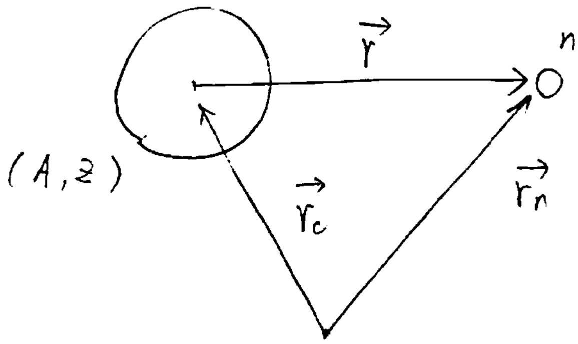
§. (γ, n) 反応

◦ 電磁場との相互作用



$$H = \frac{\vec{P}_c^{\,2}}{2Am} + \frac{\vec{P}_n^{\,2}}{2m} + V(\vec{r})$$

相対運動と重心運動に分離

$$\vec{r} = \vec{r}_n - \vec{r}_c \qquad \vec{p} = \mu\dot{\vec{r}} = \frac{Am}{A+1}(\dot{\vec{r}}_n - \dot{\vec{r}}_c)$$

$$\vec{R} = \frac{A\vec{r}_c + \vec{r}_n}{A+1} \qquad = \frac{1}{A+1}(A\vec{P}_n - \vec{P}_c)$$

$$\vec{P} = (A+1)m\dot{\vec{R}} = \vec{P}_c + \vec{P}_n$$

↓

$$H = \frac{\vec{P}^{\,2}}{2(A+1)m} + \frac{\vec{p}^{\,2}}{2\mu} + V(\vec{r}) \quad ; \mu = \frac{A}{A+1}m$$

電磁場との相互作用: $\vec{P}_c \to \vec{P}_c - \frac{Ze}{c}\vec{A}(\mathbf{r}_c, t)$

(note) $m\ddot{\mathbf{r}}_c = Ze\left[\mathbb{E}(\mathbf{r}_c, t) + \frac{1}{c}\mathbf{v}\times\mathbb{B}(\mathbf{r}_c, t)\right]$

" minimum principle "

$$H \to \frac{1}{2Am}\left(\vec{P}_c - \frac{Ze}{c}\vec{A}(r_c,t)\right)^2 + \frac{\vec{P}_n^{\,2}}{2m} + V(\vec{r}) + H_{em}$$

$$= \frac{1}{2Am}\vec{P}_c^{\,2} + \frac{\vec{P}_n^{\,2}}{2m} + V(\vec{r}) + H_{em}$$

$$\underbrace{-\frac{1}{2Am}\cdot\frac{Ze}{c}\left(\vec{P}_c\cdot\vec{A} + \vec{A}\cdot\vec{P}_c\right) + \frac{1}{2Am}\left(\frac{Ze}{c}\right)^2\vec{A}^{\,2}}_{=\ H_{int}}$$

$$H_{int} \sim -\frac{Ze}{Amc}\mathbf{A}\cdot\mathbf{P}_c$$

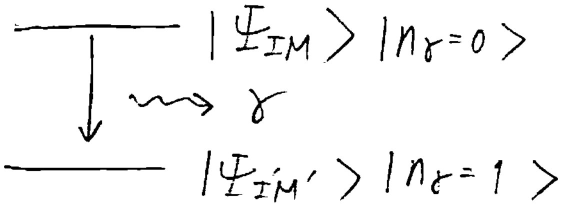
$\leftarrow$ クーロン・ゲージ

$$\nabla\cdot\mathbf{A}(\mathbf{r},t) = 0$$

$$A_0(\mathbf{r},t) = 0$$

$$\left(\begin{array}{l}\mathbb{B} = \nabla\times\mathbf{A} \\ \mathbb{E} = -\frac{1}{c}\dot{\mathbf{A}}\end{array}\right)$$

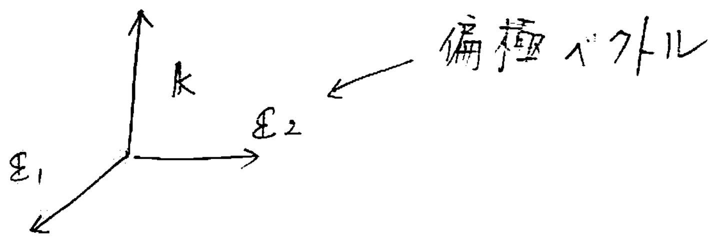
$$H_{em} = \frac{1}{8\pi}\int d\mathbf{r}\left(|\mathbb{B}|^2 + |\mathbb{E}|^2\right) = \frac{1}{8\pi}\int d\mathbf{r}\left(\frac{1}{c^2}|\dot{\mathbf{A}}|^2 + |\nabla\times\mathbf{A}|^2\right)$$



フェルミの Golden Rule：

$$T = \frac{2\pi}{\hbar}\left|\langle f | H_{int} | i \rangle\right|^2 \frac{dn}{dE}$$

○電磁場の量子化



$$A(r,t) = \sum_\alpha \int \frac{dk}{(2\pi)^3}\, c\sqrt{\frac{2\pi\hbar}{\omega}} \left[a_{k\alpha}\, \varepsilon_\alpha\, e^{ik\cdot r - i\omega t} + a_{k\alpha}^\dagger\, \varepsilon_\alpha\, e^{-ik\cdot r + i\omega t} \right]$$

(ベクトルポテンシャルをフーリエ展開して第2量子化)

○E1近似 (E1フォトンの放出)

$$e^{\pm ik\cdot r} \sim 1$$

$$\left(E_\gamma \sim \mathrm{MeV}, \quad \Updownarrow \quad k = \frac{\hbar\omega}{\hbar c} \sim \frac{1}{200}\ \mathrm{fm}^{-1} \right)$$

$\downarrow$

$$A(r,t) \sim \sum_\alpha \int \frac{dk}{(2\pi)^3}\, c\sqrt{\frac{2\pi\hbar}{\omega}} \left[a_{k\alpha}\, \varepsilon_\alpha\, e^{-i\omega t} + a_{k\alpha}^\dagger\, \varepsilon_\alpha\, e^{i\omega t} \right]$$

↑

r に依らない。

(note) $$H_{int} = -\frac{Ze}{Amc} A\cdot \mathbb{P}_c = -\frac{Ze}{Amc} A\cdot \left(\frac{A}{A+1}\mathbb{P} - \mathbb{p} \right)$$

重心固定系で考えれば $\mathbb{P} = 0$

$\Updownarrow$

$$H_{int} = \frac{Ze}{Amc} A\cdot \mathbb{p}$$

$$E = \hbar\omega = \hbar c k$$

相対運動に関係する部分のハミルトニアン：

$$h = \underbrace{\frac{\vec{P}^2}{2\mu} + V(r) + H_{em}}_{h_0} + \underbrace{\frac{ze}{Amc} A \cdot \mathbb{P}}_{H_{int}}$$

(note) 立体角 $d\hat{k} = d\Omega$ に放出されるフォトンで $(k, k+dk)$ にあるフォトンの状態数： $dn = \frac{k^2 dk}{(2\pi)^3} d\hat{k}$

$$\Downarrow$$

$$\frac{dn}{dE} = \frac{k^2}{(2\pi)^3} \frac{dk}{dE} d\hat{k} = \frac{k^2}{(2\pi)^3} \cdot \frac{1}{\hbar c} d\hat{k} = \frac{\omega^2}{(2\pi)^3 \hbar c^3} d\hat{k}$$

$$\Downarrow$$

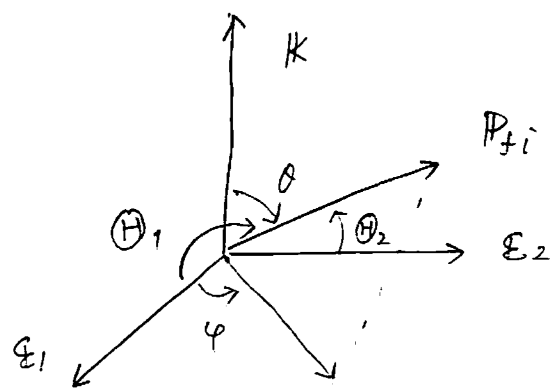
$$\frac{dT}{d\Omega} = \frac{2\pi}{\hbar} \cdot \left(\frac{ze}{Amc}\right)^2 \cdot \left(c\sqrt{\frac{2\pi\hbar}{\omega}}\right)^2$$

$$\times \sum_\alpha \left| \langle \Psi_f | \varepsilon_\alpha \cdot \mathbb{P} | \Psi_i \rangle \right|^2 \cdot \frac{\omega^2}{(2\pi)^3 \cdot \hbar c^3}$$

$$= \frac{\left(\frac{z}{A} e\right)^2 \omega}{2\pi m^2 \hbar c^3} \cdot \sum_\alpha \left| \langle \Psi_f | \mathbb{P} | \Psi_i \rangle \cdot \varepsilon_\alpha \right|^2$$

$$\sum_\alpha |\langle \Psi_f | \mathbb{P} | \Psi_i \rangle \cdot \varepsilon_\alpha|^2 = \sum_\alpha |\langle \Psi_f | \mathbb{P} | \Psi_i \rangle|^2 \cos^2 \Theta_\alpha$$

Θ_α : angle between $\mathbb{P}_{fi} = \langle \Psi_f | \mathbb{P} | \Psi_i \rangle$ and ε_α



$$\cos\Theta_1 = \sin\theta \cos\varphi$$
$$\cos\Theta_2 = \sin\theta \sin\varphi$$

integral over all possible propagation directions:

$$\sum_\alpha \int d\Omega \cos^2\Theta_\alpha = \int d\Omega \sin^2\theta = 2\pi \int_{-1}^{1} d(\cos\theta)(1-\cos^2\theta)$$
$$= 2\pi\left(2 - \frac{2}{3}\right) = \frac{8\pi}{3}$$

$$\boxed{T = \frac{4}{3} \frac{1}{\hbar c} \left(\frac{Z}{A} e\right)^2 \cdot \frac{\omega}{m^2 c^2} |\langle \Psi_f | \mathbb{P} | \Psi_i \rangle|^2}$$

(note)

$$[\mathbb{P}^2, \mathbb{r}] = -2i\hbar\, \mathbb{P}$$

$\Rightarrow$

$$\langle \Psi_f | \mathbb{P} | \Psi_i \rangle = \langle \Psi_f | \frac{1}{-2i\hbar} \cdot 2\mu \, [\underbrace{\frac{\mathbb{P}^2}{2\mu} + V(\mathbb{r})}_{= h_0}, \mathbb{r}] | \Psi_i \rangle$$

$$= \frac{i\mu}{\hbar} \langle \Psi_f | h_0 \mathbb{r} - \mathbb{r} h_0 | \Psi_i \rangle$$

$$= \frac{i\mu}{\hbar} (E_f - E_i) \langle \Psi_f | \mathbb{r} | \Psi_i \rangle$$

$$= -i\mu\omega \langle \Psi_f | \mathbb{r} | \Psi_i \rangle$$

$\Rightarrow$

$$\boxed{T = \frac{4}{3} \cdot \frac{1}{\hbar c} \left(\frac{Z}{A} e\right)^2 \cdot \frac{\mu^2 \omega^3}{m^2 c^2} \left| \langle \Psi_f | \underbrace{\mathbb{r}}_{\text{dipole operator}} | \Psi_i \rangle \right|^2}$$

(note) $[h_0, \mathbb{P}] = [V(\mathbb{r}), \mathbb{P}] = i\hbar (\nabla V)$

$\Rightarrow$

$$\langle \Psi_f | \mathbb{P} | \Psi_i \rangle = \frac{1}{E_f - E_i} \langle \Psi_f | E_f \mathbb{P} - \mathbb{P} E_i | \Psi_i \rangle$$

$$= \frac{1}{E_f - E_i} \langle \Psi_f | [h_0, \mathbb{P}] | \Psi_i \rangle$$

$$= -\frac{i}{\omega} \langle \Psi_f | \underbrace{\nabla V}_{= m\ddot{\mathbb{r}}} | \Psi_i \rangle$$

(荷電粒子は加速度運動するとphotonを出す: bremsstrahlung)

$$Y_{1\pm1} = \mp\sqrt{\frac{3}{8\pi}}\, e^{\pm i\varphi}\sin\theta \qquad Y_{11}^{*} = -Y_{1-1}$$

$$Y_{10} = \sqrt{\frac{3}{4\pi}}\cos\theta$$

◦中性子捕獲断面積

$$x = r\sin\theta\cos\varphi = r\cdot\frac{1}{2}\sqrt{\frac{8\pi}{3}}\left(-Y_{11} + Y_{1-1}\right)$$
$$y = r\sin\theta\sin\varphi = r\cdot\frac{i}{2}\sqrt{\frac{8\pi}{3}}\left(Y_{11} + Y_{1-1}\right)$$
$$z = r\cos\theta = r\cdot\sqrt{\frac{4\pi}{3}}\,Y_{10}$$

↓

$$x^2+y^2+z^2 = r^2\cdot\frac{1}{4}\frac{8\pi}{3}\left(\cancel{+Y_{11}^2}\cancel{+Y_{1-1}^2} - 2Y_{11}Y_{1-1}\cancel{-Y_{11}^2}\cancel{-Y_{1-1}^2} - 2Y_{11}Y_{1-1}\right) + r^2\cdot\frac{4\pi}{3}Y_{10}^2$$

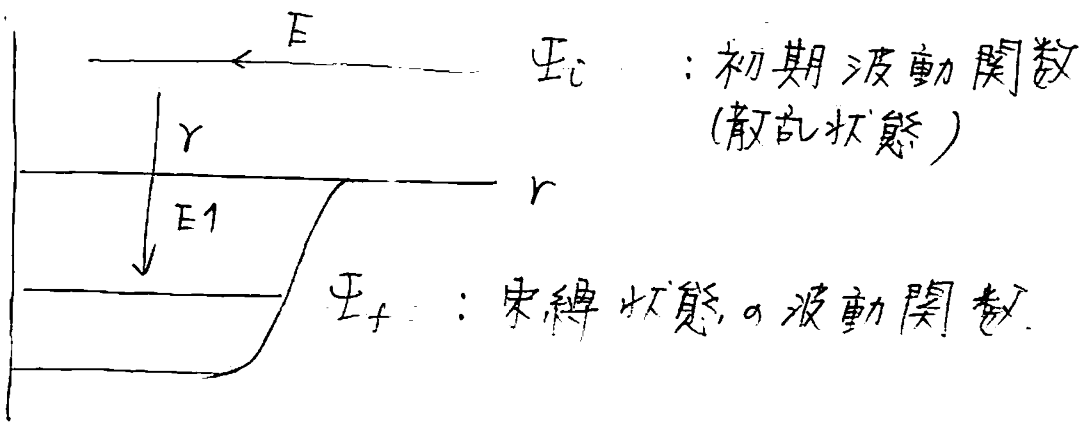
$$= \frac{4\pi}{3}\cdot r^2\left\{Y_{10}^2 - Y_{11}\cdot Y_{1-1} - Y_{1-1}\cdot Y_{11}\right\}$$

$$= \frac{4\pi}{3}\left\{(rY_{10})^2 + |rY_{11}|^2 + |rY_{1-1}|^2\right\}$$

↓

$$\sigma_{i\to f}(E) = \frac{16\pi}{9\hbar}k_\gamma^3\left(\frac{eZ}{A+1}\right)^2\cdot\frac{1}{2j_i+1}$$

$$\times\sum_{m_f}\sum_{m_i}\sum_{\mu}\left|\langle\Psi_{j_f l_f m_f}|\,rY_{1\mu}\,|\Psi_{j_i l_i m_i}(E)\rangle\right|^2$$



$$\Psi_{j_i l_i m_i}(\mathbf{r}) = \frac{u_{j_i l_i}(r)}{r} \mathcal{Y}_{j_i l_i m_i}(\hat{\mathbf{r}})$$

単位フラックスを持つ散乱状態の波動関数

$$u_{j_i l_i}(r) \xrightarrow[r\to\infty]{} \frac{1}{\sqrt{v}} \cdot \frac{i}{2k} \cdot (2l+1) \cdot i^l \sqrt{\frac{4\pi}{2l+1}}$$

$$\times \left[h_l^{(+)}(kr) - S_l h_l^{(-)}(kr) \right]$$

(note) for $\psi = \frac{1}{\sqrt{v}} e^{i\mathbf{k}\cdot\mathbf{r}}$

$$j = \frac{\hbar}{2i\mu} \cdot 2ik \cdot \frac{1}{v} = \frac{k\hbar}{\mu} \cdot \frac{1}{v} = 1$$

(note)

$$e^{i\mathbf{k}\cdot\mathbf{r}} \rightarrow \frac{i}{2k} \sum_l (2l+1) i^l \left[e^{-i(kr - \frac{l\pi}{2})} - e^{i(kr-\frac{l\pi}{2})} \right]$$

$$\times \frac{1}{r} \underbrace{P_l(\cos\theta)}_{= \sqrt{\frac{4\pi}{2l+1}} Y_{l0}(\theta)}$$

$$\Psi_{j_f l_f m_f}(\mathbf{r}) = \frac{u_{j_f l_f}(r)}{r} \mathcal{Y}_{j_f l_f m_f}(\hat{\mathbf{r}})$$

束縛状態の波動関数

(note) $\frac{1}{2j_i+1} \sum_{m_f} \sum_{m_i} \sum_{\mu} |\underbrace{\langle \mathcal{Y}_{j_f l_f m_f} | Y_{1\mu} | \mathcal{Y}_{j_i l_i m_i} \rangle}| ^2$

$$\parallel$$

$$(-)^{j_i - m_i} \frac{1}{\sqrt{3}} \langle j_f m_f \, j_i - m_i | 1 \mu \rangle$$

$$\times \langle \mathcal{Y}_{j_f l_f} \| Y_1 \| \mathcal{Y}_{j_i l_i} \rangle$$

$$= \frac{1}{2j_i+1} |\langle \mathcal{Y}_{j_f l_f} \| Y_1 \| \mathcal{Y}_{j_i l_i} \rangle|^2$$

(note) $\langle \mathcal{Y}_{j'l'} \| Y_\lambda \| \mathcal{Y}_{jl} \rangle = \delta_{l+l'+\lambda, \text{even}}$

$$\times (-)^{\frac{1}{2}+j'} \frac{\hat{j}\hat{\lambda}\hat{j}'}{\sqrt{4\pi}} \begin{pmatrix} j' & \lambda & j \\ 1/2 & 0 & -1/2 \end{pmatrix}$$

$\Rightarrow$

$$\boxed{\sigma_{i\to f}(E) = \frac{16\pi}{9\hbar} k_\gamma^3 \left(\frac{eZ}{A+1}\right)^2 \cdot \frac{1}{2j+1} |\langle \mathcal{Y}_{j_f l_f} \| Y_1 \| \mathcal{Y}_{j_i l_i} \rangle|^2 \times \left| \int_0^\infty dr \, u_{j_f l_f}(r) \cdot r \cdot u_{j_i l_i}(r) \right|^2}$$