§. Green 関数

$$G_0^{(\pm)}(\mathbf{r}, \mathbf{r}'; E) = \langle \mathbf{r} | \frac{1}{E - \hat{T} \pm i\eta} | \mathbf{r}' \rangle$$

$$= \int d\mathbf{k}' \, \langle \mathbf{r} | \mathbf{k}' \rangle \langle \mathbf{k}' | \mathbf{r}' \rangle \cdot \frac{1}{\frac{k^2\hbar^2}{2m} - \frac{k'^2\hbar^2}{2m} \pm i\eta}$$

$$= \frac{1}{(2\pi)^3} \cdot \frac{2m}{\hbar^2} \int d\mathbf{k}' \, \frac{e^{i\mathbf{k}'\cdot(\mathbf{r}-\mathbf{r}')}}{k^2 - k'^2 \pm i\eta}$$

$$= \frac{1}{(2\pi)^3} \cdot \frac{2m}{\hbar^2} \int k'^2 dk' \, d\hat{k} \, \frac{e^{ik's\cos\theta}}{k^2 - k'^2 \pm i\eta} \qquad (\vec{s} \equiv \vec{r} - \vec{r}')$$

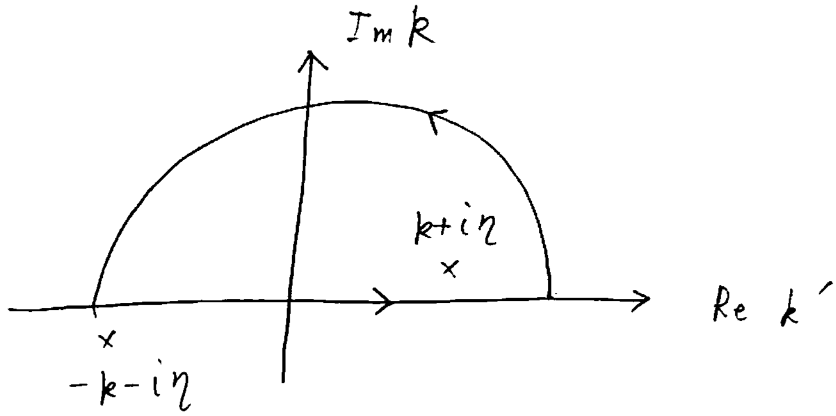
$$= \frac{1}{(2\pi)^3} \cdot \frac{2m}{\hbar^2} \int k'^2 dk' \cdot \underbrace{2\pi \int_{-1}^{1} d(\cos\theta) \, \frac{e^{ik's\cos\theta}}{k^2 - k'^2 \pm i\eta}}_{= \frac{2\pi}{k^2 - k'^2 \pm i\eta} \cdot \frac{1}{iks} \left(e^{ik's} - e^{-ik's}\right)}$$

$$= \frac{2m}{i \cdot (2\pi)^2 \hbar^2} \underbrace{\int_0^\infty \frac{k' dk'}{k^2 - k'^2 \pm i\eta} \left(\frac{e^{ik's}}{s} - \frac{e^{-ik's}}{s} \right)}_{= \int_{-\infty}^{\infty} \frac{k' dk'}{k^2 - k'^2 \pm i\eta} \cdot \frac{e^{ik's}}{s}}$$

$$= \int_{-\infty}^{\infty} \frac{k' dk'}{(k \pm i\eta + k')(k \pm i\eta - k')} \cdot \frac{e^{ik's}}{s}$$

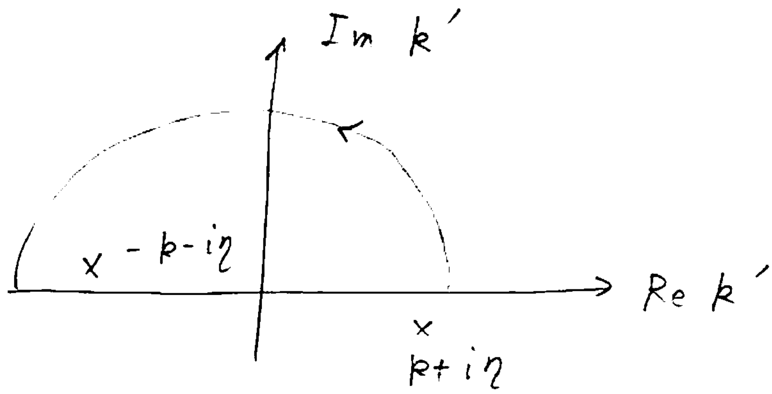
$$= -\frac{1}{2} \int_{-\infty}^{\infty} \left[\frac{1}{k + k' \pm i\eta} - \frac{1}{k \pm i\eta - k'} \right] \frac{e^{ik's}}{s} \cdot dk'$$

$\eta > 0$



$$G_0 = -\frac{2m}{i\cdot(2\pi)^2\hbar^2}\cdot\frac{1}{2}\cdot 2\pi i\,\frac{e^{iks}}{s} = -\frac{m}{2\pi\hbar^2}\cdot\frac{e^{iks}}{s}$$

$\eta < 0$



$$G_0 = -\frac{2m}{i\cdot(2\pi)^2\hbar^2}\cdot\frac{1}{2}\cdot 2\pi i\cdot\frac{e^{-iks}}{s} = -\frac{m}{2\pi\hbar^2}\cdot\frac{e^{-iks}}{s}$$

$$\boxed{G_0^{(\pm)}(\mathbf{r},\mathbf{r}';E) = -\frac{2m}{\hbar^2}\cdot\frac{1}{4\pi}\cdot\frac{e^{\pm ik|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}}$$