Note: Stark Localization

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Consider the following one-dimensional lattice model in an electric field F:

$$H = \sum_{j \in \mathbb{Z}} \left[t c_{j+1}^{\dagger} c_j + t c_j^{\dagger} c_{j+1} - F j c_j^{\dagger} c_j \right]. \tag{1}$$

The bulk part of the eigenvalue equation, with eigenvalue E, is

$$t\phi_{j+1} + t\phi_{j-1} - (Fj + E)\phi_j = 0. \tag{2}$$

Recall the recurrence relation for cylindrical functions:

$$Z_{\nu-1}(z) + Z_{\nu+1}(z) - 2\nu z^{-1} Z_\nu(z) = 0. \tag{3}$$

If we rewrite the bulk equation as

$$\phi_{j+1} + \phi_{j-1} - 2(j + E/F)(2t/F)^{-1}\phi_j = 0, \tag{4}$$

then, with $\nu = j + E/F$ and $j \in \mathbb{Z}$, the cylindrical function

$$\phi_j^{(E)} := Z_{j+E/F}(2t/F) \tag{5}$$

is a solution.

The allowed values of E, and which cylindrical function should be selected, depend on the boundary condition, normalizability, and related conditions. Reference [1] states that the wave function has “the oscillating form only in a certain interval of ℓ, and damps of exponentially both outside of it for suitably selected values of ϵ.”

References

[1] S. Katsura, T. Hatta, and A. Morita, “On the Conception of the Energy Band in the Perturbed Periodic Potential,” Progress of Theoretical Physics **5**, 330–331 (1950).