Japan-France joint seminar “Physics of dense and active disordered materials”

Crossover of scaling law as a self-similar solution : the dynamical impact of viscoelastic board

Hirokazu Maruoka

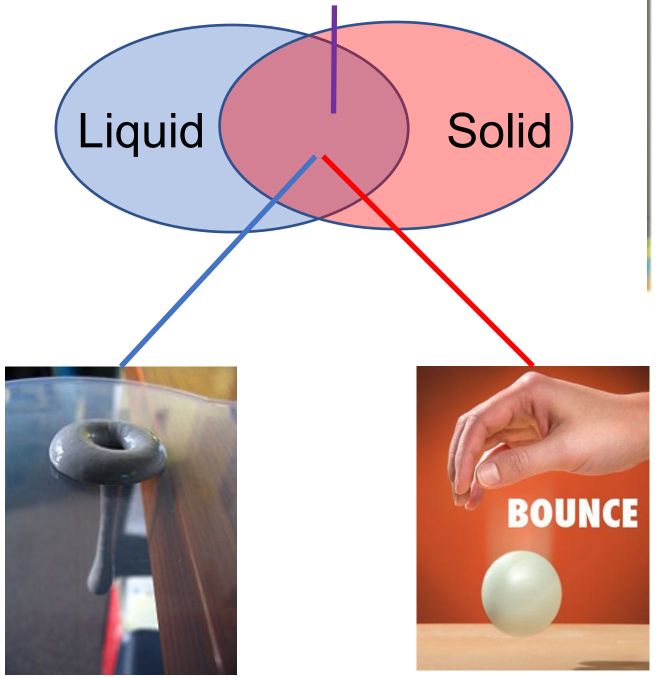
Yukawa Institute for Theoretical Physics, Kyoto University
Advanced Statistical Dynamics
Postdoctoral Researcher
Hirokazu Maruoka
address : Kitashirakawa Oiwakecho, Sakyo-ku, Kyoto
606-8502 Japan
e-mail : hirokazu.maruoka@yukawa.kyoto-u.ac.jp
TEL : +81-75-753-7063

2023/03/14

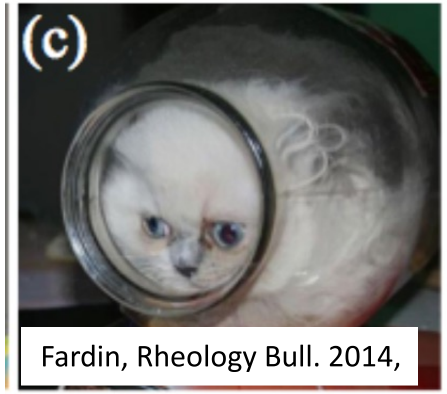
https://arxiv.org/abs/2203.00253

What is crossover of scaling law? : Soft/compex matters

- ***Mixed*** physical property

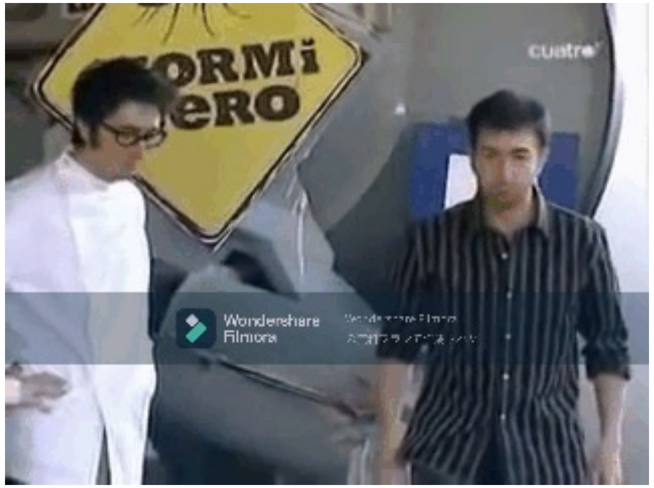


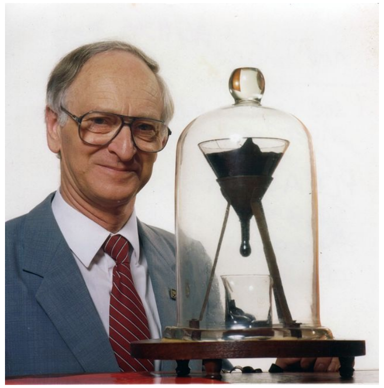
Time scale



Fardin, Rheology Bull. 2014,

- ***Scale-dependent*** behavior





$$\frac{dV}{dT} = \begin{cases} 0 & (\mathrm{De} \gg 1) \\ \frac{\pi d^4 \rho g}{128\mu}\left(1 + \frac{h}{l}\right) & (\mathrm{De} \sim 1)\,. \end{cases}$$

Parnell, T. *et al*. Eur.J.Phys. (1984)

What is crossover of scaling law?

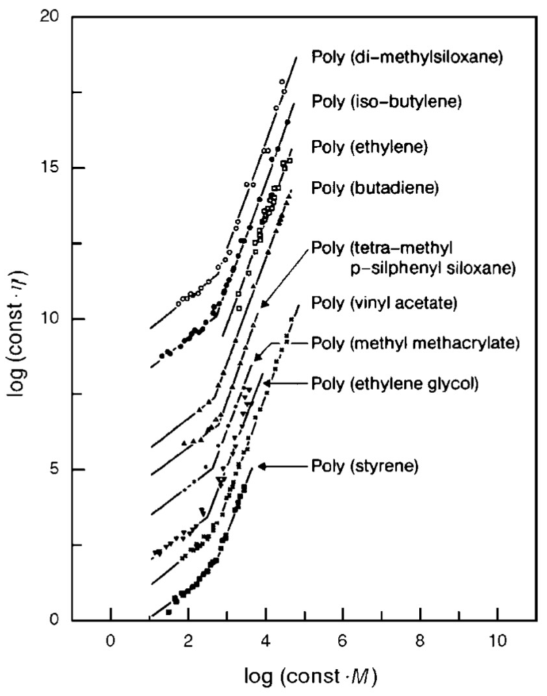
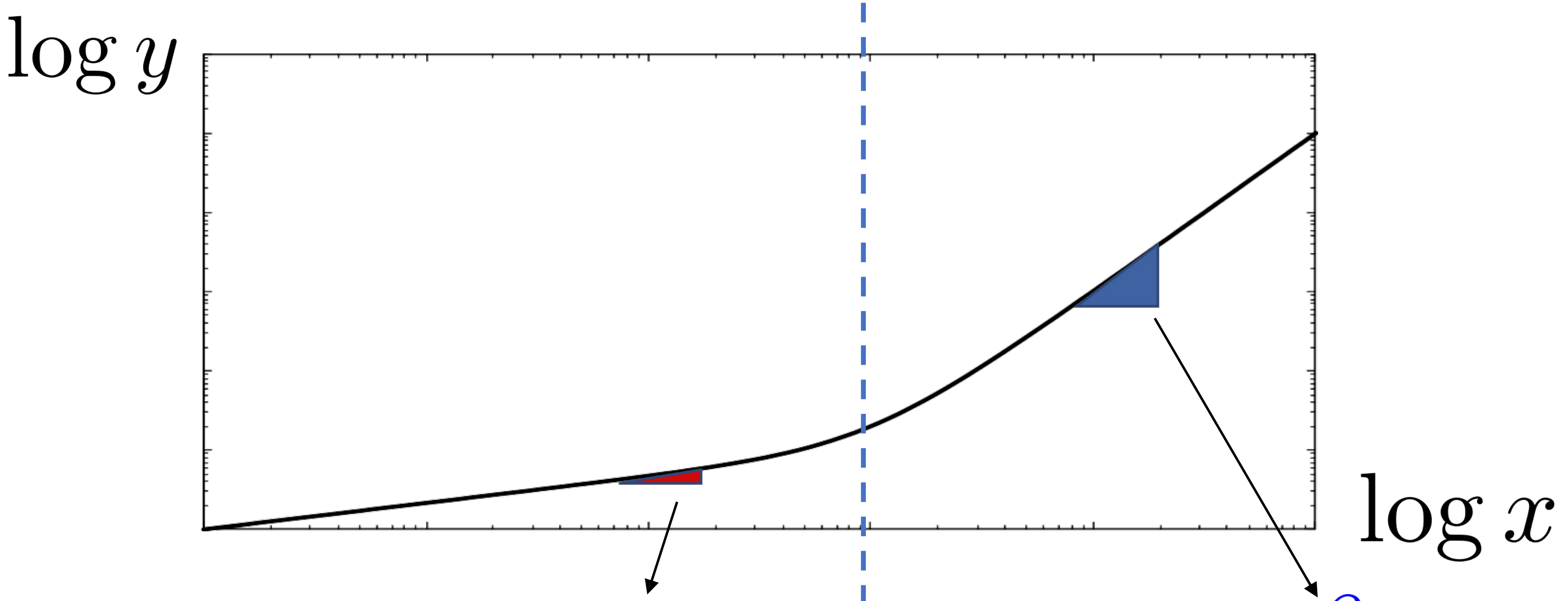


Fig. 6.13. Molecular weight dependence of the viscosity as observed for the indicated polymers. For better comparison curves are suitably shifted in horizontal and vertical directions. Data from Berry and Fox [71]

$$\eta \sim \begin{cases} M & (M < M_c) \\ M^{3.2-3.6} & (M > M_c) \end{cases}$$



$$y = Ax^{\alpha} \qquad y = Bx^{\beta}$$

x

- **Crossover of scaling is the process of transition of scaling law by continuous change of scale parameter**

Motivation of my study

- How can we characterize **scale-dependent behavi**ors and **mixing properties**?
- How can we **integrate** two different behaviors, two different scaling law?

⇩

- Self-similarity

What is crossover of scaling law in terms of self-similarity?

$$y = f(x, z)$$

$$\Pi = \frac{y}{Ax^{\alpha}}$$

$$\theta = \frac{z}{x^{\beta}}$$

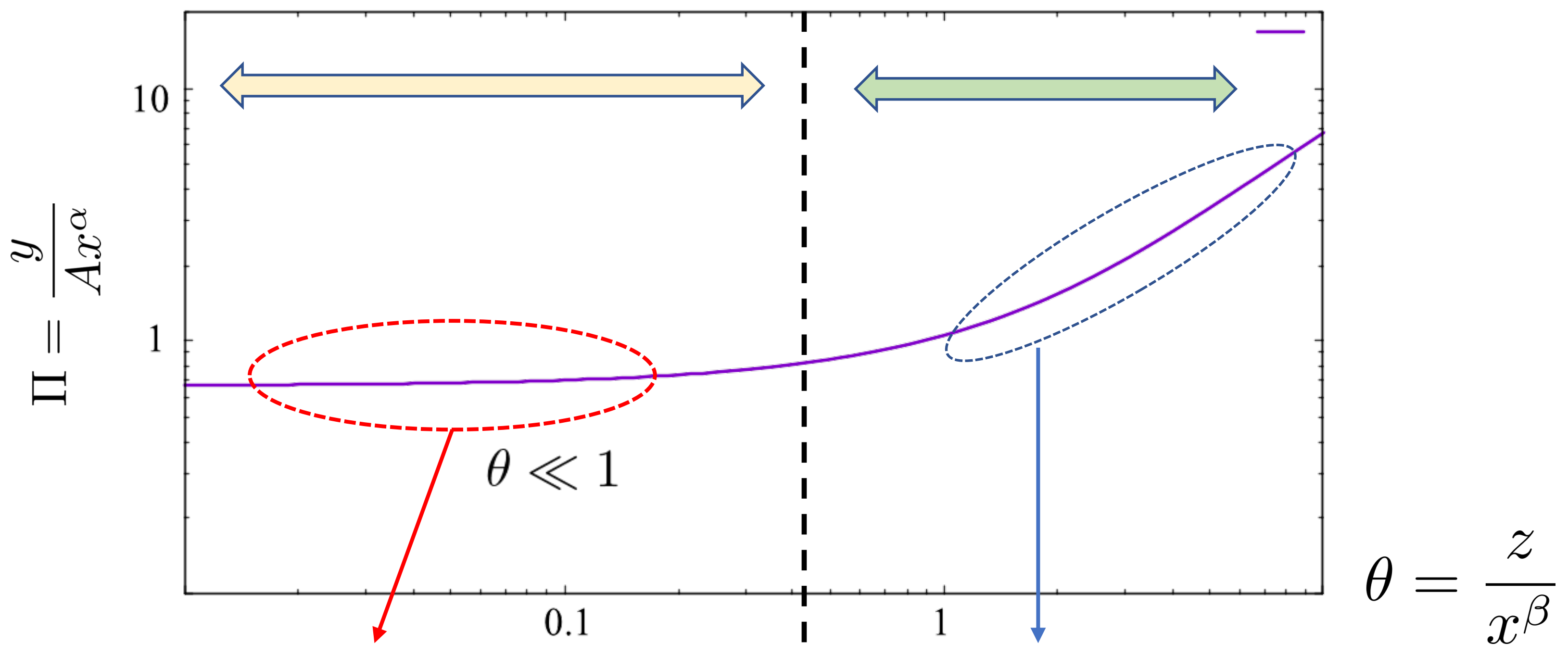
$$\boxed{\Pi = \Phi(\theta)}$$

Self-similar solution

- Scaling law
  → intermediate asymptotics
- Crossover of scaling law
  → self-similar solution

Φ idealized region

nonidealized region



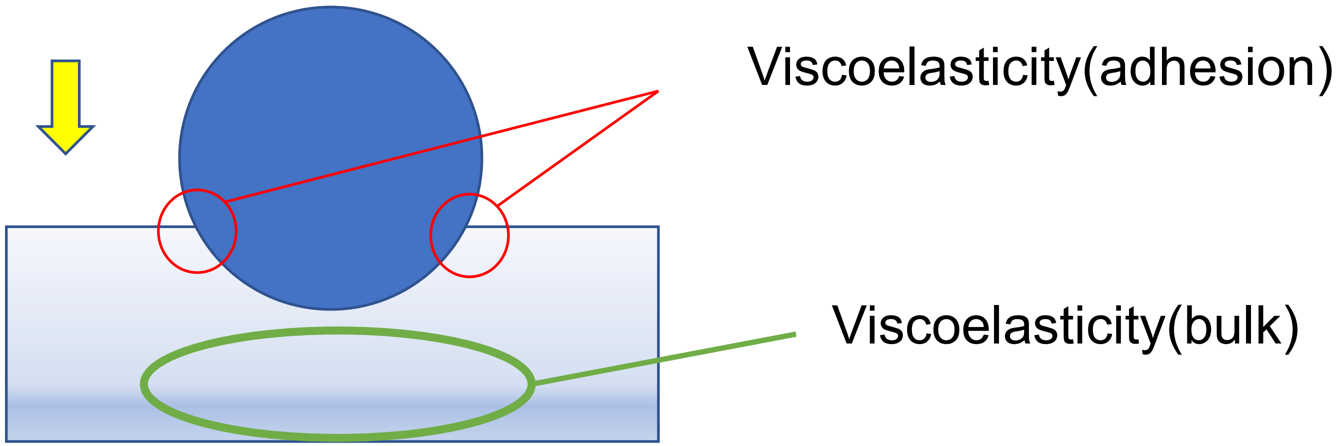
$$\Phi = \mathrm{const} \qquad \Phi \neq \mathrm{const}$$

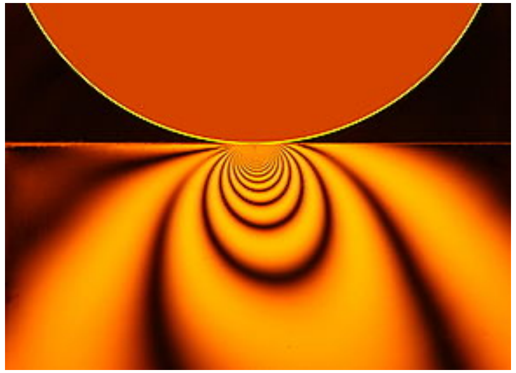
$$y = Ax^{\alpha} \longrightarrow y = Bx^{\beta}$$

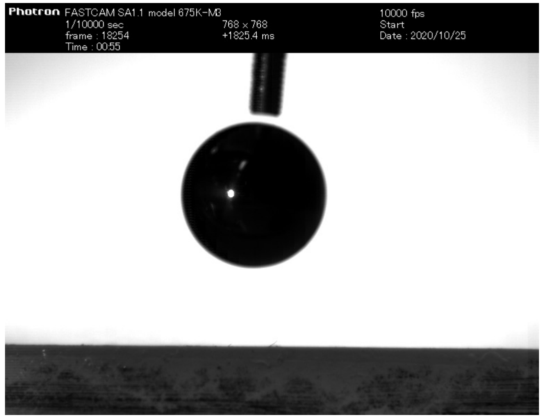
Intermediate asymptotics

Barenblatt, Scaling (CUP 2003)

Viscoelascitiy on contact mechanics

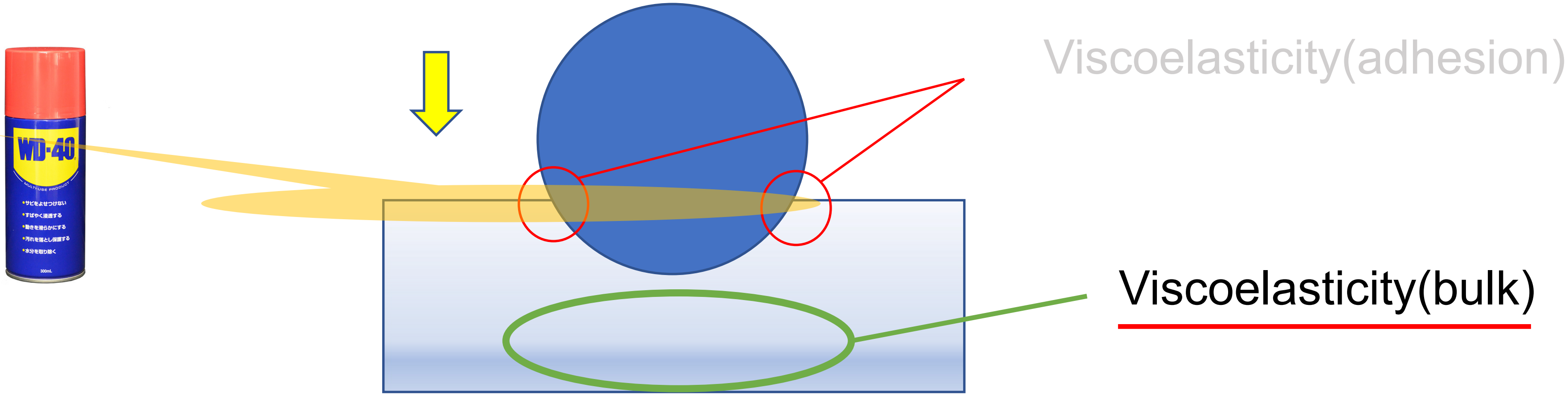


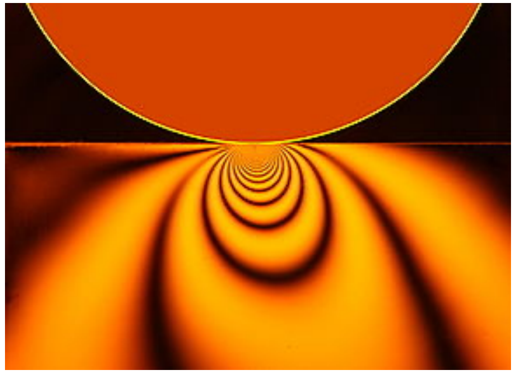


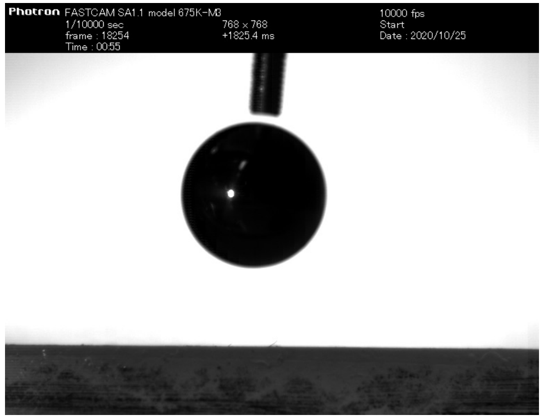


- Velocity-dependent behaviors derived from viscoelasticiy appear.
- The viscoelasticity derived from the adhesion is dominant for contact though here I focus on the viscoelasticity from bulk.

Viscoelascitiy on contact mechanics







- Velocity-dependent behaviors derived from viscoelasticiy appear.
- The viscoelasticity derived from the adhesion is dominant for contact though here I focus on the viscoelasticity from bulk.

Research Strategy

Strategy

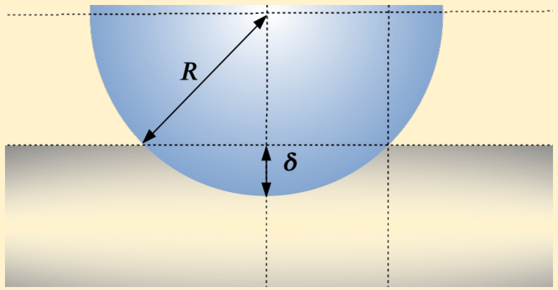
- A stability of scaling laws can be understood by intermediate asymptotics. The convergence of self-similar solution to a finite limit gives the asymptotic expression of scaling law.

$$\lim_{\theta \to 0} \Phi(\theta) = \text{const} \quad \Longleftrightarrow \quad y = Ax^{\alpha} \ \underline{(\theta \ll 1)}$$

- Incomplete convergence of scaling law generates the transition of scaling law?

$$\Phi(\theta) \neq \text{const} \quad \Longleftrightarrow \quad y = Ax^{\alpha} \longrightarrow Bx^{\gamma} \quad ?$$

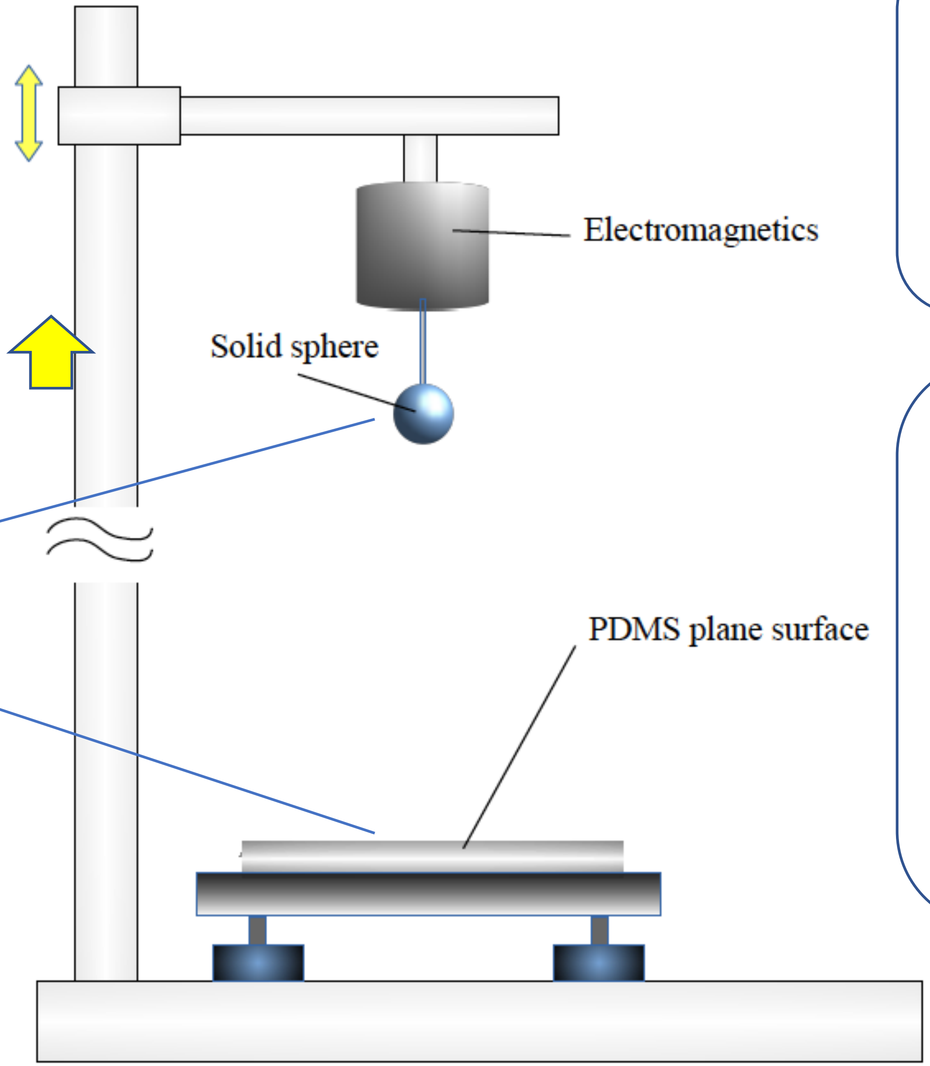
- What is a self-similar solution that describes the transition from elastic impact to viscoelastic impact?

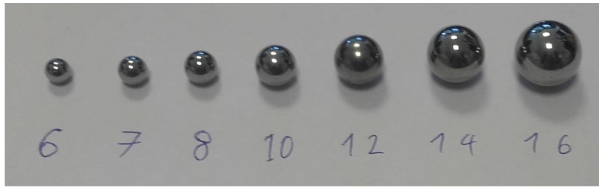


$$\Longrightarrow \quad \Psi = \Phi(Z) \quad ?$$

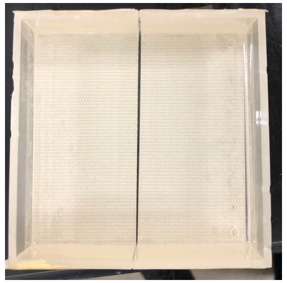
Experimental set up

High speed camera
(FASTCAM SA1.1)
Flame rate = 10000 f sec^{-1}





ρ=7800 kg/m3
R= 3.0, 4.0, 5.0, 7.0, 8.0 mm

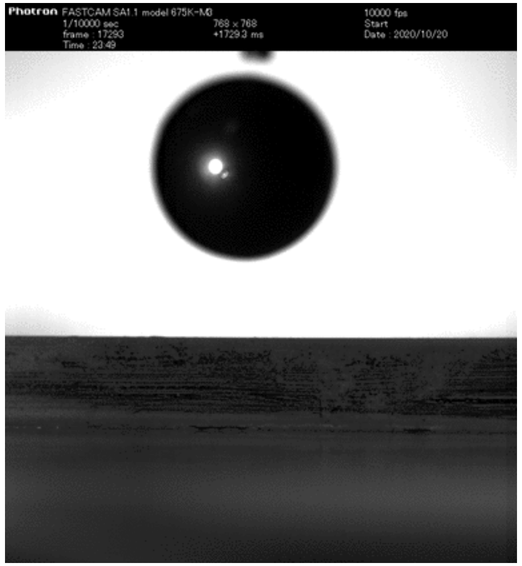


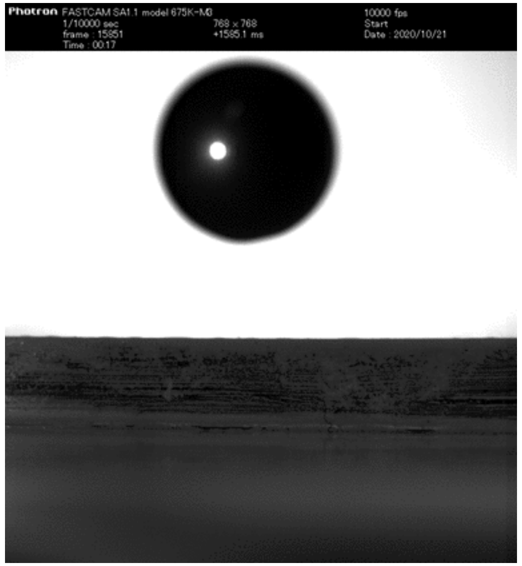
Polydimethylsiloxane

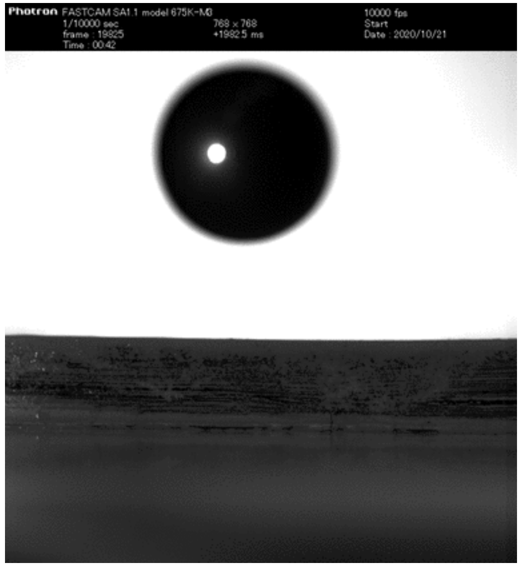
$E \simeq 0.78$ MPa
$\mu = 141$ Pa·s

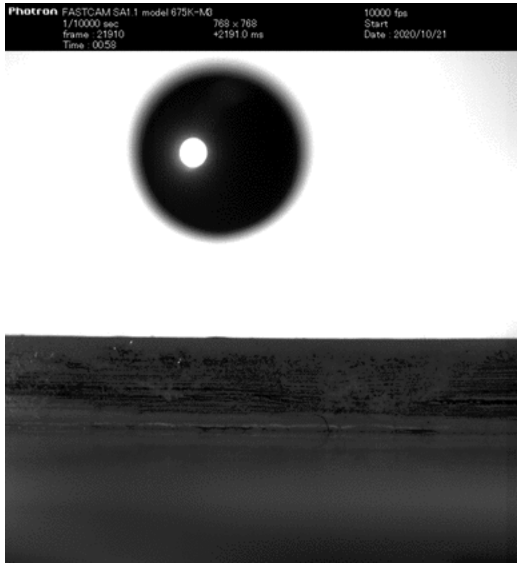
https://arxiv.org/abs/2203.00253

Dynamical impact of sphere on R = 8.0 mm









R = 8.0 mm
Vi = 390 mm/s
ρ=7800 kg/m3

Mettalic ball

R = 8.0 mm
Vi = 750 mm/s
ρ=7800 kg/m3

Mettalic ball

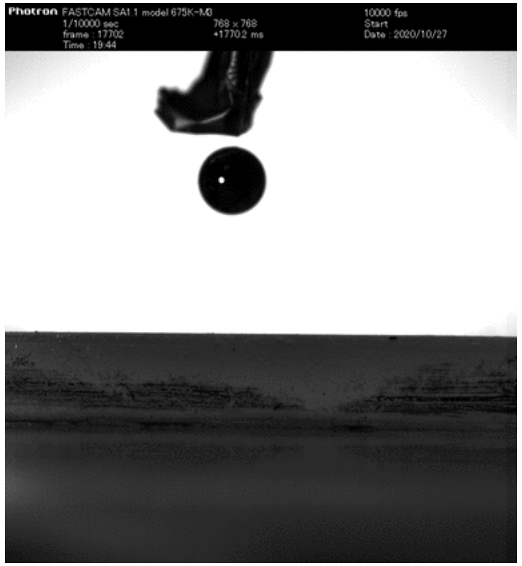
R = 8.0 mm
Vi = 1290 mm/s
ρ=7800 kg/m3

Mettalic ball

R = 8.0 mm
Vi = 2302 mm/s
ρ=7800 kg/m3

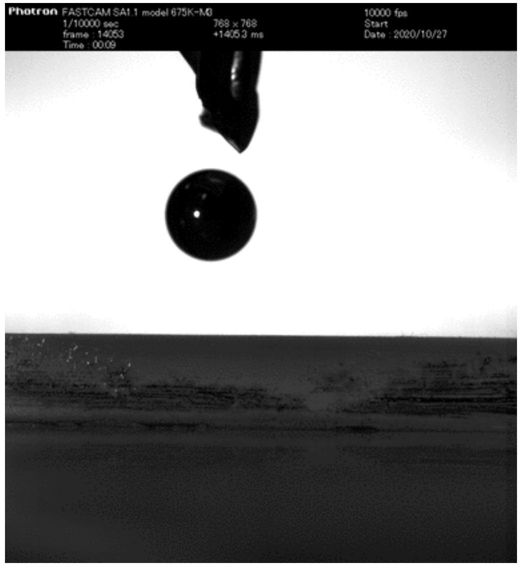
Mettalic ball

Dynamical impact of sphere on vi = 400 mm/s



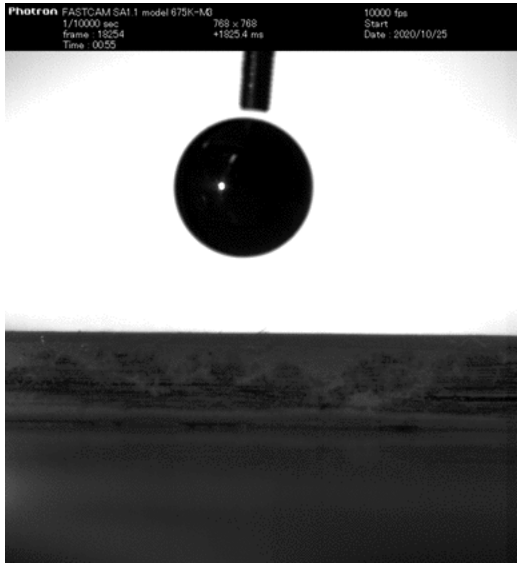
R = 3.0 mm
Vi = 471 mm/s
ρ=7800 kg/m3

Mettalic ball



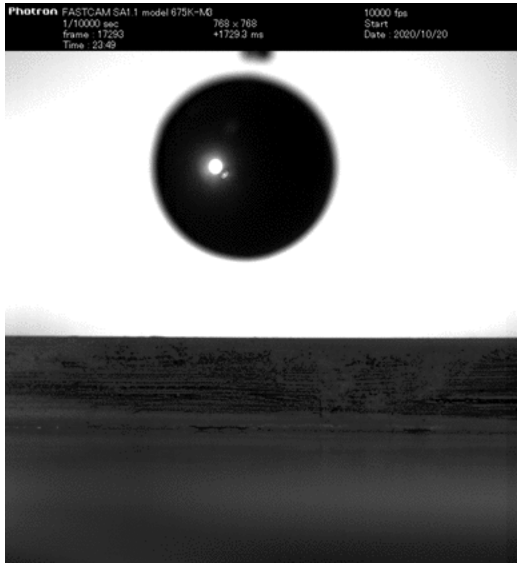
R = 4.0 mm
Vi = 430 mm/s
ρ=7800 kg/m3

Mettalic ball



R = 6.0 mm
Vi = 369 mm/s
ρ=7800 kg/m3

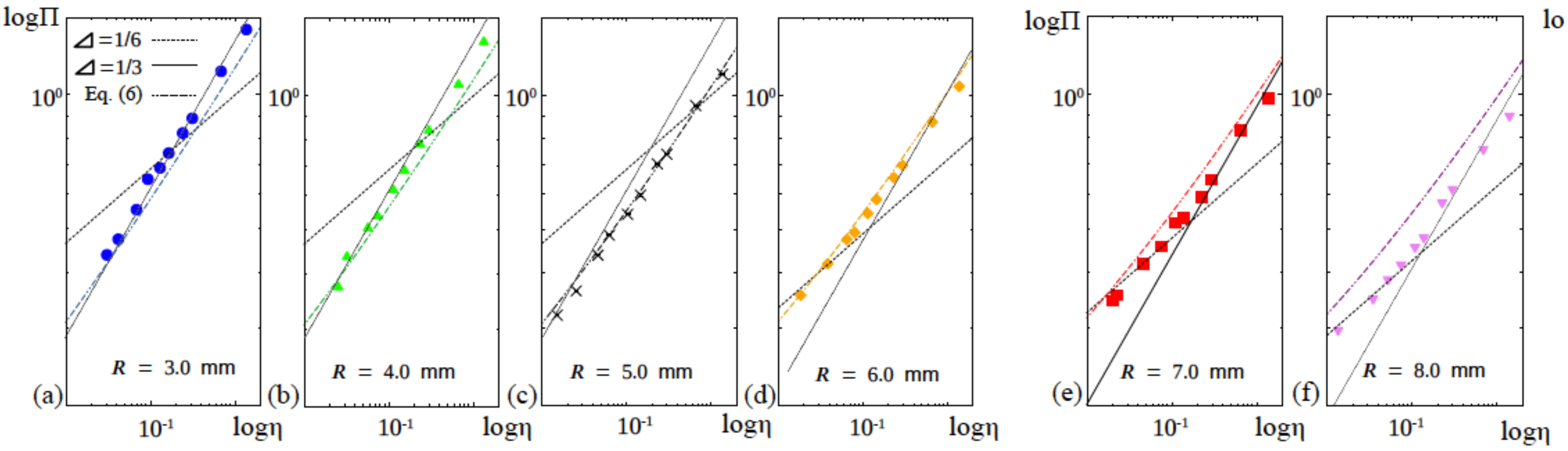
Mettalic ball



R = 8.0 mm
Vi = 390 mm/s
ρ=7800 kg/m3

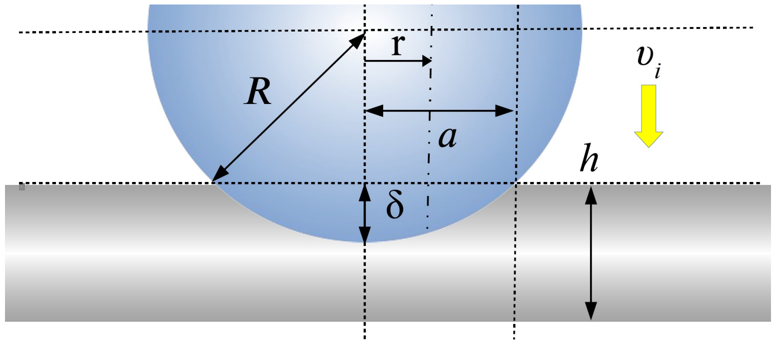
Mettalic ball

The maximum deformations and the impact velocities

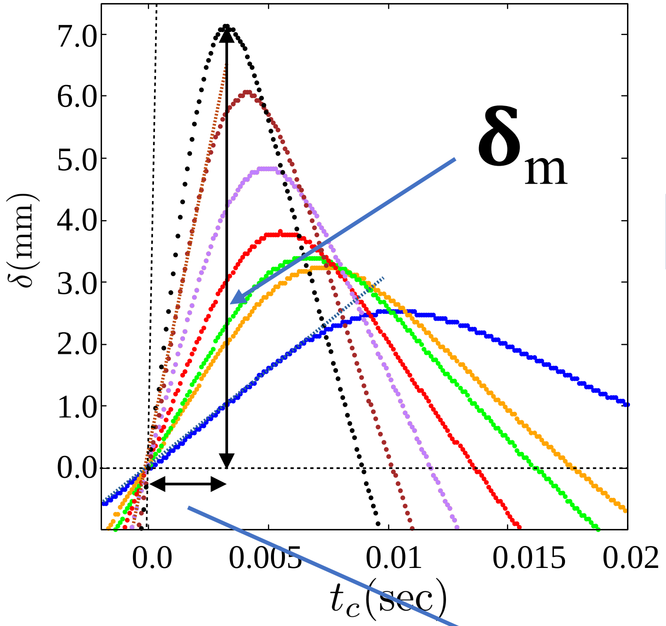


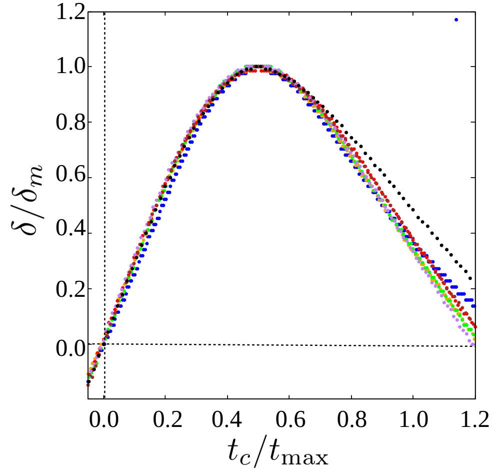
$\eta^{1/3}$: elastic impact (Chastel et al, J. Fluids Mech, 2016)

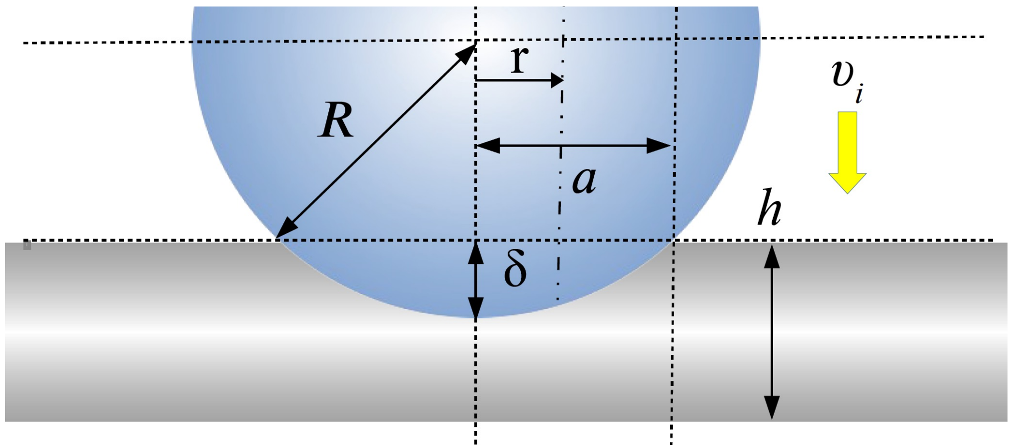
$$\Pi = \frac{\delta_m}{R} \quad \eta = \frac{\rho v_i^2}{E}$$



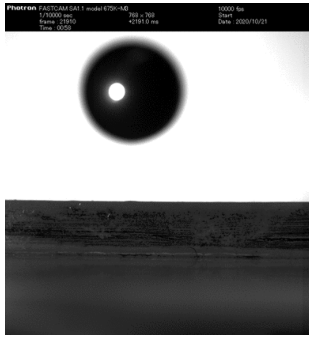
Time evolution of deformation on the impacts

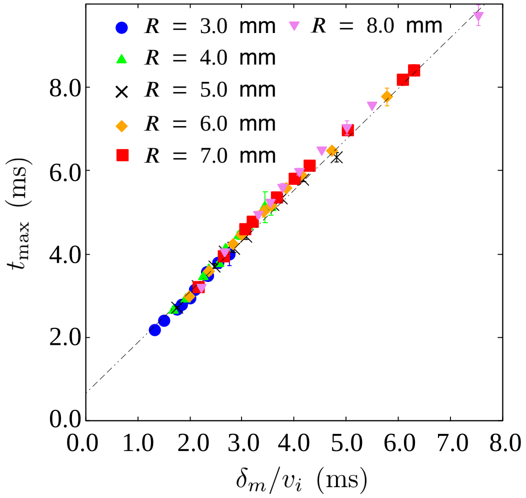






$$\Pi = \frac{\delta}{R},\ \kappa = \frac{h}{R},\ \eta = \frac{\rho v^2}{E},\ \theta = \frac{\mu}{E^{1/2}\rho^{1/2}R},$$





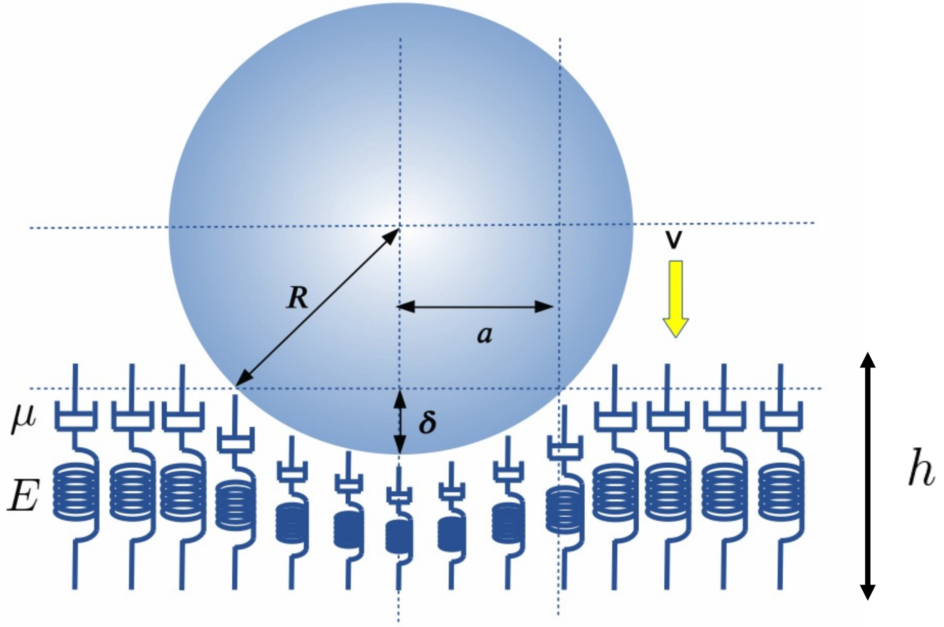
t_{max}

δ_m/v_i

R = 8.0 mm
Vi = 2302 mm/s

- The similarities of the attractors of deformations is found.
- A scaling relation between the contact times and the impact velocities.

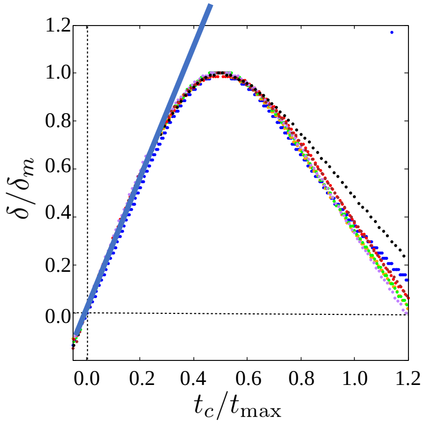
Maxwell Viscoelastic Foundation model



$$\sigma + \frac{\mu}{E}\frac{d\sigma}{dt} = \mu\frac{d\epsilon}{dt}$$

σ : stress
ϵ : strain

$$\frac{d\delta}{dt} = v_i$$



$$E_{MVF} = \frac{\phi\mu\pi R\delta_m^2}{h}\frac{d\delta}{dt}\left[1 - e^{-\frac{E}{\mu}t_c}\right]$$

https://arxiv.org/abs/2203.00253

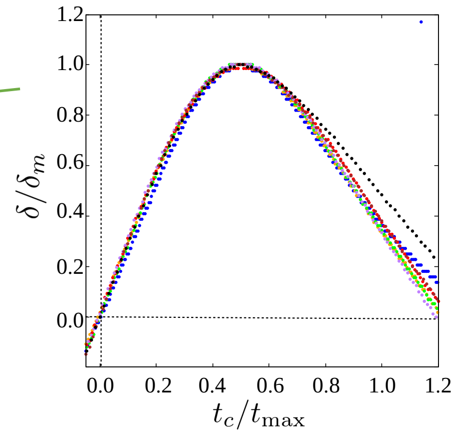
Maxwell Viscoelastic Foundation model

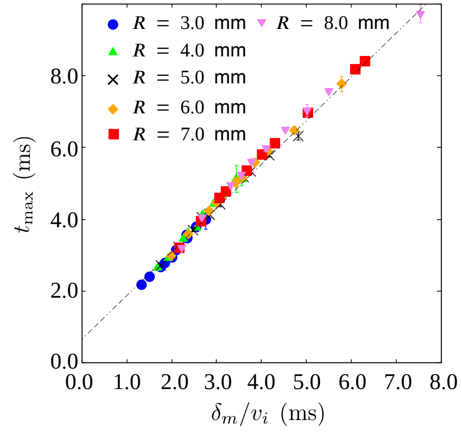
$$E_{MVF} = \frac{\phi\mu\pi R\delta_m^2}{h}\frac{d\delta}{dt}\left[1 - e^{-\frac{E}{\mu}t_c}\right]$$

$$\frac{d\delta}{dt} = v_i \qquad t_c = \frac{\delta_m}{v_i}$$

$$E_{MVF} \xrightarrow[Et_c/\mu \ll 1]{} \frac{\phi\mu\pi R\delta_m^2}{h} v_i \cdot \frac{E\delta_m}{\mu v_i} \simeq \frac{\phi E\pi R\delta_m^3}{h} \simeq E_{el}$$

$$\boxed{Z = \frac{Et_c}{\mu}}$$





- Z << 1 corresponds to elastic energy estimation
- Z ~ 1 corresponds to viscoelastic energy estimation

Step 1 : Start from a scaling law of the elastic regime

CGR solution

$$\Pi = \mathrm{const}\left(\frac{\kappa}{\phi}\right)^{1/3}\eta^{1/3}$$

$$\Pi = \frac{\delta_m}{R},\ \kappa = \frac{h}{R},\ \eta = \frac{\rho v_i^2}{E},\ \theta = \frac{\mu}{E^{1/2}\rho^{1/2}R}$$

Chastel et al., J. Fluid Mech., 2016.

Step 2 : define a dimensionless number composed of the scaling law.

$$\Psi = \frac{\phi}{\kappa}\frac{\Pi^3}{\eta}$$

How to construct a self-similar solution

$$\frac{2}{3} = \Pi^2 \theta \frac{\phi}{\kappa} \frac{1}{\eta^{1/2}} \left[1 - \exp\left(-\frac{\Pi}{\theta \eta^{1/2}} \right) \right]$$

$$\Pi = \frac{\delta_m}{R}, \; \kappa = \frac{h}{R}, \; \eta = \frac{\rho v_i^2}{E}, \; \theta = \frac{\mu}{E^{1/2} \rho^{1/2} R}$$

Step 3 : construct a self-similar solution by identifying an interfering dimensionless:

CGR sollution

$$\Psi = \frac{\phi}{\kappa} \frac{\Pi^3}{\eta}$$

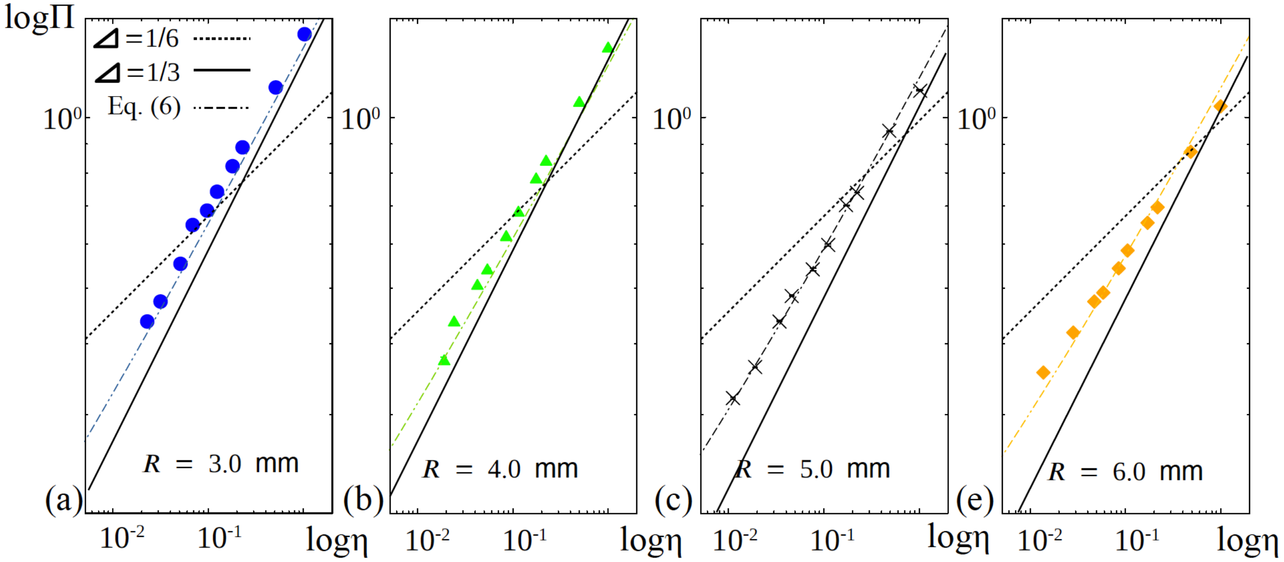
$$Z = \frac{\Pi}{\theta \eta^{1/2}}$$

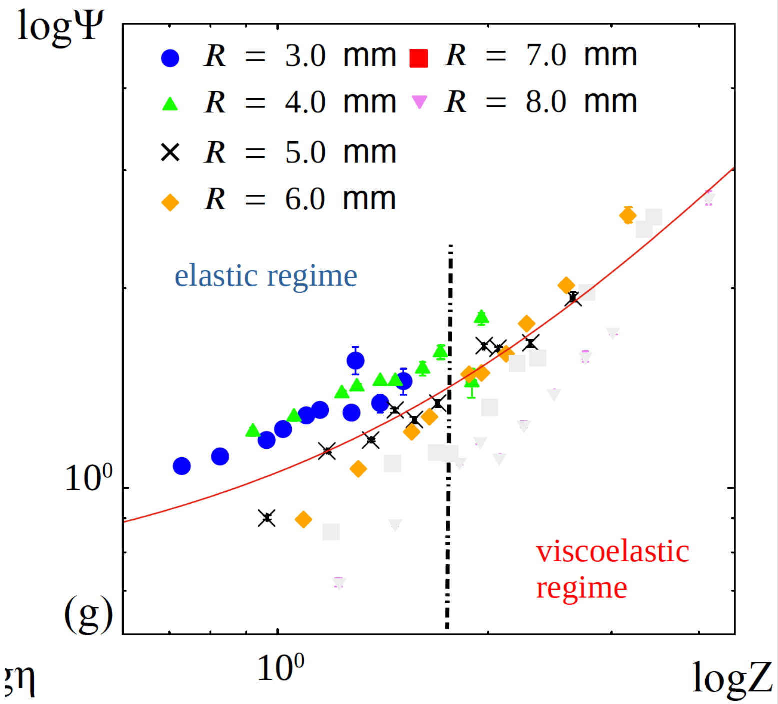
$$\Psi = \frac{2}{3} \frac{Z}{[1 - \exp(-Z)]}$$

The self-similar solution

The interfering dimensionless number: *An inverse Deborah number.*

Hierarchical structure of self-similarity





Z →

small | large

elastic | viscoelastic

$$\Psi = \frac{\phi}{\kappa}\frac{\Pi^3}{\eta} = \frac{\delta^3 E\phi}{R^2\rho v^2 h} = \frac{\text{elastic}}{\text{kinetic}}$$

$$Z = \frac{\Pi}{\theta\eta^{1/2}} = \frac{E\delta}{\mu v} = \frac{\text{viscous}}{\text{elastic}}$$

$$\Pi = \frac{\kappa}{54\phi\theta^2} + \left(\frac{\kappa^2}{486\phi^2\theta^3}\right)^{\frac{1}{3}}\eta^{\frac{1}{6}} + \left(\frac{2\kappa}{3\phi}\right)^{\frac{1}{3}}\eta^{\frac{1}{3}}$$

← Perturbation method

$$\Psi = \frac{2}{3}\frac{Z}{[1-\exp(-Z)]}$$

Conclusion

Main results in this study

- Crossover of scaling law corresponds to a self-similar solution. : There is a self-similar solution that connects different scaling laws.
- There is a scaling law for the contact time and the impact velocity experimentally, which validate the Maxwell foundation model.
- Finally, the phenomena is reduced to two dimensionless number, $\boldsymbol{\Psi}$ and Z, elasticity/kinetics and viscosity/elasticity.
- The self-similar solution naturally appears as the extension of the scaling law of the idealized region to the non-idealized region.

Future and ongoing works

- Can we find the algorithm to find the similarity variables to get data collapse/self-similar solution?
- What is the relation between phase transition phenomena?

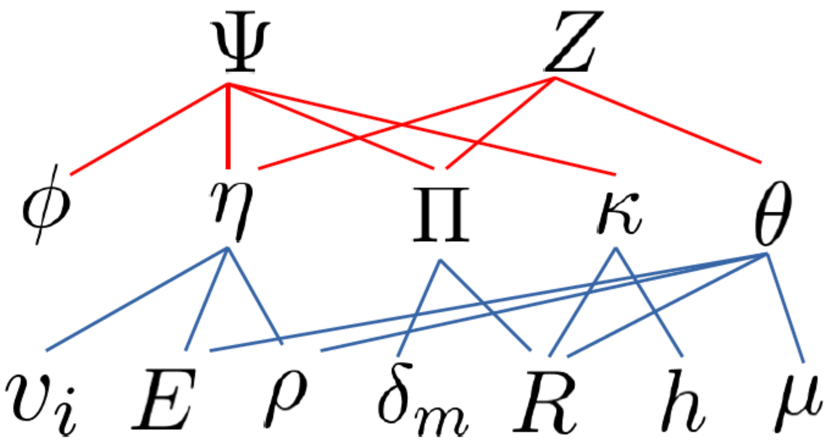
https://arxiv.org/abs/2203.00253

Scale invariance of dimensionless numbers

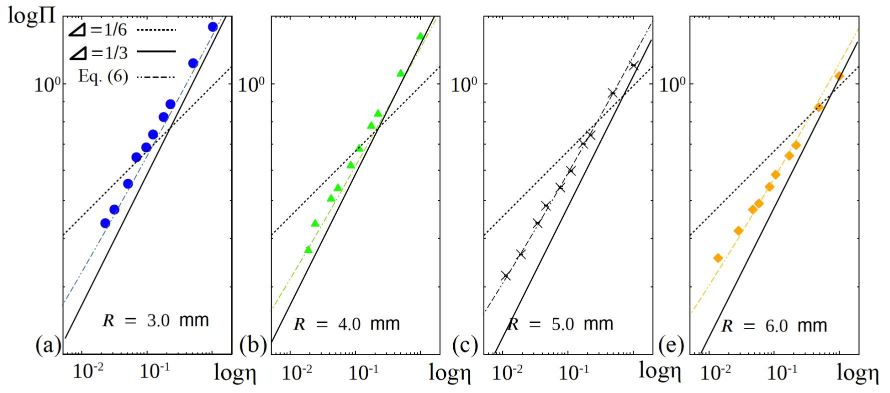
国立研究開発法人海洋研究開発機構 JAMSTEC Japan Agency for Marine-Earth Science and Technology

YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

Scale transformation $x' = A^{\alpha} x$

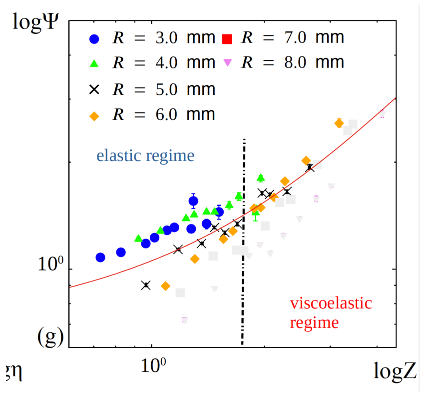


Self-similar solution	Scale invariance
$\Psi = \frac{2}{3}\frac{Z}{1-\exp(-Z)}$	×
$\frac{\Pi^3\phi}{\kappa\eta} = \frac{2}{3}\frac{\frac{\Pi}{\theta\eta^{1/2}}}{1-\exp\left(-\frac{\Pi}{\theta\eta^{1/2}}\right)}$	○
$\frac{\delta_m^3 E\phi}{R^2 h\rho v_i^2} = \frac{2}{3}\frac{\frac{E\delta_m}{\mu v_i}}{1-\exp\left(-\frac{E\delta_m}{\mu v_i}\right)}$	○



the first class

- Scaling behavior



the second class

- Dynamical balance
- Scale dependence