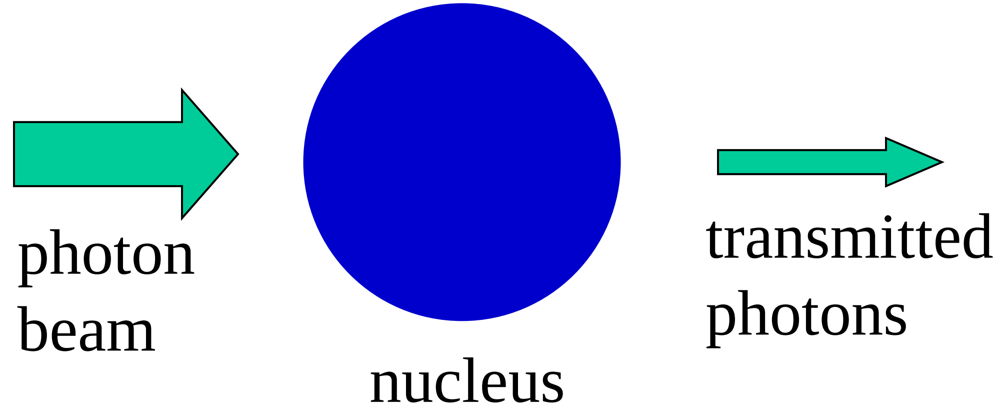
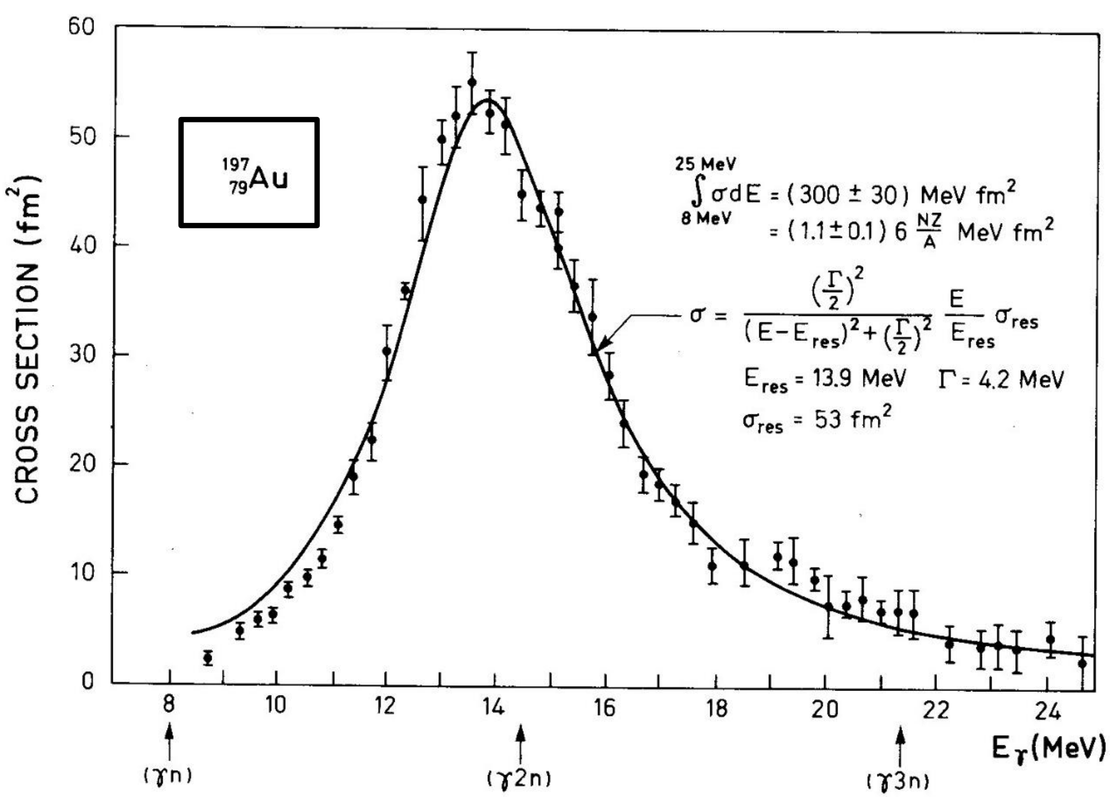
Collective Vibrations

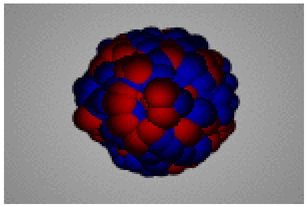


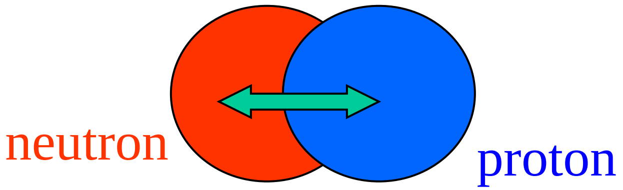
光吸収断面積

Giant Dipole Resonance (GDR)　巨大双極子共鳴



14 MeV付近に
励起状態がある





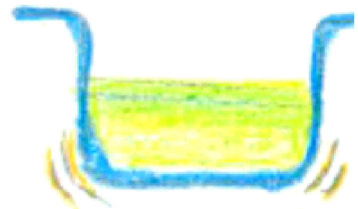
Isovector type

Sum Rule

$$|\psi\rangle = F|0\rangle$$

F（電磁場などの外場）

$$|0\rangle$$



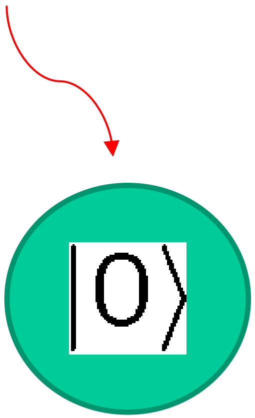
図：松柳研一氏

外場をかけて原子核をゆすってみる

Sum Rule

$$|\psi\rangle = F|0\rangle$$
$$= \sum_n |n\rangle\langle n|F|0\rangle$$

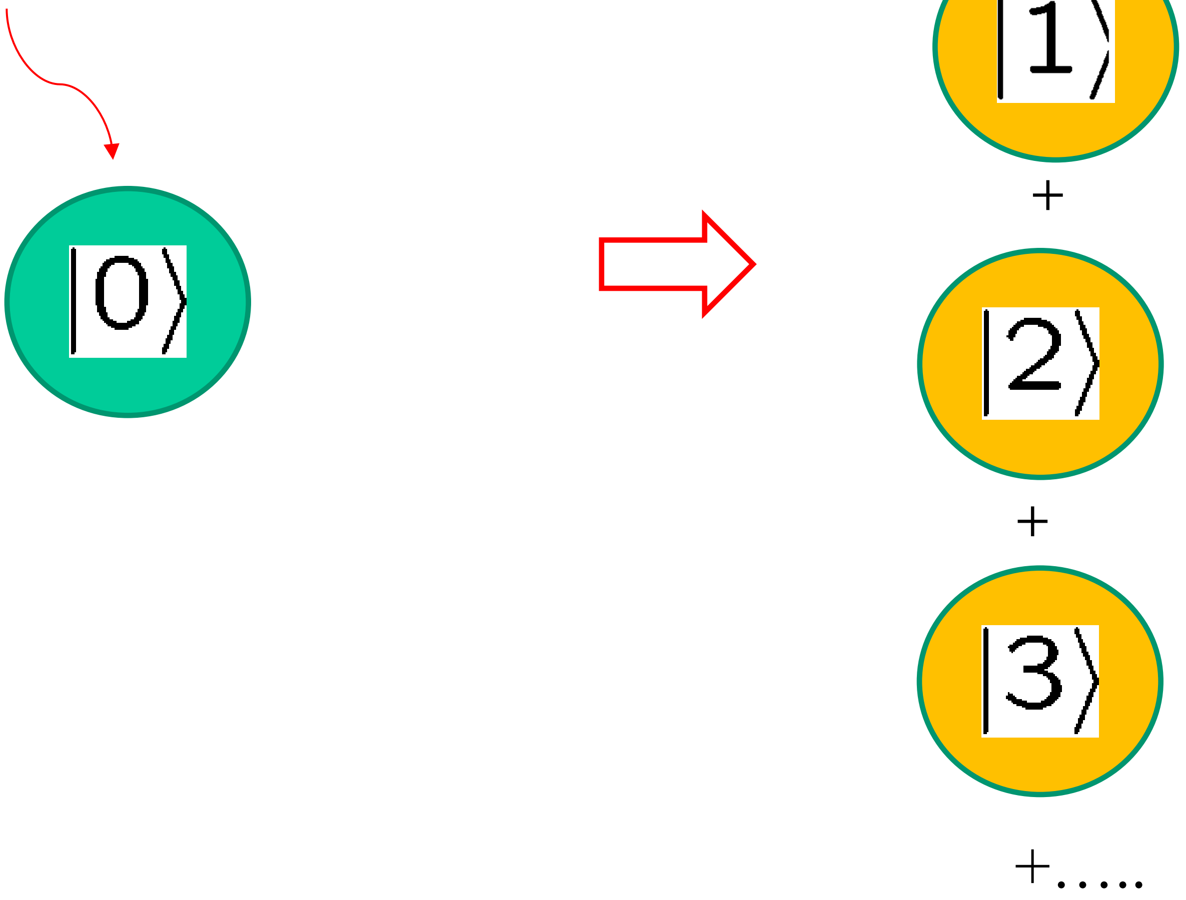
F（電磁場などの外場）



Sum Rule

$$|\psi\rangle = F|0\rangle$$
$$= \sum_n |n\rangle\langle n|F|0\rangle$$

F（電磁場などの外場）



確率

$|\langle 1|F|0\rangle|^2$

$|\langle 2|F|0\rangle|^2$

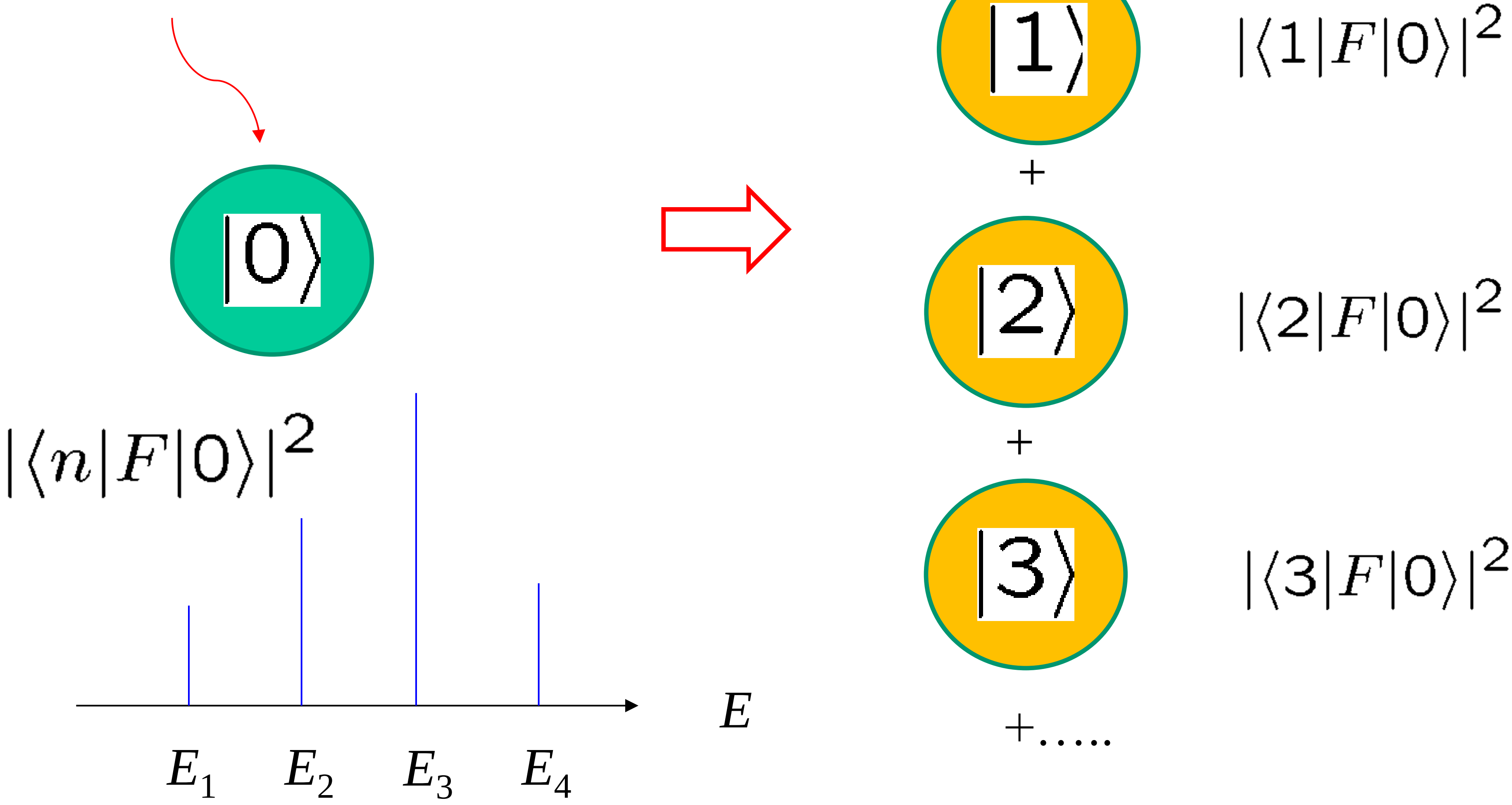
$|\langle 3|F|0\rangle|^2$

Sum Rule

Strength function
(強度関数):

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \, \delta(E_n - E_0 - E)$$

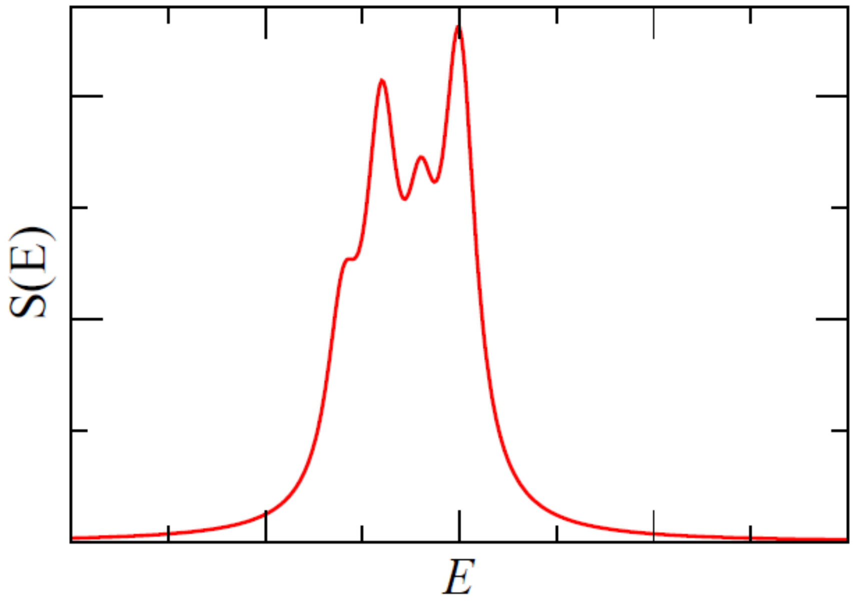
F（電磁場などの外場）



Sum Rule

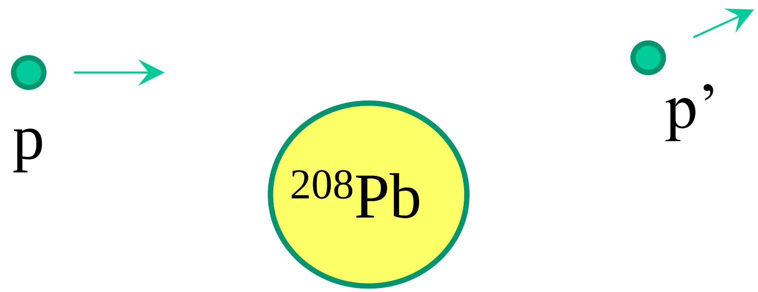
Strength function:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$

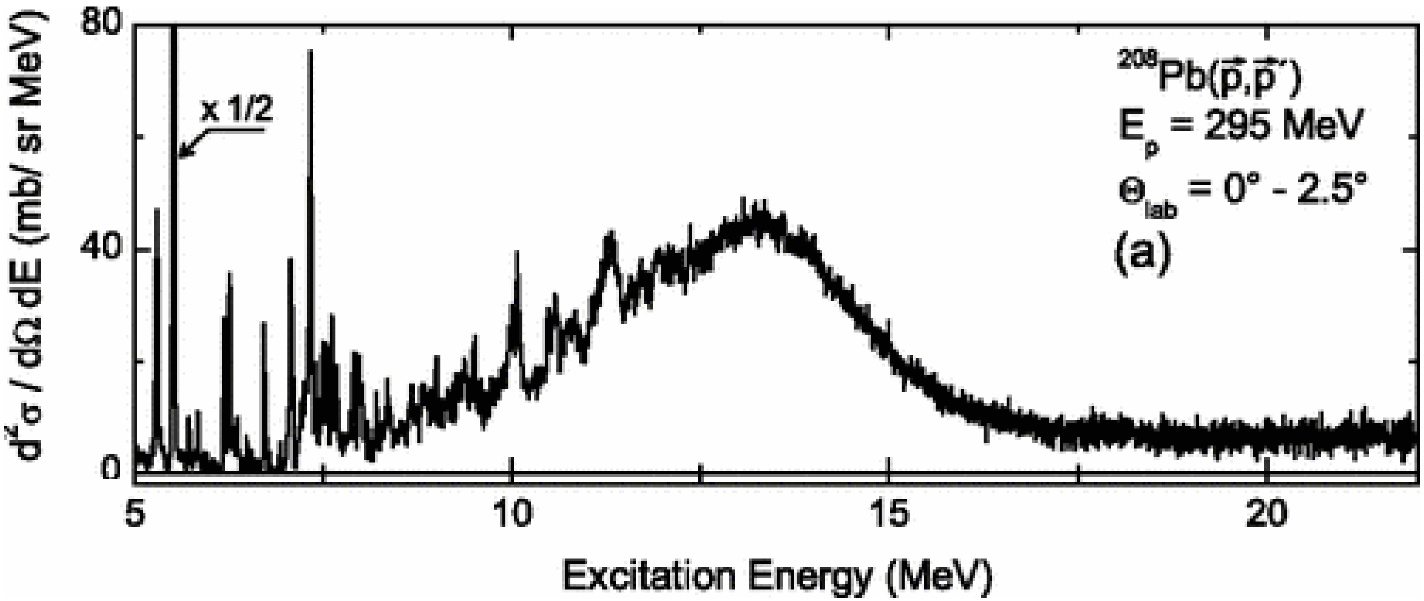


Sum Rule

例えば:



非弾性散乱(^{208}Pb の励起)のスペクトル

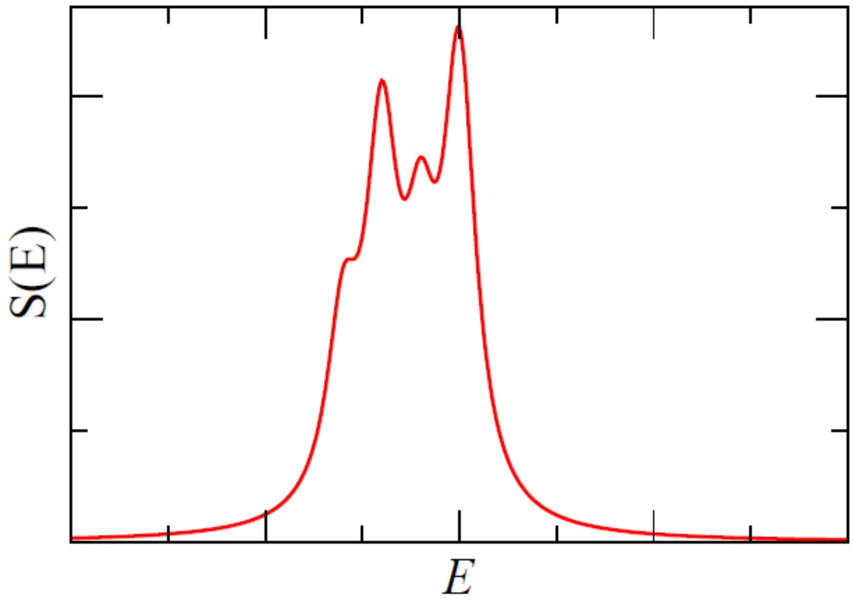


A. Tamii et al., PRL107, 062502 (2011)

Sum Rule

Strength function:

$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$



強度関数のモーメント:

$$S_k \equiv \int E^k S(E) dE$$

✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$

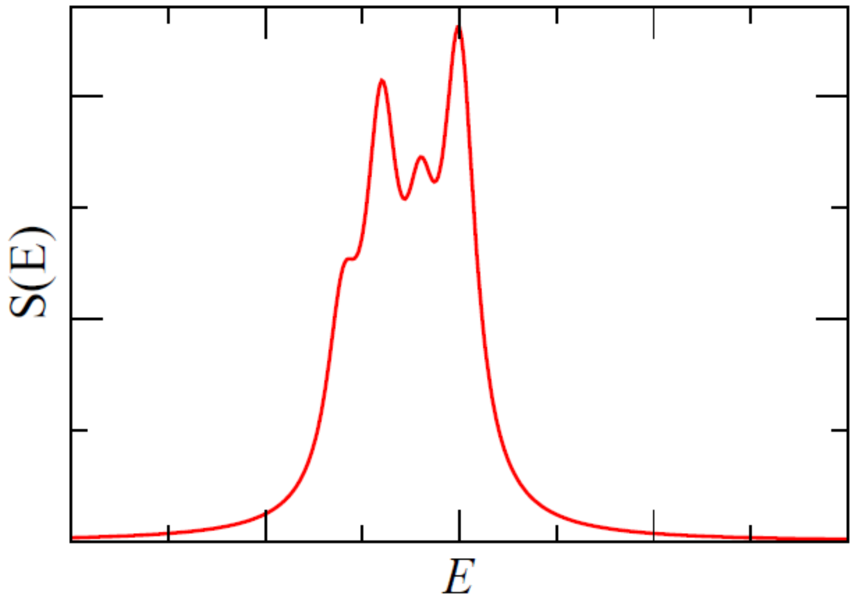
✓ energy weighted sum rule

$$S_1 \equiv \int ES(E)dE = \sum_n (E_n - E_0)|\langle n|F|0\rangle|^2$$

✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$
$$= \langle 0|F^2|0\rangle$$

F^2の基底状態期待値

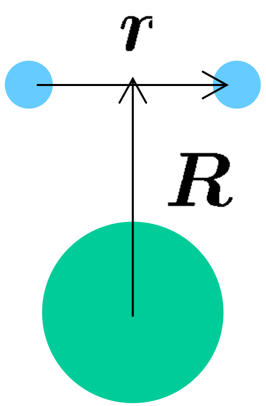


$$S(E) = \sum_n |\langle n|F|0\rangle|^2 \times\delta(E_n - E_0 - E)$$

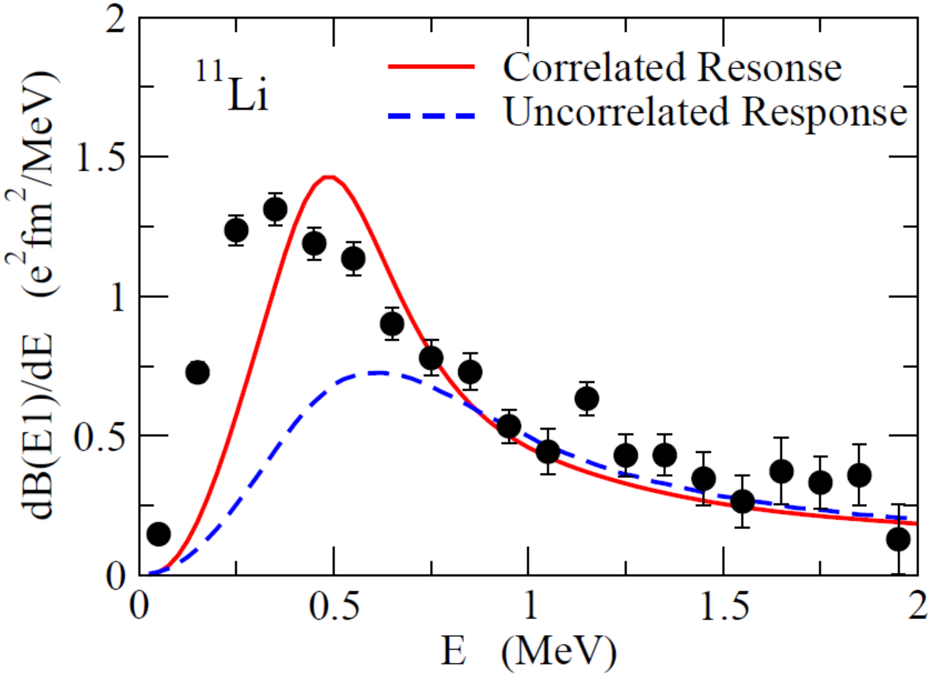
✓ non-energy weighted sum rule

$$S_0 \equiv \int S(E)dE = \sum_n |\langle n|F|0\rangle|^2$$
$$= \langle 0|F^2|0\rangle$$

F^2の基底状態期待値



cf. geometry of Borromean nuclei



experimental data:
T. Nakamura et al., PRL96('06)252502

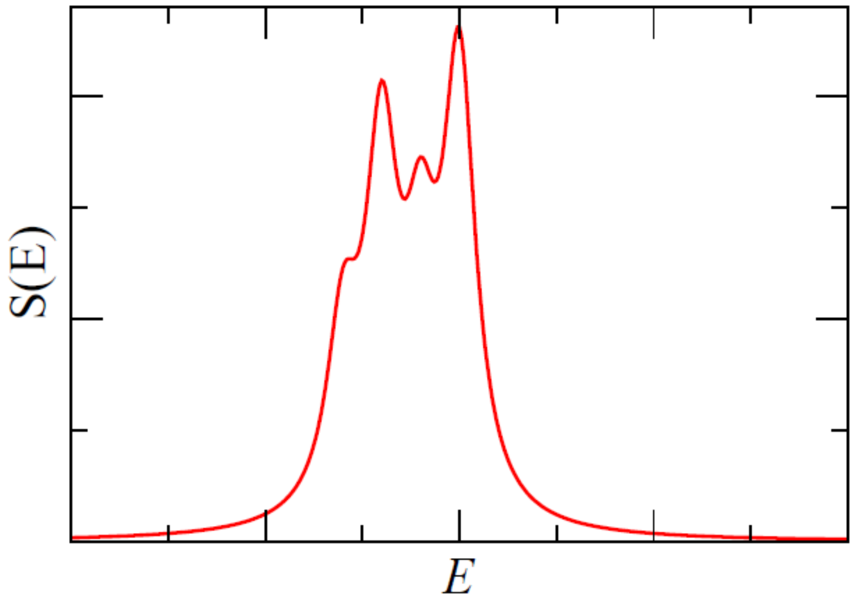
$$B(E1) = \sum_i B(E1; gs \to i)$$
$$= \frac{3}{\pi}\left(\frac{Ze}{A}\right)^2 \langle R^2\rangle$$

$$\Rightarrow \langle\theta_{nn}\rangle = 65.2^{+11.4}_{-13.0} \quad (^{11}\text{Li})$$
$$= 74.5^{+11.2}_{-13.1} \quad (^{6}\text{He})$$

K.H. and H. Sagawa,
PRC76('07)047302

✓ energy weighted sum rule

$$S_1 \equiv \int E S(E) dE$$
$$= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2$$
$$= \frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle$$



$$S(E) = \sum_\nu |\langle \nu | F | 0 \rangle|^2 \times \delta(E_\nu - E_0 - E)$$

$$\frac{1}{2} \langle 0 | [F, [H, F]] | 0 \rangle = \frac{1}{2} \langle F(HF - FH) - (HF - FH)F \rangle$$
$$= \langle FHF - E_0 F^2 \rangle$$
$$= \sum_\nu E_\nu |\langle 0 | F | \nu \rangle|^2 - E_0 \langle 0 | F^2 | 0 \rangle$$
$$= \sum_\nu (E_\nu - E_0) |\langle \nu | F | 0 \rangle|^2$$

Energy weighted sum rule:

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \frac{1}{2}\langle 0|[F,[H,F]]|0\rangle$$

For $F = F(\boldsymbol{r})$ (local operator)

$$[H,F] = \left[-\frac{\hbar^2}{2m}\nabla^2, F\right] = -\frac{\hbar^2}{2m}(\nabla^2 F + 2\nabla F\cdot\nabla)$$

$$\Rightarrow \quad [F,[H,F]] = \frac{\hbar^2}{m}(\nabla F)^2$$

$$\Rightarrow \quad S_1 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|F|0\rangle|^2 = \frac{\hbar^2}{2m}\int d\boldsymbol{r}\,\rho(\boldsymbol{r})\cdot(\nabla F)^2$$

For F=z

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle\nu|z|0\rangle|^2 = \frac{\hbar^2 N_{sys}}{2m}$$

[TRK (Thomas-Reiche-Kuhn) Sum Rule]

⬆

Model independent

$$\sigma_{\text{abs}}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

$$\rightarrow \int \sigma_{\text{abs}}(E_\gamma)dE_\gamma = \frac{2\pi^2 e^2\hbar}{mc}\cdot\frac{NZ}{A}$$

レポート問題2（〆切：12月2日（月））

１次の摂動論を用いると、エネルギー Ei にある原子核の状態 ϕ_i がエネルギー E_γ のフォトンを吸収してエネルギー E_f にある状態 ϕ_f に遷移するときの断面積は以下で与えられる。

$$\sigma_{\rm abs}(E_\gamma) = \frac{4\pi^2 e^2}{\hbar c}(E_f - E_i)|\langle\phi_f|\tilde{z}|\phi_i\rangle|^2\,\delta(E_\gamma - E_f + E_i)$$

ここで、tilde z は、重心から測った陽子の z 座標の和で、

$$\tilde{z} = \sum_p (z_p - Z_{cm}) \;=\; \sum_p \left\{ z_p - \frac{1}{A}\left(\sum_{p'} z_{p'} + \sum_n z_n\right)\right\} = \frac{NZ}{A}\left(\frac{1}{Z}\sum_p z_p - \frac{1}{N}\sum_n z_n\right)$$

で与えられる。ここで、N, Z はそれぞれ原子核の中性子数、陽子数である。TRK和則を用いて、

$$\int \sigma_{\rm abs}(E_\gamma)dE_\gamma = \frac{2\pi^2 e^2\hbar}{mc}\cdot\frac{NZ}{A}$$

となることを示せ。

Giant Dipole Resonance (GDR)

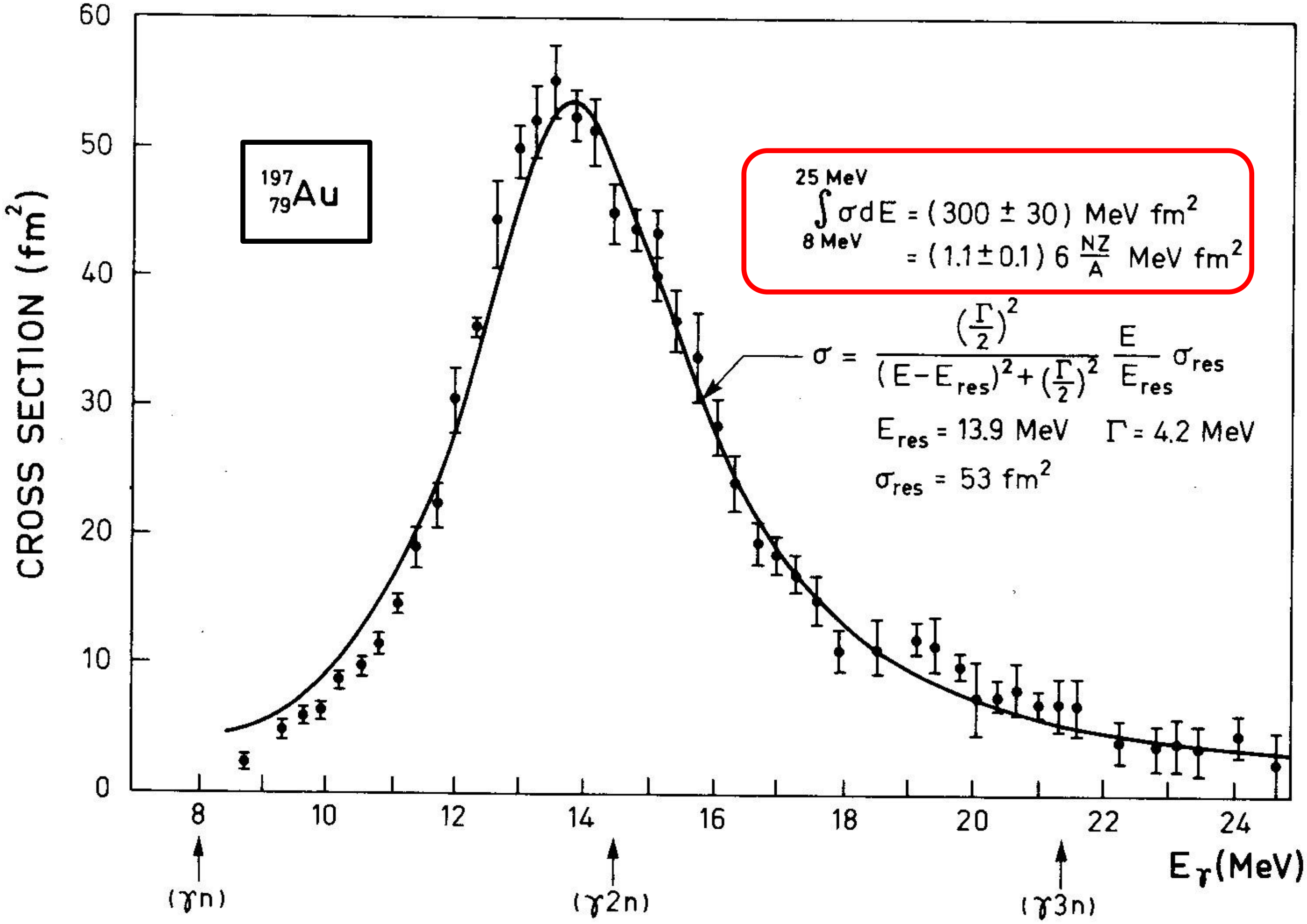


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

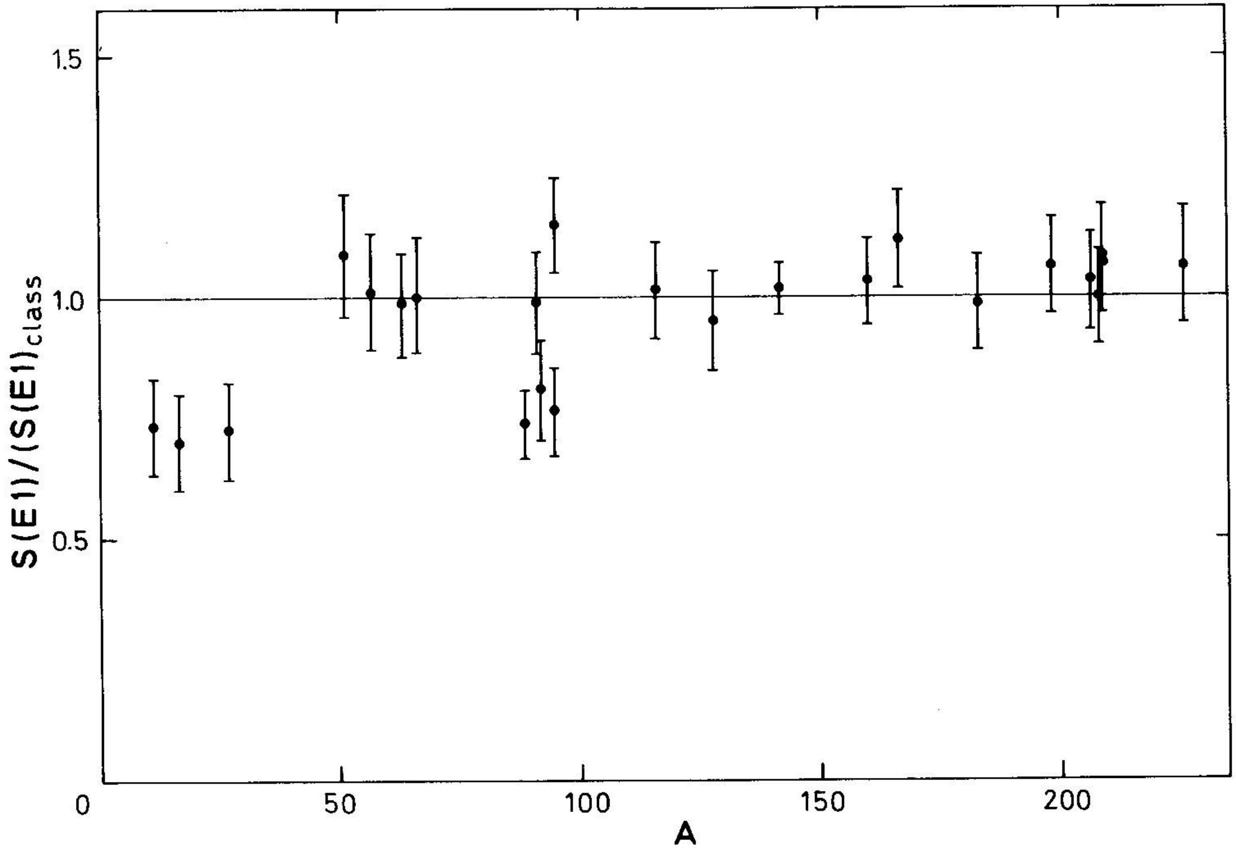


Figure 6-20 Total oscillator strength for dipole resonance. The observed total oscillator strength for energies up to 30 MeV is given in units of the classical sum rule value. For the nuclei with $A > 50$, the integrated oscillator strengths have been obtained from measurements of neutron yields produced by monochromatic γ rays (S. C. Fultz, R. L. Bramblett, B. L. Berman, J. T. Caldwell, and M. A. Kelly, in *Proc. Intern. Nuclear Physics Conference*, p. 397, ed.-in-chief R. L. Becker, Academic Press, New York, 1967). The photoscattering cross sections have been ignored, since they contribute only a very small fraction of the total cross sections. For the lighter nuclei, the yield of (γp) processes must be included and the data are from: ^{12}C and ^{27}Al (S. C. Fultz, J. T. Caldwell, B. L. Berman, R. L. Bramblett, and R. R. Harvey, *Phys. Rev.* **143**, 790, 1966); ^{16}O (Dolbilkin *et al.*, *loc.cit.*, Fig. 6-26). For the heavy nuclei ($A > 50$), other measurements have yielded total oscillator strengths that are about 20% larger than those shown in the figure (see, for example, Veyssière *et al.*, 1970).

分極率と inverse energy weighted sum rule

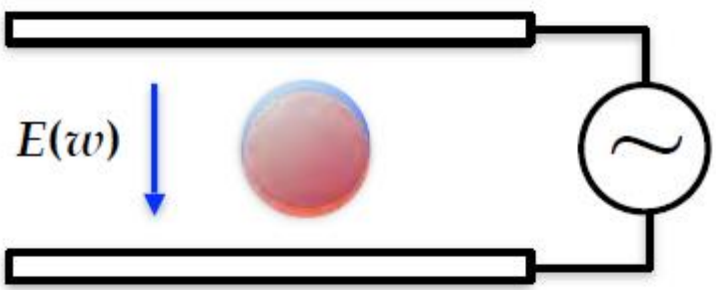


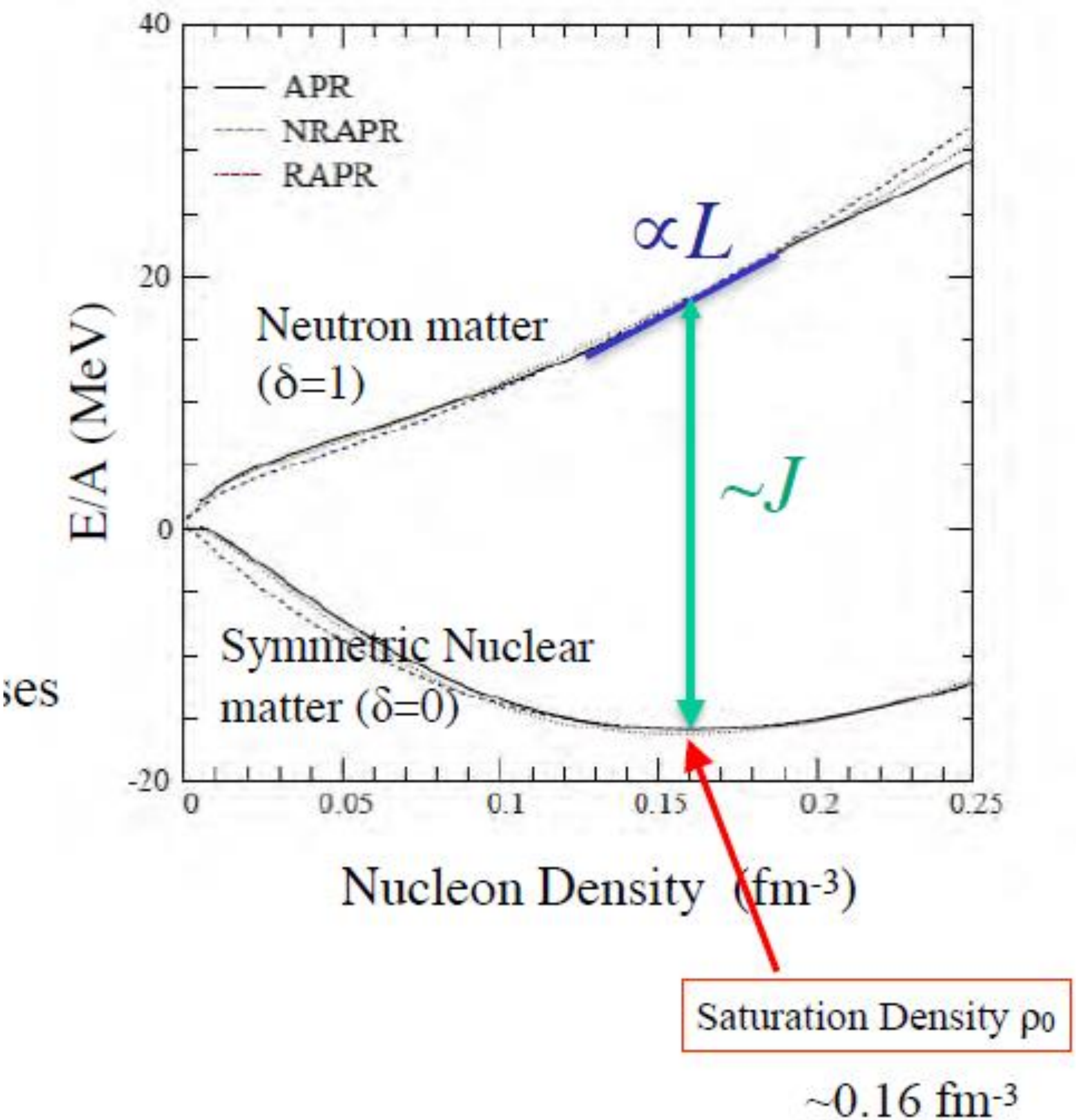
図 : 民井さん

原子核の静電分極率
→対称エネルギー

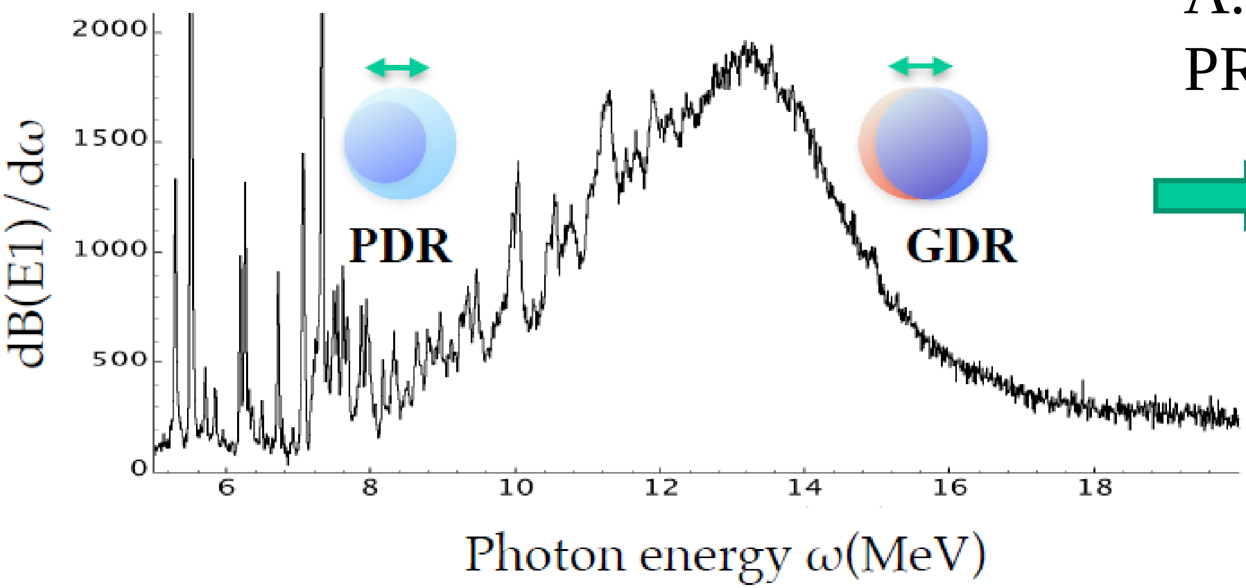
$$H = H_0 - \lambda F$$

一次の摂動論 : $|\tilde{\psi}_0\rangle = |\psi_0\rangle - \lambda \sum_{n>0} \frac{\langle\psi_n|F|\psi_0\rangle}{E_0 - E_n} |\psi_n\rangle$

$$\rightarrow \langle\tilde{\psi}_0|F|\tilde{\psi}_0\rangle = \langle\psi_0|F|\psi_0\rangle + \underbrace{2 \sum_{n>0} \frac{|\langle\psi_n|F|\psi_0\rangle|^2}{E_n - E_0}}_{\text{分極率}} \lambda$$

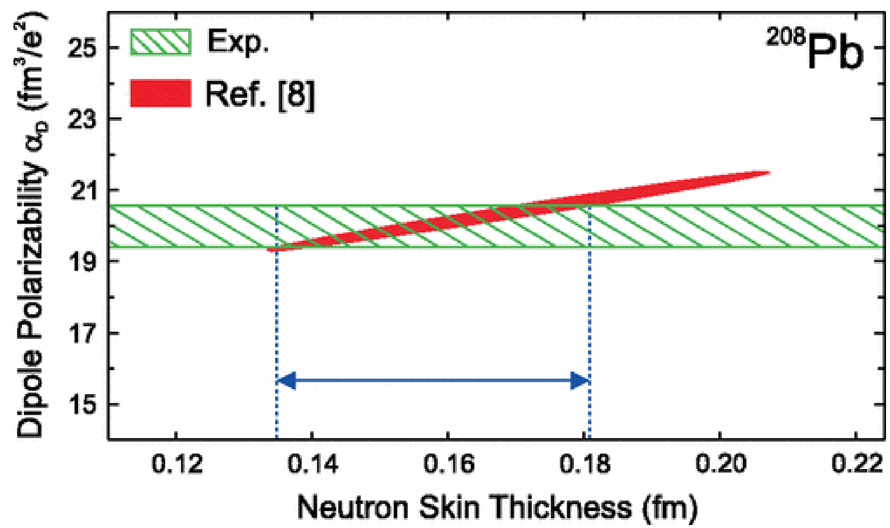

APR
NRAPR
RAPR
∝L
Neutron matter
(δ=1)
~J
Symmetric Nuclear
matter (δ=0)
ses
E/A (MeV)
Nucleon Density (fm-3)
Saturation Density ρ0
~0.16 fm-3

スライド：民井さん

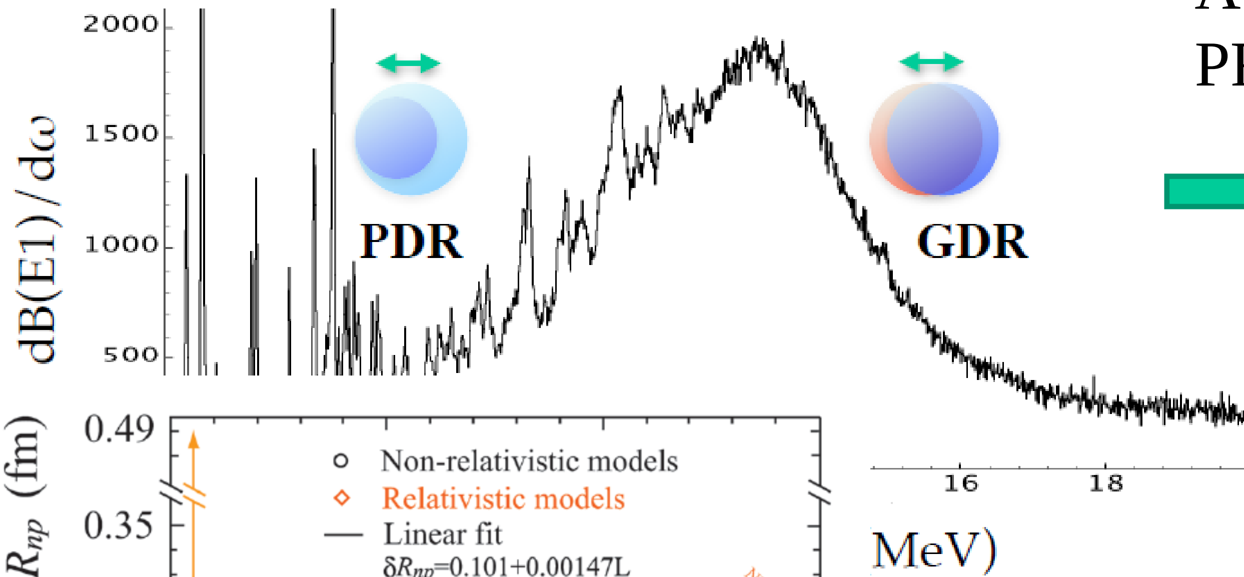


A. Tamii et al.,
PRL107, 062502 (2011)

α=20.1+/− 0.6 fm^3

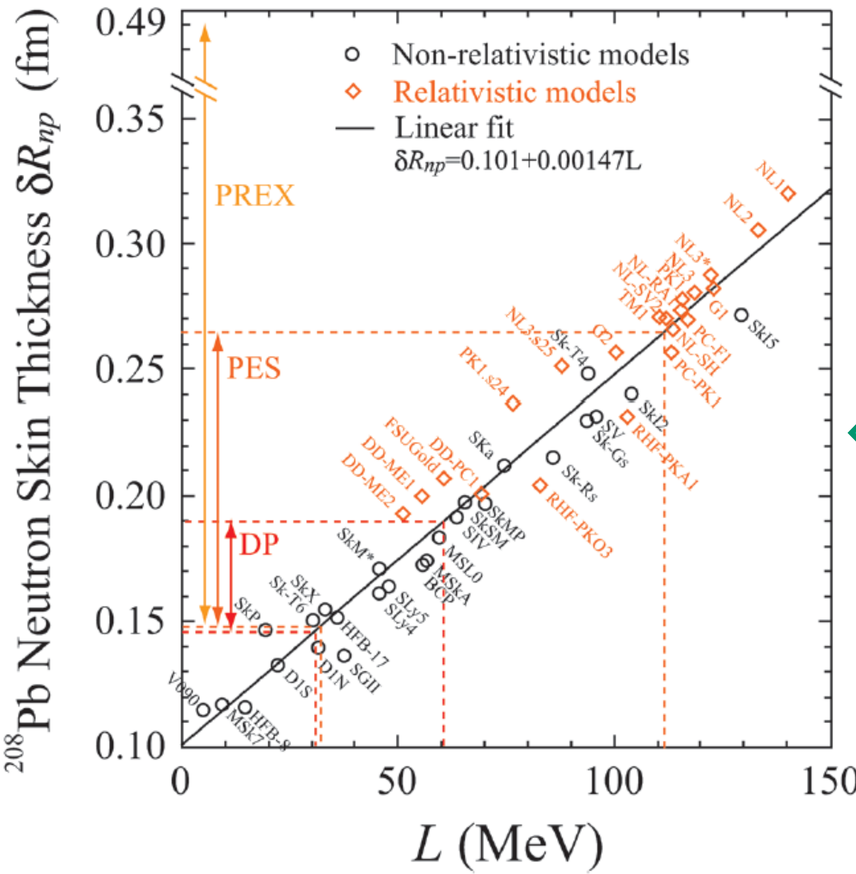


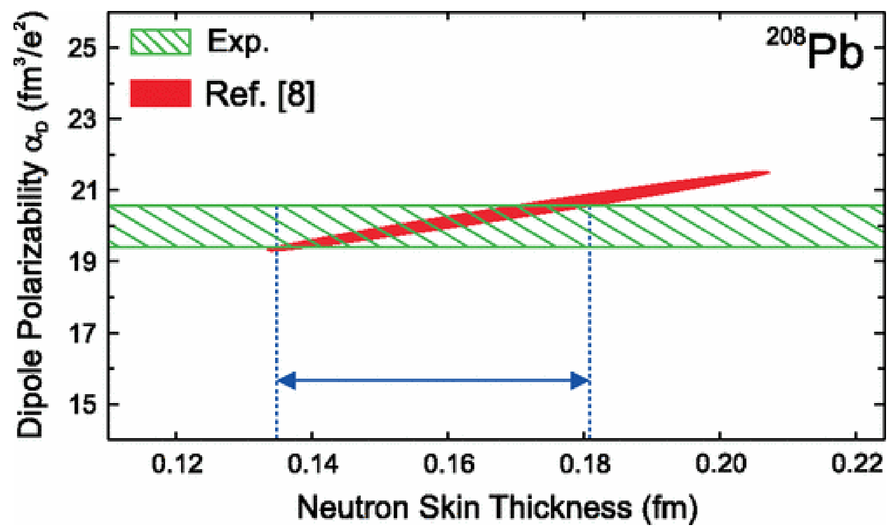
$r_{skin} = 0.156 ^{+0025}{}_{-0.021}$ fm



A. Tamii et al.,
PRL107, 062502 (2011)

$\alpha = 20.1 +/- 0.6\ \mathrm{fm}^3$





$r_{skin} = 0.156^{+0025}_{-0.021}\ \mathrm{fm}$

民井、銭廣(日本物理学会誌)

レポート問題3（〆切：12月2日（月））

1次元調和振動子

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

の n 番目の固有状態 $|n\rangle$ を考える(n=0 が基底状態)。

1) 演算子 x^2 に対して、強度関数を求めよ。n が 0 の場合、1 の場合、2以上の場合で場合分けせよ。

2) 演算子 x で遷移できる状態 $|k\rangle$ を全て書き出し(状態 k も調和振動子の固有状態)、遷移確率

$$P_{n\to k} = |\langle k|x|n\rangle|^2$$

を求めよ。

3)演算子 x に対して energy weighted sum rule

$$S_1 = \sum_k (E_k - E_n) P_{n\to k}$$

を計算し、TRK和則が成り立っていることを示せ。