Quantum Many-body Dynamics in low-energy heavy-ion reactions

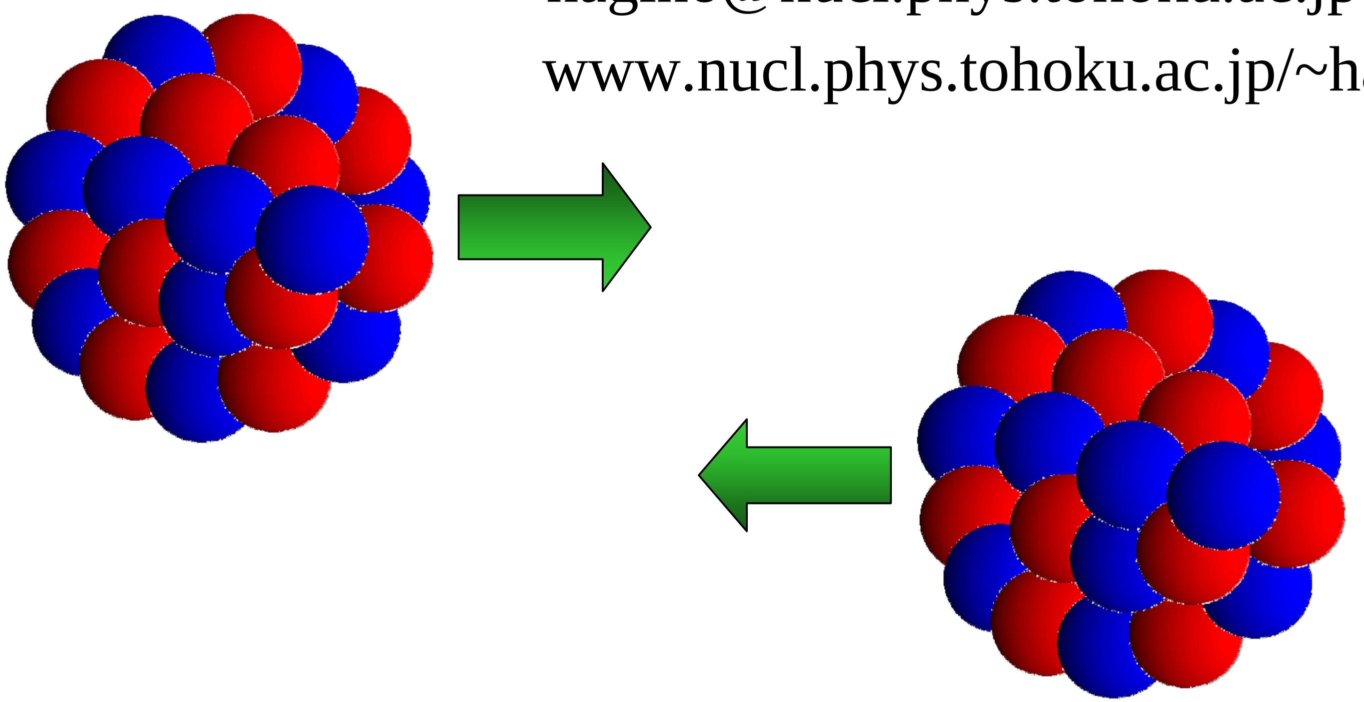
Kouichi Hagino

Tohoku University, Sendai, Japan

TOHOKU UNIVERSITY

hagino@nucl.phys.tohoku.ac.jp

www.nucl.phys.tohoku.ac.jp/~hagino



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References

Nuclear Reaction in general

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- R.A. Broglia and A. Winther, "*Heavy-Ion Reactions*"
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- P. Frobrich and R. Lipperheide, "*Theory of Nuclear Reactions*"

Heavy-ion Fusion Reactions

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- A.B. Balantekin and N. Takigawa, Rev. Mod. Phys. 70('98) 77
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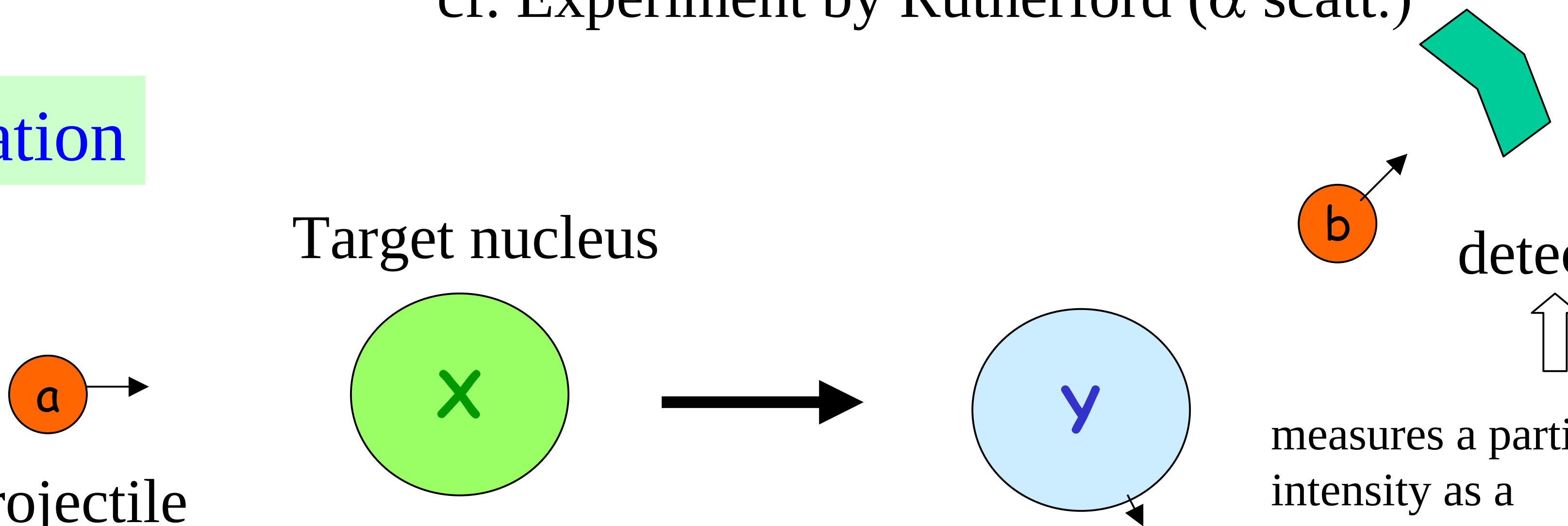
Basic of nuclear reactions

Shape, interaction, and excitation structures of nuclei ⟸ scattering expt.

cf. Experiment by Rutherford (α scatt.)

Notation



Projectile (beam)

X(a,b)Y

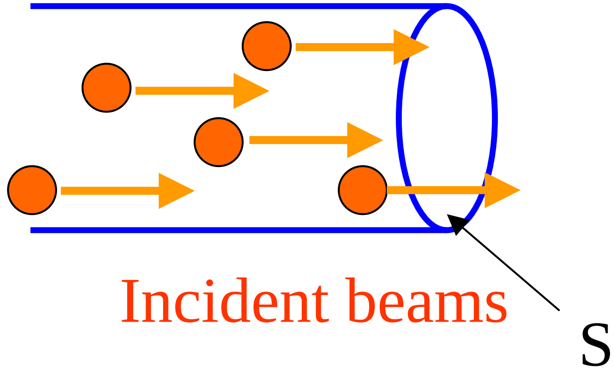
measures a particle intensity as a function of scattering angles

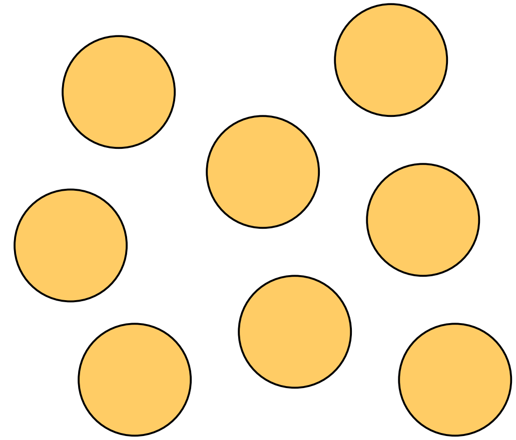
^{208}Pb(^{16}O,^{16}O)^{208}Pb : ^{16}O+^{208}Pb elastic scattering

^{208}Pb(^{16}O,^{16}O)^{208}Pb* : ^{16}O+^{208}Pb inelastic scattering

^{208}Pb(^{17}O,^{16}O)^{209}Pb : 1 neutron transfer reaction

Scattering cross sections



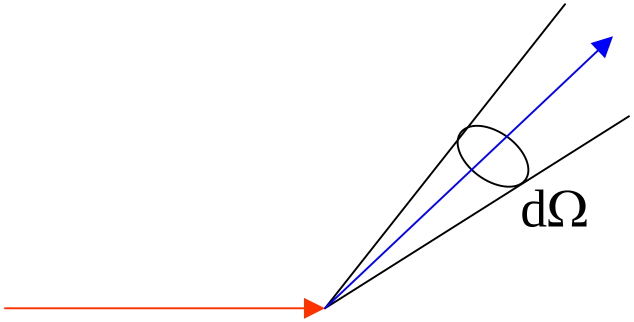


The number of reaction per unit time and per one target nucleus = σ X the number of projectile nucleus passed through a unit area per unit time

σ/S = probability for scattering to occur when a projectile nucleus in the beam collides with a target nucleus

Units: 1 barn = 10^{-24} cm^2 = 100 fm^2 (1 mb = 10^{-3} b = 0.1 fm^2)

Differential scattering cross sections:



$$\frac{d\sigma}{d\Omega}$$

Scattering Amplitude

Motion of Free particle: $$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi = \frac{\boldsymbol{k}^2\hbar^2}{2m}\psi$$

$$\psi(\boldsymbol{r}) = e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = \sum_{l=0}^{\infty}(2l+1)\, i^l j_l(kr) P_l(\cos\theta)$$

$$\rightarrow \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)i^l\left[e^{-i(kr-l\pi/2)} - e^{i(kr-l\pi/2)}\right]P_l(\cos\theta)$$

In the presence of a potential: $$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{r}) - E\right]\psi = 0$$

Asymptotic form of wave function

$$\psi(\boldsymbol{r}) \rightarrow \frac{i}{2kr}\sum_{l=0}^{\infty}(2l+1)i^l\left[e^{-i(kr-l\pi/2)} - \underline{\underline{S_l}}\, e^{i(kr-l\pi/2)}\right]P_l(\cos\theta)$$

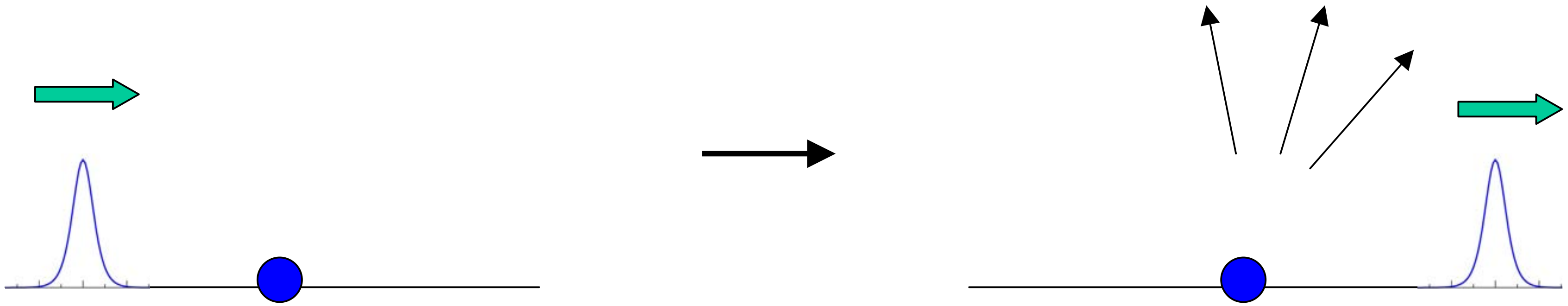
$$= e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \underbrace{\left[\sum_l (2l+1)\frac{S_l - 1}{2ik}P_l(\cos\theta)\right]}_{f(\theta)}\frac{e^{ikr}}{r}$$

$f(\theta)$ (scattering amplitude)

$$\psi(\boldsymbol{r}) \rightarrow e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + \left[\sum_l (2l+1)\,\frac{S_l - 1}{2ik} P_l(\cos\theta)\right]\frac{e^{ikr}}{r}$$

$$= e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + f(\theta)\frac{e^{ikr}}{r}$$

=(incident wave)
+ (scattering wave)

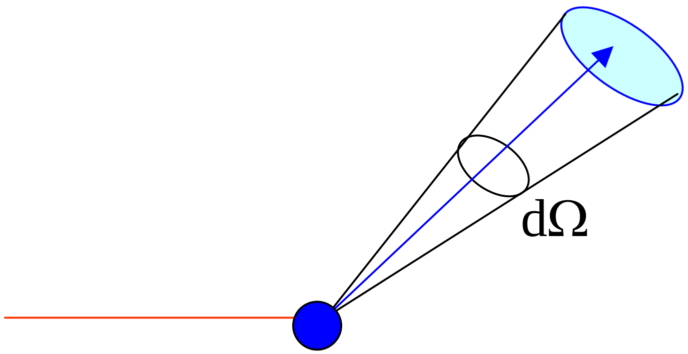


If only elastic scattering: $|S_l| = 1$ (flux conservation)

$$S_l = e^{2i\delta_l}$$

δ_l : phase shift

Differential cross section



$$\psi_{sc}(\boldsymbol{r}) \sim f(\theta)\frac{e^{ikr}}{r}$$

The number of scattered particle through the solid angle of dΩ per unit time:

$$N_{\mathrm{scatt}} = \boldsymbol{j}_{sc} \cdot \boldsymbol{e}_r \, r^2 d\Omega$$

$$\boldsymbol{j}_{sc} = \frac{\hbar}{2im}\left[\psi_{sc}^* \boldsymbol{\nabla}\psi_{sc} - c.c.\right] \sim \frac{k\hbar}{m}\frac{|f(\theta)|^2}{r^2}\boldsymbol{e}_r$$

(flux for the scatt. wave)

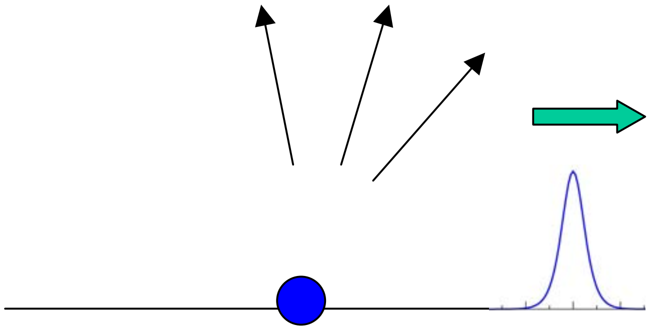
$$\boxed{\frac{d\sigma}{d\Omega} = |f(\theta)|^2}$$

$$f(\theta) = \sum_l (2l+1)\frac{S_l - 1}{2ik}P_l(\cos\theta)$$

Optical potential and Absorption cross section

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux (absorption)

Optical potential

$$\boxed{V_{\mathrm{opt}}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)}$$

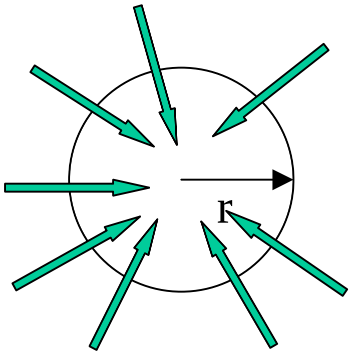
$$\Longrightarrow \quad \boldsymbol{\nabla}\cdot\boldsymbol{j} = \cdots = -\frac{2}{\hbar}W|\psi|^2$$

(note) Gauss's law

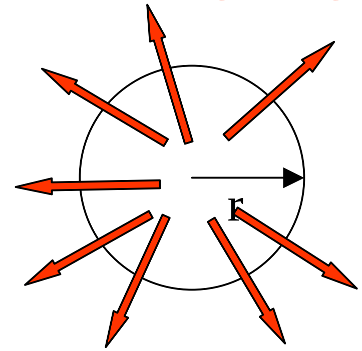
$$\int_S \boldsymbol{j}\cdot\boldsymbol{n}\,dS = \int_V \boldsymbol{\nabla}\cdot\boldsymbol{j}\,dV$$

$$\psi(\boldsymbol{r}) \to \frac{i}{2k}\sum_l (2l+1)\, i^l \,\frac{1}{r}\left[\underbrace{e^{-i(kr-l\pi/2)}}_{\psi_{\text{in}}} - \underbrace{S_l e^{i(kr-l\pi/2)}}_{\psi_{\text{out}}}\right] P_l(\cos\theta)$$

Total incoming flux



Total outgoing flux



$$j_{\text{in}}^{\text{net}} = \frac{k\hbar}{m}\cdot\frac{\pi}{k^2}\sum_l (2l+1)$$

$$j_{\text{out}}^{\text{net}} = \frac{k\hbar}{m}\cdot\frac{\pi}{k^2}\sum_l (2l+1)|S_l|^2$$

Net flux loss: $$j_{\text{in}}^{\text{net}} - j_{\text{out}}^{\text{net}} = \frac{k\hbar}{m}\cdot\frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)$$

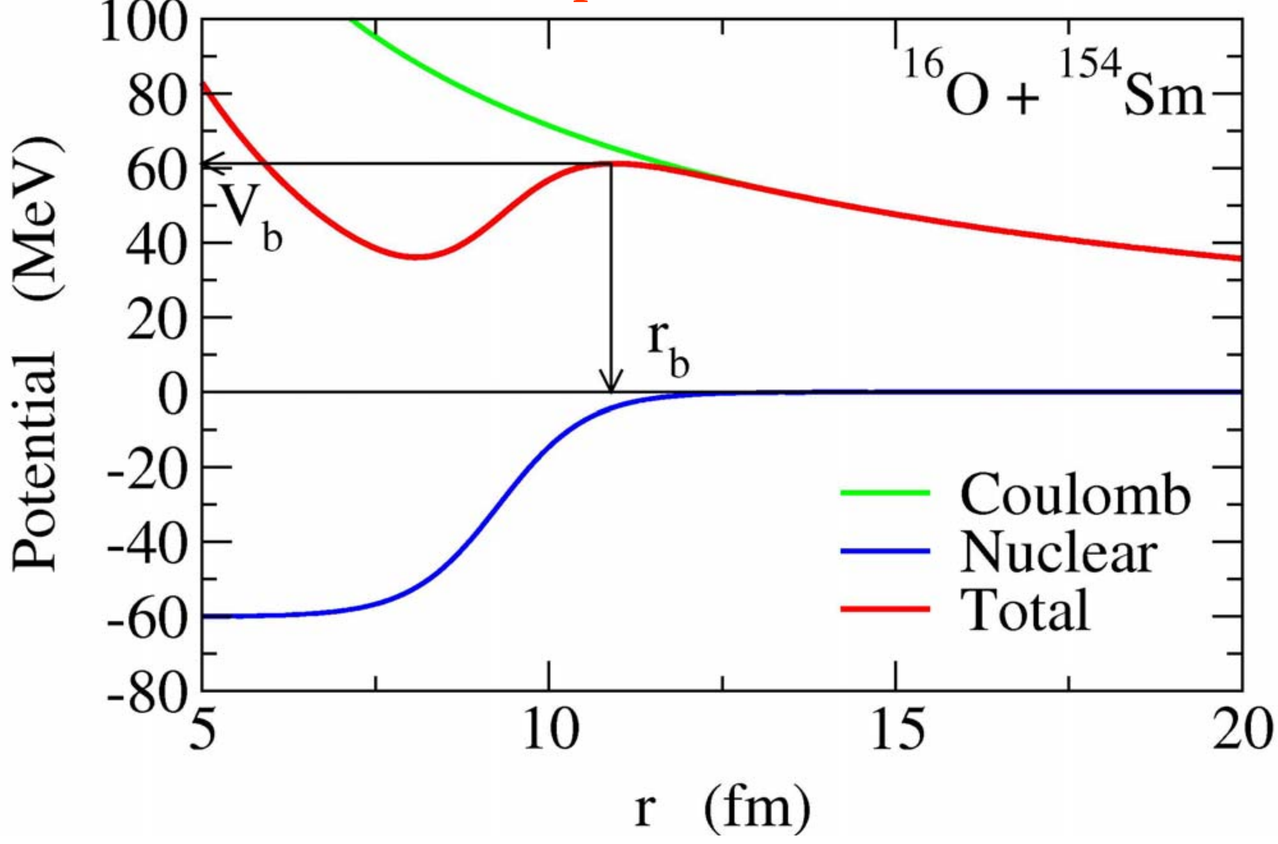
Absorption cross section:

$$\boxed{\sigma_{\text{abs}} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2)}$$

Overview of heavy-ion reactions

Heavy-ion: Nuclei heavier than ^{4}He

Inter-nucleus potential

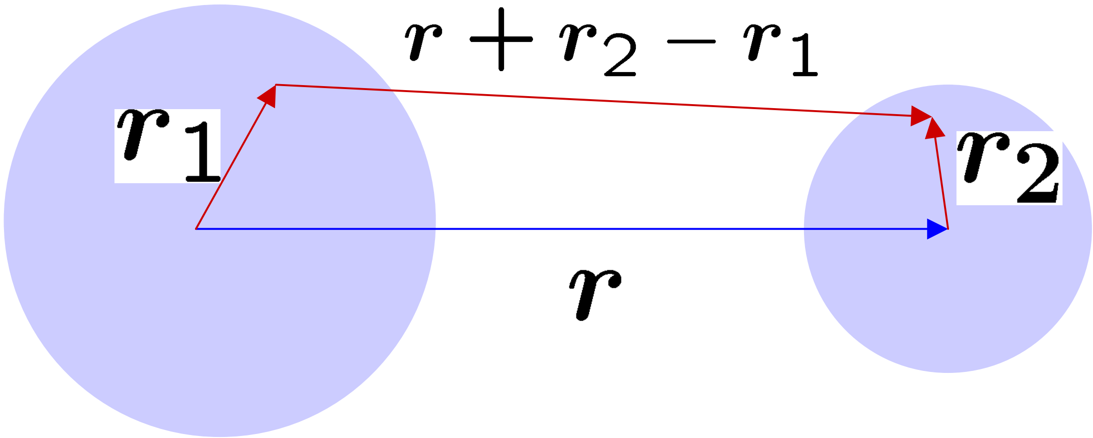


Two forces:

1. Coulomb force
   Long range, repulsive
2. Nuclear force
   Short range, attractive

Potential barrier due to the compensation between the two (Coulomb barrier)

- Double Folding Potential



$$\boxed{\begin{aligned} V_{DF}(\boldsymbol{r}) &= \int d\boldsymbol{r}_1 d\boldsymbol{r}_2\, \rho_1(\boldsymbol{r}_1)\rho_2(\boldsymbol{r}_2) \\ &\quad \times v(\boldsymbol{r}+\boldsymbol{r}_2-\boldsymbol{r}_1) \end{aligned}}$$

v : nucleon-nucleon interaction

- Phenomenological potential

$$\boxed{V_{WS}(\boldsymbol{r}) = -\frac{V_0}{1+\exp[(r-R_0)/a]}}$$

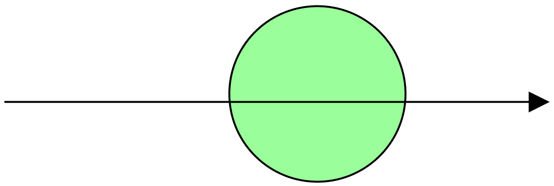
Three important features of heavy-ion reactions

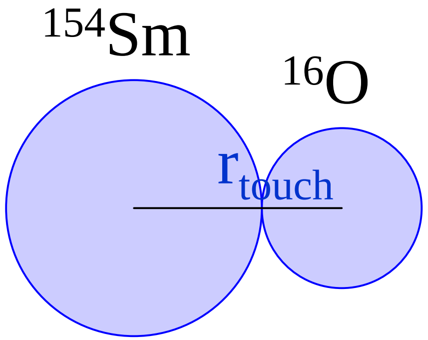
1. Coulomb interaction: important
2. Reduced mass: large → (semi-) classical picture

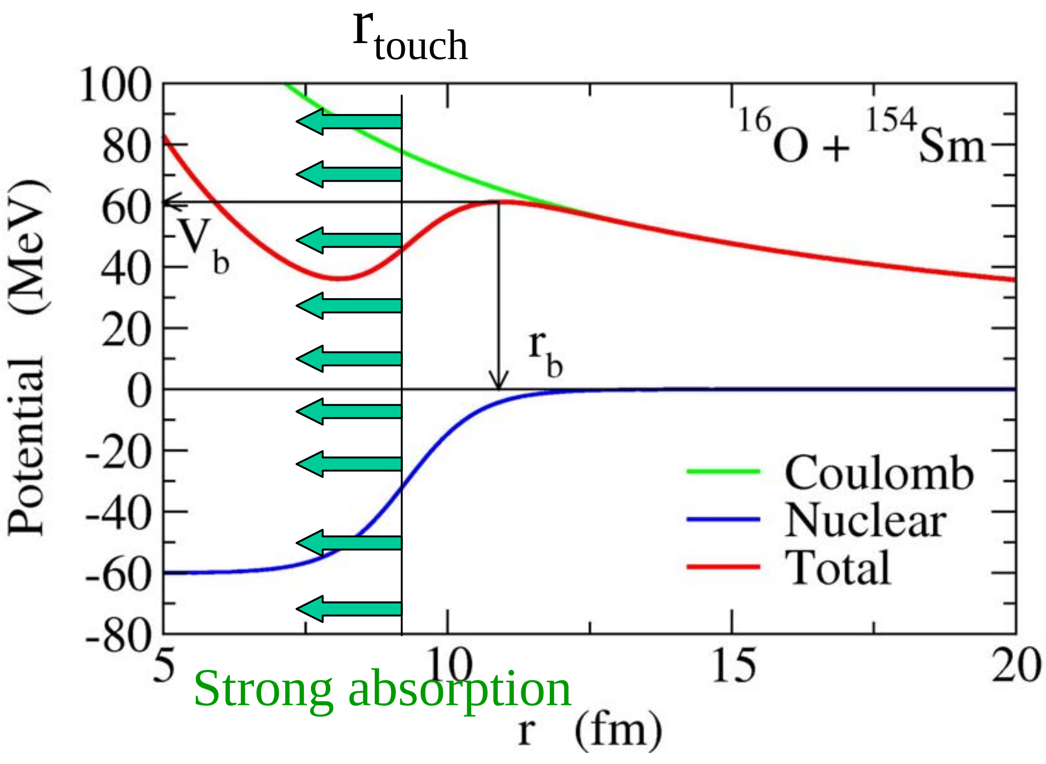
$$\mu = m_T m_P/(m_T + m_P)$$

concept of trajectory

3. Strong absorption inside the Coul. barrier



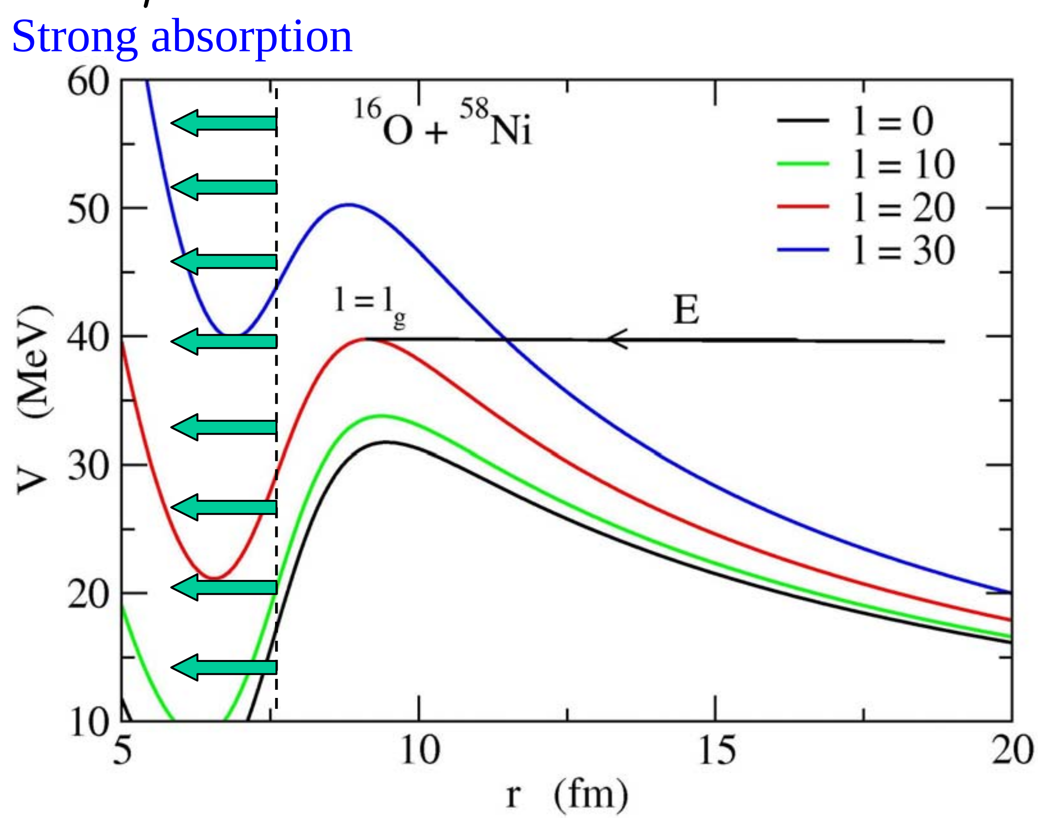




Automatic Compound nucleus formation once touched (assumption of strong absorption)

Grazing angular momentum

$$V_l(r) = V_0(r) + \frac{l(l+1)\hbar^2}{2\mu r^2}$$



Grazing angular momentum: $V_{l_g}(r_b) = E$

$l < l_g$: can access to the strong absorption region *classically*

$l \geq l_g$: the strong absorption region is *classically forbidden*

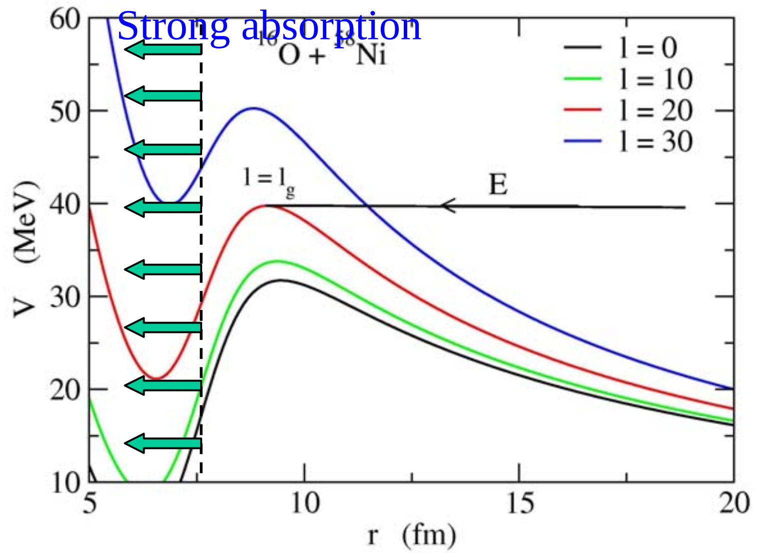
Reaction is drastically changed at $l=l_g$ for a given E

i) $l \gg l_g$ or $E \ll V_b$

Classical turning point: very far away

- Elastic scatt. due to Coul. force
  (Rutherford scatt.)
- Excitation due to the Coul. field
  (Coulomb excitation)

Low-lying collective motion



$l < l_g$: can access to the strong absorption

$l \geq l_g$: cannot access cassically

ii) $l \gtrsim l_g$

Nuclear effect: important

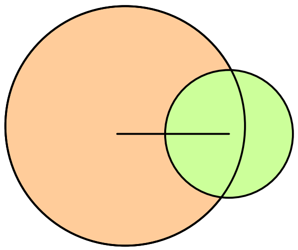
direct reaction
- Elastic scatt.
- Inelastic scatt.
- Transfer reaction

Quasi-elastic scattering

Dynamics as a many-body system

iii) $l < l_g$

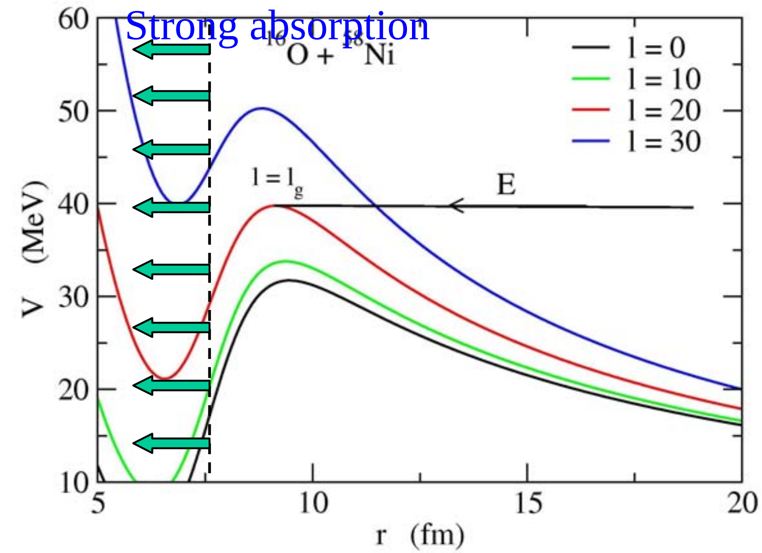
Access to the region of large overlap



- High level density (CN)
- Huge number of d.o.f.

Relative energy is quickly lost
and converted to internal energy

Formation of hot CN (fusion reaction)



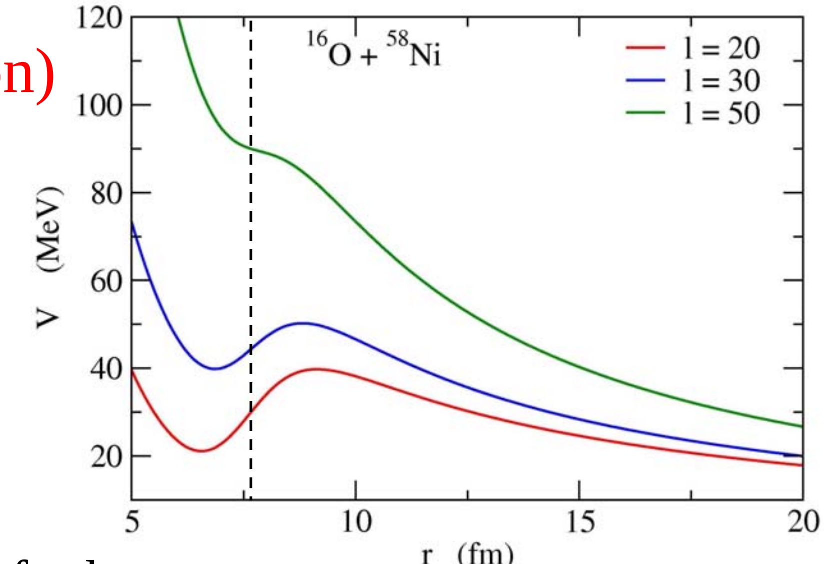
$l < l_g$: can access to the strong absorption

$l \geq l_g$: cannot access cassically

iv) In the case of $l_c < l_g$

Coul. Pocket: disappears at $l = l_g$

Reaction intermediate between
Direct reaction and fusion:
Deep Inelastic Collisions (DIC)



Scattering at relatively high energy a/o for heavy systems

Partial decomposition of reaction cross section

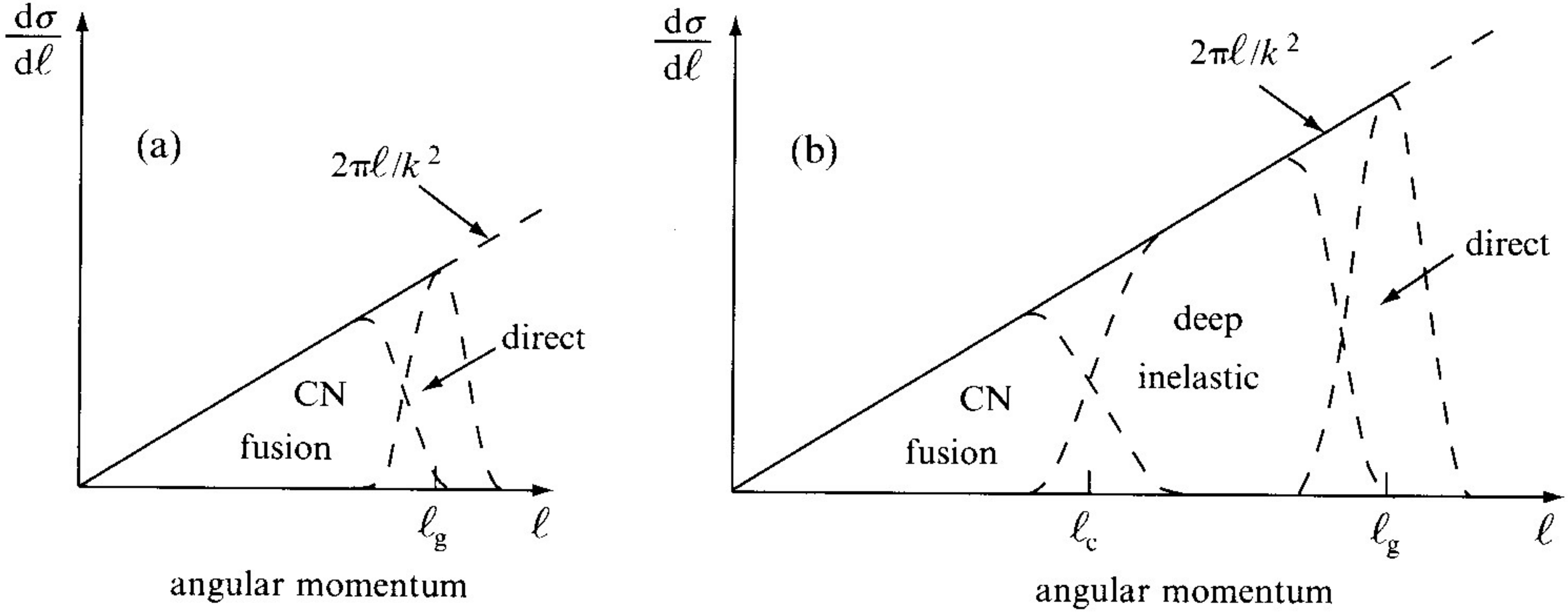
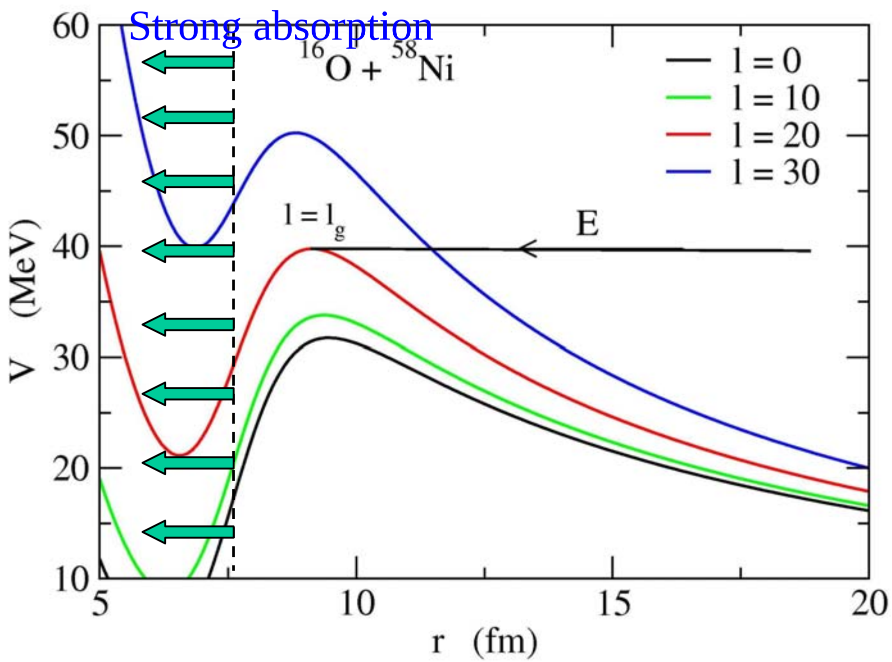
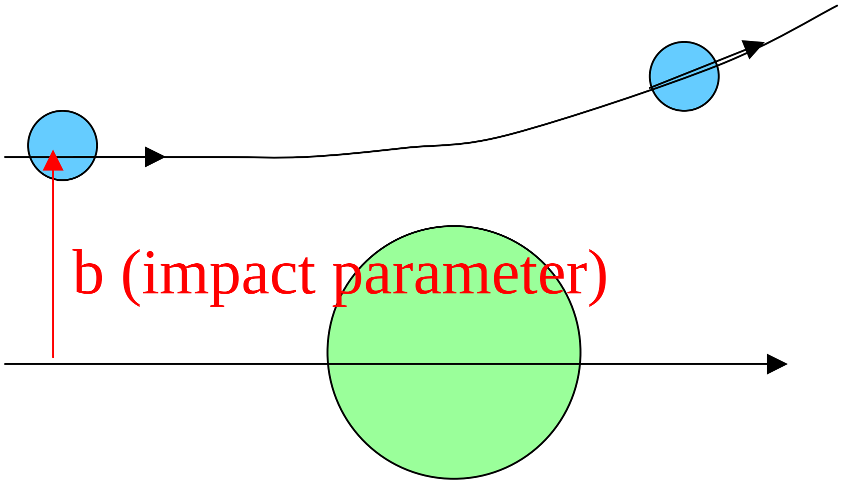


Figure 4.18 Schematic decomposition of the total heavy-ion reaction cross section into contributions from different partial waves when (a) the grazing angular momentum (quantum number ℓ_g) is below the critical angular momentum (quantum number ℓ_c) that can be carried by the compound nucleus, and (b) when ℓ_g exceeds ℓ_c. In both (a) and (b) the straight line is obtained from Equation (4.3) and the dashed areas indicate regions in which different types of heavy-ion nuclear reaction mechanisms predominate.

Taken from J.S. Lilley, "Nuclear Physics"

Classical Model for heavy-ion fusion reactions





$$l_{cl} = kb \qquad k = \sqrt{2\mu E/\hbar^2}$$

$l < l_g$: can access to the strong absorption region classically

$$\Rightarrow b_g = l_g/k$$

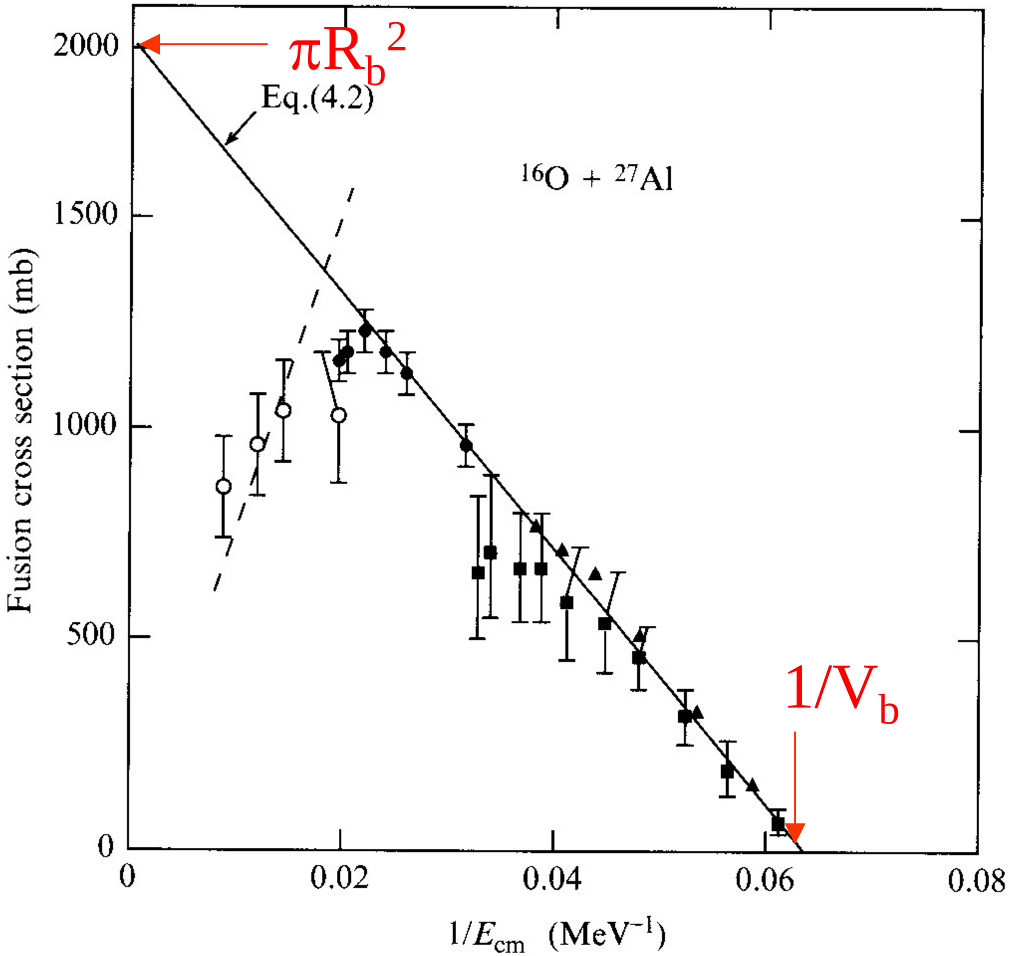
$$\boxed{\sigma^{cl} = 2\pi \int_0^{b_g} b\, db = \pi\, b_g^2}$$

$$V_b + \frac{(kb_g)^2\hbar^2}{2\mu R_b^2} = E$$

$$\boxed{\sigma^{cl}_{\mathrm{fus}}(E) = \pi R_b^2 \left(1 - \frac{V_b}{E}\right)}$$

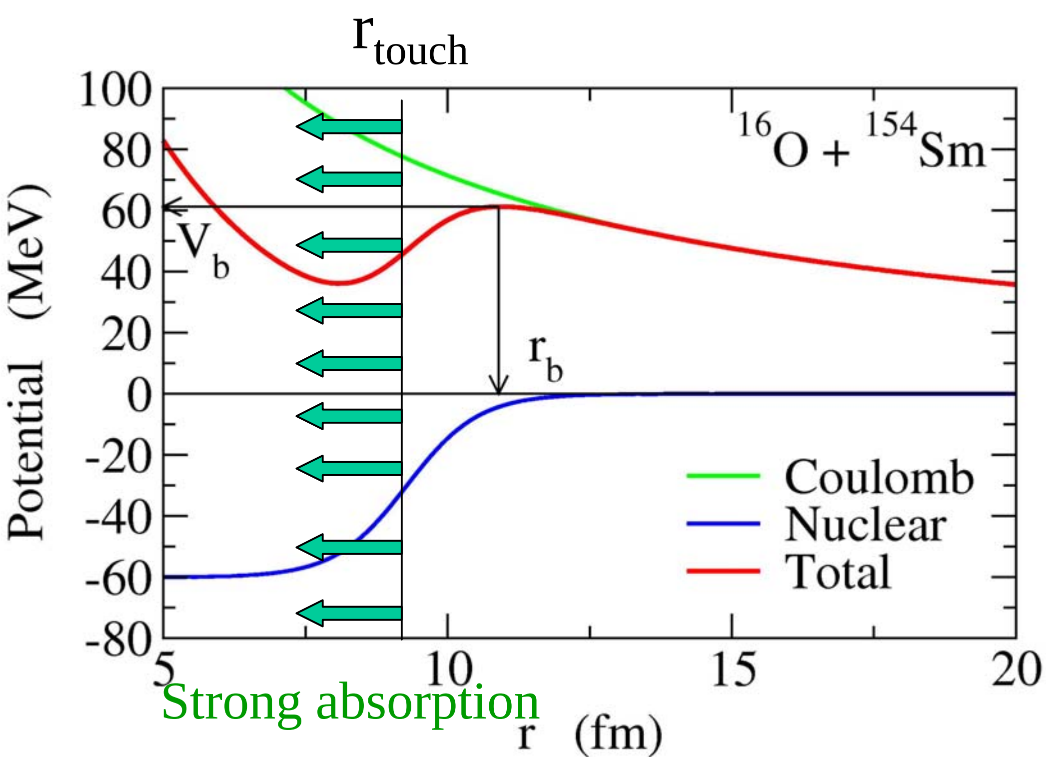
$$\sigma^{cl}_{\mathrm{fus}}(E) = \pi R_b^2\left(1 - \frac{V_b}{E}\right)$$

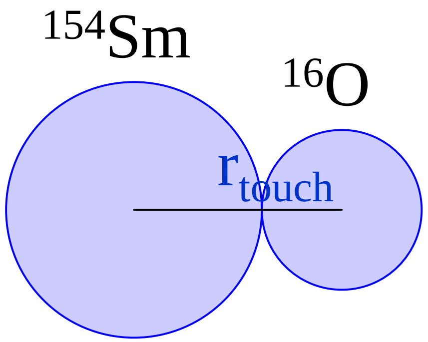
Classical fusion cross section is proportional to *1 / E*



Taken from J.S. Lilley, "Nuclear Physics"

Fusion reaction and Quantum Tunneling





Automatic CN formation once touched (assumption of strong absorption)

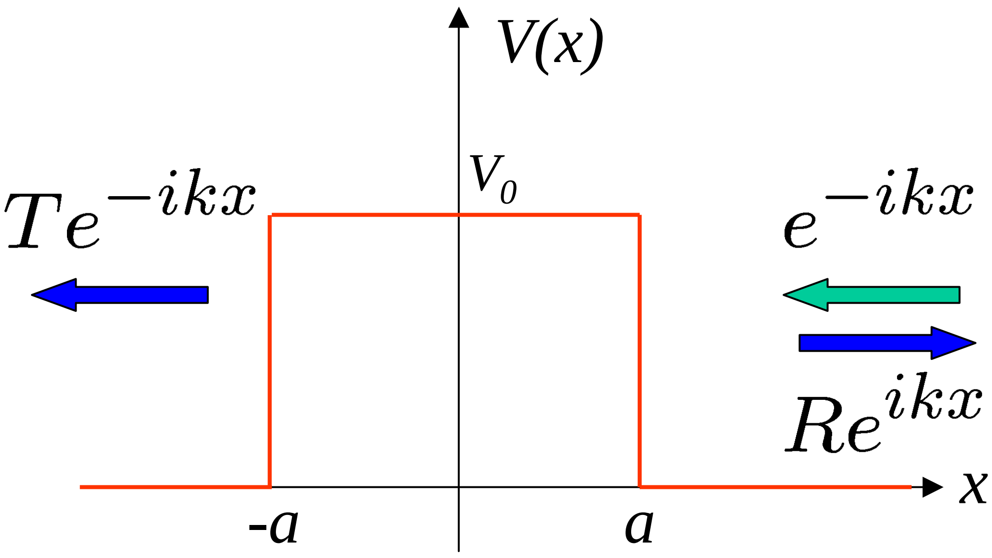
Probability of fusion
= prob. to access to r_{touch}

Penetrability of barrier

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

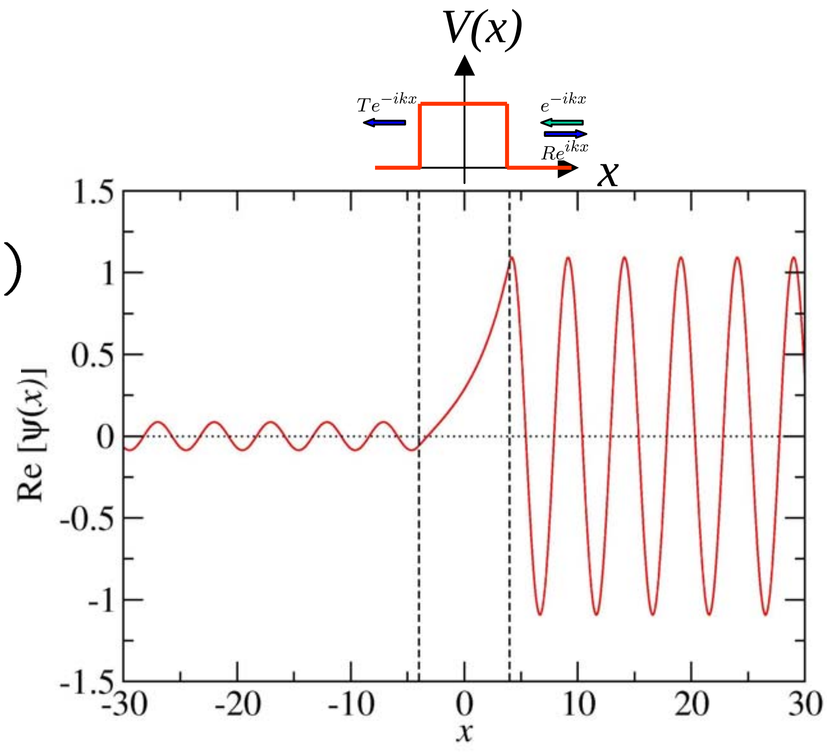
Fusion takes place by quantum tunneling at low energies!

Quantum Tunneling Phenomena



$$
\begin{aligned}
\psi(x) &= T\,e^{-ikx} && (x \le -a) \\
&= A\,e^{-\kappa x} + B\,e^{\kappa x} && (-a < x < a) \\
&= e^{-ikx} + R\,e^{ikx} && (x \ge a)
\end{aligned}
$$

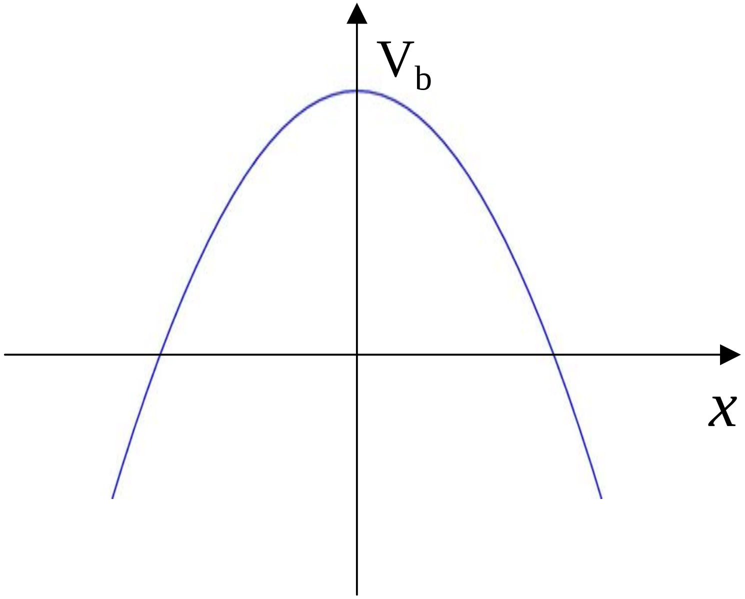
$$
\begin{aligned}
k &= \sqrt{2mE/\hbar^2} \\
\kappa &= \sqrt{2m(V_0 - E)/\hbar^2}
\end{aligned}
$$



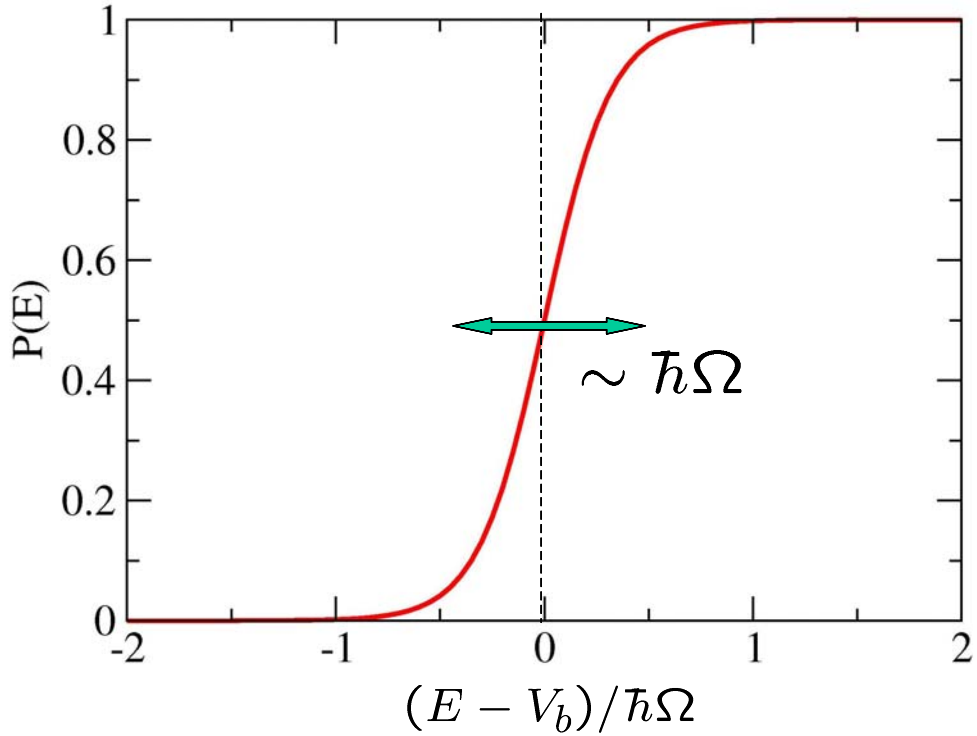
Tunnel probability: $P(E) = |T|^2$

For a parabolic barrier……

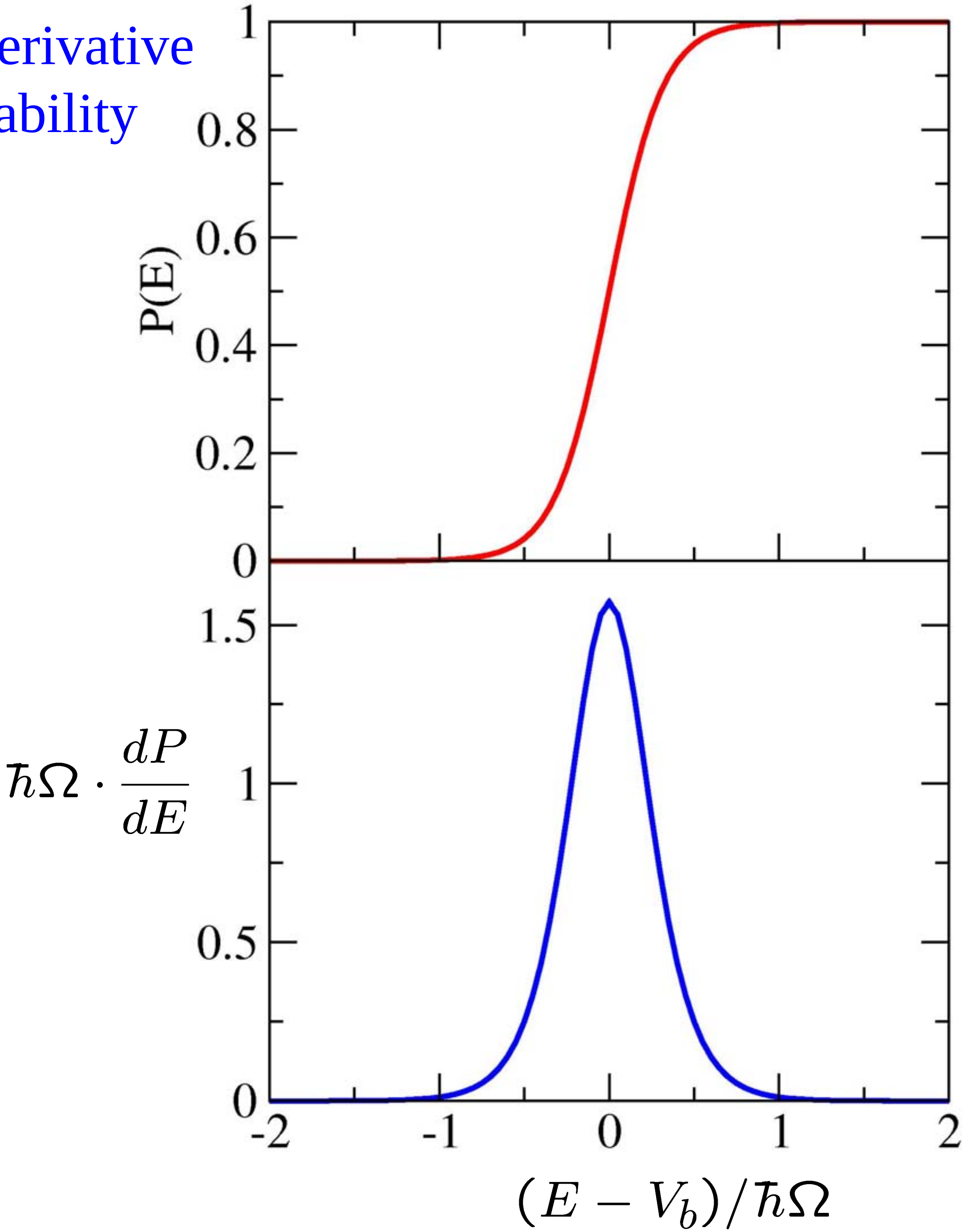
$$V(x) = V_b - \frac{1}{2} m \Omega^2 x^2$$



$$\boxed{P(E) = \frac{1}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]}}$$



Energy derivative of penetrability

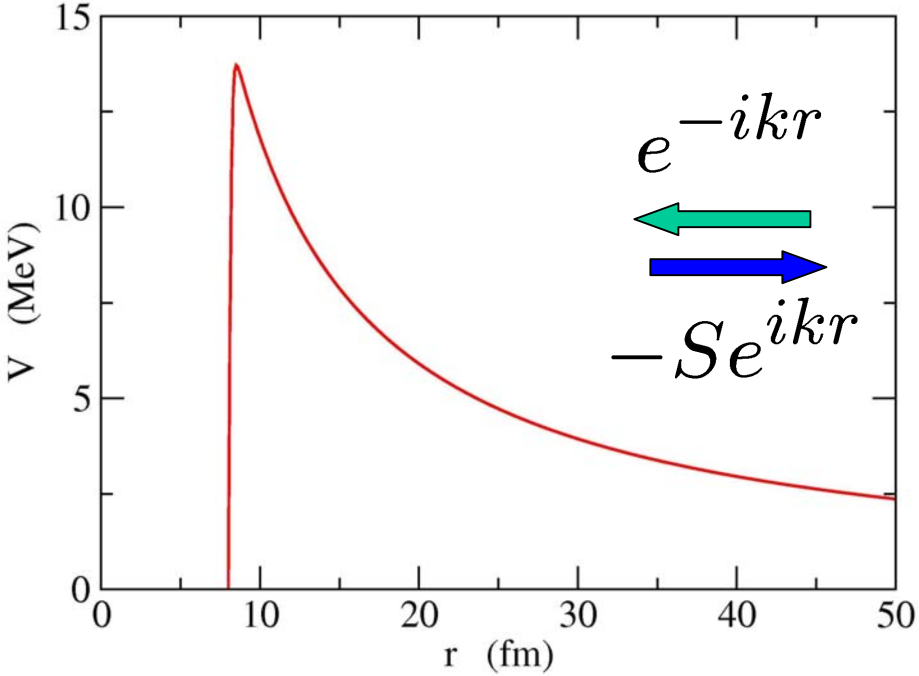


(note) Classical limit

$$P(E) = \theta(E - V_b)$$
$$dP/dE = \delta(E - V_b)$$

In the case of three-dimensional spherical potential:

$$\psi(\boldsymbol{r}) \to \frac{i}{2k}\sum_l (2l+1)\, i^l \,\frac{1}{r}\left[e^{-i(kr-l\pi/2)} - S_l e^{i(kr-l\pi/2)}\right] P_l(\cos\theta)$$



$-S_l \sim R$ (reflection coeff.) $\Longrightarrow$ $P = |T|^2 = 1 - |S_l|^2$

$$\boxed{\sigma_{\rm abs} = \frac{\pi}{k^2}\sum_l (2l+1)(1-|S_l|^2) = \frac{\pi}{k^2}\sum_l (2l+1)P_l}$$

Potential Model: its success and failure

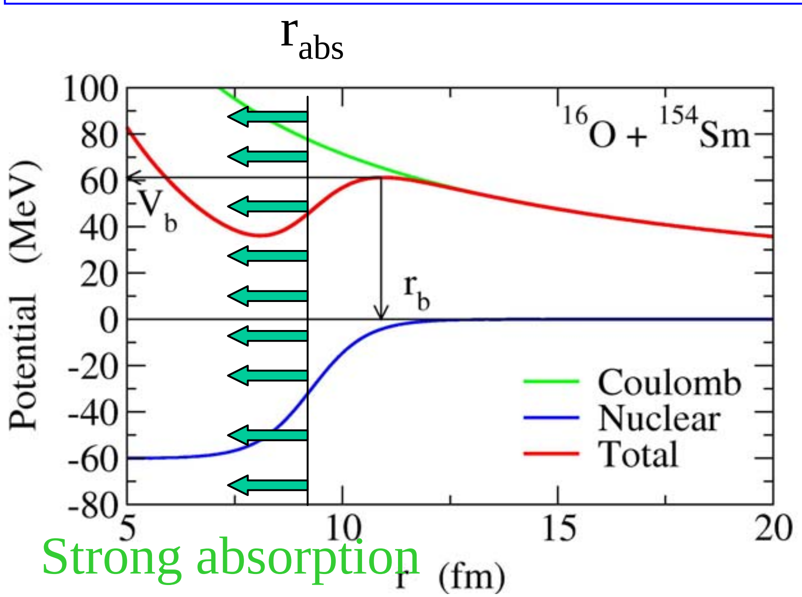
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+V(r)+\frac{l(l+1)^2}{2\mu r^2}-E\right]u_l(r)=0$$

Asymptotic boundary condition: $u_l(r) \to H_l^{(-)}(kr) - S_l\, H_l^{(+)}(kr)$

Fusion cross section: $$\sigma_{\text{fus}} = \frac{\pi}{k^2}\sum_l (2l+1)P_l$$

Mean angular mom. of CN: $$\langle l \rangle = \sum_l l(2l+1)P_l \Big/ \sum_l (2l+1)P_l$$

$$P_l = 1 - |S_l|^2$$



Strong absorption

Asymptotic boundary condition: $u_l(r) \to H_l^{(-)}(kr) - S_l\, H_l^{(+)}(kr)$

Fusion cross section: $\sigma_{\rm fus} = \frac{\pi}{k^2}\sum_l (2l+1)P_l$

$$P_l = 1 - |S_l|^2$$

Inner boundary condition:

(i) Method of Absorbing potential

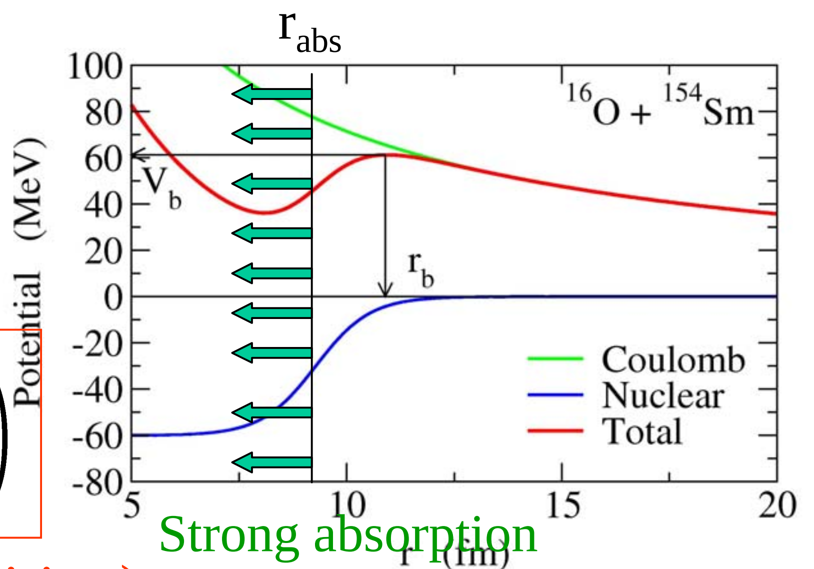
$$V(r) = V_R(r) - iW(r)$$

$$u_l(r) \sim r^{l+1}$$

(ii) IWBC

Large *W* limit

$$\boxed{u_l(r) = T_l \exp\left(-i\int_{r_{abs}}^{r} k_l(r')dr'\right)}$$



Strong absorption

(Incoming Wave Boundary Condition)

$$k_l(r) = \sqrt{2\mu/\hbar^2[E - V_R(r) - l(l+1)\hbar^2/2\mu r^2]}$$

- Requires only Real Potential (no need for imaginary part)

$$P_l = |T_l|^2$$

Wong's formula

C.Y. Wong, Phys. Rev. Lett. 31 ('73)766

$$\sigma_{\mathrm{fus}}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

i) Approximate the Coul. barrier by a parabola: $V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$

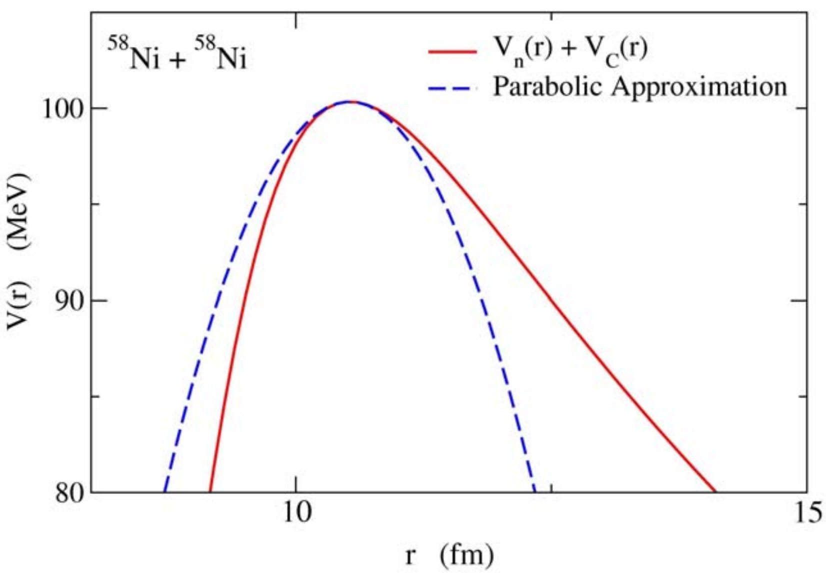
$$\Rightarrow P_0(E) = 1\Big/\left(1+\exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]\right)$$

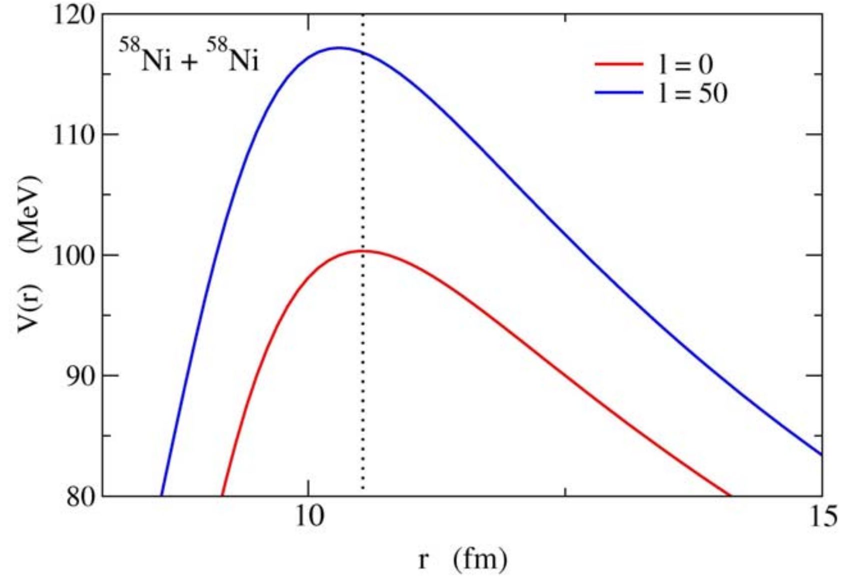
ii) Approximate P_l by P_0:

$$P_l(E) \sim P_0\left(E - \frac{l(l+1)\hbar^2}{2\mu R_b^2}\right)$$

(assume l-independent Rb and curvature)

iii) Replace the sum of l with an integral





$$\boxed{\sigma_{\text{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \log\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]}$$

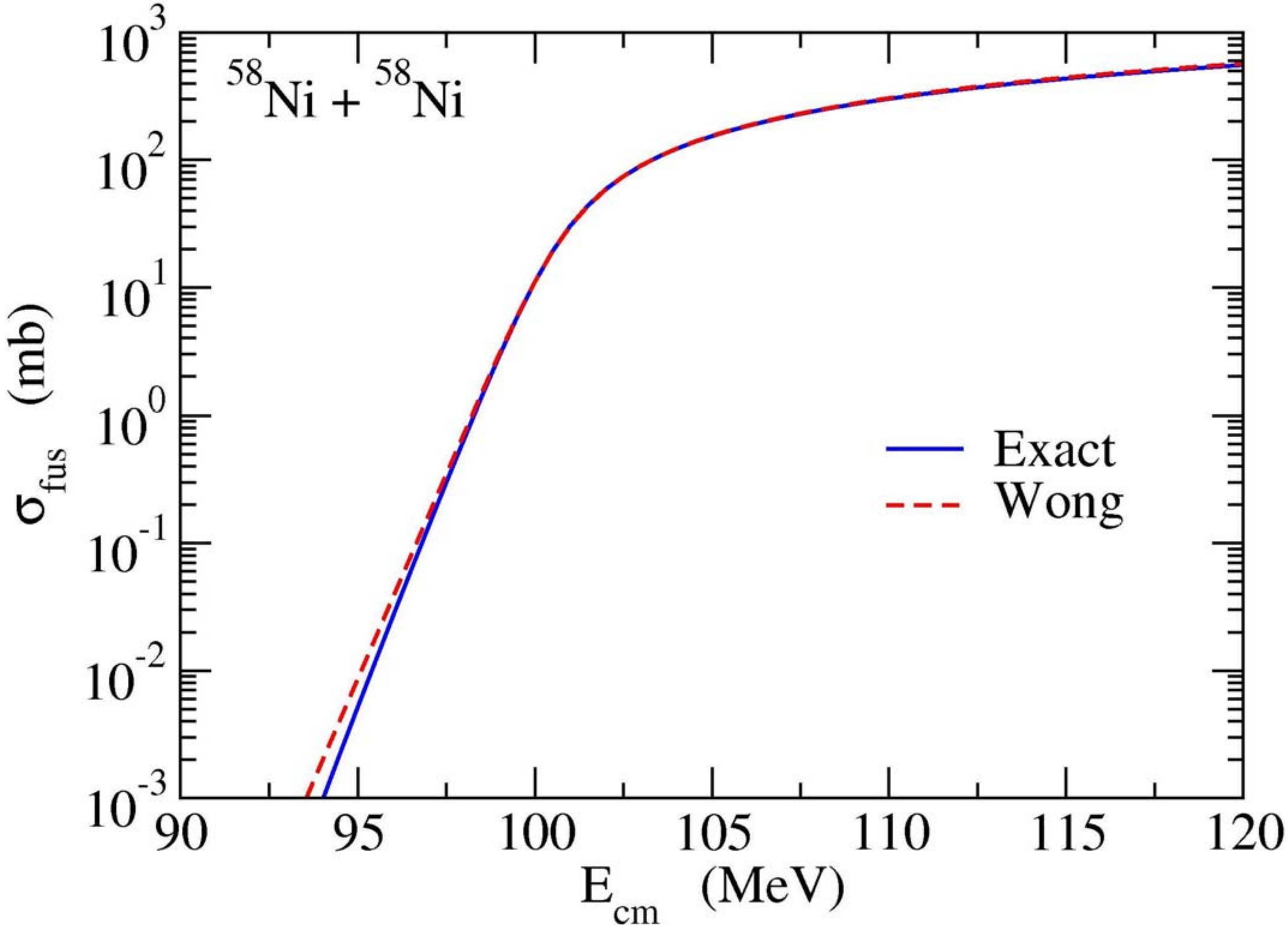
(note) For $E \gg V_b$ $\qquad 1 \ll \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)$

$$\Longrightarrow \sigma_{\text{fus}}(E) \sim \pi R_b^2 \left(1 - \frac{V_b}{E}\right) = \sigma_{\text{fus}}^{cl}(E)$$

(note)

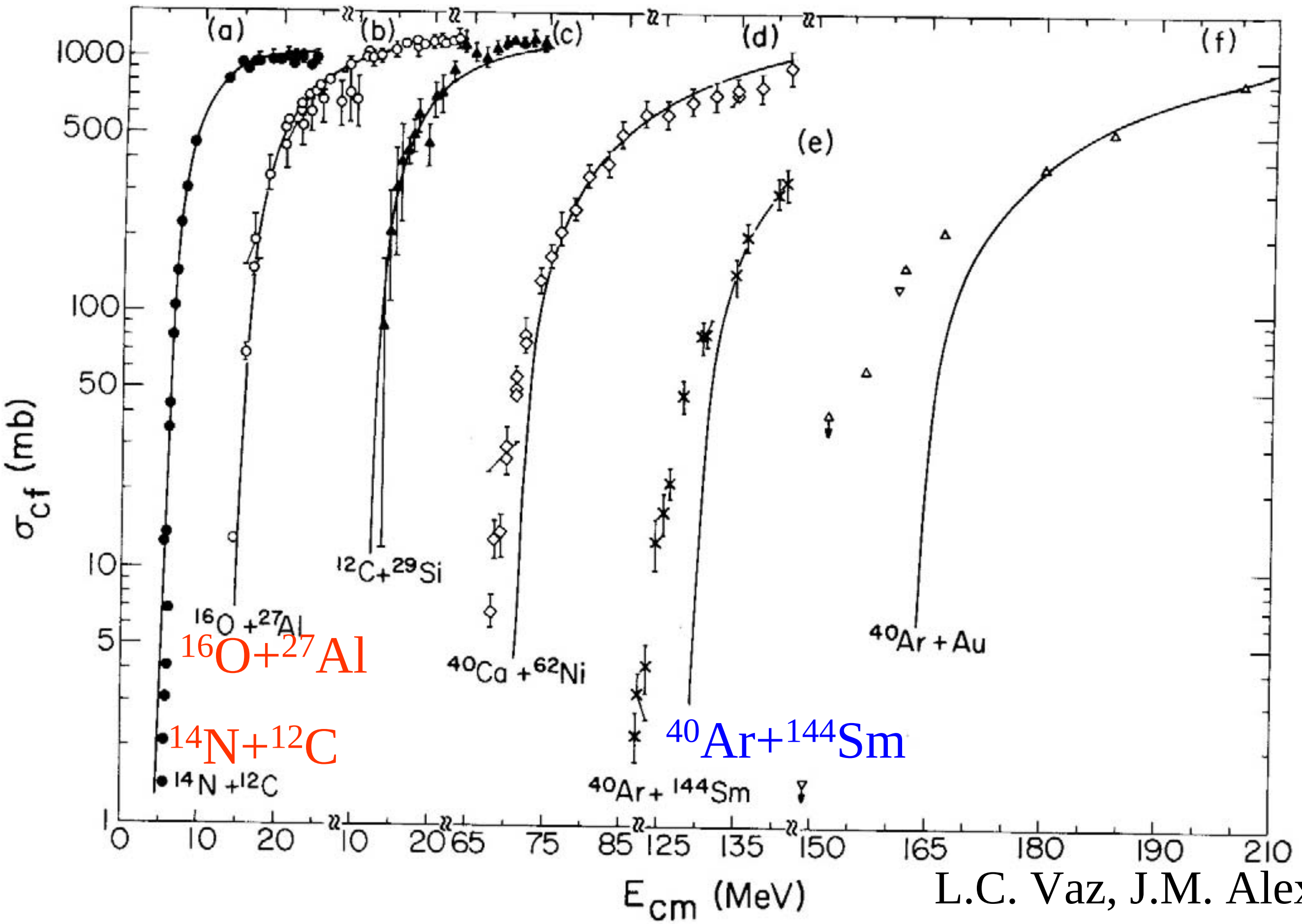
$$\boxed{\frac{d(E\sigma_{\text{fus}})}{dE} = \frac{\pi R_b^2}{1 + \exp\left[\frac{2\pi}{\hbar\Omega}(V_b - E)\right]} = \pi R_b^2 \cdot P_{l=0}(E)}$$

$$\sigma_{\mathrm{fus}}(E) = \frac{\hbar\Omega}{2E} R_b^2 \log\left[1 + \exp\left(\frac{2\pi}{\hbar\Omega}(E - V_b)\right)\right]$$



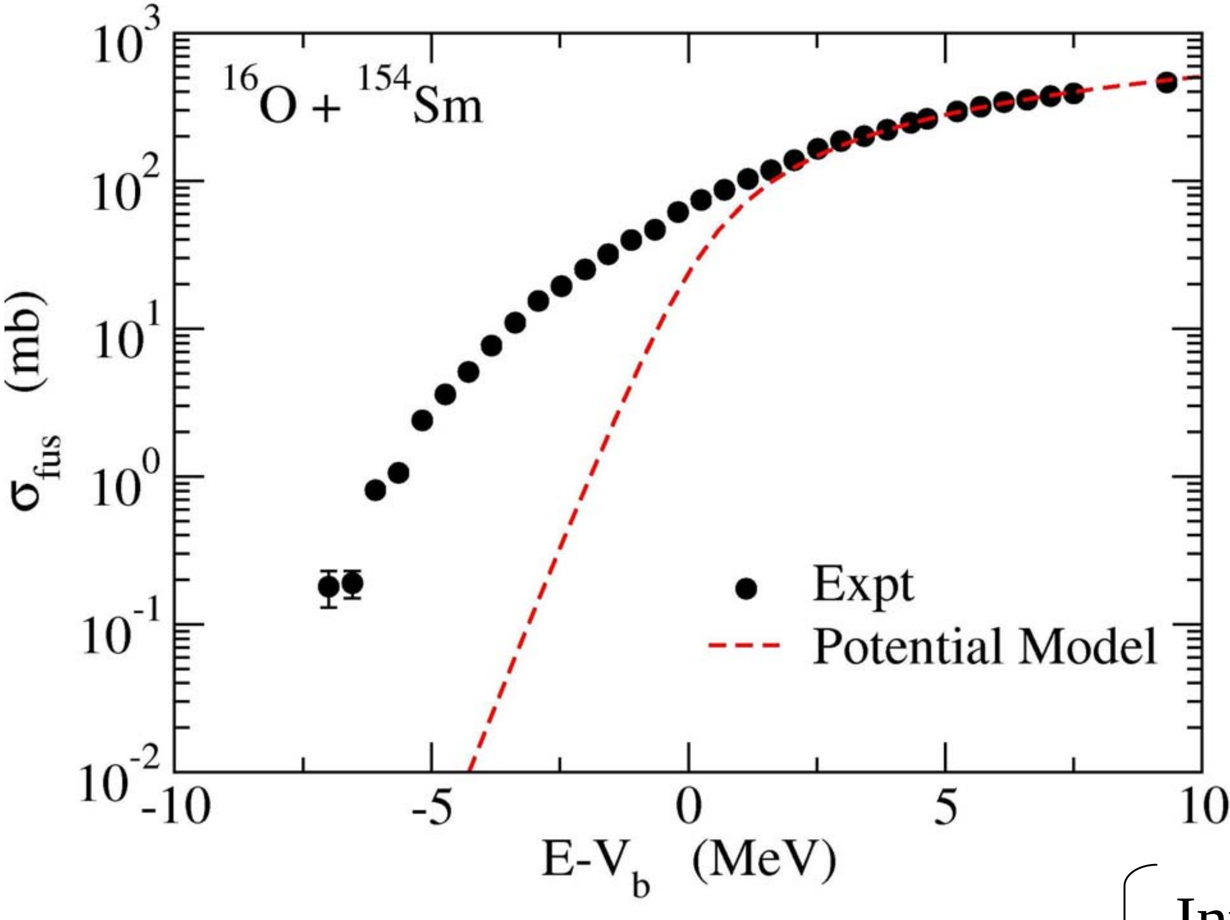
Comparison between prediction of pot. model with expt. data

Fusion cross sections calculated with a static energy independent potential



L.C. Vaz, J.M. Alexander, and G.R. Satchler, Phys. Rep. 69('81)373

- Works well for relatively light systems
- Underpredicts σ_{fus} for heavy systems at low energies



Potential model:
Reproduces the data reasonably well for
$E > V_b$

Underpredicts σ_{fus} for
$E < V_b$

What is the origin?

Inter-nuclear Potential is poorly parametrized?
Other origins?

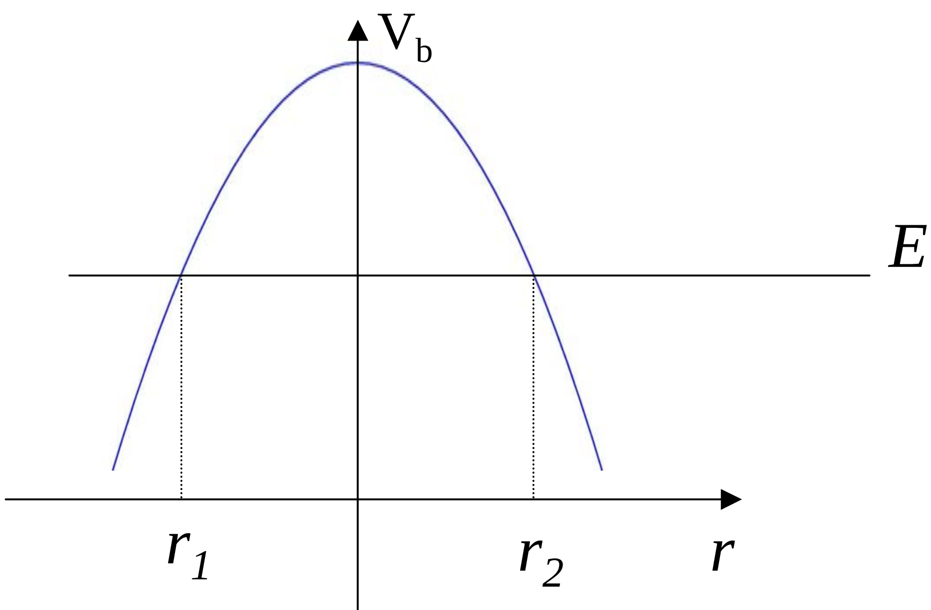
Potential Inversion

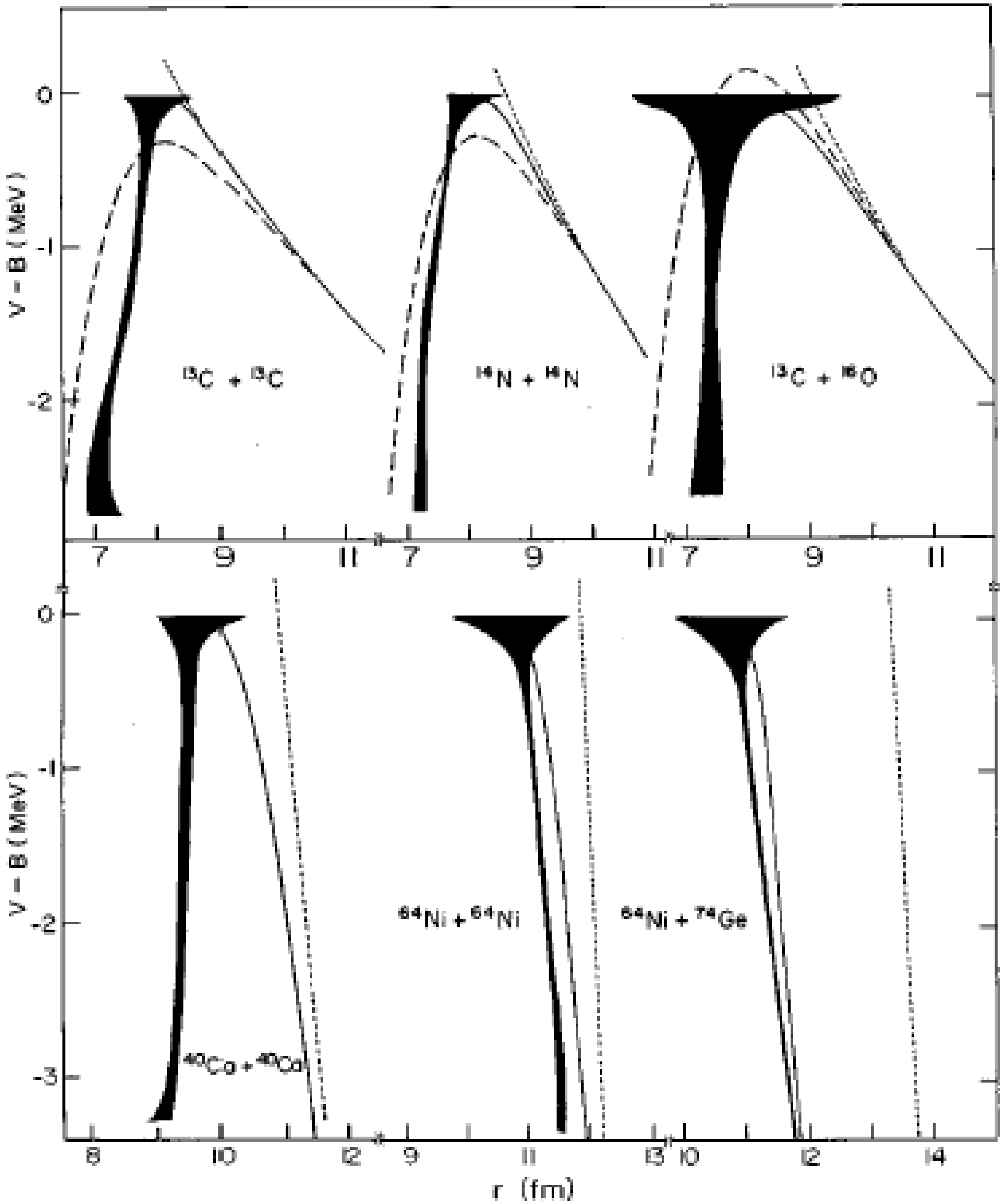
$$P_0(E) = \frac{1}{\pi R_b^2}\frac{d(E\sigma_{\text{fus}})}{dE}$$

(note)

$$P_0(E) = 1/[1+S_0(E)], \quad S_0(E) = \int_{r_1}^{r_2} dr\,\sqrt{\frac{2\mu}{\hbar^2}(V(r)-E)}$$

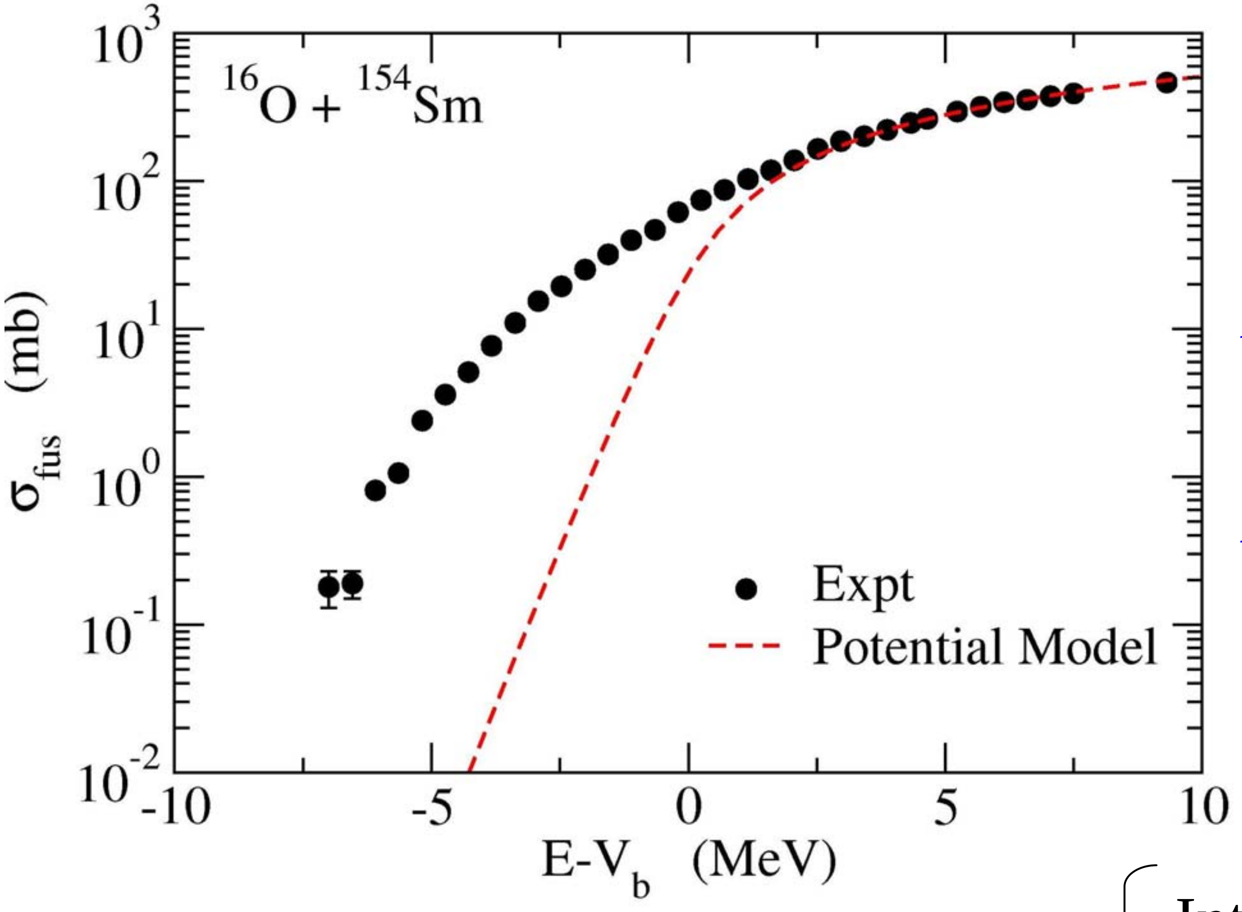
$$\boxed{t(E) \equiv r_2 - r_1 = -\frac{2}{\pi}\sqrt{\frac{\hbar^2}{2\mu}}\int_E^{V_b}\frac{\frac{dS_0(E')}{dE'}}{\sqrt{E'-E}}\,dE'}$$





A.B. Balantekin, S.E. Koonin, and J.W. Negele, PRC28('83)1565

Fusion cross sections calculated with a static energy independent potential



Potential model:

Reproduces the data reasonably well for $E > V_b$

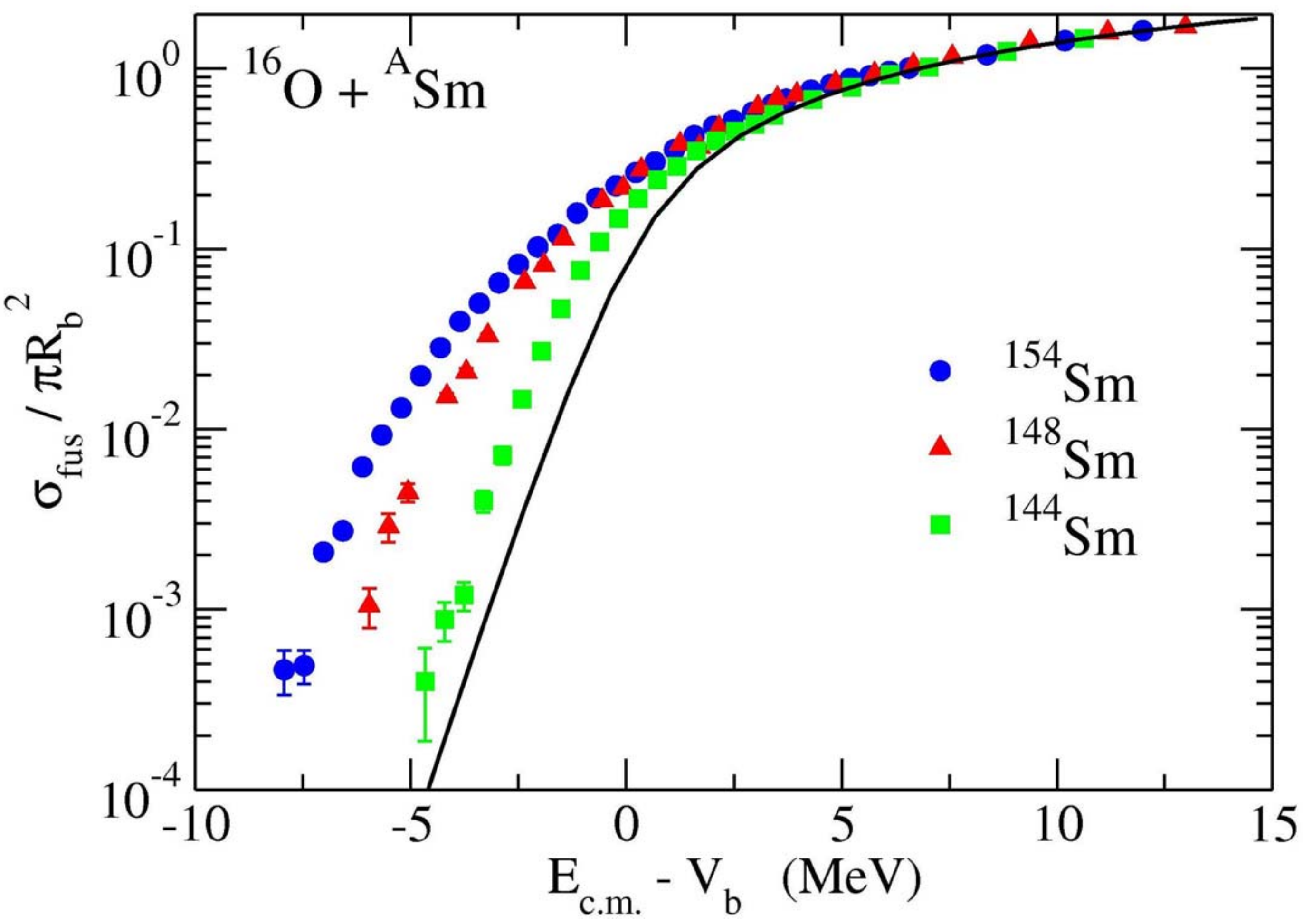
Underpredicts σ_{fus} for $E < V_b$

What is the origin?

~~Inter-nuclear Potential is poorly parametrized?~~

Other origins?

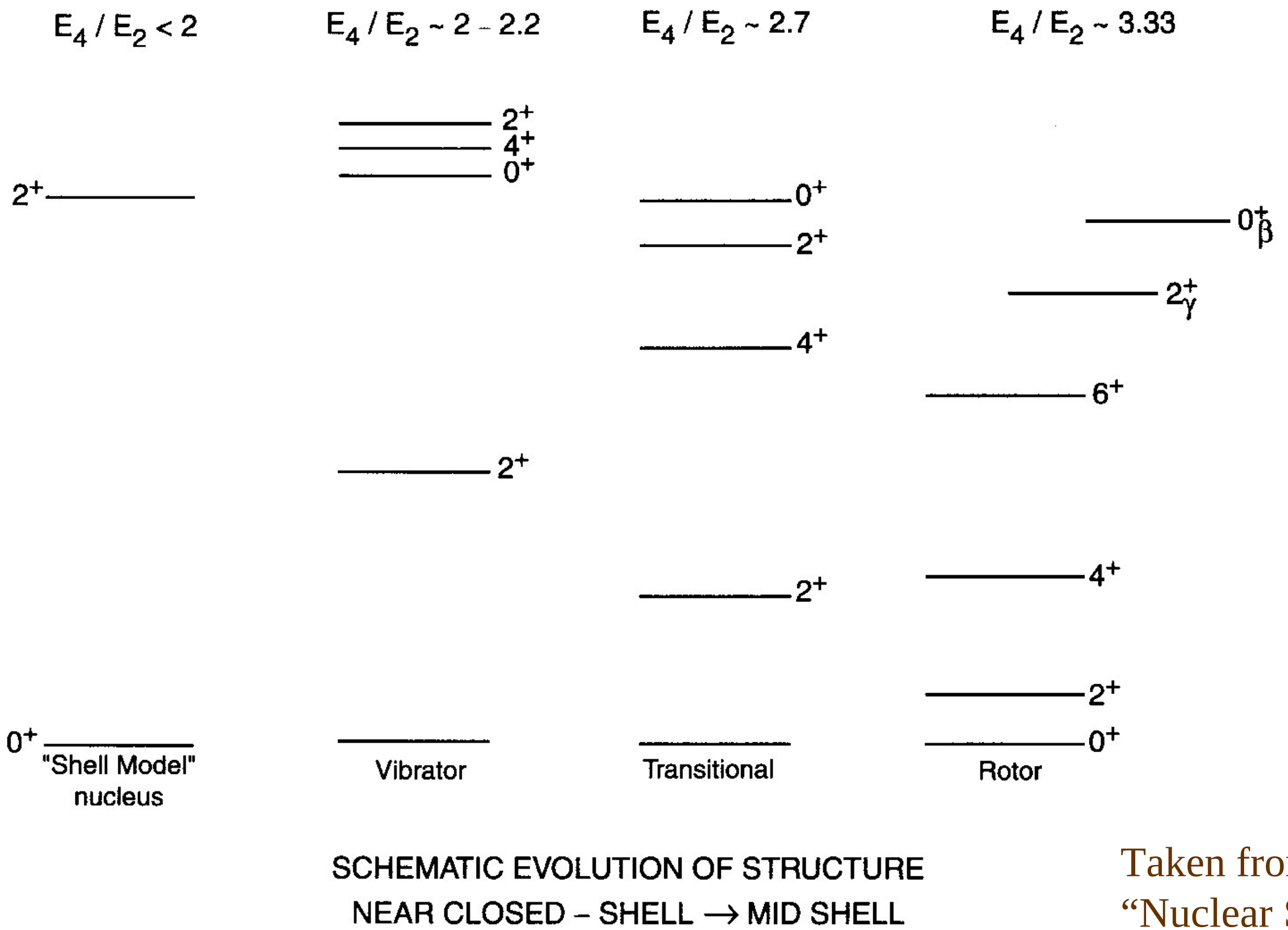
Target dependence of fusion cross section



Strong target dependence at $E < V_b$

Low-lying collective excitations in atomic nuclei

Low-lying excited states in even-even nuclei are collective excitations, and strongly reflect the pairing correlation and shell strucuture



Taken from R.F. Casten, "Nuclear Structure from a Simple Perspective"

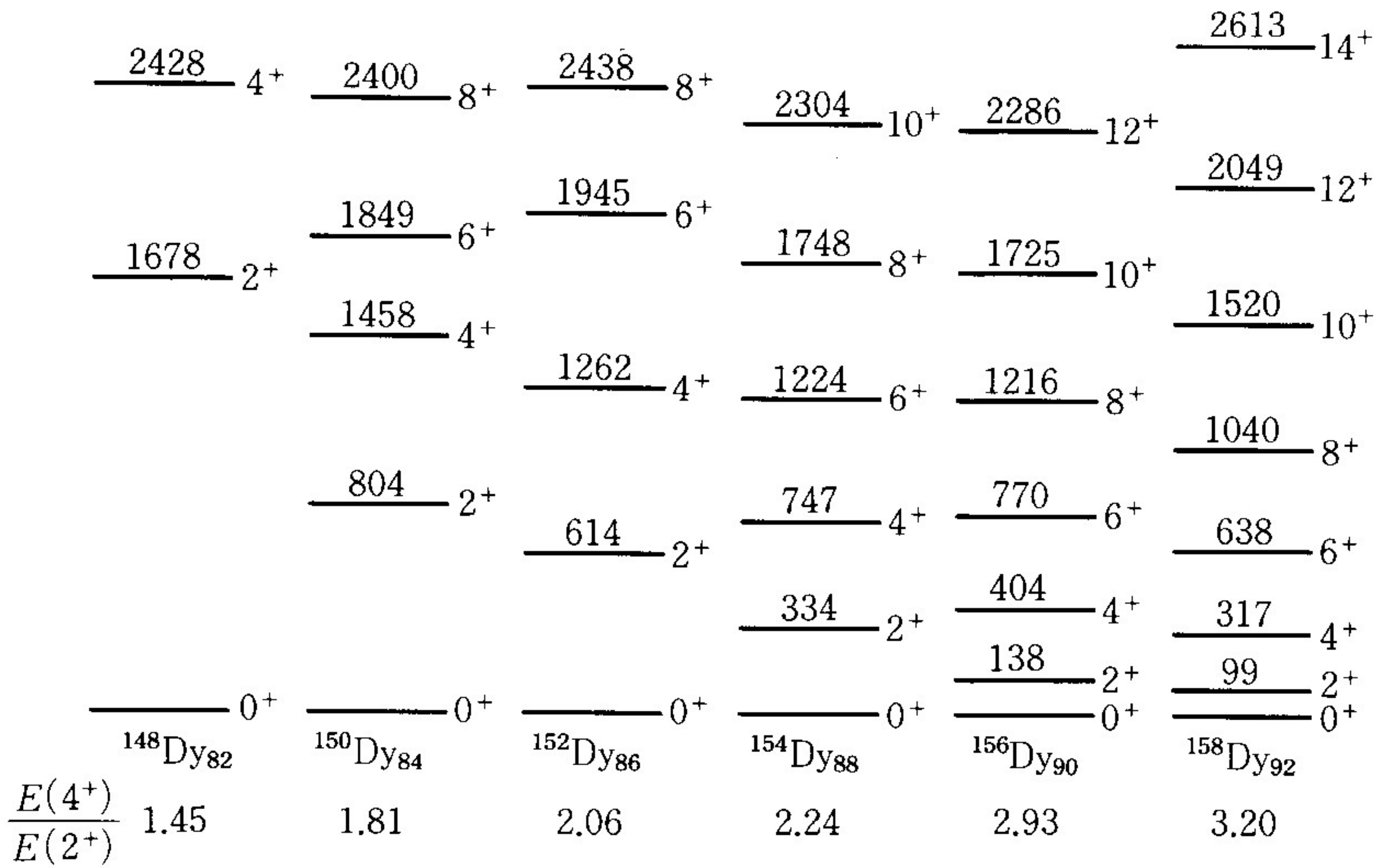
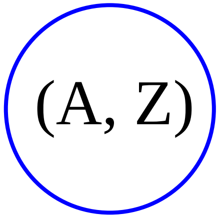


図 3-4　Dy アイソトープの低励起スペクトル．励起エネルギーの単位は **keV**.

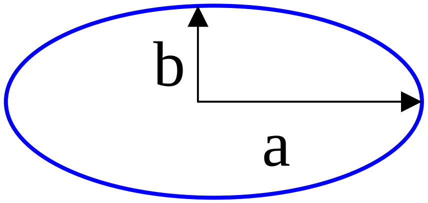
Vibrational excitations

Bethe-Weizacker formula: Mass formula based on Liquid-Drop Model



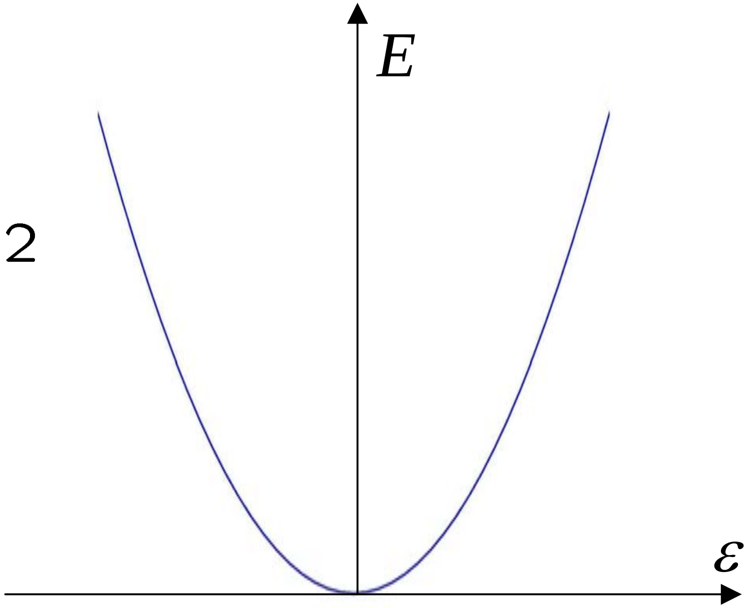
$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\rm sym} \frac{(N-Z)^2}{A}$$

For a deformed shape,



$$\begin{aligned} a &= R \cdot (1+\epsilon) \\ b &= R \cdot (1+\epsilon)^{-1/2} \end{aligned}$$

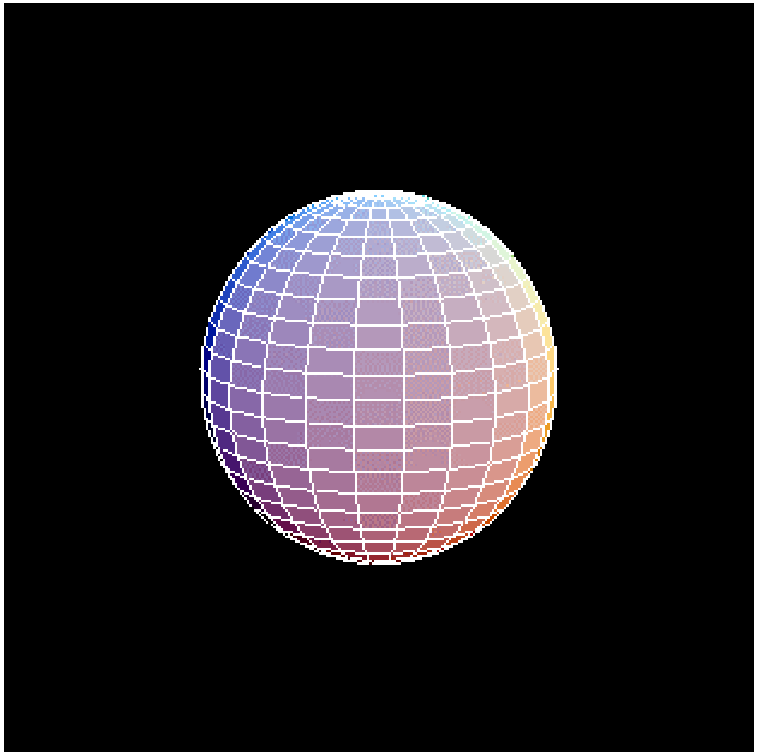
$$\begin{aligned} E_{\rm surf} &= E_{\rm surf}^{(0)} (1 + 2\epsilon^2/5 + \cdots) \\ E_C &= E_C^{(0)} (1 - \epsilon^2/5 + \cdots) \end{aligned}$$

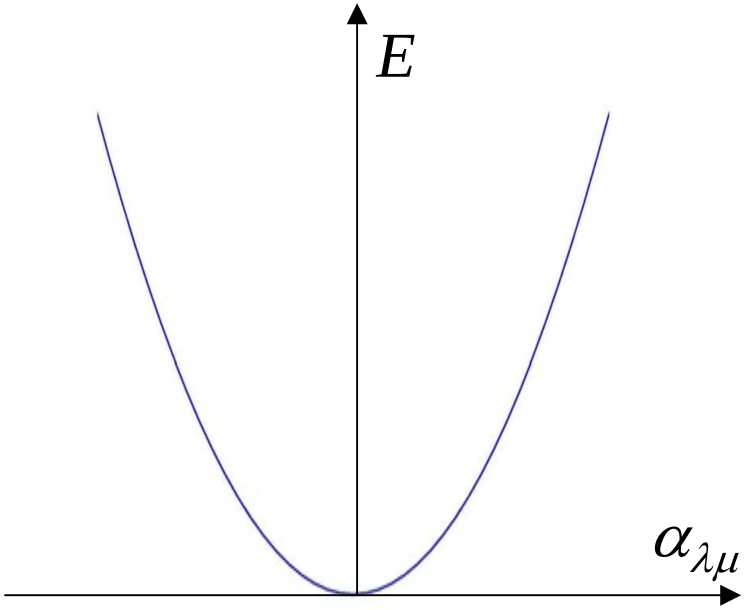


In general $R(\theta, \phi) = R_0 \left(1 + \sum_{\lambda,\mu} \alpha_{\lambda\mu} Y^*_{\lambda\mu} \right)$

$$V = \frac{1}{2} \sum_{\lambda,\mu} C_\lambda \left| \alpha_{\lambda\mu} \right|^2$$

Harmonic oscillation



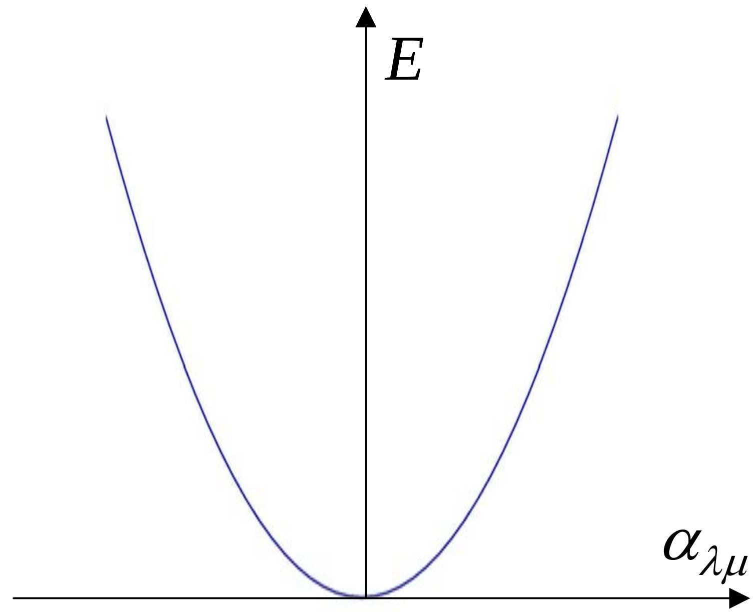


λ =2: Quadrupole vibration

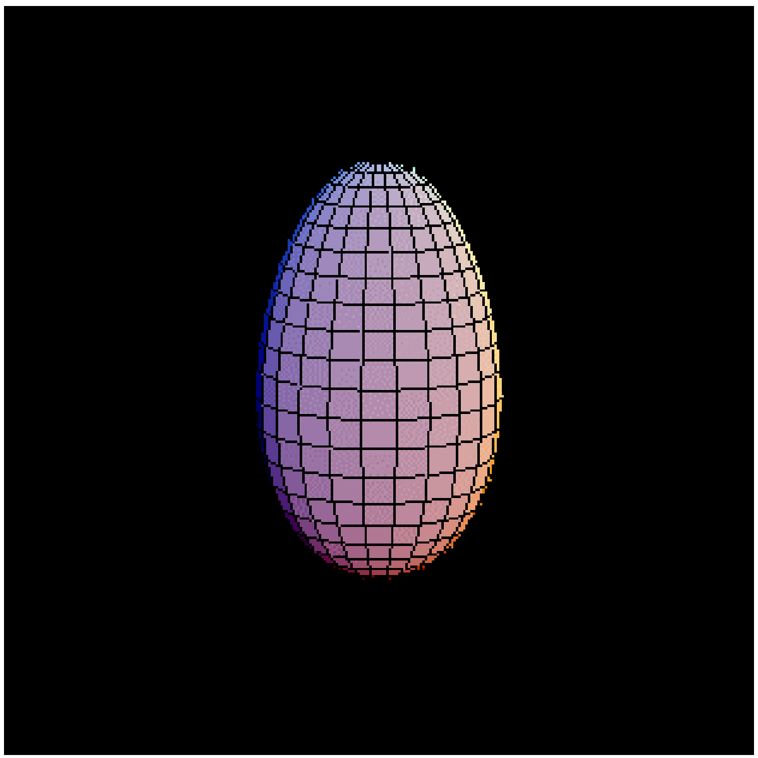
Movie: Dr. K. Arita (Nagoya Tech. U.)
http://www.phys.nitech.ac.jp/~arita/

In general $$R(\theta,\phi) = R_0\left(1 + \sum_{\lambda,\mu}\alpha_{\lambda\mu}Y^*_{\lambda\mu}\right)$$

$$V = \frac{1}{2}\sum_{\lambda,\mu}C_\lambda\,|\alpha_{\lambda\mu}|^2$$



Harmonic oscillation



λ =3: Octupole vibration

Movie: Dr. K. Arita (Nagoya Tech. U.)
http://www.phys.nitech.ac.jp/~arita/

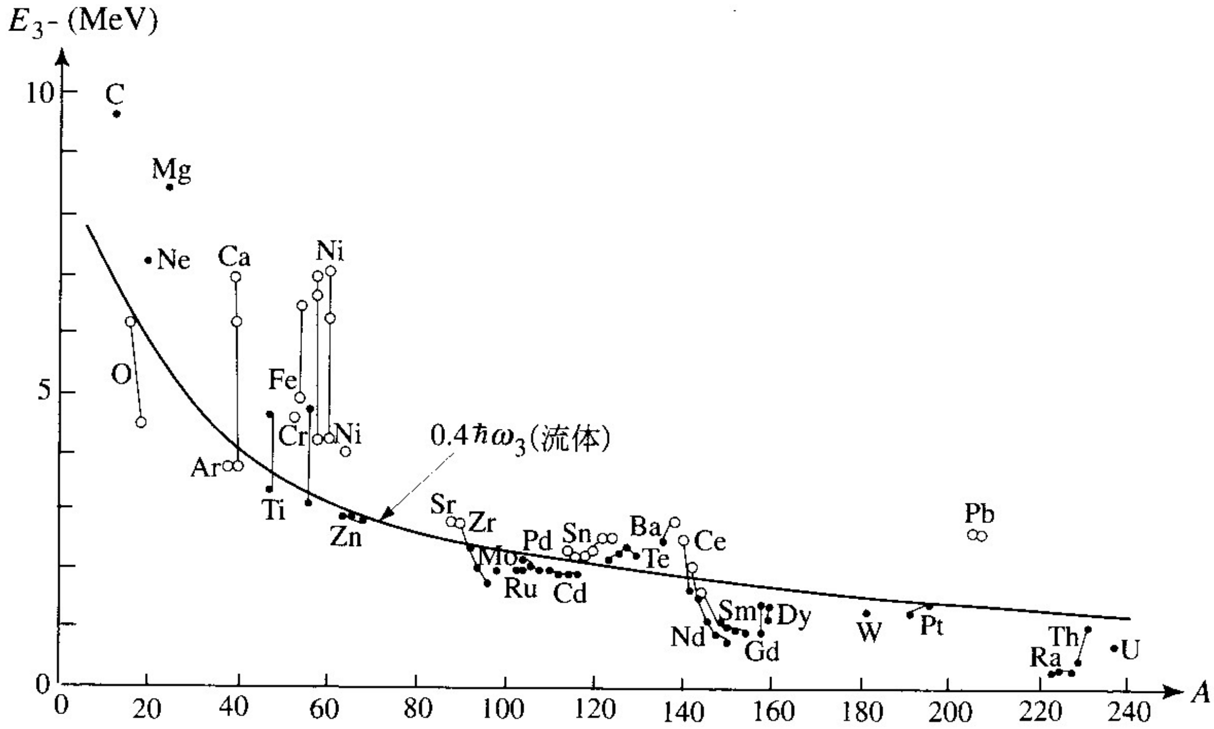


図 3.3 偶々核の第 1 励起 3^- 状態の励起エネルギー

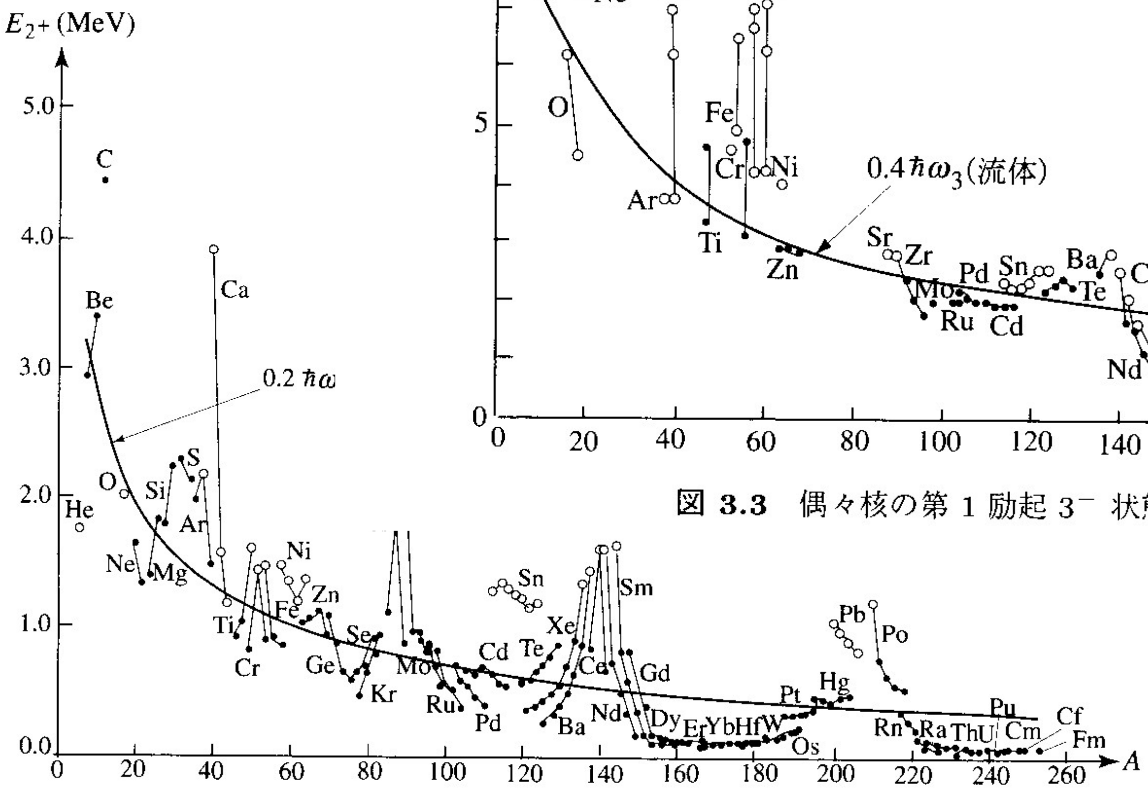
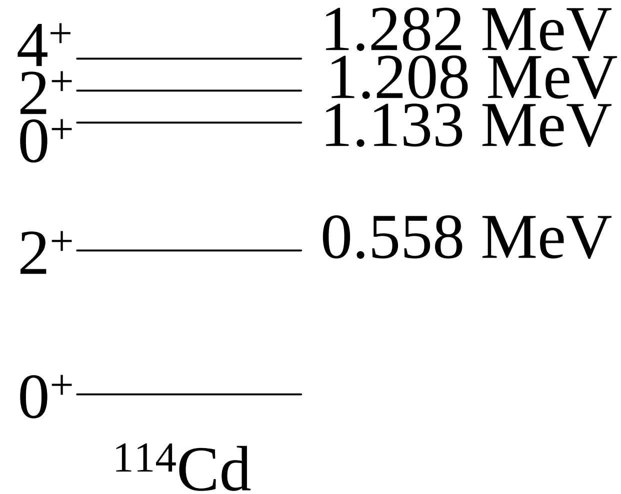


図 3.2 偶々核の第 1 励起 2^+ 状態の励起エネルギー

Double phonon states

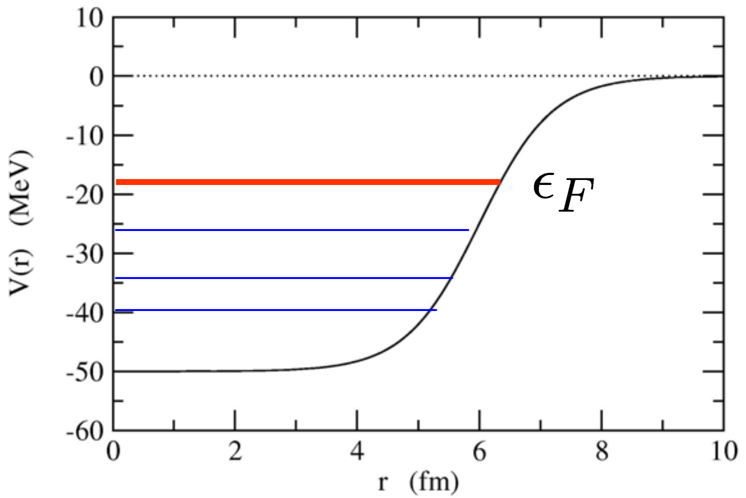


Microscopic description

⇨ Random phase approximation (RPA)

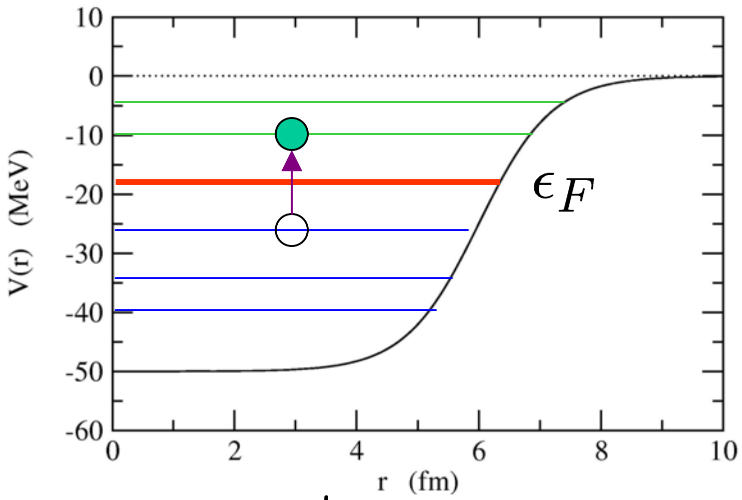
Random Phase Approximation

Hartree-Fock state



$|HF\rangle$

1 particle-1 hole (1p1h) state



$a_p^\dagger a_h|HF\rangle$

$$|\mathrm{vib}\rangle = Q^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\,a_p^\dagger a_h - Y_{ph}\,a_h^\dagger a_p\right)|0\rangle$$

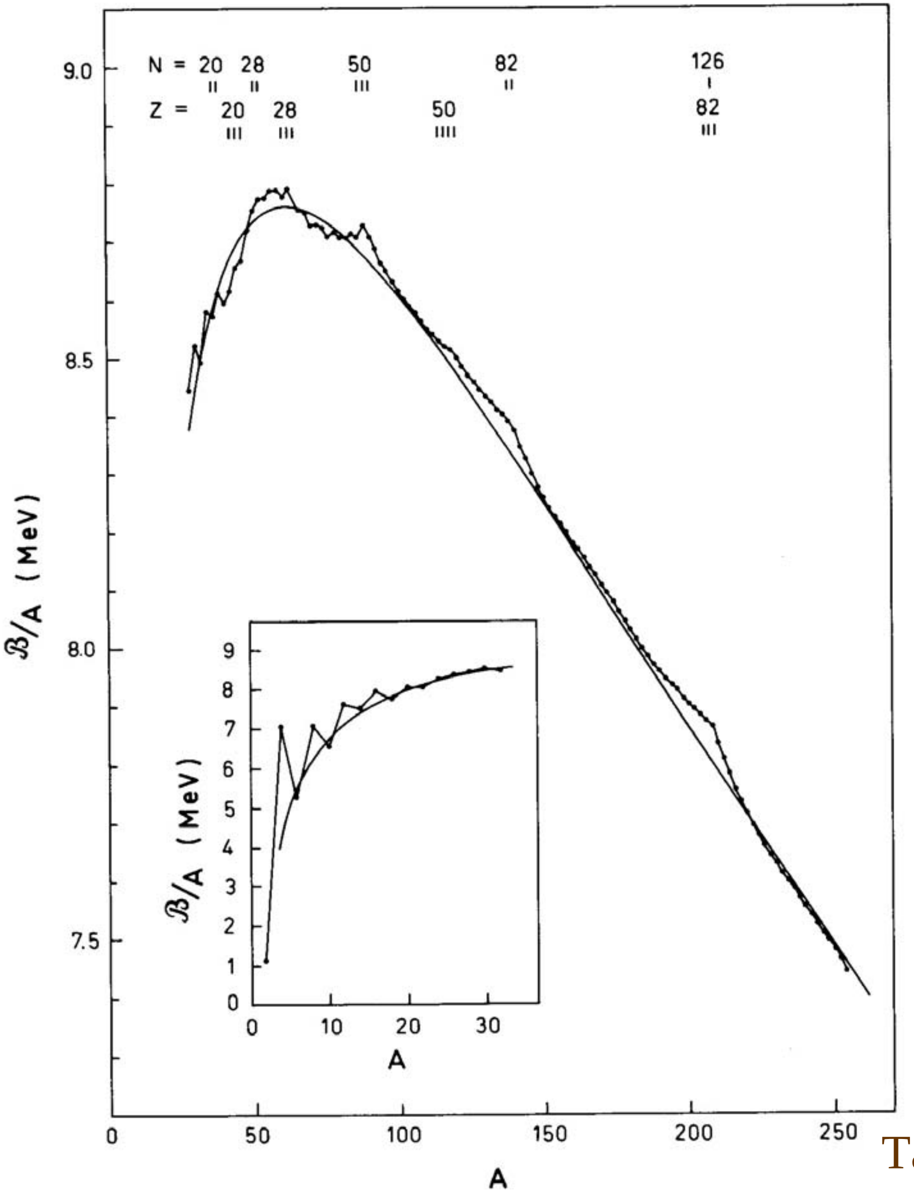
(coherent superposition of 1p1h states)

$$\Longrightarrow \quad [H, Q^\dagger] \approx \hbar\omega Q^\dagger$$

Rotational excitation

- <u>Shell energy</u>

$$B(N,Z) = B_{\text{macro}}(N,Z) + B_{\text{micro}}(N,Z)$$



- Smooth part

$$B_{\text{macro}}(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

- Fluctuation part

$$B_{\text{micro}} = B_{\text{pair}} + B_{\text{shell}}$$

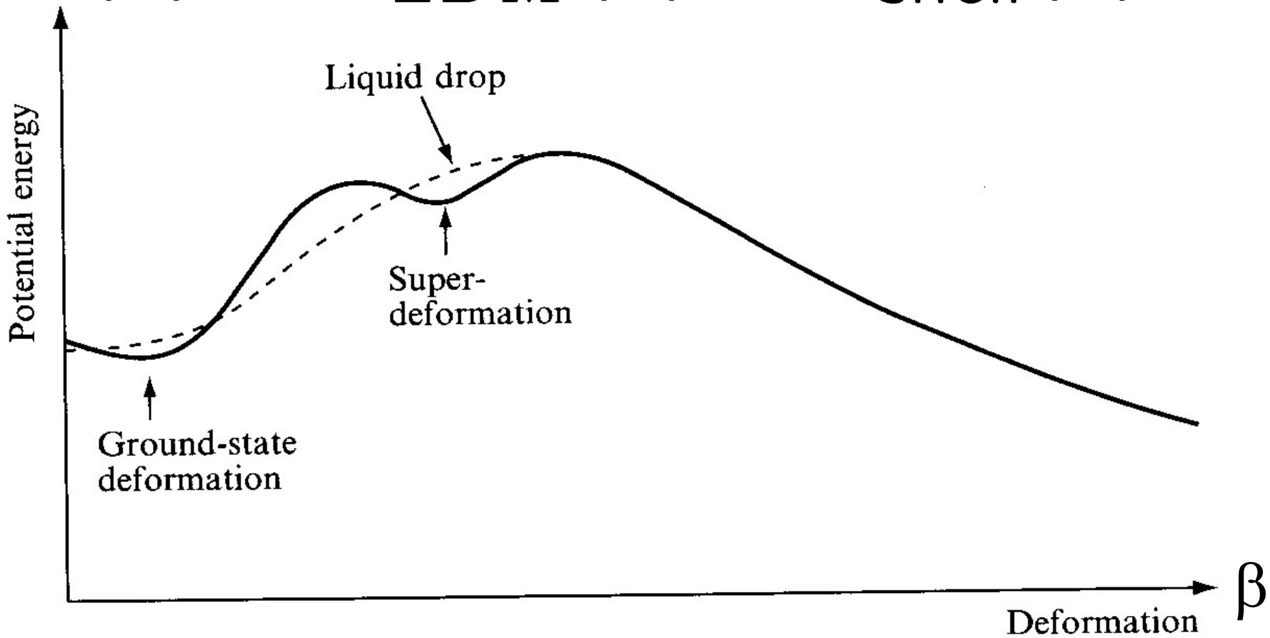
Liquid drop model:

$$B_{\text{LDM}} = B_{\text{macro}} + B_{\text{pair}}$$

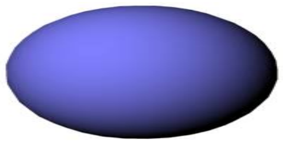
Taken from Bohr-Mottelson

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\mathrm{shell}}(\beta)$$



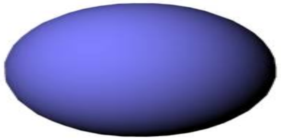
Taken from J.S. Lilley, "Nuclear Physics"



LDM only $\longrightarrow$ always spherical ground state

Shell correction $\longrightarrow$ may lead to a deformed g.s.

* Spontaneous Symmetry Breaking



Excitation spectra of ^{154}Sm

(MeV)	
0.903	8^+
0.544	6^+
0.267	4^+
0.082	2^+
0	0^+

^{154}Sm

$$\boxed{E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}}$$

cf. Rotational energy of a rigid body
(Classical mechanics)

$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

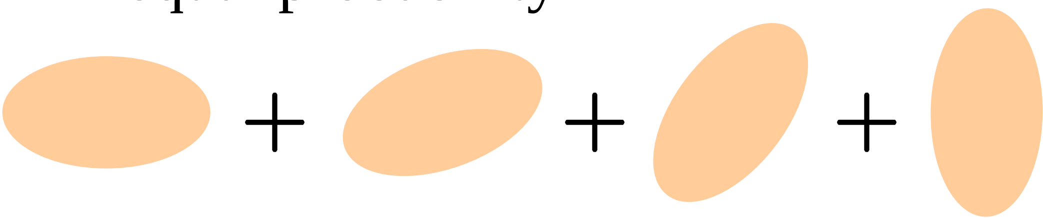
$$(I = \mathcal{J}\omega, \;\; \omega = \dot{\theta})$$

^{154}Sm is deformed

(note) What is 0^+ state (Quantum Mechanics)?

0^+: no preference of direction (spherical)

→ Mixing of all orientations with an equal probability

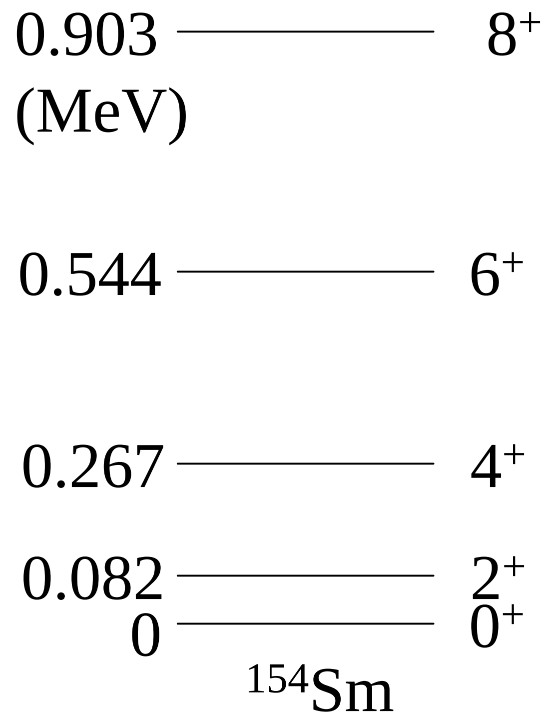


c.f. HF + Angular Momentum Projection

Evidences for nuclear deformation

- The existence of rotational bands

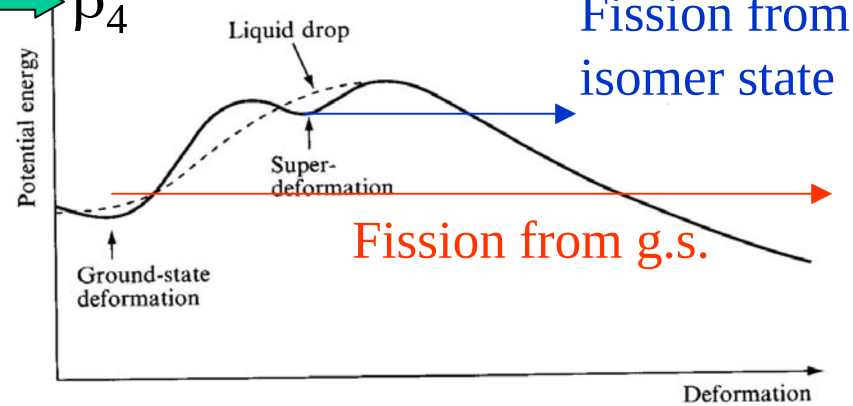
$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$



- Very large quadrupole moments (for odd-A nuclei)

$$Q = e\sqrt{\frac{16\pi}{5}}\langle\Psi_{II}|r^2Y_{20}|\Psi_{II}\rangle$$

- Strongly enhanced quadrupole transition probabilities
- Hexadecapole matrix elements ⟷ β_4
- Single-particle structure
- Fission isomers



Effect of collective excitation on σ_{fus}: rotational case

Comparison of energy scales

$$V(r) \sim V_b - \frac{1}{2}\mu\Omega^2 r^2$$

Tunneling motion: $E_{\rm tun} \sim \hbar\Omega \sim$ 3.5 MeV (barrier curvature)

Rotational motion: $E_{\rm rot} \sim E_{2+} \sim 0.08\,\mathrm{MeV}$

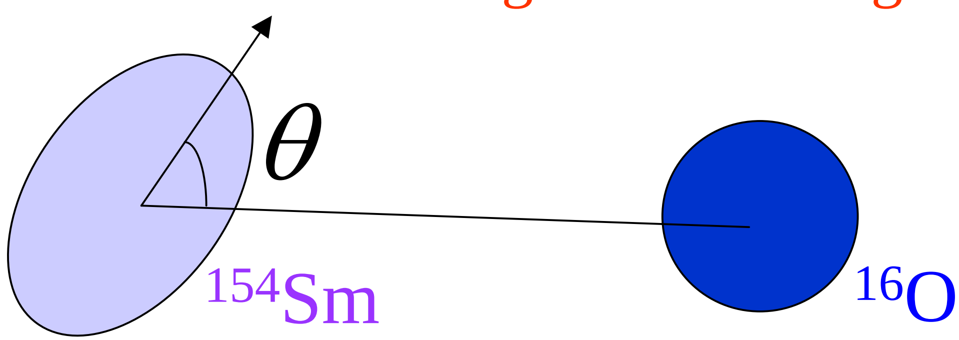
$$E_{\rm tun} \gg E_{\rm rot} = I(I+1)\hbar^2/2\mathcal{J} \to 0$$

$$\mathcal{J} \to \infty$$

The orientation angle of ^{154}Sm does not change much during fusion

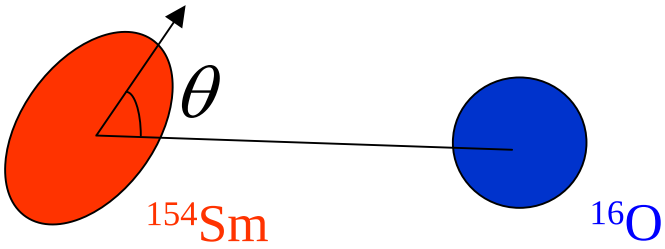
(note)

Ground state (0+ state) when reaction starts

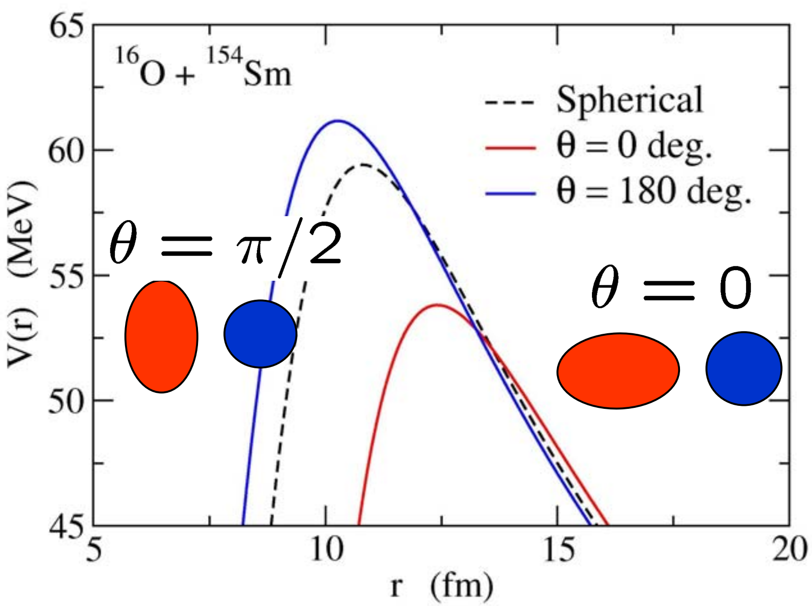


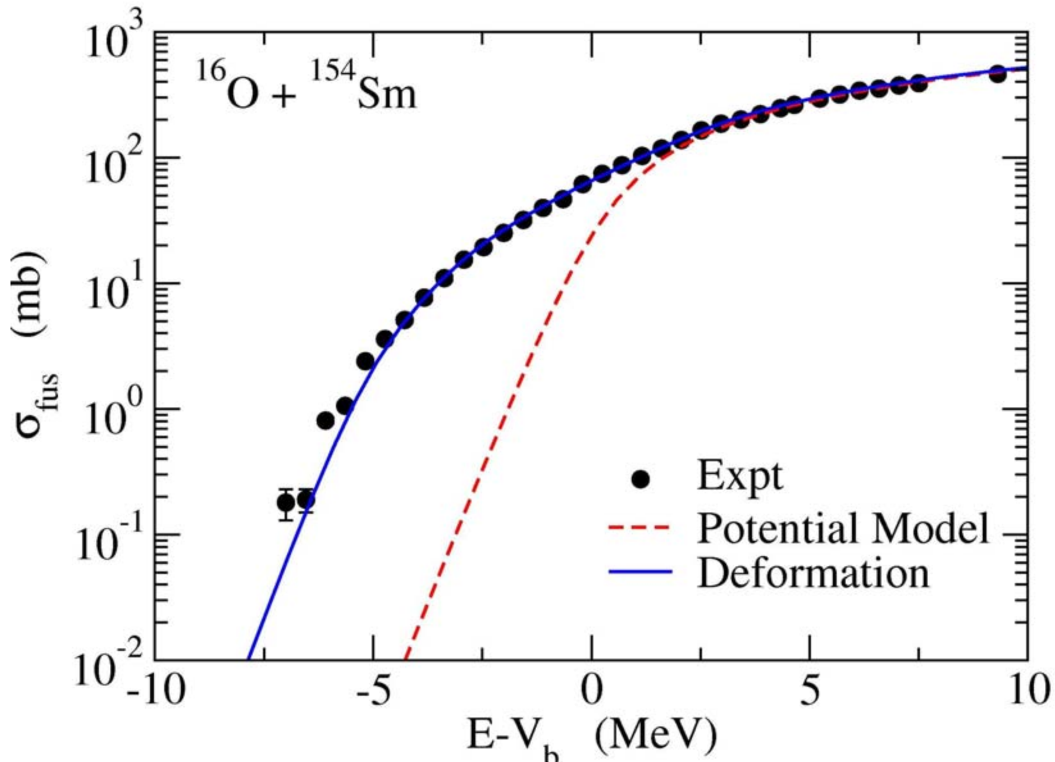
Mixing of all orientations with an equal weight

$$\boxed{\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\rm fus}(E;\theta)}$$



$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\,\sigma_{\text{fus}}(E;\theta)$$





The barrier is lowered for θ=0 because an attraction works from large distances.

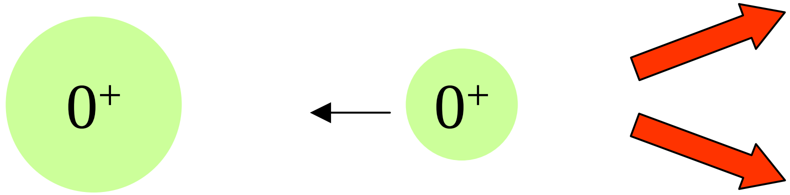
The opposite for θ=π/2. The barrier is highered as the rel. distance has to get small for the attraction to work

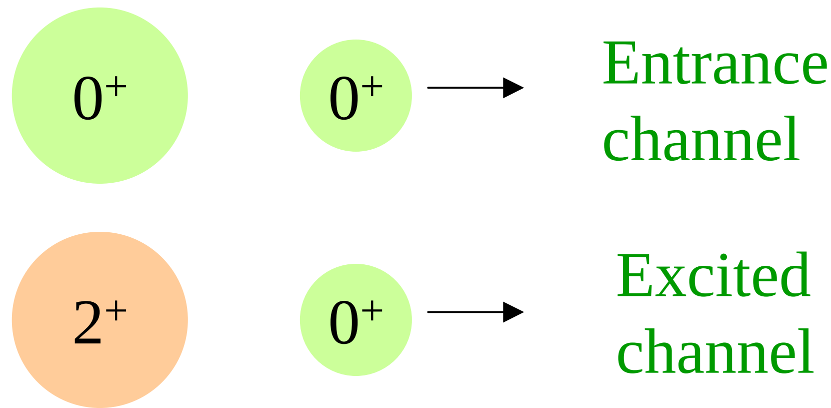
Def. Effect: enhances σ_{fus} by a factor of 10 ~ 100

Fusion: interesting probe for nuclear structure

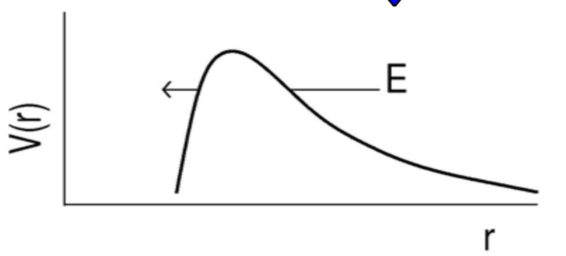
More quantal treatment: Coupled-Channels method

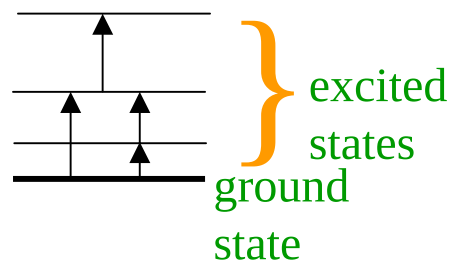
Coupling between rel. and intrinsic motions





$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)$$





$$H_0(\xi)\phi_k(\xi) = \epsilon_k\,\phi_k(\xi)$$

$$\boxed{\Psi(\boldsymbol{r},\xi) = \sum_k \psi_k(\boldsymbol{r})\phi_k(\xi)}$$

$$\begin{cases} H = -\dfrac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi) \\ \Psi(\boldsymbol{r},\xi) = \displaystyle\sum_k \psi_k(\boldsymbol{r})\phi_k(\xi) \end{cases}$$

$$H_0(\xi)\phi_k(\xi) = \epsilon_k\,\phi_k(\xi)$$

Schroedinger equation: $(H-E)\Psi(\boldsymbol{r},\xi) = 0$

$\langle\phi_k| \longrightarrow$

$$\langle\phi_k|H-E|\Psi\rangle = 0$$

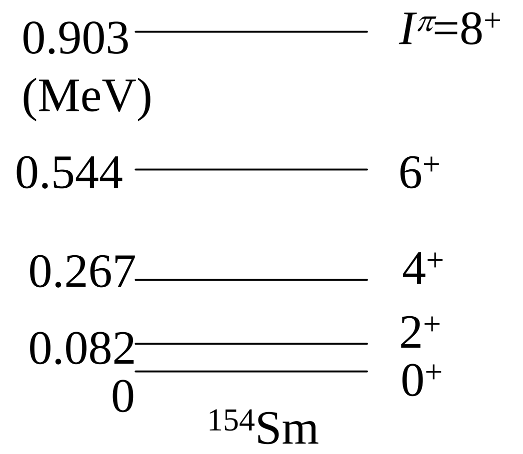
or

$$\boxed{\left[-\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + \epsilon_k - E\right]\psi_k(\boldsymbol{r}) + \sum_{k'}\langle\phi_k|V_{\rm coup}|\phi_{k'}\rangle\psi_{k'}(\boldsymbol{r}) = 0}$$

Coupled-channels equations

Angular momentum coupling

$$H_0(\xi)\phi_{nIm_I}(\xi) = \epsilon_{nI}\,\phi_{nIm_I}(\xi)$$



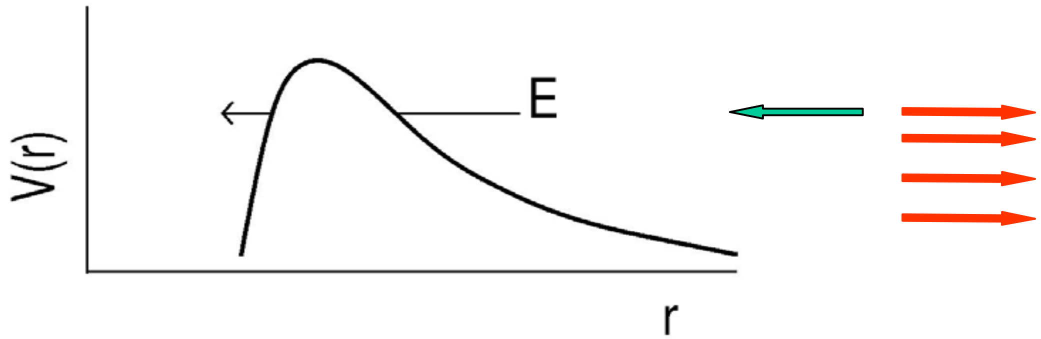
Total ang. mom.: $\boldsymbol{I} + \boldsymbol{l} = \boldsymbol{J}$

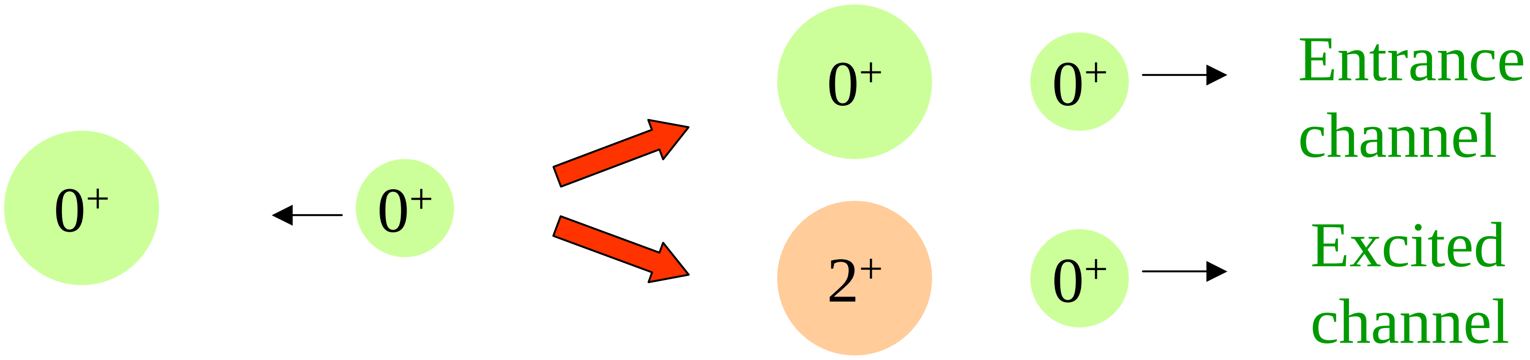
$$\Psi(\boldsymbol{r},\xi) = \sum_k \psi_k(\boldsymbol{r})\phi_k(\xi) = \sum_{n,l,I} \frac{u_{nlI}(r)}{r}[Y_l(\hat{\boldsymbol{r}})\phi_{nI}(\xi)]^{(JM)}$$

$$\langle [Y_l\phi_{nI}]^{(JM)}|H - E|\Psi\rangle = 0$$

$$\boxed{\begin{aligned}&\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_0(r) - E + \epsilon_{nI}\right]u_{nlI}(r)\\ &\quad + \sum_{n'l'I'}\langle [Y_l\phi_{nI}]^{(JM)}|V_{\rm coup}(r)|[Y_{l'}\phi_{n'I'}]^{(JM)}\rangle\, u_{n'l'I'}(r) = 0\end{aligned}}$$

Boundary condition



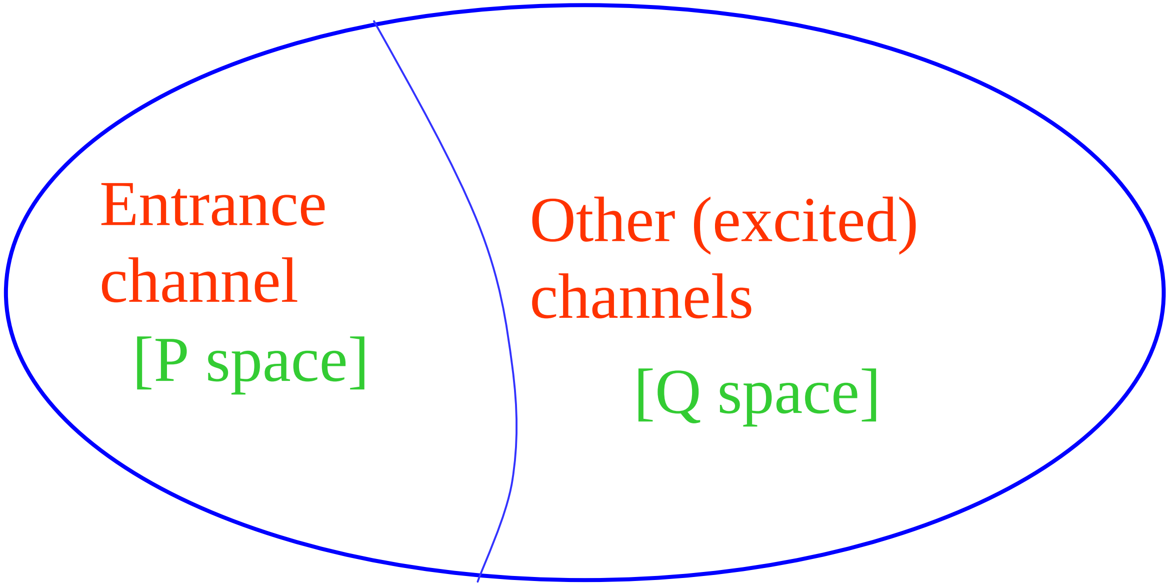


$$\Psi(\boldsymbol{r},\xi)=\sum_{n,l,I}\frac{u_{nlI}(r)}{r}[Y_l(\hat{\boldsymbol{r}})\phi_{nI}(\xi)]^{(JM)}$$

$$\boxed{u_{nlI}(r)\to H_l^{(-)}(k_{nI}r)\delta_{n,n_i}\delta_{l,l_i}\delta_{I,I_i}-\sqrt{\frac{k_0}{k_{nI}}}\,S_{nlI}\,H_l^{(+)}(k_{nI}r)}$$

$$P_l(E)=1-\sum_{nI}|S_{nlI}|^2 \qquad \sigma_{\rm fus}(E)=\frac{\pi}{k^2}\sum_l(2l+1)P_l(E)$$

(note) Dynamical Polarization Potential



Total Hilbert space considered (conceptual figure)

“Eliminate” the Q-space and project on to the P-space

Effective potential for the P-space (entrance channel) :
Dynamical Polarization Potential

Energy dependent, non-local, complex potential

Optical potential V_{opt}

Example: 2 channel problem

$$\Bigg[\underbrace{-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V_0(r)}_{\equiv \hat{h}_l}+\begin{pmatrix} 0 & F(r) \\ F(r) & \epsilon \end{pmatrix}\Bigg]\begin{pmatrix} u_0(r) \\ u_1(r) \end{pmatrix} = E\begin{pmatrix} u_0(r) \\ u_1(r) \end{pmatrix}$$

or

$$\begin{cases} \hat{h}_l\, u_0(r) + F(r)u_1(r) = E\, u_0(r) & (1) \\ \hat{h}_l\, u_1(r) + F(r)u_0(r) = (E-\epsilon)u_1(r) & (2) \end{cases}$$

(2) $\Longrightarrow$ $$u_1(r) = -\int_0^\infty dr'\, G^{(+)}(r,r';E-\epsilon)F(r')u_0(r')$$

$$G^{(+)}(r,r';E-\epsilon) = \left(\frac{1}{\hat{h}_l-(E-\epsilon)+i\eta}\right)_{r,r'}$$

$$\boxed{\hat{h}_l\, u_0(r) - F(r)\int_0^\infty dr'\, G^{(+)}(r,r';E-\epsilon)F(r')u_0(r') = E\, u_0(r)}$$

Example: 2 channel problem (cont'd)

$$\underbrace{\widehat{h}_l\, u_0(r) - F(r)\int_0^\infty dr'\, G^{(+)}(r,r';E-\epsilon)F(r')u_0(r')}_{} = E\, u_0(r)$$

$$= \int_0^\infty dr'\, V_{\mathsf{DPP}}(r,r')u_0(r')$$

$$\boxed{V_{\mathsf{DPP}}(r,r') = -F(r)\, G^{(+)}(r,r';E-\epsilon)F(r')}$$

$$\begin{aligned} G^{(+)}(r,r';E) &= \left(\frac{1}{\widehat{h}_l - E + i\eta}\right)_{r,r'} \\ &= -\frac{2\mu}{\hbar^2}\cdot\frac{f_l(kr_<)\,\tilde{h}_l^{(+)}(kr_>)}{W} \end{aligned}$$

$f_l \;\rightarrow\; \sin(kr - l\pi/2 + \delta_l)$	(regular solution)
$\tilde{h}_l \;\rightarrow\; \exp[i(kr - l\pi/2 + \delta_l)]$	(outgoing solution)
$W \;=\; f_l'\,\tilde{h}_l = f_l\,\tilde{h}_l' = k$	(Wronskian)

For a more general derivation: Feshbach formalism (see References)

Summary of Coupled-channels method

$$\begin{cases} H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi) \\ \Psi(\boldsymbol{r},\xi) = \sum_{n,l,I} \frac{u_{nlI}(r)}{r}[Y_l(\hat{\boldsymbol{r}})\phi_{nI}(\xi)]^{(JM)} \end{cases}$$

$$H_0(\xi)\phi_{nIm_I}(\xi) = \epsilon_{nI}\,\phi_{nIm_I}(\xi)$$

$$\langle [Y_l\phi_{nI}]^{(JM)}|H - E|\Psi\rangle = 0$$

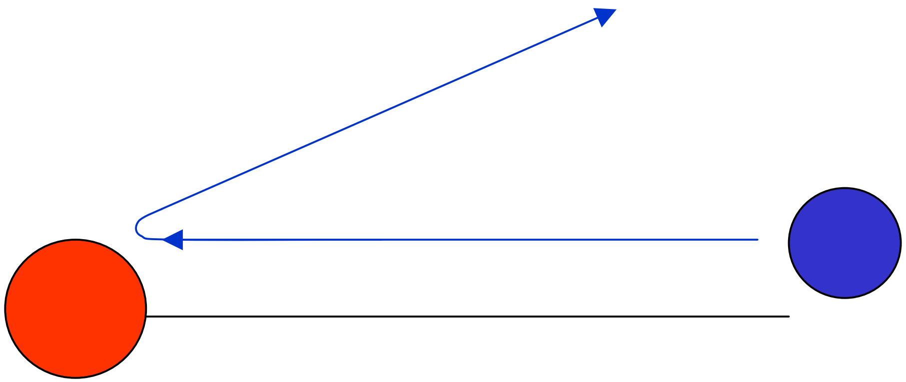
$$\boxed{\begin{aligned}&\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + \frac{l(l+1)\hbar^2}{2\mu r^2} + V_0(r) - E + \epsilon_{nI}\right] u_{nlI}(r) \\ &\quad + \sum_{n'l'I'} \langle [Y_l\phi_{nI}]^{(JM)}|V_{\rm coup}(r)|[Y_{l'}\phi_{n'I'}]^{(JM)}\rangle\, u_{n'l'I'}(r) = 0\end{aligned}}$$

$$\boxed{u_{nlI}(r) \to H_l^{(-)}(k_{nI}r)\delta_{n,n_i}\delta_{l,l_i}\delta_{I,I_i} - \sqrt{\frac{k_0}{k_{nI}}}\, S_{nlI}\, H_l^{(+)}(k_{nI}r)}$$

$$P_l(E) = 1 - \sum_{nI} |S_{nlI}|^2$$

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E)$$

Excitation structure of atomic nuclei



Excite the target nucleus via collision with the projectile nucleus

How does the targ. respond to the interaction with the proj.?

Standard approach: analysis with the coupled-channels method

- Inelastic cross sections
- Elastic cross sections
- Fusion cross sections

S-matrix S_{nlI}

Coupling Potential: Collective Model

$$R(\theta,\phi) = R_T\left(1 + \sum_\mu \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi)\right)$$

(note) coordinate transformation to the rotating frame ($\hat{\boldsymbol{r}} = 0$)

$$\sum_\mu \alpha_{\lambda\mu} Y^*_{\lambda\mu}(\theta,\phi) \to \sqrt{\frac{2\lambda+1}{4\pi}}\,\alpha_{\lambda 0}$$

➢ Vibrational case

$$\begin{cases} \alpha_{\lambda\mu} = \dfrac{\beta_\lambda}{\sqrt{2\lambda+1}}\left(a^\dagger_{\lambda\mu} + (-)^\mu a_{\lambda\mu}\right) \\ H_0 = \hbar\omega_\lambda \sum_\mu a^\dagger_{\lambda\mu} a_{\lambda\mu} \end{cases}$$

➢ Rotational case

Coordinate transformation to the body-fixed rame

$$\begin{cases} \alpha_{\lambda\mu} = \sqrt{\dfrac{4\pi}{2\lambda+1}}\,\beta_\lambda Y_{\lambda\mu}(\theta_d,\phi_d) \quad \text{(for axial symmetry)} \\ H_0 = \dfrac{I(I+1)\hbar^2}{2\mathcal{J}} \end{cases}$$

In both cases

$$\beta_\lambda = \frac{4\pi}{3Z_T R_T^\lambda}\sqrt{\frac{B(E\lambda)\uparrow}{e^2}}$$

Deformed Woods-Saxon model:

$$V_{WS}(r) = -\frac{V_0}{1+\exp[(r-R_0)/a]}$$
$$= -\frac{V_0}{1+\exp[(r-R_P-R_T)/a]}$$

$$R_T \to R_T\left(1+\sum_\mu \alpha_{\lambda\mu}Y^*_{\lambda\mu}(\theta,\phi)\right)$$

$$\boxed{V_{WS}(\boldsymbol{r}) = -\frac{V_0}{1+\exp[(r-R_0-R_T\alpha_\lambda\cdot Y_\lambda(\hat{\boldsymbol{r}}))/a]}}$$

Deformed Woods-Saxon model (collective model)

K.H., N. Rowley, and A.T. Kruppa,
Comp. Phys. Comm. 123('99)143

$$V_{\rm coup}(r,\hat{O}) = V_{\rm coup}^{(N)}(r,\hat{O}) + V_{\rm coup}^{(C)}(r,\hat{O})$$

Nuclear coupling:

$$V_{\rm coup}^{(N)}(r,\hat{O}) = -\frac{V_0}{1+\exp[(r-R_0-R_T\hat{O})/a]}$$

Coulomb coupling:

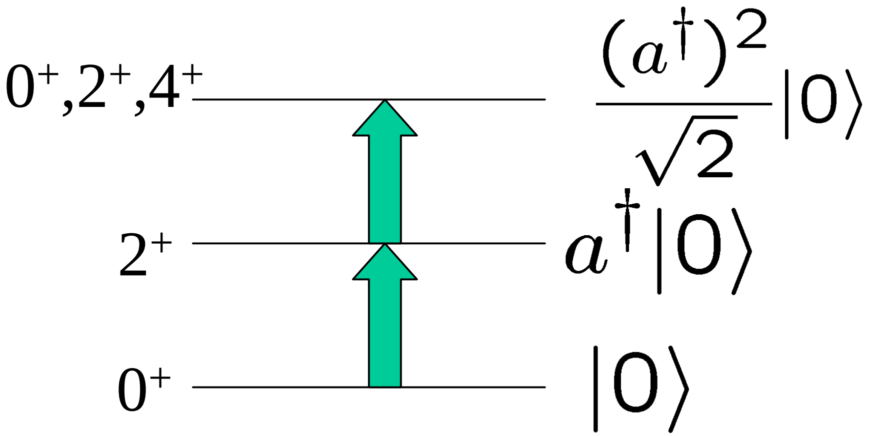
$$V_{\rm coup}^{(C)}(r,\hat{O}) = \frac{3}{2\lambda+1}Z_PZ_Te^2\frac{R_T^\lambda}{r^{\lambda+1}}\hat{O}$$

Rotational coupling: $\hat{O} = \beta Y_{20}(\theta)$

Vibrational coupling: $\hat{O} = \frac{\beta}{\sqrt{4\pi}}(a+a^\dagger)$

Vibrational coupling

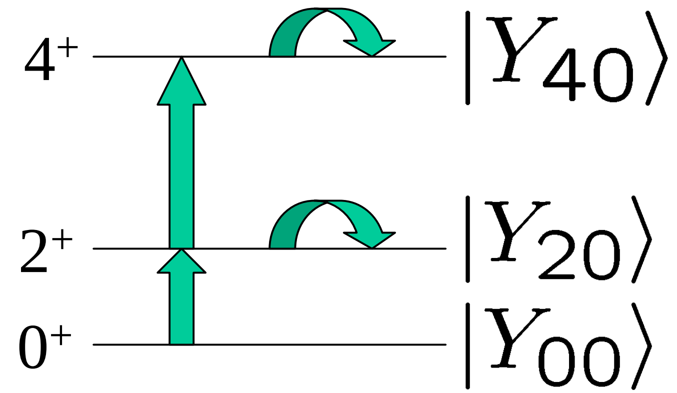
$$\hat{O} = \frac{\beta}{\sqrt{4\pi}}(a + a^\dagger)$$



$$\begin{pmatrix} 0 & F & 0 \\ F & \epsilon & \sqrt{2}F \\ 0 & \sqrt{2}F & 2\epsilon \end{pmatrix}$$

Rotational coupling

$$\hat{O} = \beta Y_{20}(\theta)$$

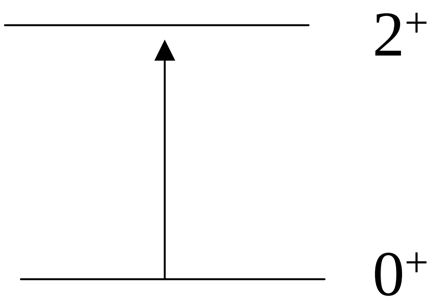


$$\begin{pmatrix} 0 & F & 0 \\ F & \epsilon + \frac{2\sqrt{5}}{7}F & \frac{6}{7}F \\ 0 & \frac{6}{7}F & \frac{10\epsilon}{3} + \frac{20\sqrt{5}}{77}F \end{pmatrix}$$

$$F = \frac{\beta}{\sqrt{4\pi}}$$

Iso-centrifugal approximation

- No-Coriolis approximation
- Rotating frame approximation

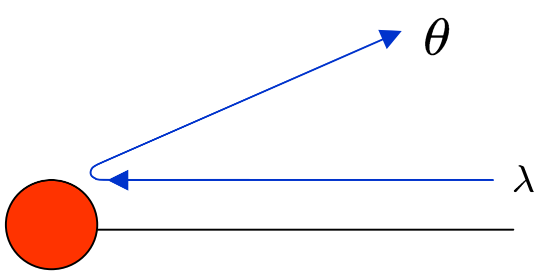


l=J-2, J, J+2

l=J

J=l+I

Truncation	Dimension
2^+	$4 \longrightarrow 2$
4^+	$9 \longrightarrow 3$
6^+	$16 \longrightarrow 4$
8^+	$25 \longrightarrow 5$



Iso-centrifugal approximation:

λ : independent of excitations

$$\boxed{\frac{l(l+1)\hbar^2}{2\mu r^2} \to \frac{J(J+1)\hbar^2}{2\mu r^2}}$$

$$V_{\rm coup}(\boldsymbol{r},\xi) = f(r)Y_\lambda(\hat{\boldsymbol{r}})\cdot T_\lambda(\xi)$$

transform to the rotating frame

$$\to \sqrt{\frac{2\lambda+1}{4\pi}}f(r)T_{\lambda 0}(\xi)$$

"Spin-less system"

Coupled-channels equations: two limiting cases

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V_0(r)-E+\epsilon_{nI}\right]u_{nlI}(r)$$
$$+\sum_{n'l'I'}\langle[Y_l\phi_{nI}]^{(JM)}|V_{\rm coup}(r)|[Y_{l'}\phi_{n'I'}]^{(JM)}\rangle\,u_{n'l'I'}(r)=0$$

$$u_{nlI}(r)\to H_l^{(-)}(k_{nI}r)\delta_{n,n_i}\delta_{l,l_i}\delta_{I,I_i}-\sqrt{\frac{k_0}{k_{nI}}}\,S_{nlI}\,H_l^{(+)}(k_{nI}r)$$

$$P_l(E)=1-\sum_{nI}|S_{nlI}|^2 \qquad \sigma_{\rm fus}(E)=\frac{\pi}{k^2}\sum_l(2l+1)P_l(E)$$

Calculate σ_{fus} by numerically solving the coupled-channels equations

Let us consider two limiting cases in order to understand (interpret) the numerical results

- ε_{nI}: very large — *Adiabatic limit*
- $\varepsilon_{nI} = 0$ — *Sudden limit*

Two limiting cases: (i) adiabatic limit

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)$$

In case where the rel. motion is much slower than the intrinsic motion

In case where the energy scale for intrinsic motion is much larger than the typical energy scale for the rel. motion

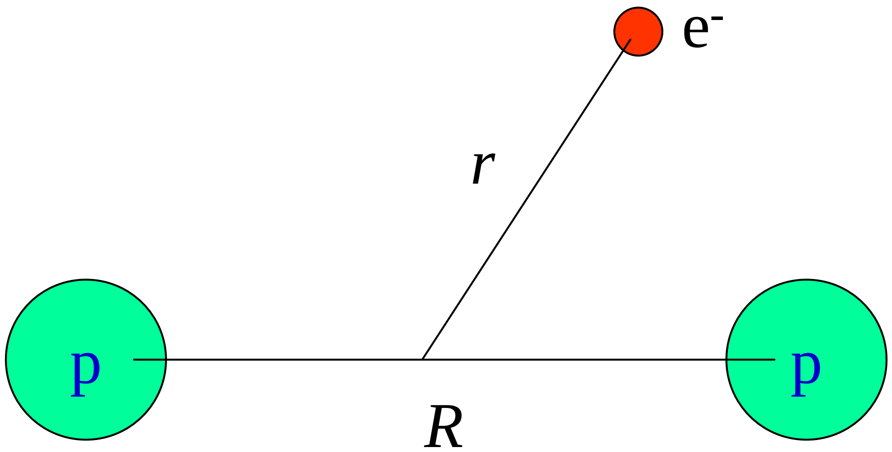
$$\hbar\Omega \ll \epsilon$$

(Barrier curvature v.s. Intrinsic excitation energy)

$$[H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)]\varphi_0(\xi;\boldsymbol{r}) = \epsilon_0(r)\,\varphi_0(\xi;\boldsymbol{r})$$

$$\boxed{H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi) \to \epsilon_0(r)}$$

c.f. Born-Oppenheimer approximation for hydrogen molecule



$$[T_R + T_r + V(r,R)]\Psi(r,R) = E\Psi(r,R)$$

1. Consider first the electron motion for a fixed *R*

$$[T_r + V(r,R)]u_n(r;R) = \epsilon_n(R)u_n(r;R)$$

2. Minimize $\varepsilon_n(R)$ with respect to *R*

Or 2’. Consider the proton motion in a potential $\varepsilon_n(R)$

$$[T_R + \epsilon_n(R)]\phi_n(R) = E\phi_n(R)$$

Adiabatic Potential Renormalization

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V_0(r) + H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)$$

When ε is large,

$$\boxed{H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi) \to \epsilon_0(r)}$$

where

$$[H_0(\xi) + V_{\rm coup}(\boldsymbol{r},\xi)]\varphi_0(\xi;\boldsymbol{r}) = \epsilon_0(r)\,\varphi_0(\xi;\boldsymbol{r})$$

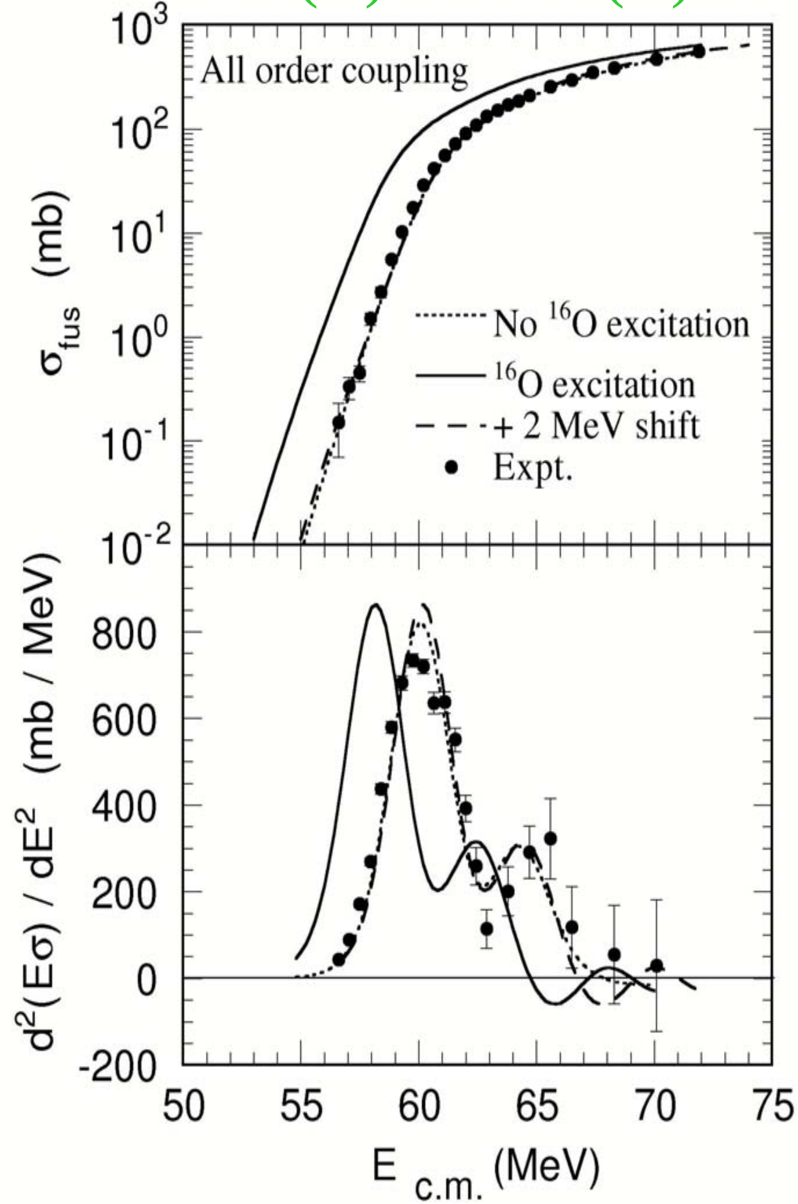
Fast intrinsic motion

→ Adiabatic potential renormalization

$$V_{\rm ad}(r) = V_0(r) + \epsilon_0(r)$$

Giant Resonances, ^{16}O(3$^-$) [6.31 MeV]

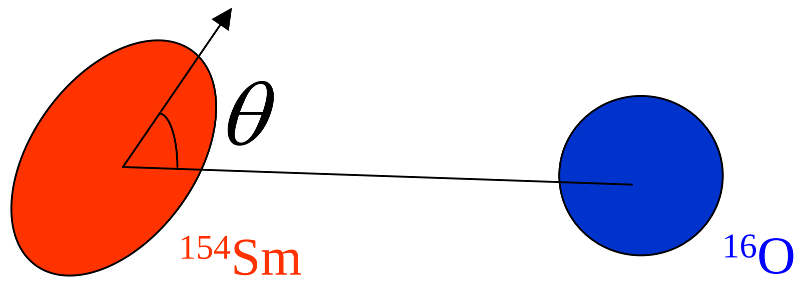
^{16}O(3$^-$) + ^{144}Sm(3$^-$)



K.H., N. Takigawa, M. Dasgupta, D.J. Hinde, J.R. Leigh, PRL79('99)2014

Two limiting cases: (ii) sudden limit

$$\epsilon \to 0$$



$$\epsilon_I = I(I+1)\hbar^2/2\mathcal{J}$$

$$\mathcal{J} \to \infty$$

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

Coupled-channels:

$$\begin{pmatrix} 0 & f(r) & 0 \\ f(r) & \frac{2\sqrt{5}}{7}f(r) & \frac{6}{7}f(r) \\ 0 & \frac{6}{7}f(r) & \frac{20\sqrt{5}}{77}f(r) \end{pmatrix} \xrightarrow{\text{diagonalize}} \begin{pmatrix} \lambda_1(r) & 0 & 0 \\ 0 & \lambda_2(r) & 0 \\ 0 & 0 & \lambda_3(r) \end{pmatrix}$$

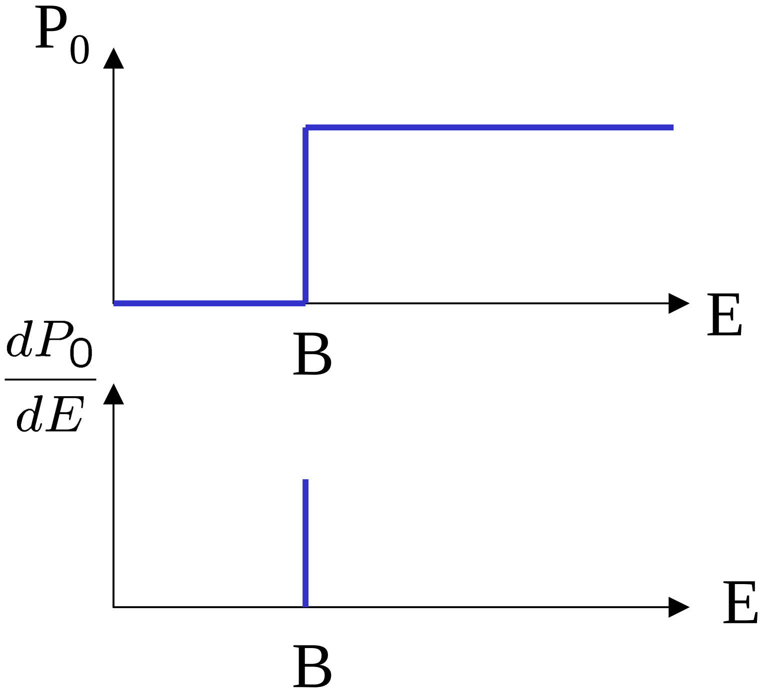
$$\Rightarrow \quad P(E) = \sum_i w_i\, P(E; V_0(r) + \lambda_i(r))$$

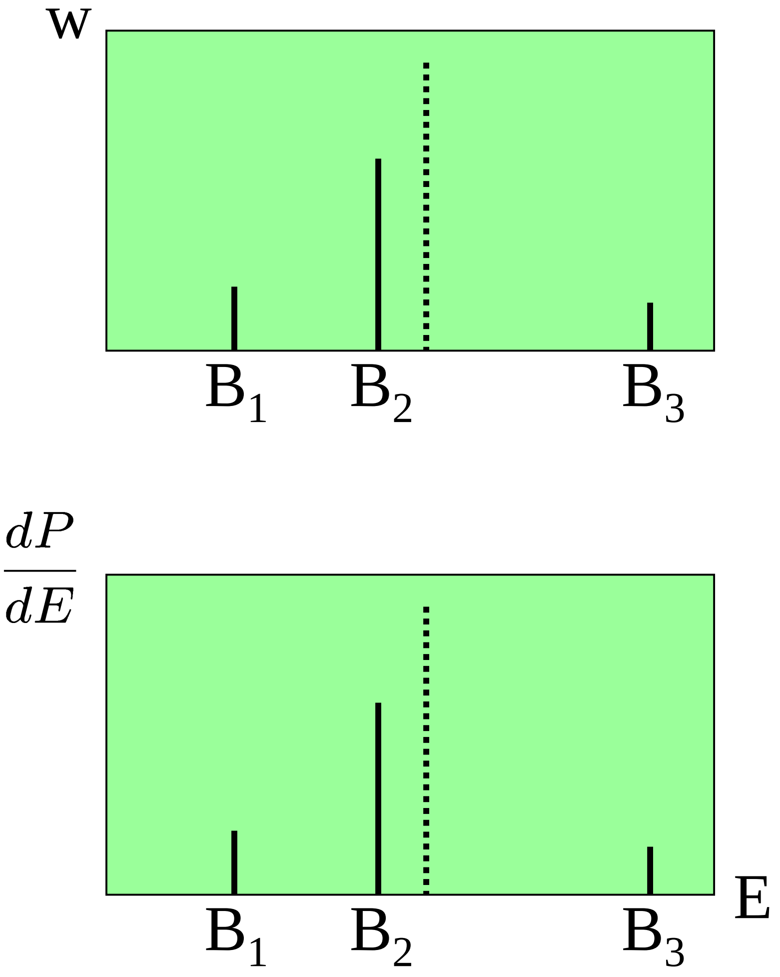
Slow intrinsic motion

$\Rightarrow$ Barrier Distribution

Barrier distribution

$$P(E) = \sum_i w_i \, P(E; V_0(r) + \lambda_i(r))$$



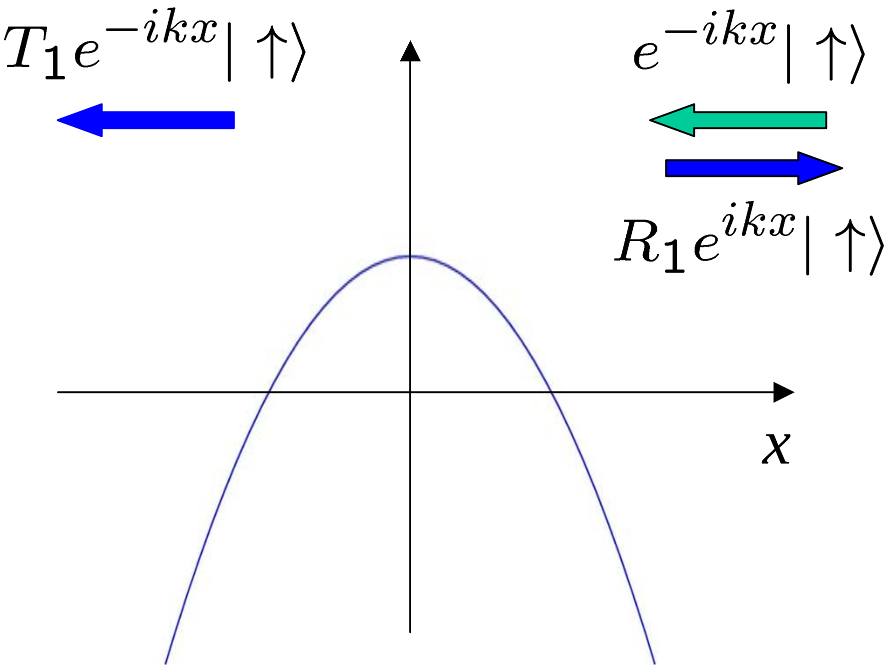


Barrier distribution: understand the concept using a spin Hamiltonian

Hamiltonian (example 1): $$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0(x) + \hat{\sigma}_z \cdot V_s(x)$$

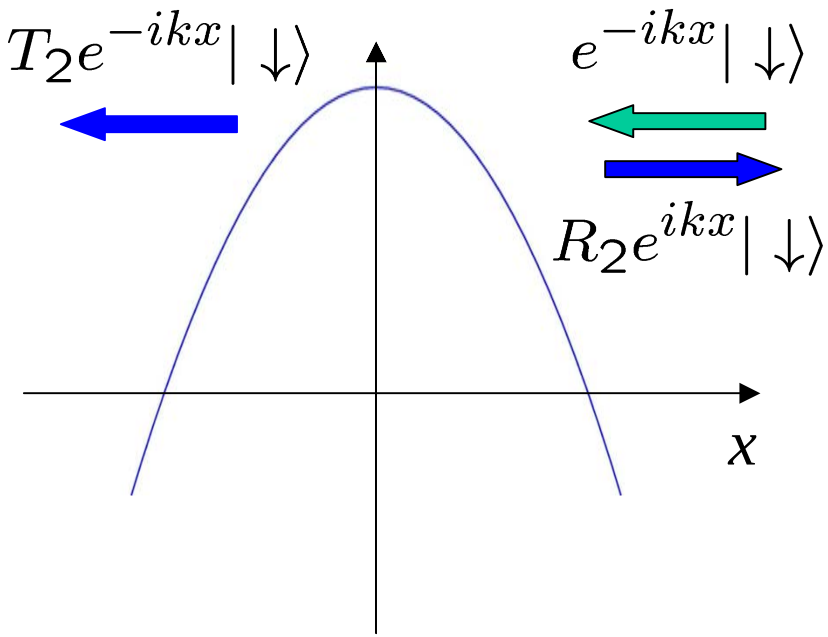
$$\hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For Spin-up



$$V_1(x) = V_0(x) + V_s(x)$$

For Spin-down



$$V_2(x) = V_0(x) - V_s(x)$$

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0(x) + \hat{\sigma}_z \cdot V_1(x)$$

Wave function (general form)

$$\Psi(x) = \psi_1(x)\,|\uparrow\rangle + \psi_2(x)\,|\downarrow\rangle = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix}$$

Asymptotic form at $x \to \pm\infty$

$$\Psi(x) \to \begin{pmatrix} C_1(e^{-ikx} + R_1 e^{ikx}) \\ C_2(e^{-ikx} + R_2 e^{ikx}) \end{pmatrix} \quad (x \to \infty)$$

$$\to \begin{pmatrix} C_1\, T_1 e^{-ikx} \\ C_2\, T_2 e^{-ikx} \end{pmatrix} \quad (x \to -\infty)$$

$$|C_1|^2 + |C_2|^2 = 1$$

(the C_1 and C_2 are fixed according to the spin state of the system)

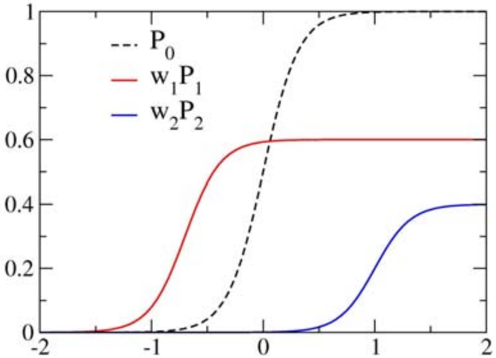
Tunnel probability = $\dfrac{(\text{flux at } x = -\infty)}{(\text{incoming flux at } x = \infty)}$

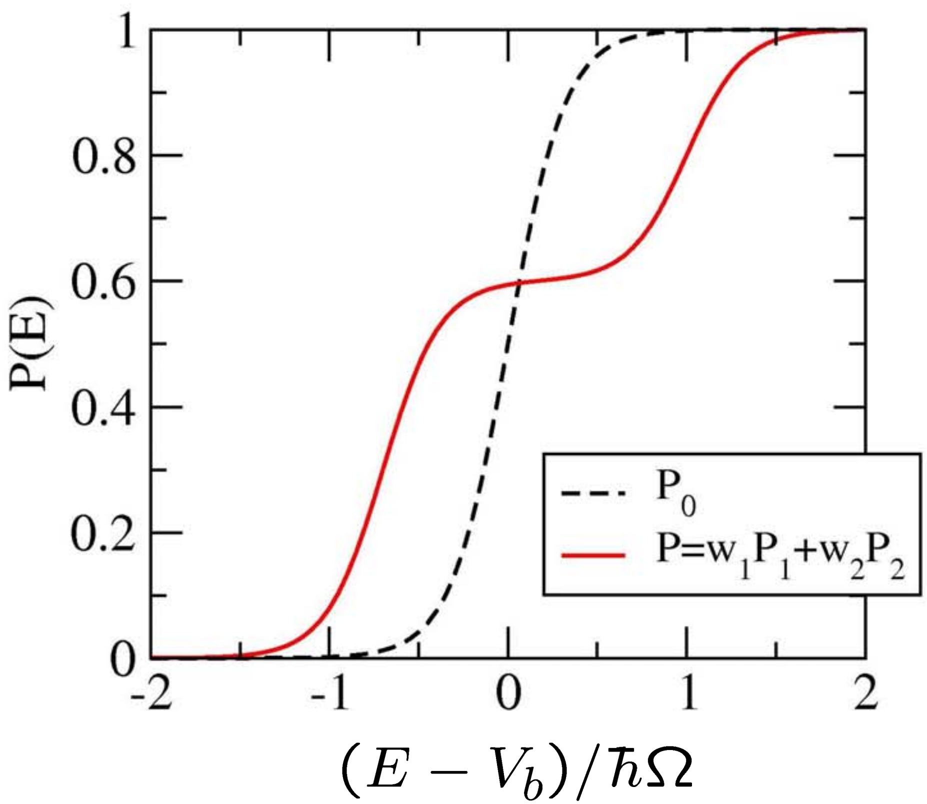
$$\boxed{\begin{aligned} P(E) &= \frac{|C_1 T_1|^2 + |C_2 T_2|^2}{|C_1|^2 + |C_2|^2} \\ &= |C_1|^2 P_1(E) + |C_2|^2 P_2(E) \equiv w_1 P_1(E) + w_2 P_2(E) \end{aligned}}$$

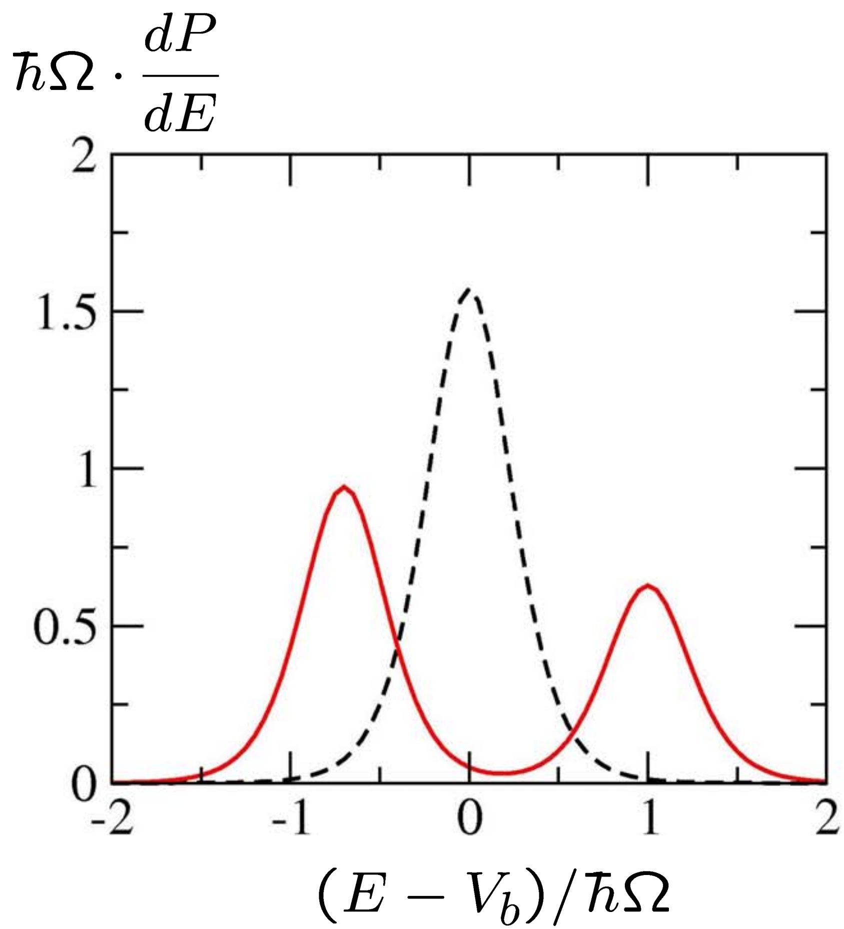
$$P(E) = w_1 P_1(E) + w_2 P_2(E)$$

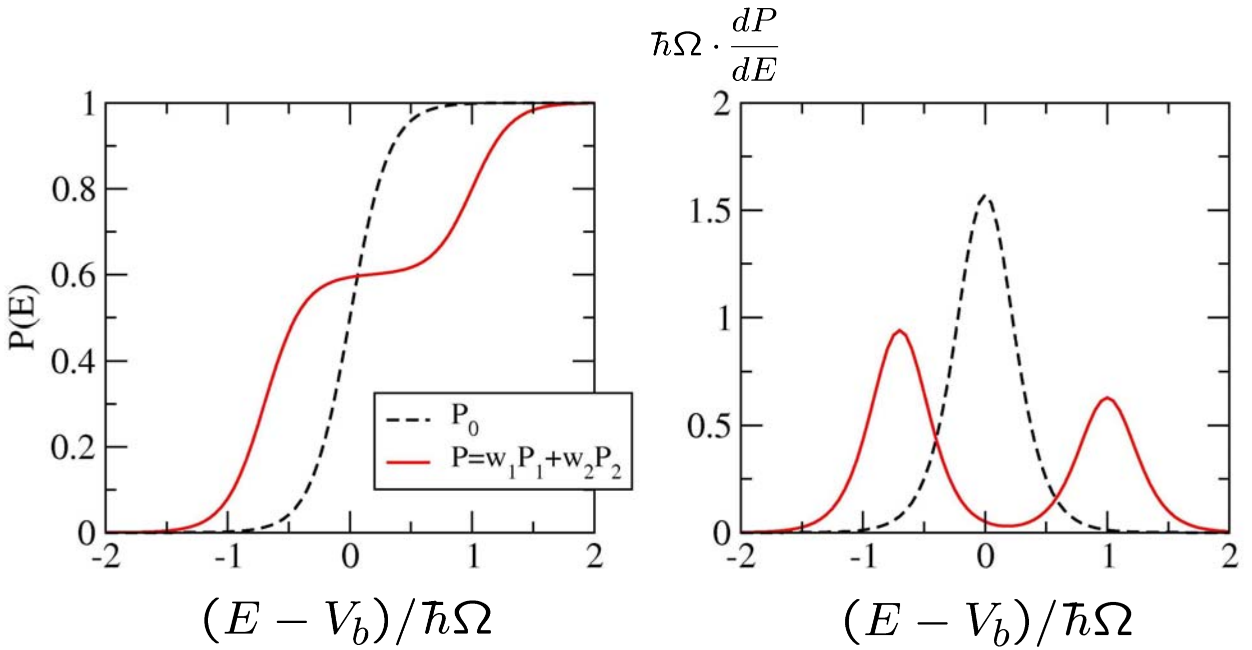
$\Longrightarrow$ Tunneling prob. is a weighted sum of tunnel prob. for two barriers

$$\begin{cases} V_1(x) = V_0(x) + V_s(x) & \Longleftarrow \quad |\uparrow\rangle \\ V_2(x) = V_0(x) - V_s(x) & \Longleftarrow \quad |\downarrow\rangle \end{cases}$$









- Tunnel prob. is enhanced at $E < V_b$ and hindered $E > V_b$
- *dP/dE* splits to two peaks → "barrier distribution"
- The peak positions of *dP/dE* correspond to each barrier height
- The height of each peak is proportional to the weight factor

$$\begin{aligned} P(E) &= w_1 P_1(E) + w_2 P_2(E) \\ \frac{dP}{dE} &= w_1 \frac{dP_1}{dE} + w_2 \frac{dP_2}{dE} \end{aligned}$$

Hamiltonian (example 2): in the case with off-diagonal components

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0(x) + \hat{\sigma}_x \cdot F(x) \qquad \hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\boxed{\begin{aligned} [\hat{t} + V_0(x)]\psi_1(x) + F(x)\psi_2(x) &= E\psi_1(x) \\ [\hat{t} + V_0(x)]\psi_2(x) + F(x)\psi_1(x) &= E\psi_2(x) \end{aligned}}$$

$$\phi_\pm(x) = [\psi_1(x) \pm \psi_2(x)]/\sqrt{2}$$

$$[\hat{t} + V_0(x) \pm F(x)]\phi_\pm(x) = E\phi_\pm(x)$$

If spin-up at the beginning of the reaction

$$\boxed{P(E) = \frac{1}{2}[P(E; V_0 + F) + P(E; V_0 - F)]}$$

Hamiltonian (example 3): more general cases

$$\begin{aligned} H &= -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0(x) - \epsilon\sigma_z + \hat{\sigma}_x \cdot F(x) \\ &= -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0(x) + \begin{pmatrix} -\epsilon & F(x) \\ F(x) & \epsilon \end{pmatrix} \end{aligned}$$

$$\Rightarrow \quad U(x)\begin{pmatrix} -\epsilon & F(x) \\ F(x) & \epsilon \end{pmatrix} U^\dagger(x) = \begin{pmatrix} \lambda_1(x) & 0 \\ 0 & \lambda_2(x) \end{pmatrix}$$

x dependent

$$\boxed{P(E) = \sum_i w_i(E)\, P(E; V_0(x) + \lambda_i(x))}$$

E dependent

K.H., N. Takigawa, A.B. Balantekin, PRC56('97)2104 $\quad w_i(E) \sim \text{constant}$

(note) Adiabatic limit: $\epsilon \to \infty \quad \Longrightarrow \quad w_i(E) = \delta_{i,0}$

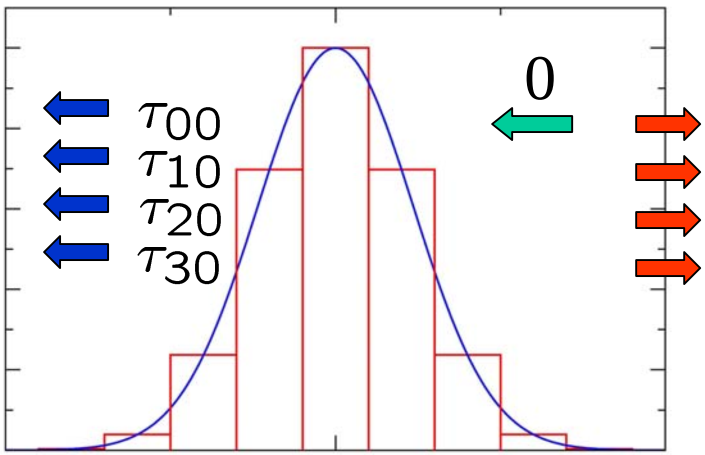
(cf.) Solving the C.C. equations with WKB approximation

WKB formula for a one dimensional barrier penetrability:

$$P_{WKB}(E) = \exp\left[-2\int_{x_0}^{x_1} dx' \sqrt{\frac{2m}{\hbar^2}(V(x') - E)}\right]$$

Generalization to a coupled-channels problem

K.H., A.B. Balantekin, Phys. Rev. A70 ('04) 032106

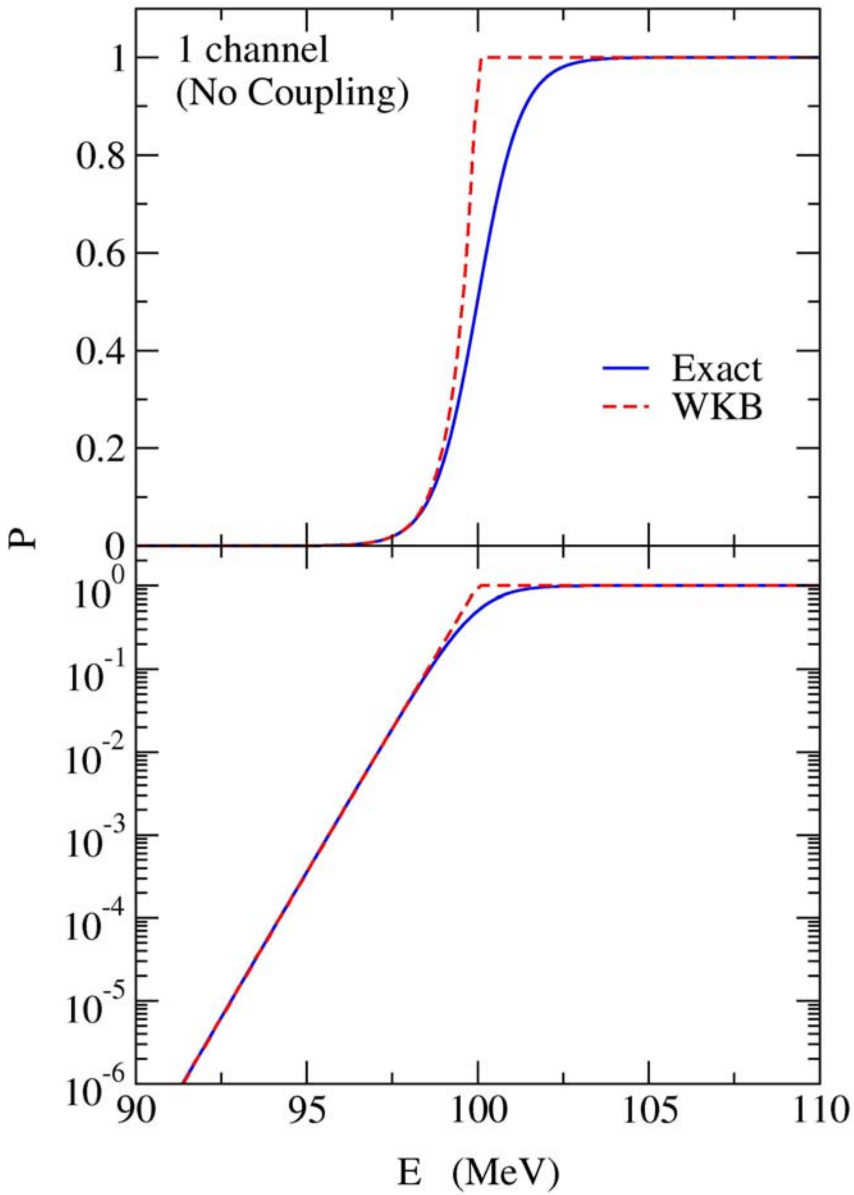


$$\boldsymbol{\tau} = \left(\prod_i e^{i\boldsymbol{q}(x_i)\Delta x}\right)$$

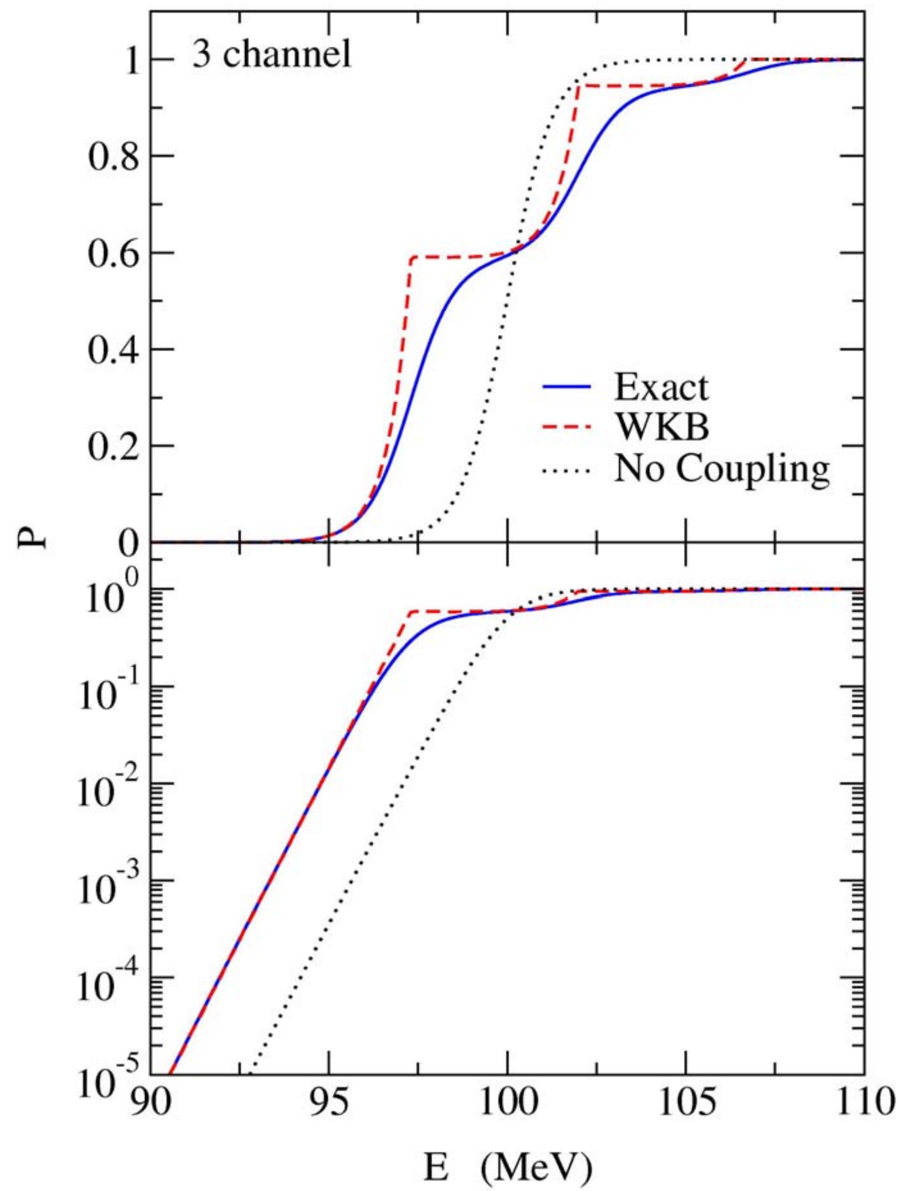
$$\boldsymbol{q}(x) = [2m(E - \boldsymbol{W}(x))/\hbar^2]^{1/2}, \qquad W_{nm}(x) = V_{nm}(x) + \epsilon_n \delta_{n,m}$$

$$\boxed{P_{WKB}(E) = \sum_n |\tau_{n0}|^2 = \sum_n \left|\left(\prod_i e^{i\boldsymbol{q}(x_i)\Delta x}\right)_{n0}\right|^2}$$

1 channel



3 channel

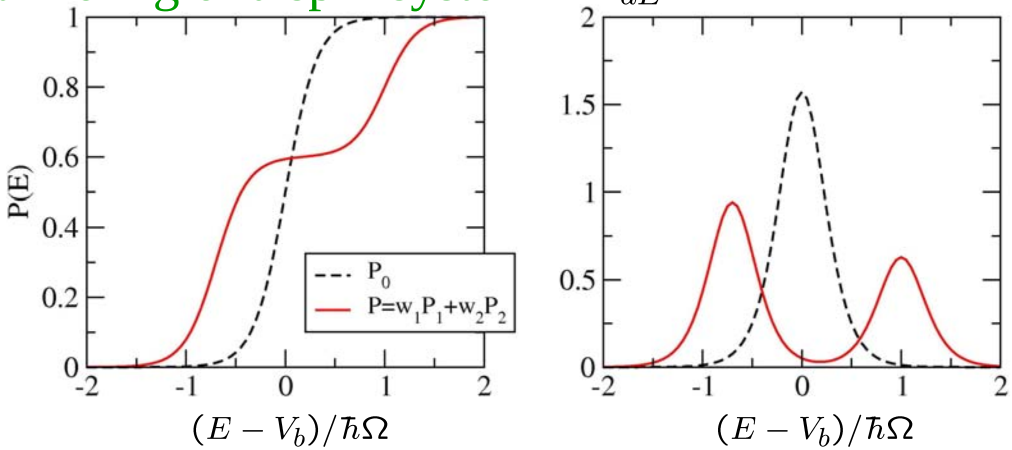


Sub-barrier Fusion and Barrier distribution method

- Fusion takes place by quantum tunneling at low energies
- C.C. effect can be understood in terms of distribution of many barriers
- σ_{fus} is given as an average over the many distributed barriers

$$\begin{aligned}\sigma_{\rm fus}(E) &= \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)\\ &= \frac{\pi}{k^2}\sum_l (2l+1)\left[\int_0^1 d(\cos\theta)P_l(E;\theta)\right]\end{aligned}$$

Tunneling of a spin system



The way how the barrier is distributed can be clearly seen by taking the energy derivative of penetrability

Can one not do a similar thing with fusion cross sections?

One important fact: experimental observable is not penetrability, but fusion cross section

$$P_{l=0}(E) \simeq \frac{1}{\pi R_b^2} \cdot \frac{d(E\sigma_{\rm fus})}{dE}$$

$$\boxed{D_{\rm fus}(E) \equiv \frac{d^2(E\sigma_{\rm fus})}{dE^2} \simeq \pi R_b^2 \frac{dP_{l=0}}{dE}}$$

(Fusion barrier distribution)

N. Rowley, G.R. Satchler, P.H. Stelson, PLB254('91)25

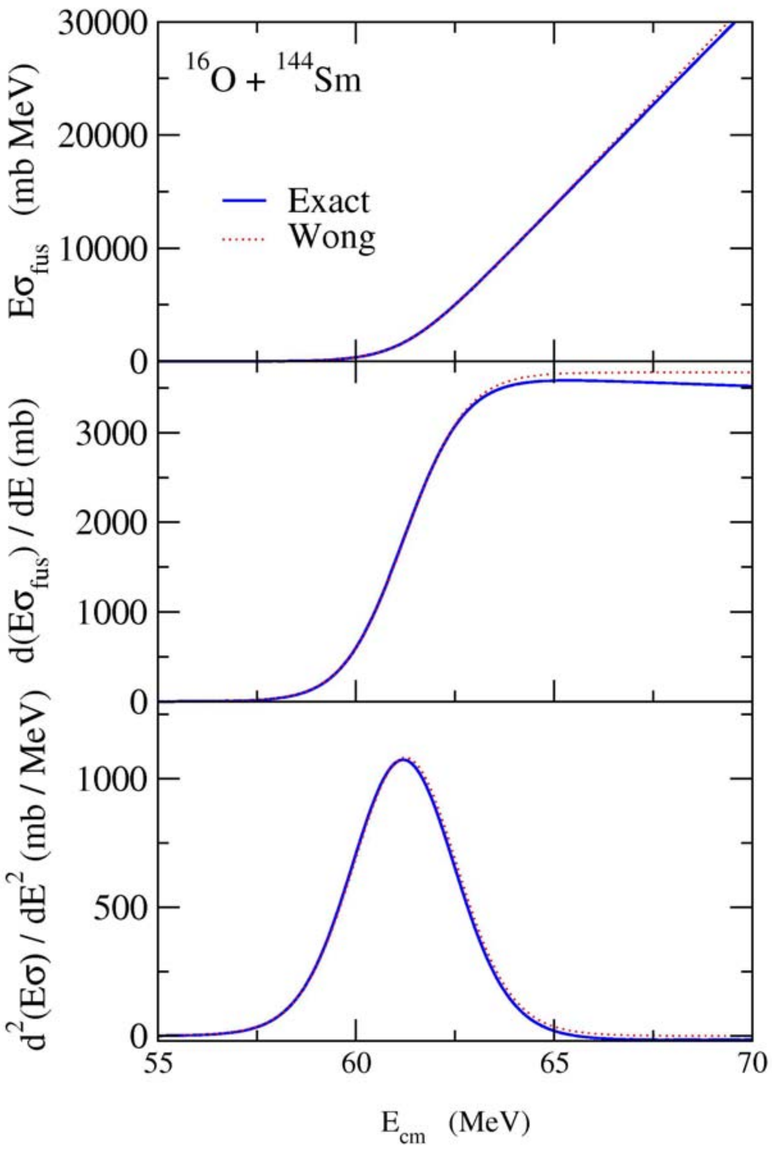
(note) Classical fusion cross section

$$\sigma_{\rm fus}^{cl}(E) = \pi R_b^2 \left(1 - \frac{V_b}{E}\right) \theta(E - V_b)$$

$$\frac{d}{dE}[E\sigma_{\rm fus}^{cl}(E)] = \pi R_b^2\, \theta(E - V_b) = \pi R_b^2\, P_{cl}(E)$$

$$\frac{d^2}{dE^2}[E\sigma_{\rm fus}^{cl}(E)] = \pi R_b^2\, \delta(E - V_b)$$

Fusion Test Function



Classical fusion cross section:

$$\sigma_{\text{fus}}^{cl}(E) = \pi R_b^2 \left(1 - \frac{V_b}{E}\right) \theta(E - V_b)$$

$$\begin{aligned} \frac{d}{dE}[E\sigma_{\text{fus}}^{cl}(E)] &= \pi R_b^2 \, \theta(E - V_b) \\ &= \pi R_b^2 \, P_{cl}(E) \\ \frac{d^2}{dE^2}[E\sigma_{\text{fus}}^{cl}(E)] &= \pi R_b^2 \, \delta(E - V_b) \end{aligned}$$

Tunneling effect

$\longrightarrow$ smears the delta function

Fusion test function:

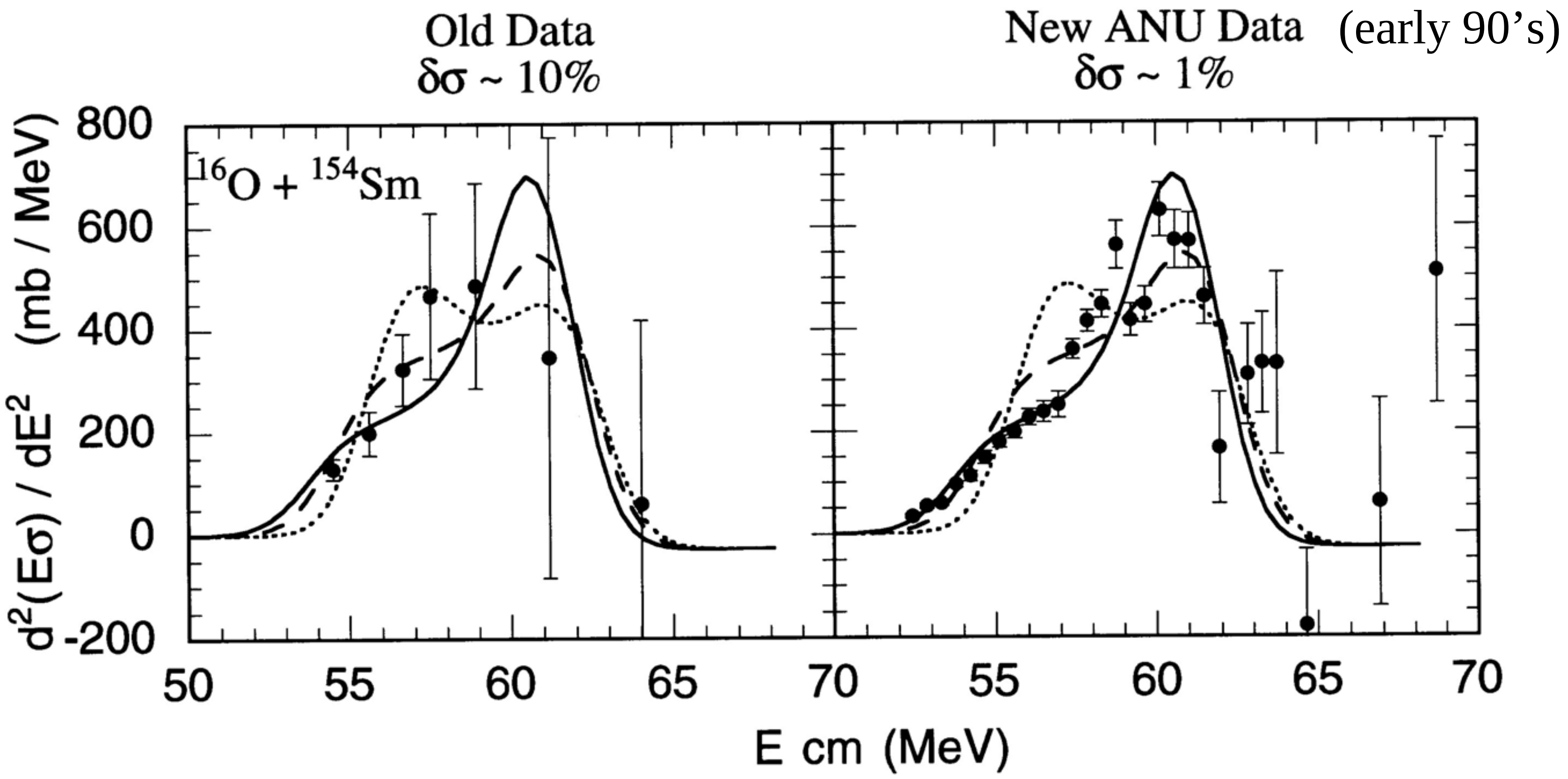
- Symmetric around $E=V_b$
- Centered on $E=V_b$
- Its integral over E is πR_b^2
- Has a relatively narrow width

$(\sim 0.56\hbar\Omega)$

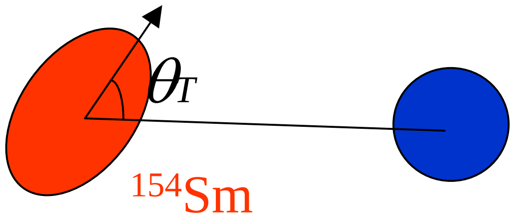
Barrier distribution measurements

Fusion barrier distribution $$D_{\text{fus}}(E) = \frac{d^2(E\sigma)}{dE^2}$$

Needs high precision data in order for the 2nd derivative to be meaningful

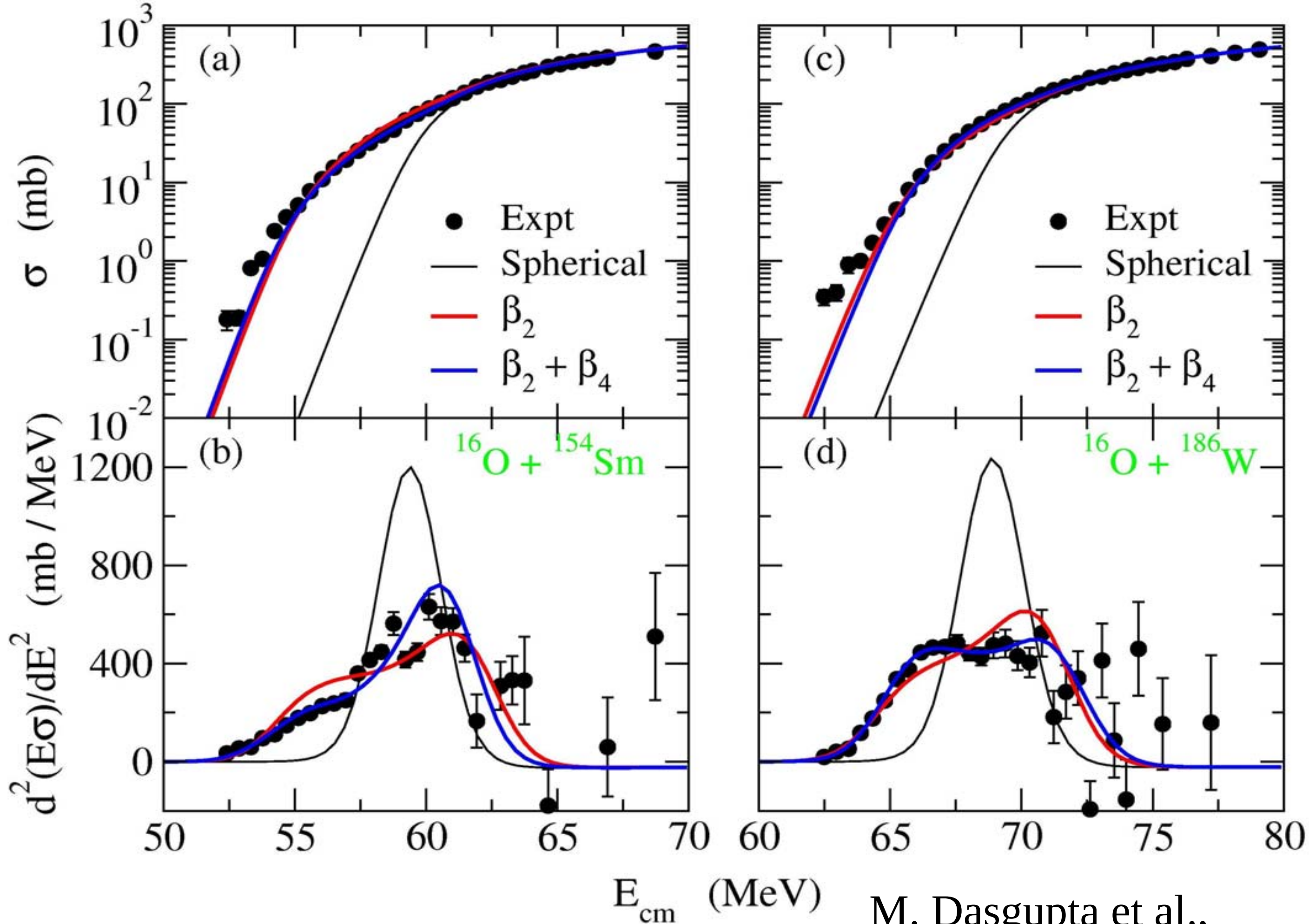


Experimental Barrier Distribution



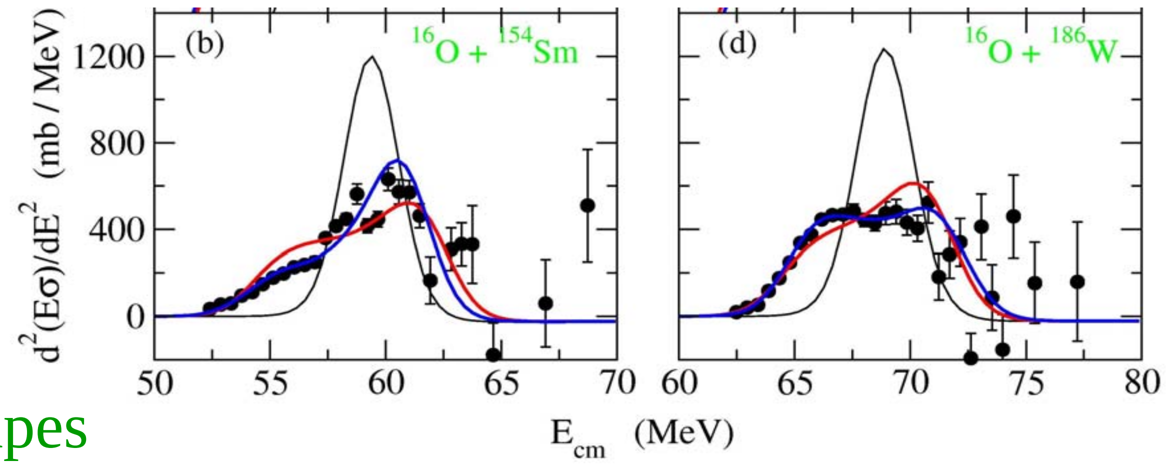
Requires high precision data

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\rm fus}(E;\theta_T)$$



M. Dasgupta et al.,
Annu. Rev. Nucl. Part. Sci. 48('98)401

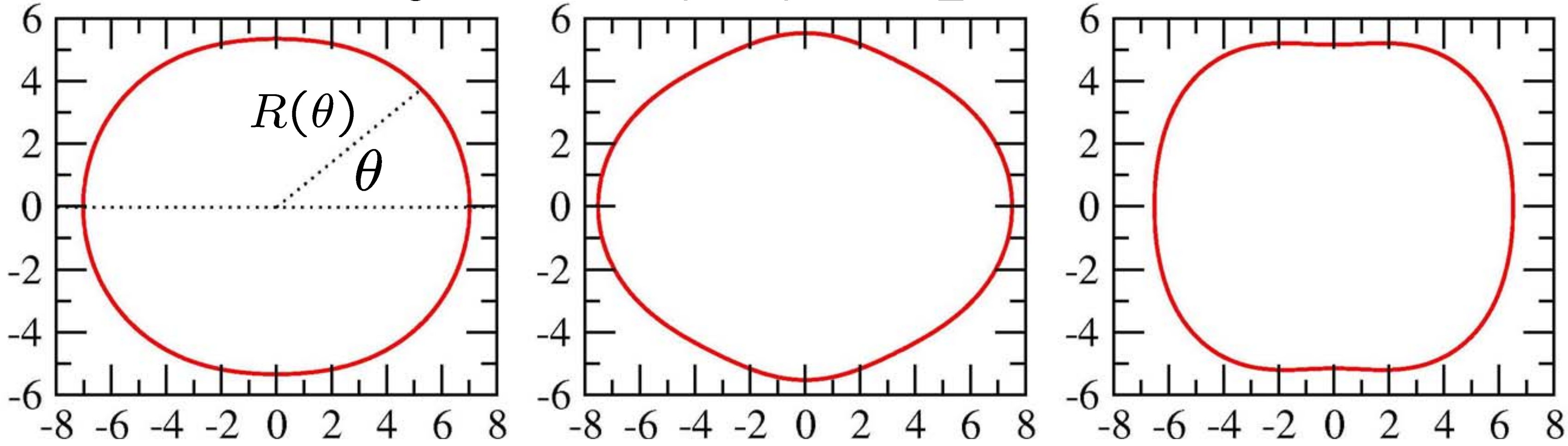
Investigate nuclear shape through barrier distribution



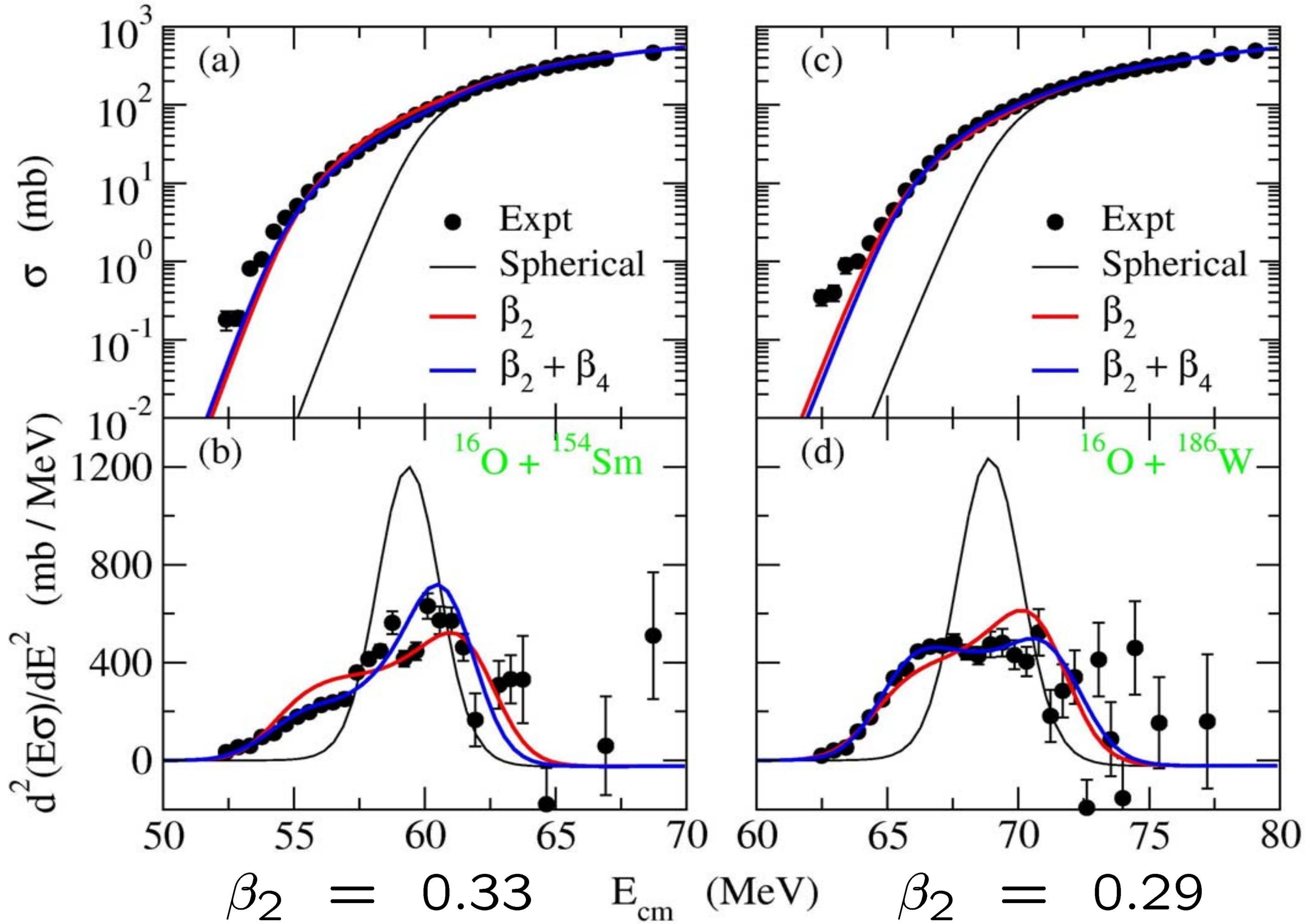
Nuclear shapes

$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

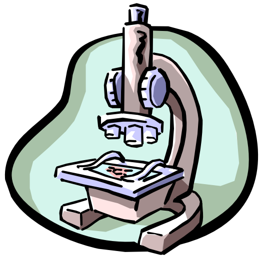
$$R_0 = 5.9 \text{ (fm)}, \quad \beta_2 = 0.3$$



$\beta_4 = 0$ $\qquad$ $\beta_4 = 0.1$ $\qquad$ $\beta_4 = -0.1$



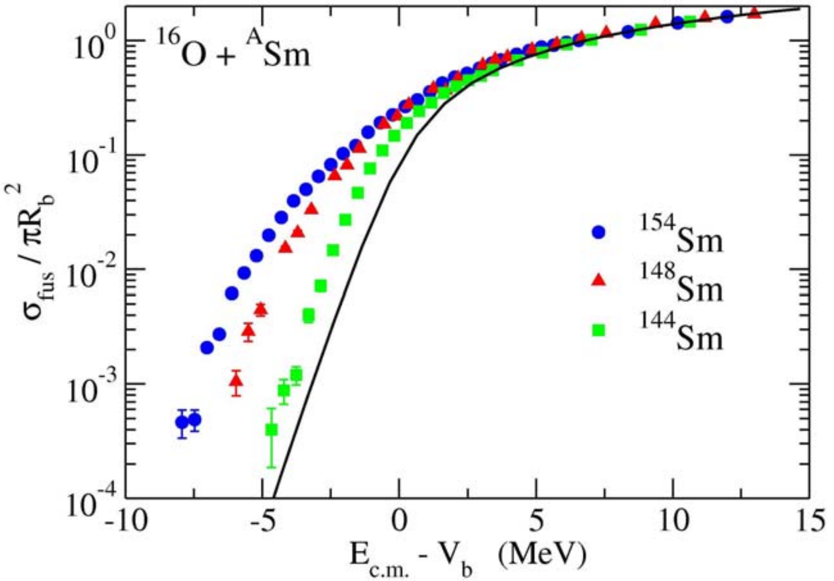
$$\beta_2 = 0.33 \qquad \beta_2 = 0.29$$
$$\beta_4 = +0.05 \qquad \beta_4 = -0.03$$



By taking the barrier distribution, one can very clearly see the difference due to β_4!

→ Fusion as a quantum tunneling microscope for nuclei

Advantage of fusion barrier distribution



Fusion Cross sections

Very strong exponential energy dependence

Difficult to see differences due to details of nuclear structure

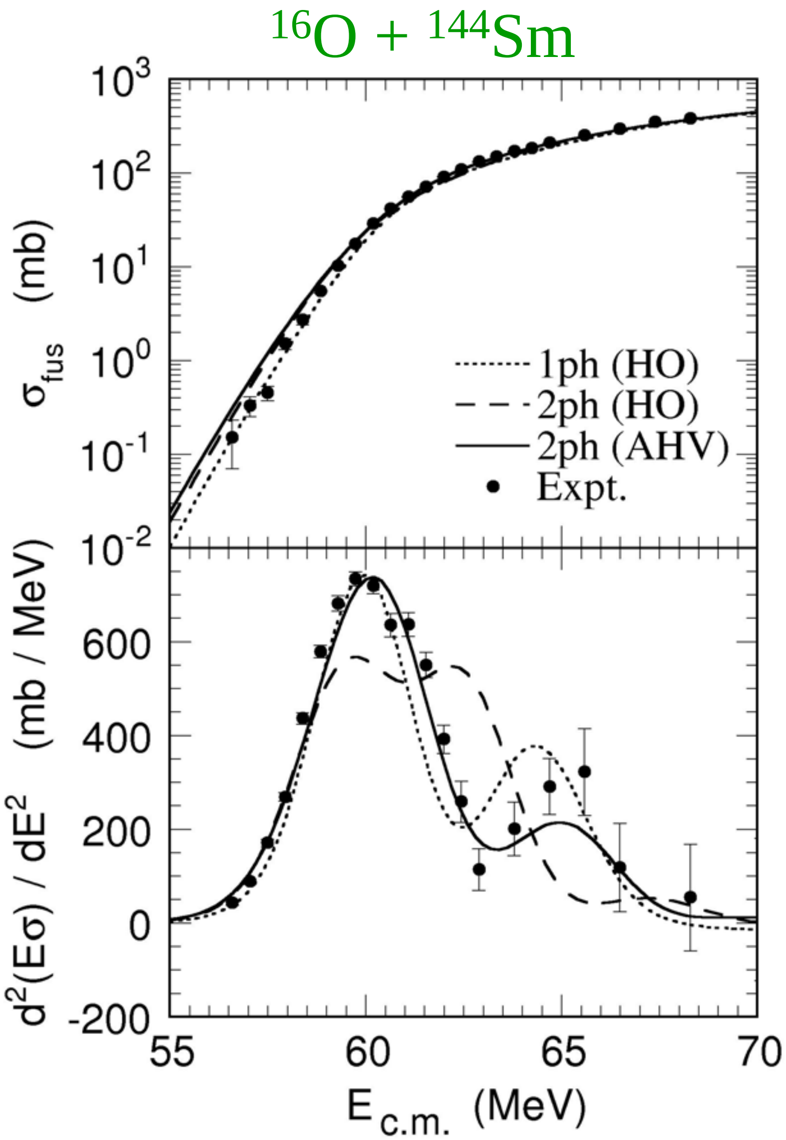
Plot cross sections in a different way: Fusion barrier distribution

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma)}{dE^2}$$

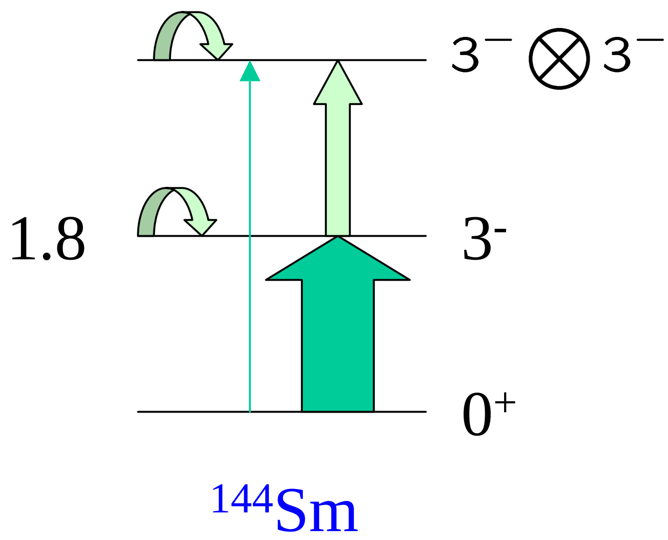
N. Rowley, G.R. Satchler, P.H. Stelson, PLB254('91)25

→ Function which is sensitive to details of nuclear structure

Example for spherical vibrational system



Anharmonicity of octupole vibration

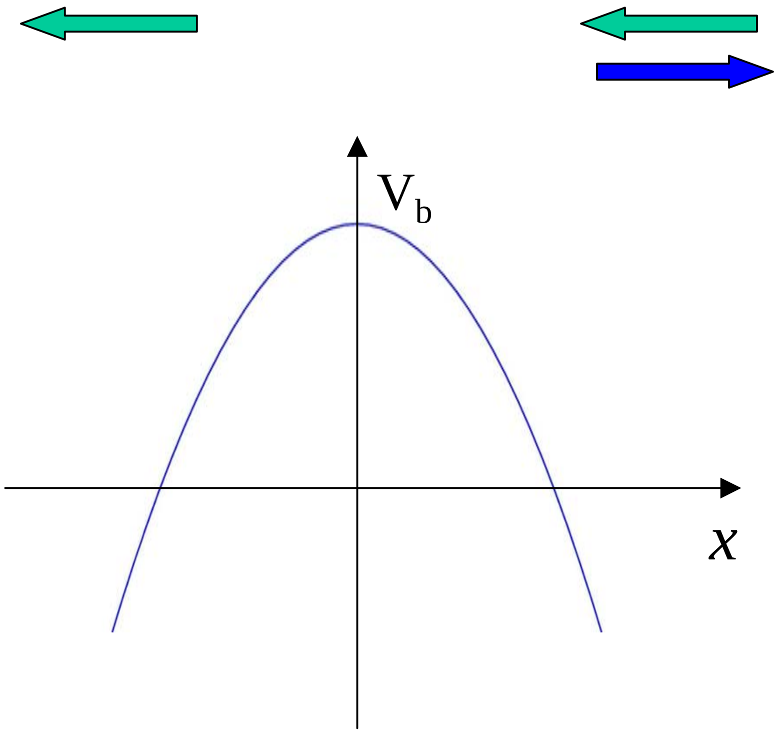


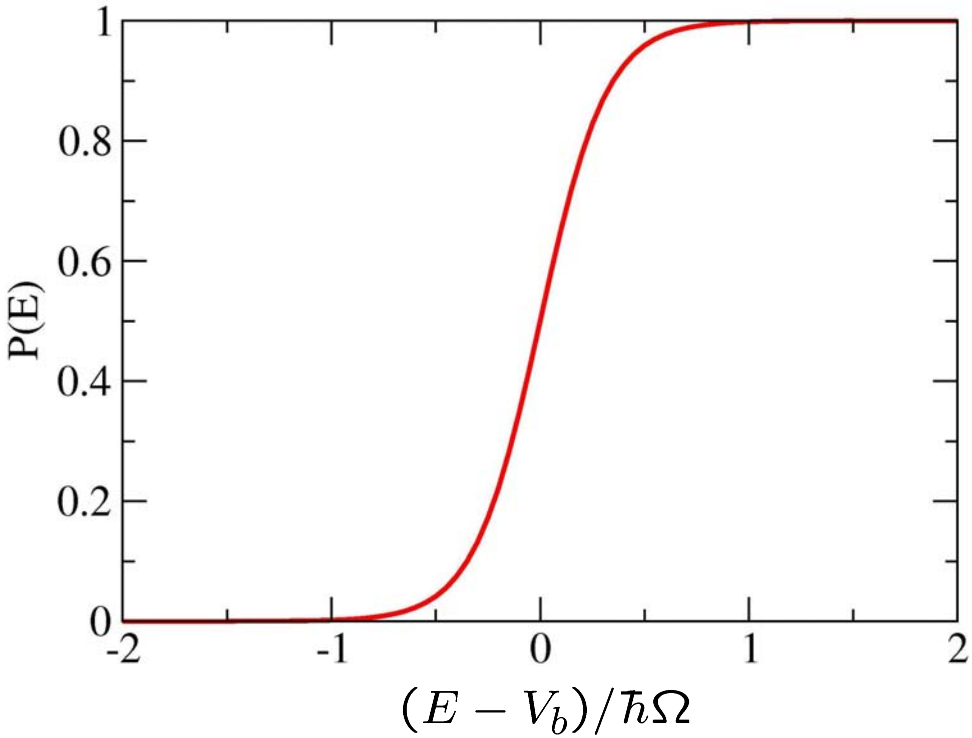
Quadrupole moment:

$$Q(3^-) = -0.70 \pm 0.02\mathrm{b}$$

K.Hagino, N. Takigawa, and S. Kuyucak, PRL79('97)2943

Quantum reflection and quasi-elastic scattering





In quantum mechanics, reflection occurs even at $E > V_b$

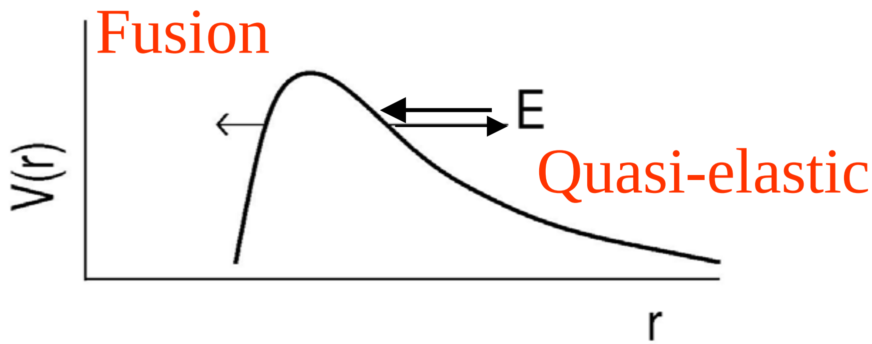
→ Quantum Reflection

$$P(E) + R(E) = 1$$

Reflection prob. carries the same information as penetrability, and barrier distribution can be defined in terms of reflection prob.

Quasi-Elastic Scattering

A sum of all the reaction processes other than fusion (elastic + inelastic + transfer + ……)

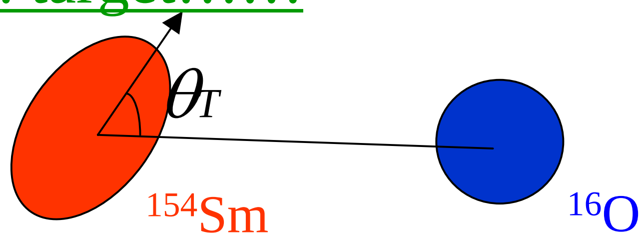


Detect all the particles which reflect at the barrier and hit the detector

Related to reflection

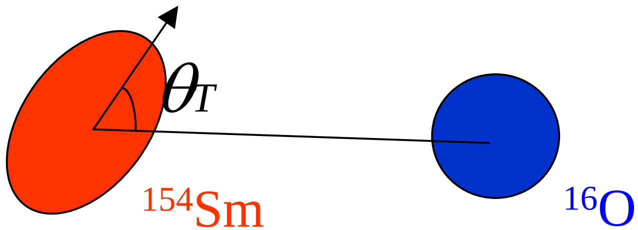
Complementary to fusion

In case of a def. target……



$$\begin{cases} \sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\rm fus}(E;\theta_T) \\ \sigma_{\rm qel}(E,\theta) = \sum_I \sigma(E,\theta) = \int_0^1 d(\cos\theta_T)\sigma_{\rm el}(E,\theta;\theta_T) \end{cases}$$

Quasi-elastic barrier distribution



$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta_T)\sigma_{\text{fus}}(E;\theta_T)$$

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

$$\sigma_{\text{qel}}(E,\theta) = \sum_I \sigma(E,\theta) = \int_0^1 d(\cos\theta_T)\sigma_{\text{el}}(E,\theta;\theta_T)$$

Quasi-elastic barrier distribution:

$$\boxed{D_{\text{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\text{qel}}(E,\pi)}{\sigma_R(E,\pi)}\right)}$$

H. Timmers et al.,
NPA584('95)190

(note) Classical elastic cross section in the limit of strong Coulomb field:

$$\sigma_{\text{el}}^{cl}(E,\pi) = \sigma_R(E,\pi)\theta(V_b - E)$$

$$\frac{\sigma_{\text{el}}^{cl}(E,\pi)}{\sigma_R(E,\pi)} = \theta(V_b - E) = R(E)$$

Quasi-elastic Test Function

Classical elastic cross section (in the limit of a strong Coulomb):

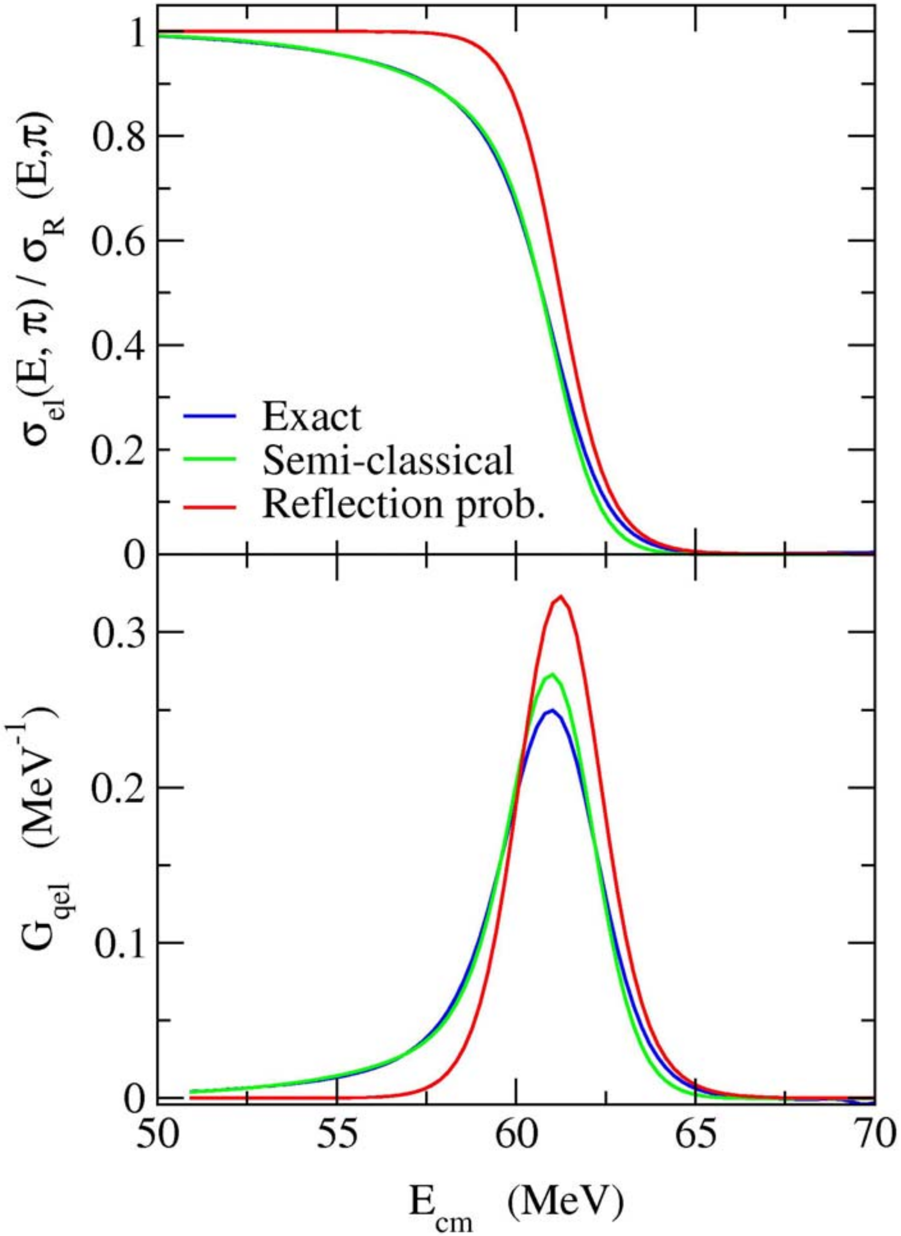
$$\sigma_{\mathrm{el}}^{cl}(E,\pi) = \sigma_R(E,\pi)\theta(V_b - E)$$

$$\boxed{\begin{aligned} \frac{\sigma_{\mathrm{el}}^{cl}(E,\pi)}{\sigma_R(E,\pi)} &= \theta(V_b - E) = R(E) \\ -\frac{d}{dE}\left(\frac{\sigma_{\mathrm{el}}^{cl}(E,\pi)}{\sigma_R(E,\pi)}\right) &= \delta(E - V_b) \end{aligned}}$$

Nuclear effects $\longleftarrow$ Semi-classical perturbation theory

$$\boxed{\frac{\sigma_{\mathrm{el}}(E,\pi)}{\sigma_R(E,\pi)} \sim \left(1 + \frac{V_N(r_c)}{ka}\frac{\sqrt{2a\pi k\eta}}{E}\right)\cdot R(E)}$$

S. Landowne and H.H. Wolter, NPA351('81)171
K.H. and N. Rowley, PRC69('04)054610



Quasi-elastic test function

$$\boxed{G_{\mathsf{qel}}(E) \equiv -\frac{d}{dE}\left(\frac{\sigma_{\mathsf{el}}(E,\pi)}{\sigma_R(E,\pi)}\right)}$$

- ➢The peak position slightly deviates from V_b
- ➢Low energy tail
- • Integral over E: unity
- • Relatively narrow width

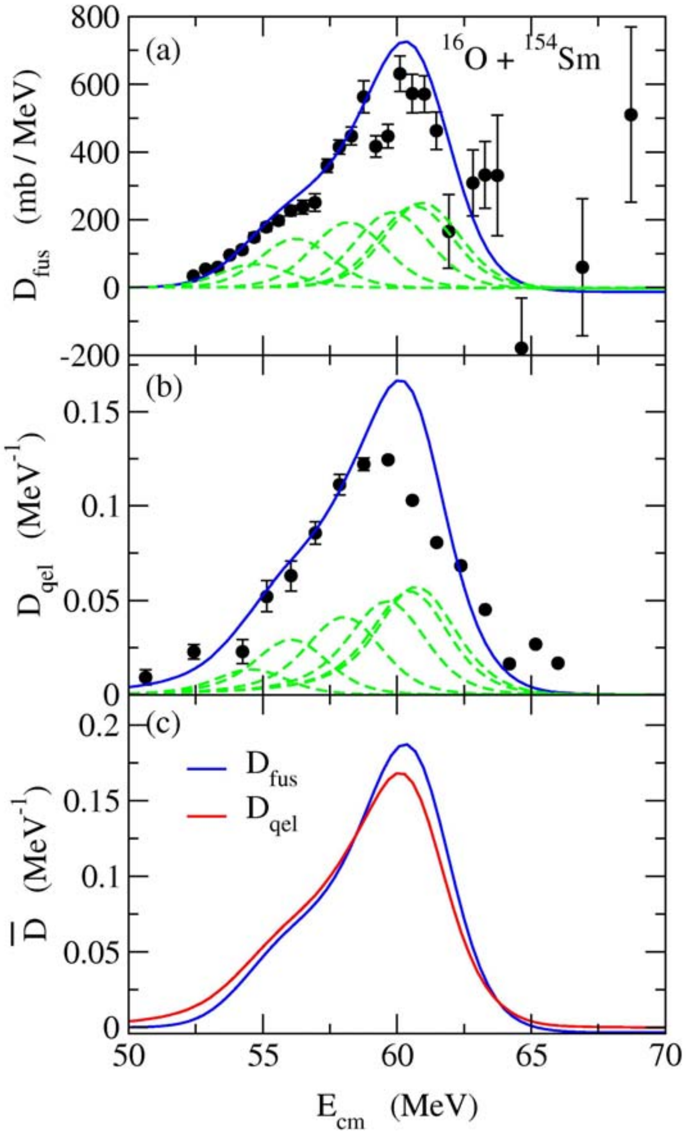
➡

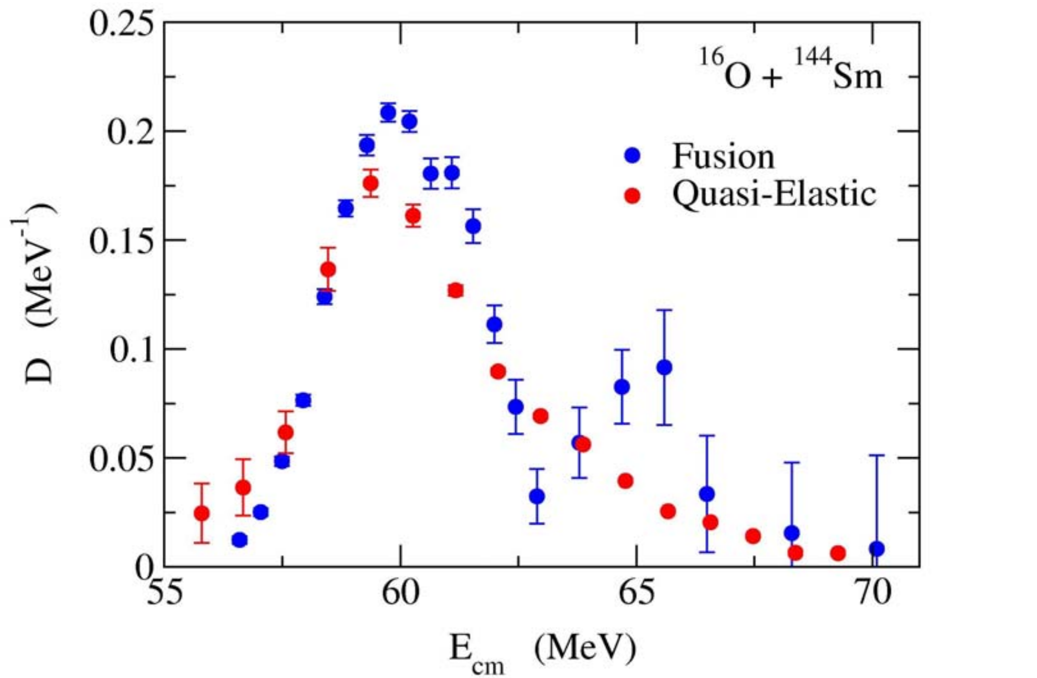
Close analog to fusion b.d.

Comparison of D_{fus} with D_{qel}

Fusion $$D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2}$$

Quasi-elastic $$D_{\rm qel}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\rm qel}(E,\pi)}{\sigma_R(E,\pi)}\right)$$





H. Timmers et al., NPA584('95)190

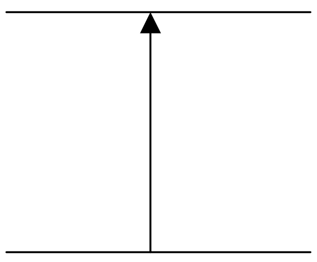
A gross feature is similar to each other

K.H. and N. Rowley, PRC69('04)054610

Comment on the iso-centrifugal approximation

Inelastic excitations: Coupled-channels method

Inherent problem $\longrightarrow$ dimensionality



2^+ ***l=J-2, J, J+2***

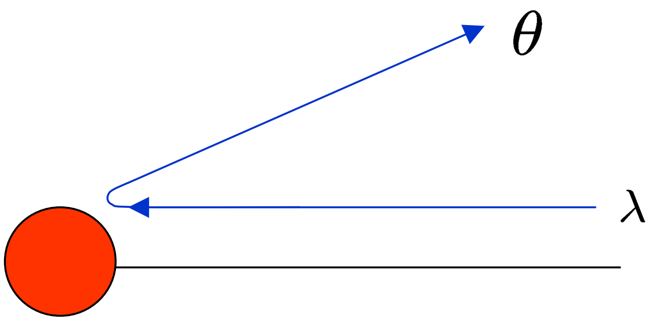
0^+ ***l=J***

J=l+I

Truncation	Dimension
2^+	4 $\longrightarrow$ 2
4^+	9 $\longrightarrow$ 3
6^+	16 $\longrightarrow$ 4
8^+	25 $\longrightarrow$ 5

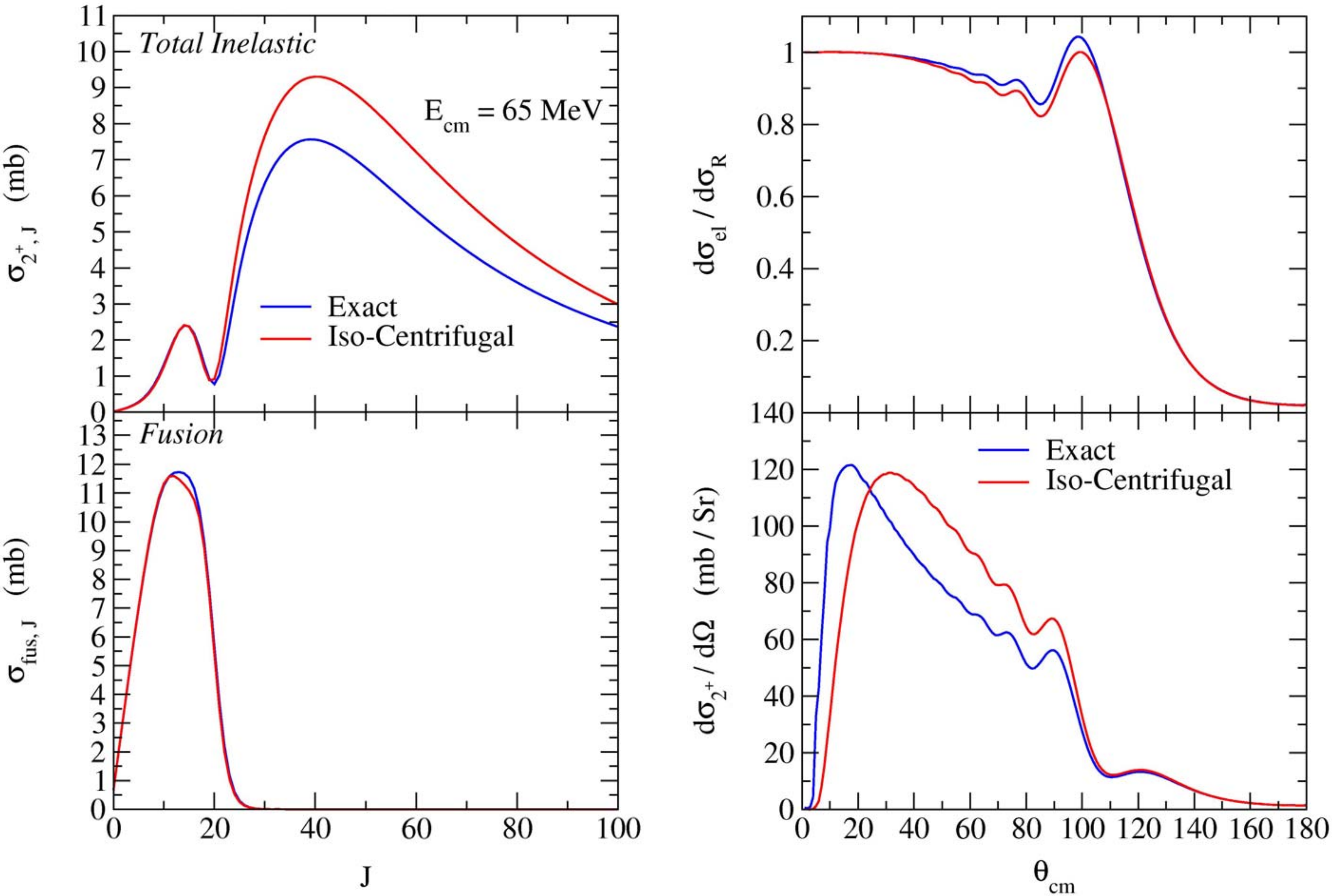
Iso-centrifugal approximation:

λ : independent of excitations



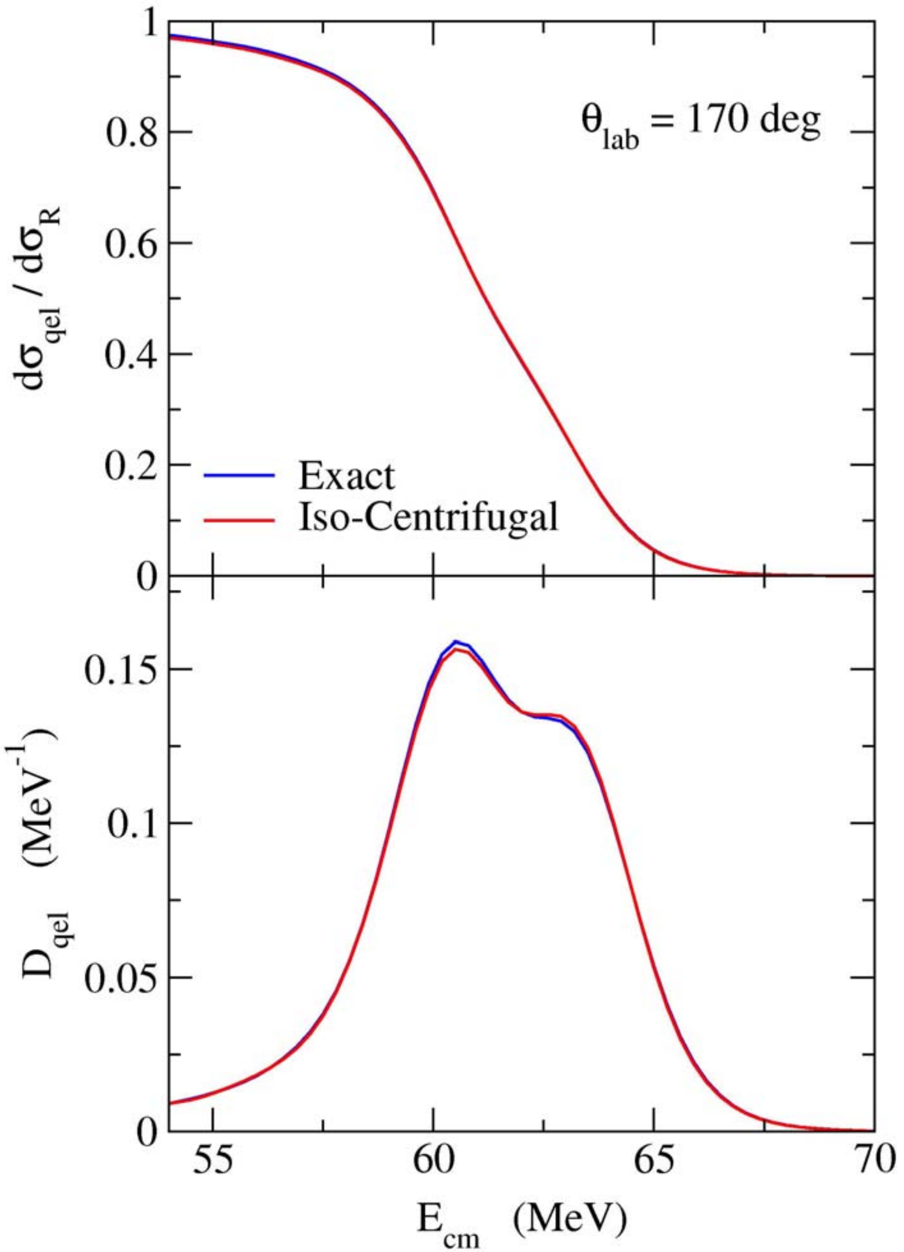
- Works well for fusion
- Not successful for scattering

$^{16}O + ^{144}Sm\ (2^{+})$



Total Inelastic
E_cm = 65 MeV
Exact
Iso-Centrifugal
Fusion
σ_2+,J (mb)
σ_fus,J (mb)
J
dσ_el / dσ_R
dσ_2+ / dΩ (mb / Sr)
θ_cm

$^{16}O + ^{144}Sm\ (2^+)$



Backward scattering

↔ Small ang. mom.

Iso-centrifugal approximation works good enough

- Simplifies C.C. calculations
- Ensures the similarities between D_{fus} and D_{qel} in CC systems

Experimental advantages for D_{qel}

$$D_{\mathrm{qel}}(E) = -\frac{d}{dE}\left(\frac{\sigma_{\mathrm{qel}}(E,\pi)}{\sigma_R(E,\pi)}\right) \qquad D_{\mathrm{fus}}(E) = \frac{d^2(E\sigma_{\mathrm{fus}})}{dE^2}$$

- less accuracy is required in the data (1st vs. 2nd derivative)
- much easier to be measured

Qel : a sum of everything

$\longrightarrow$ a very simple charged-particle detector

Fusion : requires a specialized recoil separator

to separate ER from the incident beam

ER + fission for heavy systems

- several effective energies can be measured at a single-beam energy $\longleftrightarrow$ relation between a scattering angle and an impact parameter

$$\boxed{E_{\mathrm{eff}} = 2E\,\sin(\theta/2)/[1+\sin(\theta/2)]}$$

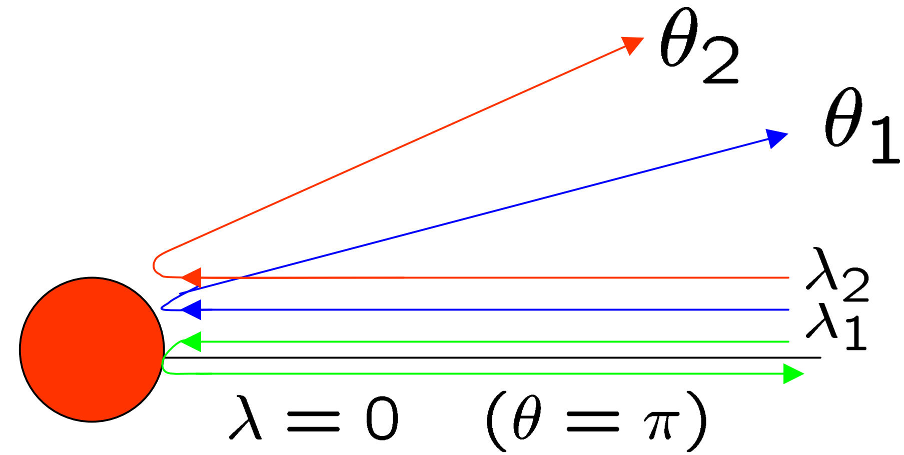
$\Rightarrow$ measurements with a cyclotron accelerator: possible

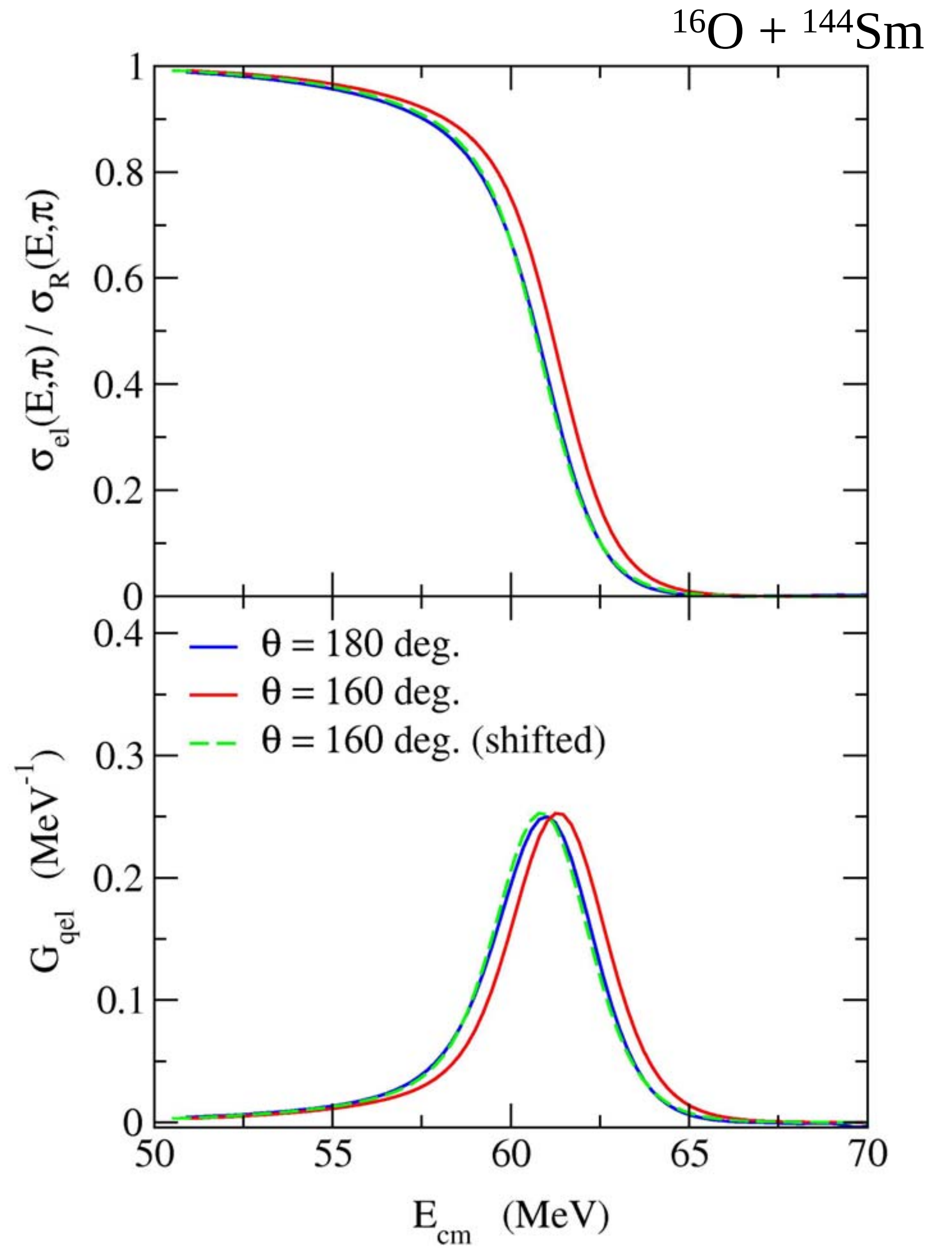
$\Rightarrow$ **Suitable for low intensity exotic beams**

Qel: will open up a possibility to study the structure of unstable nuclei

Scaling property of D_{qel}

Expt.: impossible to perform
at $\theta = \pi$
⟶ Relation among different θ?

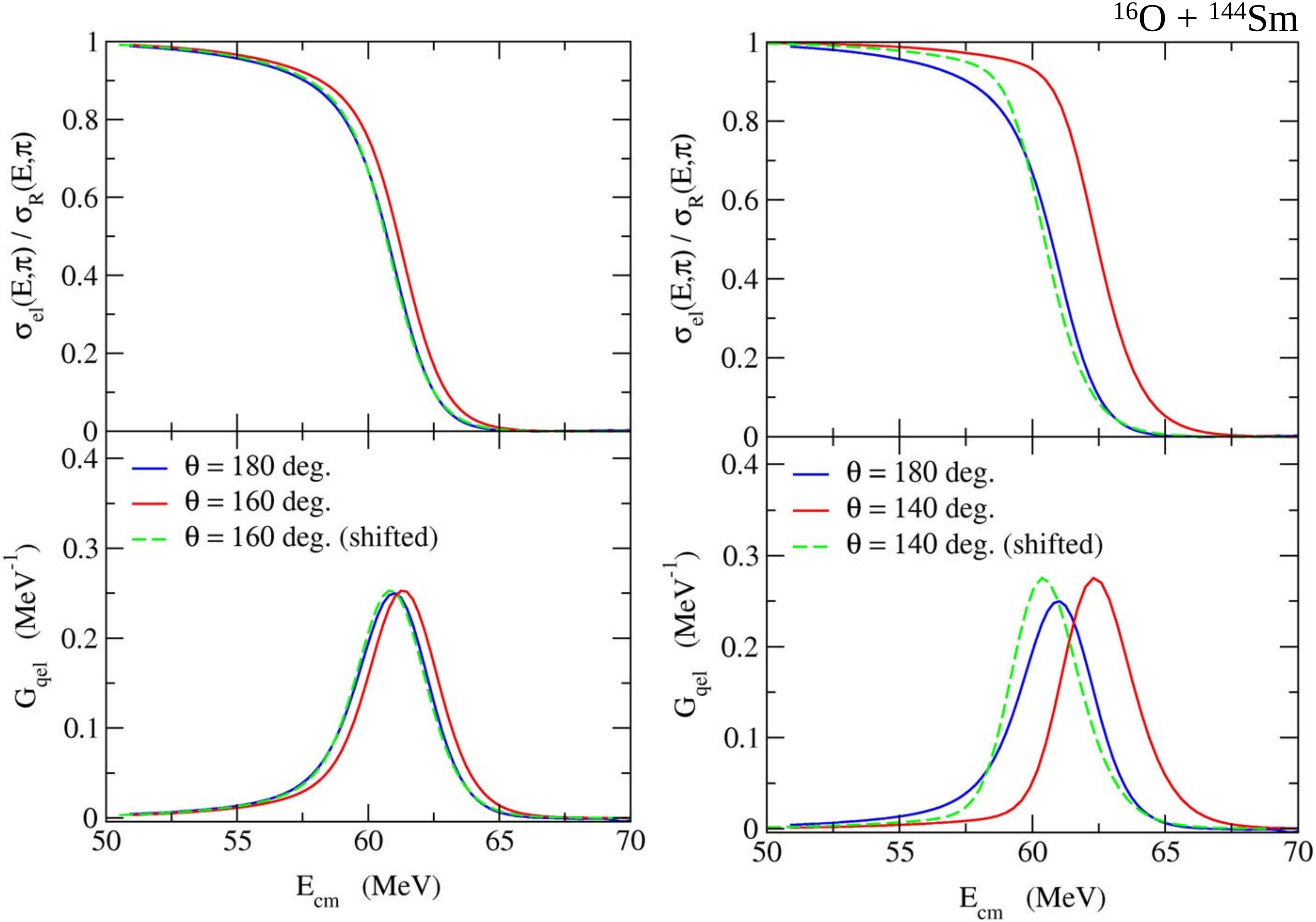




Effective energy:

$$\begin{aligned} E_{\text{eff}} &\sim E - \frac{\lambda_c^2 \hbar^2}{2\mu r_c^2} \\ &= 2E\,\frac{\sin(\theta/2)}{1+\sin(\theta/2)} \\ D_{\text{qel}}(E,\theta) &\sim D_{\text{qel}}(E_{\text{eff}},\pi) \end{aligned}$$

$$\lambda_c = \eta \cot(\theta/2)$$



^{16}O + ^{144}Sm
$\sigma_{el}(E,\pi) / \sigma_R(E,\pi)$
G_{qel} (MeV^{-1})
θ = 180 deg.
θ = 160 deg.
θ = 160 deg. (shifted)
θ = 180 deg.
θ = 140 deg.
θ = 140 deg. (shifted)
E_{cm} (MeV)

Future experiments with radioactive beams

Fusion barrier distribution: requires high precision measurements for σ_{fus}

Radioactive beams: much lower beam intensity
than beams of stable nuclei

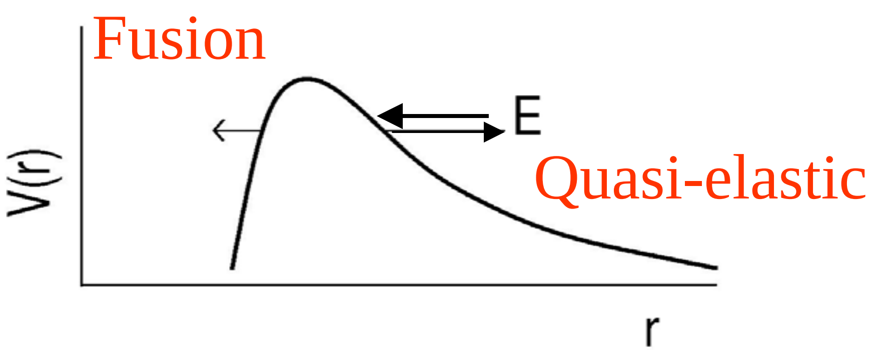
Unlikely for high precision data at this moment

Possible to extract barrier distribution in other ways?

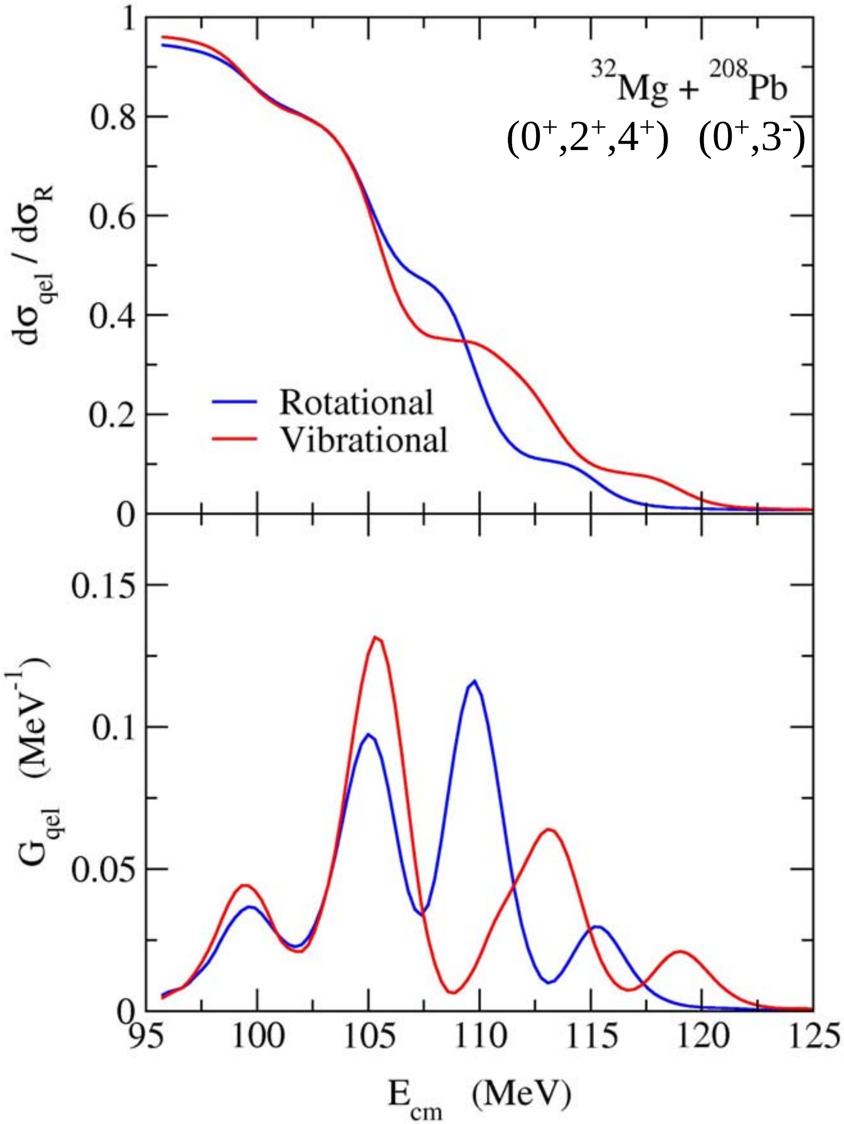
Exploit ***reflection prob.***
instead of ***penetrability***

$$\boldsymbol{P + R = 1}$$

Quasi-elastic scattering



D_{qel} measurements with radioactive beams



K.H. and N. Rowley, PRC69('04)054610

Low intensity radioactive beams:

High precision fusion measurements
$\longrightarrow$ still difficult

Quasi-elastic measurements
$\longrightarrow$ may be possible

Example: $^{32}Mg + ^{208}Pb$

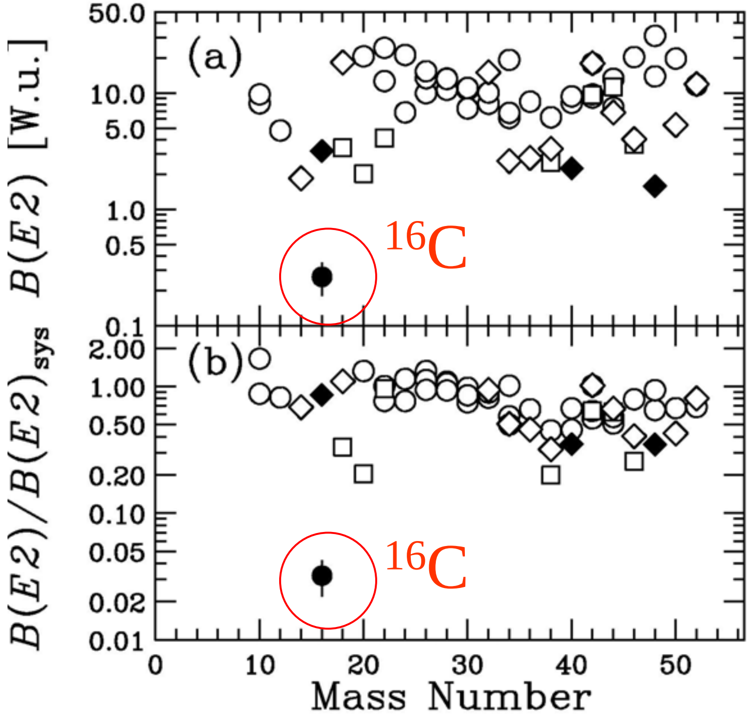
^{32}Mg: breaking of the N=20 shell?

Expt. at RIKEN and GANIL:
large B(E2) and small E_{2+}
$\longleftrightarrow$ deformation?

MF calculations $\longrightarrow$ spherical?

$E_{4+}/E_{2+} = 2.62$

Investigation of collective excitations unique to n-rich nuclei



N. Imai et al., PRL92('04)062501

Expt. at RIKEN:

E_{2+} = 1.766 MeV
B(E2) = 0.26$\pm$ 0.05W.u.
Mn/Mp=7.6$\pm$1.7

^{16}C

Recent expt: very small B(E2)

- Different (static) deformation between n and p?
- Neutron excitation of spherical nuclei?

Reference cross sections

Does break-up hinder/enhance fusion cross sections?

How to choose reference cross sections?

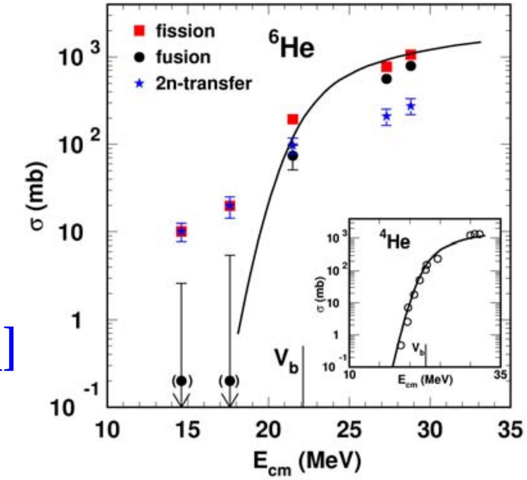
→ Fusion enhancement/hindrance compared to ***what***?

i) Comparison to tightly-bound systems

^{11}Be + ^{209}Bi ⟷ ^{10}Be + ^{209}Bi

^{6}He + ^{238}U ⟷ ^{4}He + ^{238}U

Separation between static and dynamical effects?



R. Raabe et al. Nature('04)

ii) Measurement of average fusion barrier

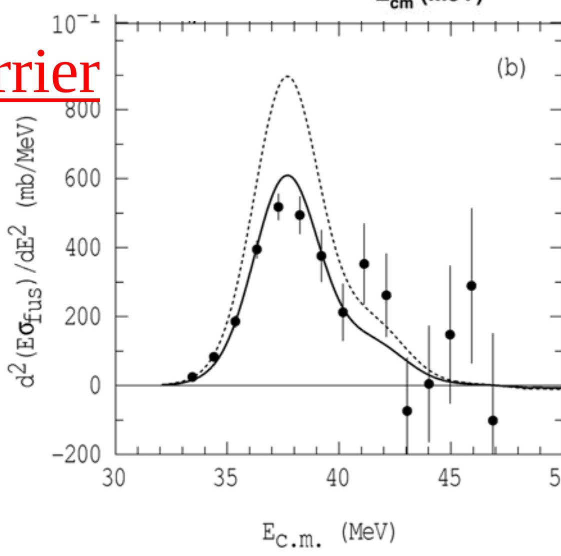
← Fusion barrier distribution

^{9}Be + ^{208}Pb

^{6,7}Li + ^{209}Bi

↕

Neutron-rich nuclei ⟶ $D_{qel}(E)$



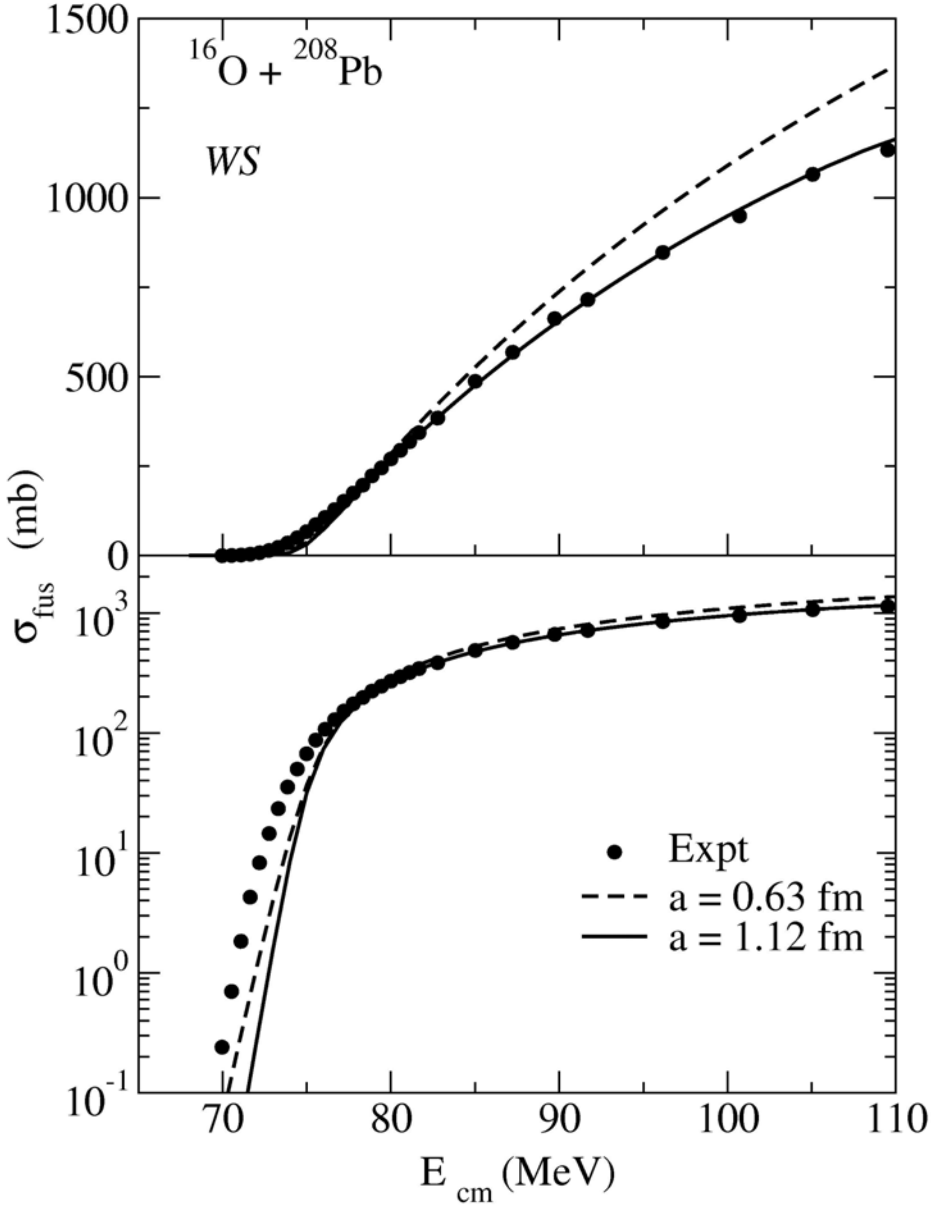
M. Dasgupta et al. PRL82('99)1395

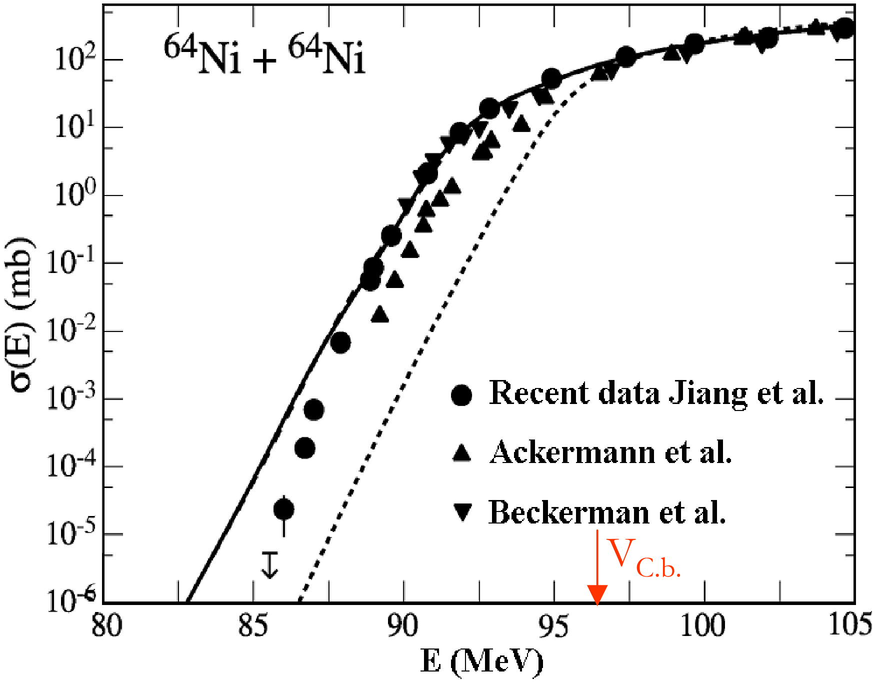
Surface diffuseness problem

$$V_N(r) = -V_0/[1+\exp((r-R_0)/a)]$$

Scattering processes: a ~ 0.63 fm

Fusion: a = 0.75 ~ 1.5 fm

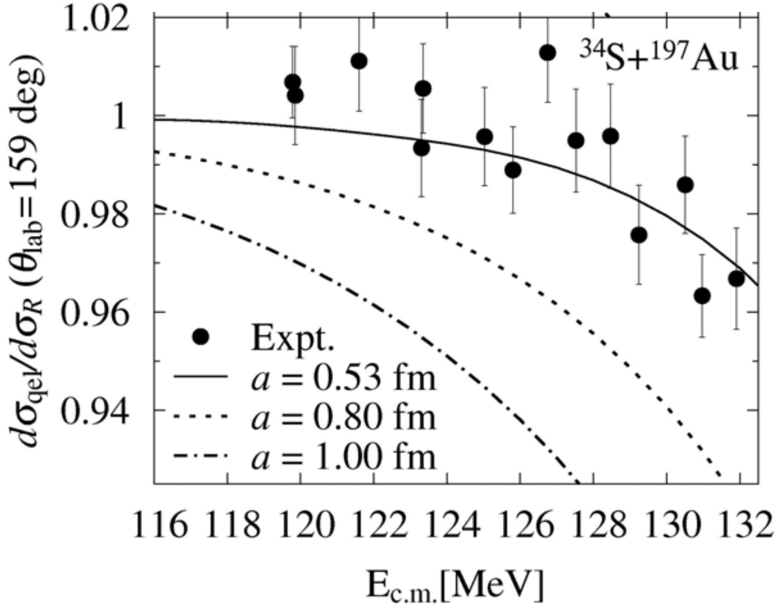


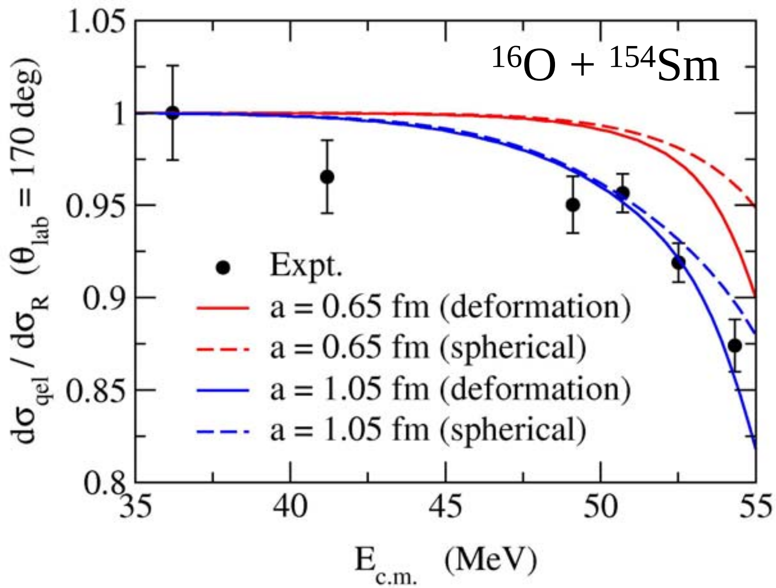


C.L. Jiang et al., PRL93('04)012701

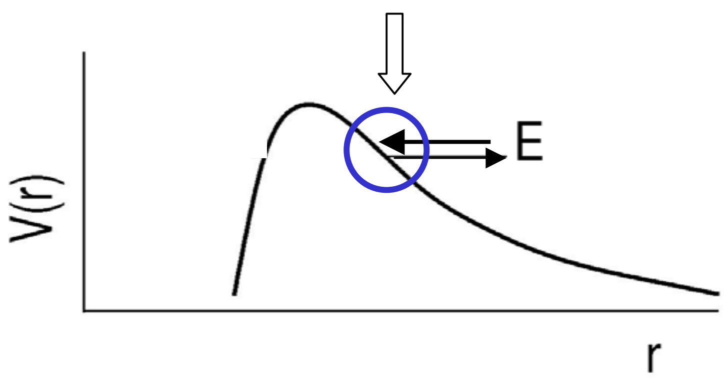
Quasi-elastic scattering at deep subbarrier energies?

K.H., T. Takehi, A.B. Balantekin, and N. Takigawa, PRC71('05) 044612
K. Washiyama, K.H., M. Dasgupta, PRC73('06) 034607



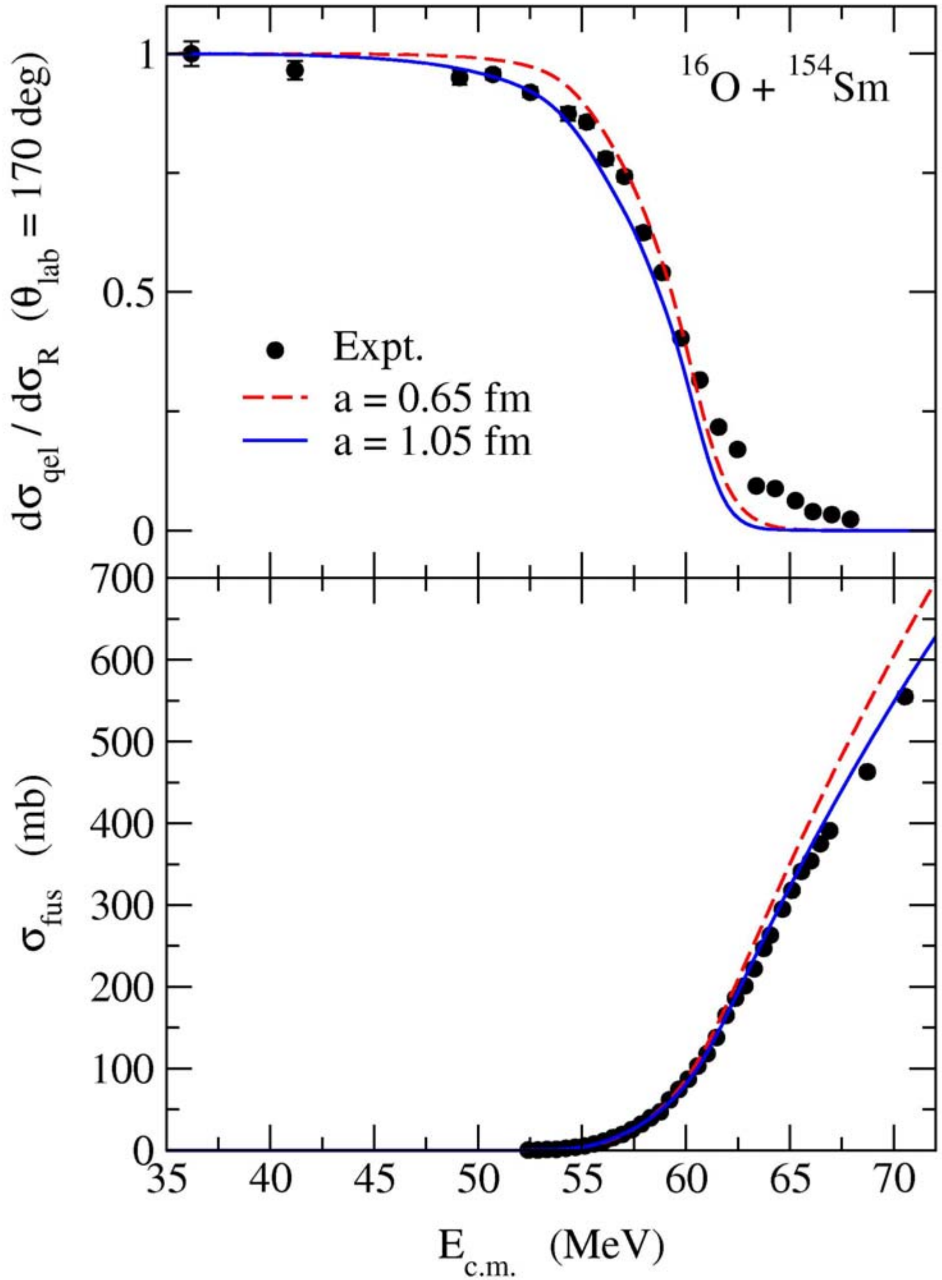


QEL at deep subbarrier energies: sensitive only to the surface region



$$\frac{\sigma_{\rm el}(E,\pi)}{\sigma_R(E,\pi)} \sim \left(1+\frac{V_N(r_c)}{ka}\frac{\sqrt{2a\pi k\eta}}{E}\right)\cdot R(E)$$

$$\sim 1+\frac{V_N(r_c)}{ka}\frac{\sqrt{2a\pi k\eta}}{E}$$



^{16}O + ^{154}Sm
Expt.
a = 0.65 fm
a = 1.05 fm
$d\sigma_{qel} / d\sigma_R$ (θ_{lab} = 170 deg)
σ_{fus} (mb)
$E_{c.m.}$ (MeV)

Application to SHE

Synthesis of superheavy elements: extremely small cross sections

Important to choose the optimum incident energy

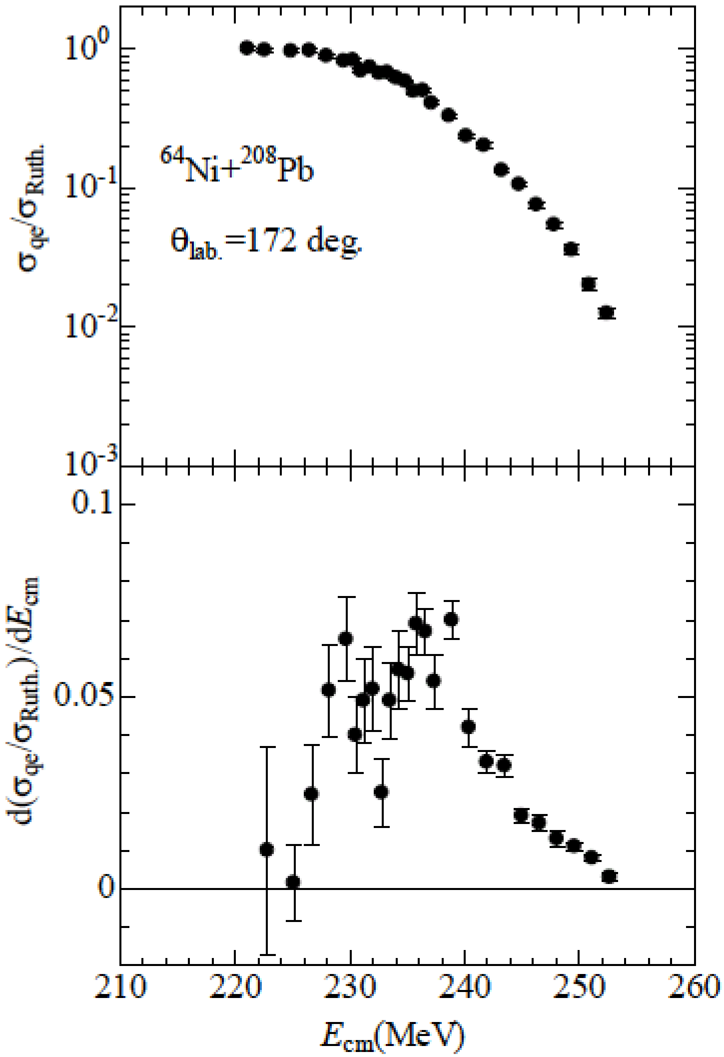
Absence of the barrier height systematics

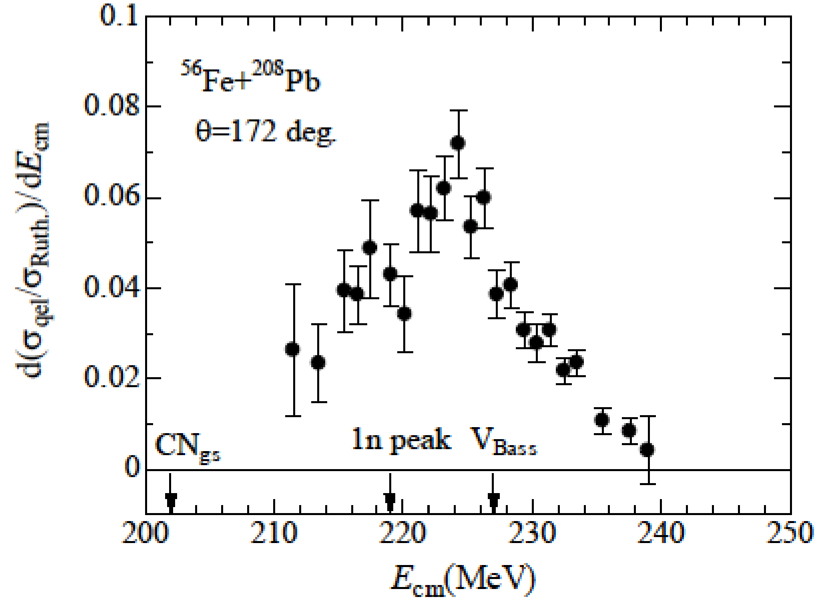
Determine the fusion barrier height for SHE using D_{qel}

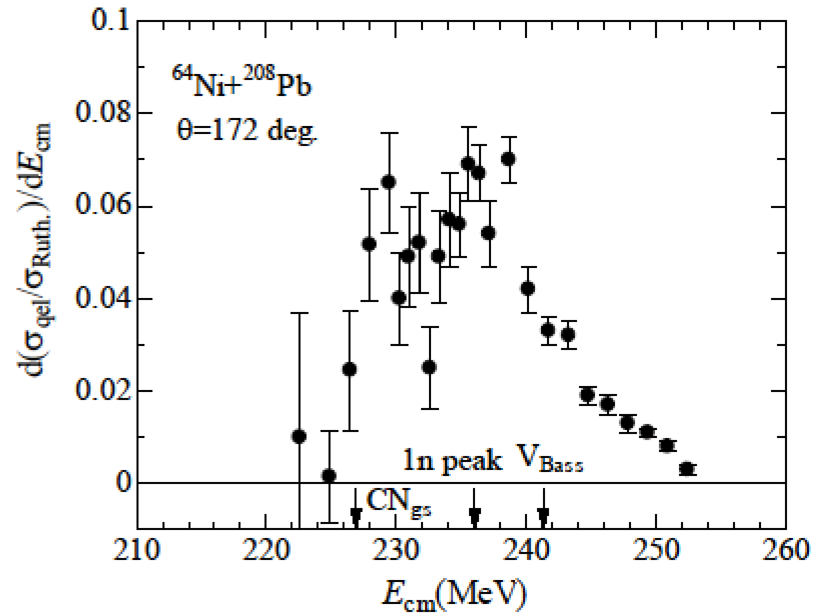
Future plan at JAERI

Cold fusion reactions: ^{50}Ti,^{54}Cr,^{58}Fe,^{64}Ni,^{70}Zn+^{208}Pb,^{209}Bi

Preliminary data

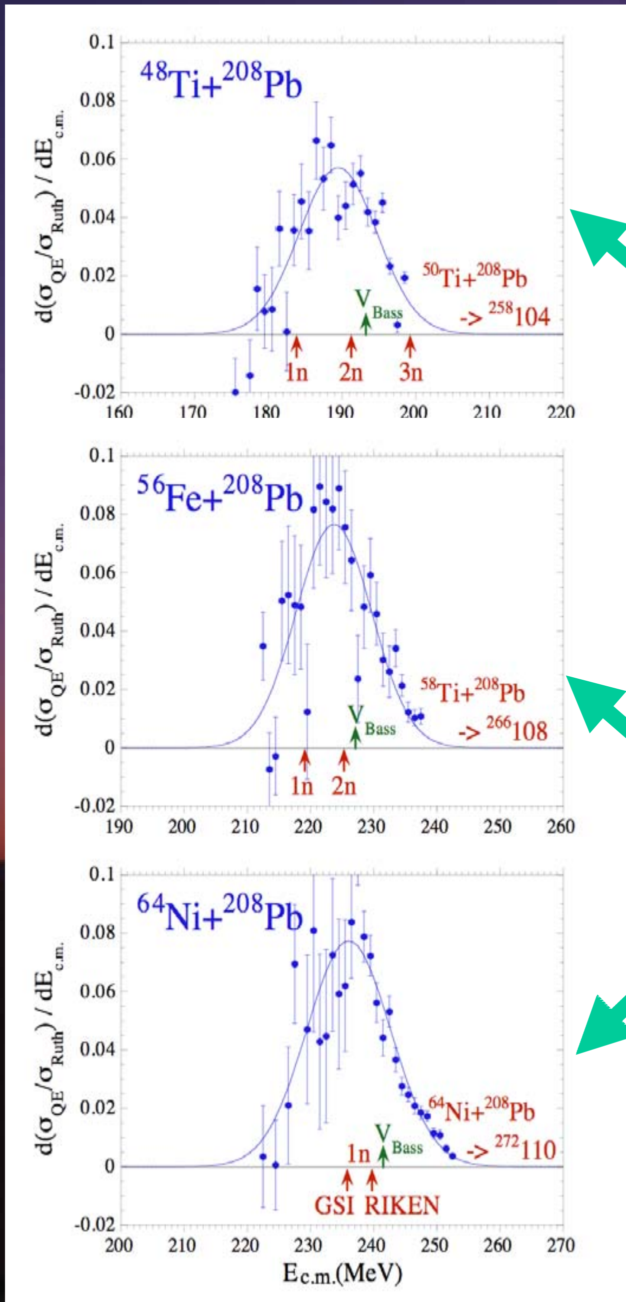




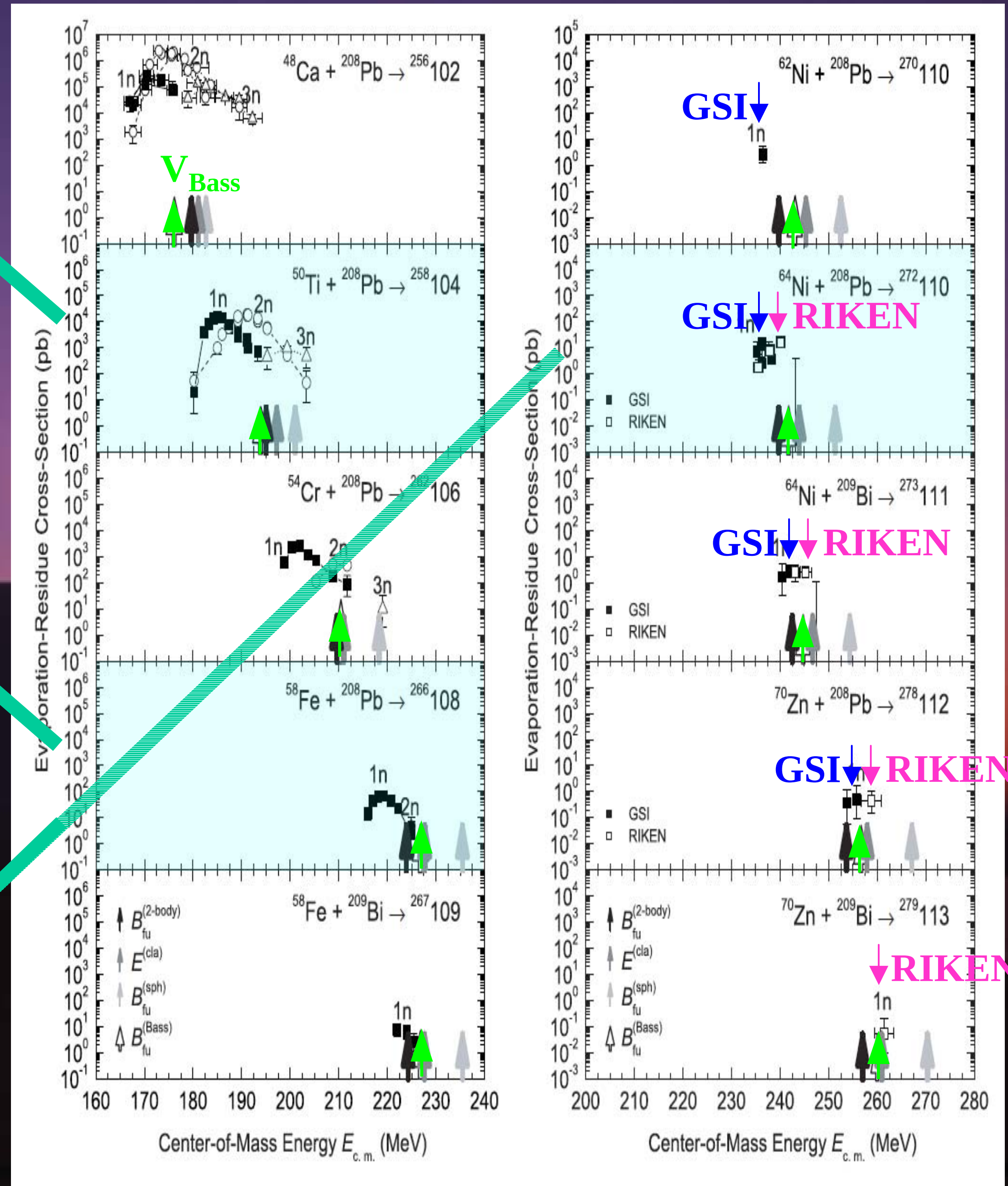


S. Mitsuoka, H. Ikezoe, K. Nishio, K. Tsuruta,
S.C. Heong, Y.X. Watanabe ('05)

Present data



Evaporation residue cross section by GSI and RIKEN



Comparison

Summary

Heavy-Ion Fusion Reactions around the Coulomb Barrier

✧Fusion and quantum tunneling

Fusion takes place by quantum tunneling

✧Basics of the Coupled-channels method

Collective excitations during fusion

✧Concept of Fusion barrier distribution

Sensitive to nuclear structure

$$D_{\mathsf{fus}}(E) = \frac{d^2(E\sigma_{\mathsf{fus}})}{dE^2}$$

✧Quasi-elastic scattering and quantum reflection

Complementary to fusion

Computer program: CCFULL

http://www.nucl.phys.tohoku.ac.jp/~hagino/ccfull.html