<u>DWBA</u>

$$\begin{cases} (T + V(r) - E)\, u_0(r) = -F(r)\, u_1(r) \\ (T + V(r) - E + \varepsilon)\, u_1(r) = -F(r)\, u_0(r) \end{cases}$$

· zero-th order solution

$$(T + V(r) - E)\, u_0(r) = 0$$

$$\text{with} \quad u_0^{(0)}(r) \to H_\ell^{(-)}(kr) - S_0 H_\ell^{(+)}(kr)$$

· first order solution

$$(T + V(r) - E_1)\, u_1^{(1)}(r) = -F(r)\, u_0^{(0)}(r)$$

$$\Updownarrow$$

$$u_1^{(1)}(r) = -\int dr' \langle r | \frac{1}{\hat{T} + V(r) - E_1 - i\eta} | r' \rangle \times F(r')\, u_0^{(0)}(r')$$

(note) $$\langle r | \frac{1}{\hat{T} + V(r) - E_1 - i\eta} | r' \rangle = \frac{2\mu}{k_1 \hbar^2} O_\ell(k_1 r_>) F_\ell(k_1 r_<)$$

$$[\hat{T} + V(r) - E_1]\, O_\ell = 0, \quad O_\ell \to H_\ell^{(+)}(k_1 r)$$

$$[\hat{T} + V(r) - E_2]\, F_\ell \doteq 0, \quad F_\ell \to -\frac{1}{2i}\left(H_\ell^{(-)}(k_1 r) - S_1 H_\ell^{(+)}\right)$$

$$= e^{i\delta_\ell} \sin(\cdots)$$

$\rightsquigarrow$

as $\quad r \to \infty$

$$u_1^{(1)}(r) \to -\frac{2\mu}{k_1\hbar^2}\, O_\ell(k_1 r) \int dr' \left(-\frac{1}{2i}\right) u_1^{(0)}(r')\, F_1(r') \times u_0^{(0)}(r')$$

$$= -\sqrt{\frac{k}{k_1}}\, S_1\, O_\ell(k_1 r)$$

$\rightsquigarrow$

$$\boxed{S_1 = -\frac{1}{2ik_1}\sqrt{\frac{k}{k_1}} \int dr\; u_1^{(0)}(r) \cdot \frac{2\mu}{\hbar^2} F_1(r)\, u_0^{(0)}(r)}$$

$$\left(-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}+V(r)-E\right)G^{(+)}(r,r')=\delta(r-r')$$

$$G^{(+)}(r,r') \sim r^{\ell+1} \qquad (r\sim 0)$$
$$\to O_\ell(kr) \qquad (r\to\infty)$$

$$G^{(+)}(r,r') = C\cdot O_\ell(kr_>)F_\ell(kr_<)$$

とすれば　$r\neq r'$ において
$(H-E)G=0$　を満たすので trivial.

比例係数は　$r=r'$ における matching できめる。

$$\int_{r'-\varepsilon}^{r'+\varepsilon}dr\left[-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}+V(r)-E\right]G^{(+)}(r,r')=\int_{r'-\varepsilon}^{r'+\varepsilon}\delta(r-r')\,dr$$
$$=1$$

$$\text{左辺}=-\frac{\hbar^2}{2m}\left[\frac{d}{dr}G^{(+)}(r,r')\Big|_{r=r'+\varepsilon}-\frac{d}{dr}G^{(+)}(r,r')\Big|_{r=r'-\varepsilon}\right]$$
$$=-\frac{\hbar^2}{2m}\cdot C\left[\underbrace{O_\ell'(kr')F_\ell(kr')-F_\ell'(kr')O_\ell(kr')}_{\parallel \atop W\ (\text{ロンスキアン})}\right]$$

$$W=(e^{ikr})'\sin kr-(\sin kr)'e^{ikr}$$
$$=ik\,e^{ikr}\sin kr-k\cos kr\cdot e^{ikr}$$
$$=\frac{k}{2}(\cancel{e^{ikr}}-e^{-ikr})e^{ikr}-\frac{k}{2}\cancel{(e^{ikr}}+e^{-ikr})e^{ikr}$$
$$=-k$$

$$C = \frac{2m}{\hbar^2 k}$$

$$\boxed{G^{(+)}(r, r') = \frac{2m}{k\hbar^2} \, O_\ell(kr_>) \, F_\ell(kr_<)}$$