集団励起の微視的理論

原子核の励起状態

- ✓ 一粒子励起（一つの核子が励起に関与）
- ✓ 集団励起（多くの核子が集団として励起に関与）

集団励起を微視的に理解してみる
（集団励起をミクロに見てみるとどうなっているのか?）

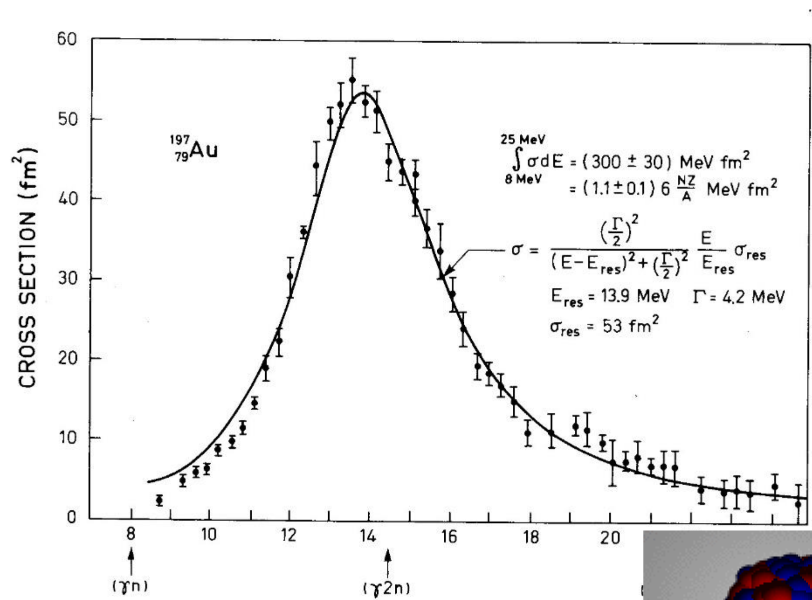
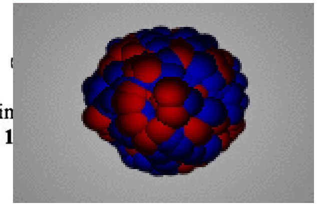
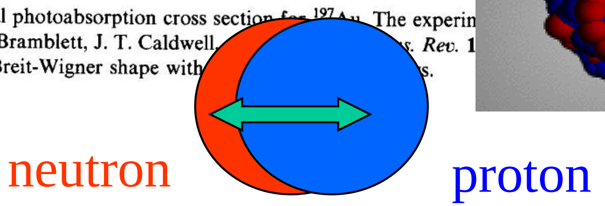


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experim… S. C. Fultz, R. L. Bramblett, J. T. Caldwell, … s. *Rev.* 1… solid curve is of Breit-Wigner shape with … s.

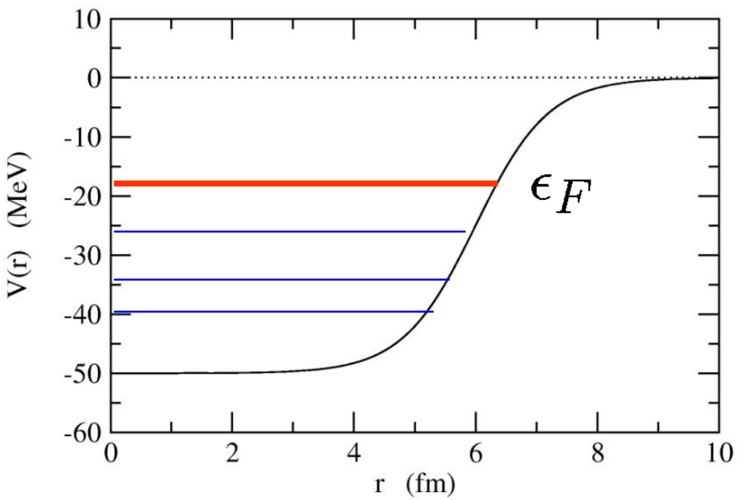




集団励起の例：巨大双極子共鳴

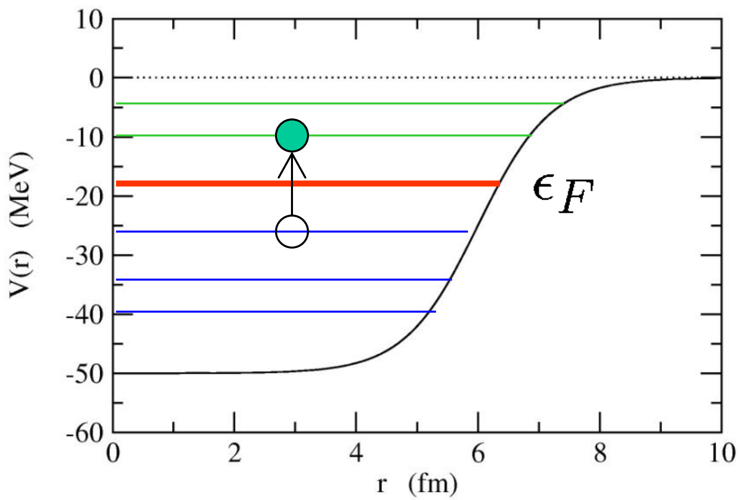
Particle-Hole excitations

Hartree-Fock state



$$|HF\rangle = \prod_h a_h^\dagger |0\rangle$$

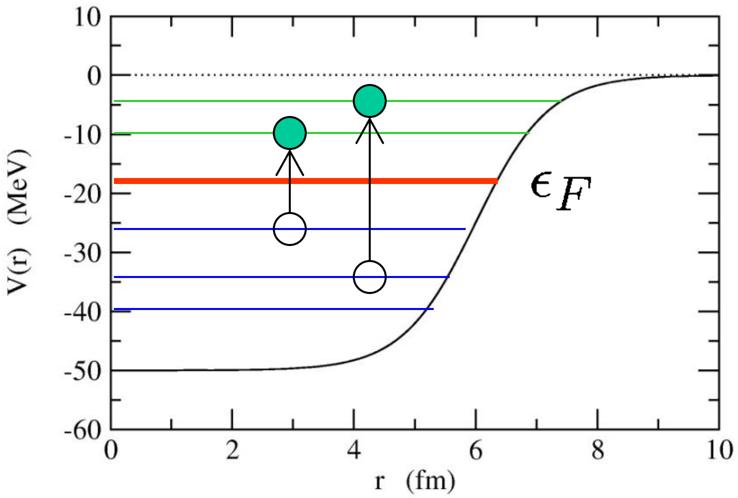
1 particle-1 hole (1p1h) state



$$a_p^\dagger a_h |HF\rangle$$

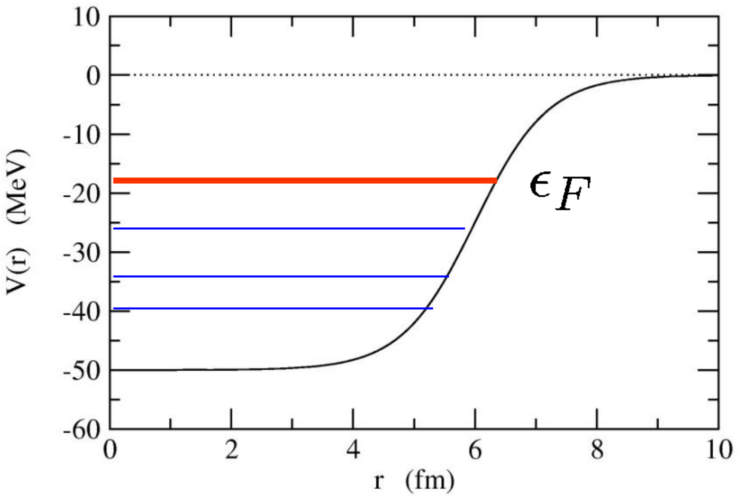
2 particle-2 hole (2p2h) state

$$a_p^\dagger a_{p'}^\dagger a_h a_{h'} |HF\rangle$$



Tamm-Dancoff Approximation

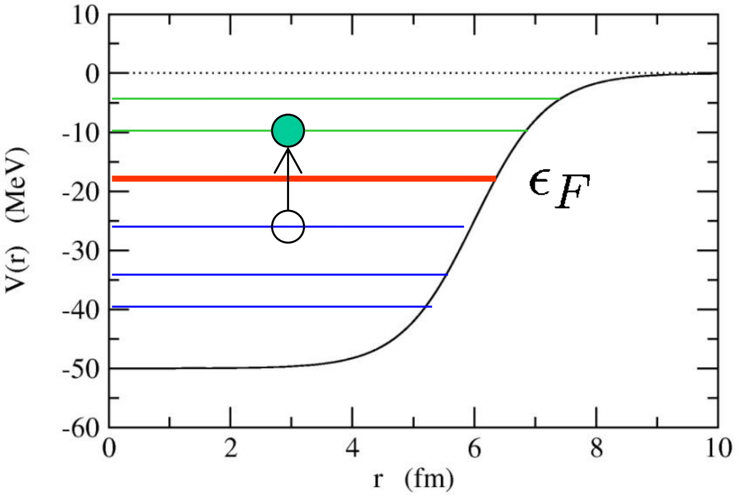
基底状態: $|HF\rangle = \prod_h a_h^\dagger |0\rangle$



励起状態:

$$|\nu\rangle = Q_\nu^\dagger |HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h |HF\rangle$$

$$\equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$$



（1p1h 状態の重ね合わせ
: 2p2h以上は寄与しないと仮定）

Tamm-Dancoff Approximation

Assume: $|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle$

$$\equiv \sum_{ph} X_{ph}|ph^{-1}\rangle$$

(superposition of 1p1h states)

$$H|\nu\rangle = E_\nu|\nu\rangle$$

$$\sum_{p'h'} H_{ph,p'h'}X_{p'h'} = E_\nu X_{ph}$$

$$H_{ph,p'h'} = \langle ph^{-1}|H|p'h'^{-1}\rangle$$
$$= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

residual interaction

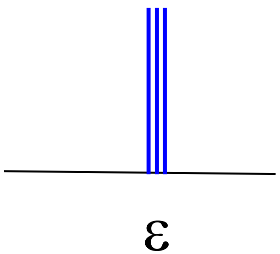
Tamm-Dancoff equation;　1p1h の空間でハミルトニアンを対角化

TDA on a schematic model

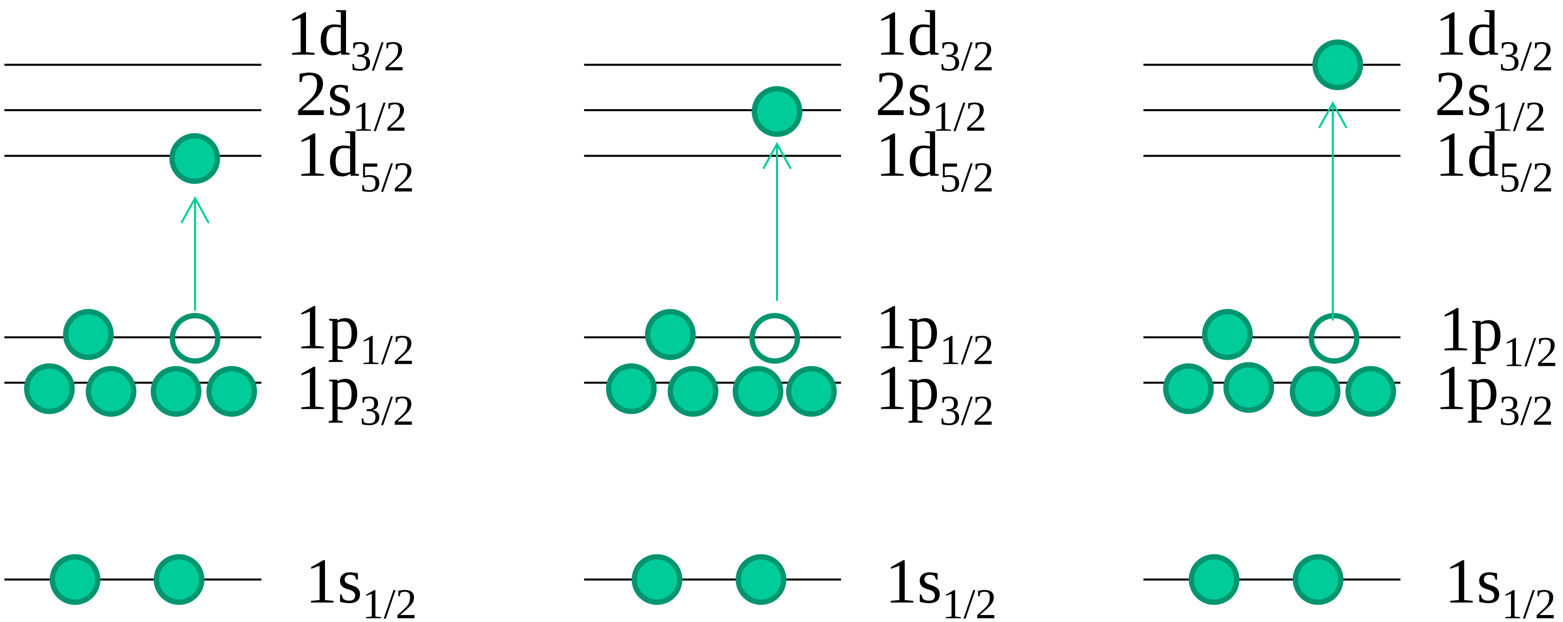
$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \quad \langle ph'|\bar{v}|hp'\rangle = g$$

for 3 ph configurations:



（例えば）



TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \qquad \langle ph'|\bar{v}|hp'\rangle = g$$

for 3 ph configurations:

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization:

$$\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$$

TDA on a schematic model

$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = (\epsilon+3g) \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

全ての状態が同位相で寄与
＝コヒーレントな重ね合わせ

他の固有状態:

$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

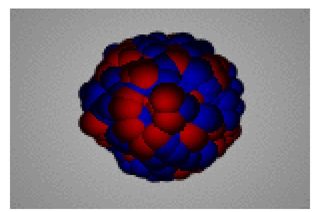
$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

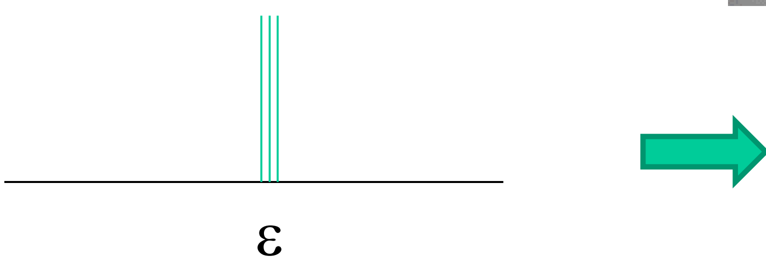
位相がそろっていない

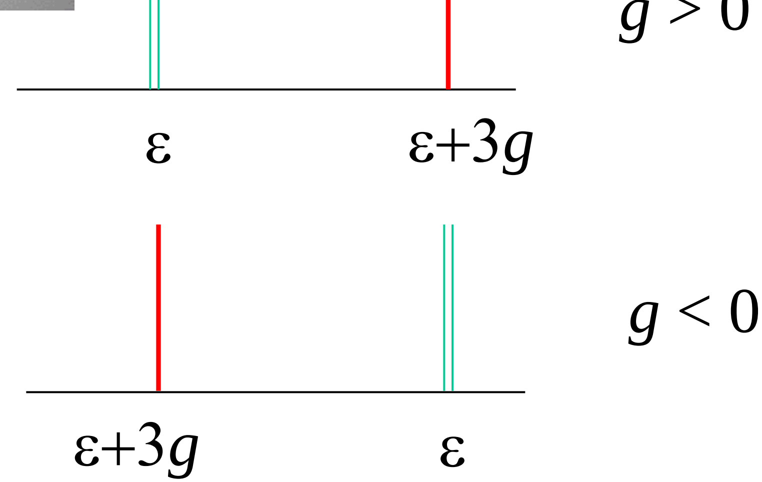
TDA on a schematic model

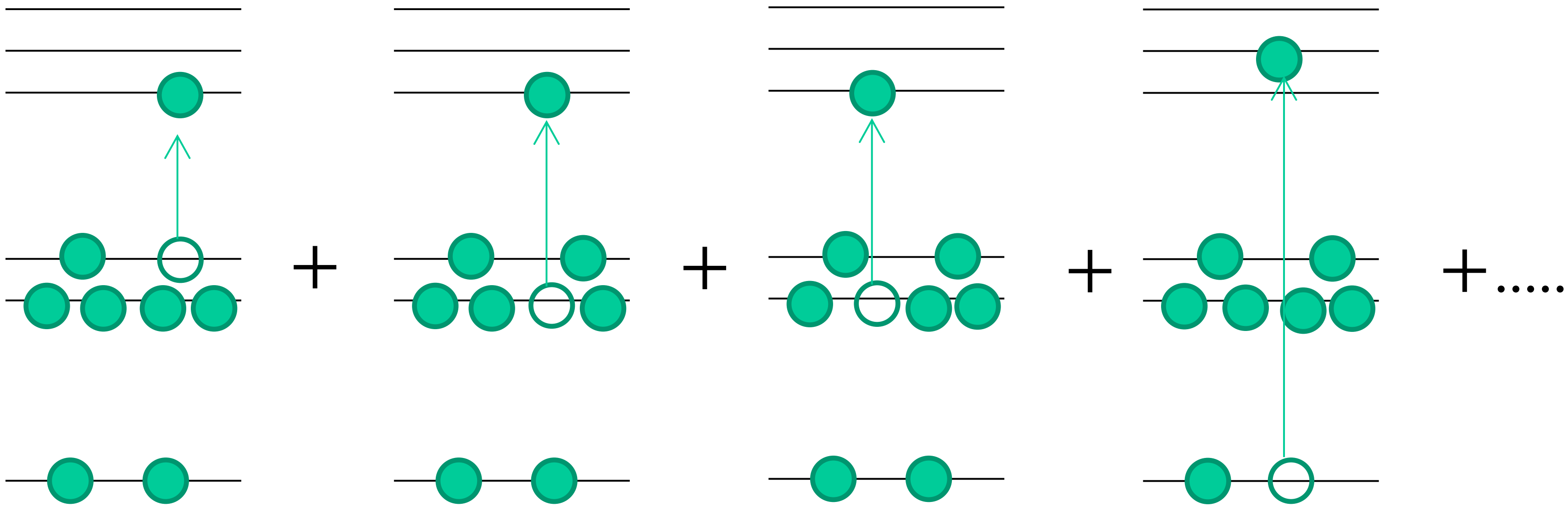
$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization: $\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$









複数の粒子・空孔状態をコヒーレントに重ね合わせることによって多数の核子が励起に関与していることを表現する

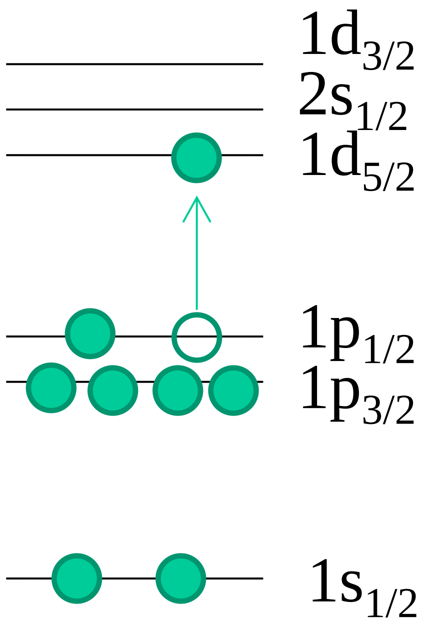
$$|\nu\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h |HF\rangle \equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$$

$$\Rightarrow \left|\left\langle \nu \left| \sum_{ph} f_{ph} a_p^\dagger a_h \right| 0 \right\rangle\right|^2 = \left(\sum_{ph} f_{ph} X_{ph}\right)^2$$

干渉項がすべて同符号で寄与

原子核の励起状態

- ✓ 一粒子励起（一つの核子が励起に関与）
- ✓ 集団励起（多くの核子が集団として励起に関与）



一粒子励起の例

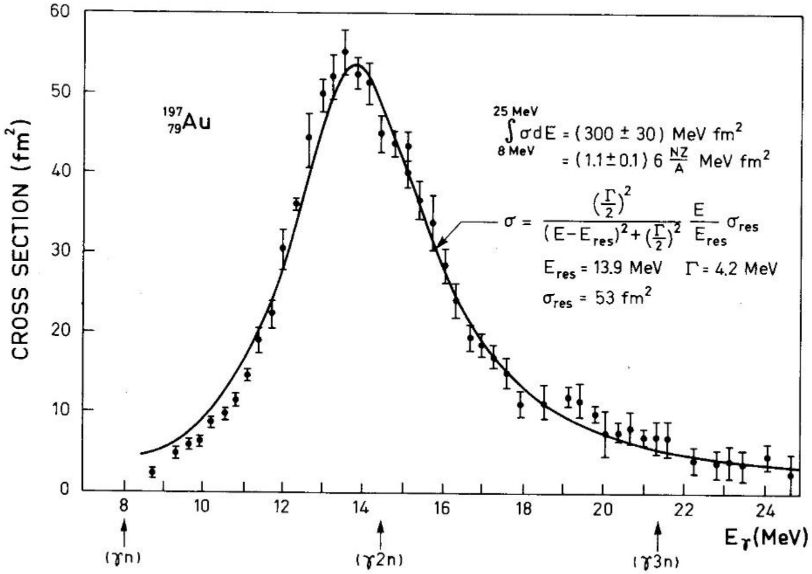
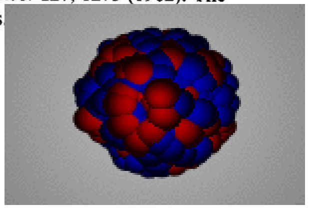
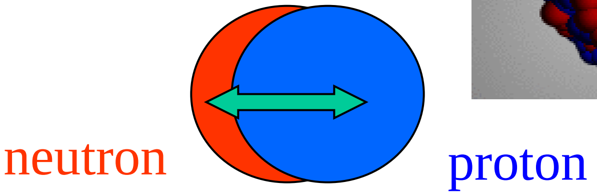


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.





集団励起の例：巨大双極子共鳴

Giant Dipole Resonance (GDR) 巨大双極子共鳴

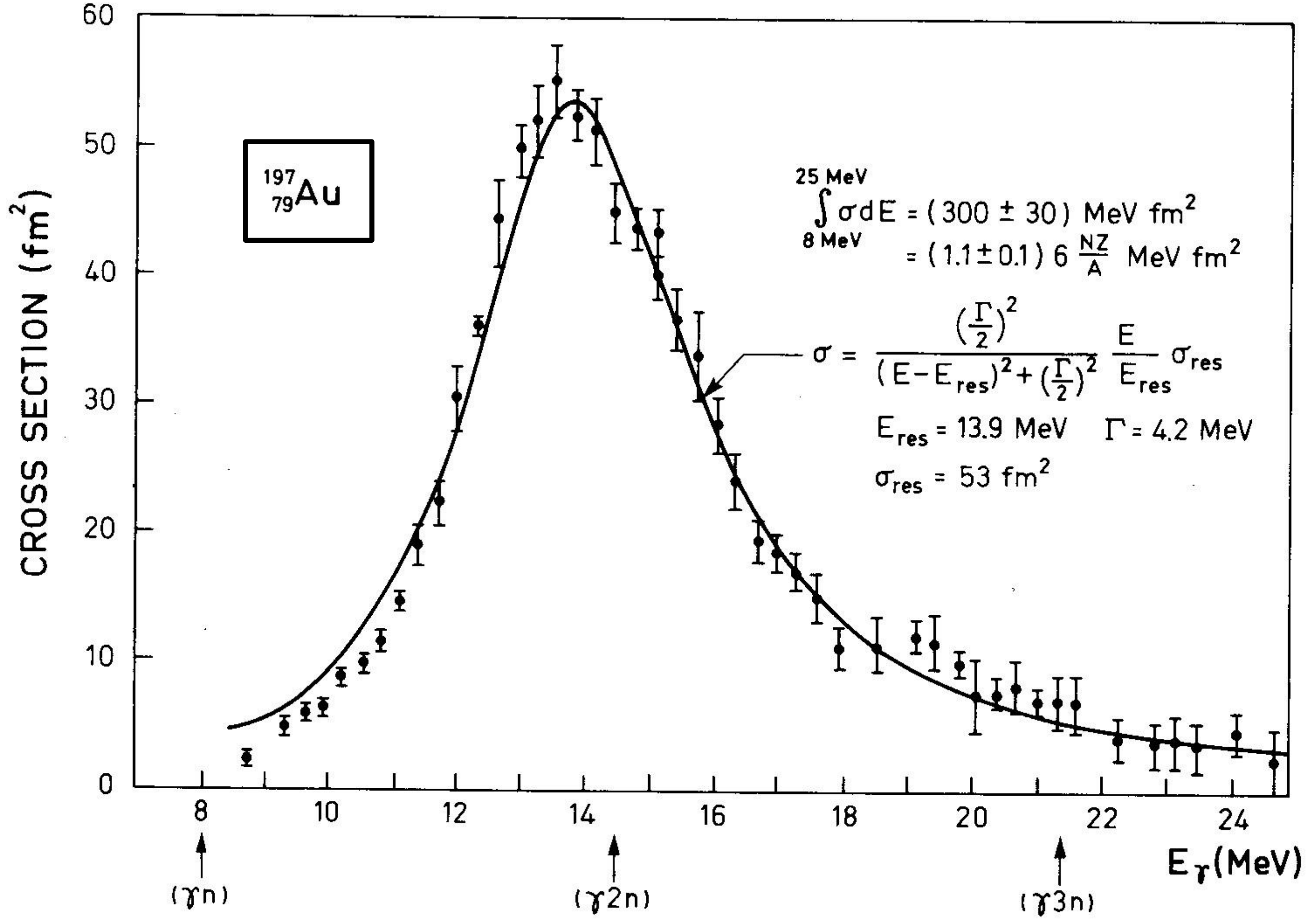


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experimental data are from S. C. Fultz, R. L. Bramblett, J. T. Caldwell, and N. A. Kerr, *Phys. Rev.* **127**, 1273 (1962). The solid curve is of Breit-Wigner shape with the indicated parameters.

cf. $41 \times 197^{-1/3} = 7.05$ MeV → 14 MeV

Iso-scalar type modes: $E < \epsilon_{ph} \rightarrow \lambda < 0$ (attractive)

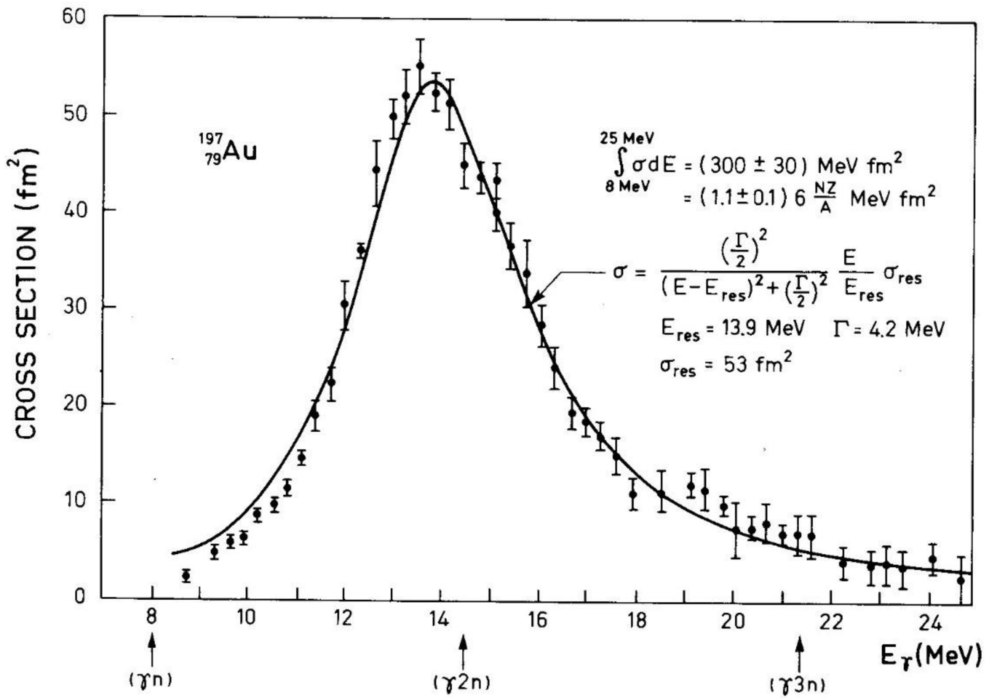
Iso-vector type modes: $E > \epsilon_{ph} \rightarrow \lambda > 0$ (repulsive)

Experimental systematics:

IV GDR: $E \sim 79 A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 41 A^{-1/3}$

IS GQR: $E \sim 65 A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 82 A^{-1/3}$

(note) single particle potential: $\hbar\omega \sim 41 A^{-1/3}$ (MeV)



^{197}Au

$$E_{\rm GDR} = 14 \ \ (\mathrm{MeV})$$

$$\epsilon_{ph} \sim 41 \cdot 197^{-1/3}$$

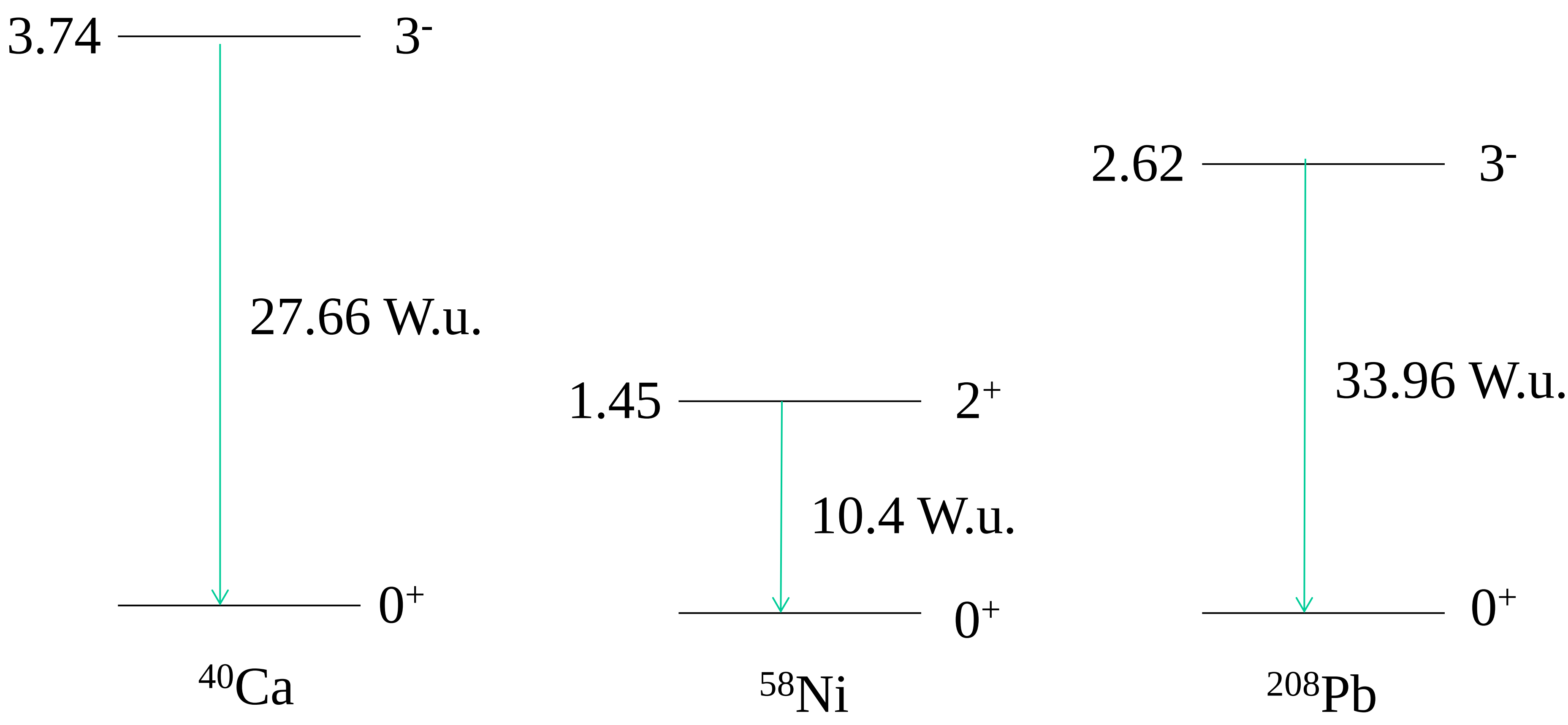
$$\sim 7 \ \ (\mathrm{MeV})$$

どれだけの核子が励起に関与しているのか?

Single-particle transition (Weisskopf unit)

$$B(E\lambda : I_i \to I_{gs}) = \frac{(1.2A^{1/3})^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 \qquad (e^2\mathrm{fm}^{2\lambda})$$

exp data:



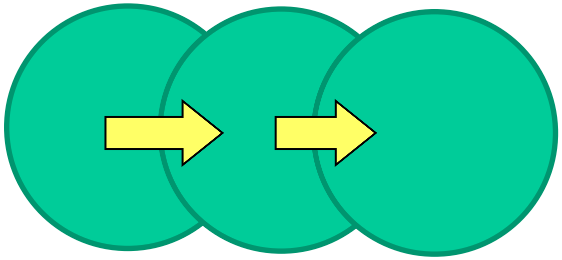
Spurious motion and RPA

Mean-Field Approximation ⟺ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero energy mode (Nambu-Goldstone mode)



does not require an extra energy → zero energy mode

A drawback of TDA:

Zero modes appear at finite excitation energies.

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|0\rangle \qquad \text{(TDA)}$$

A better approximation:

the random phase approximation (RPA)

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

規格化:

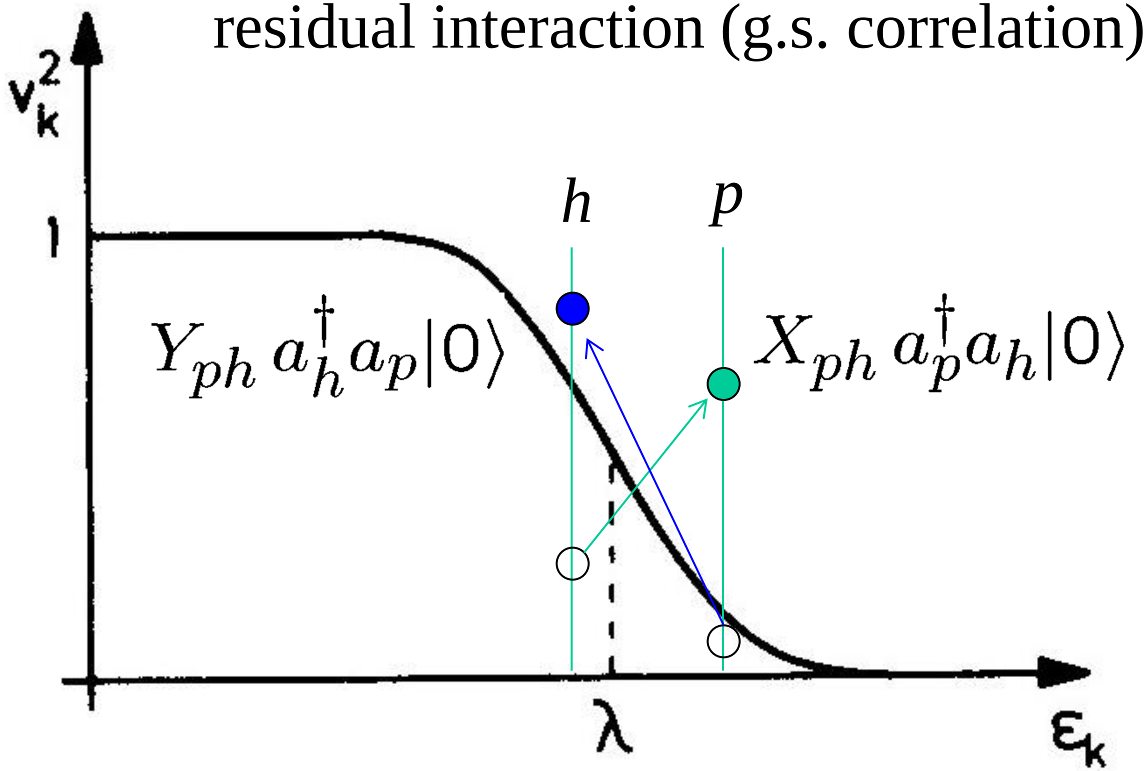
$$\langle\nu|\nu\rangle = 1 \to \sum_{ph}\left(|X_{ph}|^2 - |Y_{ph}|^2\right) = 1$$

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

smearing of Fermi surface due to the residual interaction (g.s. correlation)



A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\,a_p^\dagger a_h - Y_{ph}\,a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

$$\Downarrow$$

$$\langle HF|[\delta Q, [H, Q_\nu^\dagger]]|HF\rangle = E_\nu \langle HF|[\delta Q, Q_\nu^\dagger]|HF\rangle$$

→ coupled equations for *X* and *Y*

$$\delta Q = a_h^\dagger a_p, \quad a_p^\dagger a_h$$

$$\langle HF|[\delta Q,[H,Q^\dagger_\nu]]|HF\rangle = E_\nu \langle HF|[\delta Q,Q^\dagger_\nu]|HF\rangle$$

$$Q^\dagger_\nu = \sum_{ph} X_{ph}\, a^\dagger_p a_h - Y_{ph}\, a^\dagger_h a_p \qquad \delta Q = a^\dagger_h a_p, \quad a^\dagger_p a_h$$

RPA equation:

$$\begin{aligned} \sum_{p'h'} A_{ph,p'h'} X_{p'h'} + B_{ph,p'h'} Y_{p'h'} &= E_\nu X_{ph} \\ \sum_{p'h'} B^*_{ph,p'h'} X_{p'h'} + A^*_{ph,p'h'} Y_{p'h'} &= -E_\nu Y_{ph} \end{aligned}$$

$$\begin{aligned} A_{ph,p'h'} &= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle \\ B_{ph,p'h'} &= \langle pp'|\bar{v}|hh'\rangle \end{aligned}$$

or

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E_\nu \begin{pmatrix} X \\ Y \end{pmatrix}$$

Spurious motion in RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero mode (Nambu-Goldstone mode)

$$[H, \hat{O}] = 0$$

RPA

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

$\hat{O}$ is a solution of RPA with E=0

$$Q^\dagger = \hat{O} = \sum_{ph} (O_{ph} a_p^\dagger a_h + O_{hp} a_h^\dagger a_p)$$

(note) $Q_{\mathrm{TDA}}^\dagger = \sum_{ph} O_{ph} a_p^\dagger a_h$ ⇒ $[H, Q_{\mathrm{TDA}}^\dagger] \neq 0$

Spurious motion in RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero mode (Nambu-Goldstone mode)

RPA $$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

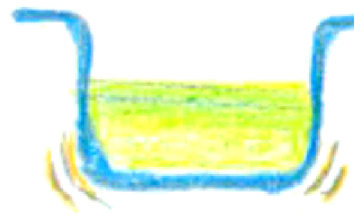
if $[H, \hat{O}] = 0$

Then $\hat{O}$ is a solution of RPA with E=0

The physical solutions are completely separated out from the spurious modes.

他のRPAの定式化

- 線形応答理論

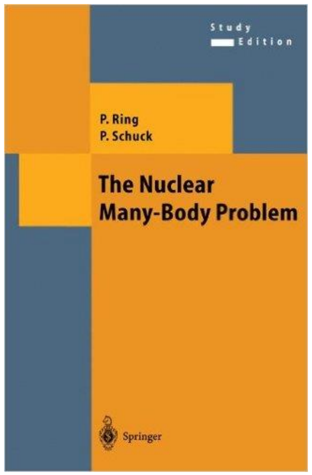


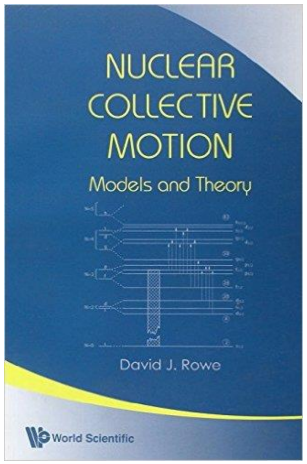
外場で原子核を揺すった時に、
原子核がどのように応答するか摂動論
を使って議論する
→固有モードを見つける

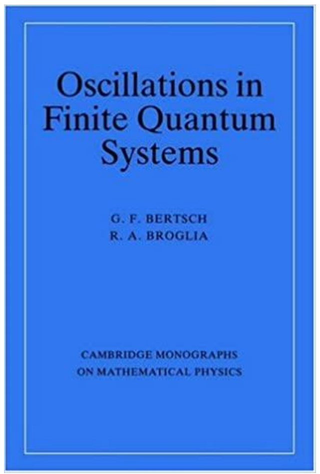
- 時間に依存するハートリー・フォック(TDHF)方程式を線形化

$$i\hbar\dot{\rho}(t) = [h[\rho], \rho] \qquad \longleftarrow \qquad \rho(t) = \rho_0 + \delta\rho(t)$$

詳しくは：







Comparison between Skyrme-(Q)RPA calculation and exp. data

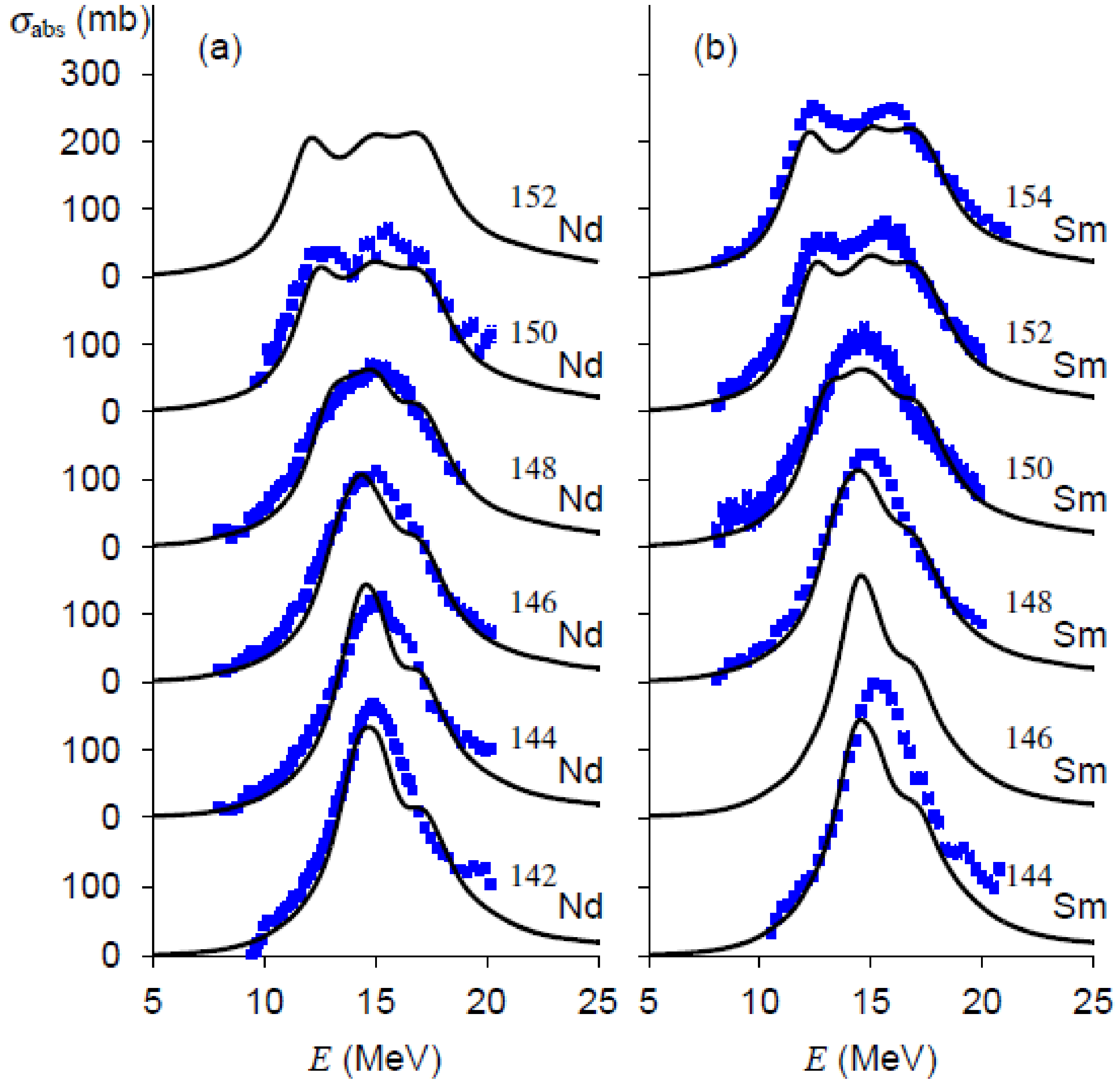
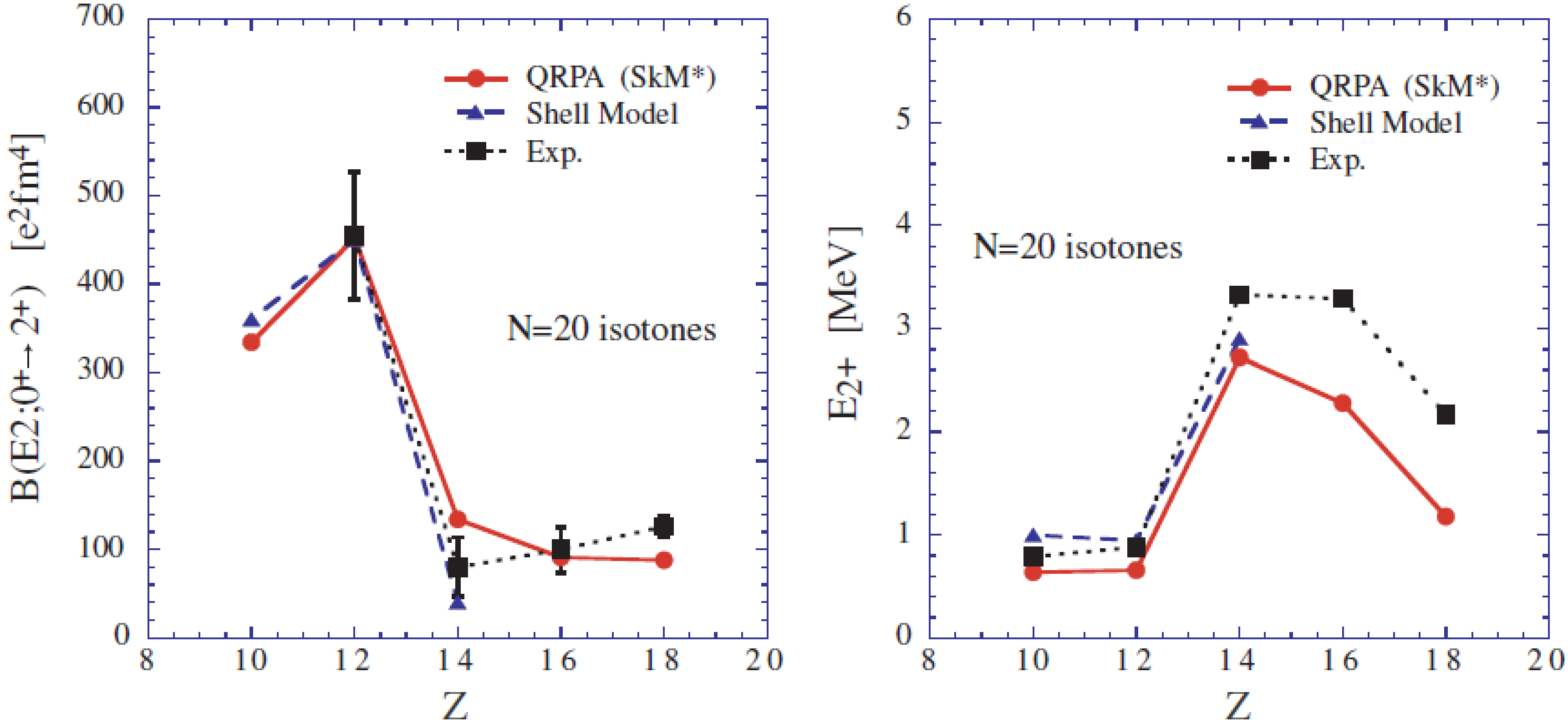


photo-absorption cross section (GDR)

K. Yoshida and T. Nakatsukasa, PRC83('11)021304



M. Yamagami and Nguyen Van Giai, PRC69 ('04) 034301

TDA on a separable interaction

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\overline{v}|hp'\rangle$$

Separable interaction: $\langle ph'|\overline{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$

Note: a separable interaction

$$H\psi = E\psi; \qquad \psi = \sum_i C_i \phi_i$$

$$\longrightarrow \quad \sum_j H_{ij} C_j = E C_i; \qquad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose $\quad H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j \quad$ (separable form)

$$(\epsilon_i - E) C_i + \lambda f_i^* \underbrace{\sum_j f_j C_j}_{\equiv T} = 0$$

$$C_i = -\lambda \frac{T f_i^*}{\epsilon_i - E}$$

$$T = -\lambda \sum_j \frac{|f_j|^2}{\epsilon_j - E} T \quad \longrightarrow \quad \boxed{\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}}$$

(separable interaction)

$$H\psi = E\psi; \qquad \psi = \sum_i C_i \phi_i$$

$$\longrightarrow \quad \sum_j H_{ij} C_j = EC_i; \qquad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose $\quad H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j \quad \longrightarrow \quad \boxed{\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}}$

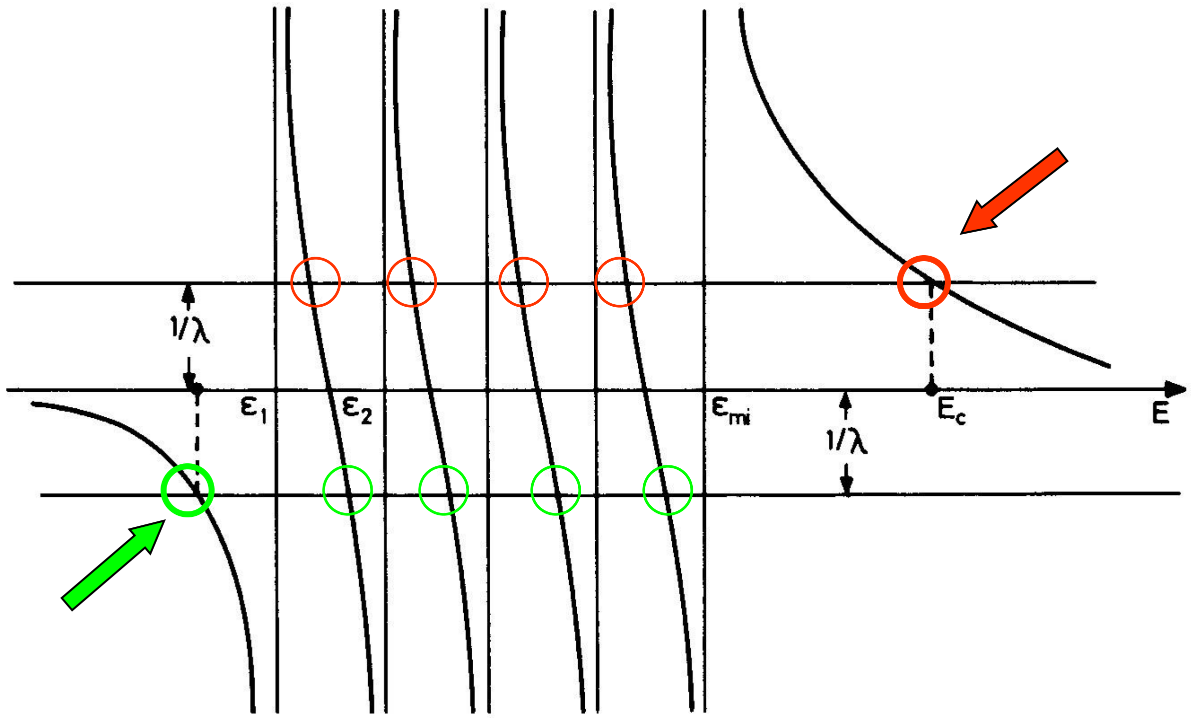
$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \lambda D_{ph} D^*_{p'h'}$$

$$\longrightarrow \quad \boxed{\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}}$$

(TDA dispersion relation)

Graphical solutions

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$



(if λ > 0)

(if λ < 0)

Figure 8.4. Graphical solution of Eq. (8.18).

(note) in the degenerate limit: $\epsilon_{ph} \sim \epsilon$

$$E = \epsilon + \lambda \sum_{ph} |D_{ph}|^2, \quad |\nu\rangle = \sum_{ph} D_{ph} a_p^\dagger a_h |HF\rangle$$

coherent superpositon of 1p1h states

RPA on a schematic model

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\begin{aligned} A_{ph,p'h'} &= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle \\ B_{ph,p'h'} &= \langle pp'|\bar{v}|hh'\rangle \end{aligned}$$

Separable interaction:

$$\begin{aligned} \langle ph'|\bar{v}|hp'\rangle &= \lambda D_{ph} D^*_{p'h'} \\ \langle pp'|\bar{v}|hh'\rangle &= \lambda D_{ph} D_{p'h'} \end{aligned}$$

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

Cf. TDA dispersion relation: $\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$

RPA on a schematic model

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$$
$$\langle pp'|\bar{v}|hh'\rangle = \lambda D_{ph} D_{p'h'}$$

Separable interaction:

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

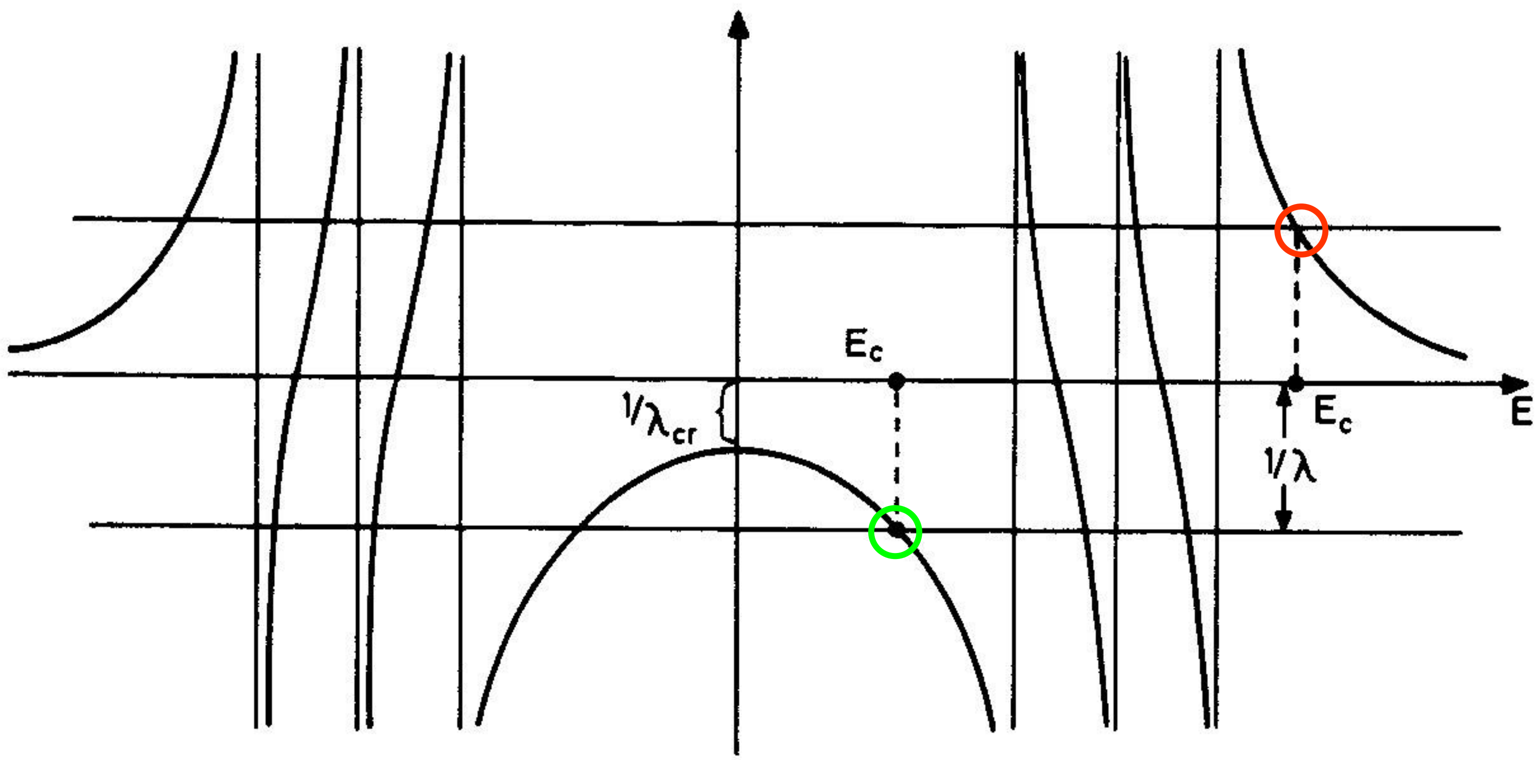


Figure 8.11. Graphical solution of the dispersion relation (8.135).

RPA on a schematic model

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$$
$$\langle pp'|\bar{v}|hh'\rangle = \lambda D_{ph} D_{p'h'}$$

Separable interaction:

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E-\epsilon_{ph}} - \frac{|D_{ph}|^2}{E+\epsilon_{ph}}$$

(RPA dispersion relation)

$\epsilon_{ph} = \epsilon$ のとき、

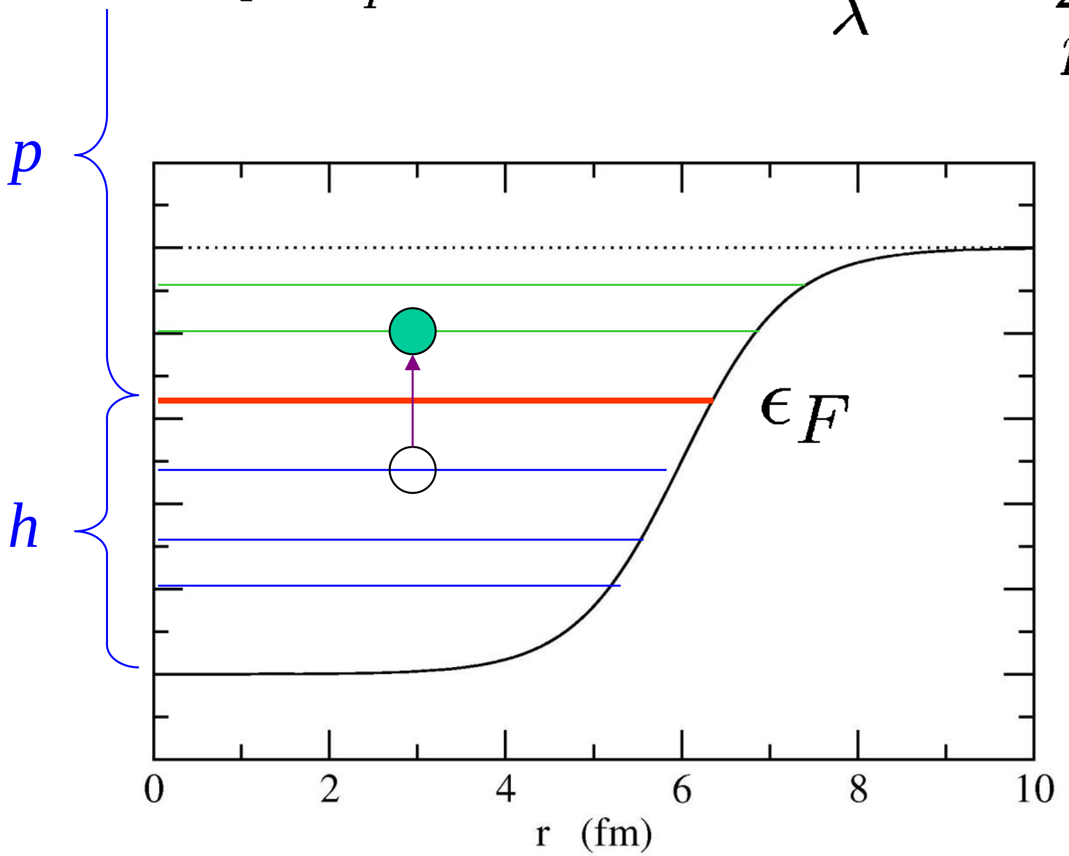
$$\frac{1}{\lambda} = \sum_{ph} |D_{ph}|^2 \left(\frac{1}{E-\epsilon} - \frac{1}{E+\epsilon}\right) = \sum_{ph} |D_{ph}|^2 \frac{2\epsilon}{E^2-\epsilon^2}$$

$$\longrightarrow \quad E^2 = \epsilon^2 + 2\epsilon\lambda \sum_{ph} |D_{ph}|^2$$

λ が負（引力）だと、どこかで $E^2 < 0$ となる

Continuum Excitations

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'} \quad \Rightarrow \quad \frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E}$$



h: all the occupied (bound) states

p: the bound excited states + continuum states

$$\frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E} = -\sum_{ph} \langle\phi_h|D^\dagger|\phi_p\rangle \frac{1}{\epsilon_p - \epsilon_h - E} \langle\phi_p|D|\phi_h\rangle$$

(note) $\hat{h}\phi_p = \epsilon_p \phi_p$

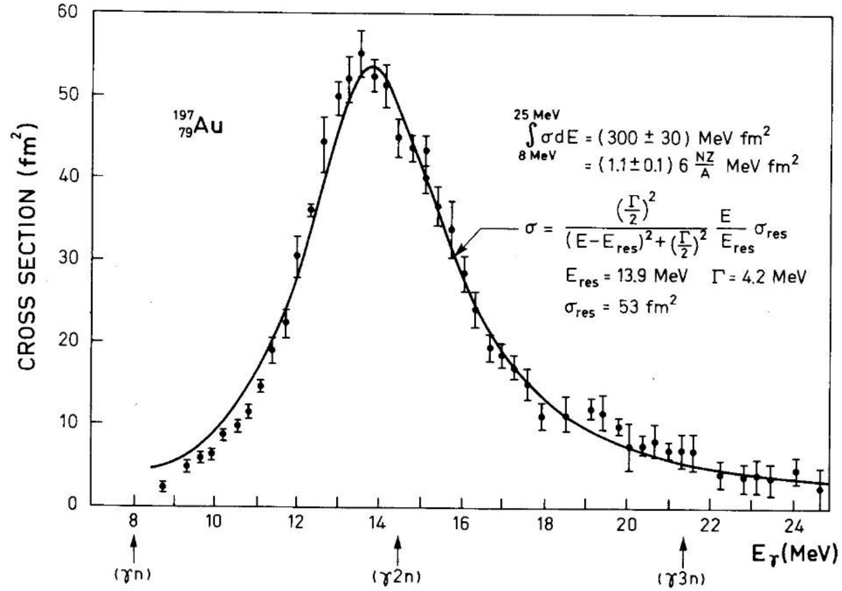
$$\frac{1}{\lambda} = -\sum_{ph} \langle\phi_h|D^\dagger \frac{1}{\hat{h} - \epsilon_h - E} |\phi_p\rangle\langle\phi_p|D|\phi_h\rangle$$

$$1 = \sum_i |\phi_i\rangle\langle\phi_i| = \sum_p |\phi_p\rangle\langle\phi_p| + \sum_h |\phi_h\rangle\langle\phi_h|$$

$$\frac{1}{\lambda} = -\sum_h \langle\phi_h|D^\dagger \frac{1}{\hat{h} - \epsilon_h - E} \left[1 - \sum_{h'} |\phi_{h'}\rangle\langle\phi_{h'}|\right] D|\phi_h\rangle$$

particle 状態の和がなくなった→連続状態もすべて自動的に入る

巨大共鳴の幅



i) 連続状態との結合（粒子放出）

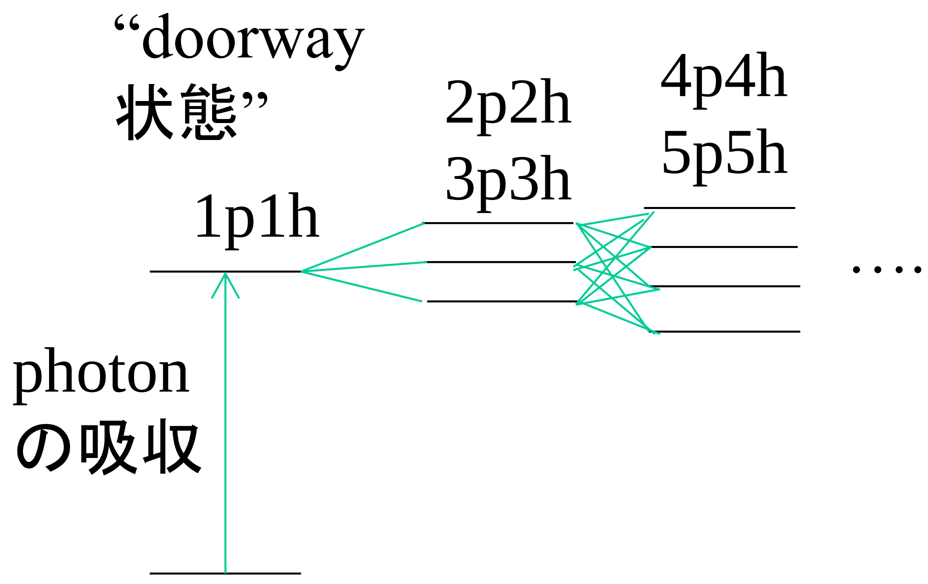
escape width $\Gamma^{\uparrow}$

continuum RPA

ii) より複雑な状態との結合

spreading width $\Gamma^{\downarrow}$

軽い核を除き
幅の主成分



1体演算子

レポート問題4（〆切：12月2日（月））

RPA のA 行列、B行列が

$$A = \begin{pmatrix} \epsilon + g & g \\ g & \epsilon + g \end{pmatrix}, \qquad B = \begin{pmatrix} g & g \\ g & g \end{pmatrix}$$

で与えられているときに RPA方程式を

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ Y_1 \\ Y_2 \end{pmatrix} = E \begin{pmatrix} X_1 \\ X_2 \\ Y_1 \\ Y_2 \end{pmatrix}$$

解き、（正の）固有値2つを求めよ。ただし、ε は正とする。

g が負のとき、その大きさをゼロから大きくしていくと、ここで求めた2つの解のどちらかはどこかで 0 になる。そのときの g を求めよ。
g がさらに小さくなると解は複素数になるが、それは物理的にどういう状況に対応するか考察せよ。