

$$\left(-\frac{\hbar^2}{2m} \nabla^2 + \underbrace{V(r)}_{\text{擾動}} - E\right) \psi(r) = 0$$

散乱問題にあつては

5.2. Born Approximation

$$\left(\frac{V(r)}{E} \ll 1\right)$$

$$\psi_i(r) = e^{i\vec{p}_i \cdot \vec{r} / \hbar} \quad \xrightarrow{\text{遷移}} \quad \psi_f(r) = e^{i\vec{p}_f \cdot \vec{r} / \hbar}$$

$$\left(\frac{p_i^2}{2m} = \frac{p_f^2}{2m} = E\right)$$

Fermi's Golden Rule:

$$R_{i \rightarrow f} = \frac{2\pi}{\hbar} \int \frac{d^3 p_f}{(2\pi\hbar)^3} |M_{fi}|^2 \delta\left(\frac{p_f^2}{2m} - \frac{p_i^2}{2m}\right)$$

$$M_{fi} = \langle \psi_f | V | \psi_i \rangle \quad \xrightarrow{\vec{p}_i \oplus \vec{p}_f}$$

$$= \int d\vec{r} \psi_f^*(\vec{r}) V(\vec{r}) \psi_i(\vec{r})$$

$$= \int d\vec{r} e^{i(\vec{p}_i - \vec{p}_f) \cdot \vec{r} / \hbar} V(\vec{r})$$

$$= \int d\vec{r} e^{-i\vec{q} \cdot \vec{r}} V(\vec{r})$$

$$\vec{q} = (\vec{p}_f - \vec{p}_i) / \hbar \quad : \text{momentum transfer}$$

$$= \tilde{V}(\vec{q}) \quad (\text{フーリエ変換})$$

↓

$$R_{i \rightarrow f} = \frac{2\pi}{\hbar} \int \frac{p_f^2 dp_f d\Omega}{(2\pi\hbar)^3} |\tilde{V}(\vec{q})|^2 \delta\left(\frac{p_f^2}{2m} - E\right)$$

$$= \frac{2\pi}{\hbar} \cdot \frac{1}{(2\pi\hbar)^3} \cdot \int m p_f d\left(\frac{p_f^2}{2m}\right) d\Omega |\tilde{V}(\vec{q})|^2 \delta\left(\frac{p_f^2}{2m} - E\right)$$

$$= \frac{m p_i}{4\pi^2 \hbar^4} \int d\Omega |\tilde{V}(\vec{q})|^2$$

$$\frac{1}{2} m v^2 = \frac{\hbar^2 k^2}{2m}$$

$$\Downarrow v = \frac{\hbar k}{m}$$

$$= \frac{\hbar}{2cm} (\psi^* \nabla \psi - \psi \nabla \psi^*)$$

$$j_{in} = \frac{\hbar k}{m} = v$$

\Downarrow

$$d\sigma = \frac{1}{4\pi^2 \hbar^2} \frac{1}{|v_{rel}|} \frac{m^2}{\hbar^2} d\Omega |\tilde{V}(\vec{g})|^2$$

\Downarrow

$$\boxed{\frac{d\sigma}{d\Omega} = \frac{m^2}{4\pi^2} \left| \frac{1}{\hbar^2} \tilde{V}(\vec{g}) \right|^2}$$

例)

$$V(\vec{r}) = \int \frac{\rho(r')}{|\vec{r} - \vec{r}'|} dr' \cdot z_p e$$

(note)

$$\nabla^2 V = -4\pi \rho \cdot z_p e$$

\downarrow

$$\tilde{V}(\vec{g}) = \int d\vec{r} e^{-i\vec{g} \cdot \vec{r}} V(\vec{r})$$

$$= \frac{1}{-i\vec{g}} e^{-i\vec{g} \cdot \vec{r}} V(\vec{r}) \Big|_{-\infty}^{\infty} + \frac{1}{i\vec{g}} \int e^{-i\vec{g} \cdot \vec{r}} \nabla V(\vec{r}) d\vec{r}$$

$$= \frac{1}{g^2} \nabla V \Big|_{-\infty}^{\infty} - \frac{1}{g^2} \int e^{-i\vec{g} \cdot \vec{r}} \nabla^2 V(\vec{r}) d\vec{r}$$

$$= + \frac{4\pi}{g^2} \int e^{-i\vec{g} \cdot \vec{r}} \rho(\vec{r}) d\vec{r} \cdot z_p e$$

\Downarrow

$$\frac{d\sigma}{d\Omega} = \frac{4m^2}{g^4 \hbar^4} |F(\vec{g})|^2 \cdot z_p^2 e^2$$

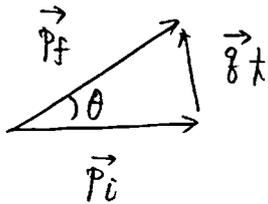
$$F(\vec{g}) = \int e^{-i\vec{g} \cdot \vec{r}} \rho(\vec{r}) d\vec{r}$$

$$\int_{-1}^1 d(\cos\theta) P_l(\cos\theta) P_l'(\cos\theta) \frac{P_l^2}{2l+1} = E$$

$$= \frac{2}{2l+1} f_{ll'}$$

$$P_0(\cos\theta) = 1$$

(note)



$$g_k = 2 P_i \sin \frac{\theta}{2}$$

$$= 2 \cdot \sqrt{2mE} \sin \frac{\theta}{2}$$

↓

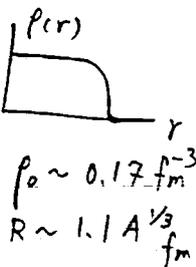
$$\frac{d\sigma}{d\Omega} = \frac{4m^2}{16 \cdot 4m^2 E^2} \frac{1}{\sin^4 \frac{\theta}{2}} |F(\vec{g})|^2 \cdot z_p^2 e^2$$

$$= \frac{1}{(4E \sin^2 \frac{\theta}{2})^2} |F(\vec{g})|^2 \cdot z_p^2 e^2$$

形状因子
"form factor"

→ 電荷密度 ρ 決定

(note) $e^{i\vec{g} \cdot \vec{r}} = \sum_{l=0}^{\infty} (2l+1) i^l j_l(g r) P_l(\cos\theta)$



↓ for $\rho(\vec{r}) = \rho(r)$

$$F(\vec{g}) = 2\pi \int_{-1}^1 d(\cos\theta) \int_0^{\infty} r^2 dr \sum_{l=0}^{\infty} (2l+1) i^l j_l(g r) P_l(\cos\theta) \times \rho(r)$$

$$= 4\pi \int_0^{\infty} r^2 dr \left(\frac{j_0(g r)}{g r} \rho(r) \right)$$

(note) if $\rho(r) = z_T e^{-r}$ $\rightsquigarrow F(\vec{g}) = z_T e$

$$\downarrow \frac{d\sigma}{d\Omega} = \left(\frac{z_p z_T e^2}{4E \sin^2 \frac{\theta}{2}} \right)^2$$

(古典的な「フォード」
散乱式と同じ)