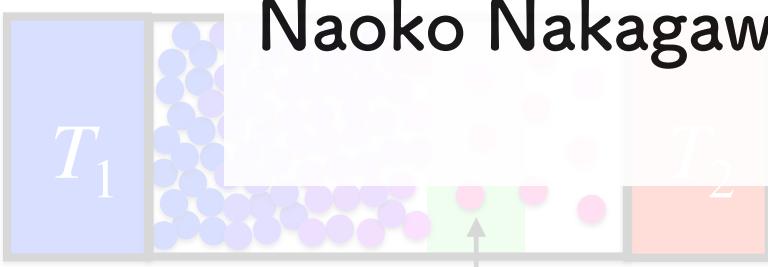
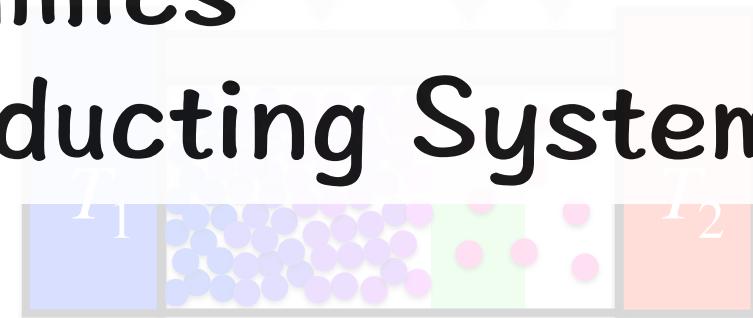


Global Thermodynamics for Heat Conducting Systems



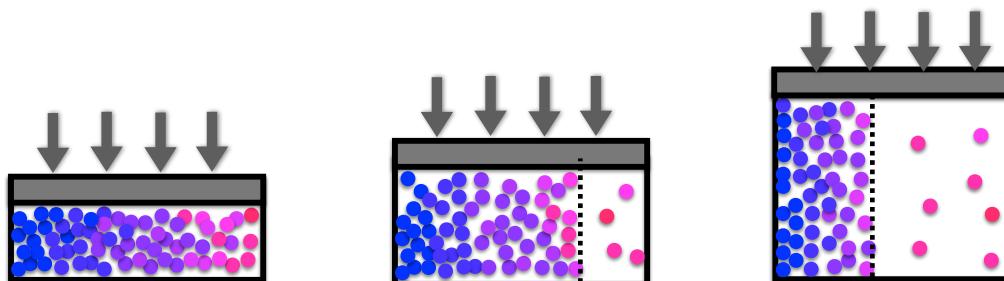
Naoko Nakagawa (Ibaraki Univ., Japan)

in collaboration with Shin-ichi Sasa

Japan-France joint seminar "Physics of dense and active disordered materials"

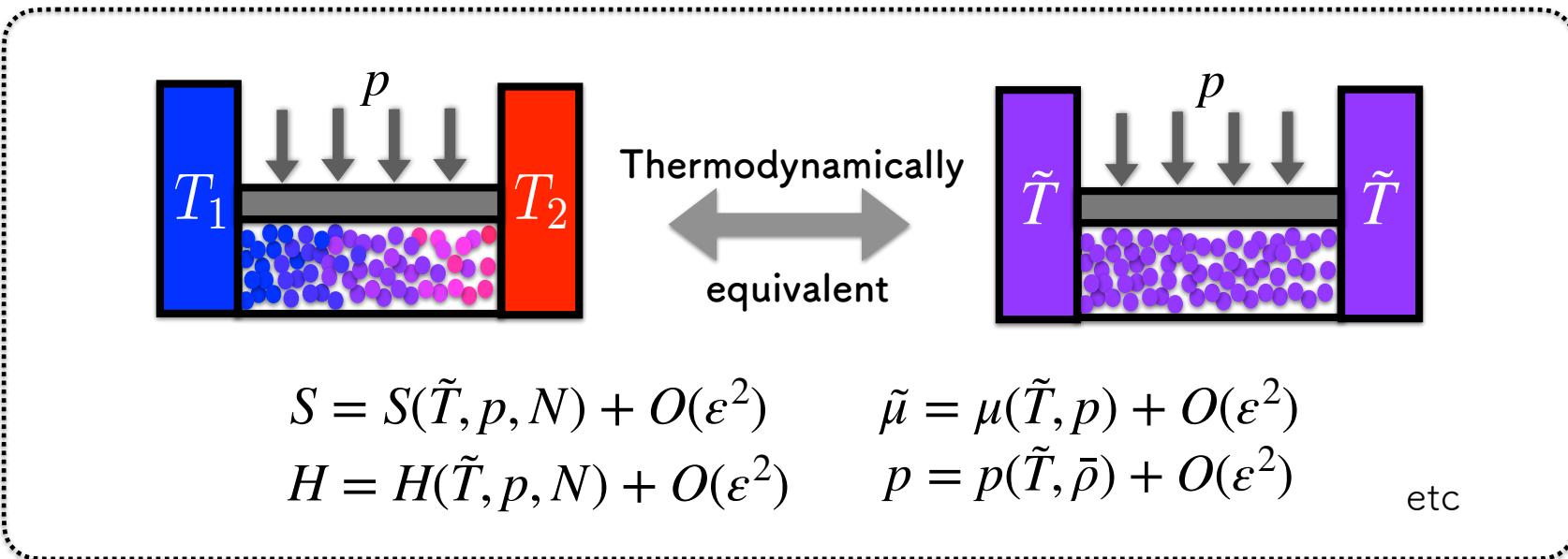
Extension of Entropies to Heat Conduction

Which is the steady state ?



Need a GLOBAL viewpoint for Heat Conducting systems

Global thermodynamic structure with Global temperature



Thermodynamic relations in linear response regime

$$dH = \tilde{T}dS + Vdp + \tilde{\mu}dN + O(\epsilon^2)$$

$$G = H - \tilde{T}S + pV + O(\epsilon^2)$$

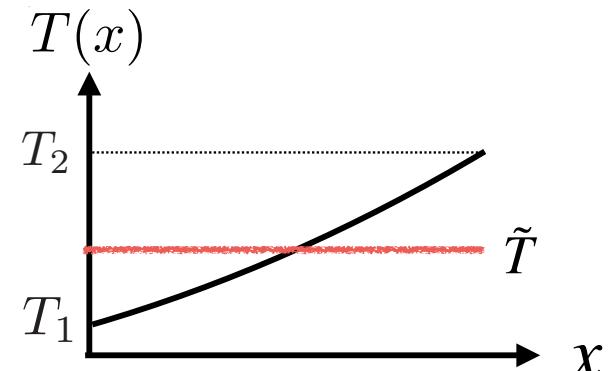
$$G = \tilde{\mu}N$$

The degree of non-equilibrium

$$\epsilon = \frac{|T_2 - T_1|}{T_1}$$

Global Temperature

$$\tilde{T} = \frac{\int_0^{L_x} \rho(x)T(x)dx}{\int_0^{L_x} \rho(x)dx}$$



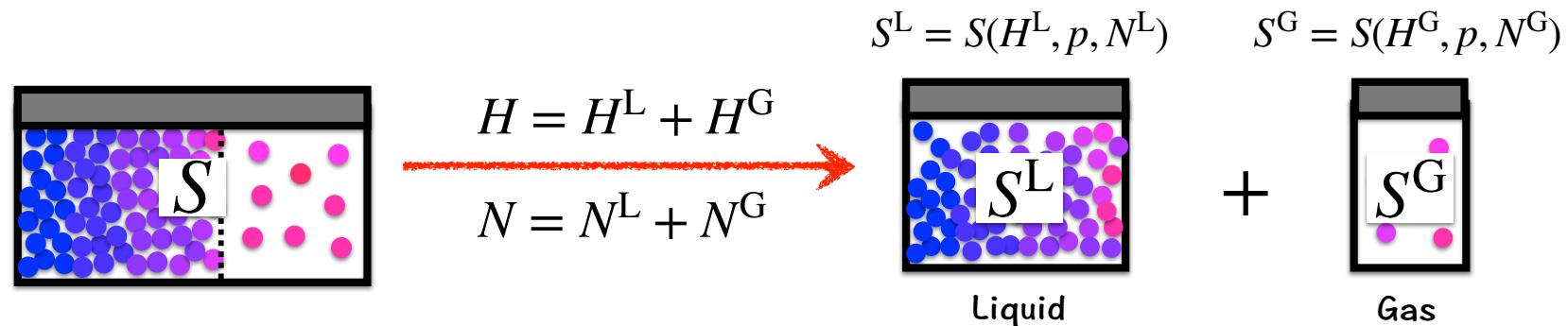
$$\langle K \rangle = \frac{3N}{2} \tilde{T}$$

mean kinetic energy of the system

$$\tilde{T} = \frac{T_1 + T_2}{2} + O(\epsilon^2)$$

mean temperature

Additivity ?



Question

$S = S^L + S^G$? Thermodynamic additivity?

No ! $\tilde{T}^L = \tilde{T}^G$

to be answered

$$S = S^L + S^G + \underline{\phi\psi}$$

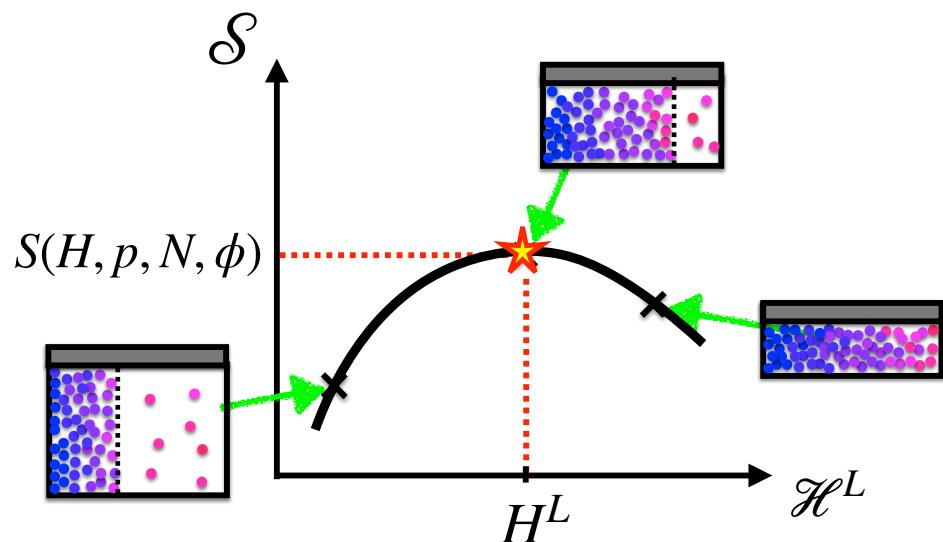
unknown intensive quantity $\phi \propto J$

unknown extensive quantity Ψ

Simultaneous Extension of Thermodynamic and Variational Entropies

$$\mathcal{S}(\mathcal{H}^L, \mathcal{N}^L; H, p, N, \phi) = S^L + S^G + \underline{\phi\psi}$$

$$S(H, p, N, \phi) = S^L + S^G + \underline{\phi\Psi}$$



$$S(H, p, N, \phi) = \max_{\mathcal{H}^L, \mathcal{N}^L} \mathcal{S}(\mathcal{H}^L, \mathcal{N}^L; H, p, N, \phi)$$

$$(H^L, N^L) = \arg \max_{\mathcal{H}^L, \mathcal{N}^L} \mathcal{S}(\mathcal{H}^L, \mathcal{N}^L; H, p, N, \phi)$$

$$H^L = H^L(H, p, N, \phi) \quad N^L = N^L(H, p, N, \phi)$$

Conditions

(1) Thermodynamic variables $(H, p, N, \underline{\phi})$

(2) $S(H, p, N, \phi) = \max_{\mathcal{H}^L, \mathcal{N}^L} \mathcal{S}(\mathcal{H}^L, \mathcal{N}^L; H, p, N, \underline{\phi})$

$$(3) \quad dS = \frac{dH}{\tilde{T}} - \frac{V}{\tilde{T}}dp - \frac{\tilde{\mu}}{\tilde{T}}dN + \underline{\Psi d\phi} \quad \tilde{T} = \frac{\int \rho(x)T(x) dx}{\int \rho(x) dx}$$

$$\frac{1}{\tilde{T}} = \left(\frac{\partial S}{\partial H} \right)_{p, N, \phi} \quad \frac{V}{\tilde{T}} = - \left(\frac{\partial S}{\partial p} \right)_{H, N, \phi} \quad \frac{\tilde{\mu}}{\tilde{T}} = - \left(\frac{\partial S}{\partial N} \right)_{H, p, \phi} \quad \Psi = \left(\frac{\partial S}{\partial \phi} \right)_{H, p, N}$$

Results

Unique extension of Entropy

Variational function $\mathcal{S}(\mathcal{H}^L, \mathcal{N}^L; H, p, N, \phi) = S^L + S^G + \underline{\phi\psi}$

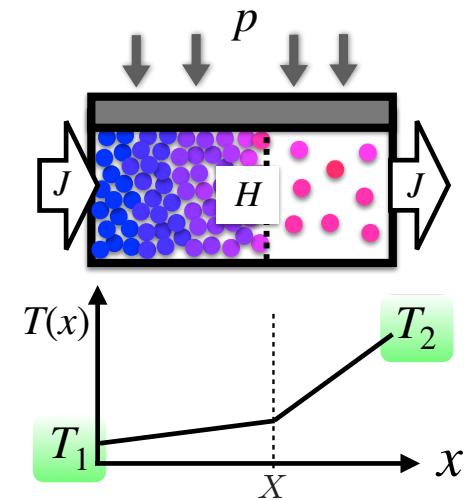
$$\phi = \frac{T_2 - T_1}{T_c(p)} \quad \psi = \frac{\mathcal{H}^G \mathcal{N}^L - \mathcal{H}^L \mathcal{N}^G}{2NT_c(p)}$$

$$S(H, p, N, \phi) = \max_{\mathcal{H}^L, \mathcal{N}^L} \mathcal{S}(\mathcal{H}^L, \mathcal{N}^L; H, p, N, \phi)$$

Thermodynamic function $S = S^L + S^G + \phi\Psi$

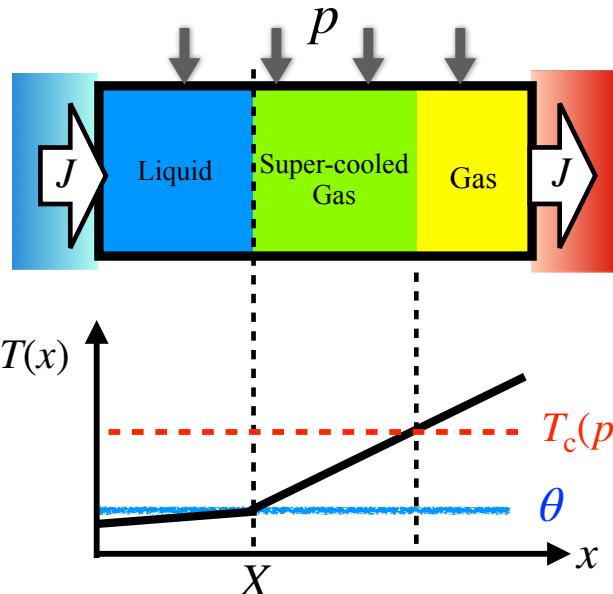
$$\Psi = \frac{H^G N^L - H^L N^G}{2NT_c(p)} \longrightarrow \Psi = \frac{\hat{q}(p)}{T_c(p)} \frac{N^L N^G}{2N}$$

$$\hat{q}(p) = \hat{h}^G(p) - \hat{h}^L(p) \quad \text{latent heat}$$

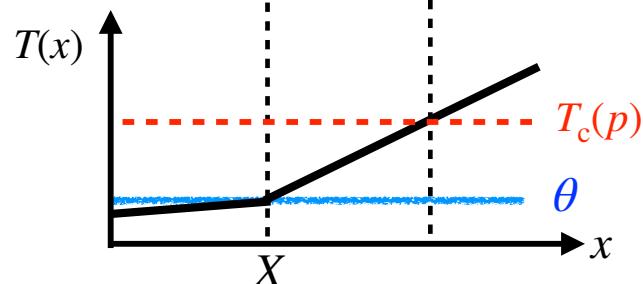


Interface temperature θ and meta-stable states

$$\theta - T_c(P) = \frac{X(L_x - X)}{2L_x} \left[|J| \left(\frac{1}{\kappa^G} - \frac{1}{\kappa^L} \right) - \frac{|T_2 - T_1|}{L_x} \frac{V}{N} (\rho^L - \rho^G) \right]$$



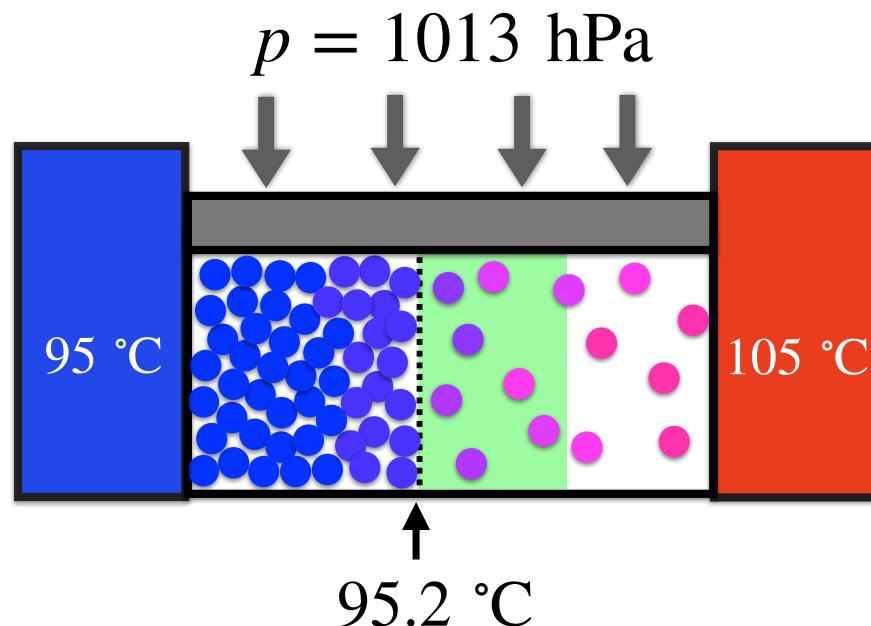
$\kappa^{L/G}$ heat conductivity of liquid or gas
 $\rho^{L/G}$ density of liquid or gas



Heat flow stabilizes meta-stable local states.

Prediction from Global Thermodynamics

H₂O in atmospheric pressure



Summary of Global Thermodynamics

	Variational function	Thermodynamic relation
constant enthalpy (H, p, N, ϕ)	$\mathcal{S} = L_y L_z \int_0^{L_x} s(x) dx + \phi \Psi$	$dS = \frac{dH}{\tilde{T}} - \frac{V}{\tilde{T}} dp - \frac{\tilde{\mu}}{\tilde{T}} dN + \Psi d\phi$
constant pressure (\tilde{T}, p, N, ϕ)	$\mathcal{G} = L_y L_z \int_0^{L_x} \mu(x) \rho(x) dx$	$dG = -S d\tilde{T} + V dp + \tilde{\mu} dN - \tilde{T} \Psi d\phi$
constant volume (\tilde{T}, V, N, ϕ)	$\mathcal{F} = L_y L_z \int_0^{L_x} f(x) dx$	$dF = -S d\tilde{T} - p dV + \tilde{\mu} dN - \tilde{T} \Psi d\phi$

Steady States

$$\theta - T_c(P) = \frac{X(L_x - X)}{2L_x} \left[|J| \left(\frac{1}{\kappa_c^G} - \frac{1}{\kappa_c^L} \right) - \frac{|T_2 - T_1|}{L_x} \frac{V}{N} (\rho_c^L - \rho_c^G) \right]$$

Numerical Verification in the Next Talk!

Experimental studies are now in progress.

Global Temperature

$$\tilde{T} = \frac{\int_V \rho(\mathbf{r}) T(\mathbf{r}) dr}{\int_V \rho(\mathbf{r}) dr}$$

intensive and extensive variables

$$\phi = \frac{T_2 - T_1}{T_c(p)} \quad \Psi = \frac{\hat{q}(p)}{T_c(p)} \frac{N^L N^G}{2N}$$

