The Large Charge Expansion in QFT

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Kavli Asian Winter School | Kyoto | 6 to 8 December 2023

Selected Topics in the Large Quantum Number Expansion, Phys.Rept. 933 (2021) 1-66, arXiv:2008.03308
Resurgence of the large-charge expansion, JHEP 05 (2021) 035, arXiv:2102.12488

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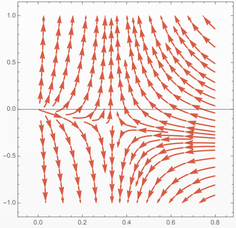
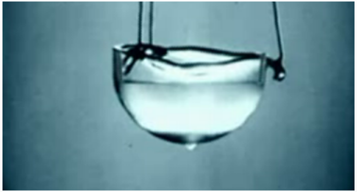
WHY ARE WE HERE? CONFORMAL FIELD THEORIES

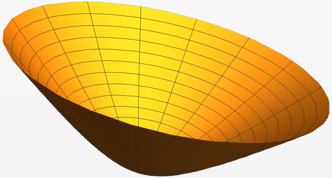
extrema of the RG flow



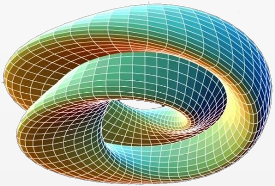
critical phenomena



quantum gravity



string theory



WHY ARE WE HERE? CONFORMAL FIELD THEORIES ARE HARD

Most conformal field theories (CFTs) lack nice limits where they become simple and solvable.

No parameter of the theory can be dialed to a simplifying limit.

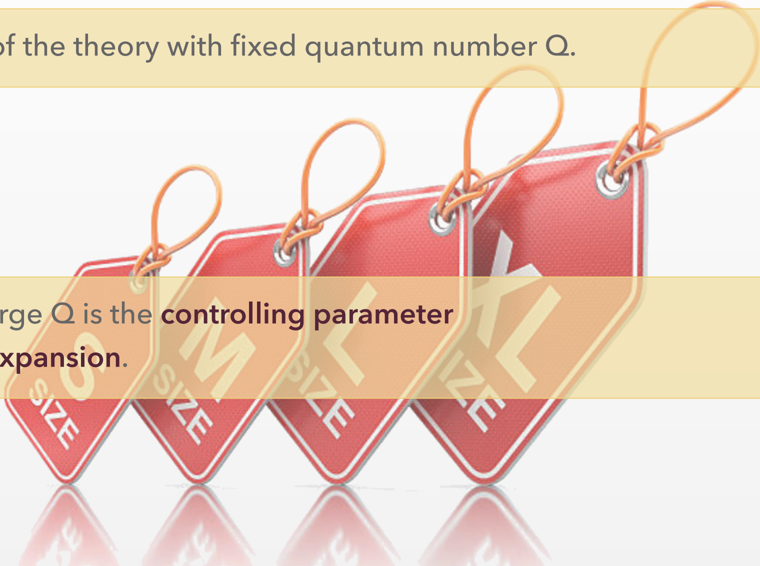
WHY ARE WE HERE? CONFORMAL FIELD THEORIES ARE HARD

In presence of a **symmetry** there can be **sectors of the theory** where anomalous dimension and OPE coefficients simplify.

THE IDEA

Study **subsectors** of the theory with fixed quantum number Q.

In each sector, a large Q is the **controlling parameter** in a **perturbative expansion**.



NOT AN ORIGINAL IDEA

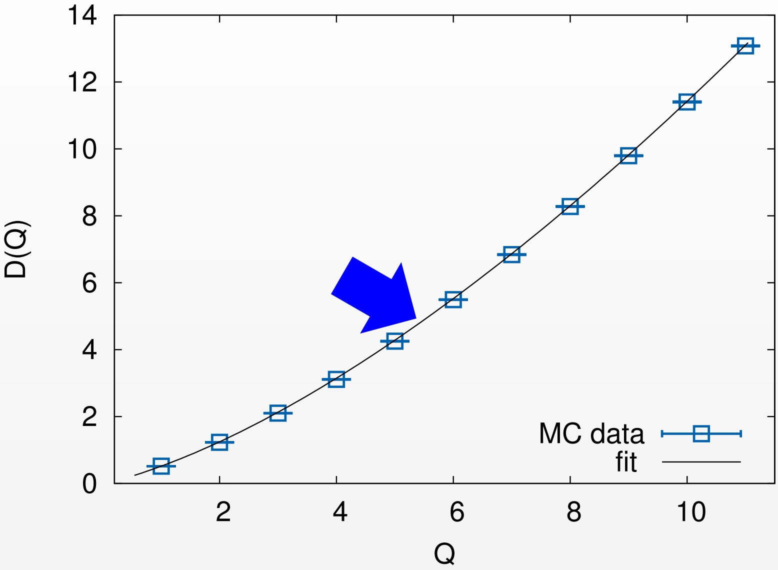
CONCRETE RESULTS

We consider the $O(N)$ **vector model in three dimensions**. In the IR it flows to a **conformal fixed point** [Wilson & Fisher].

We find an explicit formula for the **dimension of the lowest primary at fixed charge**:

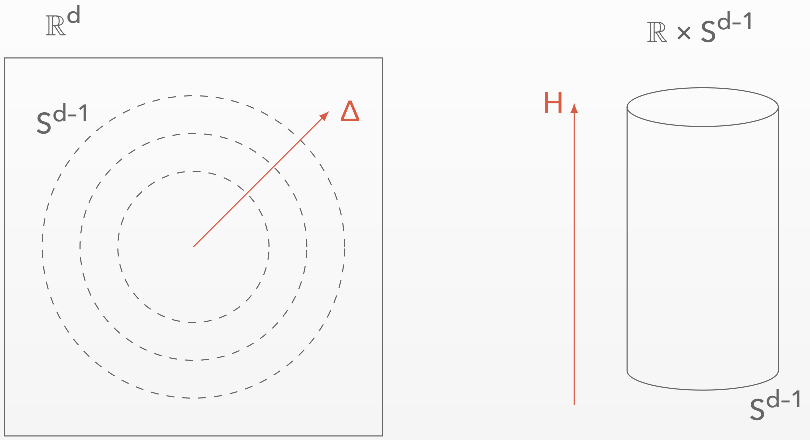
$$\Delta_Q = \frac{c_{3/2}}{2\sqrt{\pi}} Q^{3/2} + 2\sqrt{\pi} c_{1/2} Q^{1/2} - 0.094 + \mathcal{O}\left(Q^{-1/2}\right)$$

SUMMARY OF THE RESULTS: O(2)



STATE-OPERATOR CORRESPONDENCE

The anomalous dimension on $\mathbb{R}^d$ is the energy in the cylinder frame.



Protected by conformal invariance: a well-defined quantity.

SCALES

We want to write a **Wilsonian effective action.**

Choose a cutoff Λ, separate the fields into high and low frequency φ_H, φ_L and do the path integral over the high-frequency part:

$$e^{iS_\Lambda(\varphi_L)} = \int \mathcal{D}\varphi_H \, e^{iS(\varphi_H,\varphi_L)}$$

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SCALES

- We look at a finite box of typical **length R**
- The U(1) charge Q fixes a **second scale $\rho^{1/2} \sim Q^{1/2}/R$**

$$\frac{1}{R} \ll \Lambda \ll \rho^{1/2} \sim \frac{Q^{1/2}}{R} \ll \Lambda_{UV}$$

For $\Lambda \ll \rho^{1/2}$ the **effective action is weakly coupled and under perturbative control** in powers of ρ^{-1}.

NON-LINEAR SIGMA MODEL

In a generic theory™, picking the lowest state of fixed charge induces a spontaneous symmetry breaking.
The low-energy physics is described by a **Goldstone field** χ.

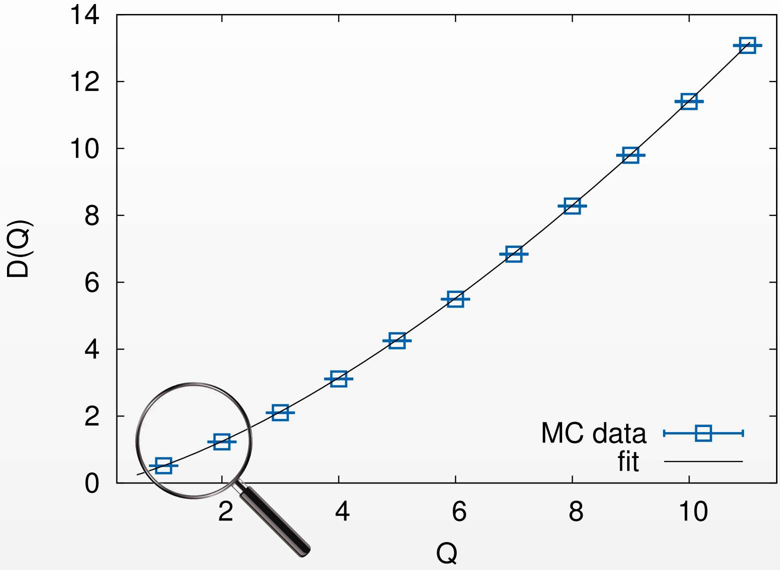
Using conformal invariance, the most general action must take the form

$$L[\chi] = k_{3/2}(\partial_\mu \chi \partial^\mu \chi)^{3/2} + k_{1/2} R (\partial_\mu \chi \partial^\mu \chi)^{1/2} + \dots$$

These are the leading terms in the expansion around the classical solution $\chi = \mu t$. All other terms are suppressed by powers of $1/Q$.

The **energy of the lowest state** for this action is the **conformal dimension of the lowest operator** of given charge Q.

TOO GOOD TO BE TRUE?



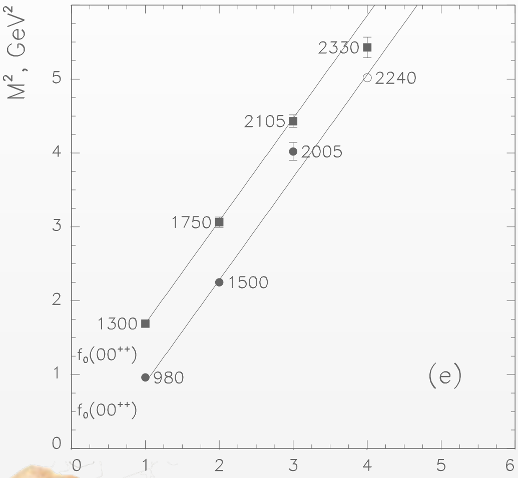
TOO GOOD TO BE TRUE?

Think of **Regge trajectories**.

The prediction of the theory is

$$m^2 \propto J\left(1 + \mathcal{O}\left(J^{-1}\right)\right)$$

but experimentally everything works so well at small J that String Theory was invented.



TOO GOOD TO BE TRUE?

The unreasonable effectiveness

of the **large charge expansion**.

SELECTED TOPICS IN THE LARGE CHARGE EXPANSION

- O(2) model [Hellerman, DO, Reffert, Watanabe] [Monin, Pirtskhalava, Rattazzi, Seibold]
- fermions [Komargodski, Mezei, Pal, Raviv-Moshe] [Antipin, Bersini, Panopoulos] [Hellerman, Dondi, Kalogerakis, Moser, DO, Reffert]
- holography [Loukas, DO, Reffert, Sarkar] [de la Fuente] [Guo, Liu, Lu, Pang] [Giombi, Komatsu, Offertaler]
- large N [Álvarez-Gaumé, DO, Reffert] [Giombi, Hyman]
- ε double-scaling [Badel, Cuomo, Monin, Rattazzi] [Arias-Tamargo, Rodriguez-Gomez, Russo] [Antipin, Bersini, Sannino, Wang, Zhang] [Jack, Jones]
- non-relativistic CFTs [Kravec, Pal] [Hellerman, Swanson] [Favrod, DO, Reffert] [DO, Reffert, Pellizzani] [Hellerman, DO, Reffert, Pellizzani, Swanson]
- $\mathcal{N} = 2$ [Hellerman, Maeda] [Hellerman, Maeda, DO, Reffert, Watanabe] [Bourget, Rodriguez-Gomez, Russo] [Grassi, Komargodski, Tizzano] [Cremonesi, Lanza, Martucci]
- bootstrap [Jafferis, Zhiboedov]
- resurgence [Dondi, Kalogerakis, DO, Reffert] [Antipin, Bersini, Sannino, Torres] [Watanabe]

THESE LECTURES

The EFT for the $O(n)$ model in $2 + 1$ dimensions

THESE LECTURES

The EFT for the O(n) model in 2 + 1 dimensions

Justify and prove all my claims from first principles in the large-N model

THESE LECTURES

The EFT for the $O(n)$ model in $2 + 1$ dimensions

Justify and prove all my claims from first principles in the large-N model

Use resurgence for the large-charge EFT

PARENTAL

ADVISORY

EXPLICIT CONTENT