

# Coset Decomposition of a Finite Group

Ken Shiozaki

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Let  $H$  and  $G$  be finite groups. The statement that  $H$  is a subgroup of  $G$  is equivalent, as a statement about sets, to  $H \subset G$ . We want to compute the left coset decomposition

$$G = \prod_{a=1}^{|G/H|} h_a H. \quad (1)$$

Here

$$h_a H = \{h_a h \mid h \in H\}. \quad (2)$$

Since the above decomposition is also a decomposition as sets, the left coset decomposition can be computed as follows.

- Check that  $H \subset G$ .
- Set  $G_1 = G$ .
- Repeat the following  $|G/H|$  times: choose one element  $h_i \in G_i$ . Compute the set  $G_{i+1} = G_i \setminus h_i H$ .
- Check that  $G_{|G/H|+1}$  is the empty set.

Notice that one can choose  $h_1 = e$ . The right coset decomposition

$$G = \prod_{a=1}^{|G/H|} H h_a \quad (3)$$

is computed in the same way.