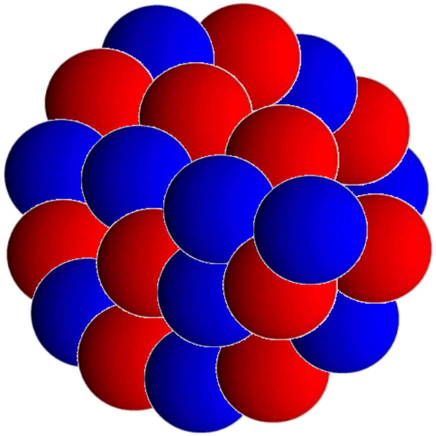
原子核反応論

- 原子核物理：核子多体系としての原子核の振る舞い

  ← 核子間相互作用から理解する

➢ 静的な振る舞い：原子核構造論



✓ 基底状態の性質 （質量、大きさ、形など）
✓ 励起状態の性質

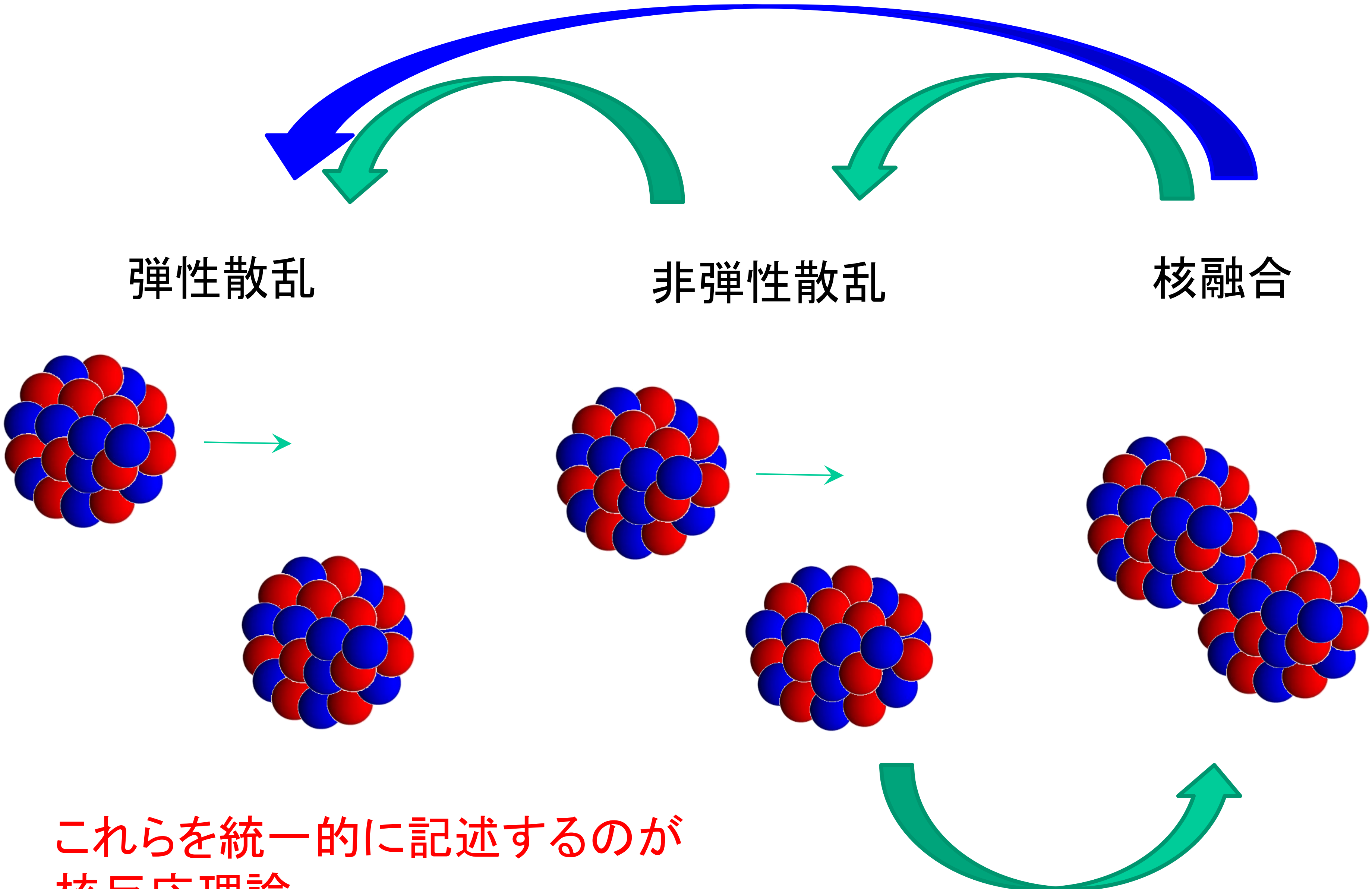
➢ ダイナミックス：原子核反応論

原子核は複合粒子
✓ 豊富な反応様式
✓ 核構造と核反応の織り成す様々なインタープレイ

✓ 弾性散乱
✓ 非弾性散乱
✓ 核子移行反応
✓ 核融合反応

✓ 基底状態の性質（質量、大きさ、形など）
✓ 励起状態の性質

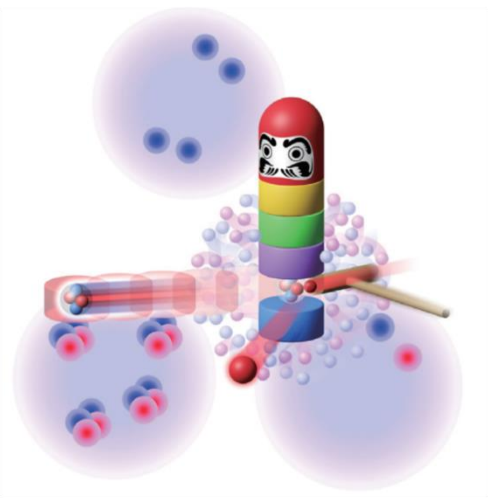
量子多体系のダイナミックス（原子核反応）



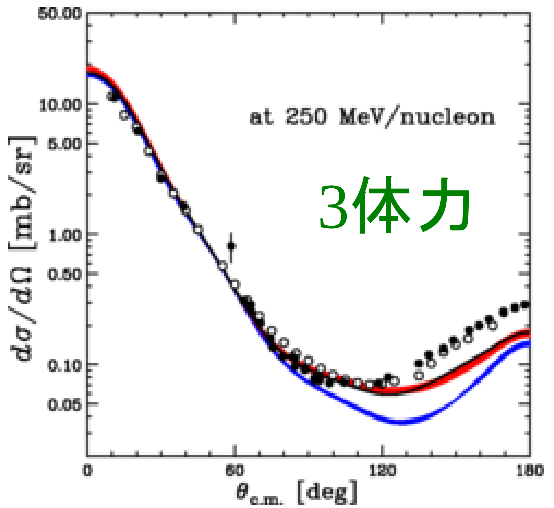
これらを統一的に記述するのが
核反応理論

原子核反応の2つの側面

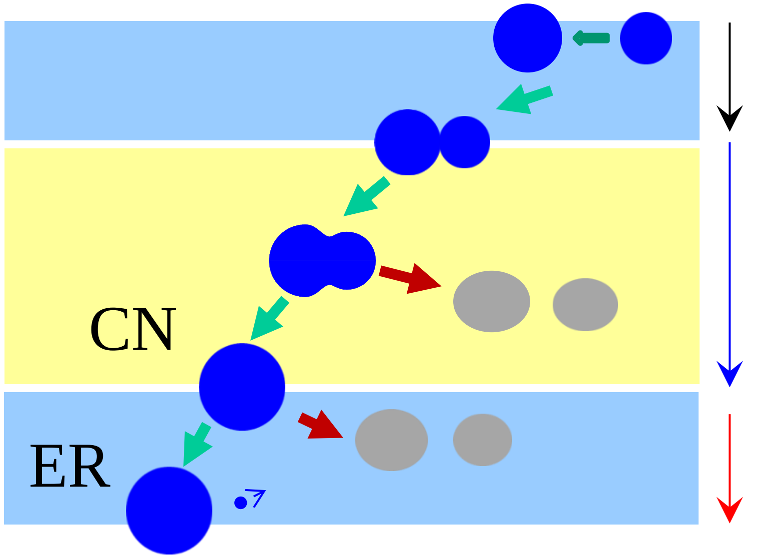
✓ ツールとしての原子核反応



緒方さんのHPより

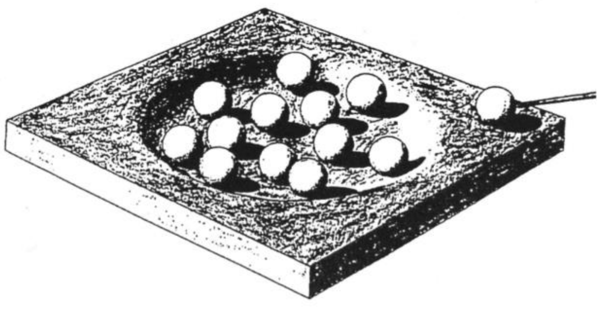


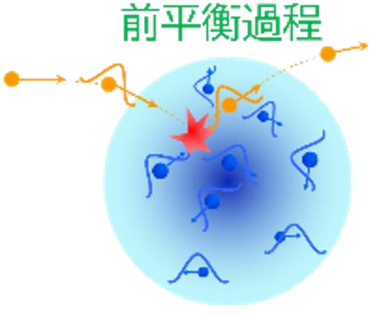
K. Sekiguchi et al., PRC89('14)064007



超重元素生成反応

✓ 反応ダイナミックス自体としての面白み



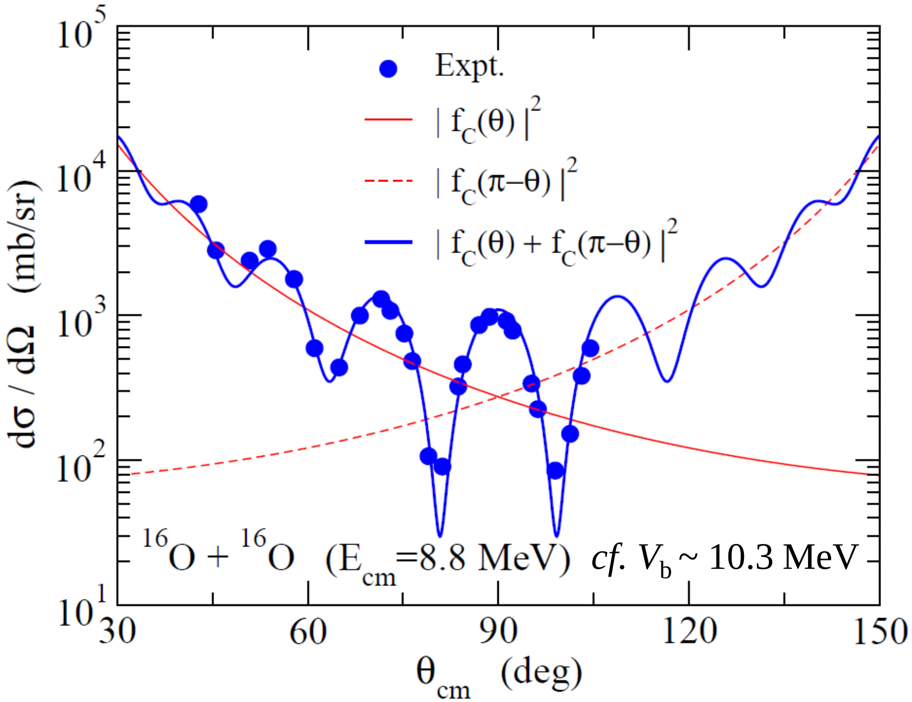


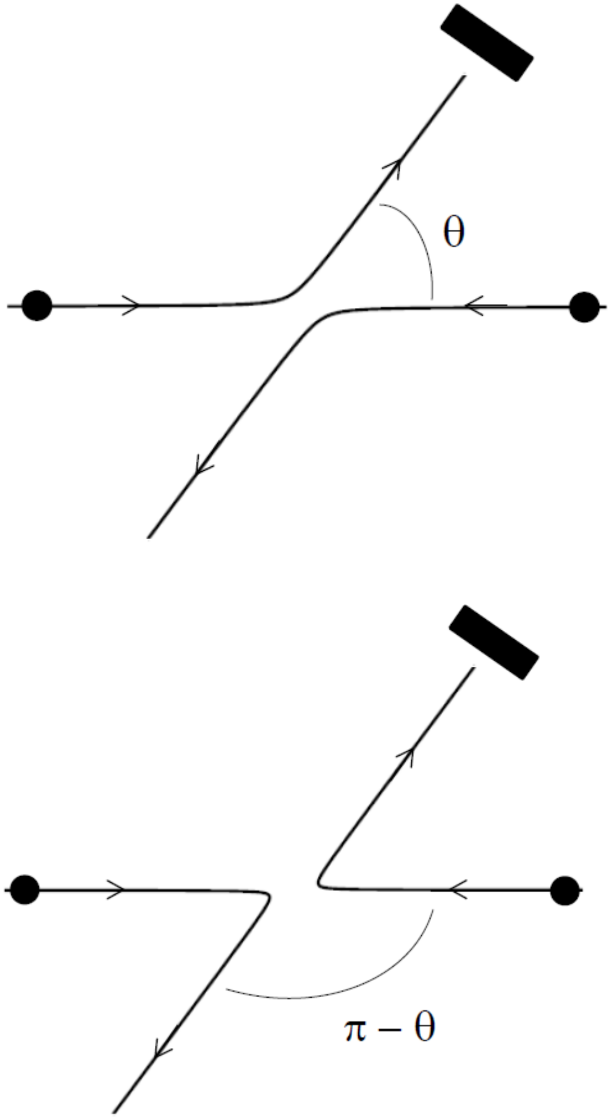
緒方さんのHPより

核反応は宝の山：核反応に見られる量子性

例）モット散乱：同種粒子の散乱

$$\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi - \theta)|^2$$





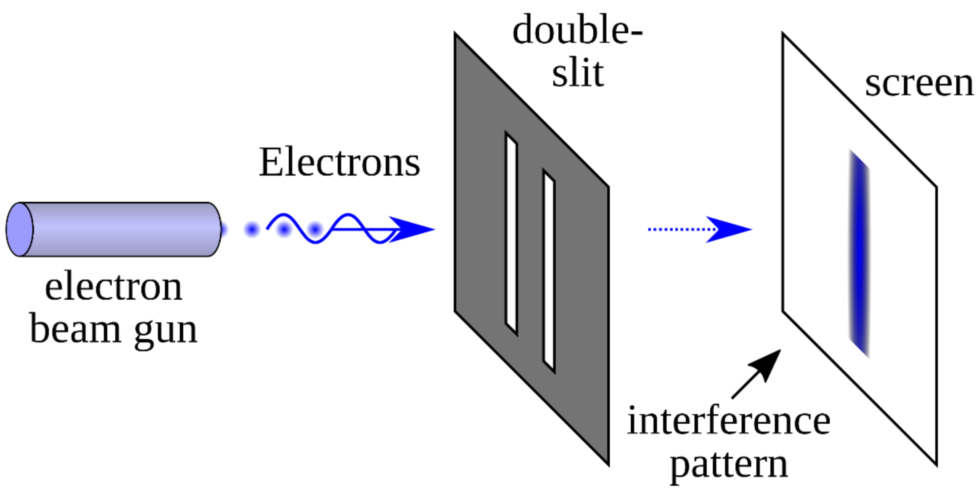
expt: D.A. Bromley et al., Phys. Rev. 123 ('61)878

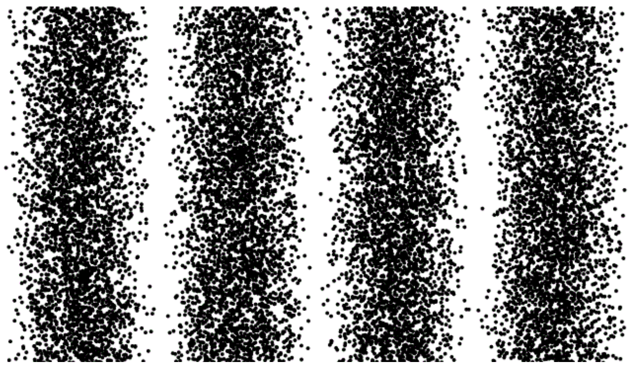
Manifestation of Quantum Nature in Nuclear Reactions

a superposition principle $\psi = \alpha\psi_1 + \beta\psi_2$

$$\to |\psi|^2 = |\alpha\psi_1|^2 + |\beta\psi_2|^2 + \underbrace{(\alpha\psi_1)^*(\beta\psi_2) + (\alpha\psi_1)(\beta\psi_2)^*}_{\text{interference}}$$

when two processes are in principle indistinguishable
$\to$ take square after adding two amplitudes

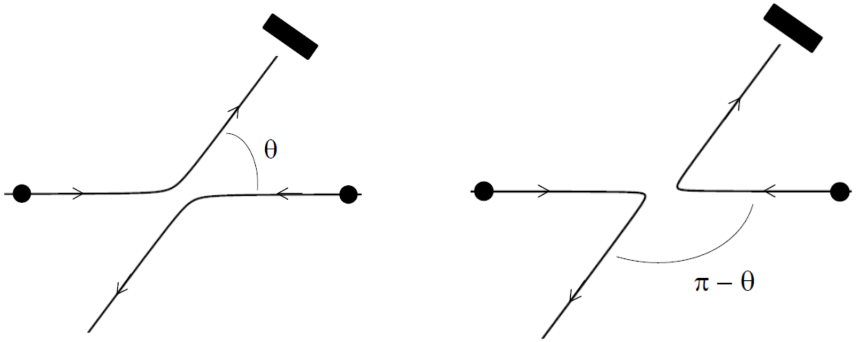




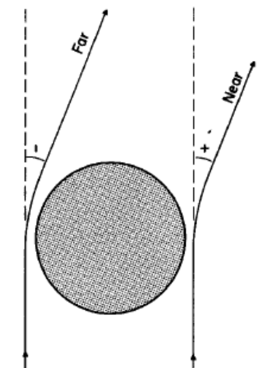
Wikipedia

Interference phenomena in Nuclear Reactions

(i) Mott Scattering

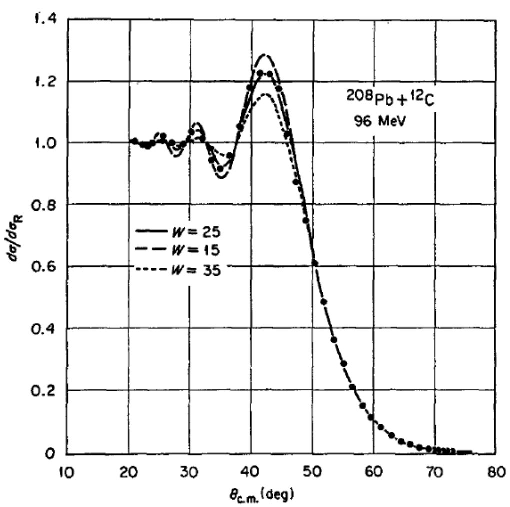


(iii) Near-far interference



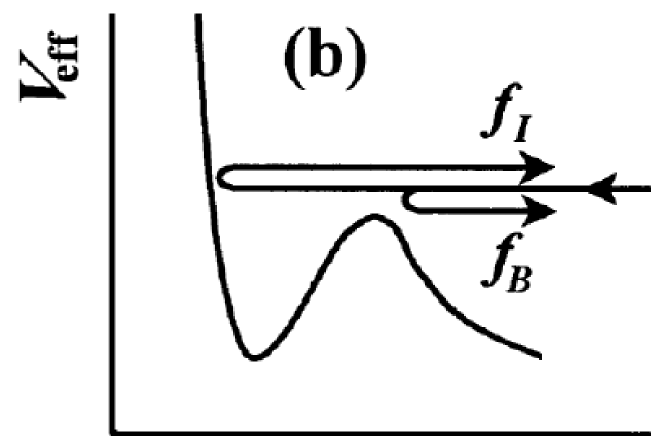
$$f(\theta) = f_{\text{near}}(\theta) + f_{\text{far}}(\theta)$$

(ii) Nuclear-Coulomb interference



$$f(\theta) = f_C(\theta) + f_N(\theta)$$

(iv) barrier-wave internal-wave interference



Interference phenomena in Nuclear Reactions

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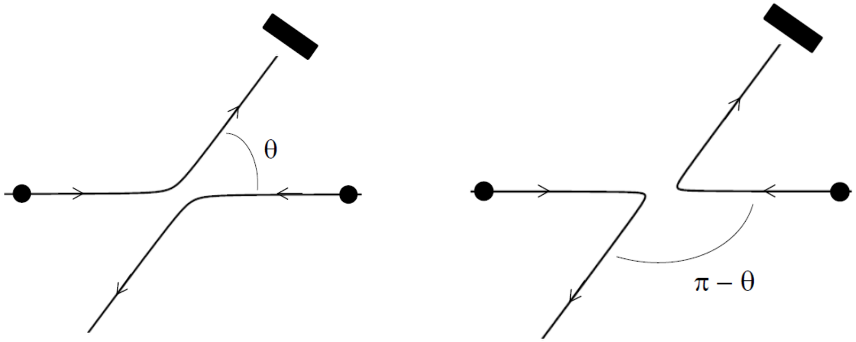
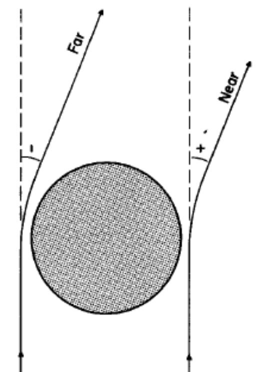


(iii) Near-far interference



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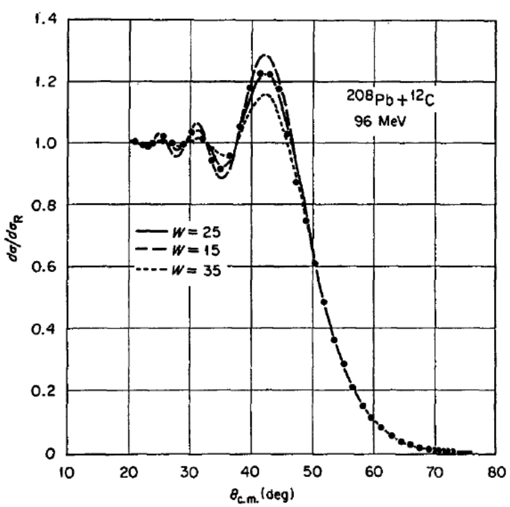

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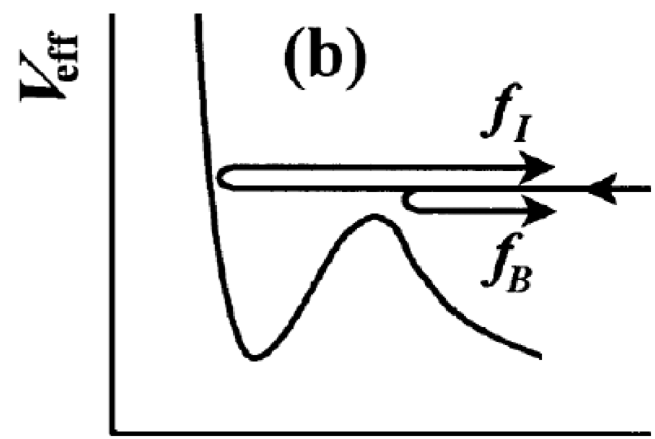
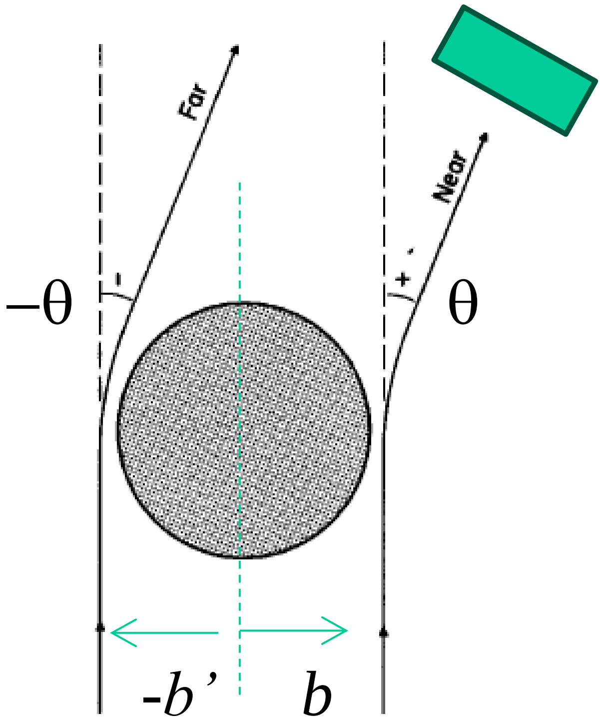
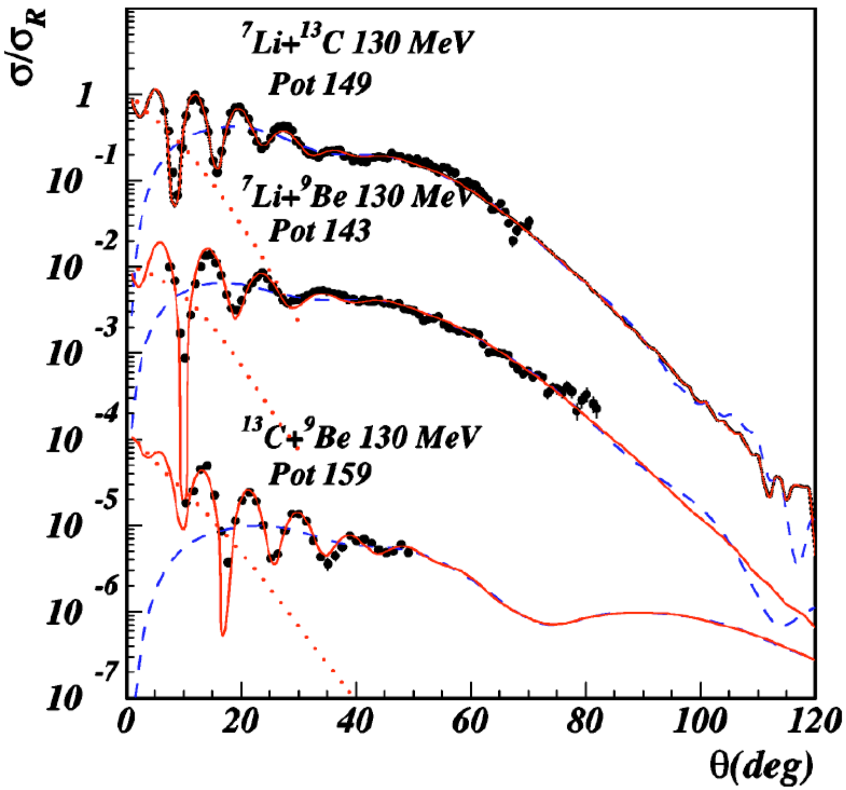


➢ near side - far side interference

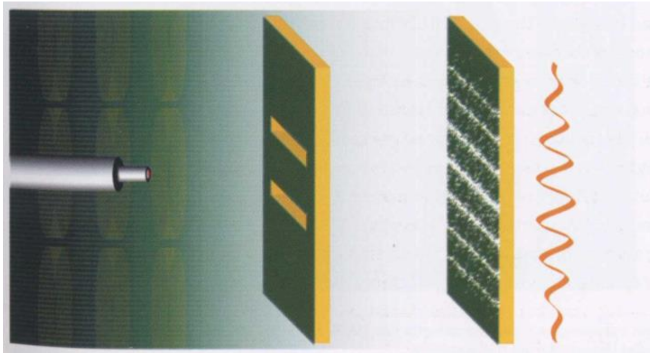




F. Carstoiu et al., PRC70 ('04) 054610

R.C. Fuller, PRC12('75)1561
N. Rowley and C. Marty,
NPA266('76)494
M.S. Hussein and K.W. McVoy,
Prog. in Part. and Nucl. Phys.
12 ('84)103

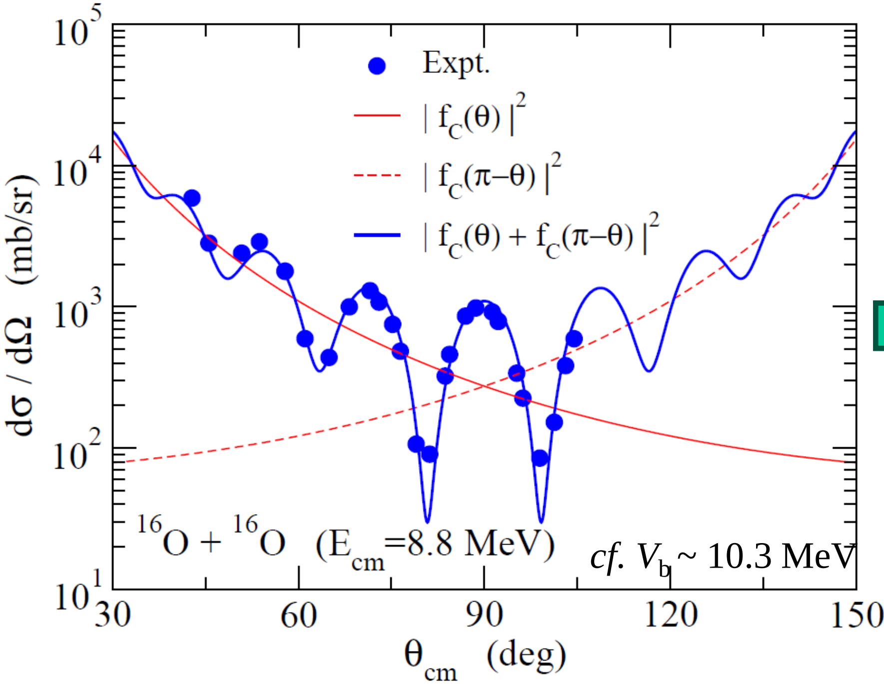
2重井戸問題との類似性



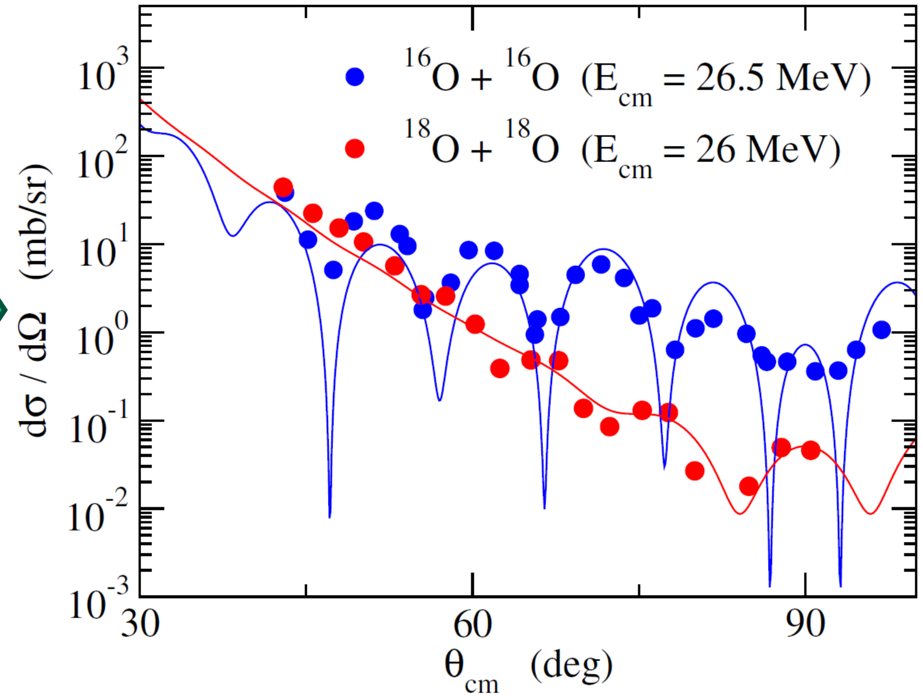
$^{16}O+^{16}O$ system

$V_b \sim 10.3$ MeV

$E < V_b$（クーロン散乱のみ）



$E > V_b$（クーロン＋核力散乱）



expt: D.A. Bromley et al., Phys. Rev. 123 (‘61)878

光学ポテンシャル計算

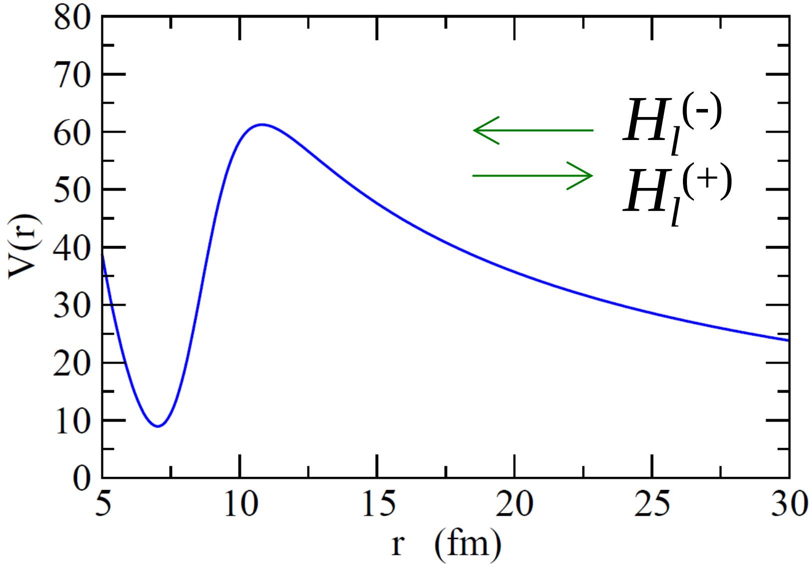
$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{l(l+1)\hbar^2}{2\mu r^2}+V(r)\underline{-iW(r)}-E\right]u_l(r)=0$$

an imaginary part → absorption

$$u_l(r)\to H_l^{(-)}(kr)-S_lH_l^{(+)}(kr),\quad r\to\infty$$

$$f(\theta)=f_C(\theta)+\sum_l e^{2i\sigma_l}(2l+1)\frac{S_l-1}{2ik}P_l(\cos\theta)$$

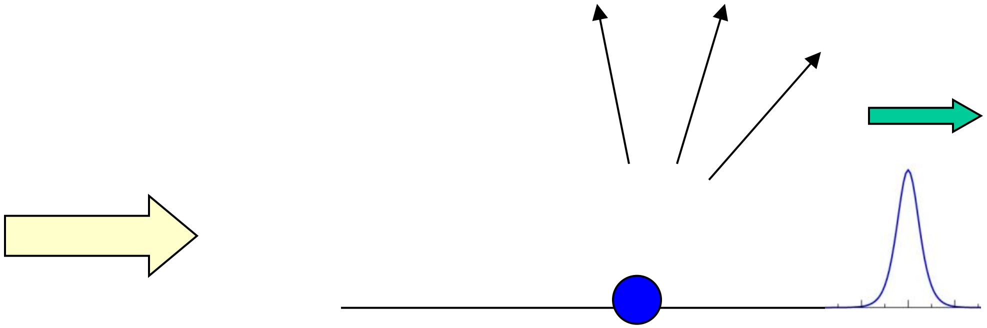
$$\frac{d\sigma}{d\Omega}=|f(\theta)|^2$$



光学ポテンシャル計算

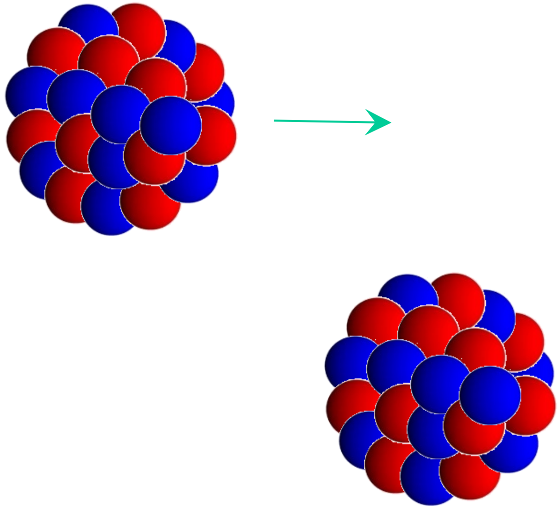
Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)

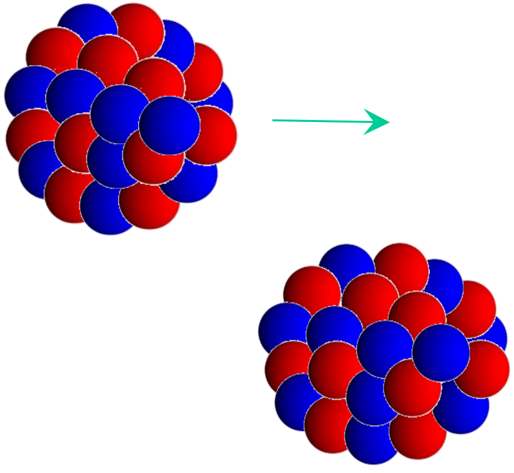


Loss of incident flux (absorption)

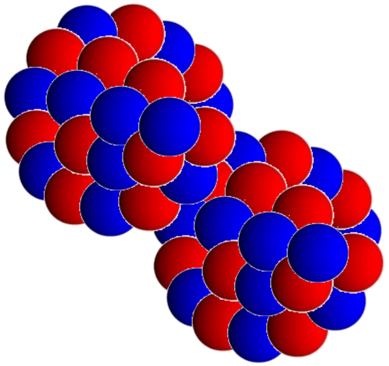
弾性散乱



非弾性散乱



核融合



How to choose $V_0(r)$? : Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
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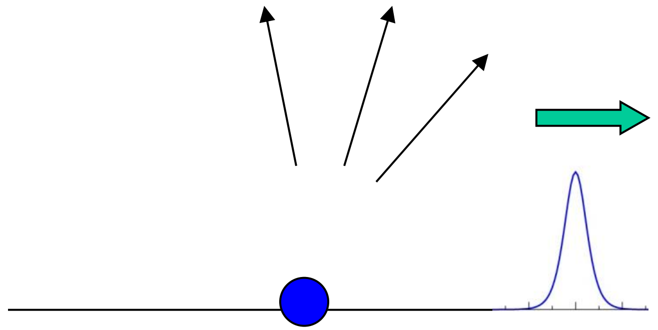

Loss of incident flux (absorption)

Optical potential

$$V_{\mathrm{opt}}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)$$

How to choose $V_0(r)$? : Optical model

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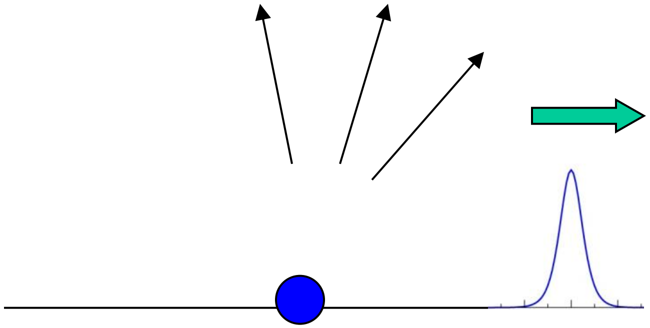


Loss of incident flux (absorption)

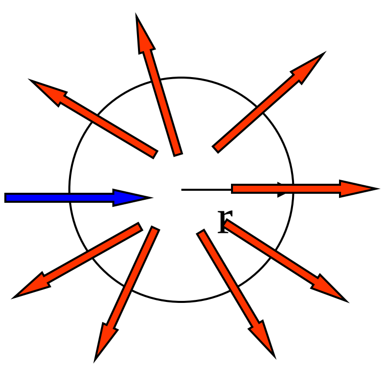
Optical potential

$$\boxed{V_{\rm opt}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)}$$

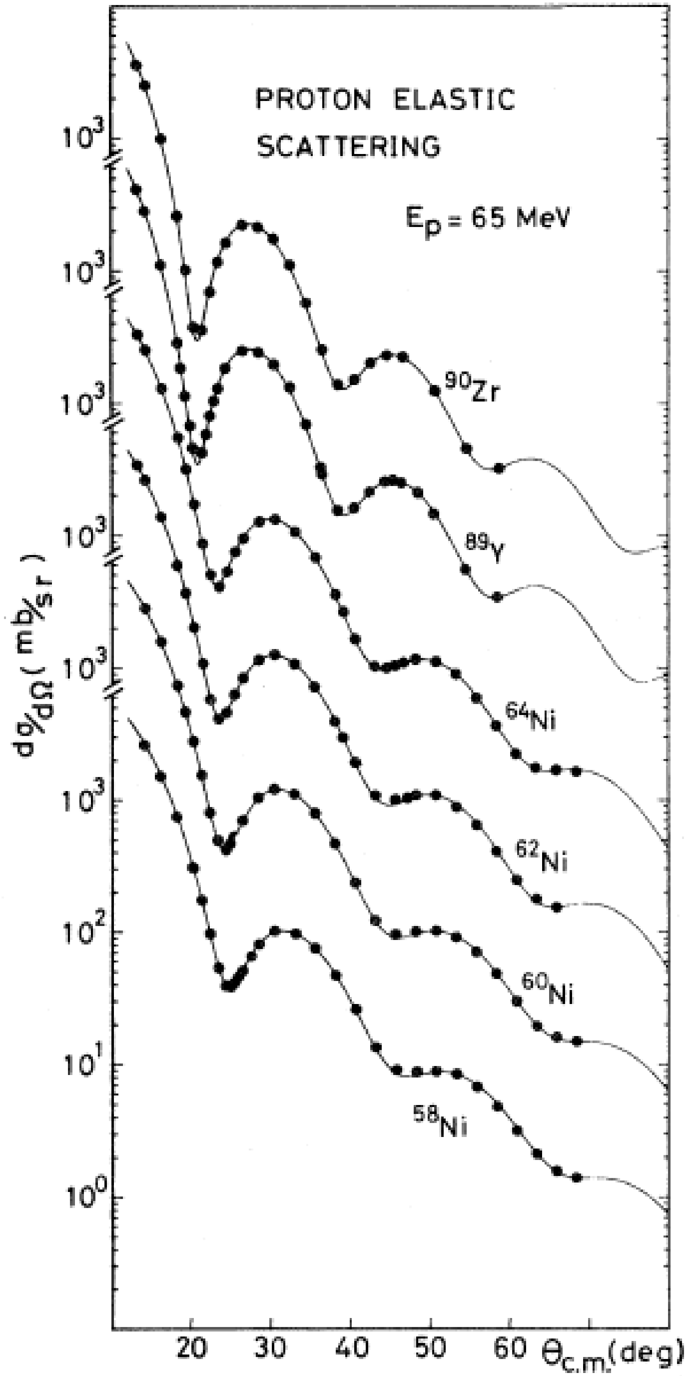
$$\Longrightarrow \quad \boldsymbol{\nabla}\cdot\boldsymbol{j} = \cdots = -\frac{2}{\hbar}W|\psi|^2$$

(note) Gauss's theorem

$$\int_S \boldsymbol{j}\cdot\boldsymbol{n}\,dS = \int_V \boldsymbol{\nabla}\cdot\boldsymbol{j}\,dV$$



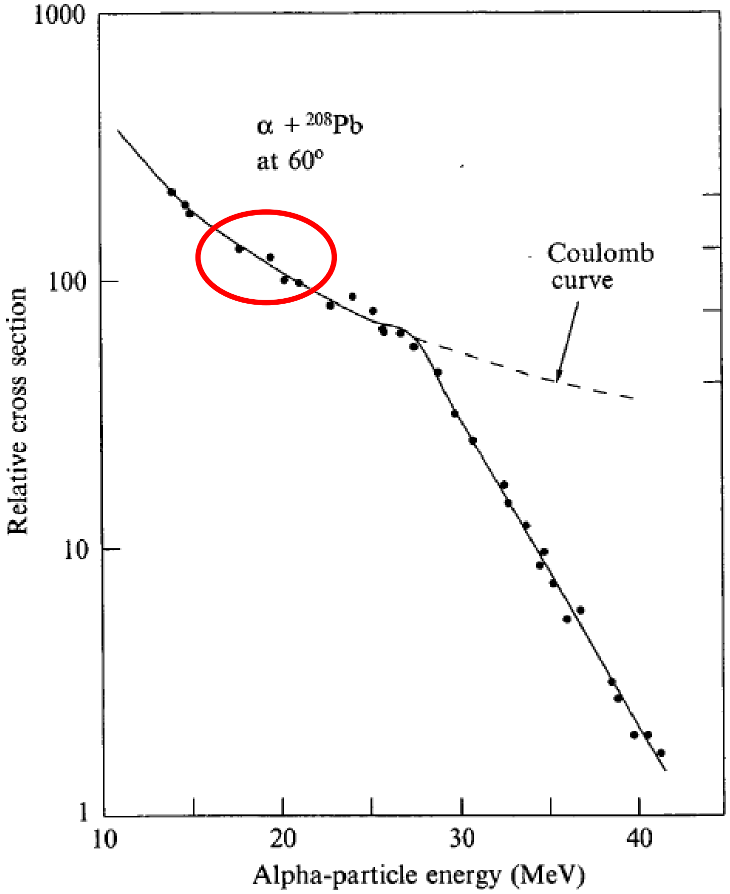
$$\left(-\frac{\hbar^2}{2\mu}\nabla^2+\frac{Z_PZ_Te^2}{r}+V_{\rm opt}(r)-E\right)\psi(\boldsymbol{r})=0$$

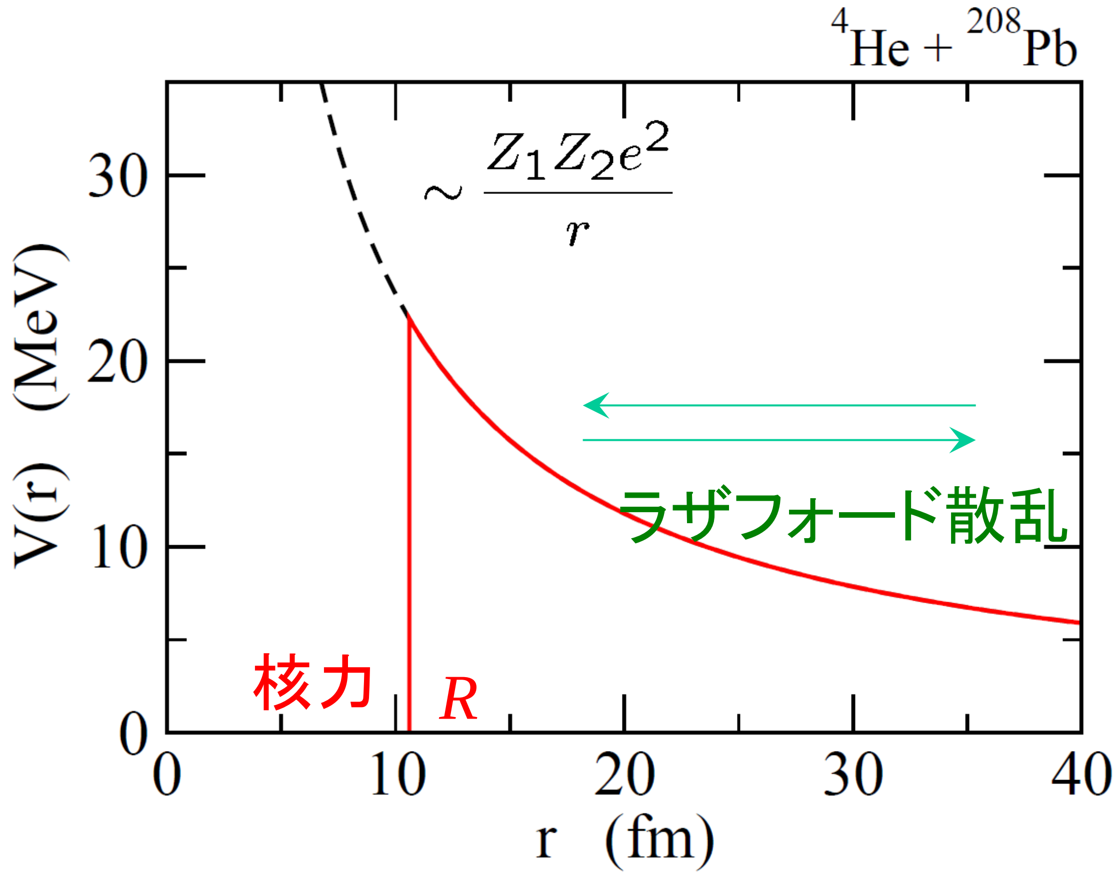


Woods-Saxon + volume &surface imaginary parts

H. Sakaguchi et al., PRC26 (1982) 944

原子核の吸収から原子核の大きさを見る





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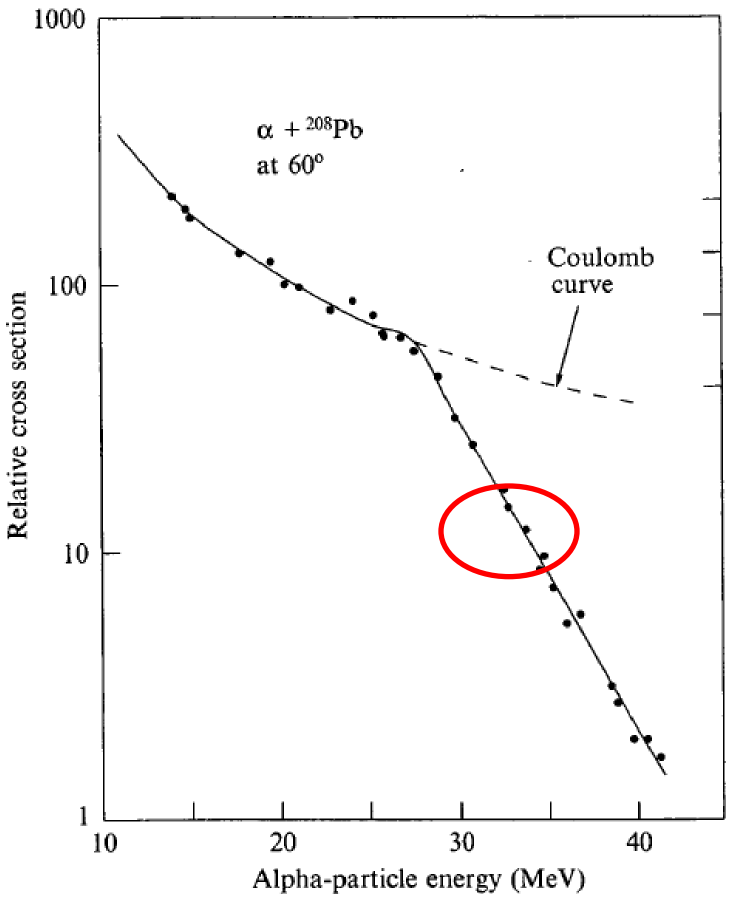
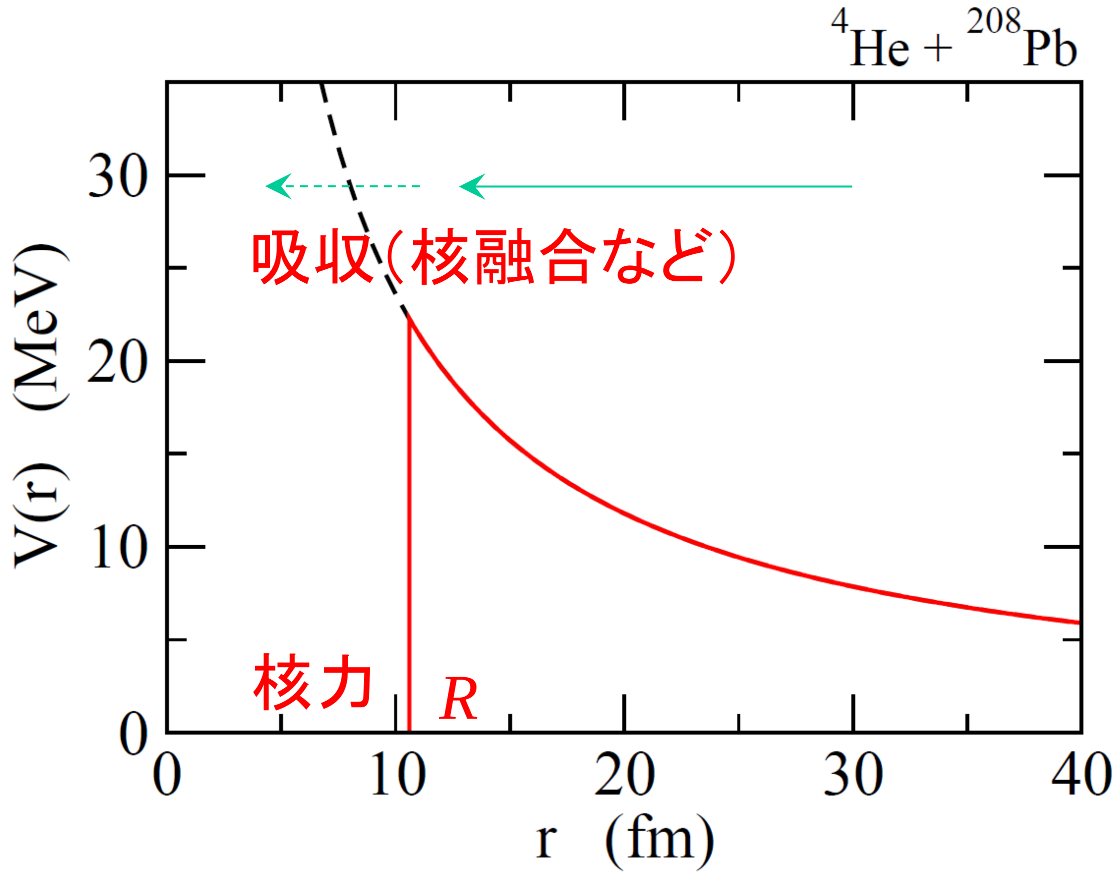




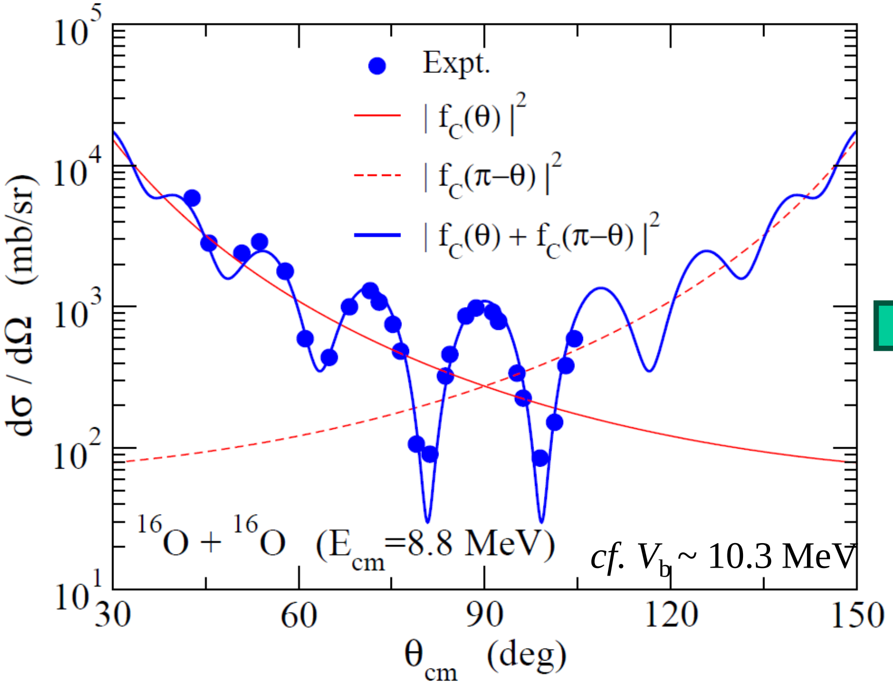
$\rightarrow R \sim 1.41\, A^{1/3} + 2.11$ fm

(核力のレンジや密度分布のテールの効果により
実際の半径はもう少し小さい)

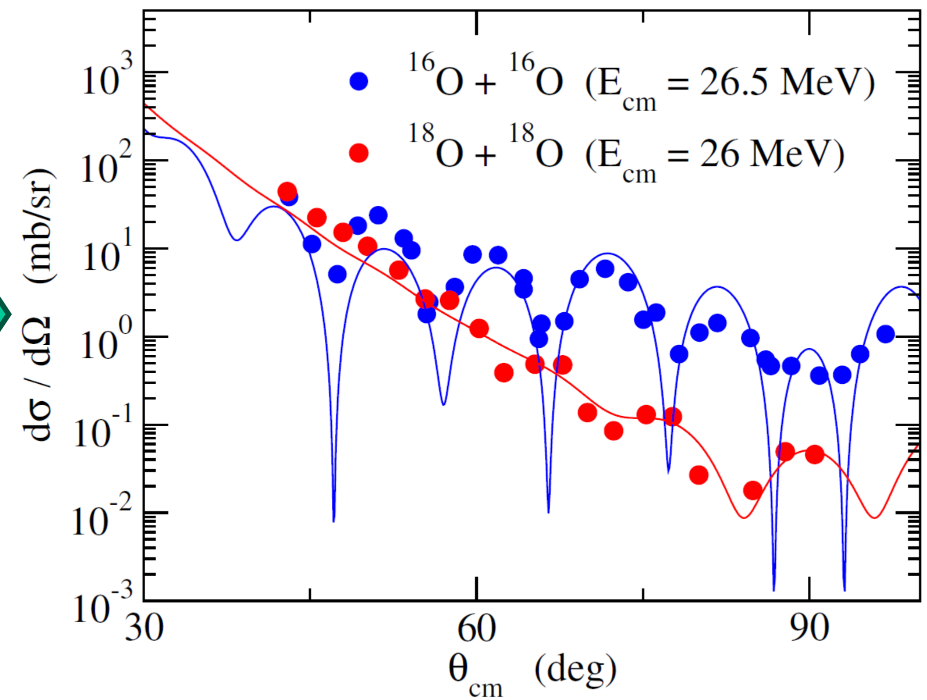
$^{16}O+^{16}O$ system

$V_b \sim 10.3$ MeV

$E < V_b$（クーロン散乱のみ）



$E > V_b$（クーロン＋核力散乱）



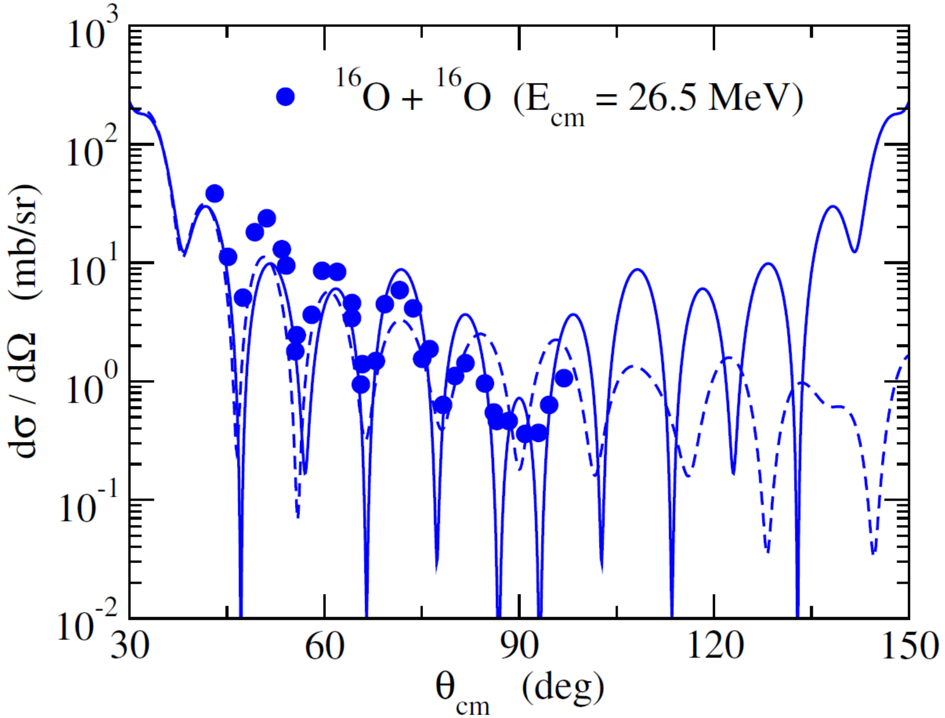
expt: D.A. Bromley et al., Phys. Rev. 123 (‘61)878

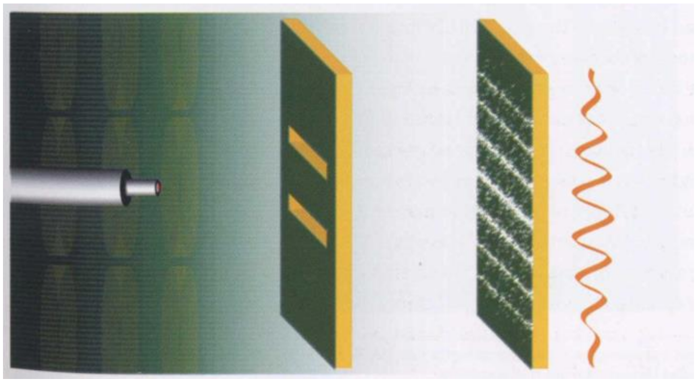
干渉の起源

対称化しなくても強い振動

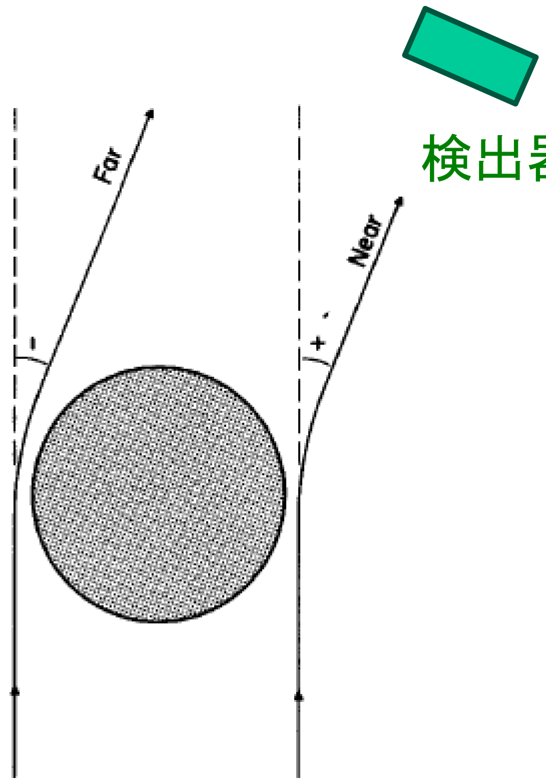
↓

✓ 対称化による干渉はマイナー
✓ near-sideとfar-side の干渉が主



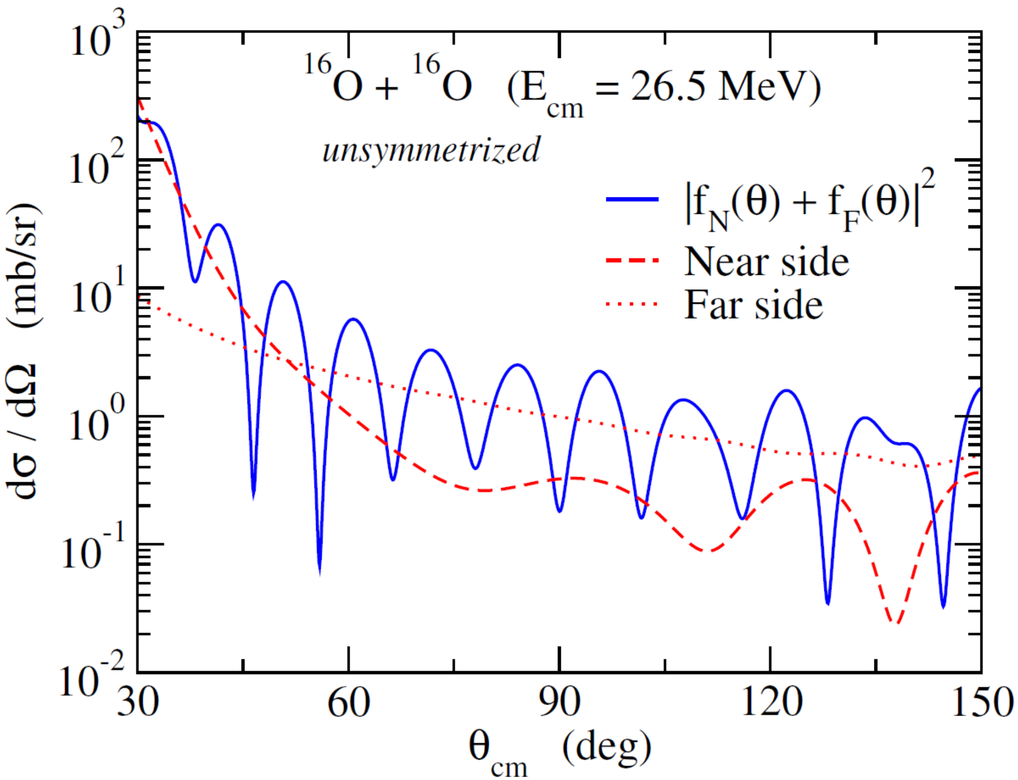


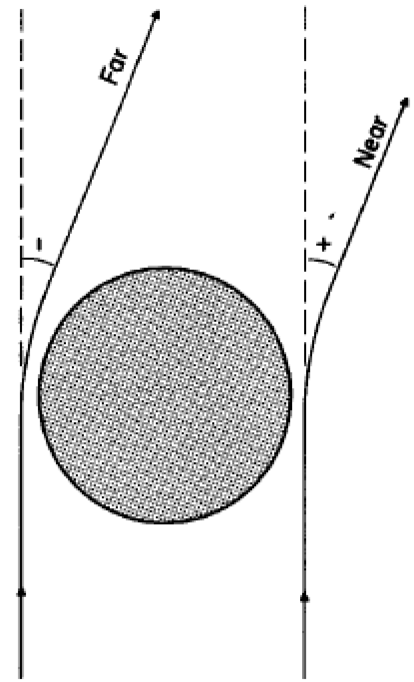
cf. 二重スリット



M.S. Hussein and K.W. McVoy, PPNP 12 (‘84)103.

near side-far side interference





$$P_l(\cos\theta) \to \frac{1}{2}\left[P_l(\cos\theta) \mp i\frac{2}{\pi}Q_l(\cos\theta)\right]$$

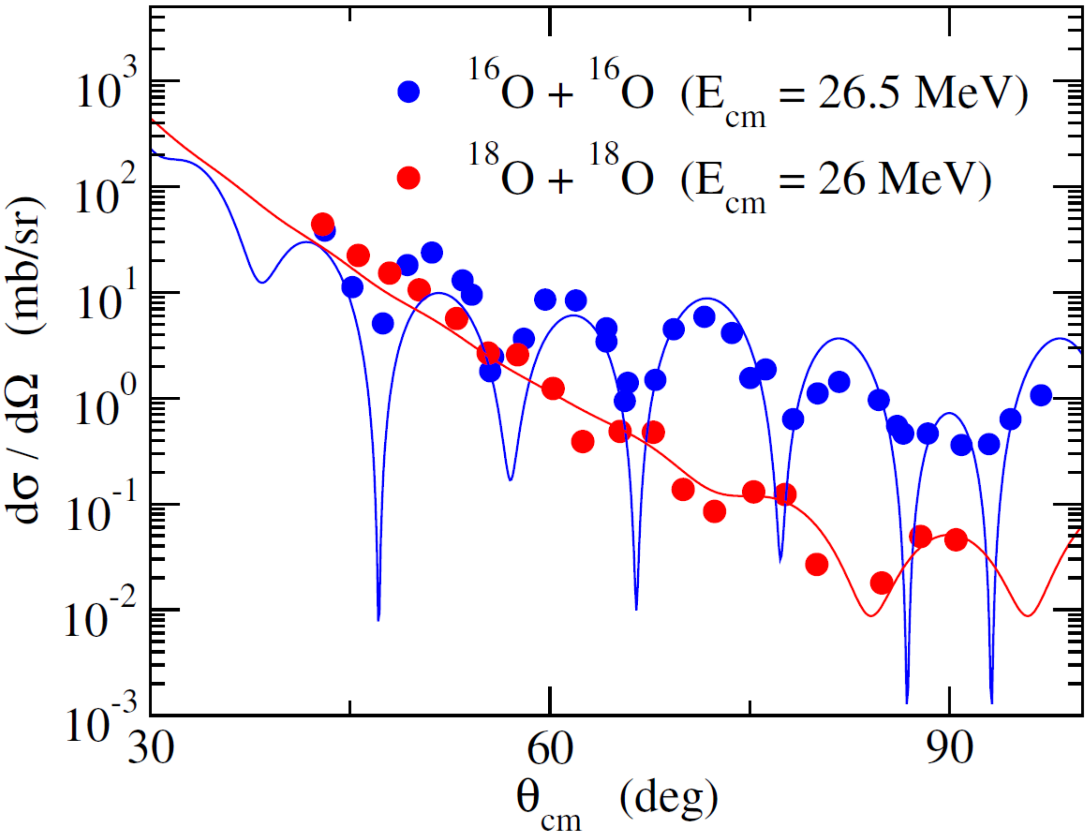
F

N

Q_l: Legendre function of the second kind

R.C. Fuller, PRC12, 1561 (1975)

Comparison between $^{16}O+^{16}O$ and $^{18}O+^{18}O$



^{16}O, ^{18}O: I^{π} (g.s.) = 0^{+}
(both are bosons)

$V_b \sim 10.3$ MeV

$\longrightarrow \quad E_{cm} \sim 2.5\, V_b$

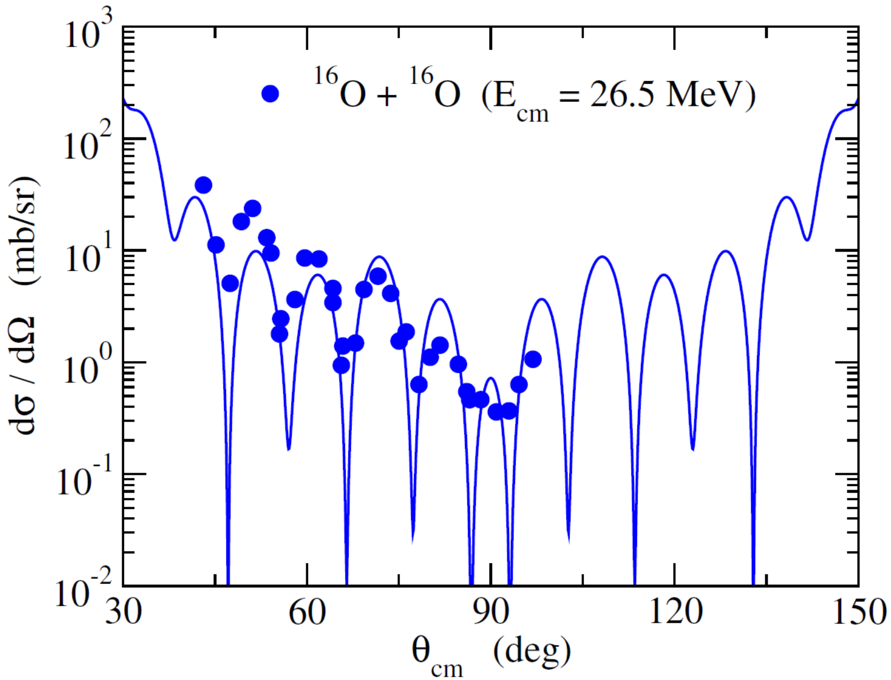
$^{18}O+^{18}O$: much less pronounced interference pattern

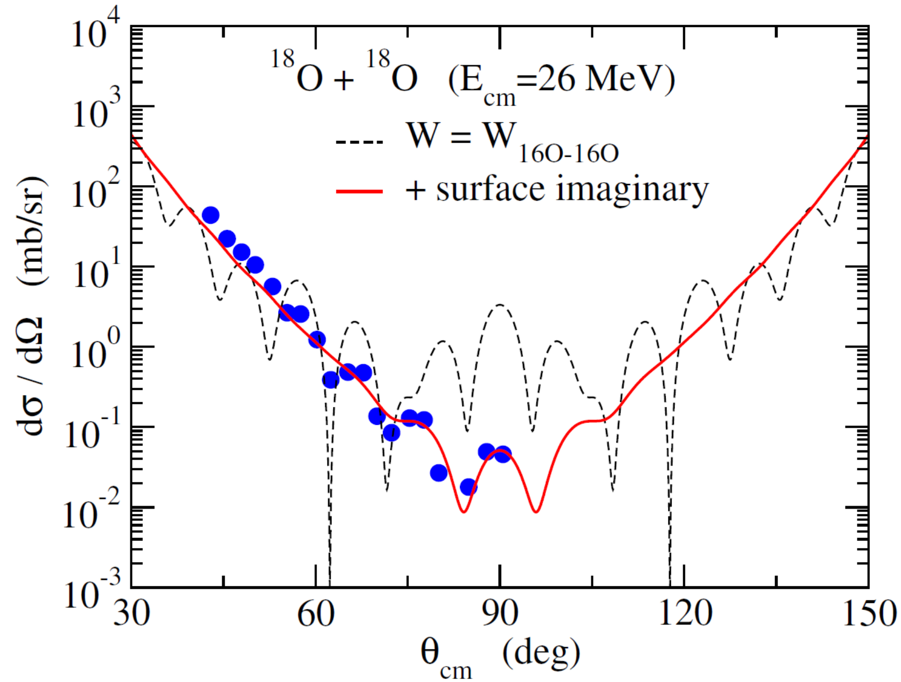
$^{18}O = ^{16}O$ (double closed shell) + 2n

$\rightarrow$ stronger coupling to environment

manifestation of (environmental) decoherence

光学ポテンシャル計算



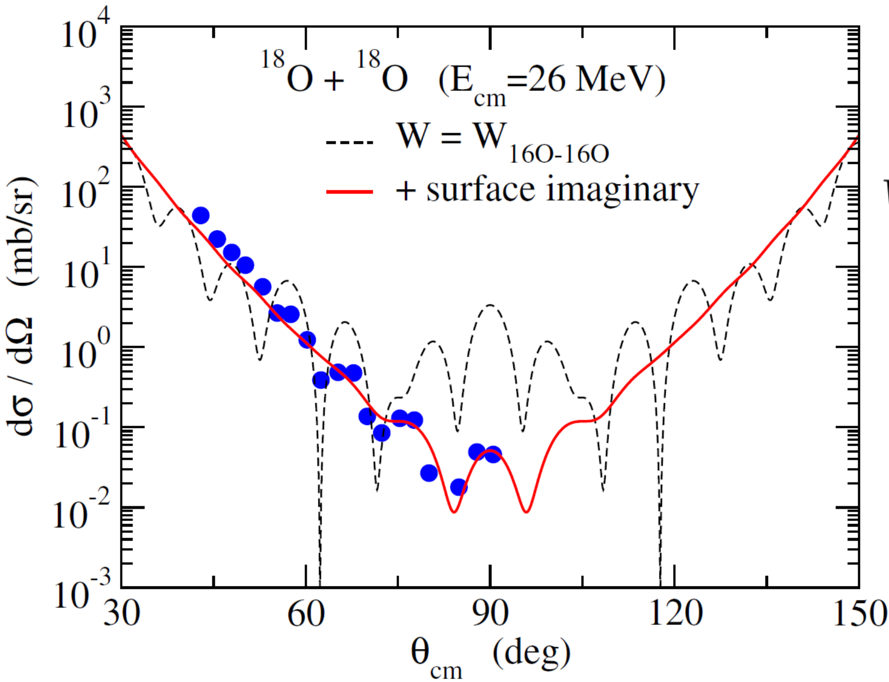


深い WS^2 型ポテンシャルによるフィット

同じポテンシャルで ^{18}O+^{18}O はフィットできず

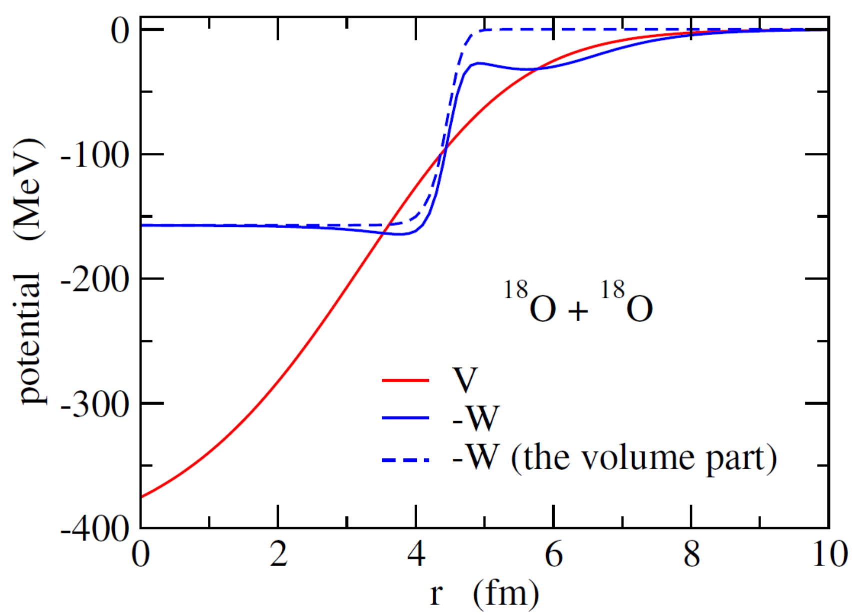
→吸収を強くする必要あり（ここでは表面型吸収ポテンシャルを導入）

Optical potential model calculation



$$V(r) = -\frac{V_0}{(1+\exp[(r-R)/a])^2}$$

$$W(r) = \frac{W_0}{(1+\exp[(r-R_W)/a_W])^2} + \frac{d}{dr}\frac{W_{0S}}{1+\exp[(r-R_{WS})/a_{WS}}$$

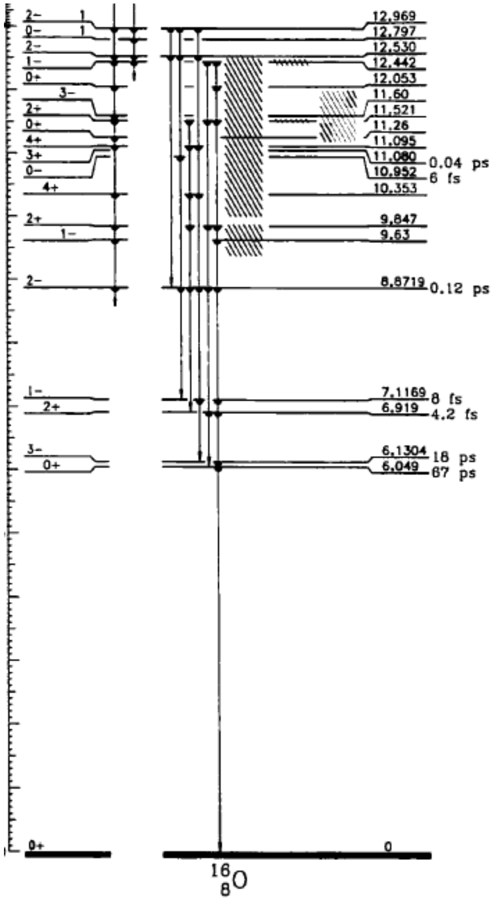


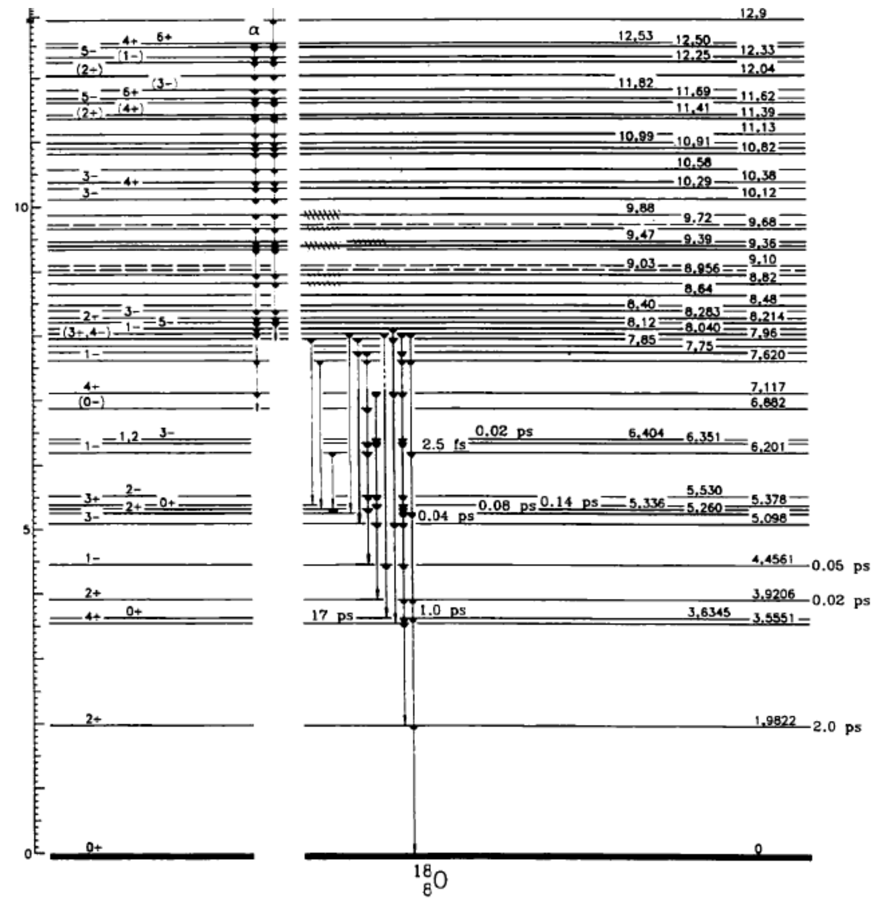
However, the same opt. pot. does not fit ^{18}O+^{18}O

↓

need to increase *W*
(with a surface imaginary pot.)

Spectra up to E* = 13 MeV





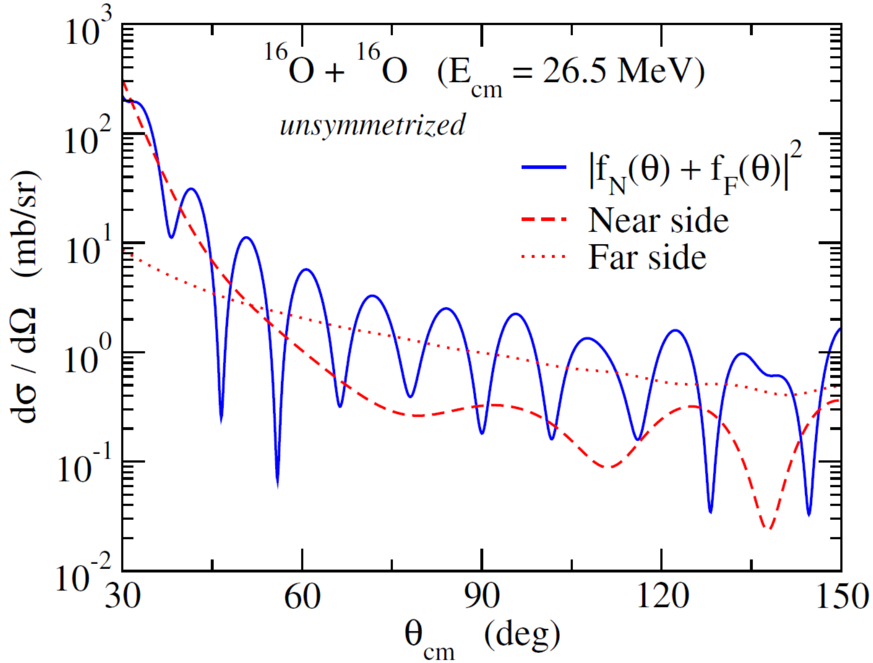
^{16}O

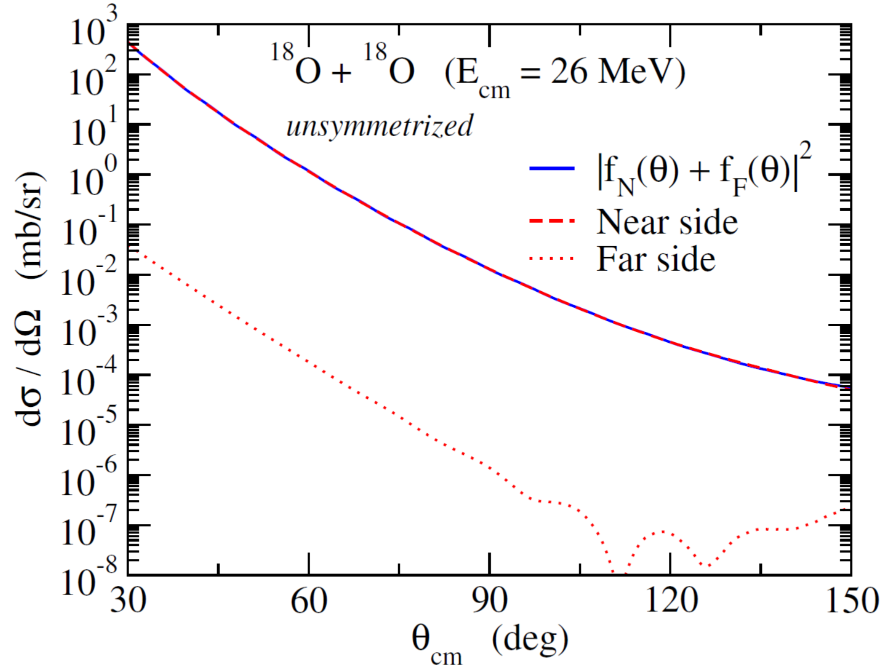
20 levels

^{18}O

56 levels

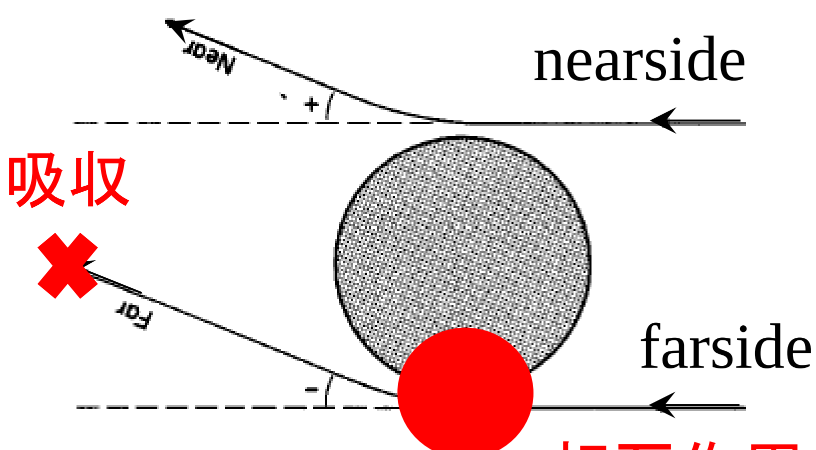
cf. the number of oepn channels, F. Haas and Y. Abe, PRL46('81)1667

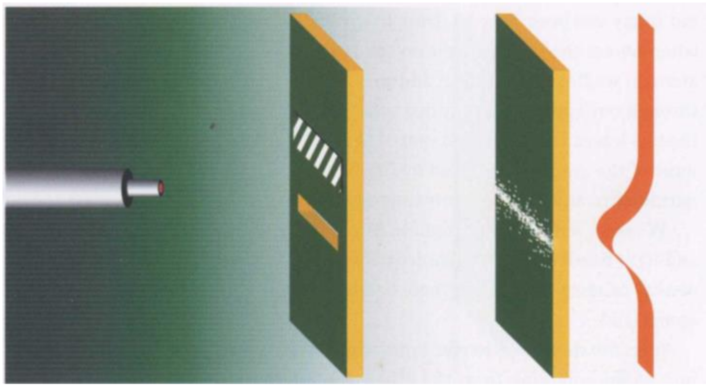




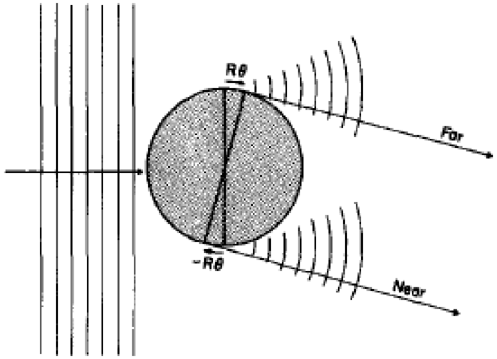
強い吸収のため、^{18}O+^{18}Oでは far-side 成分が大きく減衰
→干渉パターンがほとんど消えている

cf. 一重スリット

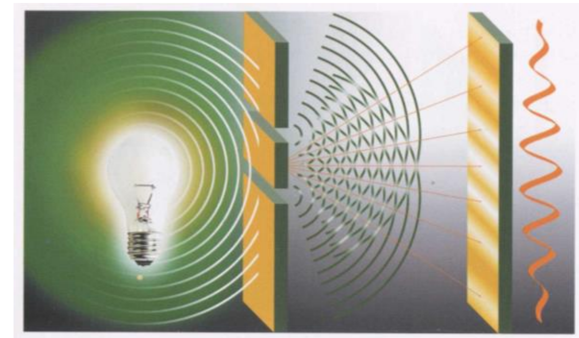




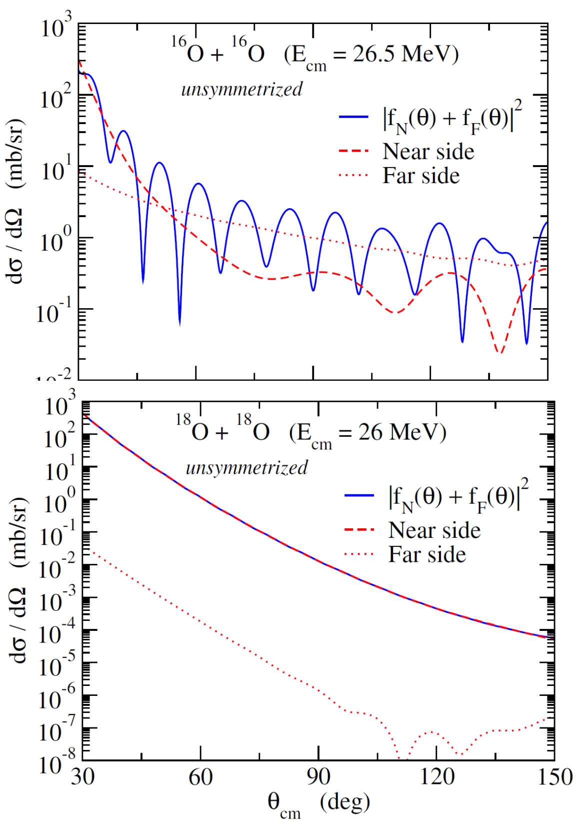
analogy to the double slit problem

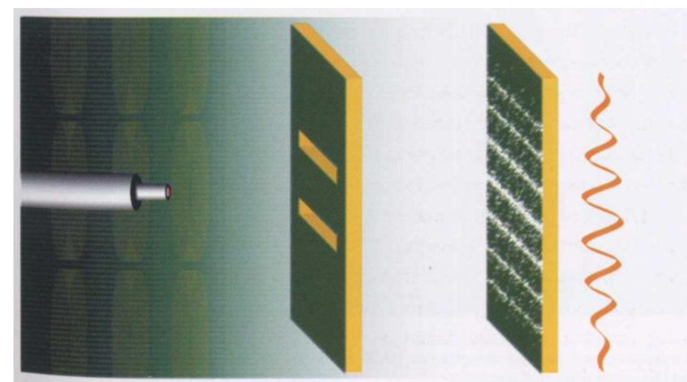


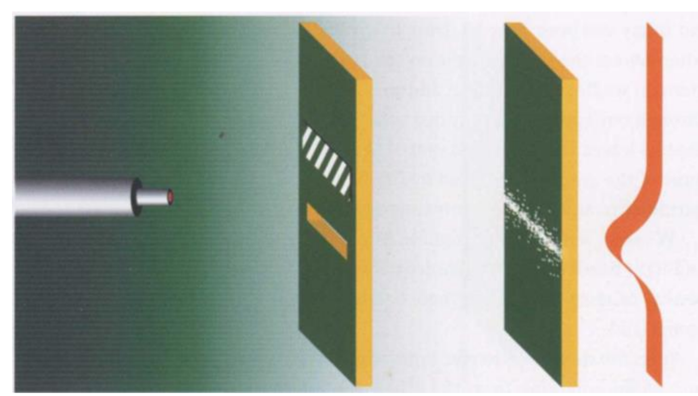
M.S. Hussein and K.W. McVoy,
Prog. in Part. and Nucl. Phys. 12 (‘84)103



J. Al-Khalili, “Quantum”







Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

cf. Experiment by Rutherford (α scatt.)

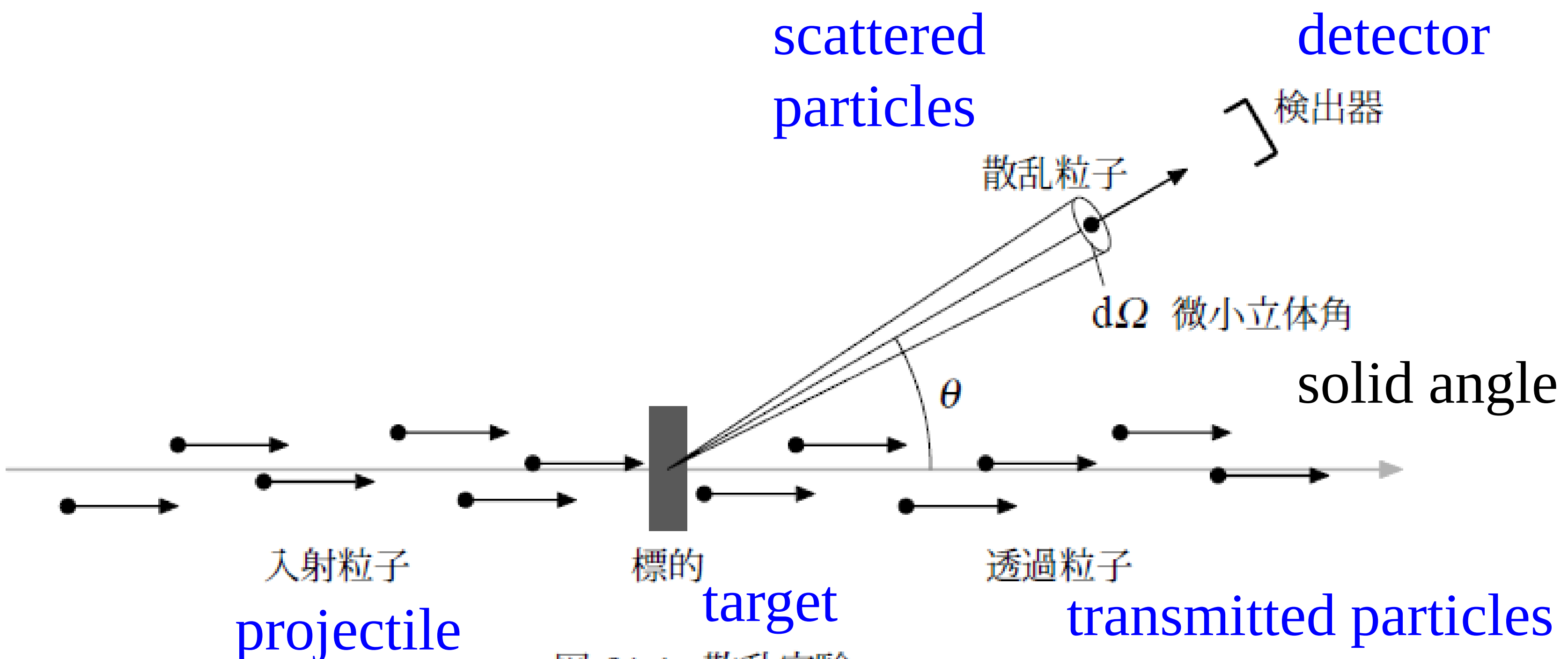


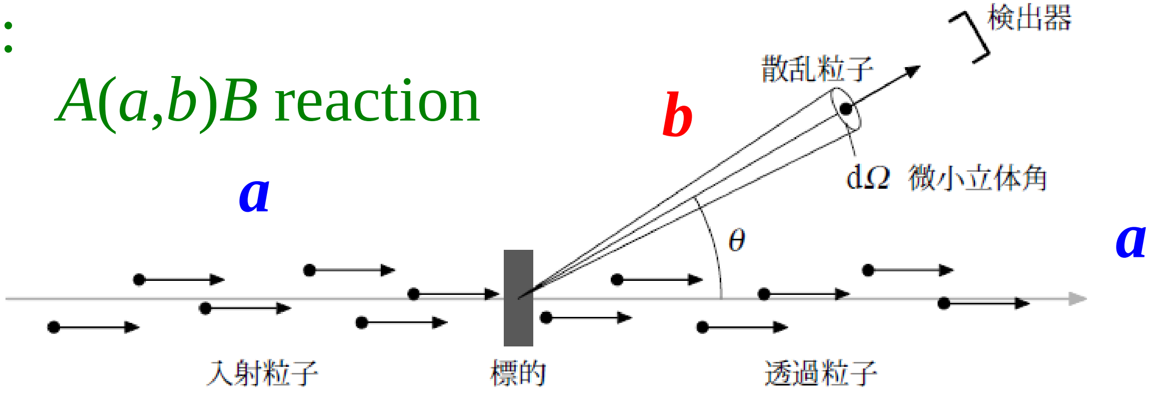
図 21.1: 散乱実験

http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

K. Muto (TIT)

notation:

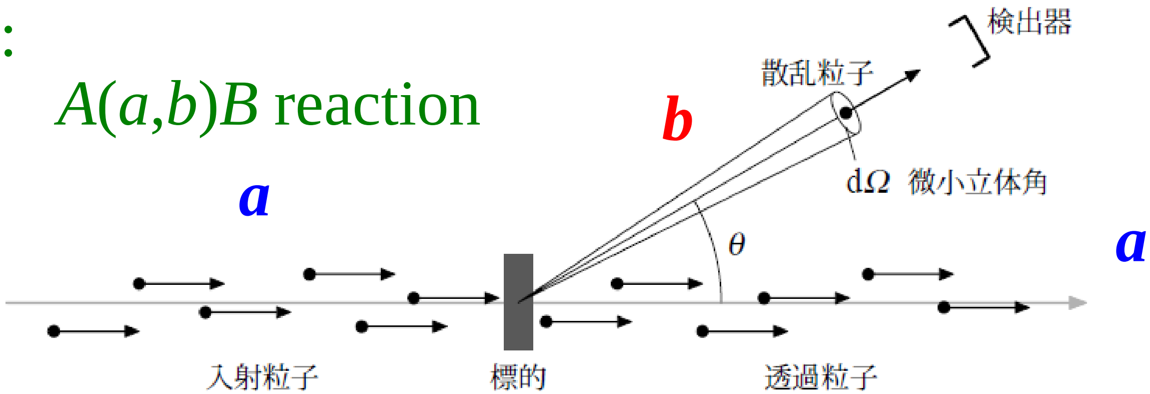
$A(a,b)B$ reaction



$A \rightarrow B$

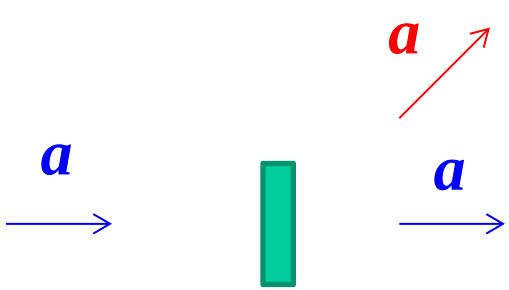
notation:

$A(a,b)B$ reaction



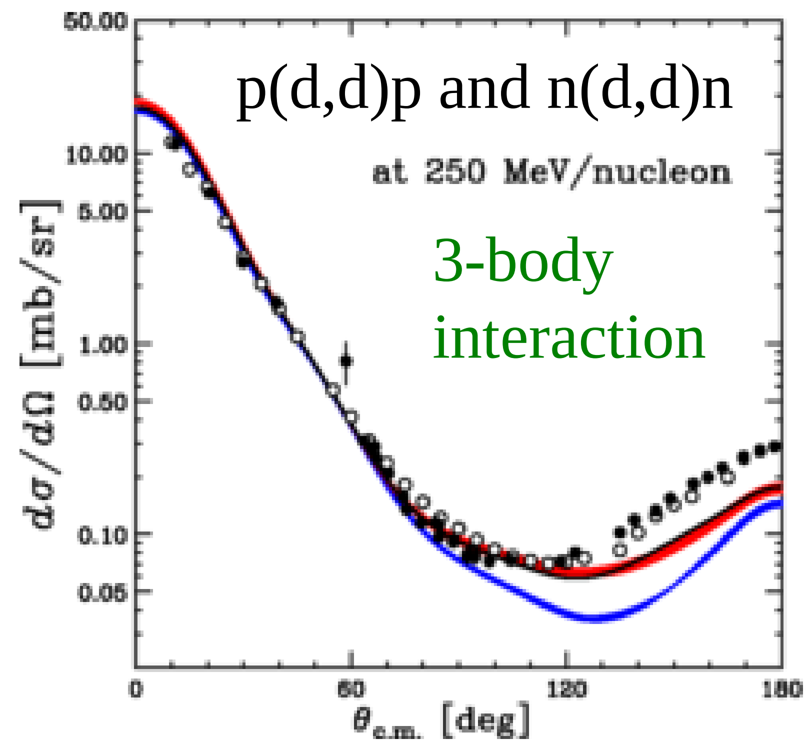
$A \rightarrow B$

✓elastic scattering



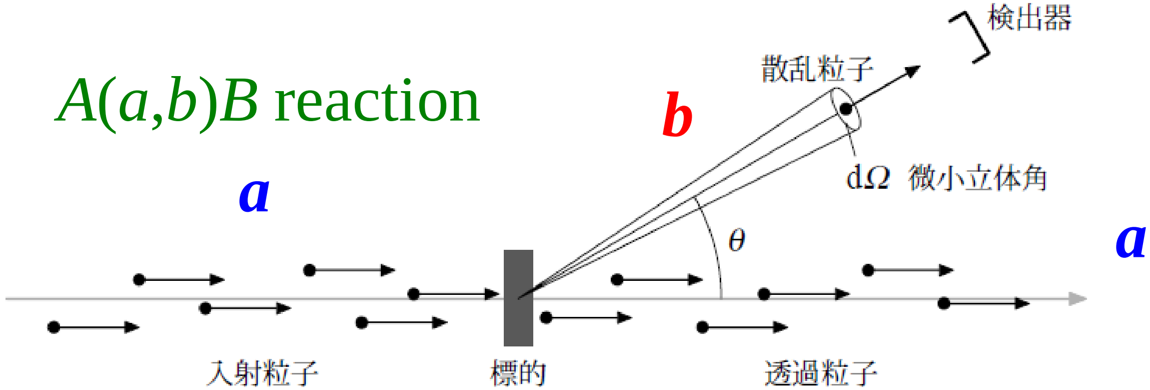
$A_{gs} \rightarrow A_{gs}$

fundamental interaction between a and A



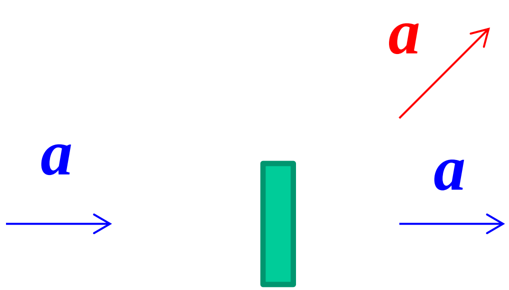
K. Sekiguchi et al., PRC89('14)064007

$A(a,b)B$ reaction



$A \rightarrow B$

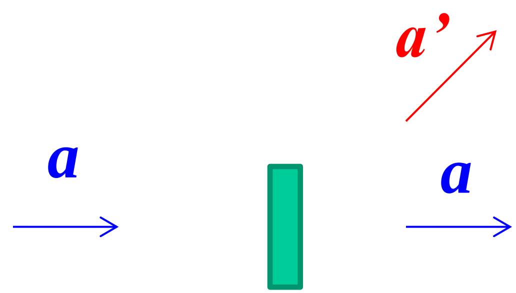
✓elastic scattering



$A_{gs} \rightarrow A_{gs}$

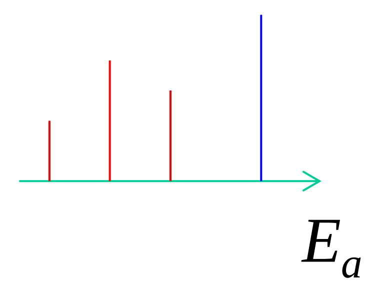
fundamental interaction between a and A

✓inelastic scattering



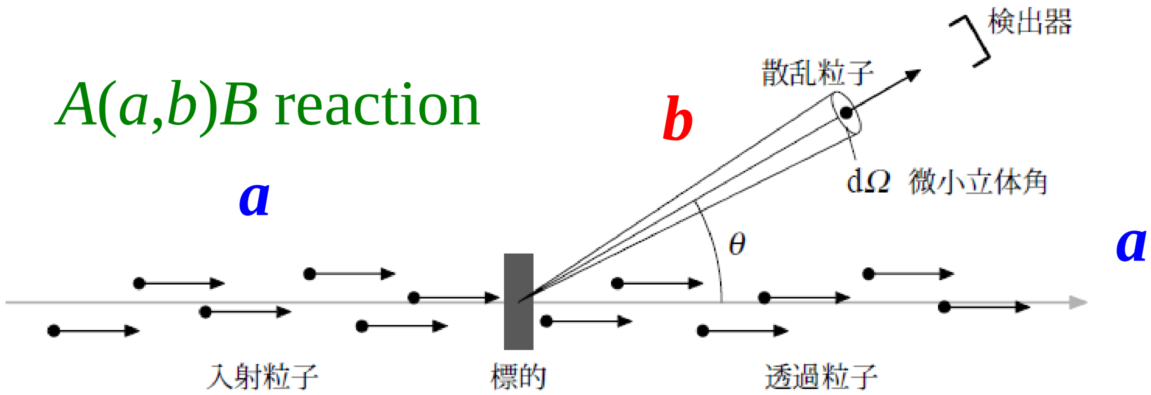
$A_{gs} \rightarrow A^*$

excitation spectrum of a nucleus A



励起状態の
✓ エネルギー
✓ 角運動量

$A(a,b)B$ reaction



$A \rightarrow B$

transfer reactions

✓transfer reaction
(pick-up reaction)

^{17}O

^{16}O $\longrightarrow$ ^{16}O $\longrightarrow$

$^{208}\mathrm{Pb}_{gs} \rightarrow {}^{207}\mathrm{Pb}$

level schem of ^{207}Pb

✓transfer reaction
(stripping reaction)

^{16}O

^{17}O $\longrightarrow$ ^{17}O $\longrightarrow$

$^{208}\mathrm{Pb}_{gs} \rightarrow {}^{209}\mathrm{Pb}$

level schem of ^{209}Pb

✓fusion reaction

$(a+A)$ $\longrightarrow$

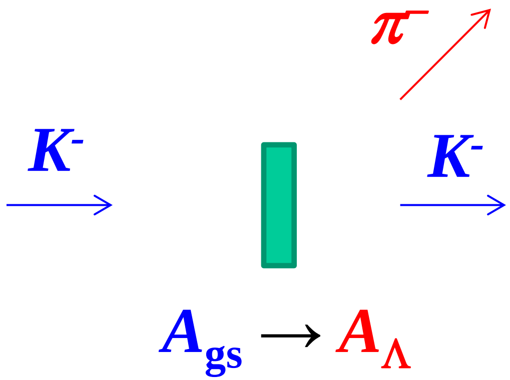
a $\longrightarrow$ $\longrightarrow$ a

$A_{gs} \rightarrow X$

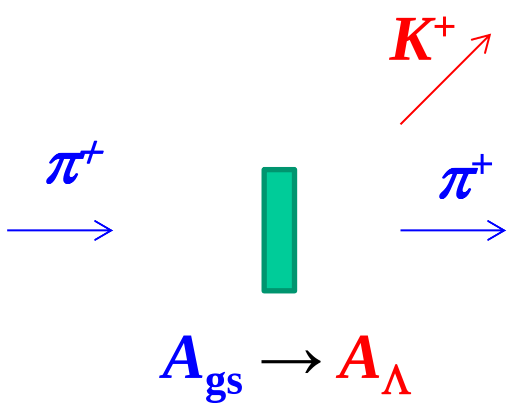
- interaction between a and A
- structure of a and A
- 未知核の生成

hypernucleus production reactions

✓ (K^-,π^-) reaction

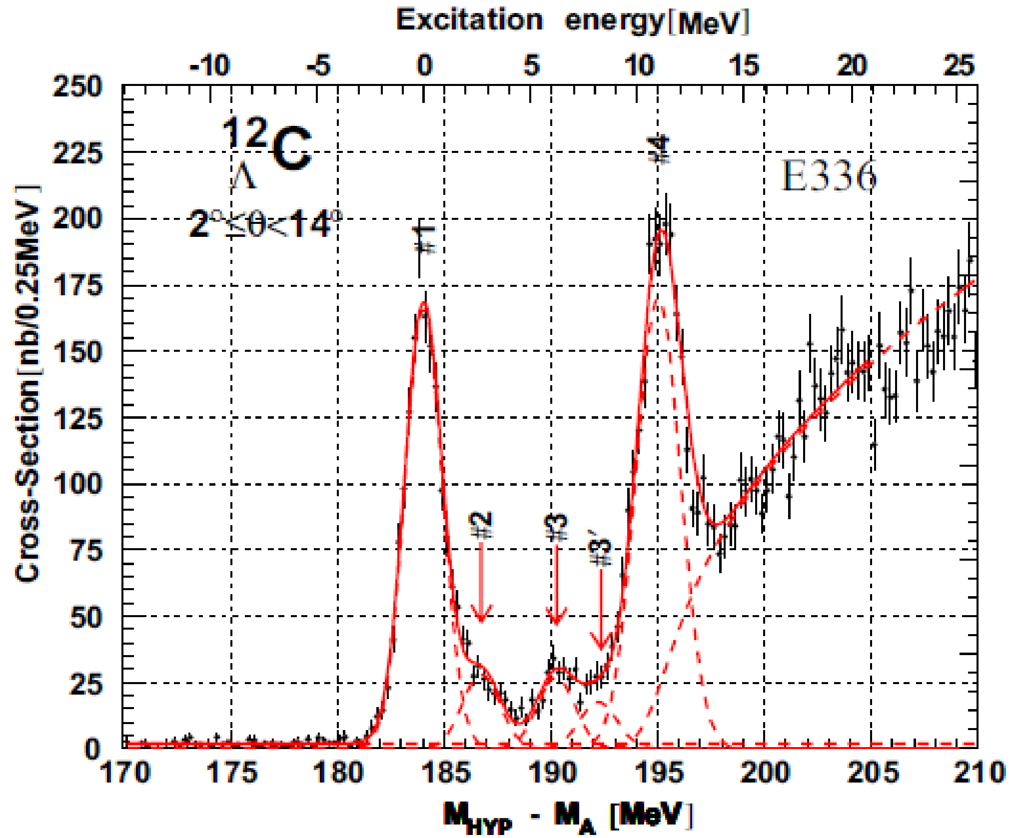


✓ (π^+,K^+) reaction



excitation spectrum
of a hypernucleus A_Λ

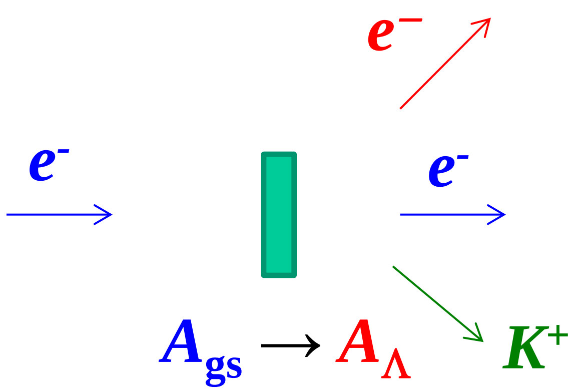
$^{12}C\ (\pi^+,K^+)\ ^{12}_{\Lambda}C$ reaction



O. Hashimoto and H. Tamura,
Prog. in Part. and Nucl. Phys. 57 ('06)564

"reaction spectroscopy"

✓(e,e'K$^+$) reaction

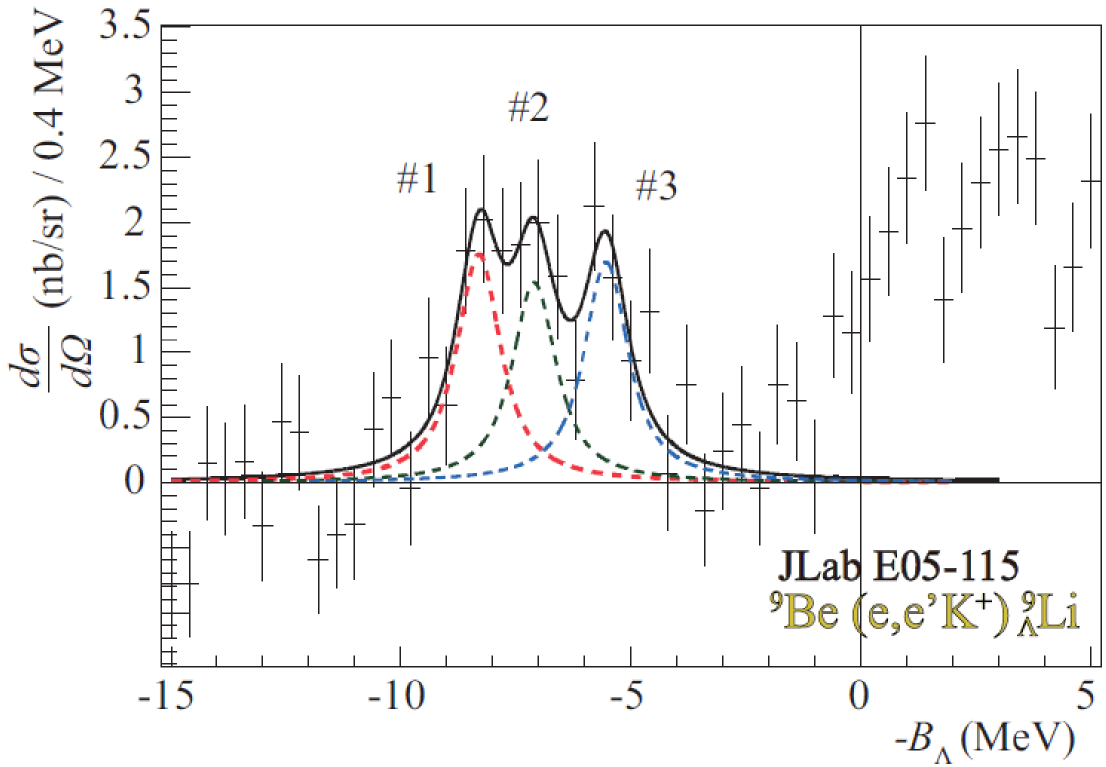


S.N. Nakamura et al.,
PRL110('13)012502

T. Gogami,
Ph.D. Thesis (Tohoku U.)
2014

K.N. Suzuki, T. Gogami et al.,
PTEP2022 (2022) 013D01

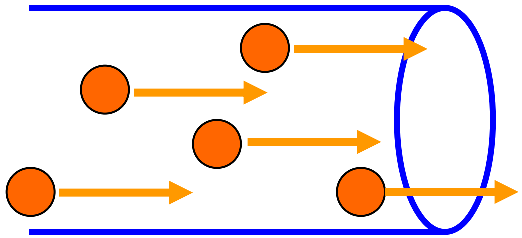
^{9}Be(e,e'K$^+$) $^9_\Lambda$Li



T. Gogami et al., PRC103('21)L041301

^{3}He(e,e'K$^+$) nnΛ

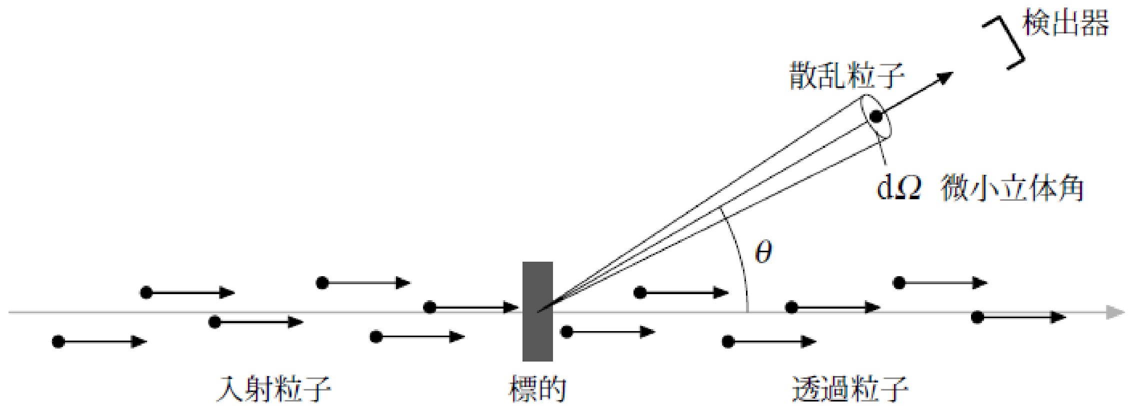
Cross sections



incident beam

flux = the number of particles
crossing unit area
per unit time

$$j = \rho_{\mathrm{P}} \cdot v$$

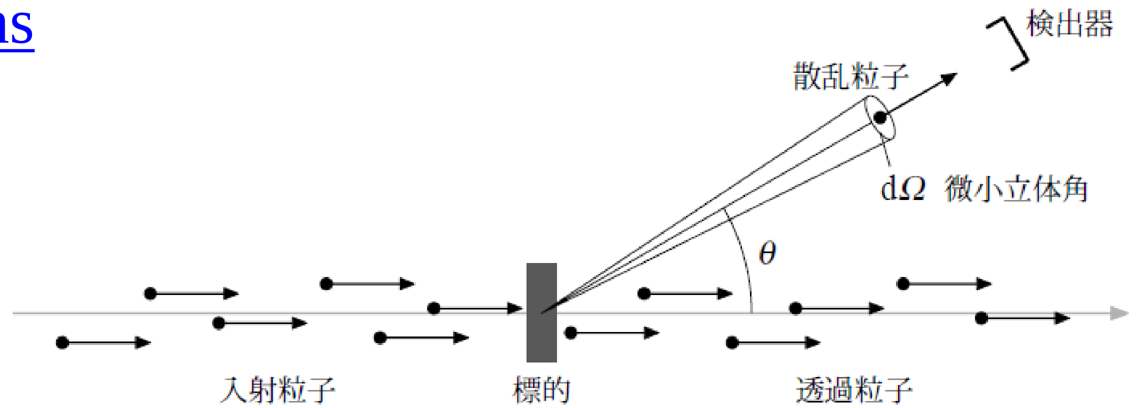


event rate (the number of event per unit time per target nucleus)
: proportional to the incident flux

cross section

$$\longrightarrow \quad R = N_{\mathrm{T}} \cdot \sigma \cdot j$$

Cross sections



event rate (the number of event per unit time per target nucleus)

: proportional to the incident flux

cross section

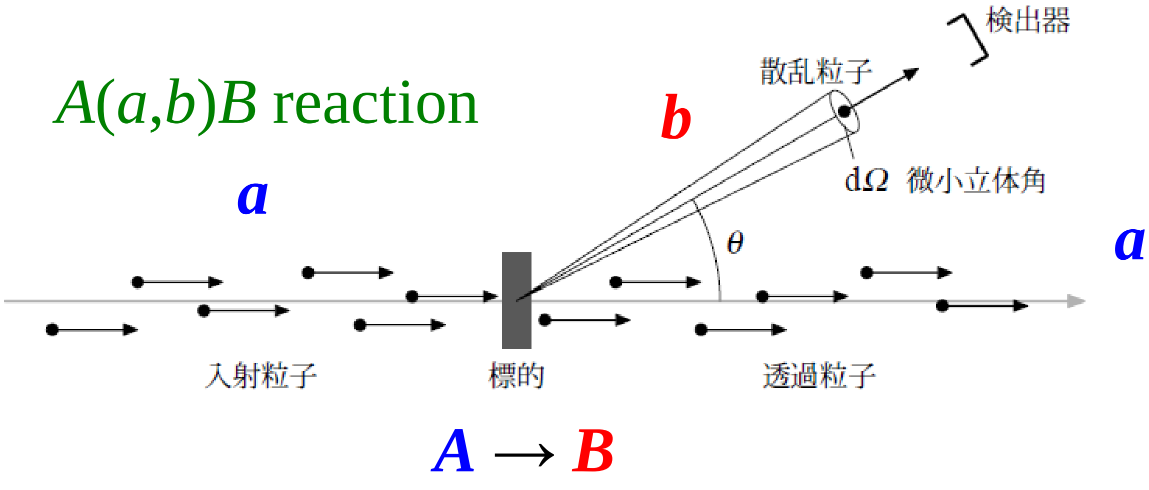
$$\longrightarrow \quad R = N_T \cdot \sigma \cdot j$$

differential cross sections (angular distribution)

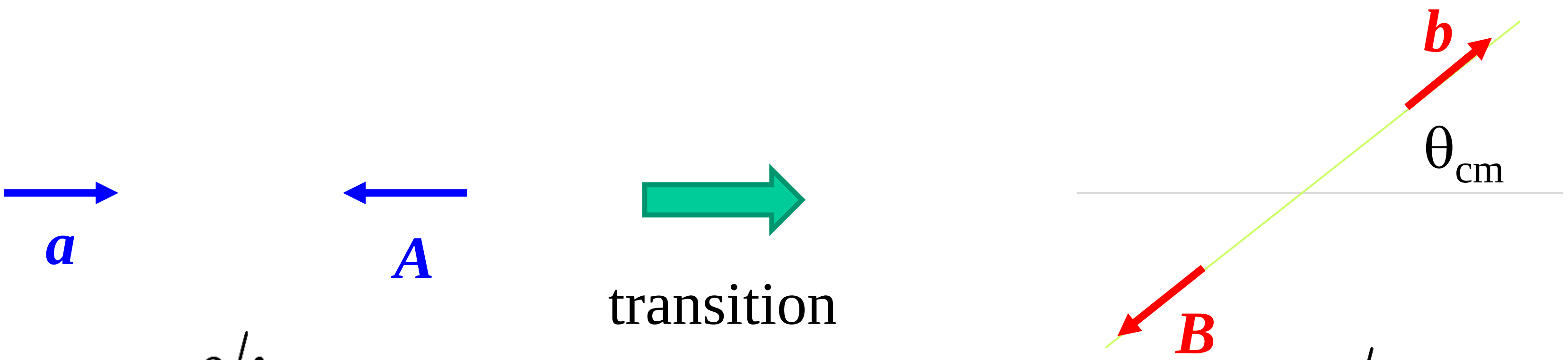
$$dR(\theta, \phi) = N_T \cdot \boxed{\frac{d\sigma}{d\Omega}} \cdot j \cdot d\Omega, \qquad \sigma = \int d\Omega \frac{d\sigma}{d\Omega}$$

units: 1 barn = 10^{-24} cm^2 = 100 fm^2 (1 mb = 10^{-3} b = 0.1 fm^2)

断面積（量子力学）



center of mass frame



$$\psi_i \qquad \psi_f$$

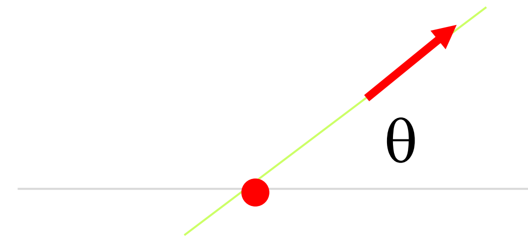
$$\boxed{\frac{d\sigma}{d\Omega} = \frac{R}{j_{\rm in}}}$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

$V(r)$



$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

transition rate for elastic scattering:

$$\begin{aligned} W_{fi} &= \frac{2\pi}{\hbar}\int\frac{d\boldsymbol{p}_f}{(2\pi\hbar)^3}\left|\langle\psi_f|V|\psi_i\rangle\right|^2\delta(E_f - E_i) \\ &= \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega\left|\tilde{V}(\boldsymbol{q})\right|^2 \end{aligned}$$

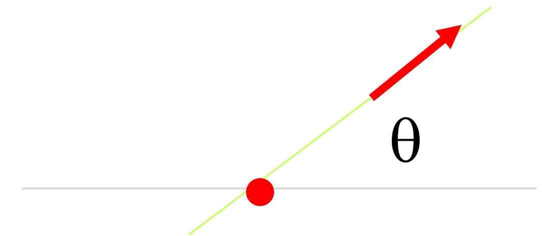
$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r}e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar}V(r) \equiv \int d\boldsymbol{r}e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(r)$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

V(r)



$$W_{fi} = \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega \left|\tilde{V}(\boldsymbol{q})\right|^2$$

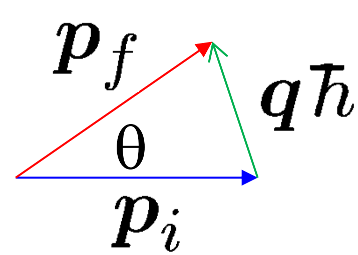
momentum transfer

$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r} e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar}V(r) \equiv \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(r)$$

incident flux: $j_{\text{inc}} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \boxed{\frac{\mu^2}{4\pi^2\hbar^4}\left|\tilde{V}(\boldsymbol{q})\right|^2}$$

$$= \frac{d\sigma}{d\Omega}$$

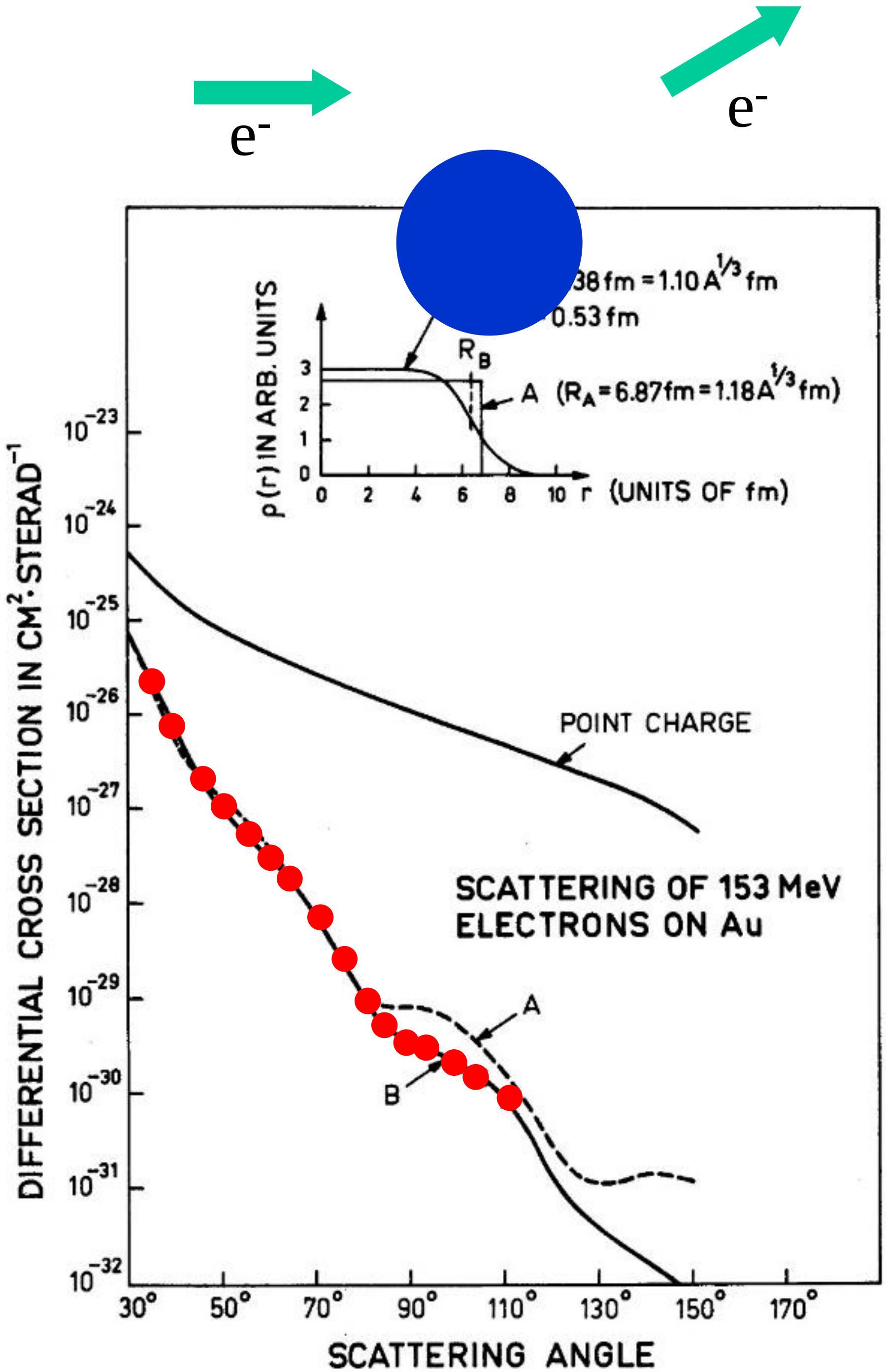


$$q\hbar = 2p_i \sin\frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho_{\text{ch}}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{(4E\sin^2\theta/2)^2} |F(\boldsymbol{q})|^2 = \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega}\right) |F(\boldsymbol{q})|^2$$



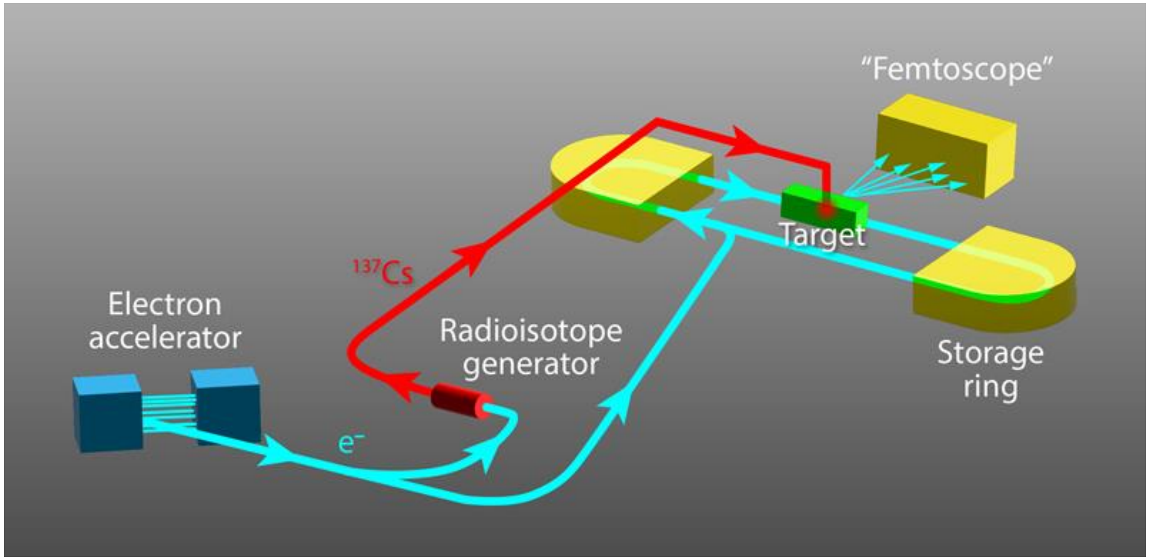
Form factor

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \rho_{\text{ch}}(\boldsymbol{r})\, d\boldsymbol{r}$$

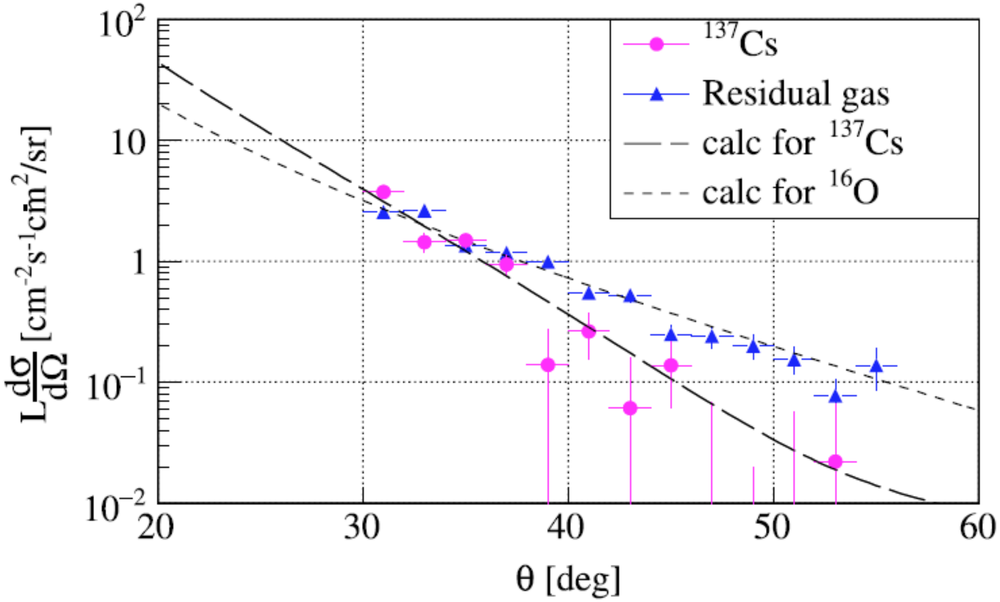
* relativistic correction:

$$\frac{d\sigma_{\text{Ruth}}}{d\Omega} \rightarrow \frac{d\sigma_{\text{Mott}}}{d\Omega} = \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \left(1 - \frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right) \sim \frac{d\sigma_{\text{Ruth}}}{d\Omega} \cdot \cos^2\frac{\theta}{2} \qquad (v \rightarrow c)$$

cf. electron scattering off unstable nuclei (SCRIT)



^{137}Cs ($T_{1/2}$=30.08 y)



K. Tsukada et al.,
PRL131, 092502 (2023)

レポート問題6（〆切：12月2日（月））

電子と原子核の相互作用が

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho_{\mathrm{ch}}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

で与えられているとする。ここで、ρ_{ch} は原子核の電荷密度で

$$\int d\boldsymbol{r}\, \rho_{\mathrm{ch}}(\boldsymbol{r}) = Z$$

と規格化されているとする。ボルン近似を用いて弾性散乱の断面積を求め、

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4E \sin^2 \theta/2} \right)^2 |F(\boldsymbol{q})|^2$$

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\, \rho_{\mathrm{ch}}(\boldsymbol{r})\, d\boldsymbol{r}$$

となることを示せ。

レポート問題7（〆切：12月2日（月））

qが小さいところで電子散乱の形状因子 $F(q)$ を決めることにより、原子核の荷電半径

$$\langle r^2 \rangle = \frac{1}{Z} \int d\boldsymbol{r}\, r^2 \rho_{\mathrm{ch}}(\boldsymbol{r})$$

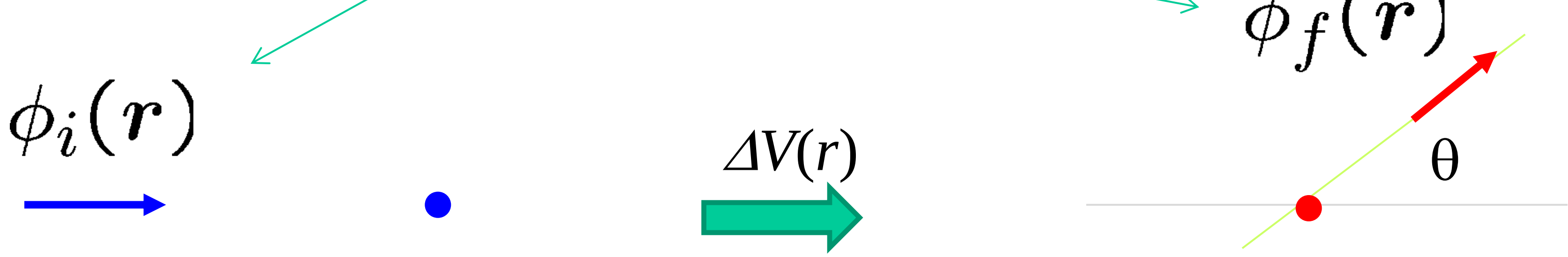
を求められることを示せ。

Distorted Wave Born approximation (DWBA)

$$\left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}^2+\underbrace{V(r)}_{\text{perturbation}}-E\right)\psi(\boldsymbol{r})=0$$

$$\Rightarrow\quad\left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}^2+\underbrace{V_0(r)}_{\text{光学ポテンシャル}}+\underbrace{V(r)-V_0(r)}_{\text{perturbation}}-E\right)\psi(\boldsymbol{r})=0$$

"distorted waves"



✓inelastic scattering
✓transfer reactions