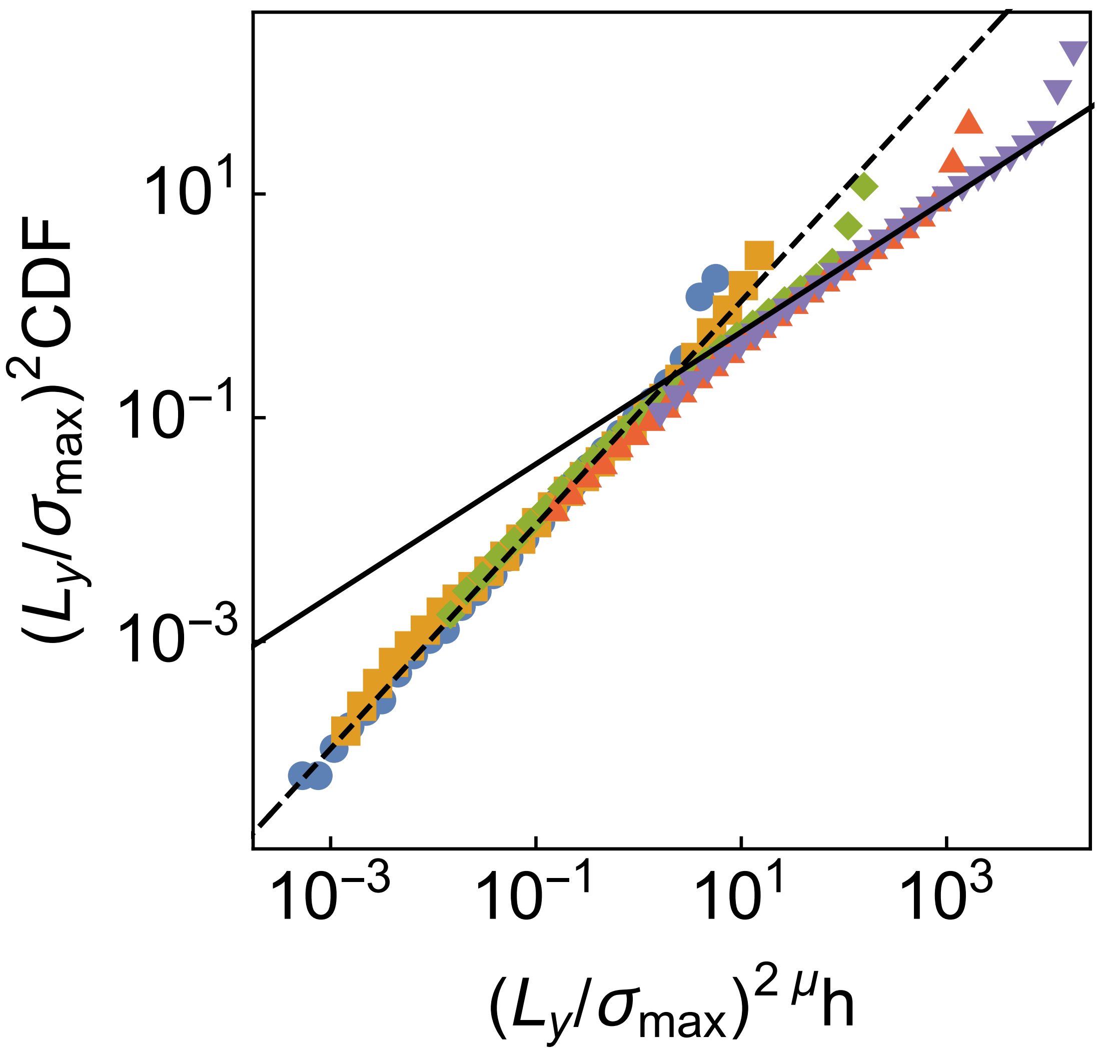
For small *Ly*, we observe $\delta c \sim \delta\varphi$.

Scaling of the radial distribution

Cumulative distribution function

$$\mathrm{CDF}(h) = \int_0^h dh' g(h') \sim h g(h)$$





For small *Ly*, we observe *g(h) ~ h^0*

Summary

2d and higher dimensions

Isostatic at jamming
(# of constraints = # of degrees of freedom)

of the excess constraints

$$\delta z \sim \delta c \sim \delta\varphi^{1/2}$$

Radial distribution function

$$g(h) \sim h^{-\gamma}$$

Quasi 1d

Isostatic at jamming
(# of constraints = # of degrees of freedom)

of the excess constraints

$$\delta c \sim \delta\varphi$$

Radial distribution function

$$g(h) \sim h^{0}$$

These results confirmed $1 < d_{UP} < 2$

Unsolved Questions

- What is the precise value of d_{UP}?
- What physical mechanism determines d_{UP}?

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科研費 KAKENHI

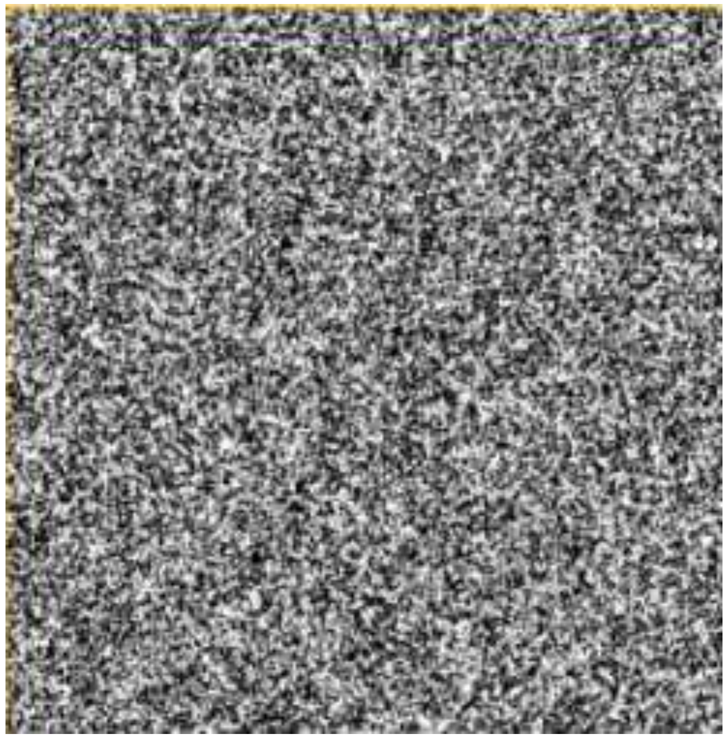
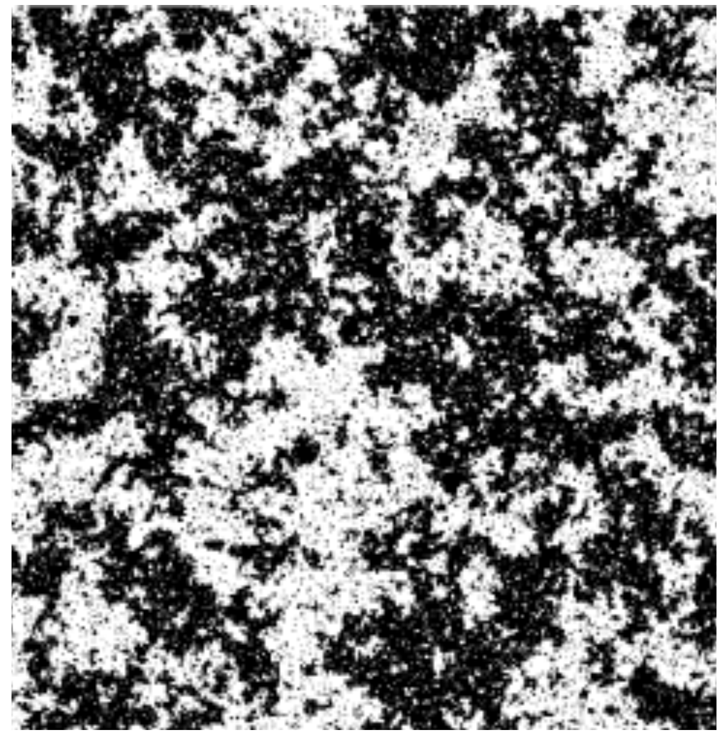
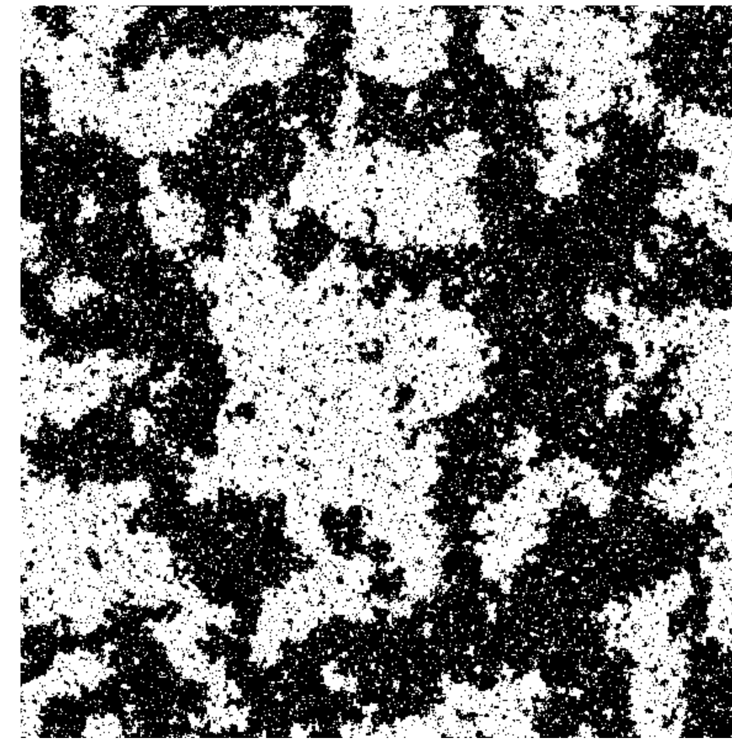
Fluctuation near Jamming

JCP **158**, 056101 (2023)

Harukuni Ikeda (Gakushuin Univ.)

Motivation

Classical Ising model

$T > T_c$	$T = T_c$	$T < T_c$
		
	Critical fluctuation!	

Ginzburg Criterion

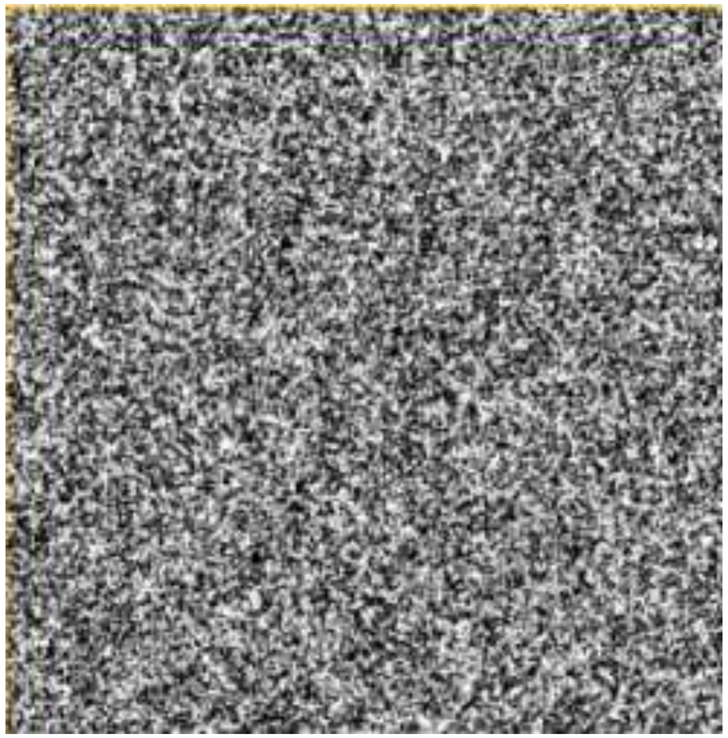
$d < d_{upper}$ $\langle \phi \rangle \langle \phi \rangle \ll \langle \phi \phi \rangle$ → **MFT fail**

$d > d_{upper}$ $\langle \phi \rangle \langle \phi \rangle \gg \langle \phi \phi \rangle$ → **MFT exact**

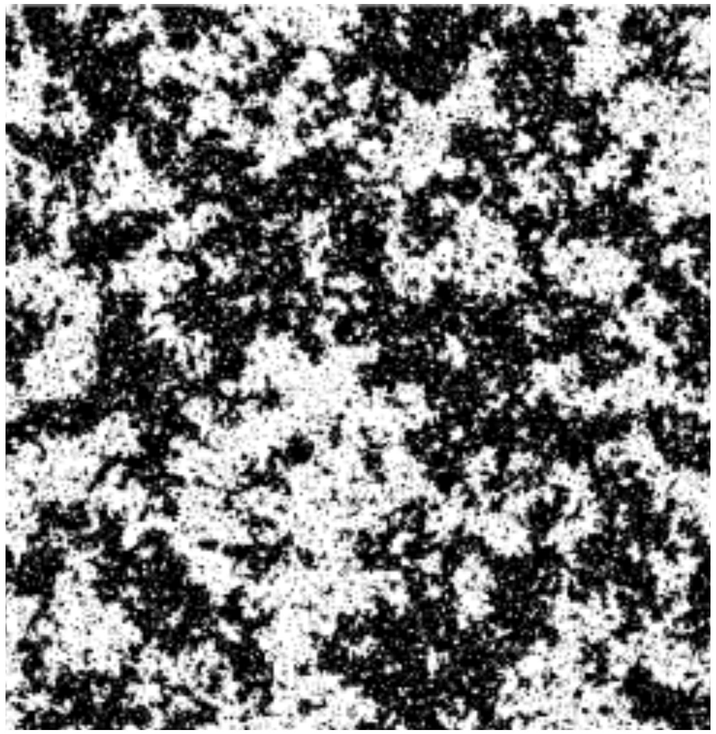
Motivation

Classical Ising model

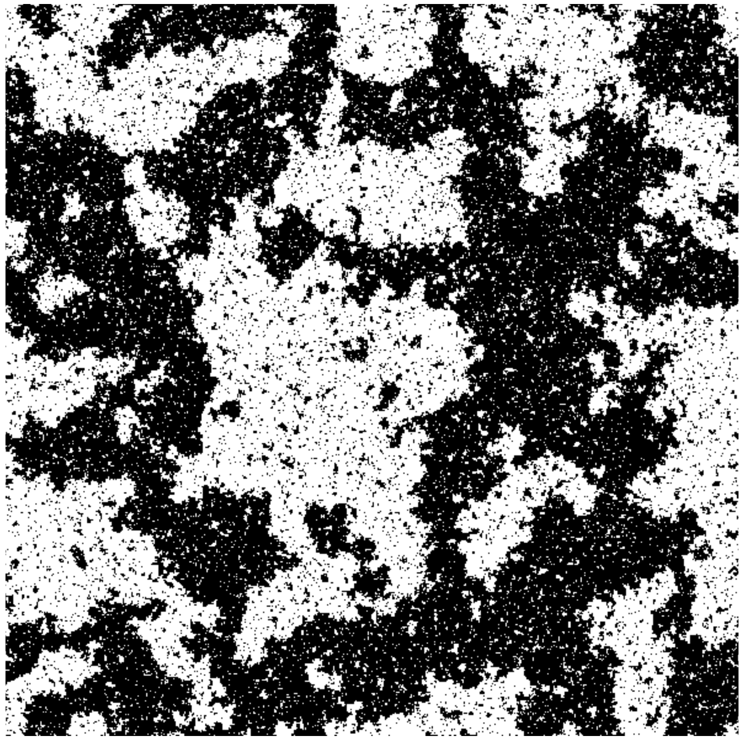
$T > T_c$



$T = T_c$



$T < T_c$



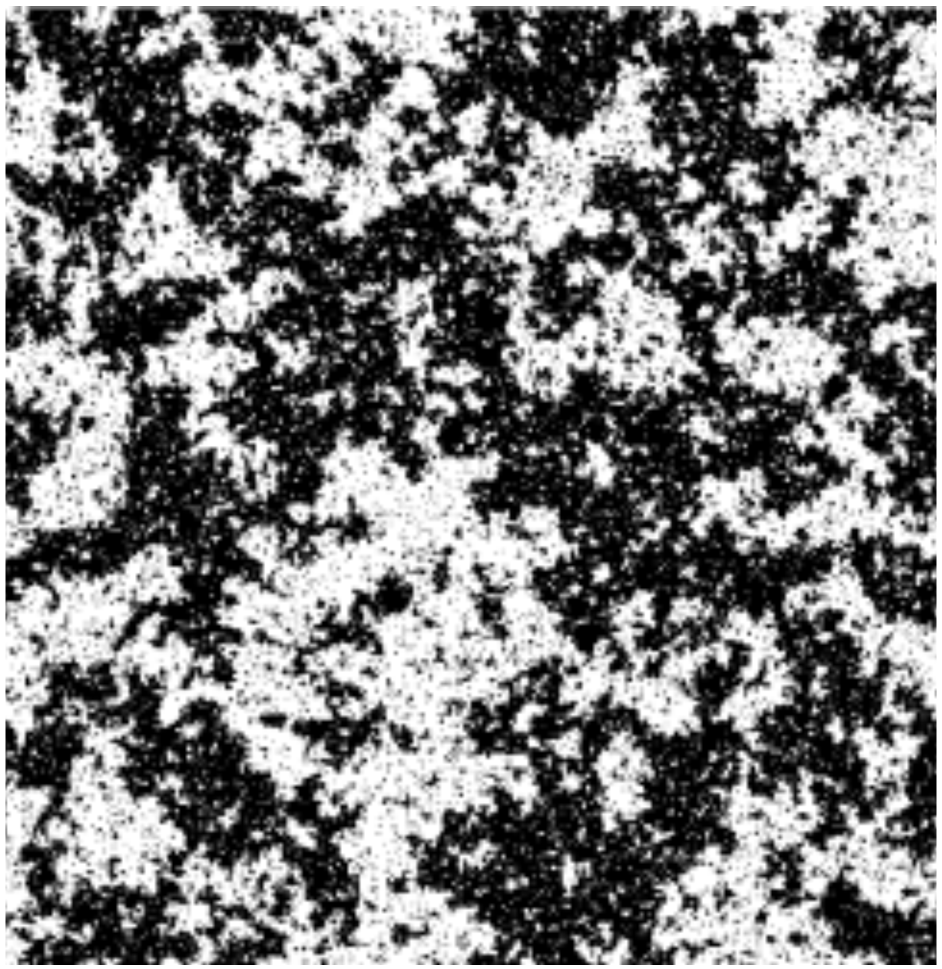
Critical fluctuation!

- Does the critical fluctuation appear at the jamming transition point?
- Can we construct the “Ginzburg criteria” for jamming?

Method

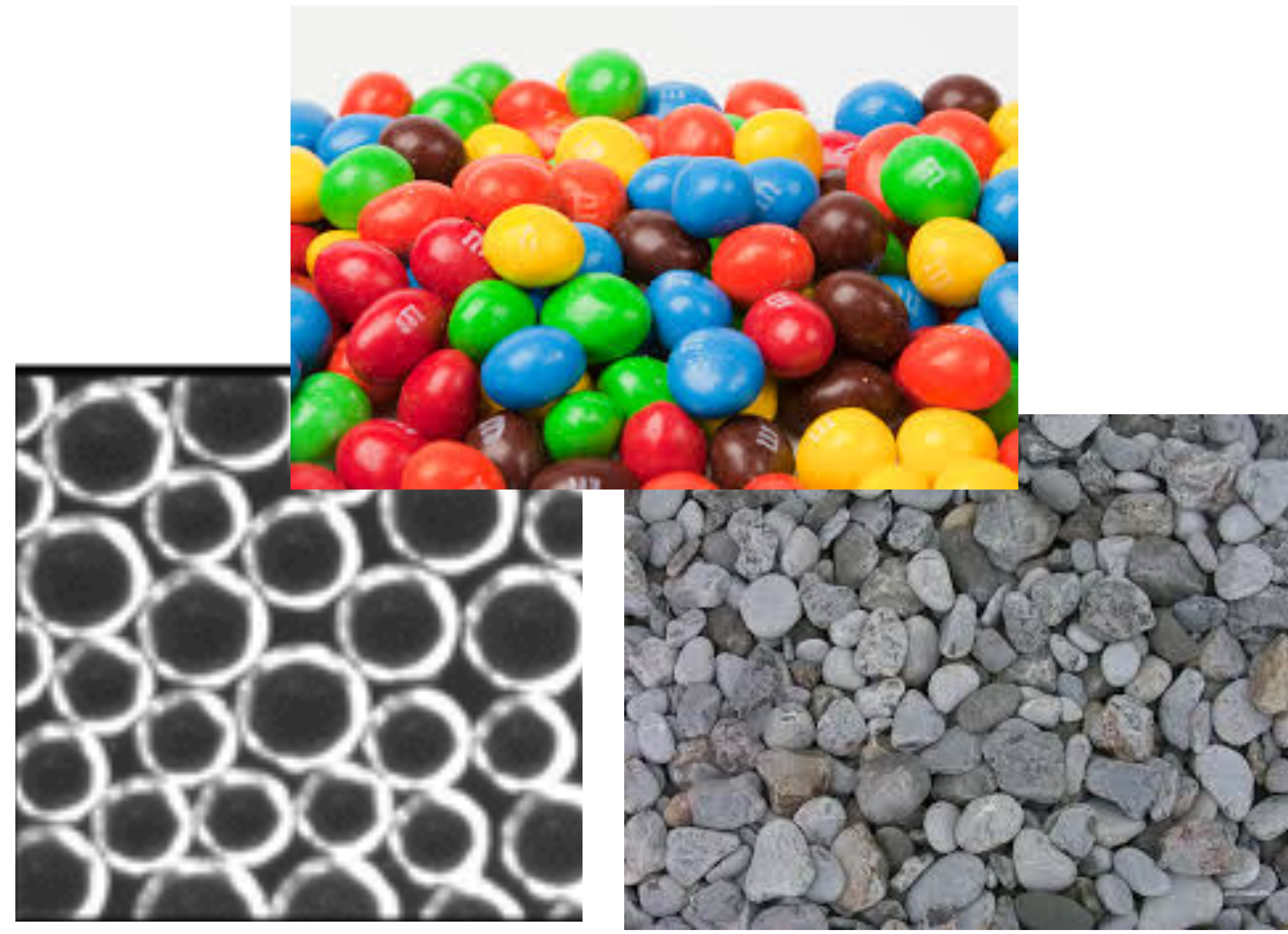
Sample to sample fluctuation

Ising model



Transition occurs at T>0
There are thermal fluctuations.

Jamming



Transition occurs at T=0
No thermal fluctuations
↓
Sample to sample fluctuation
for 10^3 samples with different IC

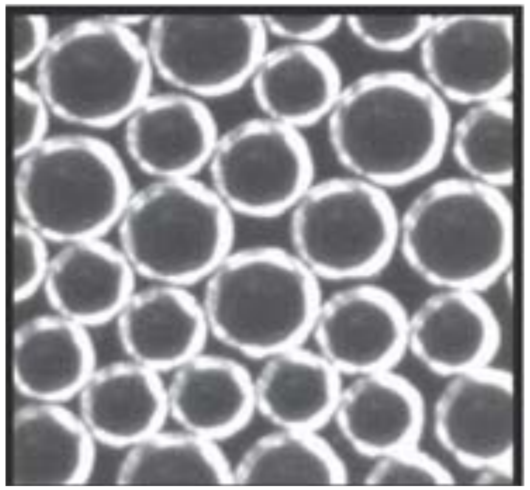
Method

Model

Numerical simulations for frictionless spherical particles in d=2

Harmonic potential

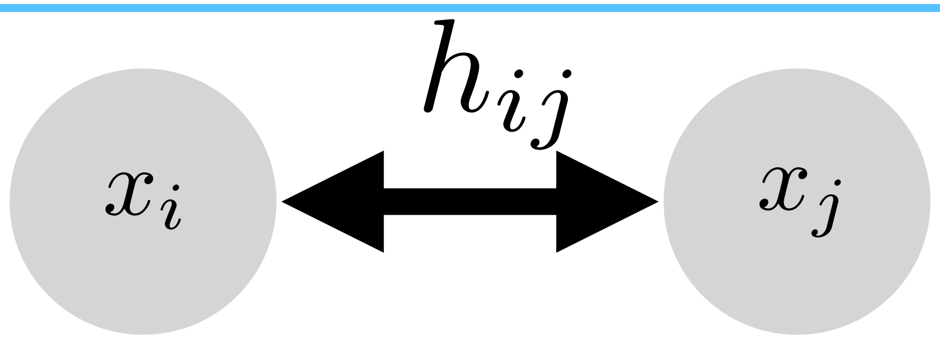
$$E = \sum_{i<j} v(h_{ij}) = \frac{\varepsilon}{2} \sum_{i<j} h_{ij}^2 \theta(-h_{ij})$$



Wet foams

Gap function

$$h_{ij} = |x_i - x_j| - R_i - R_j$$

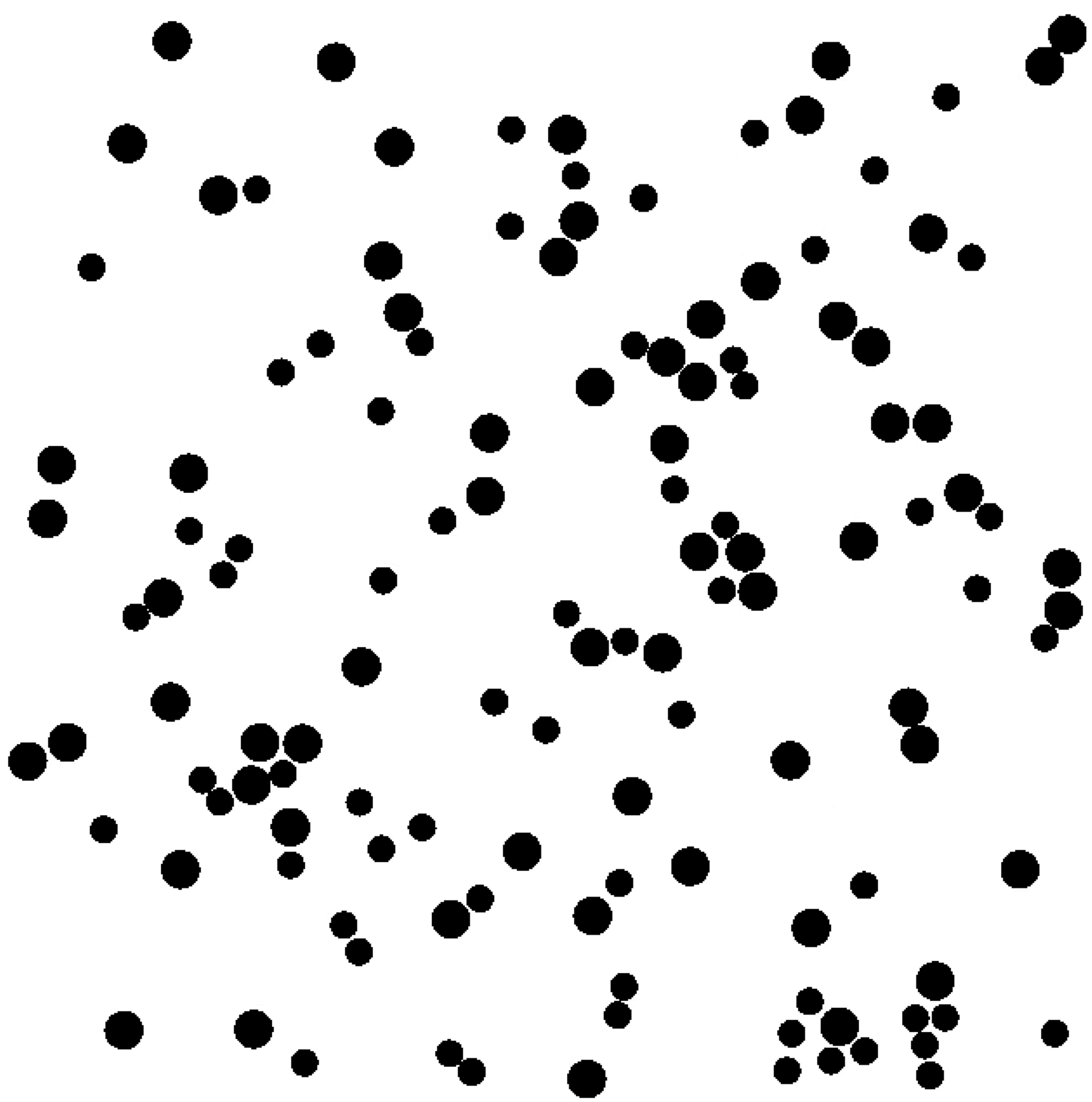


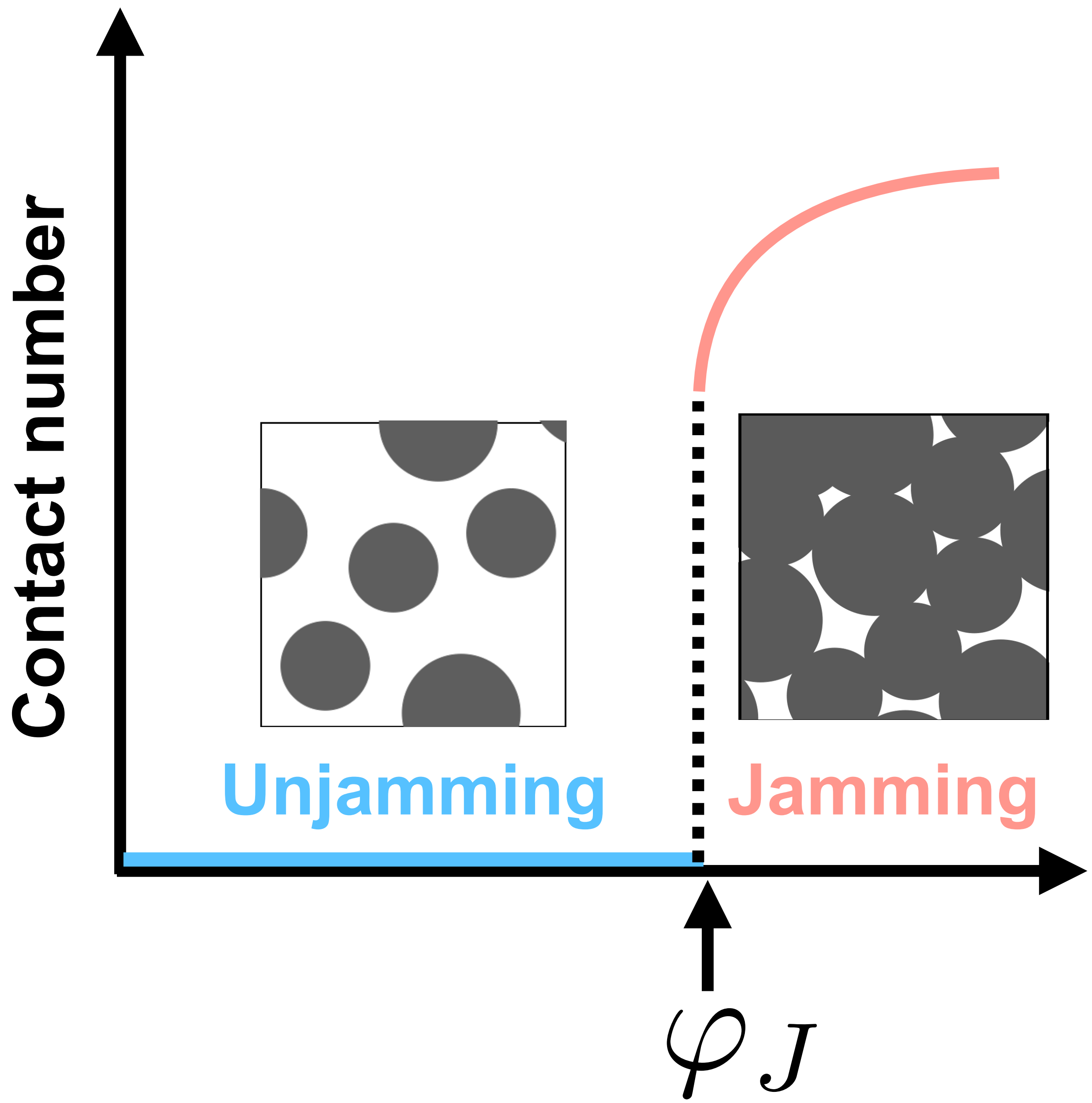
Method

Algorithm

1. Generate a random initial configuration.
2. Increase density *φ -> φ+δφ.*
3. Minimize energy.
4. Repeat 2-3.

C. O'Hern *et al.* (2003)

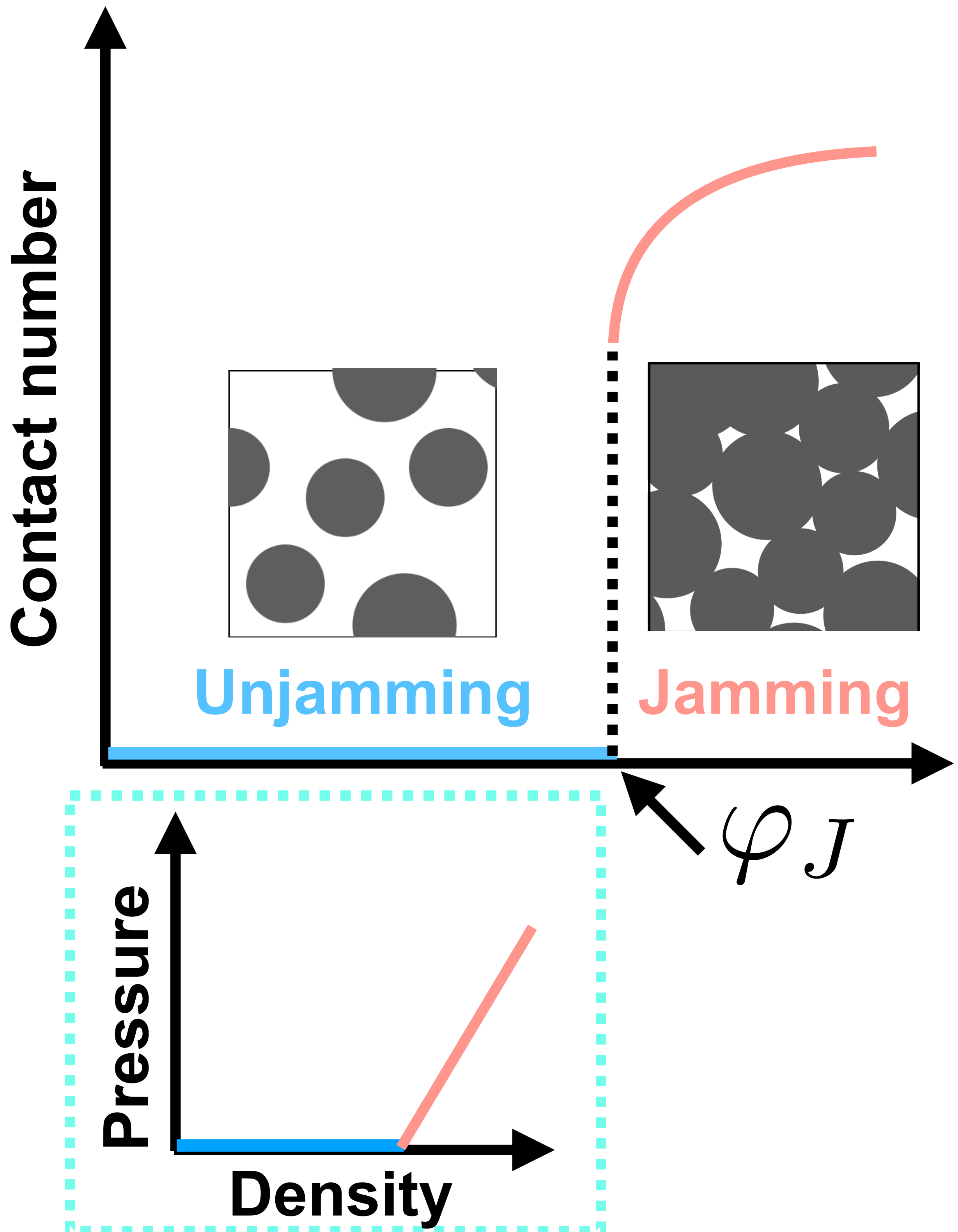




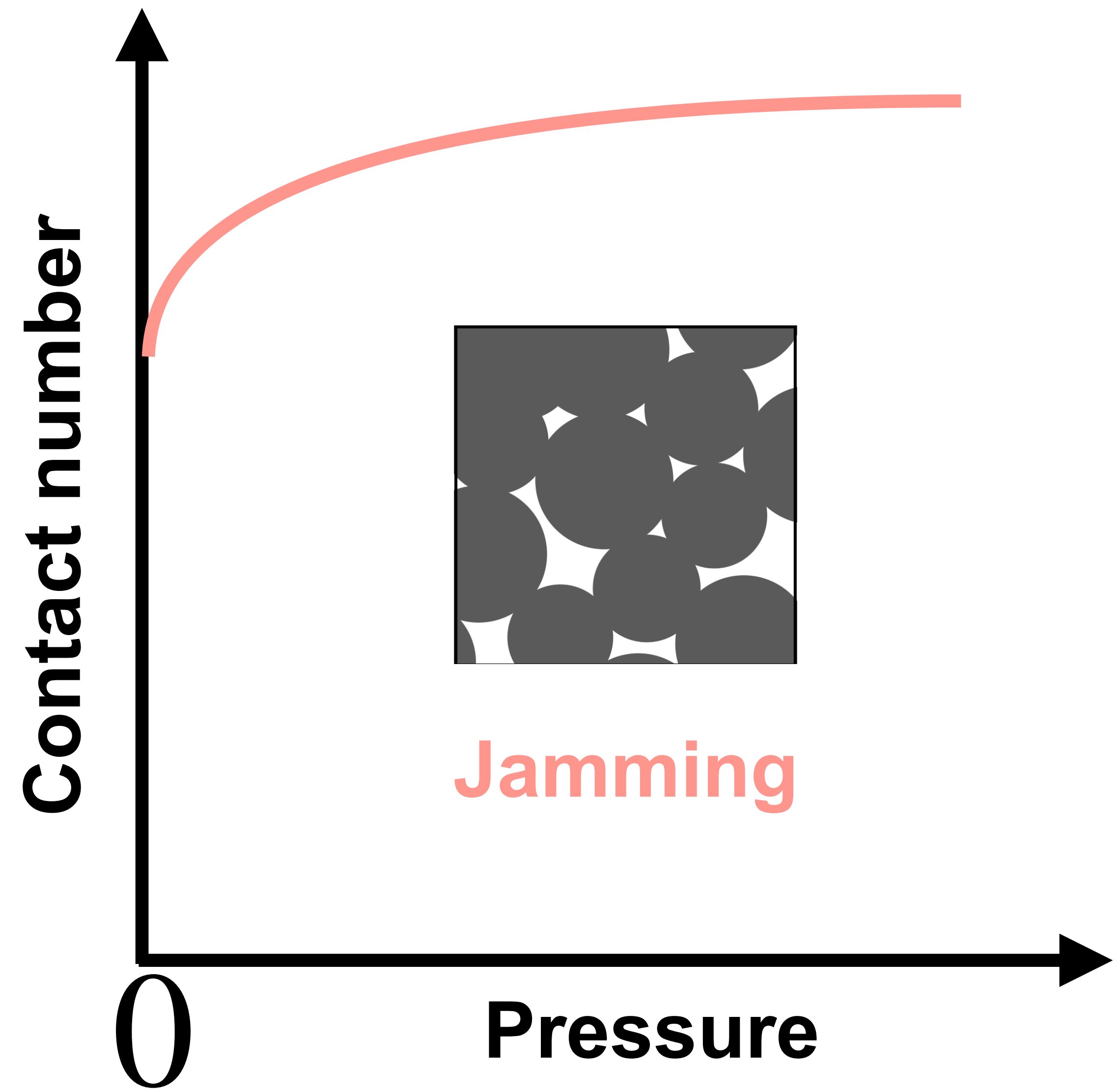
Method

Control parameter

Density control



Pressure control

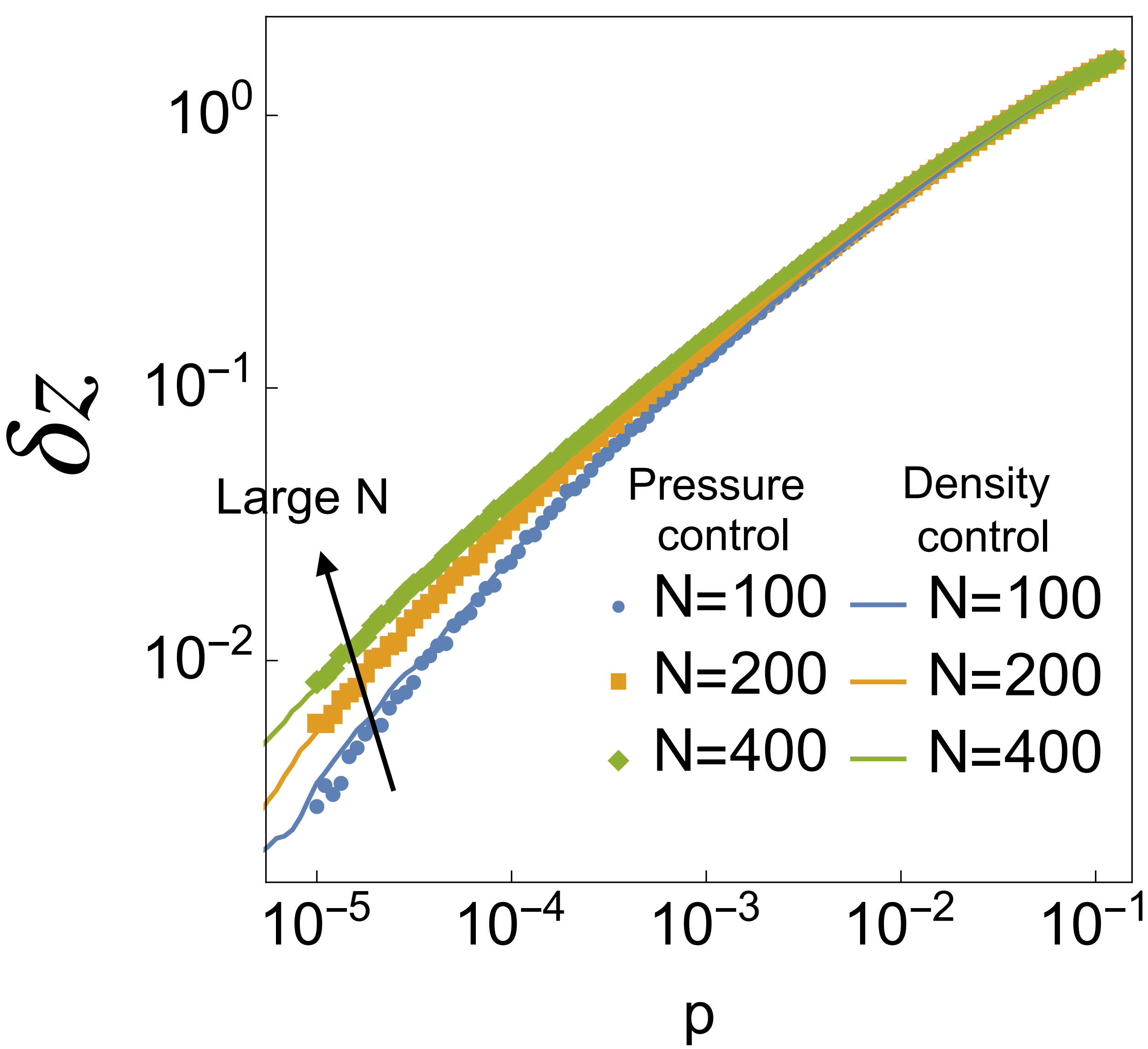


Compress/decompress the system until
the system's pressure reaches the target pressure.

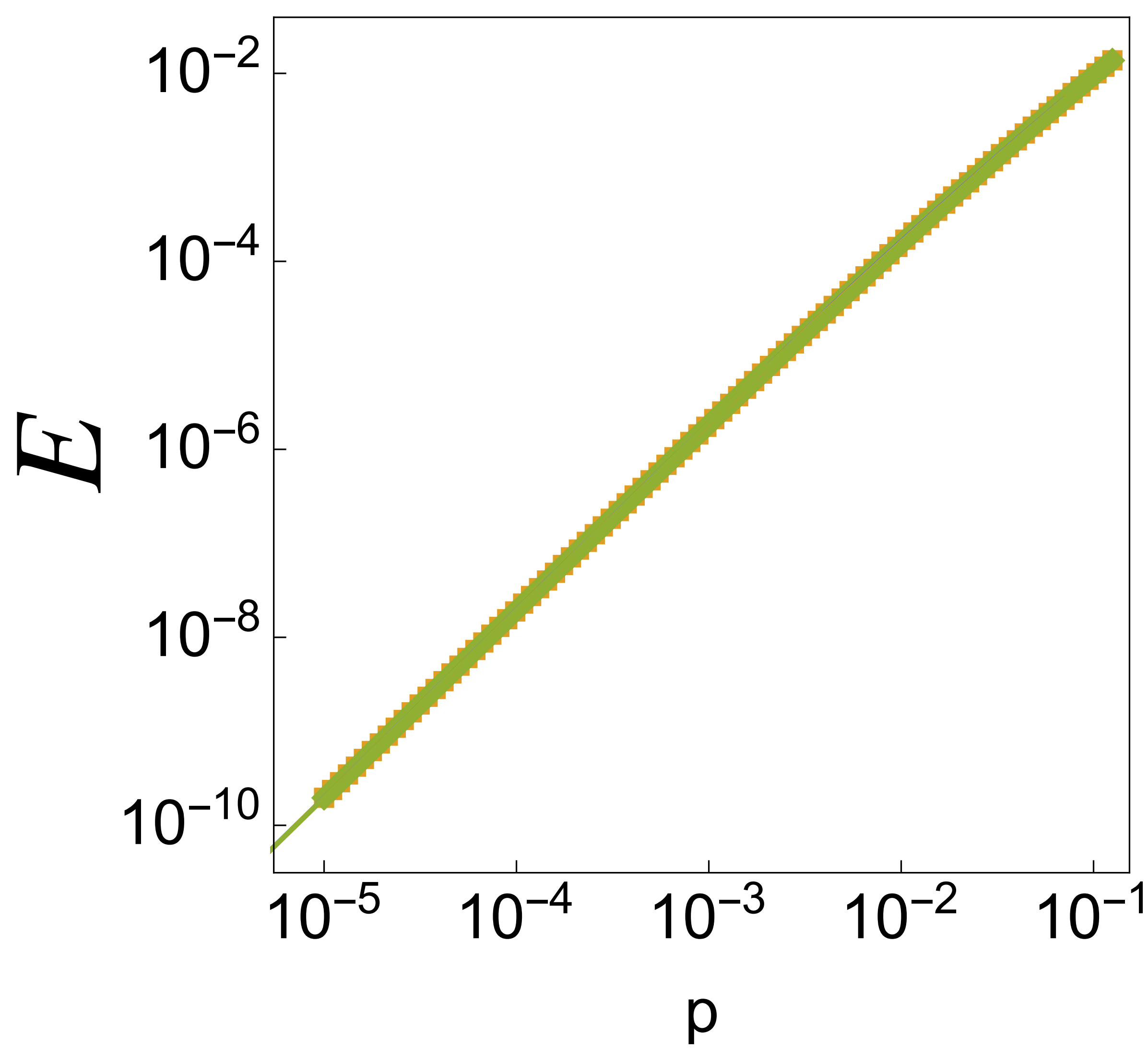
Results

Mean values

Contact number



Energy



*Results of density control are plotted as a function of the average pressure at each density.

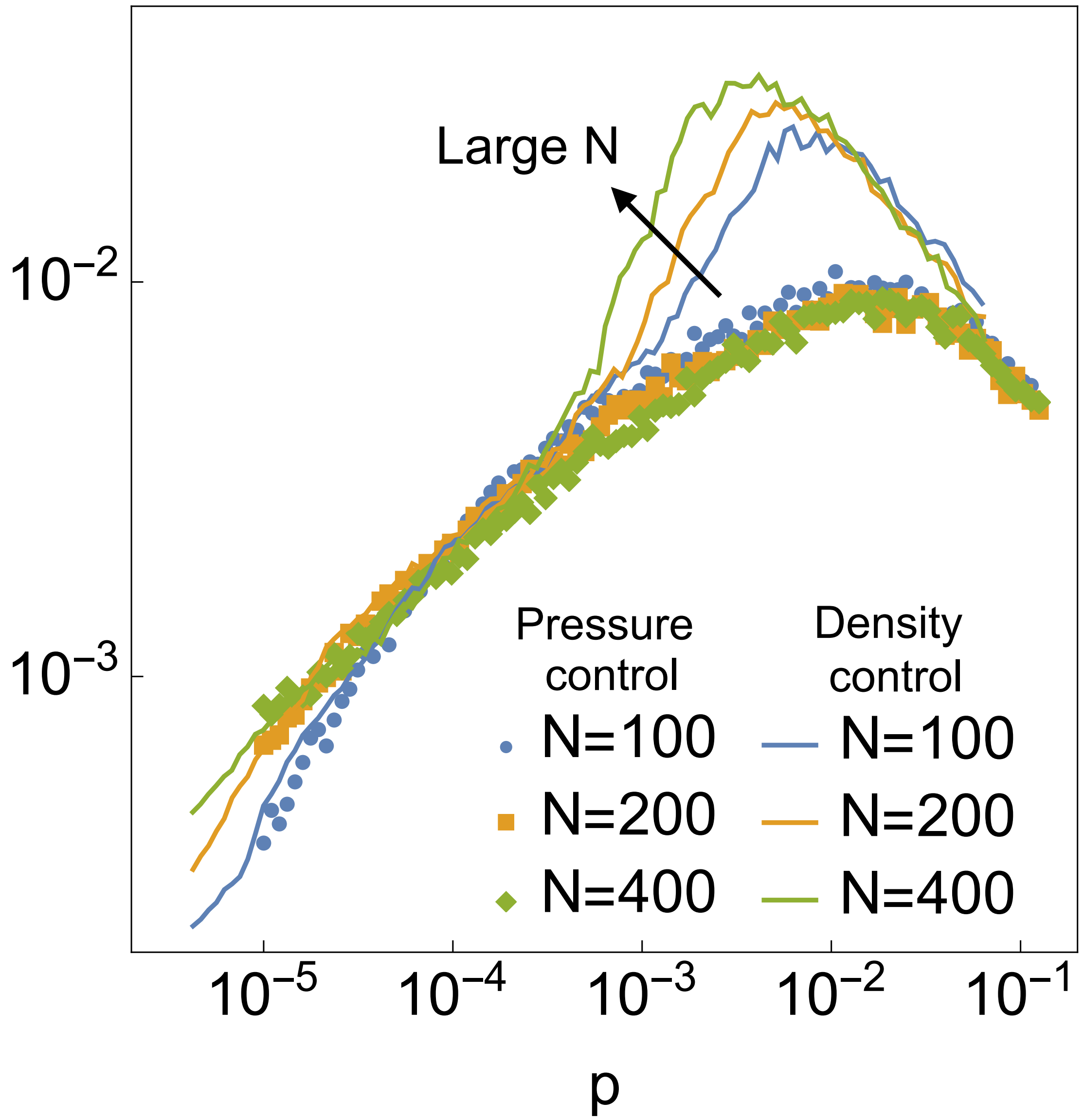
Mean values do not depend on the control parameters!

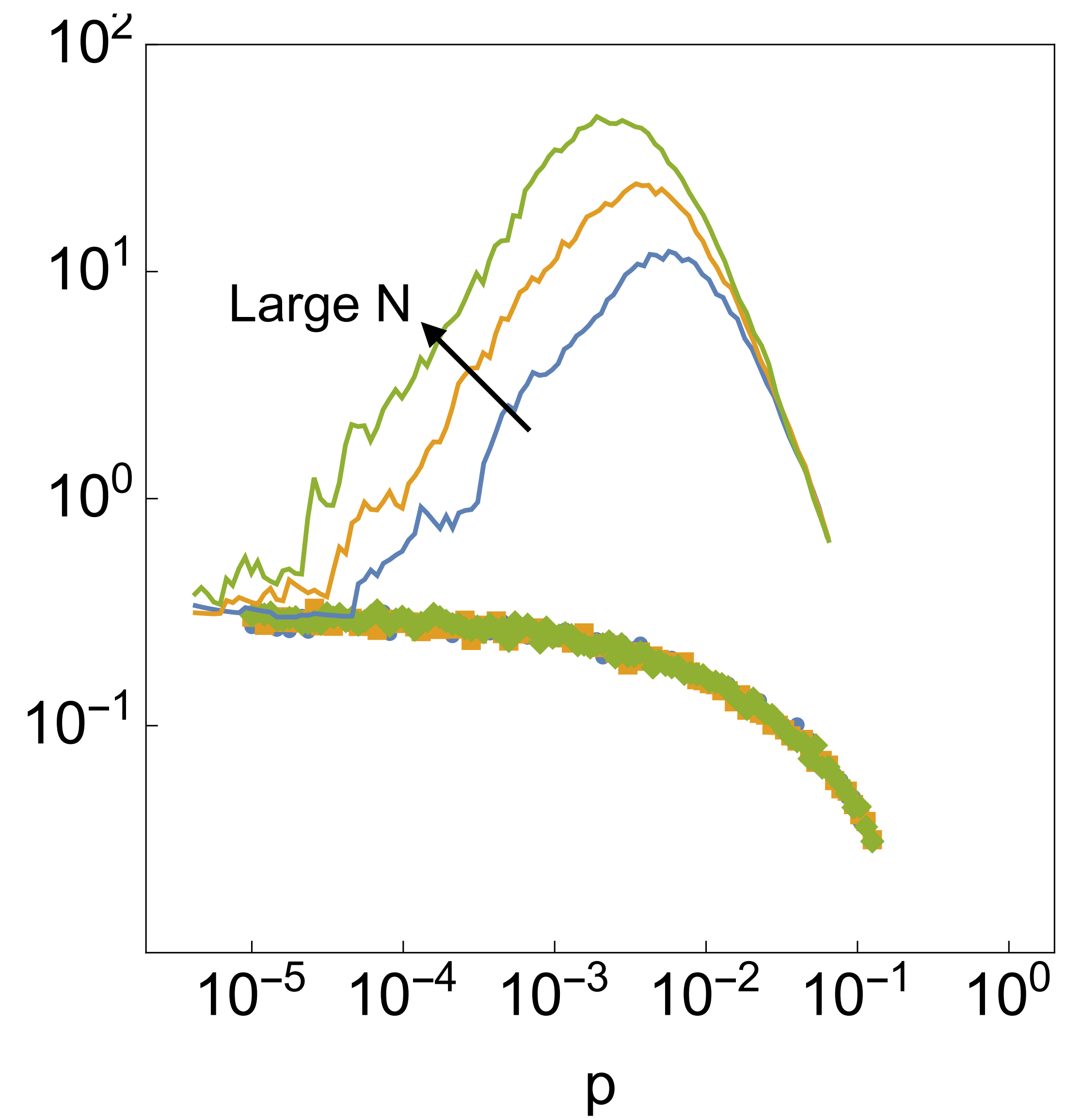
Results

Sample to sample fluctuation

$$\chi_z = N\langle \delta z^2 \rangle / \langle z \rangle^2$$

$$\chi_E = N\langle \delta E^2 \rangle / \langle E \rangle^2$$





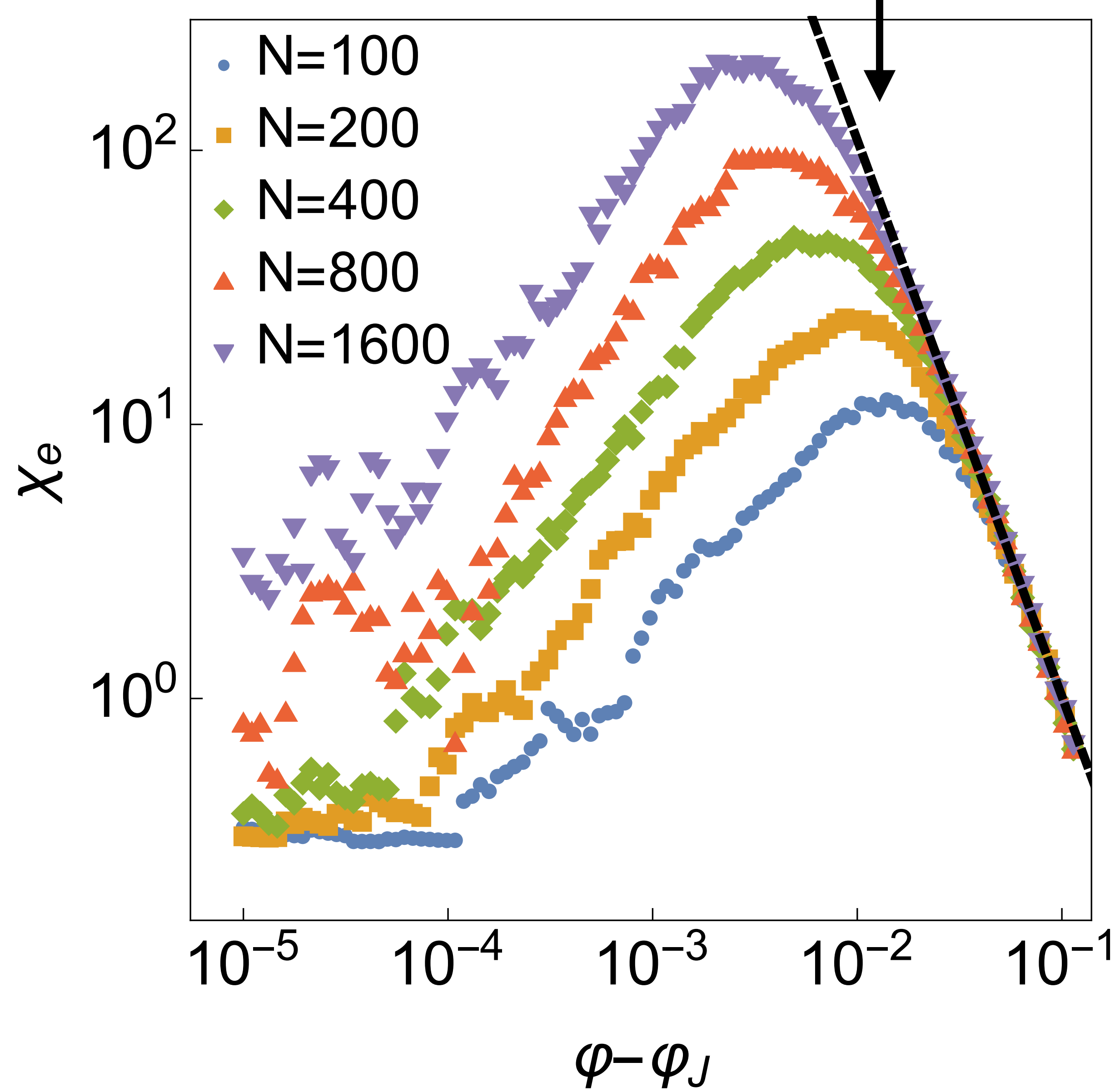
Differences are qualitative rather than quantitative

- Pressure control: do not diverge
- Density control: diverge

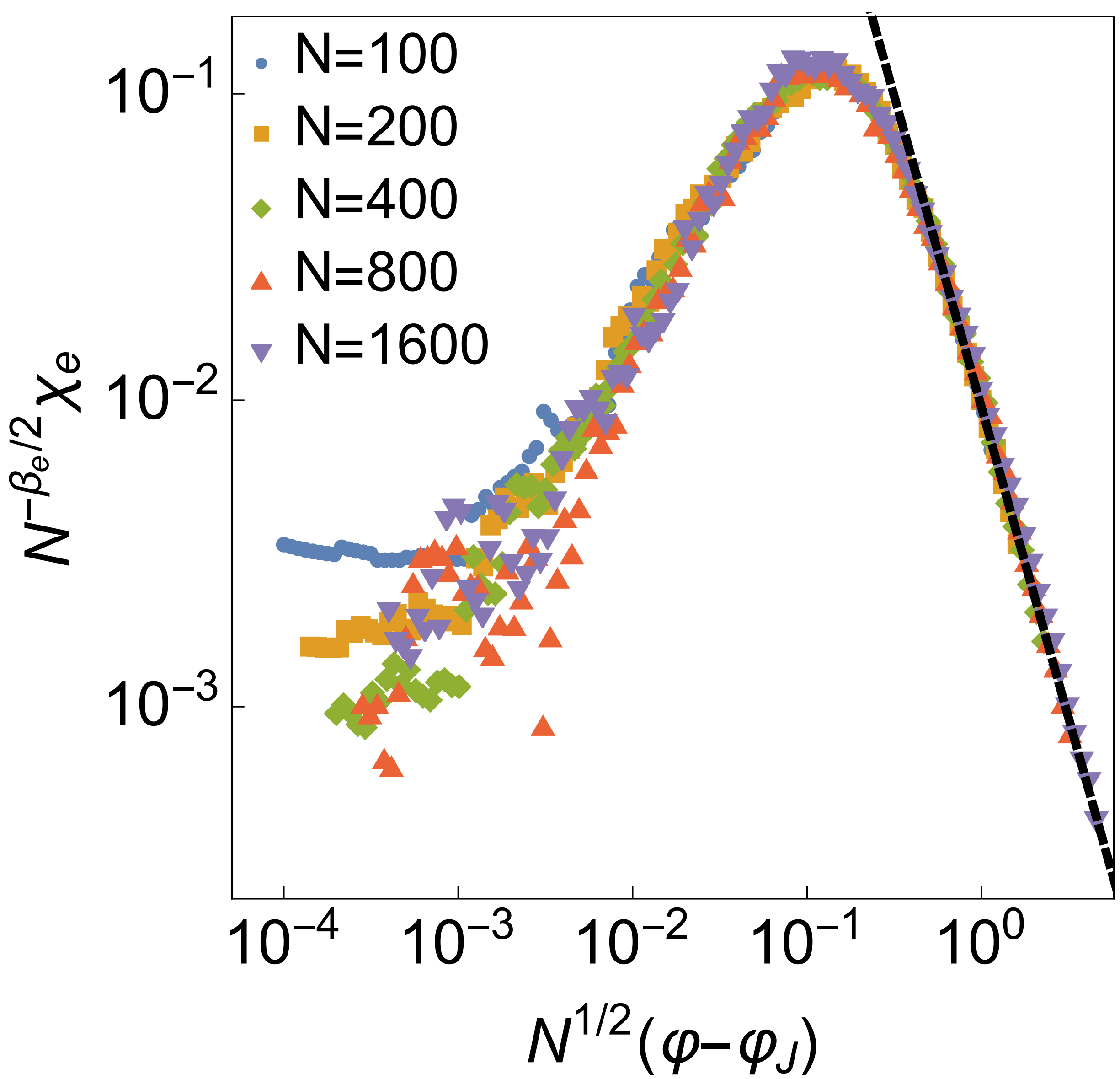
Results

Finite size scaling

$x^{-\beta_e}$, $\beta_e = 2.03$



Scaling plot

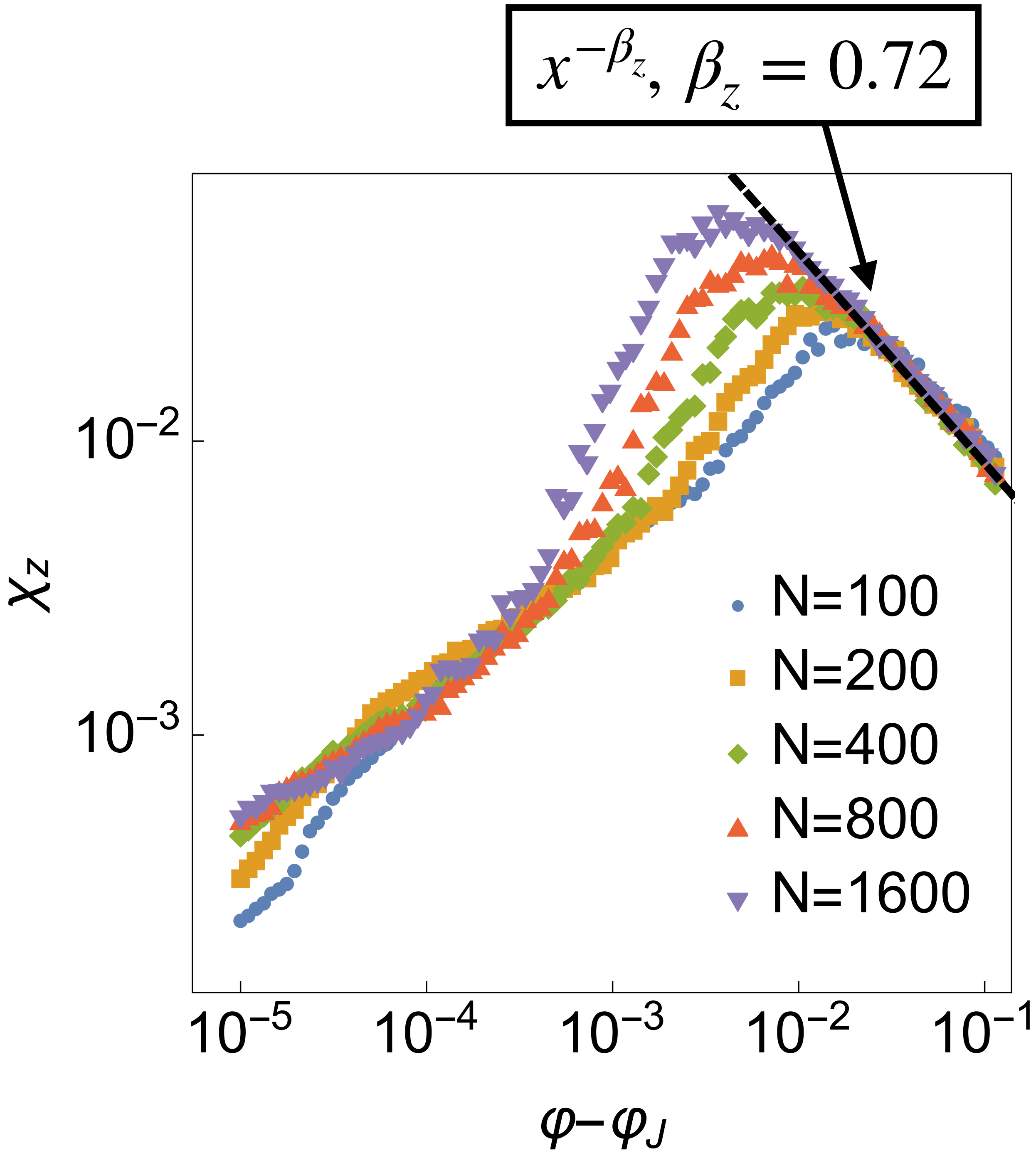


Correlated particles

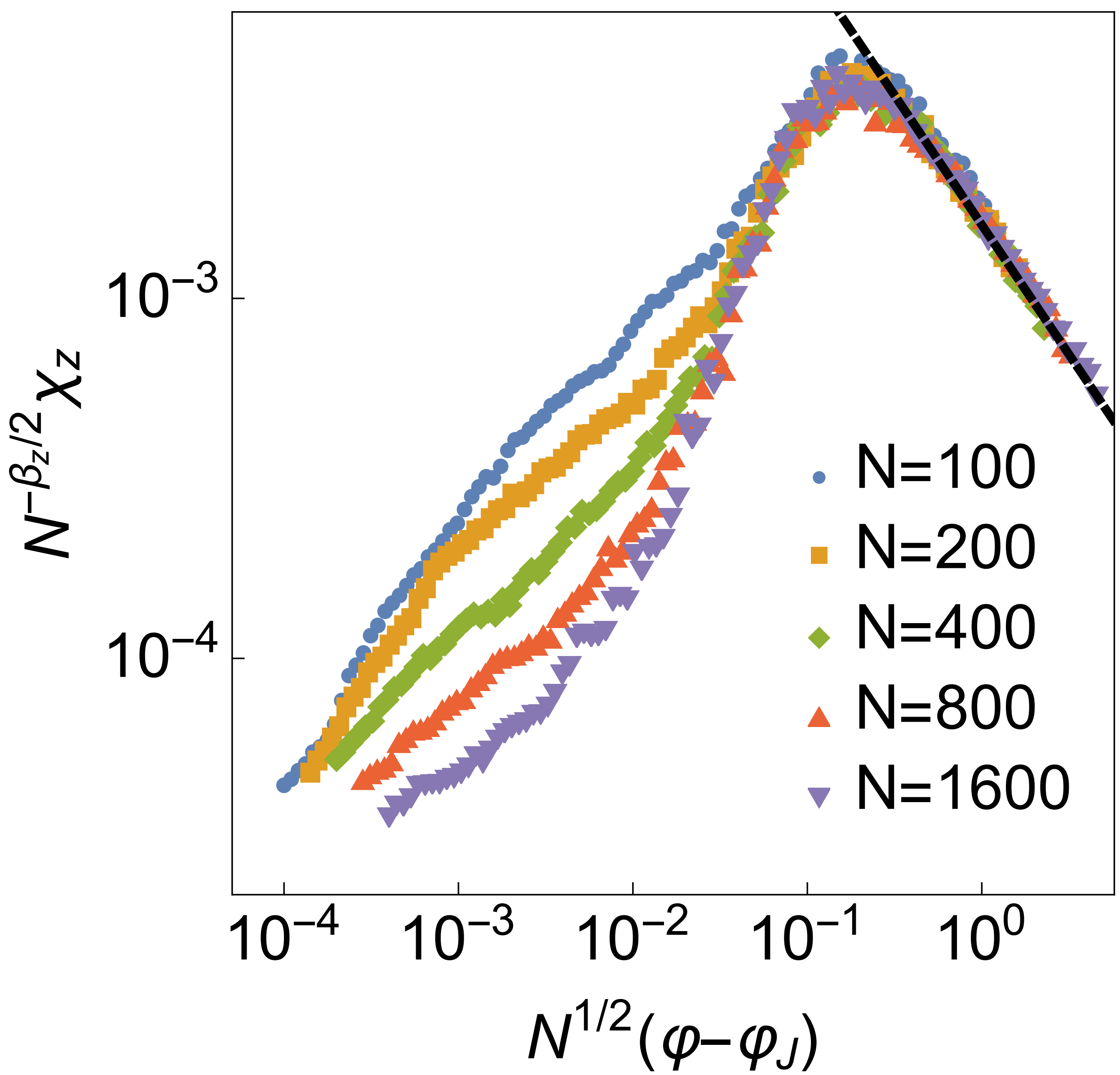
$$N_{cor} \sim \delta\varphi^{-2}$$

Results

Finite size scaling



Scaling plot



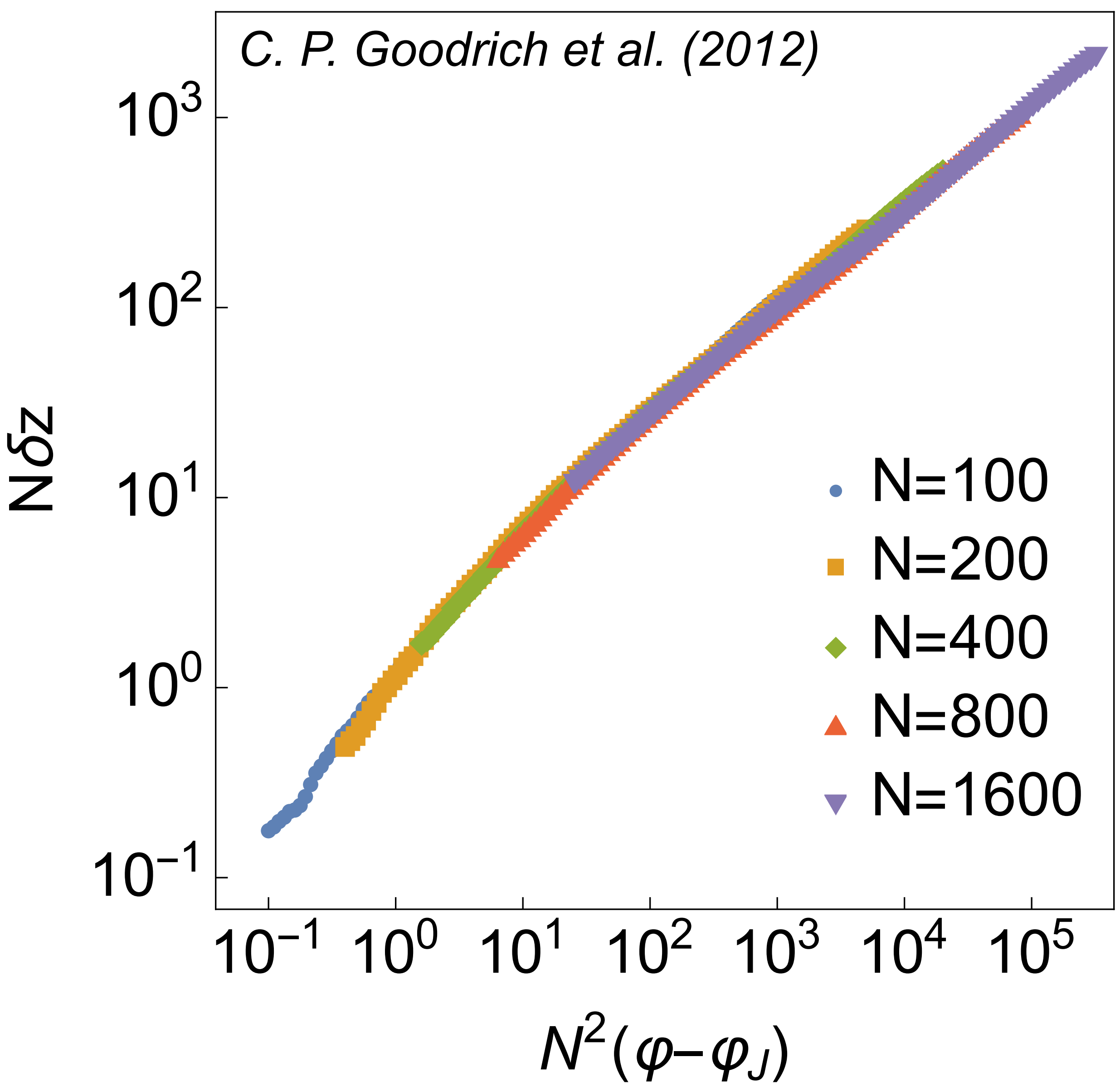
Correlated particles

$$N_{cor} \sim \delta\varphi^{-2}$$

Results

Finite size scaling

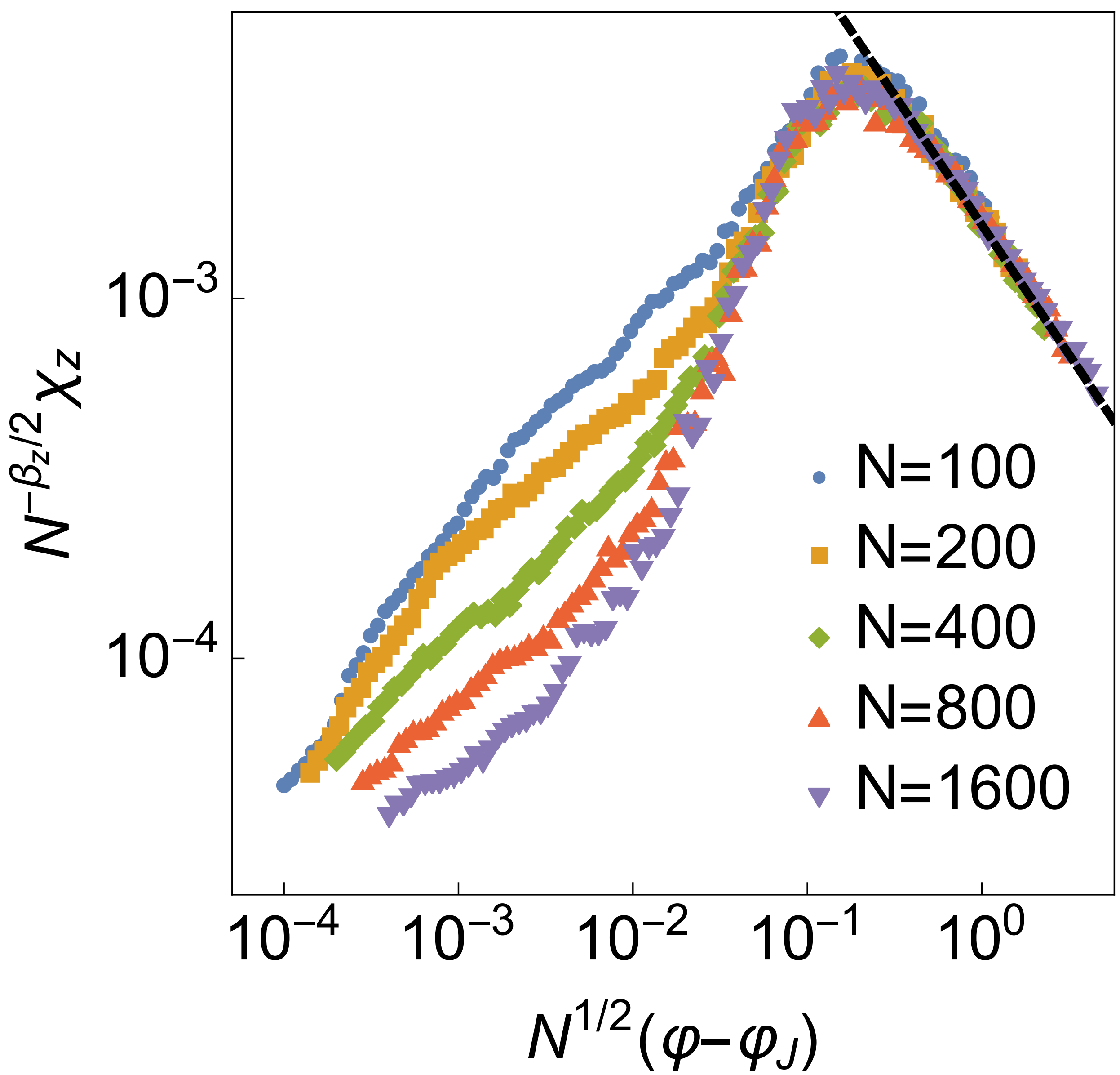
Scaling plot for mean-value



Correlated particles

$$N_{\rm cor}^{\rm mean} \sim \delta\varphi^{-1/2}$$

Scaling plot for fluctuation



Correlated particles

$$N_{\rm cor}^{\rm fluc} \sim \delta\varphi^{-2}$$

Theory

Phenomenological model

Pressure

Mean value

$$< p > = A\delta\varphi$$

Mean value + fluctuation

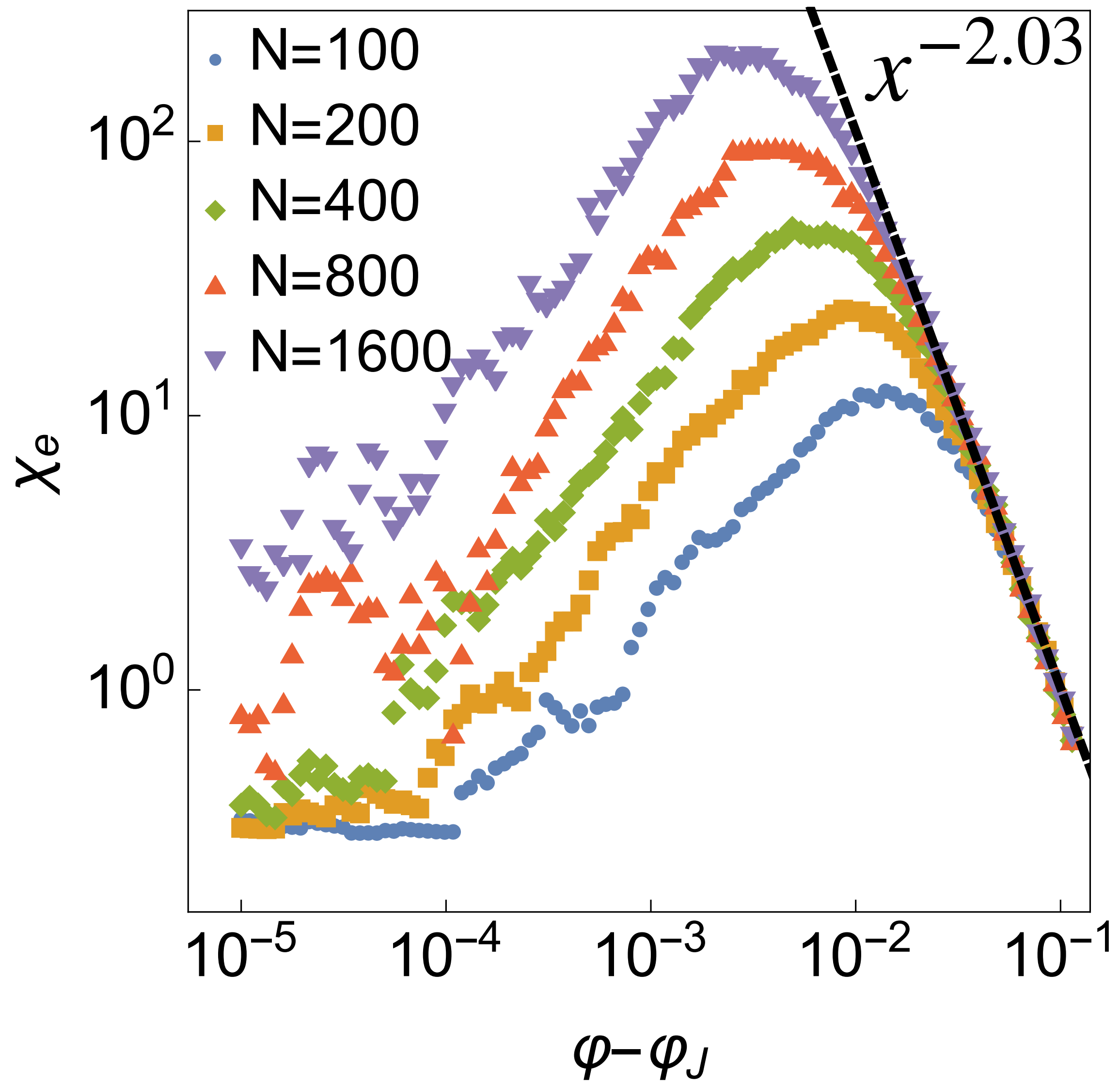
$$p = A\delta\varphi + \xi,\ \langle \xi^2 \rangle = \Delta / N$$

Energy

$$E \propto p^2 = (A\delta\varphi + \xi)^2 \approx A^2\delta\varphi^2 + 2A\delta\varphi\xi$$

$$\langle E \rangle = A^2\delta\varphi^2,\ \langle \delta E^2 \rangle = 4A^2\delta\varphi^2$$

$$\chi_e = \frac{\langle \delta E^2 \rangle}{\langle E \rangle^2} \propto \delta\varphi^{-2}$$



Theory

Phenomenological model

Pressure

Mean value

$$< p > = A\delta\varphi$$

Mean value + fluctuation

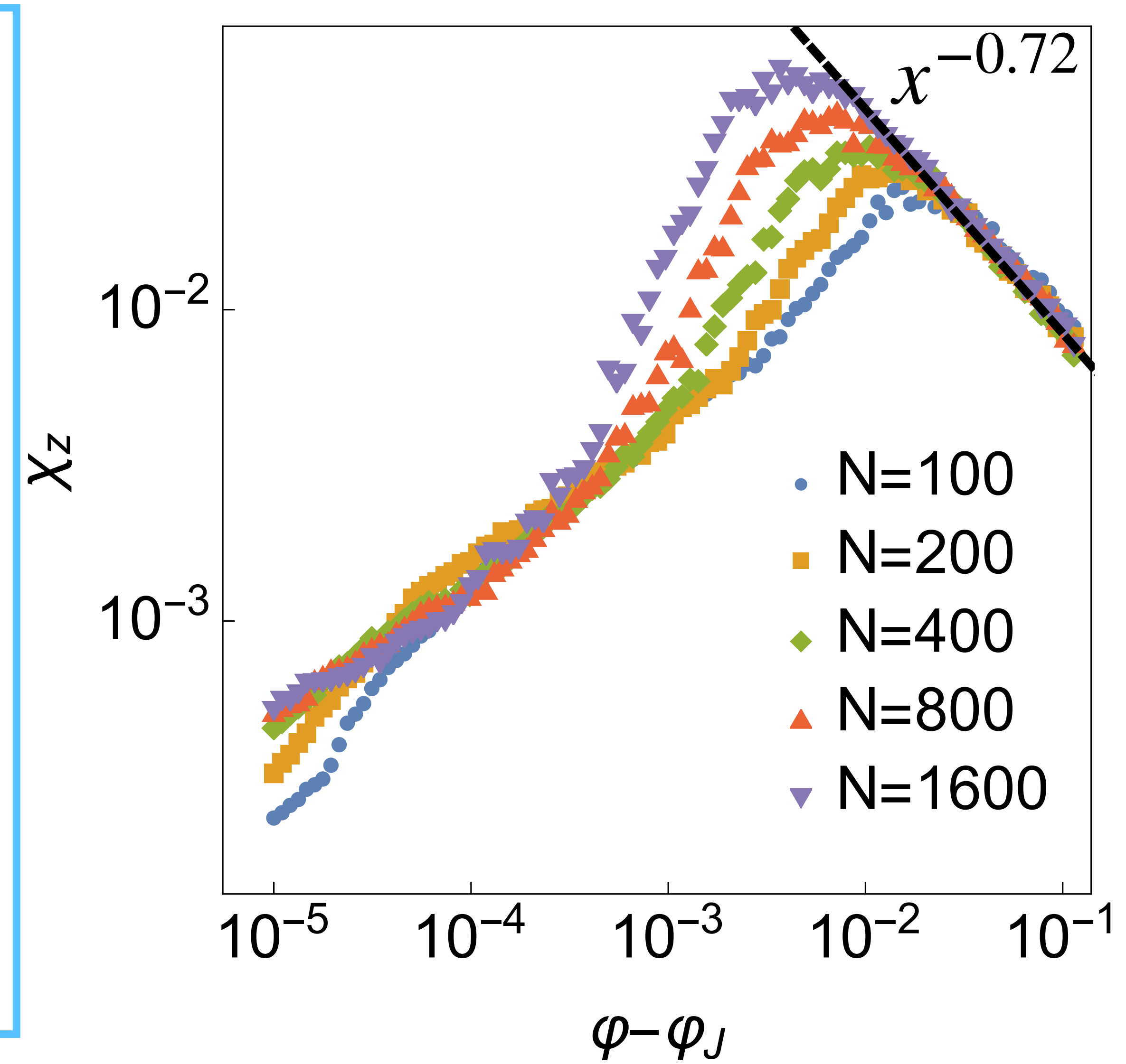
$$p = A\delta\varphi + \xi,\ \langle \xi^2 \rangle = \Delta / N$$

Contact number

$$z - z_J \propto p^{1/2} \rightarrow z = z_J + (A\delta\varphi + \xi)^{1/2}$$

$$\langle z \rangle \approx z_J,\ \langle \delta z^2 \rangle = A^{-1/2}\delta\varphi^{-1/2}$$

$$\chi_z = \frac{\langle \delta z^2 \rangle}{\langle z \rangle^2} \propto \delta\varphi^{-1/2}$$



Summary

- Fluctuations of the pressure control do not diverge.
- Fluctuations of the density control do diverge.
- Correlated volume of the fluctuation diverges as $N_{\rm cor} \sim \delta\varphi^{-2}$, different from that of the mean value $N_{\rm cor} \sim \delta\varphi^{-1/2}$
  C. P. Goodrich et al. (2012)

All those results can not be explained by the current mean-field theory.

Future Work

Can we construct the mean-field theory for the sample to sample fluctuation?

cf: Random Field Ising model $\langle m^2 \rangle_{sample} \gg \langle m^2 \rangle_{thermal}$

-> Higher upper critical dimension d_u = 6 than the pure Ising d_u=4