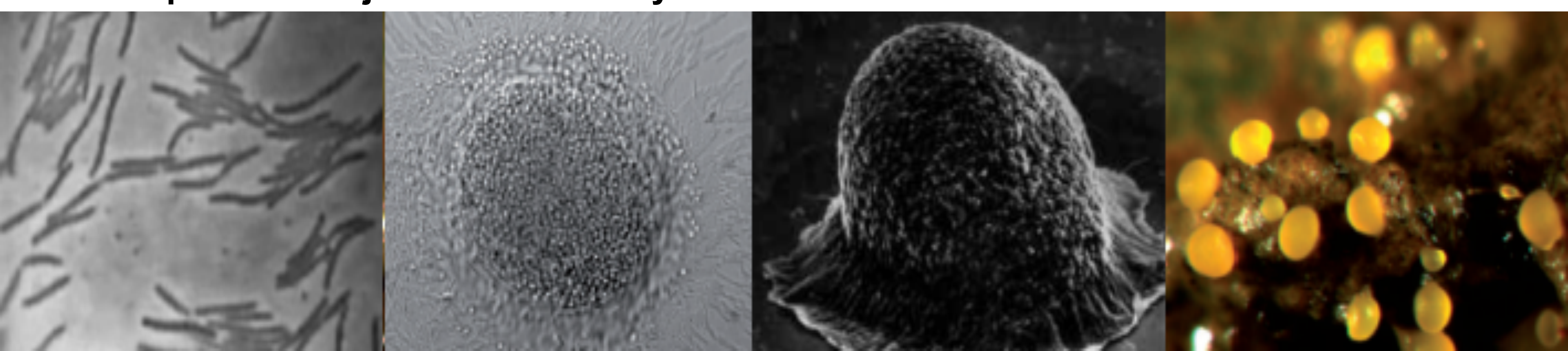
Disordered Active Matter:
On how time-independent disorder affects the motion of self-propelled particles

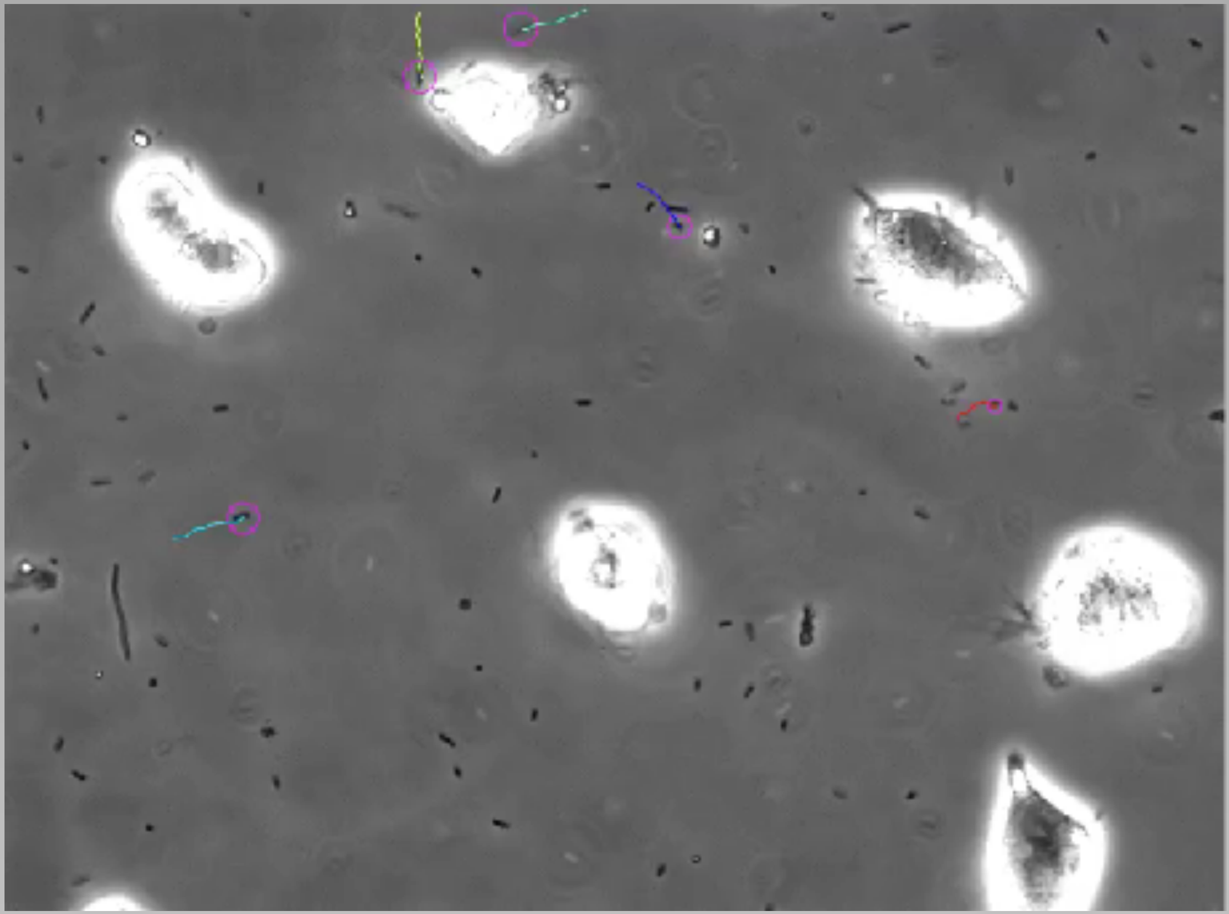
Fernando Peruani

LPTM, CY Cergy Paris University

Japan-France joint seminar "Physics of dense and active disordered materials"



Active particles in the presence of quenched disorder

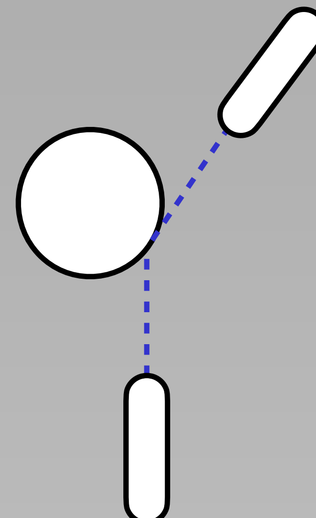


Otte, Perez-Ipiña, Czerucka, Peruani, Nat. Comm. 2021

Example of active matter in heterogeneous medium:

Bacterial trajectories are deflected each time that they hit an obstacles (here a human epithelial cell).

Active particles in the presence of quenched disorder



during the time the bacterium is in contact with the obstacle, it experiences a torque!

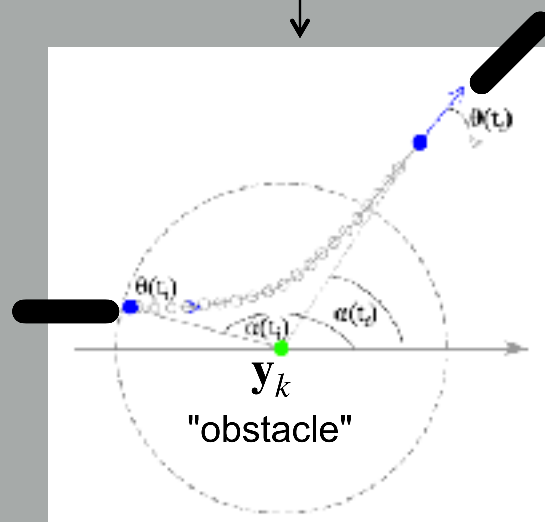
(and note that the speed remains roughly constant)

Active Particle moving in the presence of quenched disorder

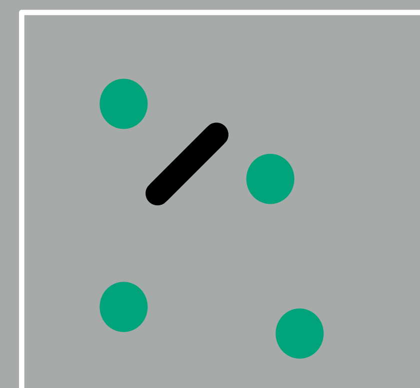
(a minimal model)

$$\dot{\mathbf{x}}_i = v_o \mathbf{V}(\theta_i)$$
$$\dot{\theta}_i = h(\mathbf{x}_i, \theta_i) + \eta \xi_i(t)$$

$$\mathbf{V}(\theta_i) = \cos(\theta_i)\,\hat{x} + \sin(\theta_i)\,\hat{y}$$



medium with obst.



turning strength

$$h(\mathbf{x}_i, \theta_i) = \frac{\gamma}{n(\mathbf{x}_i)} \sum_{|\mathbf{x}_i - \mathbf{y}_k| < R} f(|\mathbf{x}_i - \mathbf{y}_k|) \sin(\alpha_{k,i} - \theta_i)$$

# of obstacles inside R

$$\mathbf{x}_i - \mathbf{y}_k = r_{k,i}\,(\cos\alpha_{k,i}, \sin\alpha_{k,i})$$

Chepizhko, Peruani, PRL (2013)

Active Particle moving in the presence of quenched disorder
(a minimal model)

with $h = 0$

$$D = \frac{v_0^2}{\eta^2} = \frac{v_0^2}{2D_\theta}$$

where $D_\theta = \frac{\eta^2}{2}$

$$\dot{\mathbf{x}}_i = v_o \mathbf{V}(\theta_i)$$

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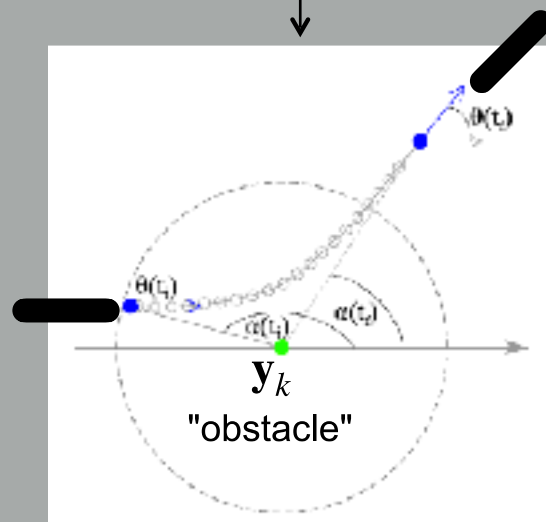

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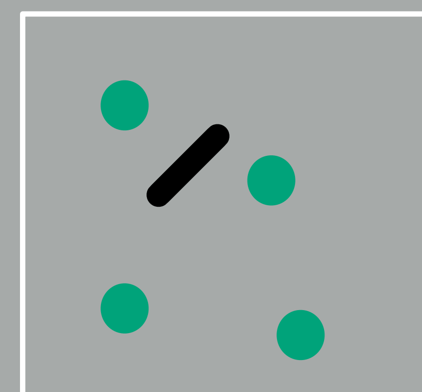

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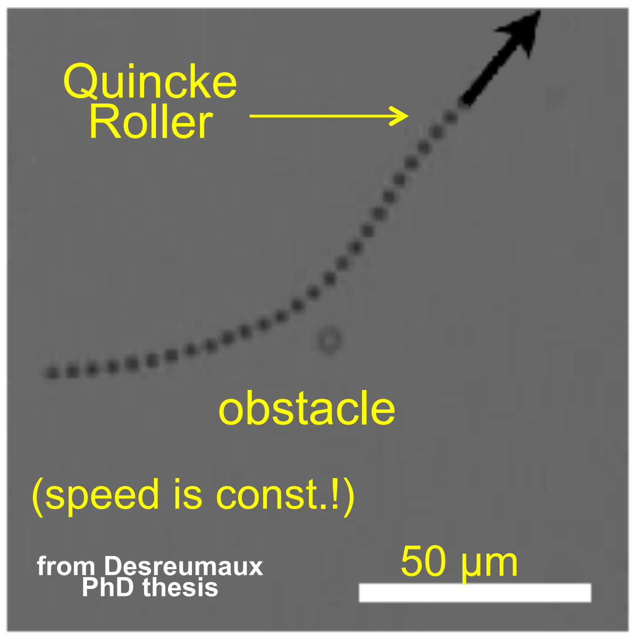
$$\mathbf{x}_i - \mathbf{y}_k = \Gamma_{k,i} (\cos \alpha_{k,i}, \sin \alpha_{k,i})$$

Chepizhko, Peruani, PRL (2013)

Active Particle moving in the presence of quenched disorder

(a minimal model)



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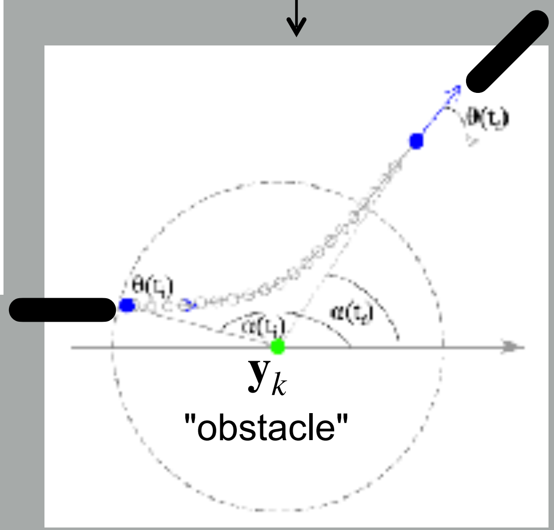

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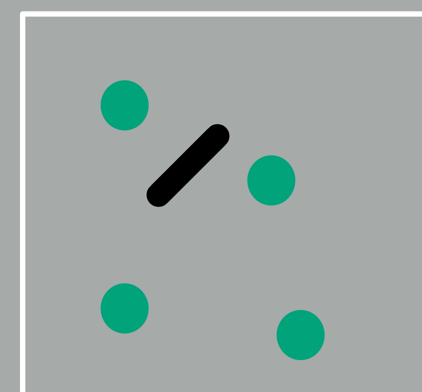

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Chepizhko, Peruani, PRL (2013)

An active “Lorentz” gas

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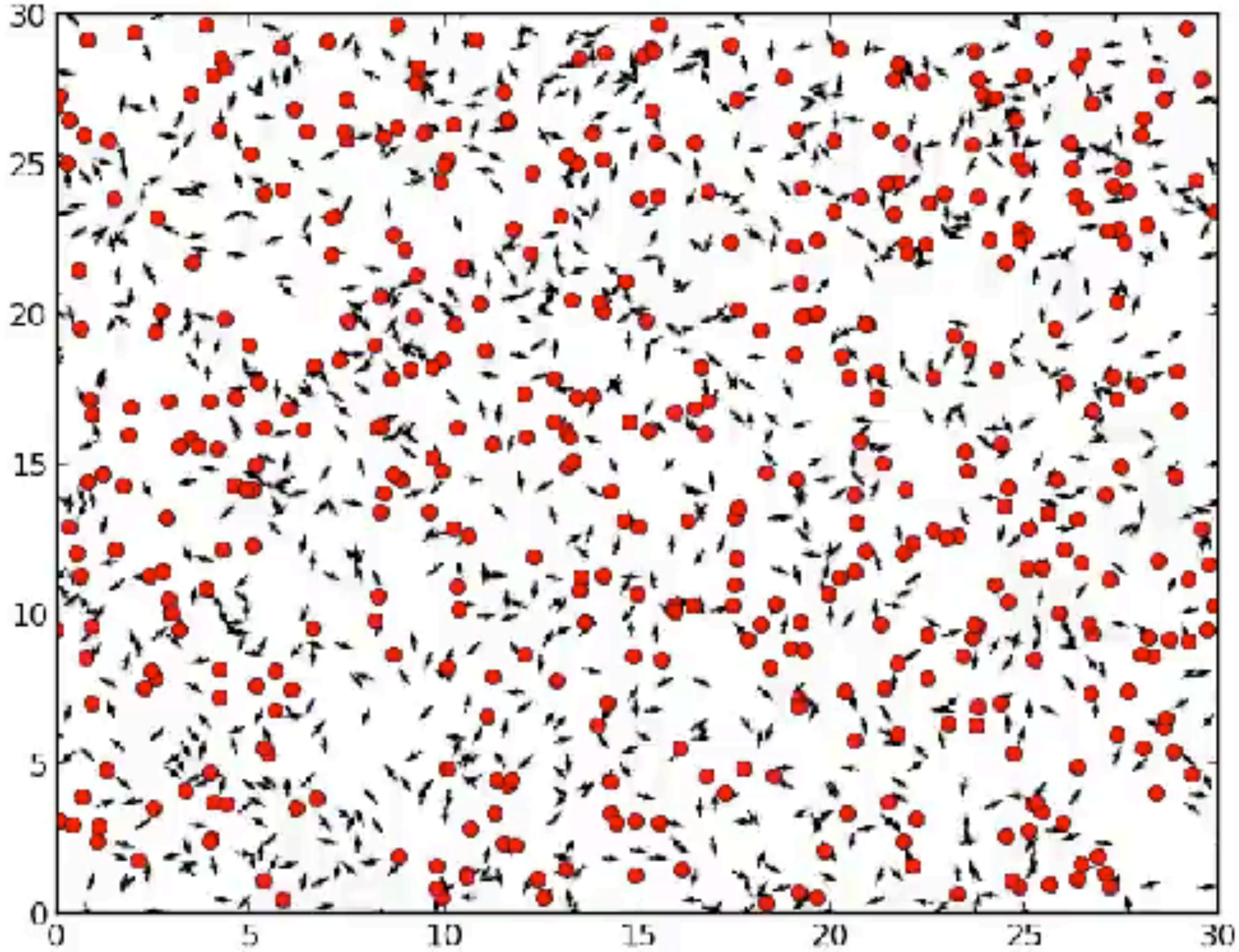
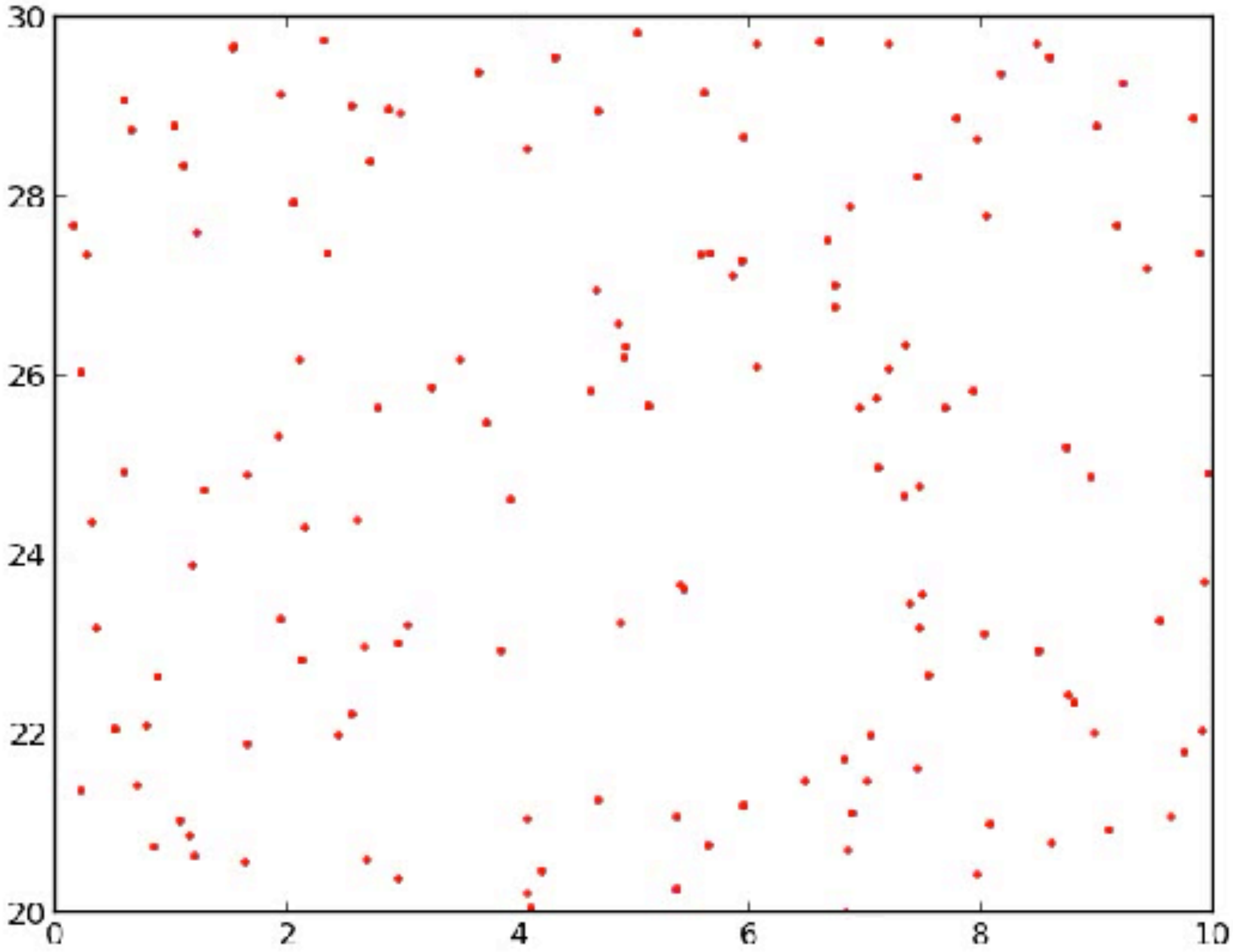


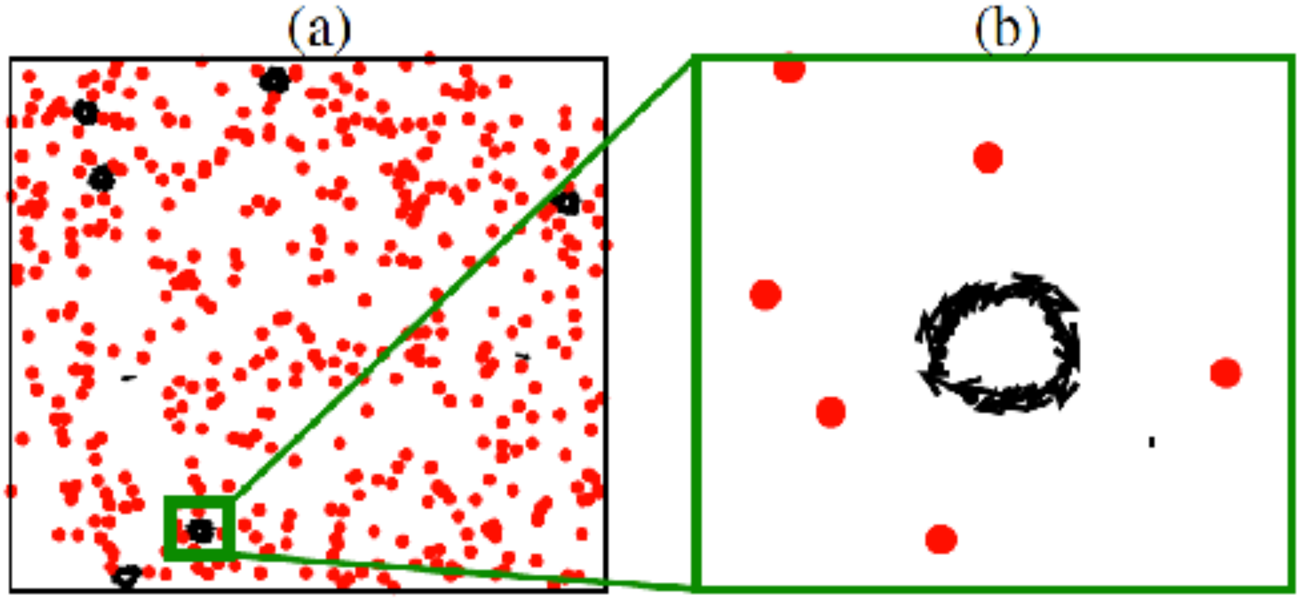
Starting from a random initial condition, traps emerge. The time that particles spend in a particular trap depends on the particular configuration of obstacles.

An active "Lorentz" gas



Looking at a small part of a large system at real time

An active “Lorentz” gas



These traps are closed stable orbits that are found by the active particles in the landscape of obstacles.

Remember that noise is present. These orbits are not absorbing, and particle can escape.

An active "Lorentz" gas

- **How can we understand the observed behavior?**

We look for a coarse-grained description of the problem in terms of $p(\mathbf{x}, \theta, t)$ whose temporal evolution can be expressed as:

$$\partial_t p + v_0 \nabla . [\mathbf{V}(\theta) p] = D_\theta \partial_{\theta\theta} p + F[p(\mathbf{x}, \theta, t), \rho_o(\mathbf{x})]$$

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Interactions with obstacles

Chepizhko, Peruani, PRL (2013)

An active “Lorentz” gas

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Interactions with obstacles

$$D_\theta = \eta^2/2$$

In absence of interactions, we will see that we recover Fürth’s formula for MSD, and a diffusion coefficient:

$$D_{x_o} = v_0^2/(2D_\theta)$$

An active "Lorentz" gas

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Interactions with obstacles

There are two limiting cases where F[.] can be specified:

- **Low density (LD) approximation**
- **High density (HD) approximation**

(with respect obstacle density)

An active "Lorentz" gas

• **Low density (LD) approximation**

We assume that → $D_\theta^{-1} v_0 >> \rho_o^{-1/2}$

This implies that in between obstacles we can assume that particles move ballistically.

If we look at time-scales larger than R/v0, we can assume that the interactions with obstacles can be described as sudden jumps in the moving direction of the particle:

$$F[p] = -\lambda(\rho_o)\, p(x,\theta,t) + \int_0^{2\pi} d\theta' T(\theta,\theta') p(x,\theta',t) \approx \frac{\lambda(\rho_o)\epsilon_\theta^2}{6} \partial_{\theta\theta} p \,, \qquad (5)$$

Top-hat simplification: $T(\theta,\theta';\mathbf{x}) \simeq \lambda(\rho_o) T(\theta,\theta') \approx [\lambda(\rho_o)/(2\epsilon_\theta)]\, \Theta(\epsilon_\theta - |\theta-\theta'|)$

where $\lambda(\rho_o) \approx v_o \rho_o \sigma_o$ and $\sigma_o = 2R$

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• **Low density (LD) approximation**

Under all these assumptions, we arrive to the following asymptotic equation for ρ(x,t):

$$\partial_t \rho = \nabla . \left[\frac{v_0^2}{2\tilde{D}_\theta} \nabla \rho \right]$$

↓

$$D_x = v_0^2 / \left[2 \left(D_\theta + \Lambda_0 \rho_o \right) \right]$$

↓

$$\Lambda_0 = v_0 \sigma_0 \epsilon_\theta^2 / 6$$

An active "Lorentz" gas

- **High density (HD) approximation**

At high obstacle densities, active particles sense always several obstacle around them, which forces us to leave the Boltzmann for the Fokker-Planck approach.

At high obstacle densities, the interaction with obstacles can then be modeled as:

$$F = \partial_0 \left[I p(\mathbf{x}, \theta, t) \right]$$

where I is the average "torque" felt by a particle at (x,θ), which takes the form:

$$I = \frac{\gamma}{n(\mathbf{x})} \sum_j \sin(\theta - \alpha_j) = \frac{\gamma \Gamma(\mathbf{x})}{n(\mathbf{x})} \sin(\theta - \psi(\mathbf{x}))$$

Since obstacles are located at random, this is equivalent to summing n(x) random vectors!

$$I \sim \sin(\theta - \psi(\mathbf{x}))/\sqrt{n}$$ where $n \approx \pi R^2 \rho_o$

An active "Lorentz" gas

• **High density (HD) approximation**

After performing the moment expansion procedure used for the LD approximation, we arrive to:

$$
\begin{aligned}
\partial_t \rho &= \frac{v_0^2}{2D_\theta}\nabla^2\rho - \frac{\gamma v_0}{2D_\theta R\sqrt{\pi\rho_0}}\nabla.\left[(\cos(\psi),\sin(\psi))\rho\right] \\
&= \frac{v_0^2}{2D_\theta}\nabla^2\rho - \frac{\gamma v_0}{2D_\theta R\sqrt{\pi\rho_0}}\nabla.\left[\frac{\rho\nabla\rho_o(\mathbf{x})}{||\nabla\rho_o(\mathbf{x})||}\right], \quad (11)
\end{aligned}
$$

From this we learn two important things:

- **The convective term dominates the asymptotic dynamics of ρ.**
- **As the density of obstacles increases, the diff. coeff. increase.**

$$
D_x \sim 1/\left[\rho_c - \Lambda_1\rho_o\right]
$$

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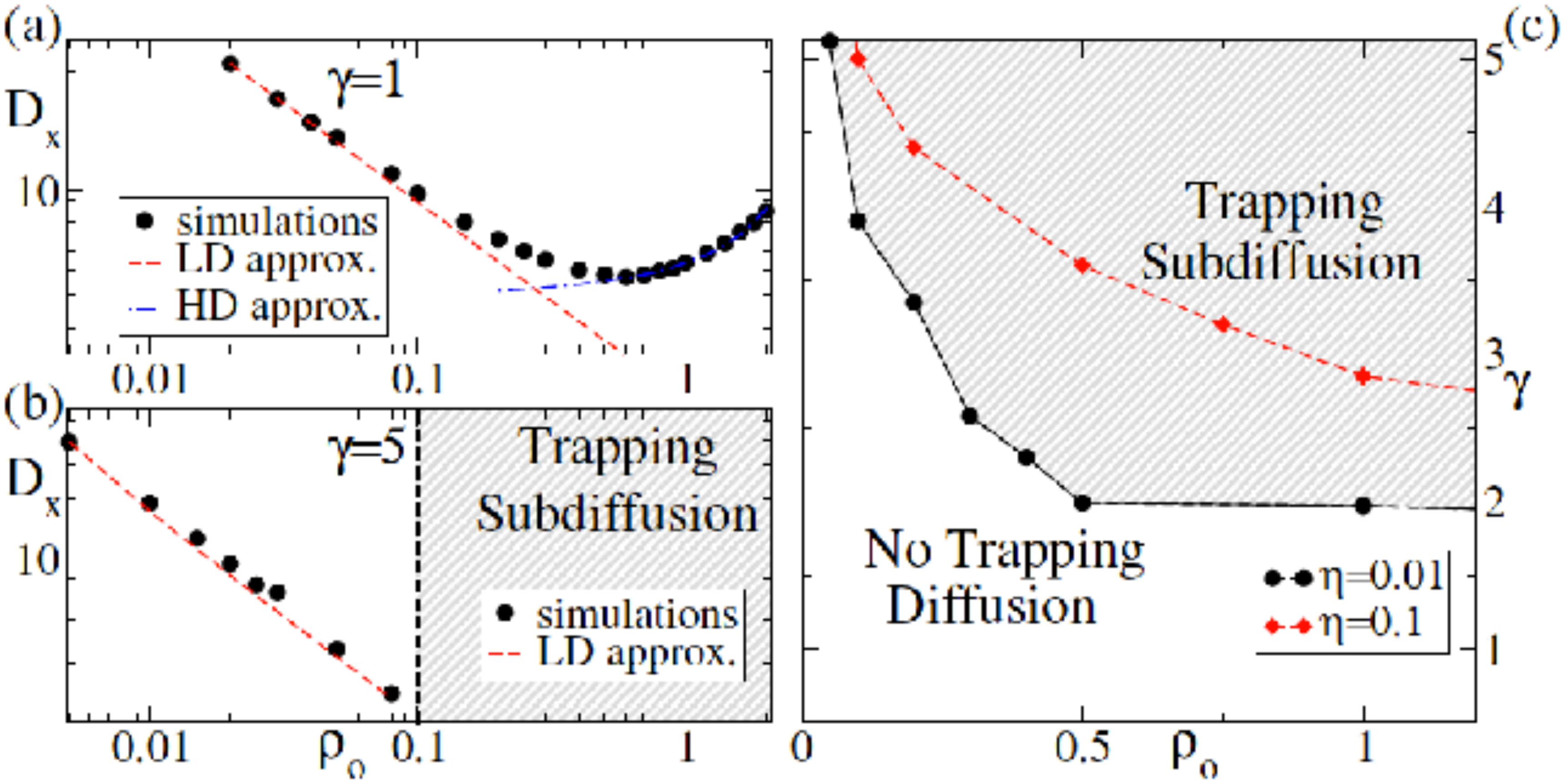
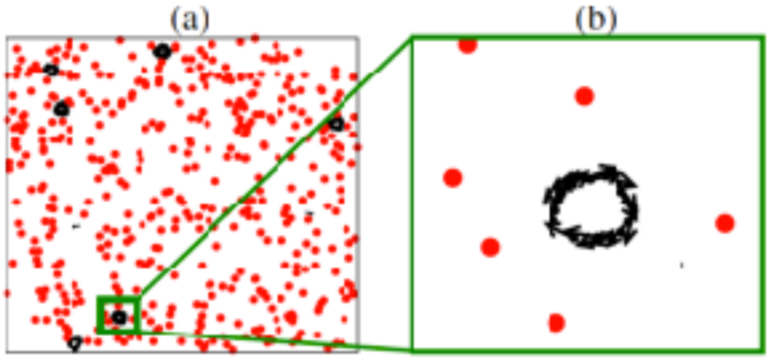
- **Comparison between the LD and HD and simulations**



- **Why do we observe subdiffusion?**

$$\sigma^2(t) = \langle \mathbf{x}^2(t) \rangle$$

If we imagine an array of traps, the MSD has to be proportional to the number of jumps made according to the CLT.



$$\sigma^2(t) \propto n_J(t)$$

→ The problem reduces to know how many jumps our particle performs during t !

If every trap is characterized by an average $<\tau_t>$, this is simply $t/<\tau_t>$

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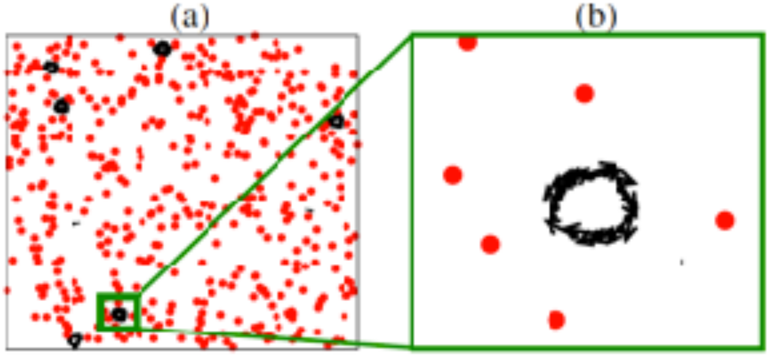

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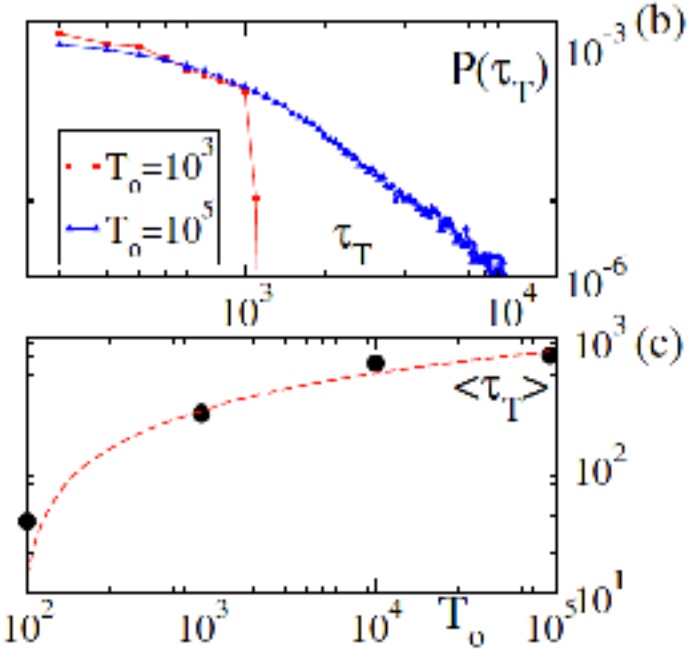
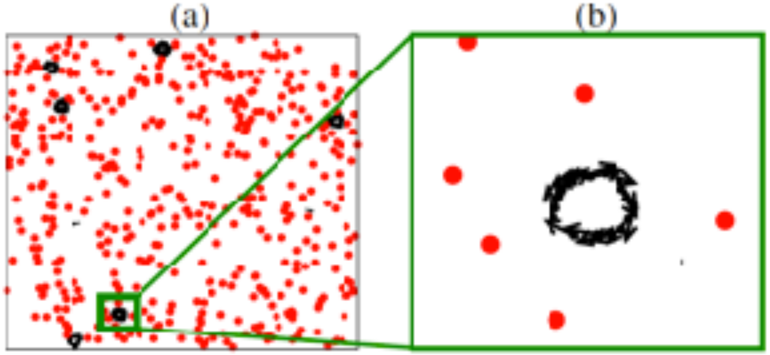
- **Measurement on** $<\tau_t>$

$<\tau_t>$ depends on the observation time !!!



An active "Lorentz" gas



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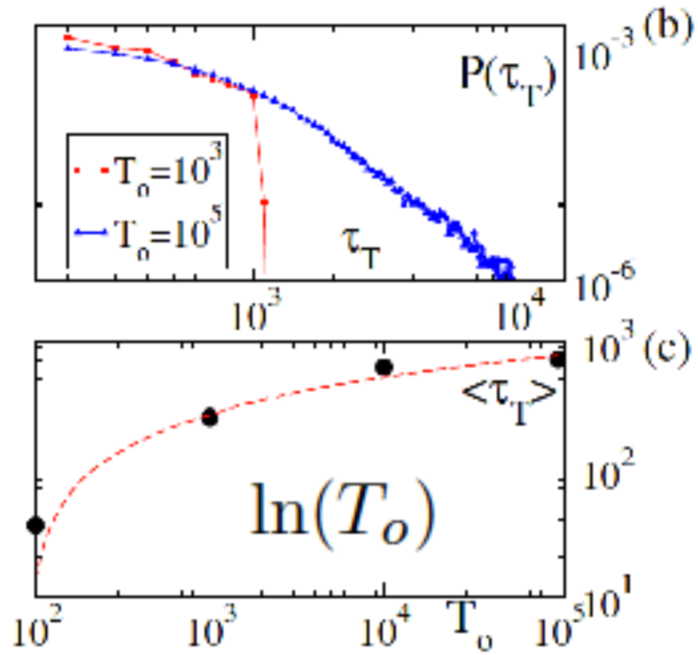

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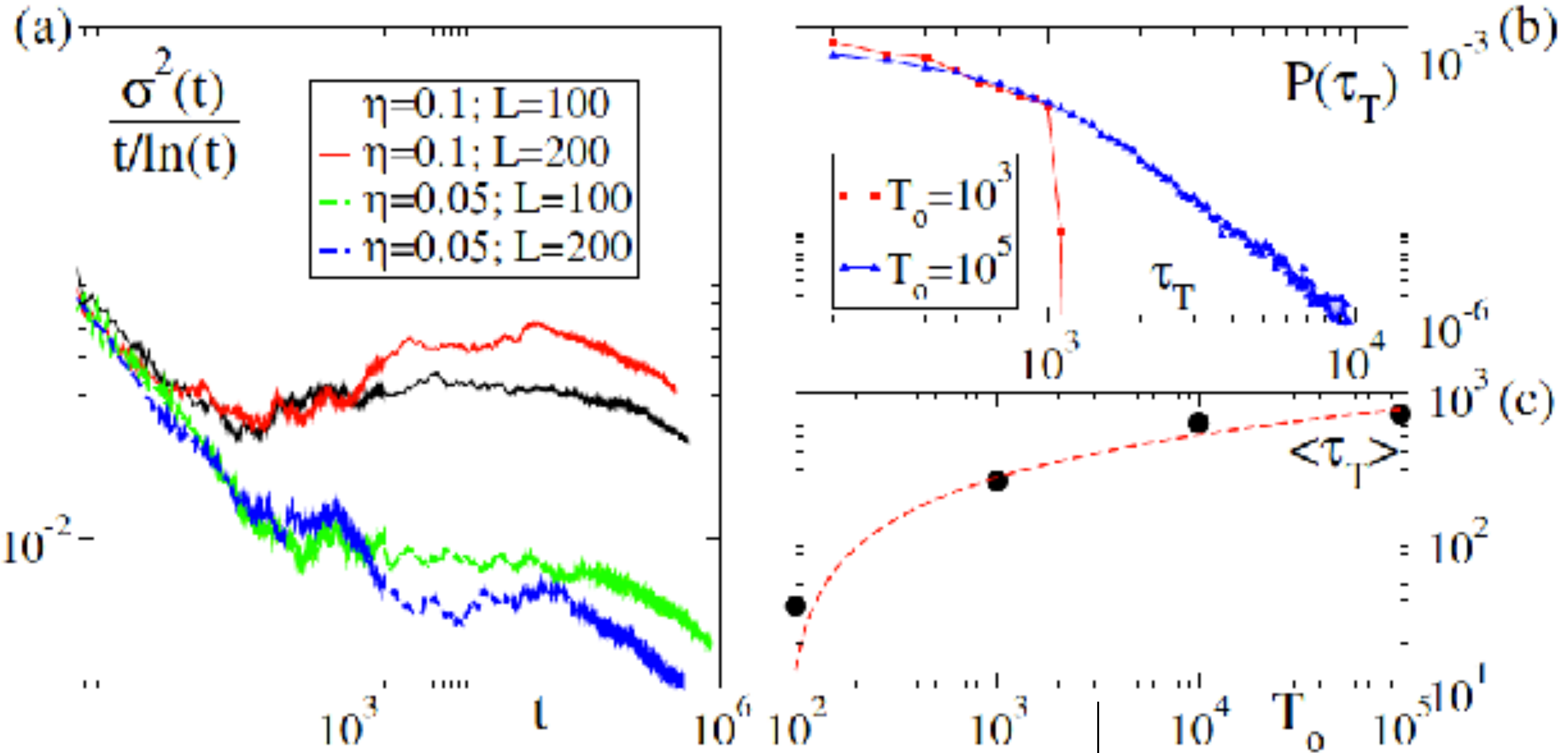

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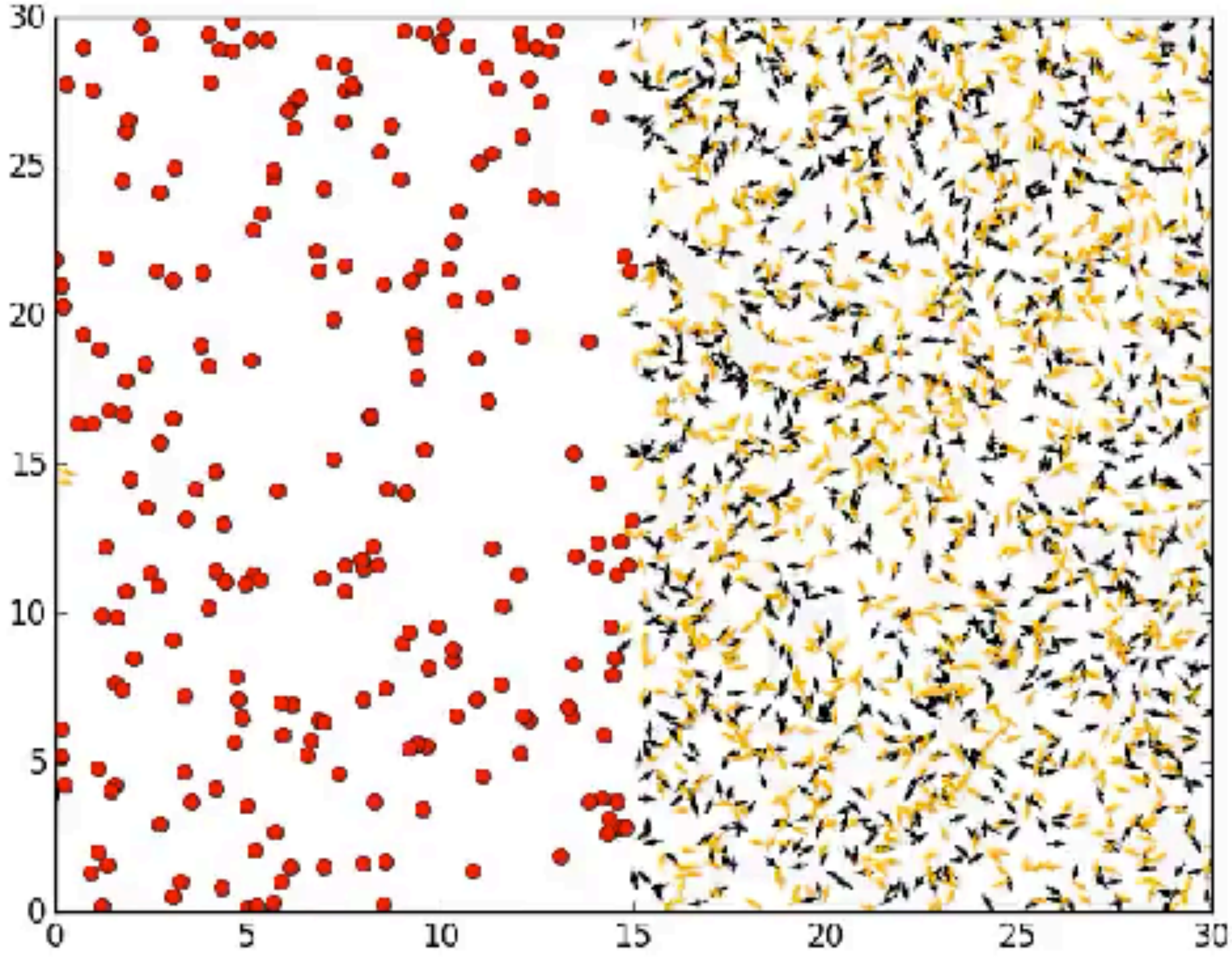
- **Filtering**

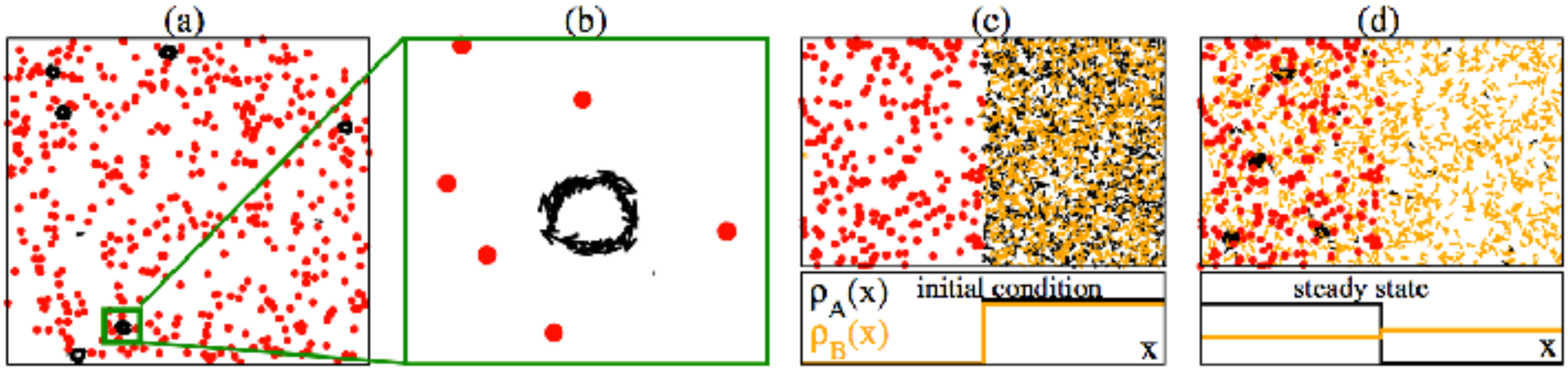
$\lambda_{\mathbf{A}} > \lambda_B$



An active "Lorentz" gas

• **<u>We can use the traps to fabricate a filter of SPPs!</u>**

$\lambda_A > \lambda_B$



Results:

- The diffusion coefficient exhibits a minimum with obstacle density (small λ)
- Spontaneous trapping of particles (moving at constant speed) occurs
- Trapping leads to subdiffusive behavior
- These findings can be used to fabricate a filter of active particles!

Chepizhko, Peruani, PRL (2013)

Adding particle-particle interactions:

Does the environment have an impact on the collective behavior?

Chepizhko, Altmann, Peruani, PRL (2013)

- **<u>A minimal continuum time SPP model with obstacles:</u>**

$$\dot{\mathbf{x}}_i = v_0 \mathbf{V}(\theta_i)$$

$$\dot{\theta}_i = g(\mathbf{x}_i)\left[\frac{\gamma_b}{n_b(\mathbf{x}_i)} \sum_{|\mathbf{x}_i - \mathbf{x}_j| < R_b} \sin(\theta_j - \theta_i)\right] + h(\mathbf{x}_i) + \eta \xi_i(t),$$

$$h(\mathbf{x}_i) = \begin{cases} \frac{\gamma_o}{n_o(\mathbf{x}_i)} \sum_{|\mathbf{x}_i - \mathbf{y}_k| < R_o} \sin(\alpha_{k,i} - \theta_i) & \text{if } n_o(\mathbf{x}_i) > 0 \\ 0 & \text{if } n_o(\mathbf{x}_i) = 0 \end{cases},$$

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Continuum time version of Vicsek model

$$h(\mathbf{x}_i) = \begin{cases} \frac{\gamma_o}{n_o(\mathbf{x}_i)} \sum_{|\mathbf{x}_i - \mathbf{y}_k| < R_o} \sin(\alpha_{k,i} - \theta_i) & \text{if } n_o(\mathbf{x}_i) > 0 \\ 0 & \text{if } n_o(\mathbf{x}_i) = 0 \end{cases},$$

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Collective motion in heterogeneous media

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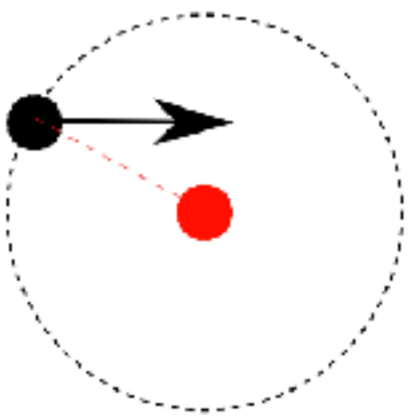
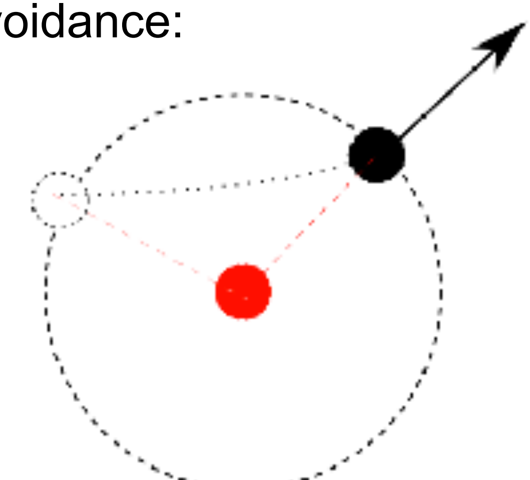
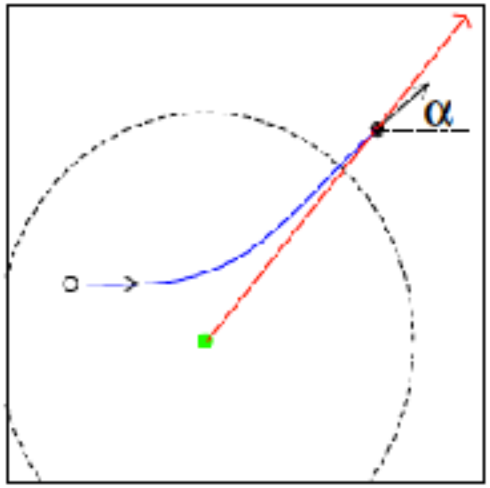
Obstacle avoidance:







Collective motion in heterogeneous media

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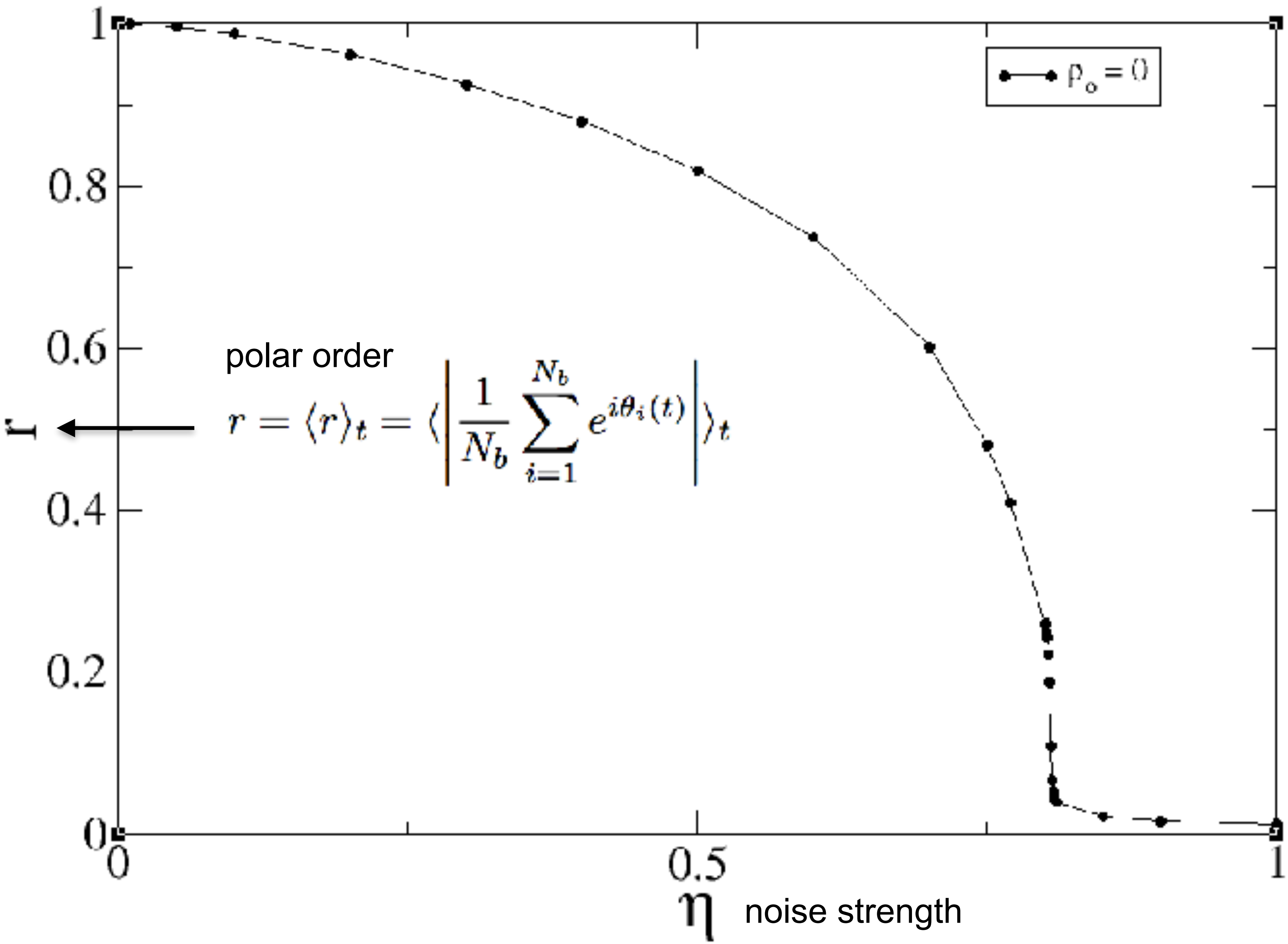

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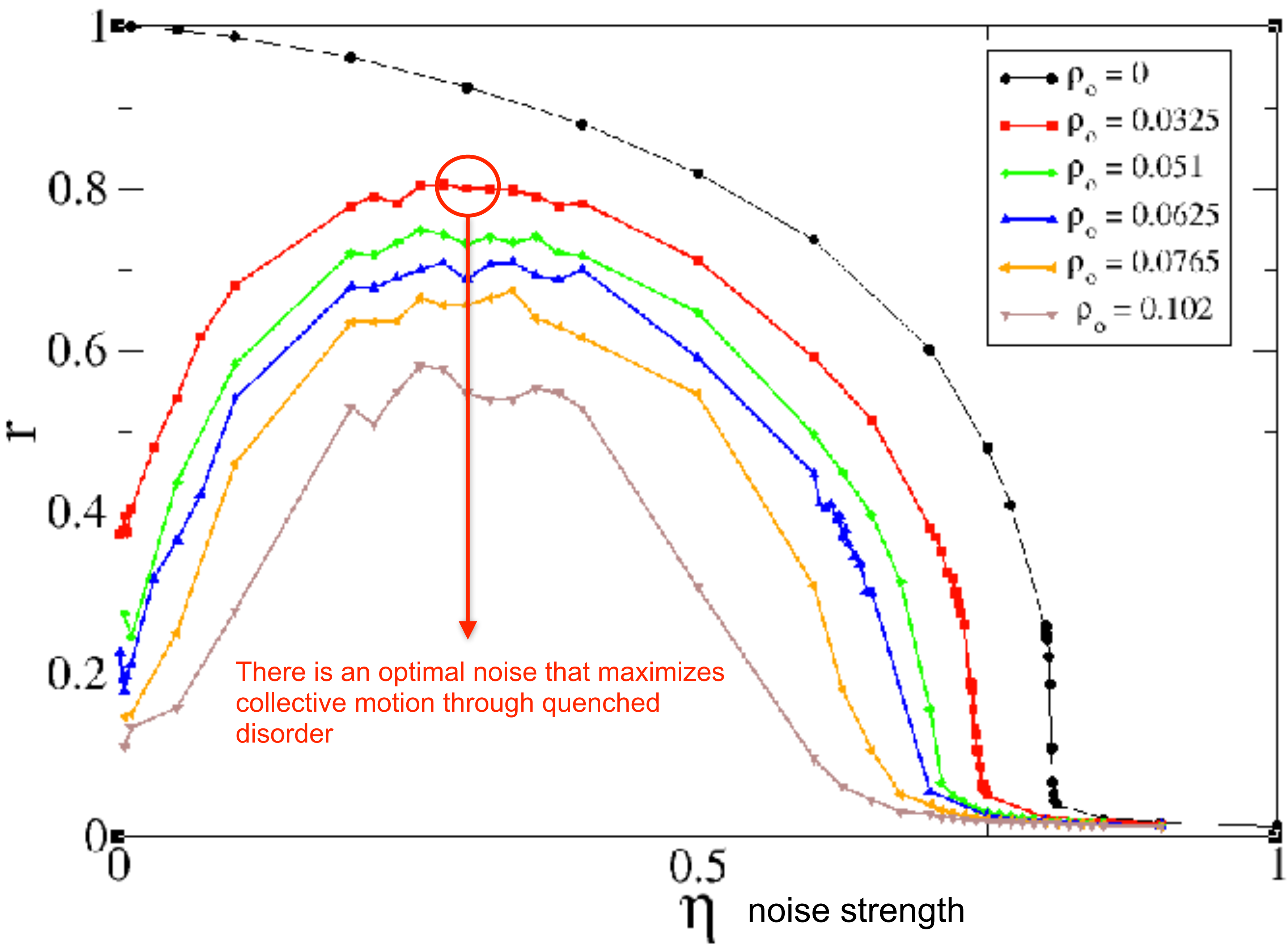

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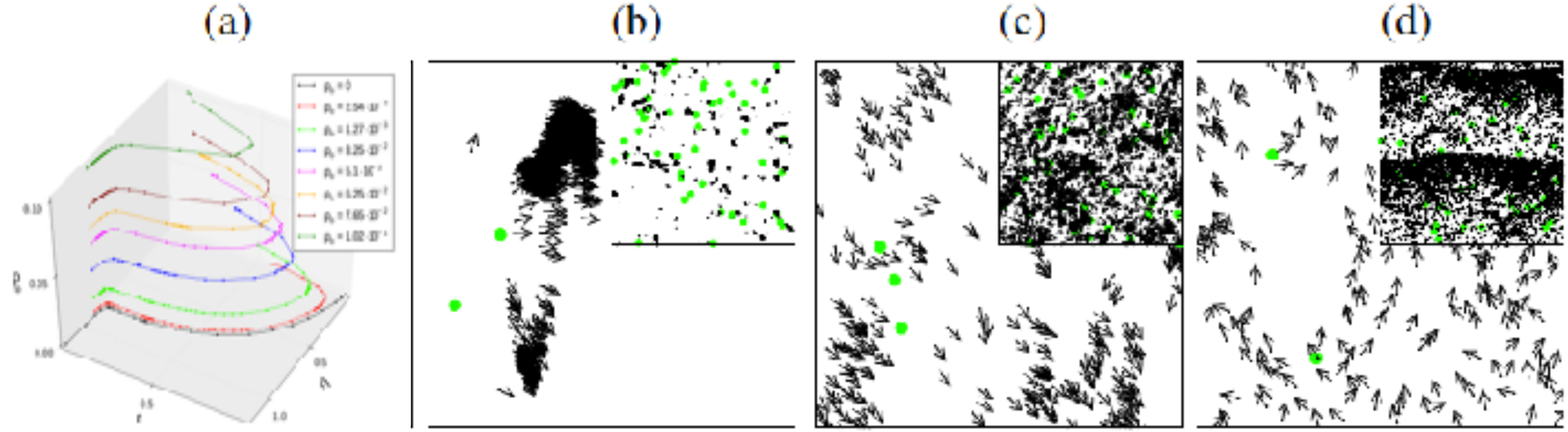
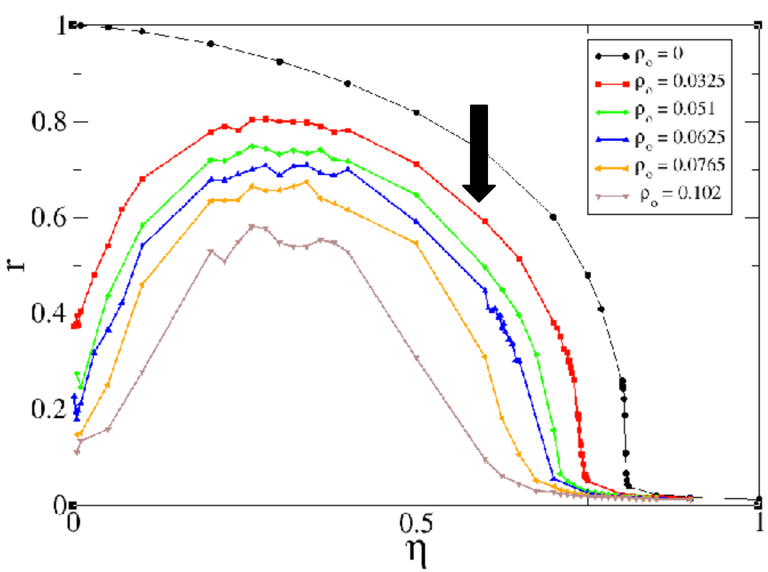
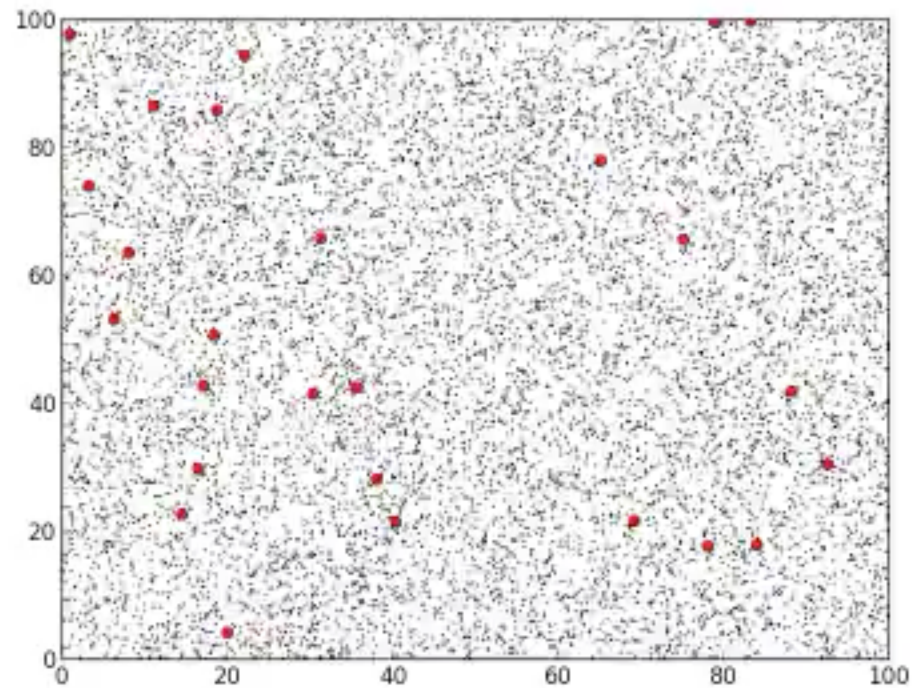
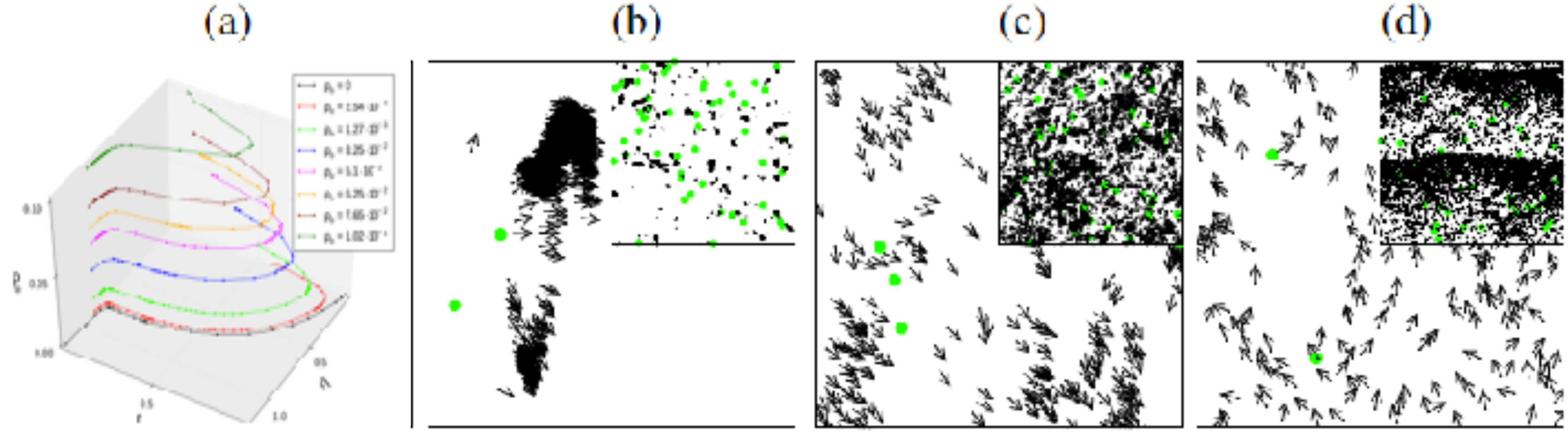


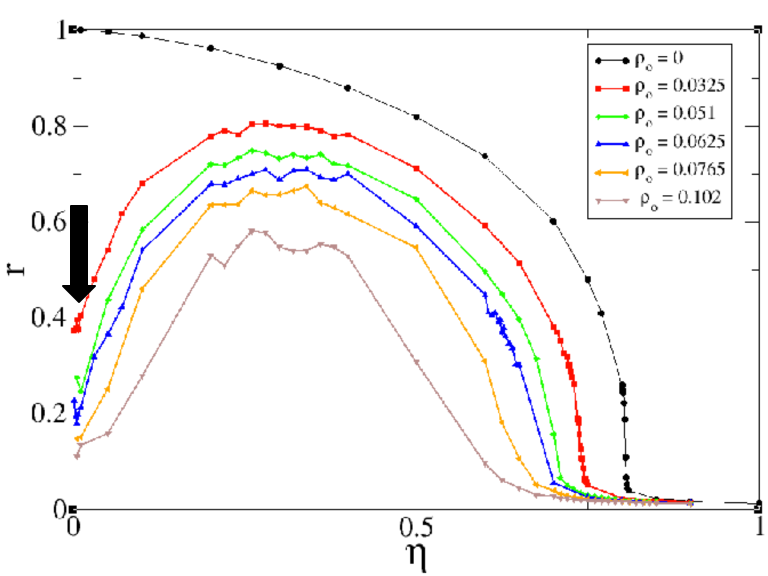


N=10000



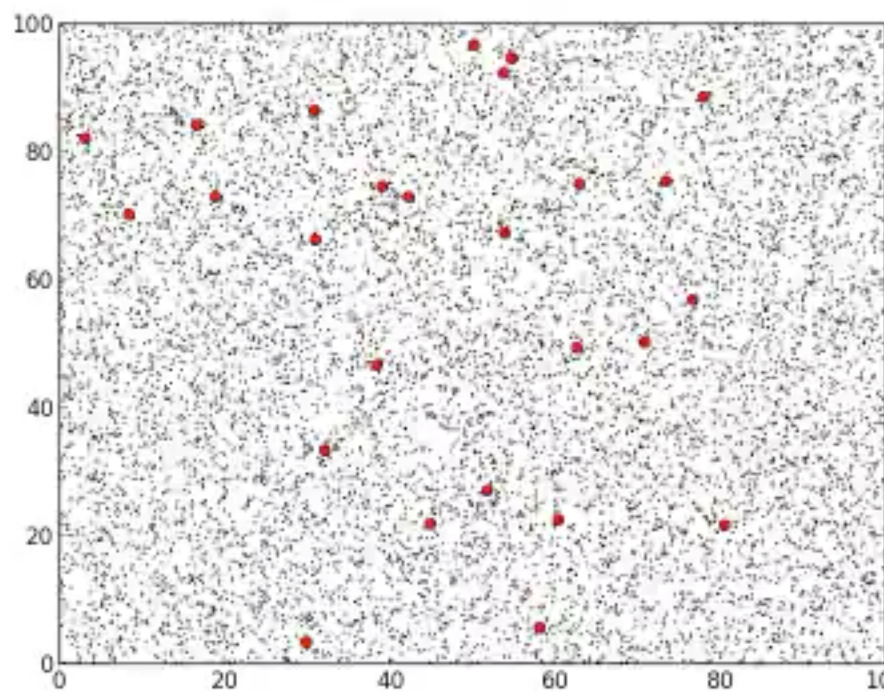


(a)
(b)
(c)
(d)



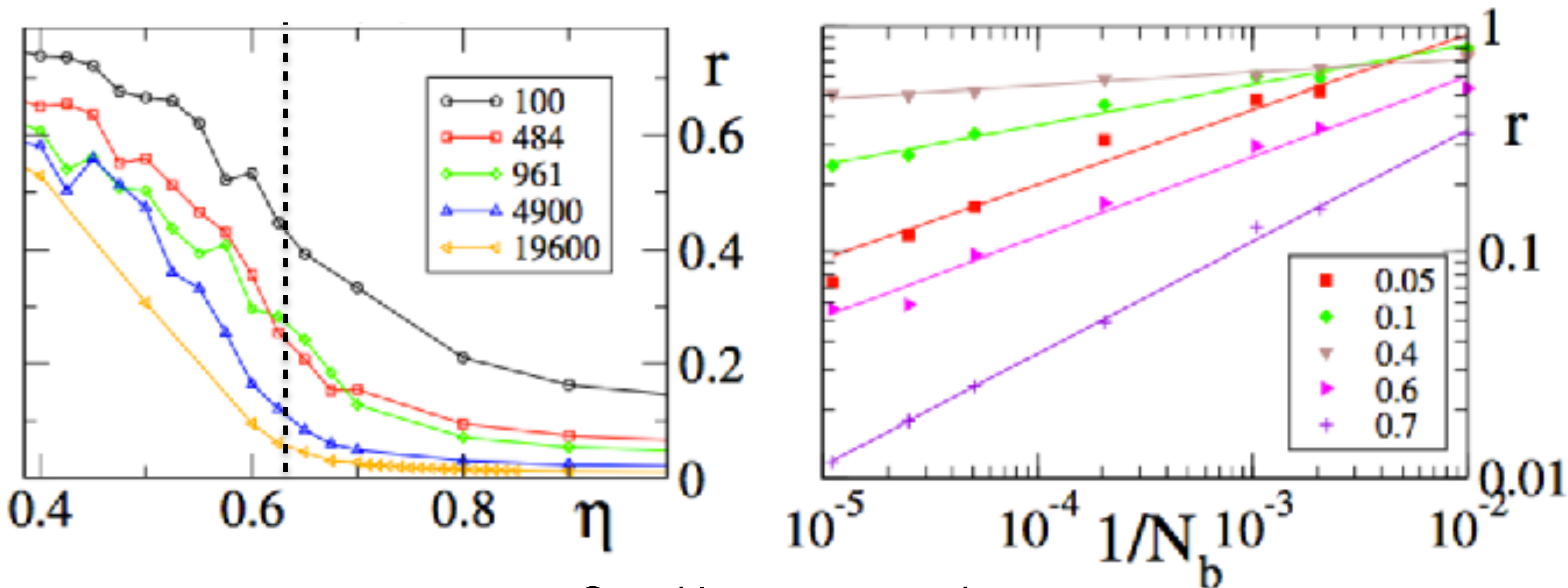
ρo = 0
ρo = 0.0325
ρo = 0.051
ρo = 0.0625
ρo = 0.0765
ρo = 0.102
r
η
0
0.2
0.4
0.6
0.8
1
0.5

N=10000



0
20
40
60
80
100

• Long-range order vs. quasi long-range order:



Quasi long-range order

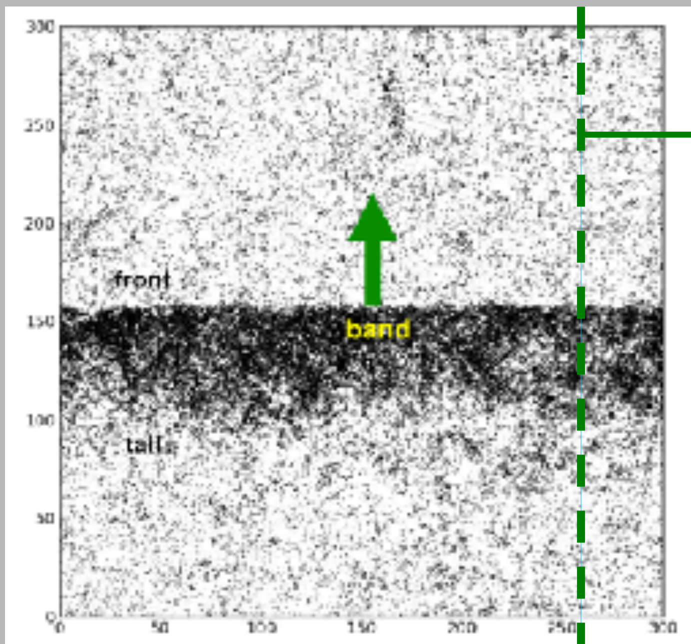
$$r \propto N_b^{-\nu(D_\theta,\rho_o)}$$

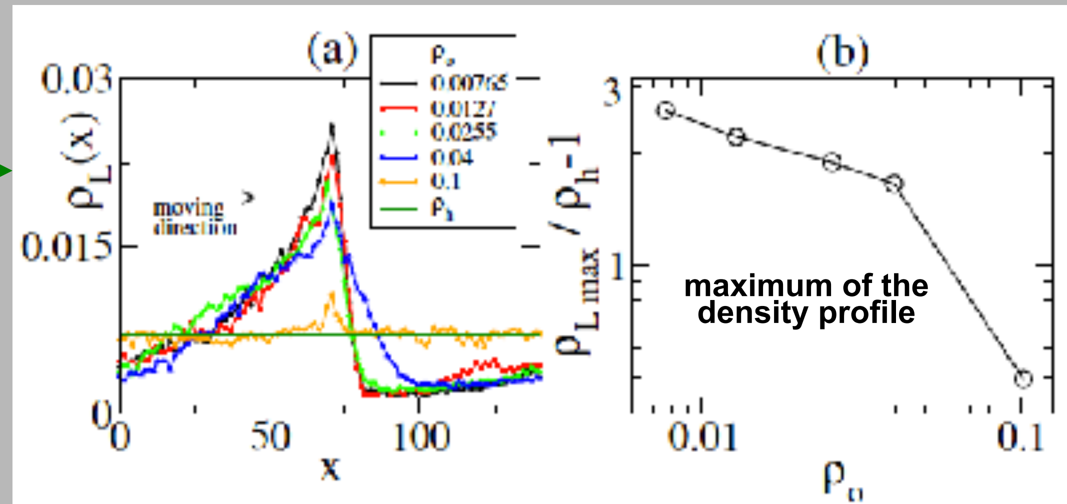
Context:

In the XY model in 2D, order is QLRO, while in the Vicsek model (in 2D) is LRO. Here, we are finding that in the presence of quenched disorder, the emergent polar order becomes QLRO!

Chepizhko, Altmann, Peruani, PRL (2013)

Bands disappear as the density is increased:

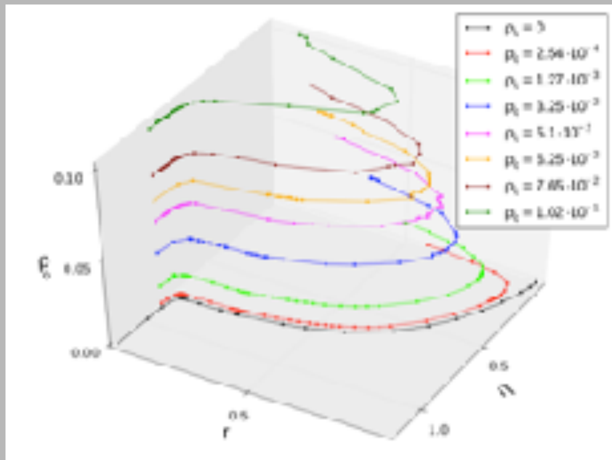


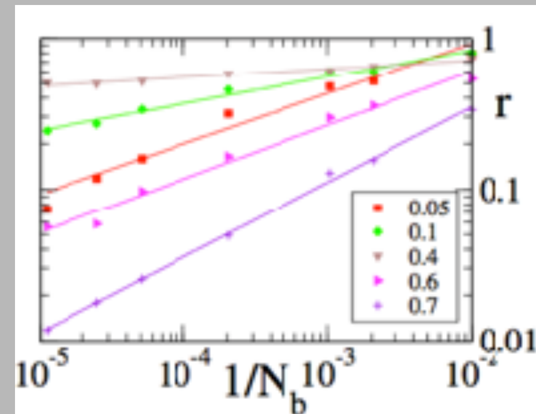


as before this occurs due to…

$$\partial_t \rho = \frac{v_0^2}{2D_\theta}\nabla^2\rho - \frac{\gamma v_0}{2D_\theta R\sqrt{\pi\rho_0}}\nabla.\left[(\cos(\psi),\sin(\psi))\rho\right]$$
$$= \frac{v_0^2}{2D_\theta}\nabla^2\rho - \frac{\gamma v_0}{2D_\theta R\sqrt{\pi\rho_0}}\nabla.\left[\frac{\rho\nabla\rho_o(\mathbf{x})}{||\nabla\rho_o(\mathbf{x})||}\right], \quad (11)$$

Chepizhko, Peruani, EPJ-ST (2015)
Chepizhko, Altmann, Peruani, PRL (2013)

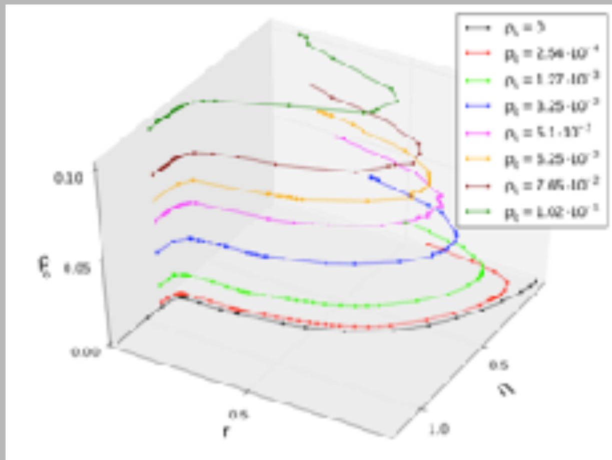


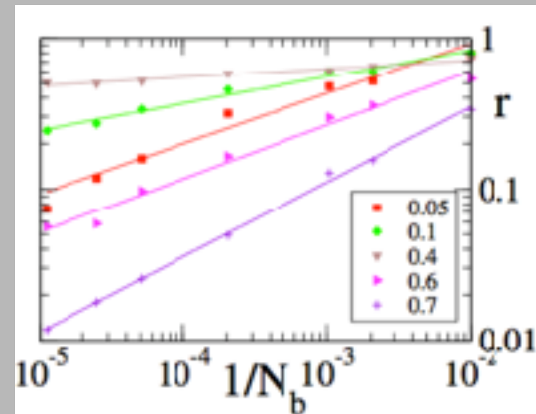


Results:

- There is an optimal noise that maximizes collective motion
- Due to the presence of “obstacles”, the system exhibits QLRO and bands disappear

Chepizhko, Peruani, EPJ-ST (2015)
Chepizhko, Altmann, Peruani, PRL (2013)





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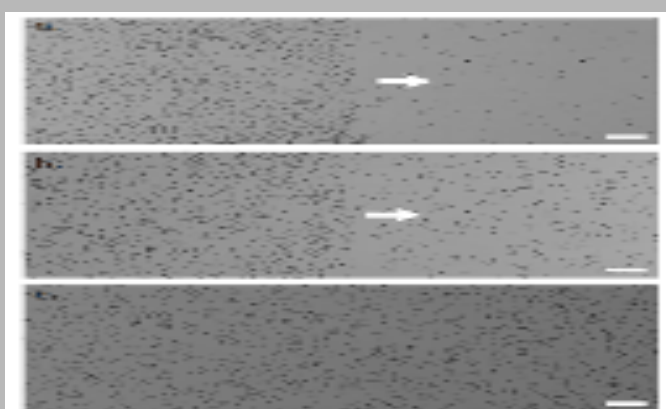
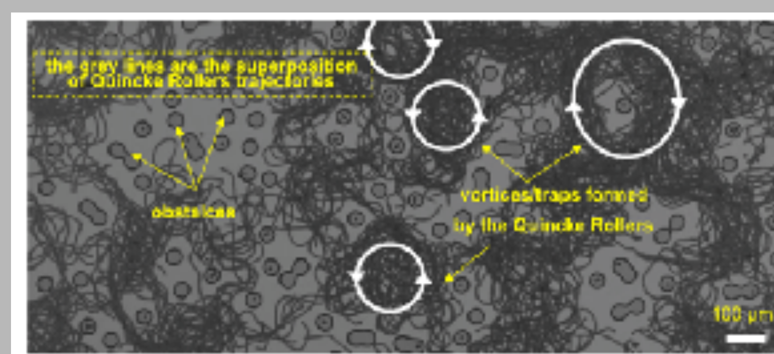
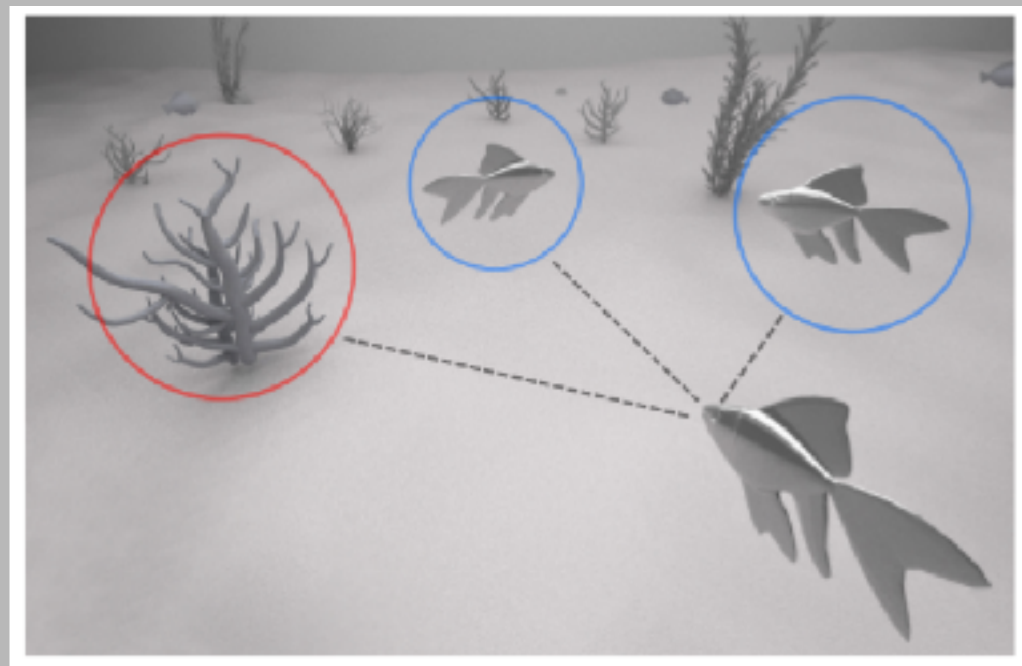




predictions observed in experiments: Briccard, Desreumaux et al. Nature Phys (2016)

Chepizhko, Peruani, EPJ-ST (2015)
Chepizhko, Altmann, Peruani, PRL (2013)

Flocking through disorder with topological interactions



$$\dot{\mathbf{x}}_i = v_0 \mathbf{V}(\theta_i)$$

$$\dot{\theta}_i = g_i(n_{o,i}) \left[\frac{\gamma}{n_{b,i}} \sum_{j \in \mathrm{TN}} \sin(\theta_j - \theta_i) \right] + \frac{\gamma}{n_{o,i}} \sum_{s \in \mathrm{TN}} \sin(\alpha_{s,i} - \theta_i) + \eta \xi_i(t)$$

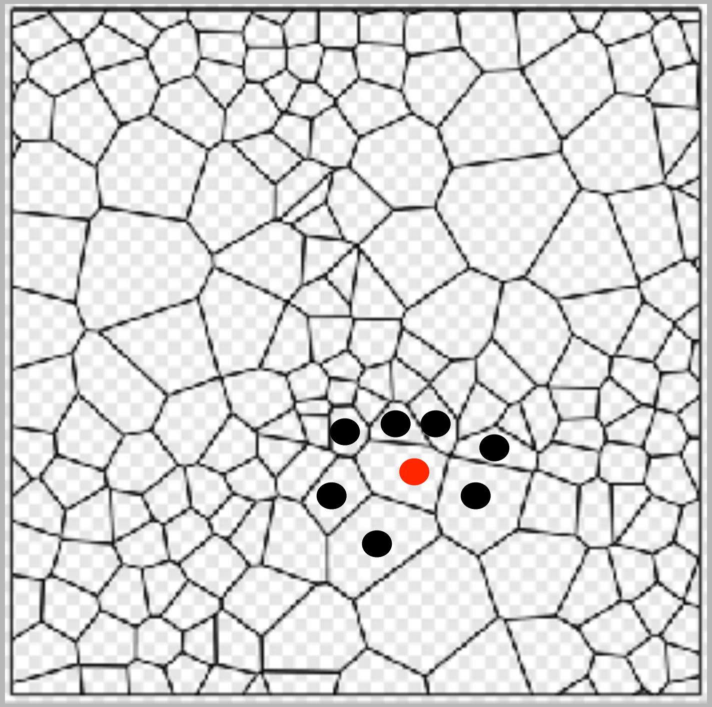
topo neighbor particles

topo neighbor obstacles

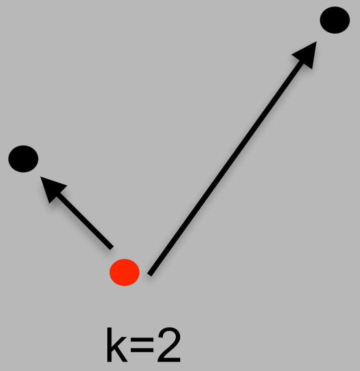
Rahmani, Peruani, Romanczuk, PloS Comp Biol (2020); Comm. Physics (2021)

Flocking through disorder with topological interactions

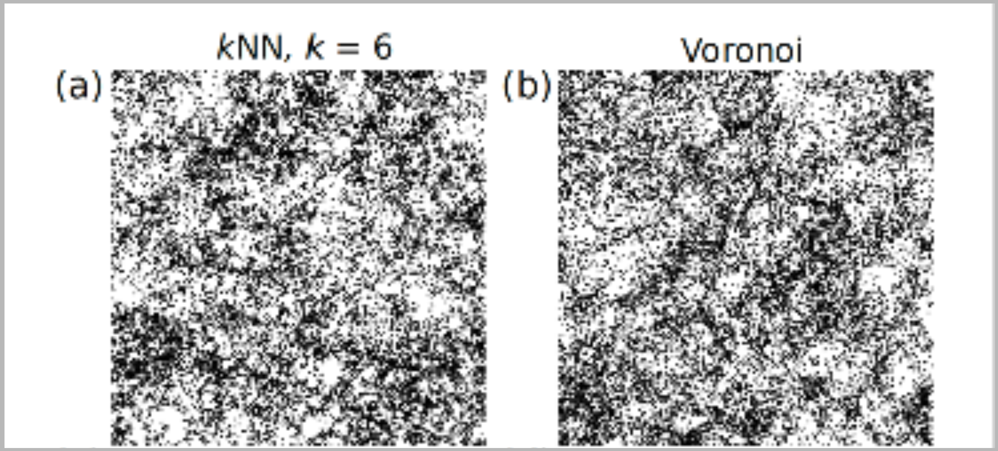
Voronoi neighbors



Knn neighbors

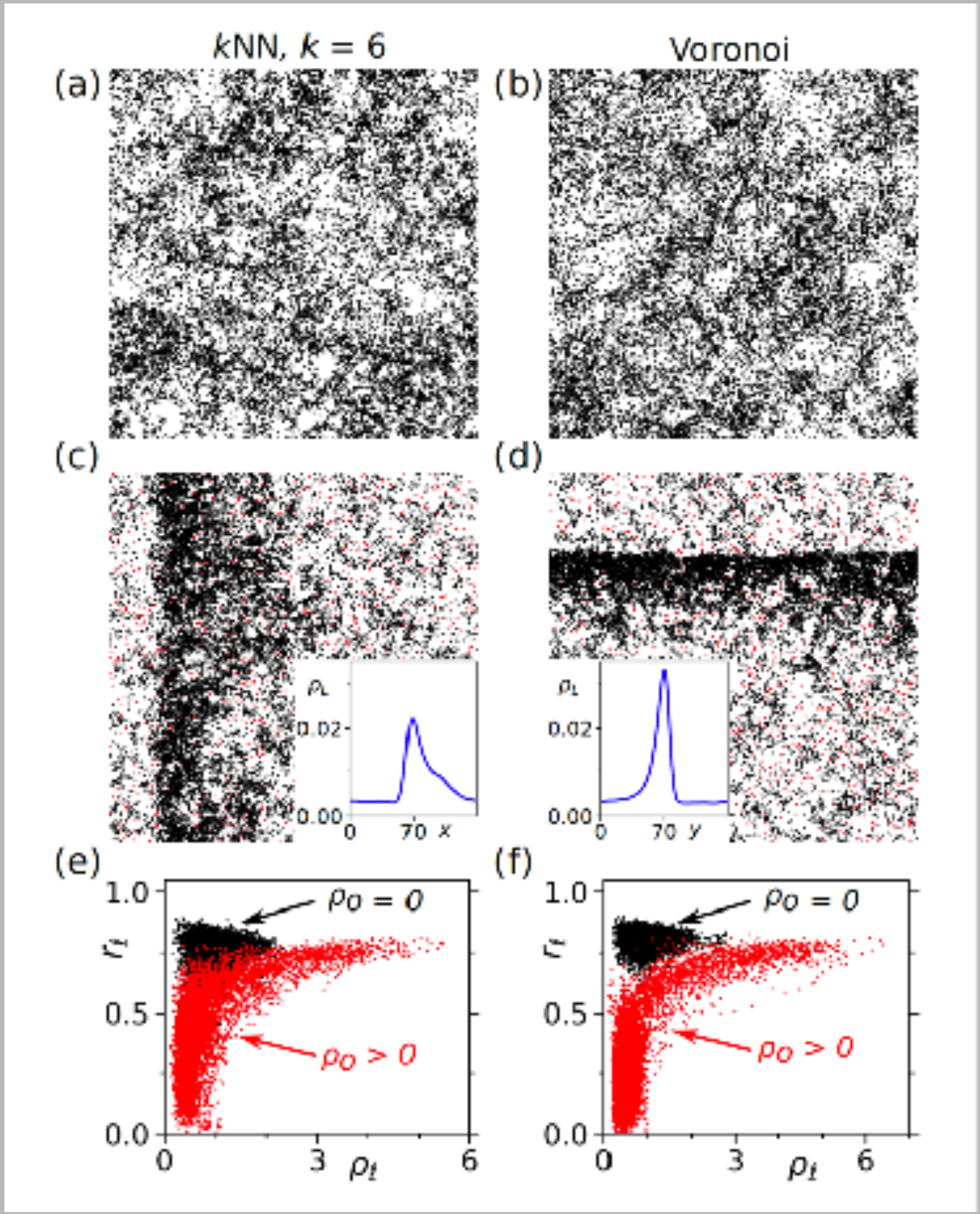


Flocking through disorder with topological interactions:



Rahmani et al. Comm Phys (2021)

Flocking through disorder with topological interactions:

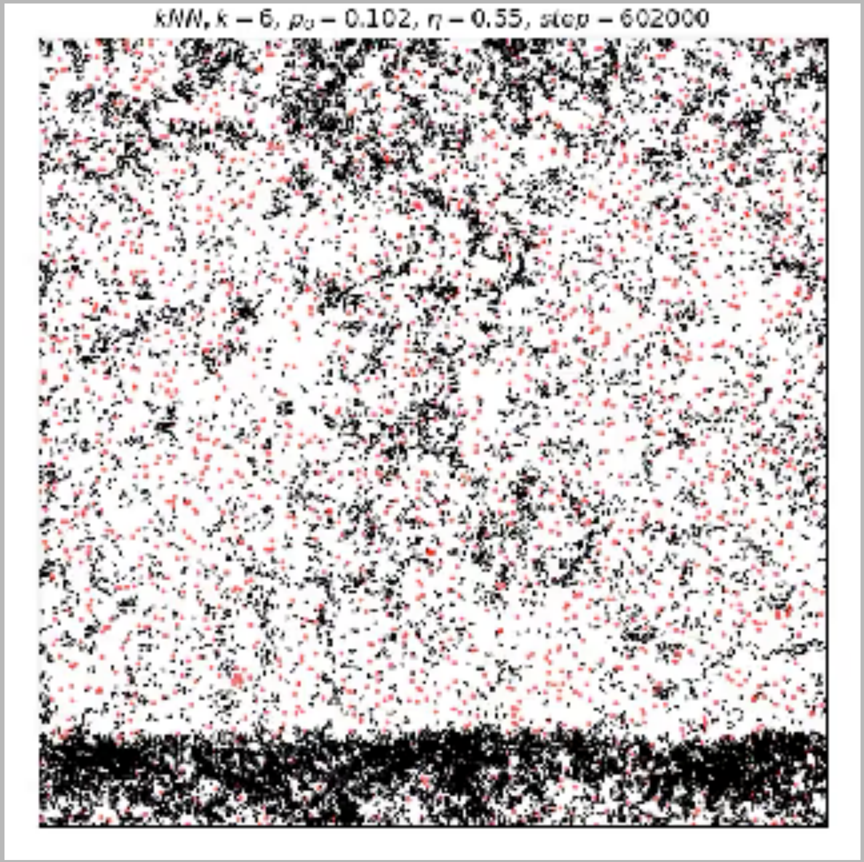


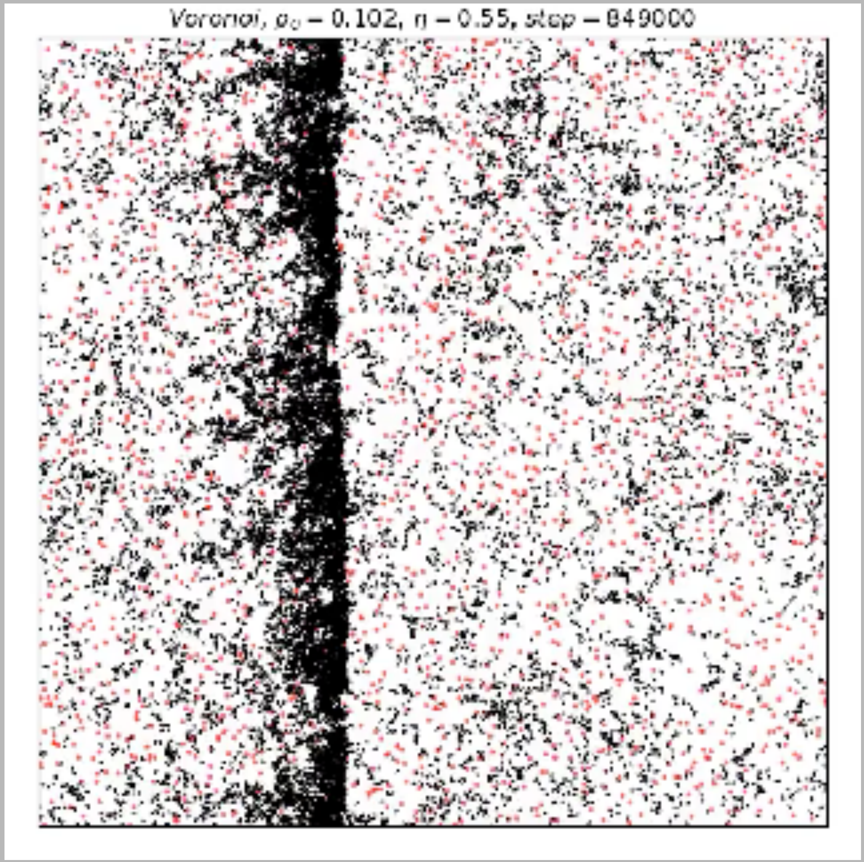
a coupling
local order-local density
emerges

Rahmani et al. Comm Phys (2021)

Flocking through disorder with topological interactions:

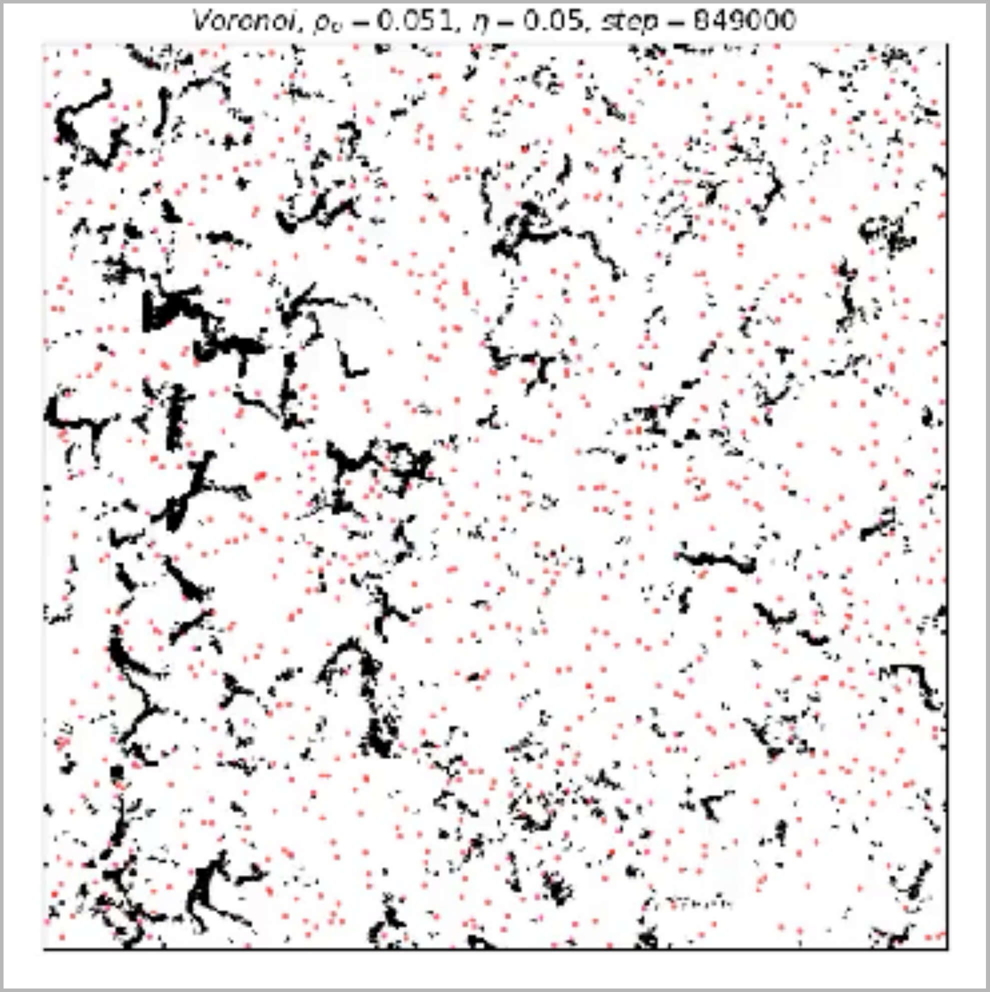
quenched disorder promotes the formation of bands!





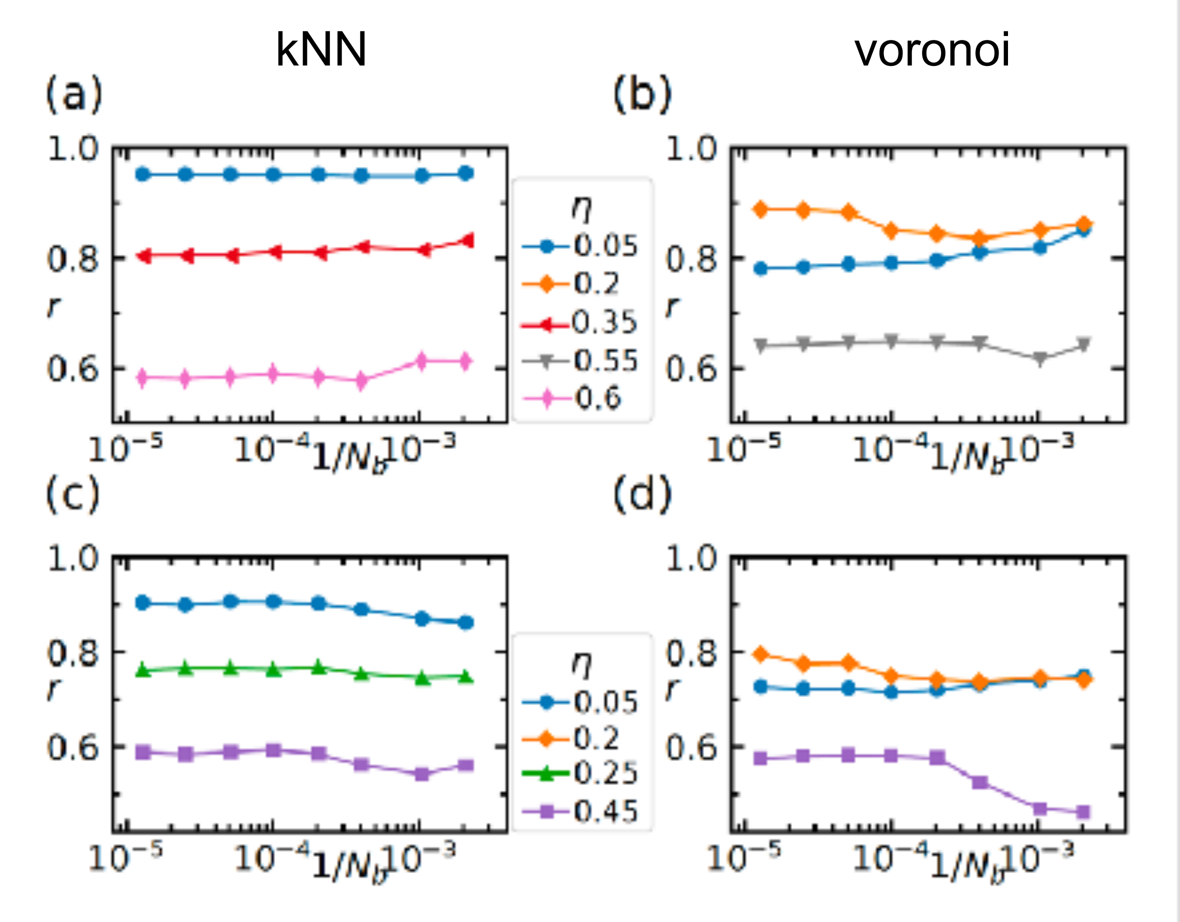
Rahmani. Peruani, Romanczuk Comm. Physics (2021)

Flocking through disorder with topological interactions:



Rahmani. Peruani, Romanczuk Comm. Physics (2021)

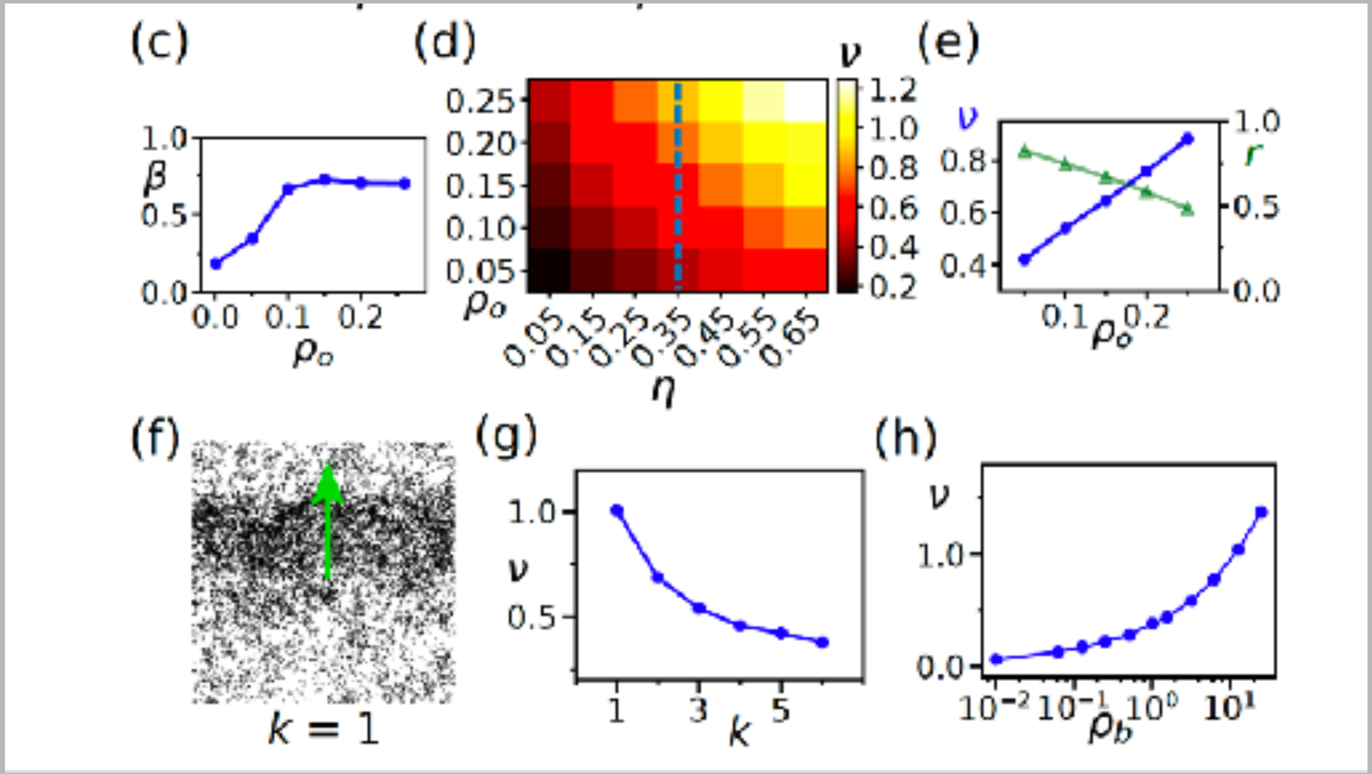
Flocking through disorder with topological interactions:



in contrast to the metric case, order is long-range!

Rahmani. Peruani, Romanczuk Comm. Physics (2021)

Rewiring of the underlying interaction network

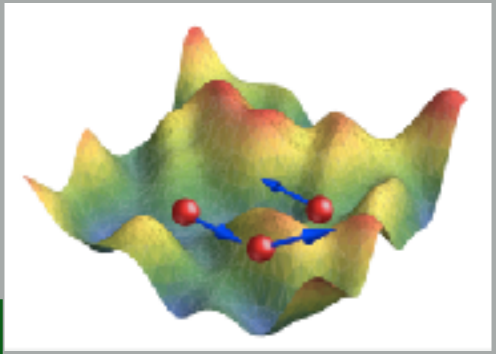


rewiring explains the emergence of the coupling order-density

rewiring also occurs in the absence of obstacles. For k=1, we can observe bands

Rahmani. Peruani, Romanczuk Comm. Physics (2021)

Hot vs Cold Active Matter



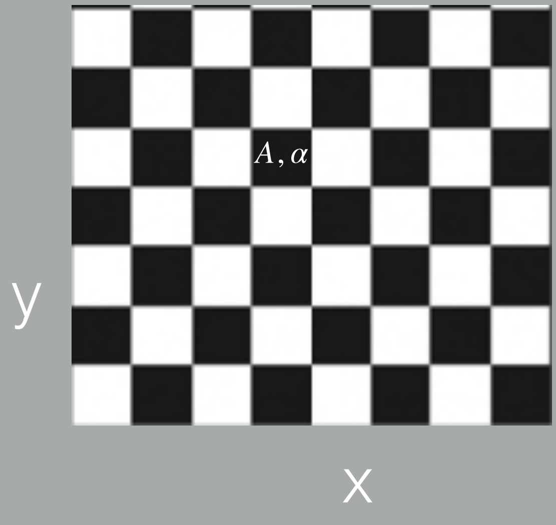
$$\dot{\mathbf{x}}_i = v_0 \mathbf{V}(\theta_i), \qquad \dot{\theta}_i = I_q(\mathbf{x}_i, \theta_i) + R_s(\mathbf{x}_i, \theta_i),$$

$$I_q(\mathbf{x}_i, \theta_i) = \frac{1}{n(\mathbf{x}_i)} \sum_{|\mathbf{x}_i - \mathbf{x}_j| < 1} \sin(q(\theta_j - \theta_i)),$$

$$R_{RT}(\mathbf{x}_i, \theta_i) = A\Gamma(\mathbf{x}_i),$$

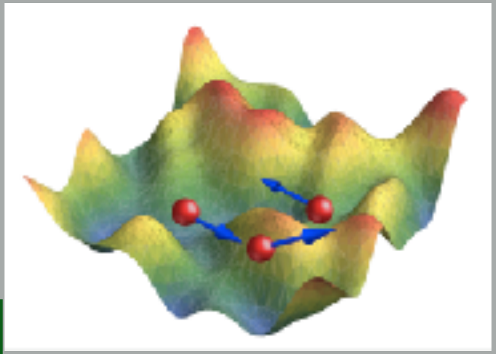
$$R_{RF}(\mathbf{x}_i, \theta_i) = A \sin(\alpha(\mathbf{x}_i) - \theta_i),$$

$$R_{RS}(\mathbf{x}_i, \theta_i) = A \sin(2(\alpha(\mathbf{x}_i) - \theta_i)),$$



Peruani, Aranson PRL (2018)

Hot vs Cold Active Matter



$$\dot{\mathbf{x}}_i = v_0 \mathbf{V}(\theta_i), \qquad \dot{\theta}_i = I_q(\mathbf{x}_i, \theta_i) + R_s(\mathbf{x}_i, \theta_i),$$

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$$R_{RS}(\mathbf{x}_i, \theta_i) = A \sin\left(2(\alpha(\mathbf{x}_i) - \theta_i)\right),$$

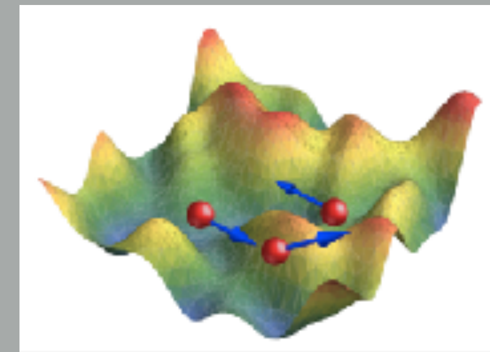
When the equations of motion exhibit a **Hamiltonian structure**, then particles trajectories cannot fall (asymptotically) into a trap [by Poincaré-Bendixon Theorem], and thus motion is **diffusive**.

On the other hand, when the equations of motion display a **dissipative structure**, traps emerge in the system and particles asymptotically fall into **traps**.

Peruani, Aranson PRL (2018)

Hot vs Cold Active Matter

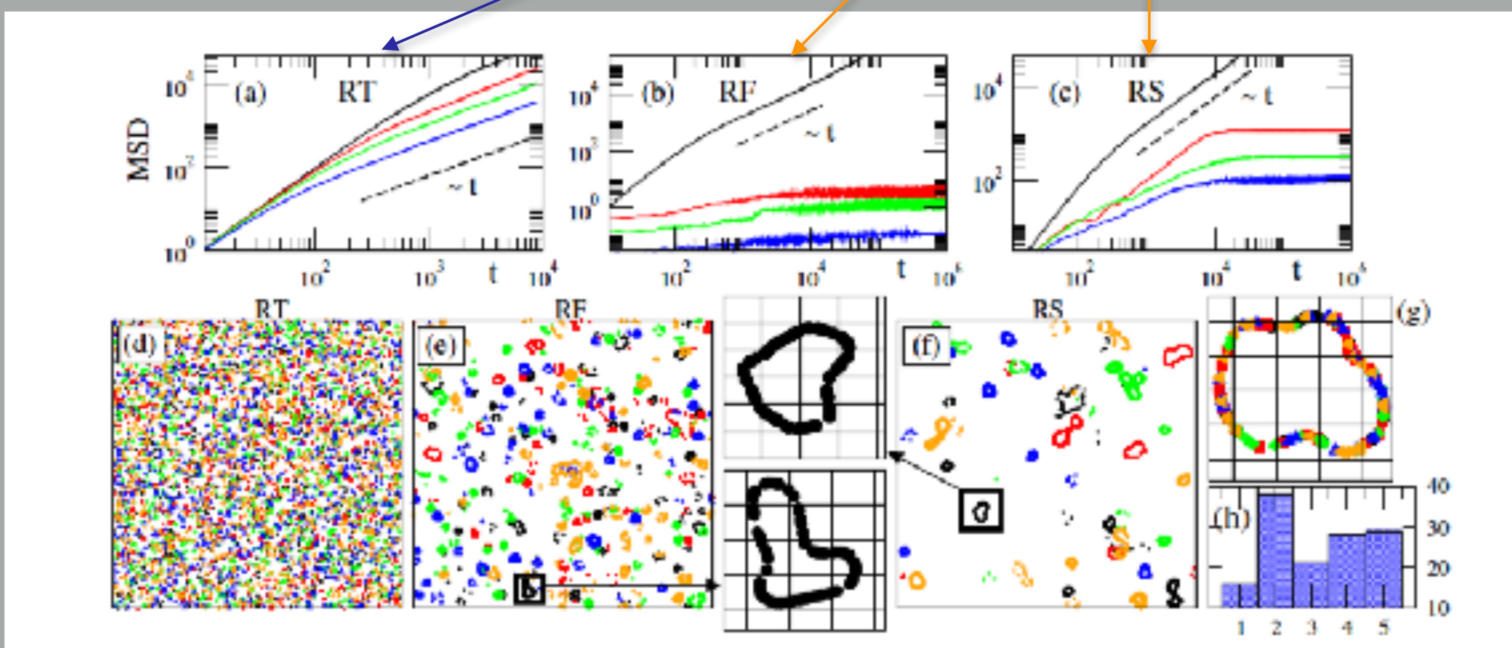
When do we have a Hamiltonian structure?



$$\dot{\mathbf{x}}_i = v_0 \mathbf{V}(\theta_i), \qquad \dot{\theta}_i = R_s(\mathbf{x}_i, \theta_i)$$

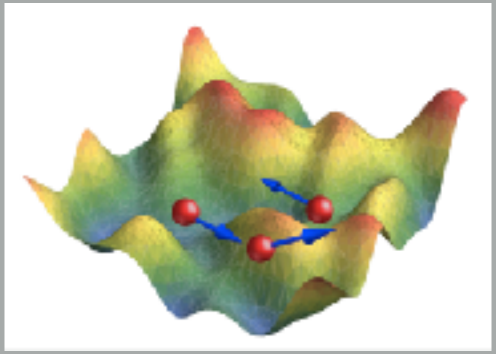
$$R_{\mathrm{RT}}(\mathbf{x}_i, \theta_i) = A\Gamma(\mathbf{x}_i), \quad \rightarrow \text{yes}$$

$$R_{\mathrm{RF}}(\mathbf{x}_i, \theta_i) = A \sin(\alpha(\mathbf{x}_i) - \theta_i),$$
$$R_{\mathrm{RS}}(\mathbf{x}_i, \theta_i) = A \sin(2(\alpha(\mathbf{x}_i) - \theta_i)), \quad \rightarrow \text{no}$$



Peruani, Aranson PRL (2018)

Hot vs Cold Active Matter



When do we have a Hamiltonian structure?

$$\ddot{\mathbf{x}} = \dot{\mathbf{x}} \times (-A\Gamma(\mathbf{x})\mathbf{z}_0)$$ for RT

What is the diffusion coefficient here?

$$\langle \theta_i^2(t)\rangle = 2A^2 \int_0^t dt' \int_0^{t'} dt'' \langle \Gamma(\mathbf{u}_0 v_0 t'')\Gamma(\mathbf{u}_0 v_0 t')\rangle \approx 2\frac{A^2\langle\Gamma(\mathbf{x})^2\rangle_c \Delta_x}{v_0} t = 2T_{\mathrm{sh}} t.$$

$$D_\theta = T_{\mathrm{sh}} = \frac{A^2\Delta_x}{3v_0}$$

shaking temp.

diffusion coefficient

$$D_{\mathrm{tr}} = \frac{v_0^2}{2D_\theta} = \frac{3v_0^3}{2A^2\Delta_x}$$

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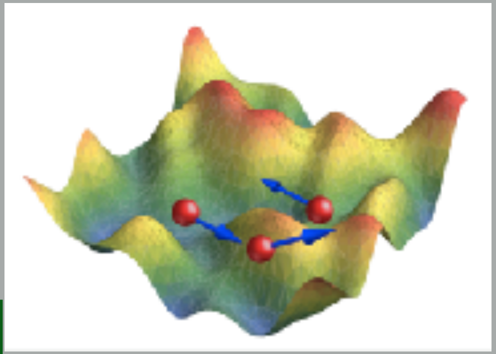
Hot vs Cold Active Matter



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$$R_{RS}(\mathbf{x}_i, \theta_i) = A\sin(2(\alpha(\mathbf{x}_i) - \theta_i)),$$

When do we find a dissipative structure?

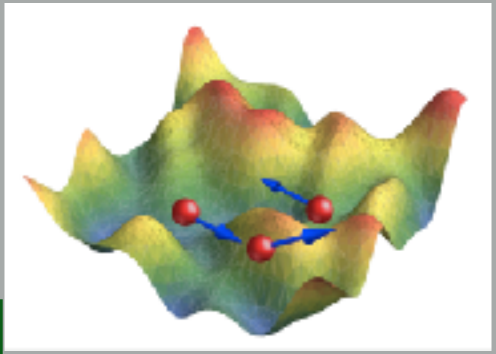
$$\ddot{\mathbf{x}}_i = -\dot{\mathbf{x}}_i \times A\Gamma(\mathbf{x}_i)\mathbf{z}_0 + \frac{1}{v_0^2 n(\mathbf{x}_i)} \dot{\mathbf{x}}_i \times \sum_{|\mathbf{x}_i - \mathbf{x}_j| < 1} \dot{\mathbf{x}}_i \times \dot{\mathbf{x}}_j$$

interactions

Peruani, Aranson PRL (2018)

Hot vs Cold Active Matter

$$\dot{\mathbf{x}}_i = v_0 \mathbf{V}(\theta_i), \qquad \dot{\theta}_i = I_q(\mathbf{x}_i, \theta_i) + R_s(\mathbf{x}_i, \theta_i),$$



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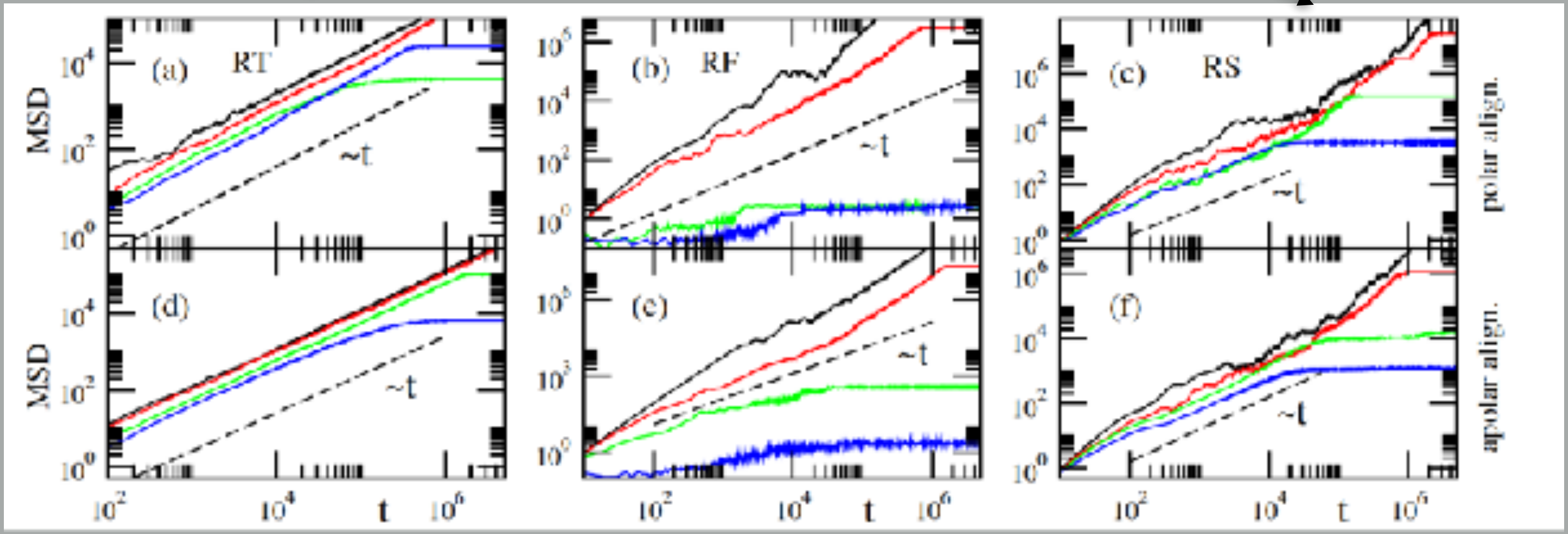


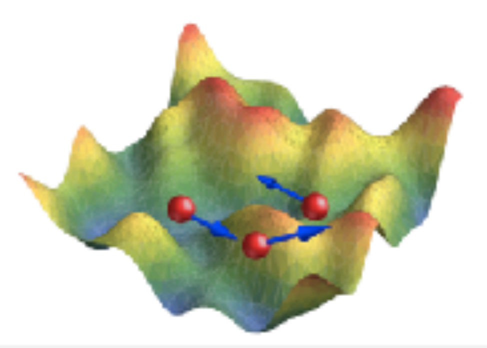
And for interacting active particles, we always find trapping!

Peruani, Aranson PRL (2018)

Hot vs Cold Active Matter

Hamiltonian structure -> diffusive-like behavior

Dissipative structure -> trapping



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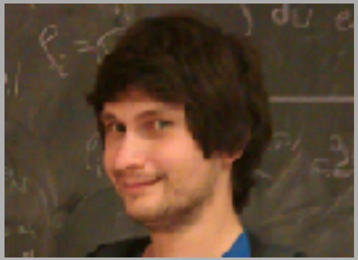
acknowledgements

Aranson

from PennState

Chepizhko



from Univ. Innsbruck

Romanczuk

Humb. Univ.

Rahmani

CYU

References

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Thanks for you attention!