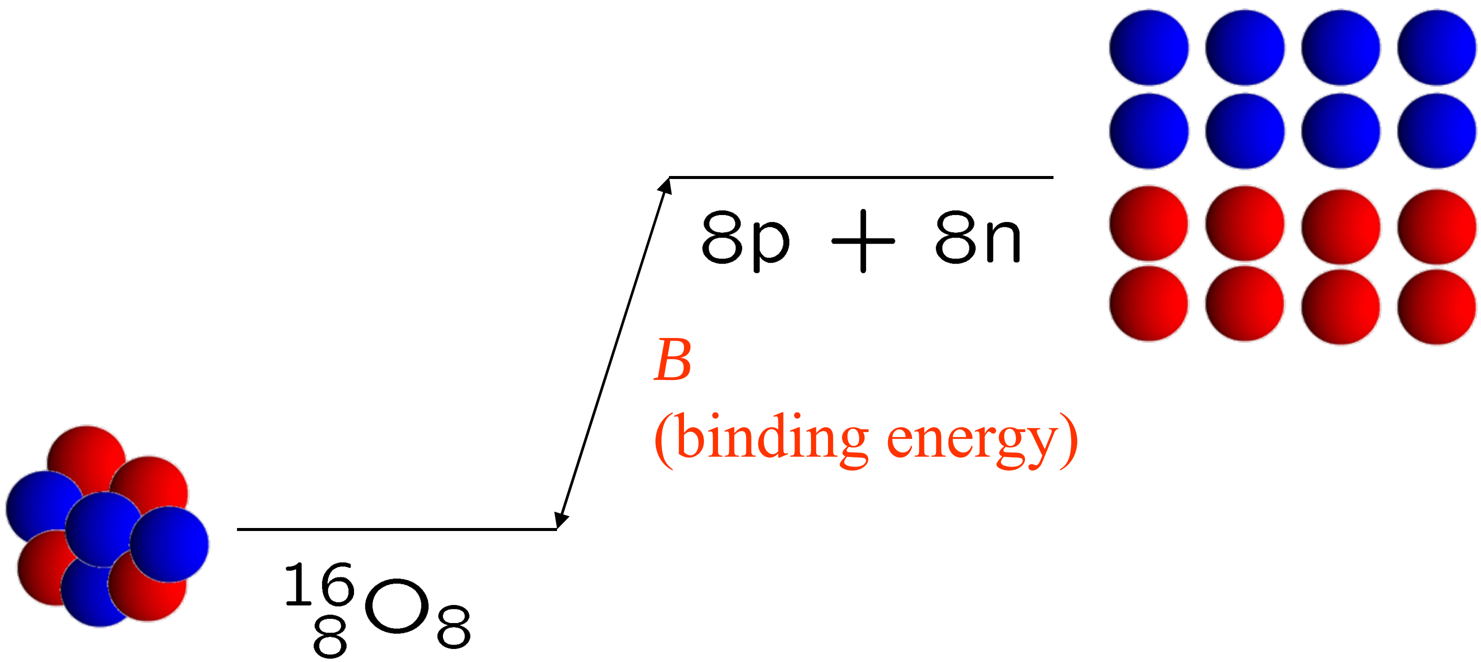
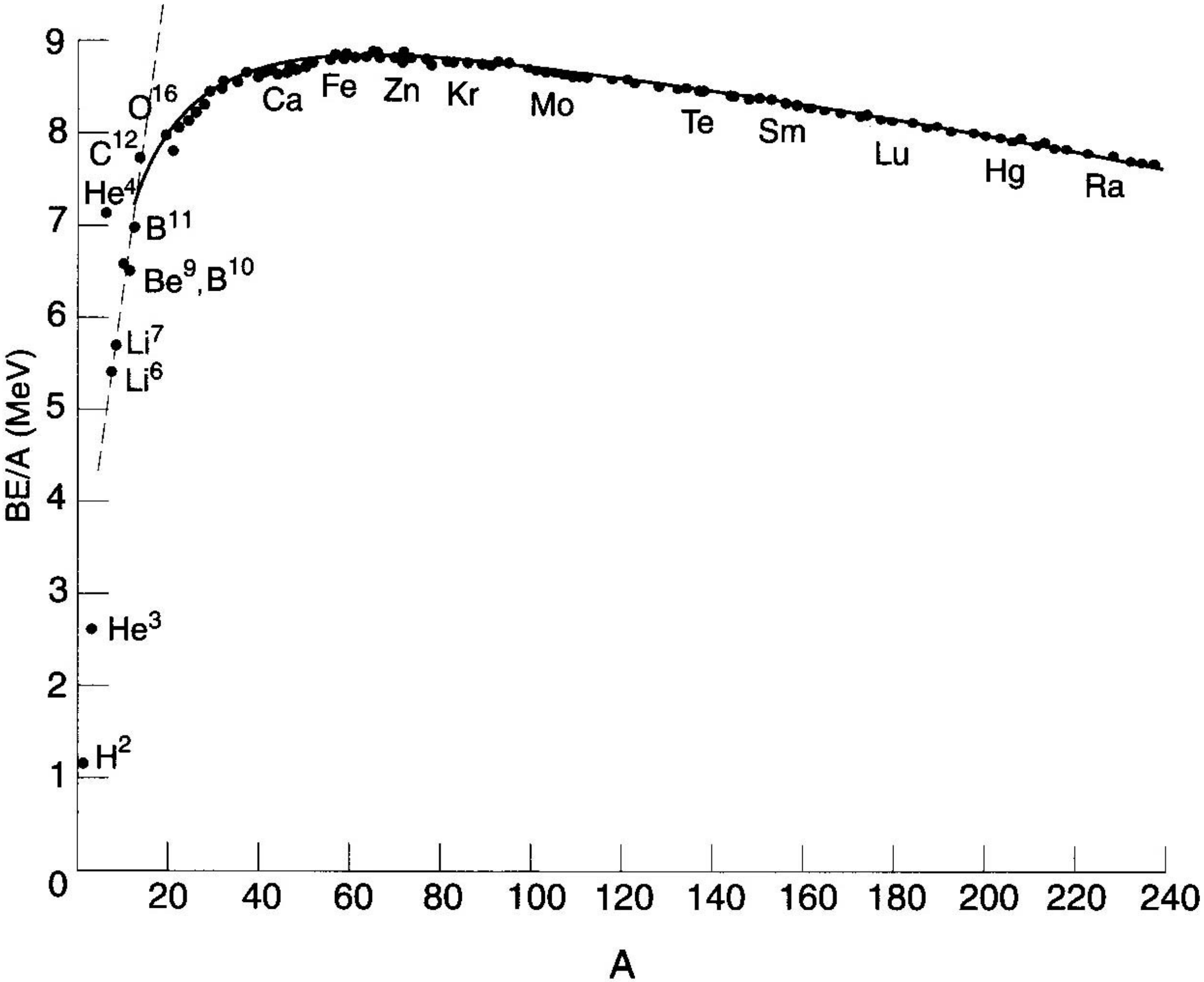
原子核の変形：液滴模型と殻補正

原子核の束縛エネルギー

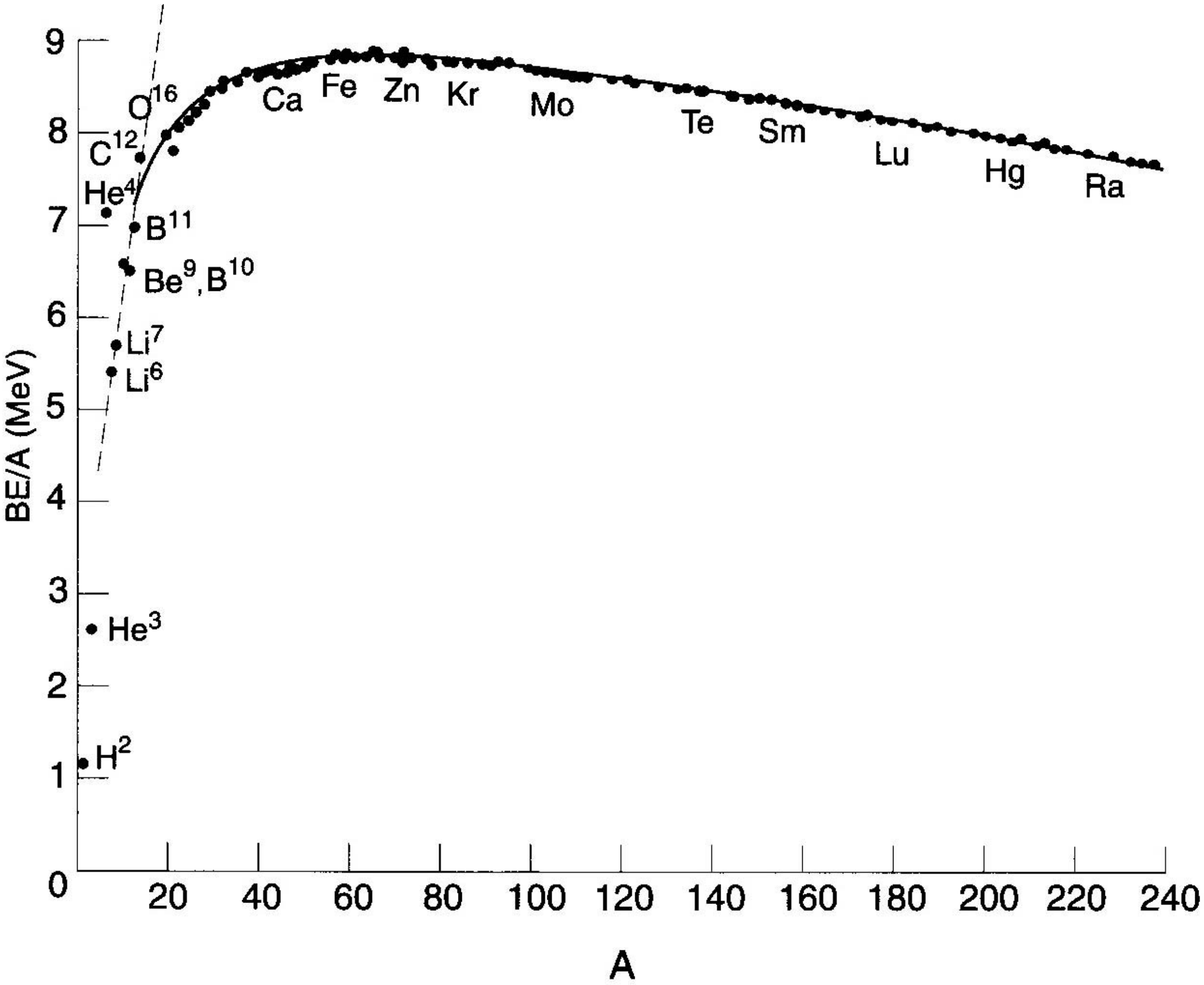


$$m(N,Z)c^2 = Zm_pc^2 + Nm_nc^2 - B$$



半経験的質量公式（液滴模型）

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$

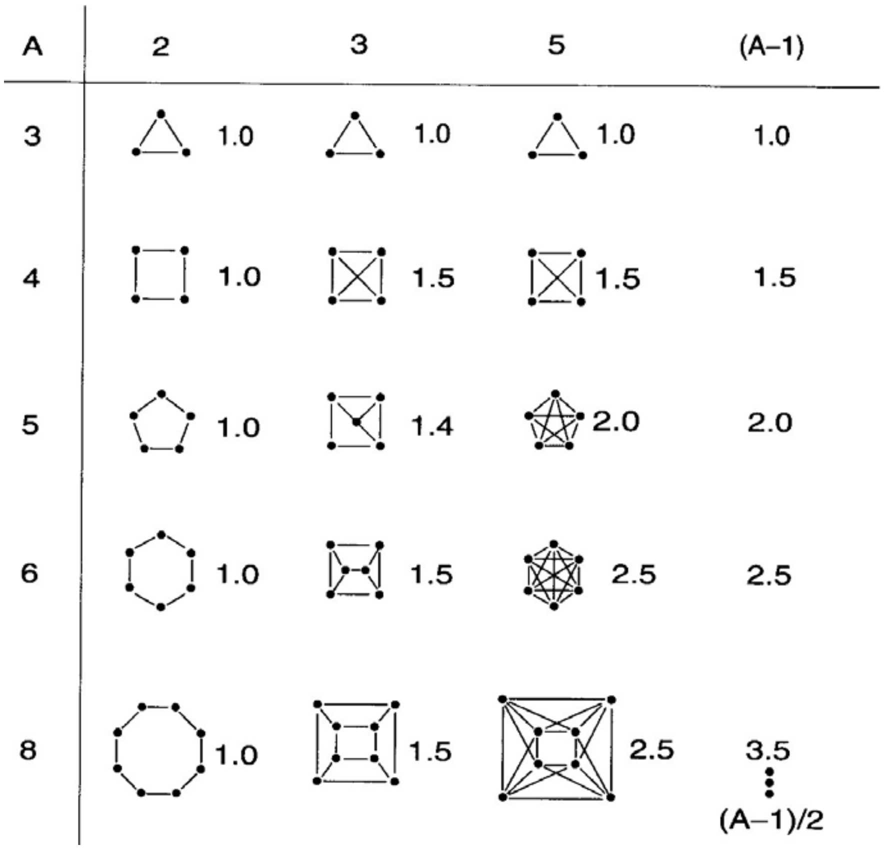


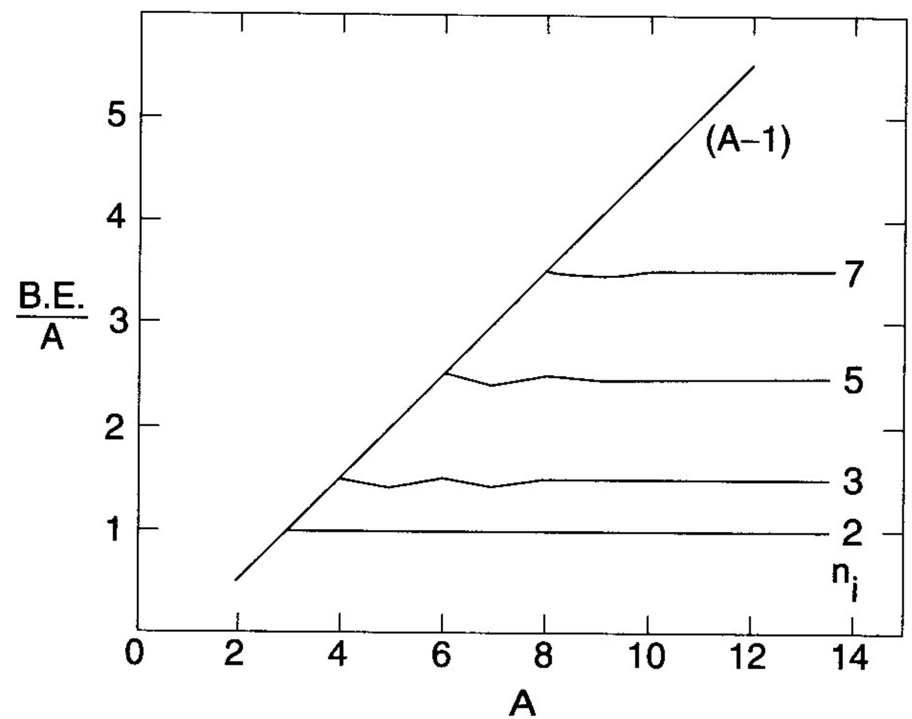
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range nuclear force

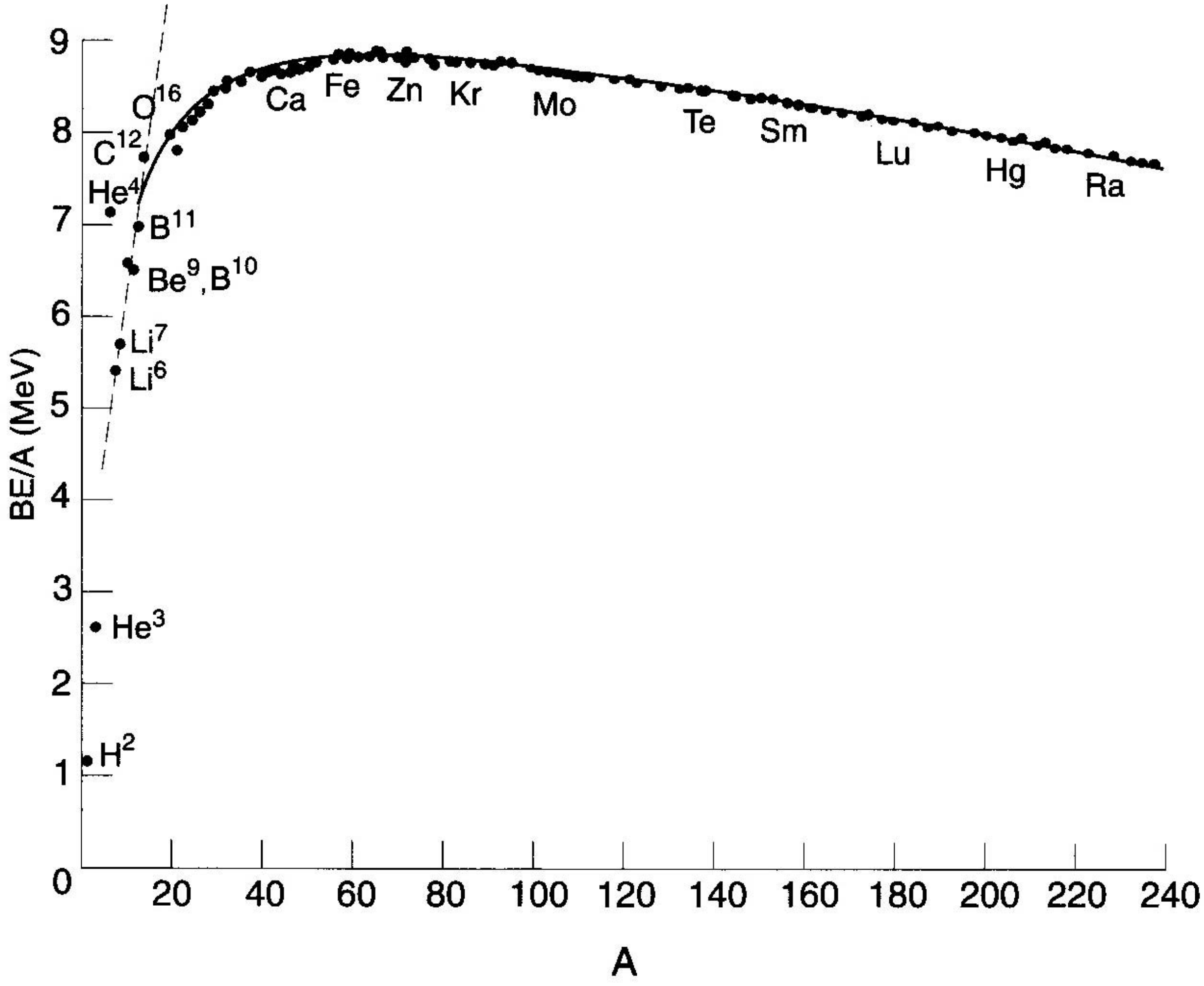
Long vs short range interaction

Long range force: $B \propto A(A-1)/2$ ⤼ $B/A \propto A$

Short range force: saturation

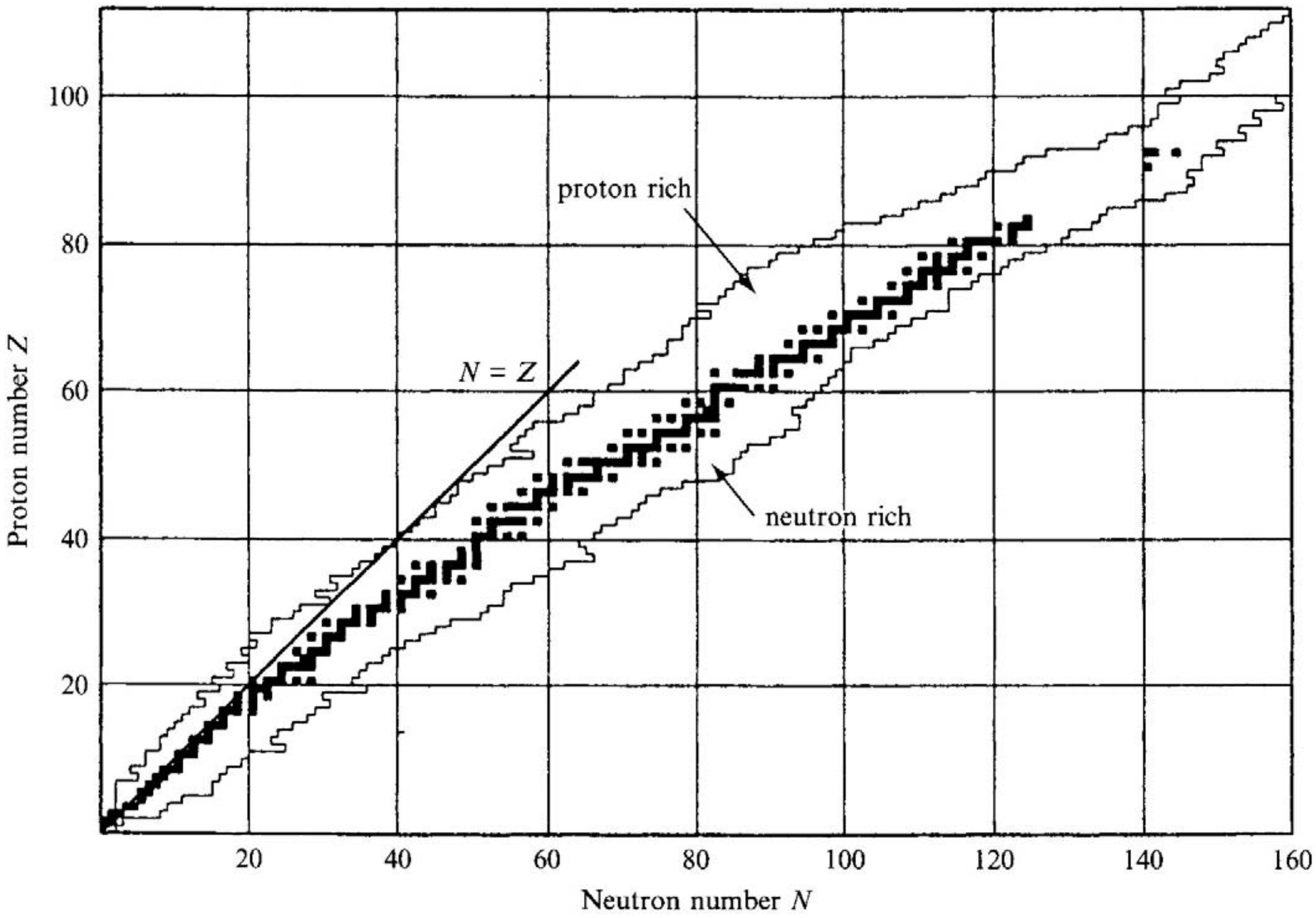




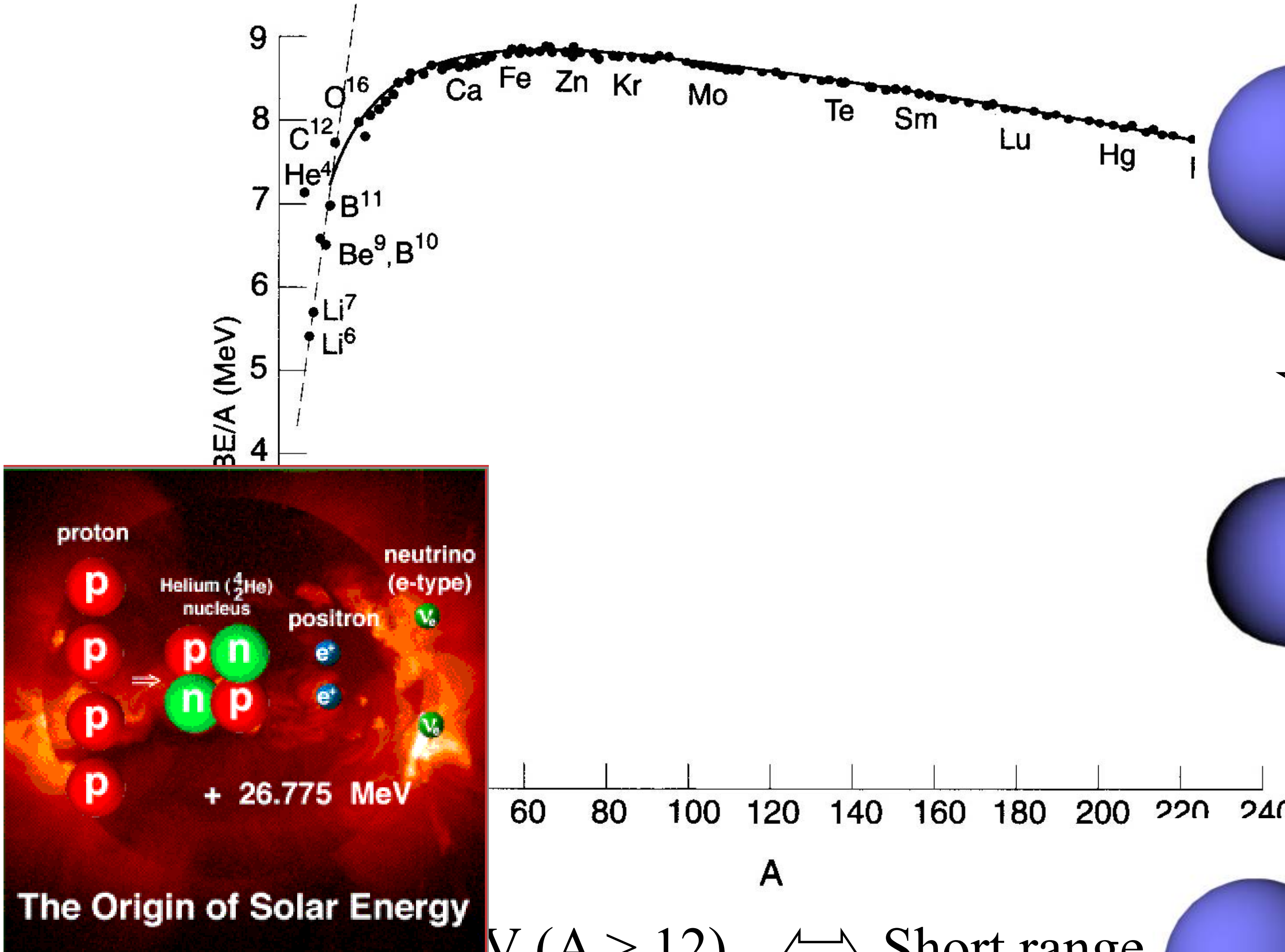


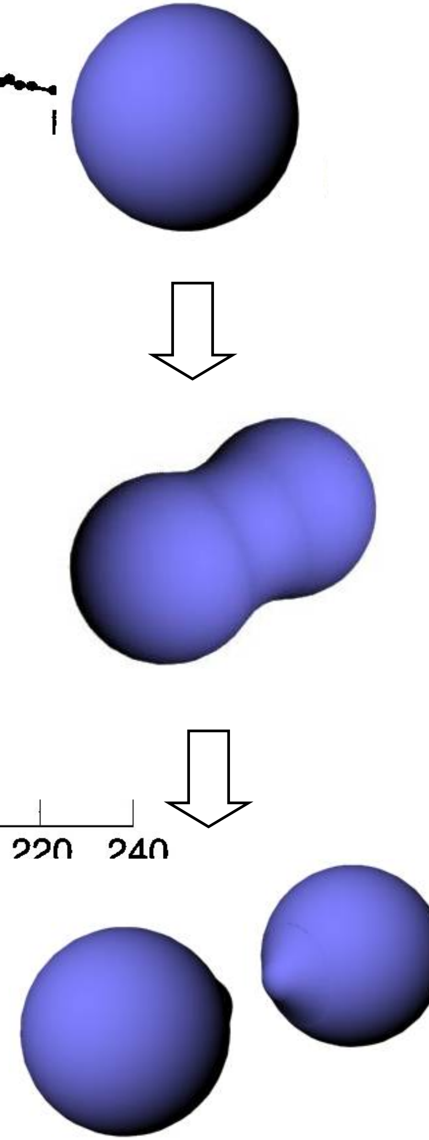
1. $B(N,Z)/A \sim 8.5$ MeV ($A > 12$) $\Longleftrightarrow$ Short range nuclear force
2. Effect of Coulomb force for heavy nuclei

Nuclear Chart



Stable nuclei: $N \geq Z$





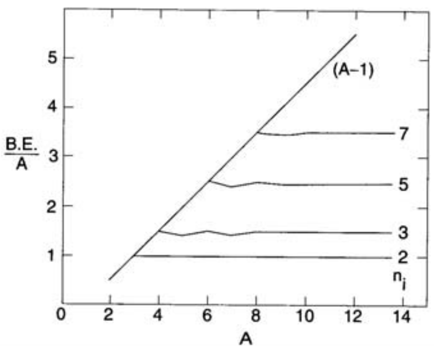
1. B(N,Z)/A ~ 8.5 MeV (A > 12) ⟺ Short range
2. Effect of Coulomb force for heavy nuclei
3. Fusion for light nuclei
4. Fission for heavy nuclei

Semi-empirical mass formula

(Bethe-Weizacker formula: Liquid-drop model)

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$

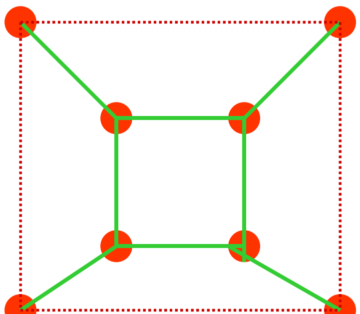
- Volume energy: $a_v A$



$$R_0 \sim 1.1 \times A^{1/3} \rightarrow \; V \propto A$$
$$S \propto A^{2/3}$$

- Surface energy: $-a_s A^{2/3}$

A nucleon near the surface interacts with fewer nucleons.



$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A}$$

- Coulomb energy: $-a_C\, Z^2/A^{1/3}$

$$E_C = \frac{3}{5}\frac{Z^2 e^2}{R_C} \quad \text{for a uniformly charged sphere}$$

- Symmetry energy: $-a_{\text{sym}}\,(N-Z)^2/A$

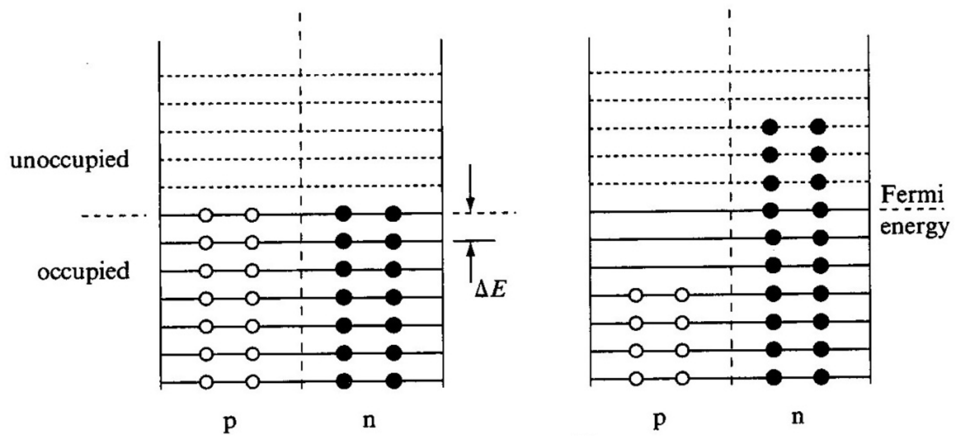
Potential energy $\quad v_{nn} = v_{pp} = v, \quad v_{np} \sim 2v$

a nucleon interacting with nuclear matter:

$$N(v_{nn}N/A + v_{pn}Z/A) + Z(v_{pn}N/A + v_{pp}Z/A) = \frac{v}{2}(3A - (N-Z)^2/A)$$

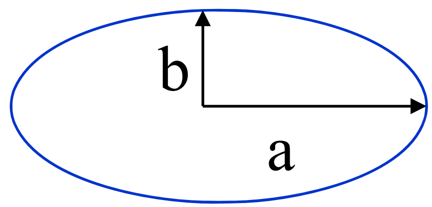
Kinetic energy

Pauli exclusion principle



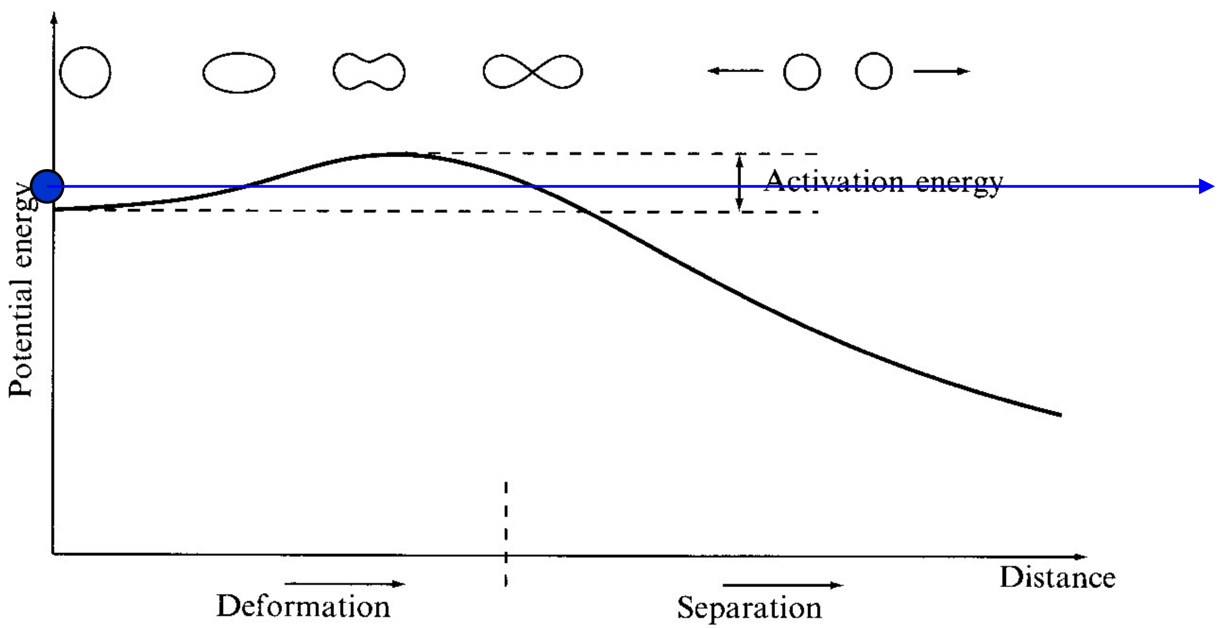
核分裂障壁

$$B(N,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_{\mathrm{sym}} \frac{(N-Z)^2}{A}$$



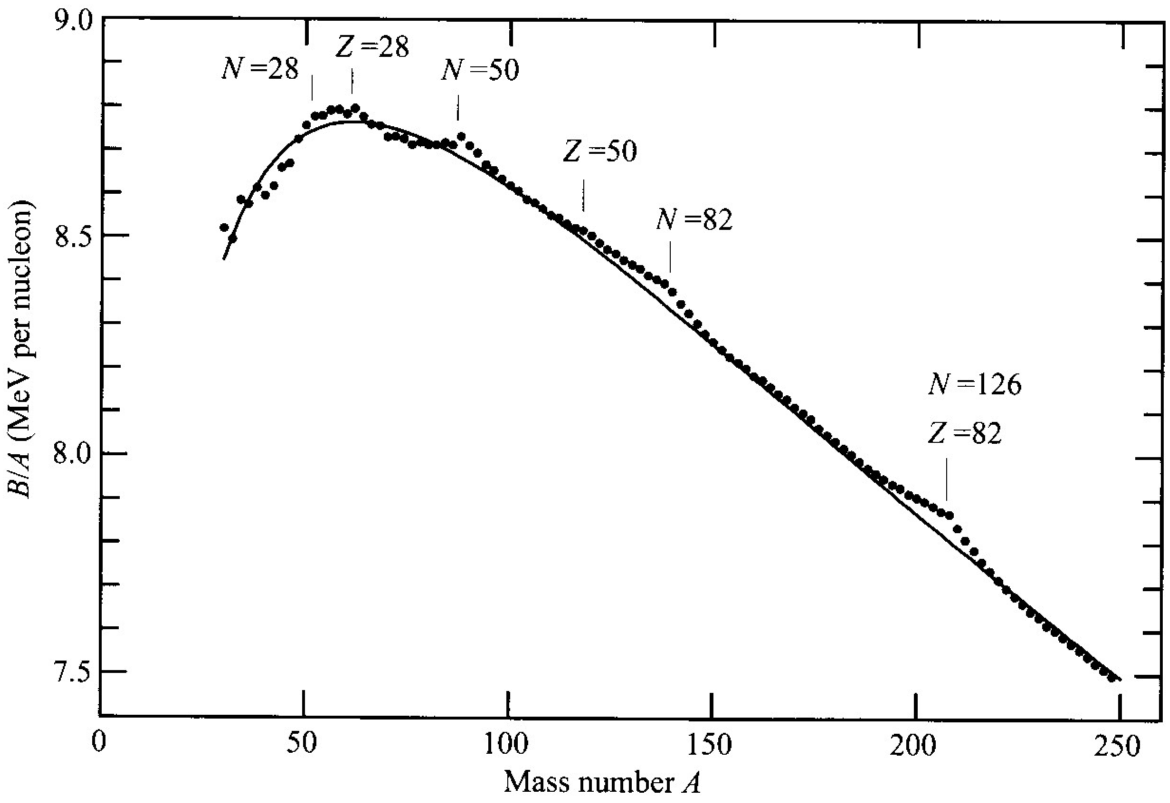
$$\begin{aligned} a &= R \cdot (1+\epsilon) \\ b &= R \cdot (1+\epsilon)^{-1/2} \end{aligned}$$

$$\begin{aligned} E_{\mathrm{surf}} &= E^{(0)}_{\mathrm{surf}} (1 + 2\epsilon^2/5 + \cdots) \\ E_C &= E^{(0)}_C (1 - \epsilon^2/5 + \cdots) \end{aligned}$$



Quantum tunneling

Shell Energy



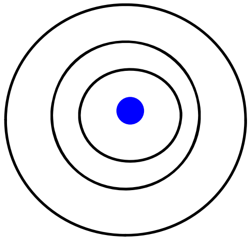
Extra binding for N,Z = 2, 8, 20, 28, 50, 82, 126 (magic numbers)

⇨ Very stable

$^{4}_{2}He_{2}, ^{16}_{8}O_{8}, ^{40}_{20}Ca_{20}, ^{48}_{20}Ca_{28}, ^{208}_{82}Pb_{126}$

(note) Atomic magic numbers (Noble gas)

He (Z=2), Ne (Z=10), Ar (Z=18), Kr (Z=36), Xe (Z=54), Rn (Z=86)

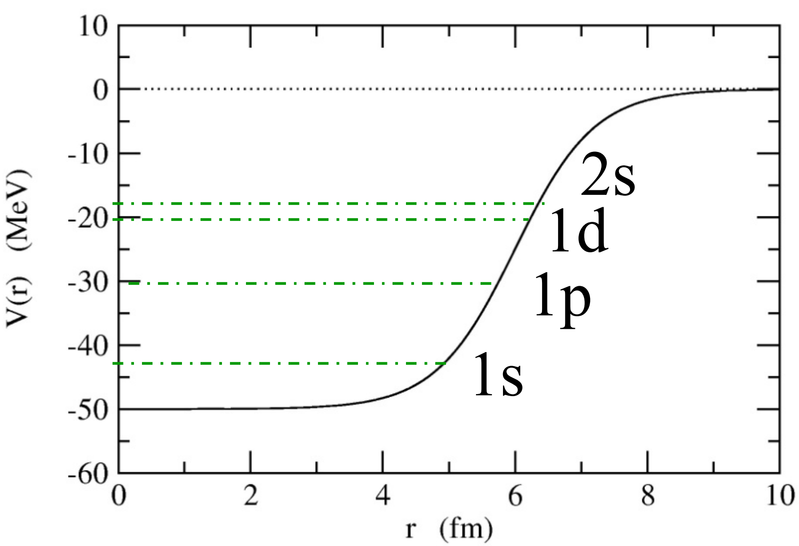


Shell structure

Similar attempt in nuclear physics: independent particle motion in a potential well

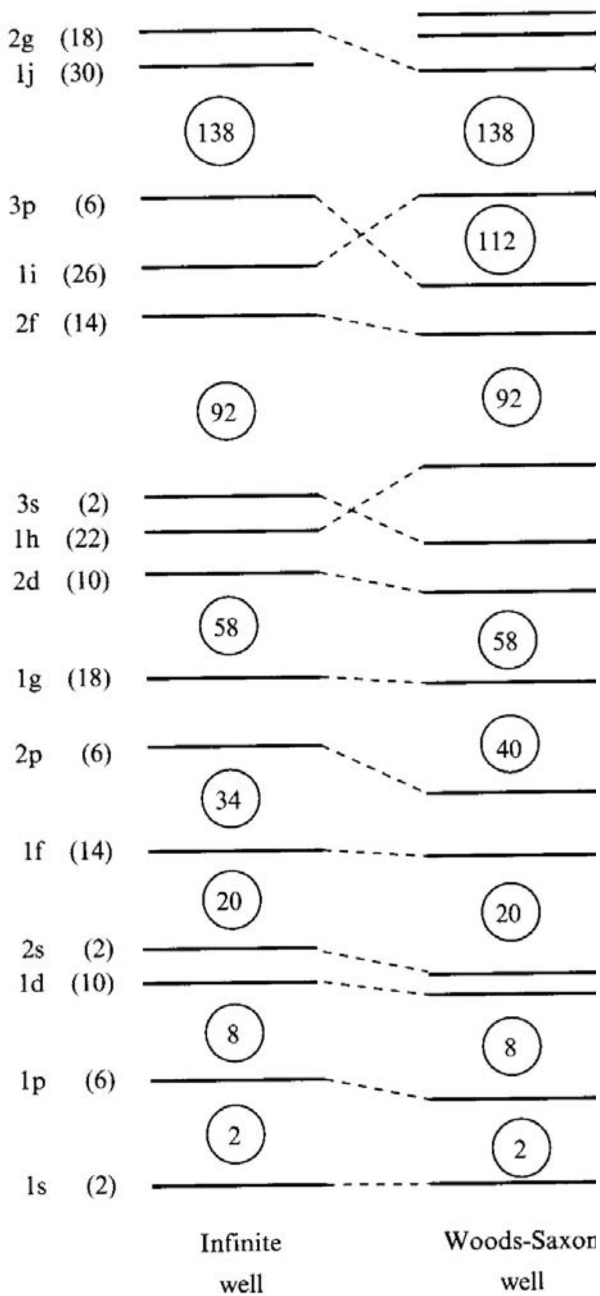
Woods-Saxon potential

$$V(r) = -V_0/[1 + \exp((r - R_0)/a]$$



$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$\psi(\boldsymbol{r}) = \frac{u_l(r)}{r} Y_{lm}(\hat{\boldsymbol{r}}) \cdot \chi_{m_s}$$

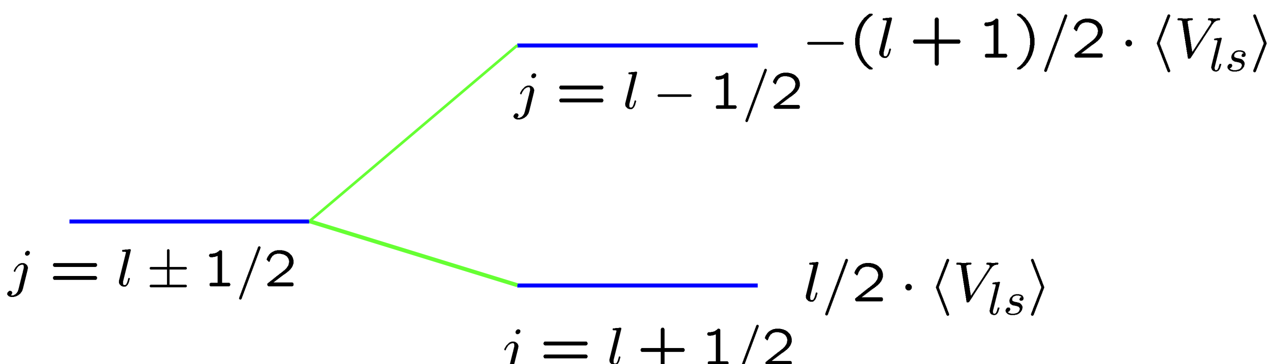


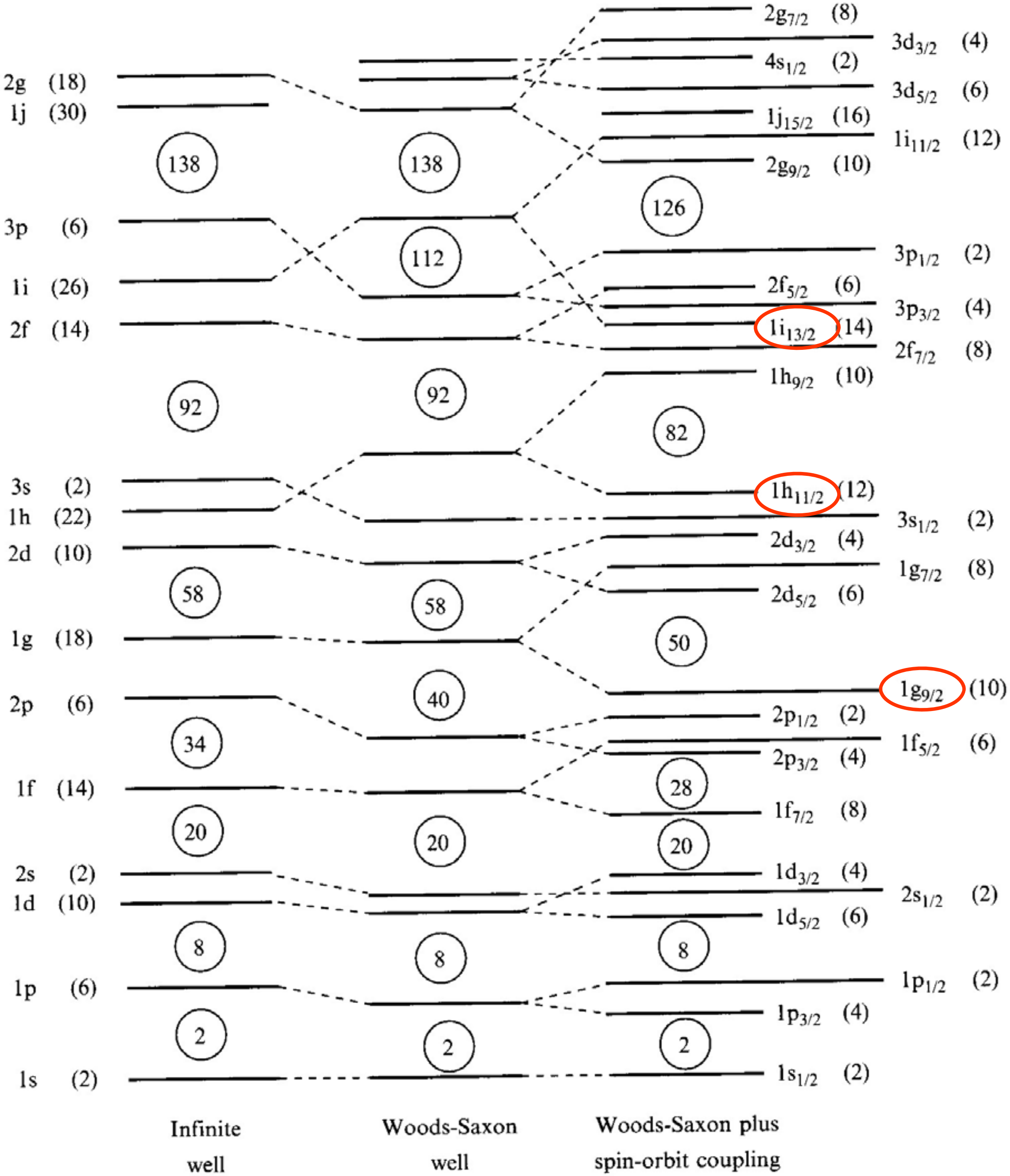
Woods-Saxon itself does not provide the correct magic numbers (2,8,20,28, 50,82,126).

Meyer and Jensen (1949):
Strong spin-orbit interaction

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(r) + V_{ls}(r)\boldsymbol{l}\cdot\boldsymbol{s} - \epsilon\right]\psi(\boldsymbol{r}) = 0$$

$$V_{ls}(r) \sim -\lambda \frac{1}{r}\frac{dV}{dr} \qquad (\lambda > 0)$$



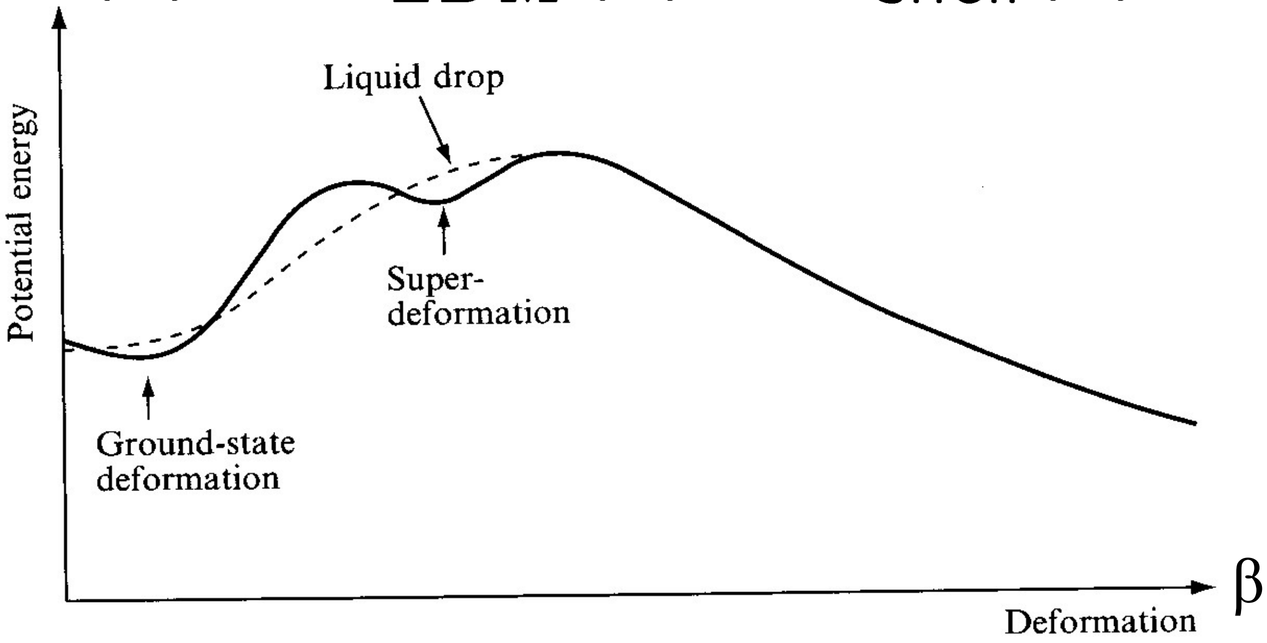


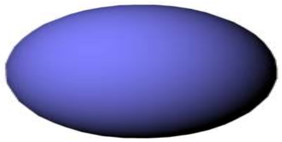
intruder states
unique parity states

Nuclear Deformation

Deformed energy surface for a given nucleus

$$E(\beta) = E_{LDM}(\beta) + E_{\mathsf{shell}}(\beta)$$



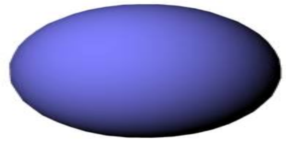


LDM only → always spherical ground state

Shell correction → may lead to a deformed g.s.

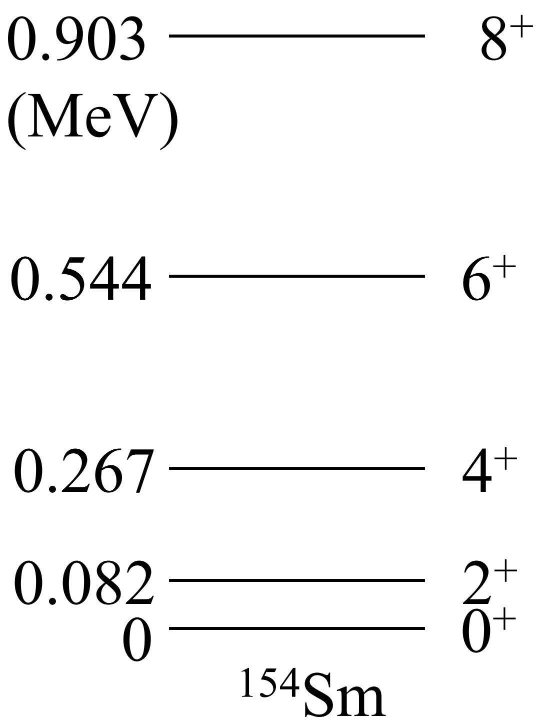
* Spontaneous Symmetry Breaking

Nuclear Deformation



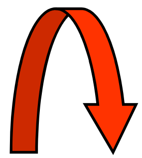
cf. Rotational energy of a rigid body
(Classical mechanics)

Excitation spectra of ^{154}Sm



$$E = \frac{1}{2}\mathcal{J}\omega^2 = \frac{I^2}{2\mathcal{J}}$$

$$(I = \mathcal{J}\omega, \ \ \omega = \dot{\theta})$$



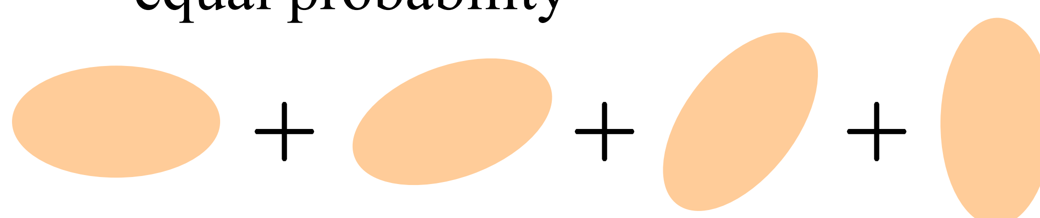
^{154}Sm is deformed

$$E_I \sim \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

(note) What is 0^+ state (Quantum Mechanics)?

0^+: no preference of direction (spherical)

→ Mixing of all orientations with an equal probability



c.f. HF + Angular Momentum Projection

Nobel Prize in Physics 2008

“for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics”

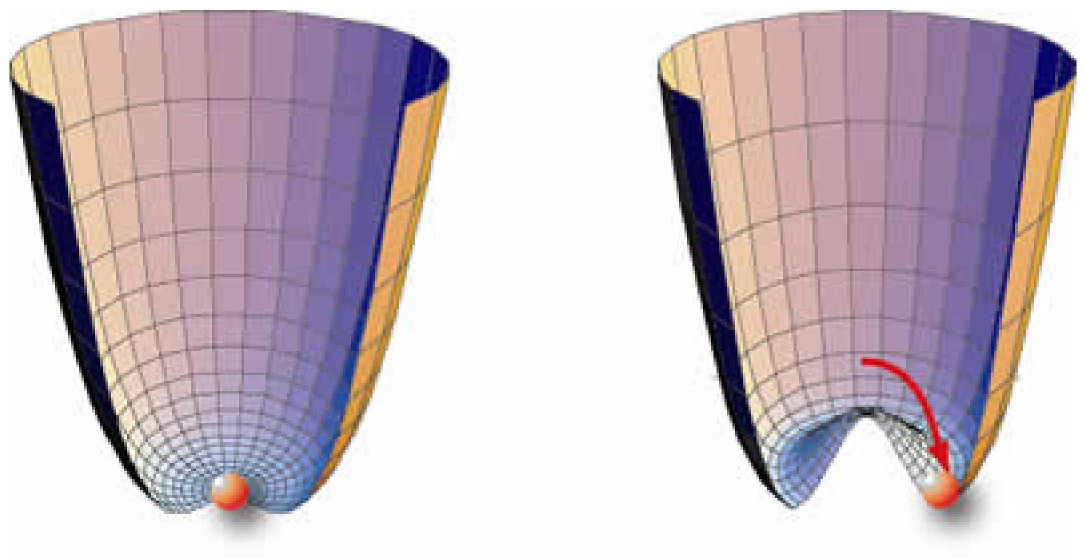
Yoichiro Nambu

“for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature”

Kobayashi-Maskawa

対称性の自発的破れ

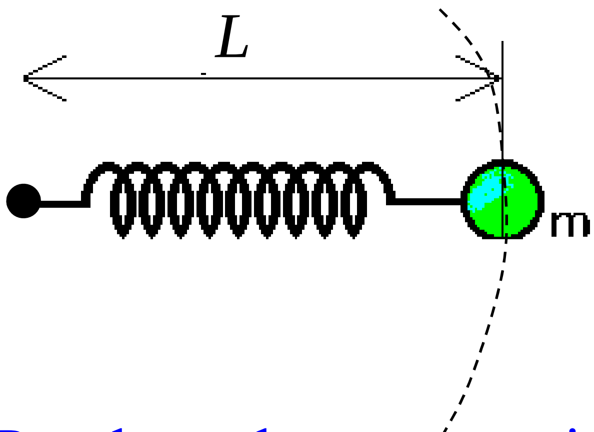
ハミルトニアンが持つ対称性を、真空が持たない（破る）。



対称性を回復するように
南部・ゴールドストン・モード（ゼロ・モード）
が発生

(note) rigid rotation of mechanical systems

E.R. Marshalek, Ann. of Phys. 53('69) 569



$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}k(\sqrt{x^2+y^2} - L)^2$$

Random phase approximation:

- Small oscillation around equillibrium

$$V(x,y) \sim V(x_0,y_0) + \frac{1}{2}\sum_{i,j}(\partial_i\partial_j V)(x_i - x_{i0})(x_j - x_{j0})$$

- All degrees of freedom are treated equally

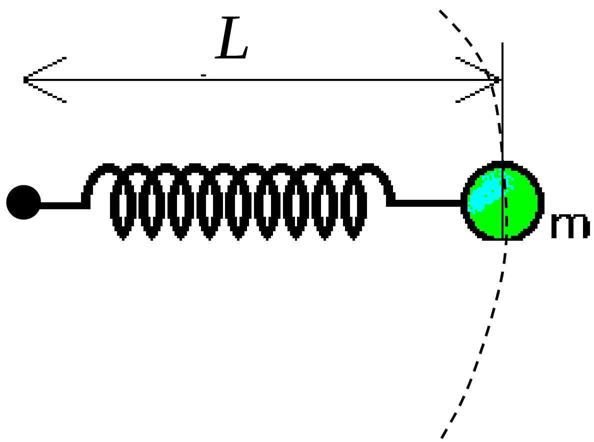
Treat *x* and *y* on the same footing
(work with the Cartesian coordinate)

i) "Spherical" case (L=0)

$$H = \frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{1}{2}k(x^2 + y^2)$$

$$\Longrightarrow \omega_x = \omega_y = \sqrt{k/m}$$

ii) "Deformed" case ($L \neq 0$)



$$x_0 = L, \;\; y_0 = 0$$

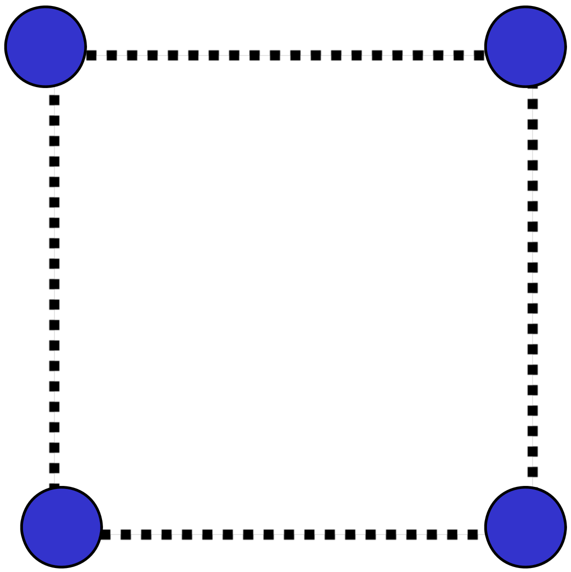
⟸ Spontaneous Symm. Breaking

(note) $$\frac{\partial^2}{\partial x^2}(r - L)^2 = \frac{2}{r^3}(r^3 - Ly^2)$$
$$\frac{\partial^2}{\partial y^2}(r - L)^2 = \frac{2}{r^3}(r^3 - Lx^2)$$

$$H \sim \frac{p_x^2}{2m} + \frac{1}{2}kx^2 + \frac{p_y^2}{2m}$$

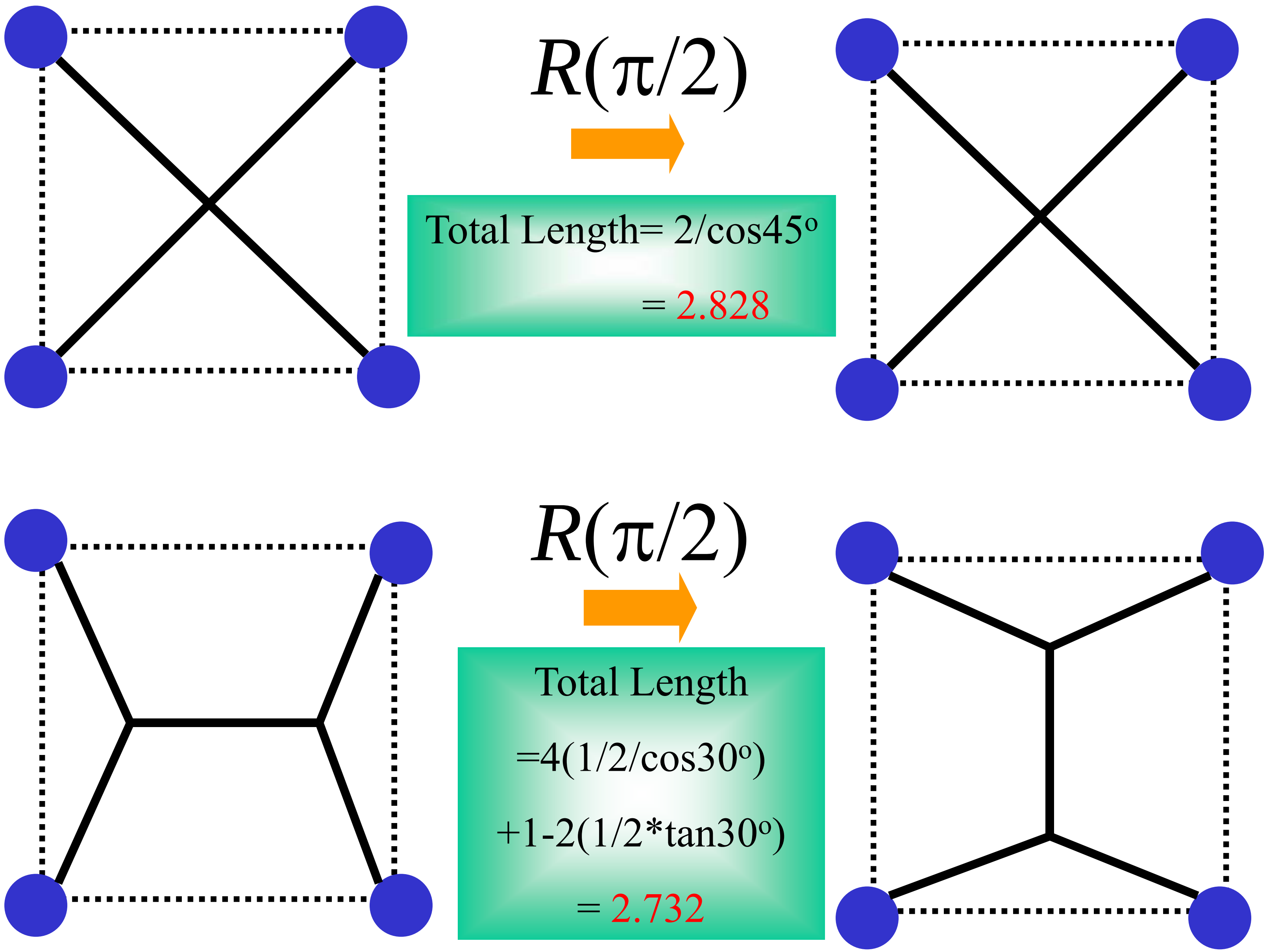
$$\omega_x = \sqrt{k/m}, \;\; \underline{\omega_y = 0}$$

A warm up



正方形の4頂点を全長が最小になるように線で結ぶには？

スライド：小池武志氏（東北大学）



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