集団励起の微視的理論

原子核の励起状態

- ✓ 一粒子励起（一つの核子が励起に関与）
- ✓ 集団励起（多くの核子が集団として励起に関与）

集団励起を微視的に理解してみる
（集団励起をミクロに見てみるとどうなっているのか?）

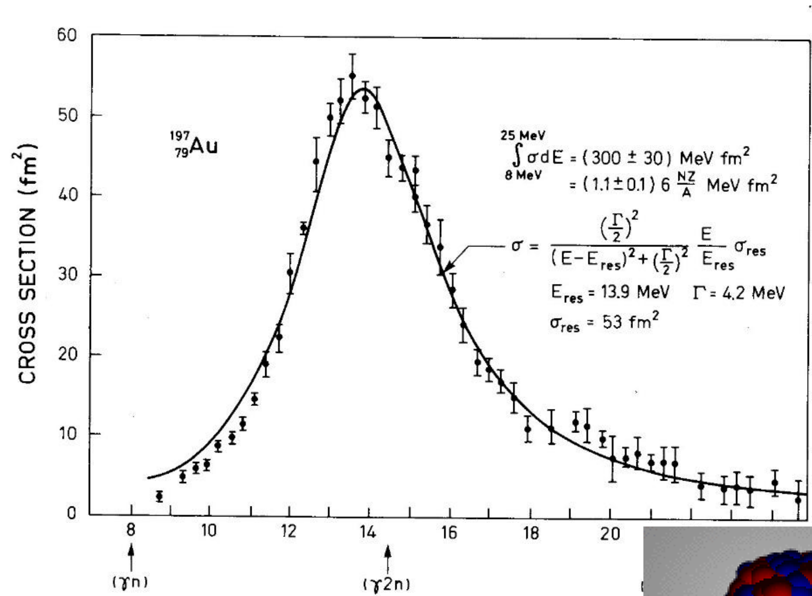
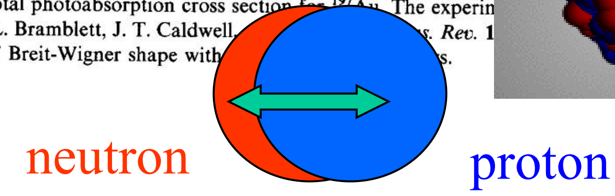
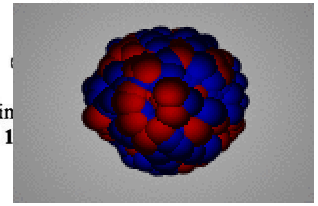


Figure 6-18 Total photoabsorption cross section for ^{197}Au. The experim... S. C. Fultz, R. L. Bramblett, J. T. Caldwell ... Rev. 1... solid curve is of Breit-Wigner shape with ...

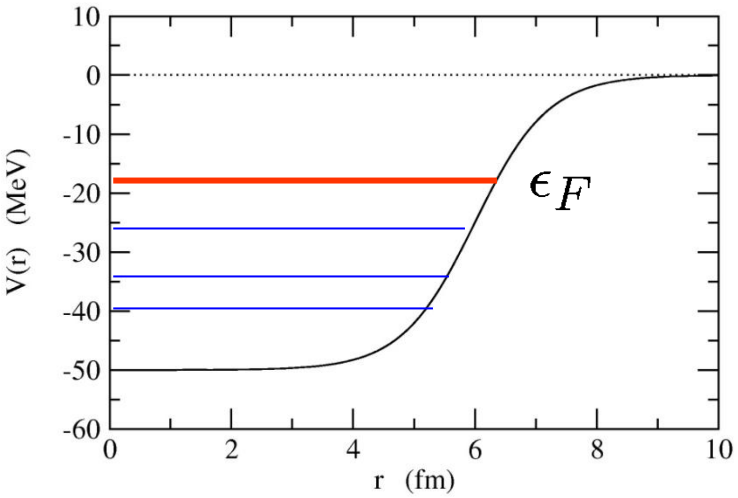




集団励起の例：巨大双極子共鳴

Tamm-Dancoff Approximation

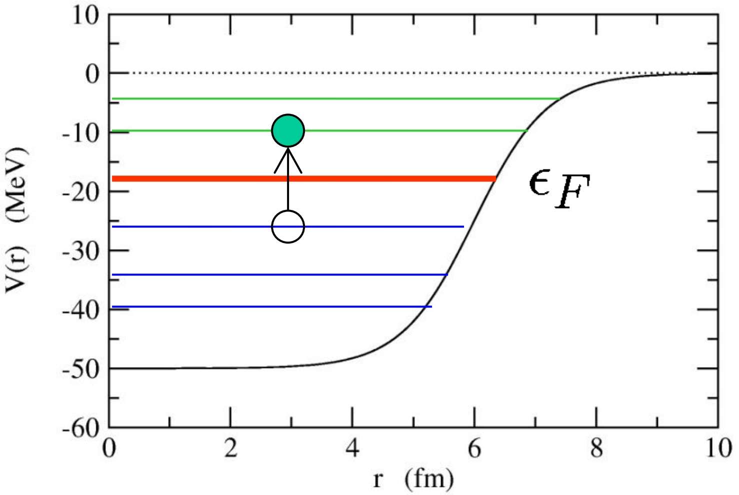
基底状態: $|HF\rangle = \prod_h a_h^\dagger|0\rangle$



励起状態:

$$|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle$$

$$\equiv \sum_{ph} X_{ph}|ph^{-1}\rangle$$



（1p1h 状態の重ね合わせ
:2p2h以上は寄与しないと仮定）

Tamm-Dancoff Approximation

Assume: $|\nu\rangle = Q_\nu^\dagger|HF\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h|HF\rangle$

$$\equiv \sum_{ph} X_{ph}|ph^{-1}\rangle$$

$$H|\nu\rangle = E_\nu|\nu\rangle$$

(superposition of 1p1h states)

$$\sum_{p'h'} H_{ph,p'h'}X_{p'h'} = E_\nu X_{ph}$$

$$H_{ph,p'h'} = \langle ph^{-1}|H|p'h'^{-1}\rangle$$
$$= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

residual interaction

Tamm-Dancoff equation; 1p1h の空間でハミルトニアンを対角化

TDA on a schematic model

$$H_{ph,p'h'} = (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle$$

$$\epsilon_p - \epsilon_h = \epsilon, \qquad \langle ph'|\bar{v}|hp'\rangle = g$$

for 3 ph configurations:

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$\rightarrow$ Diagonalization:

$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = (\epsilon+3g) \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

←位相が揃っている

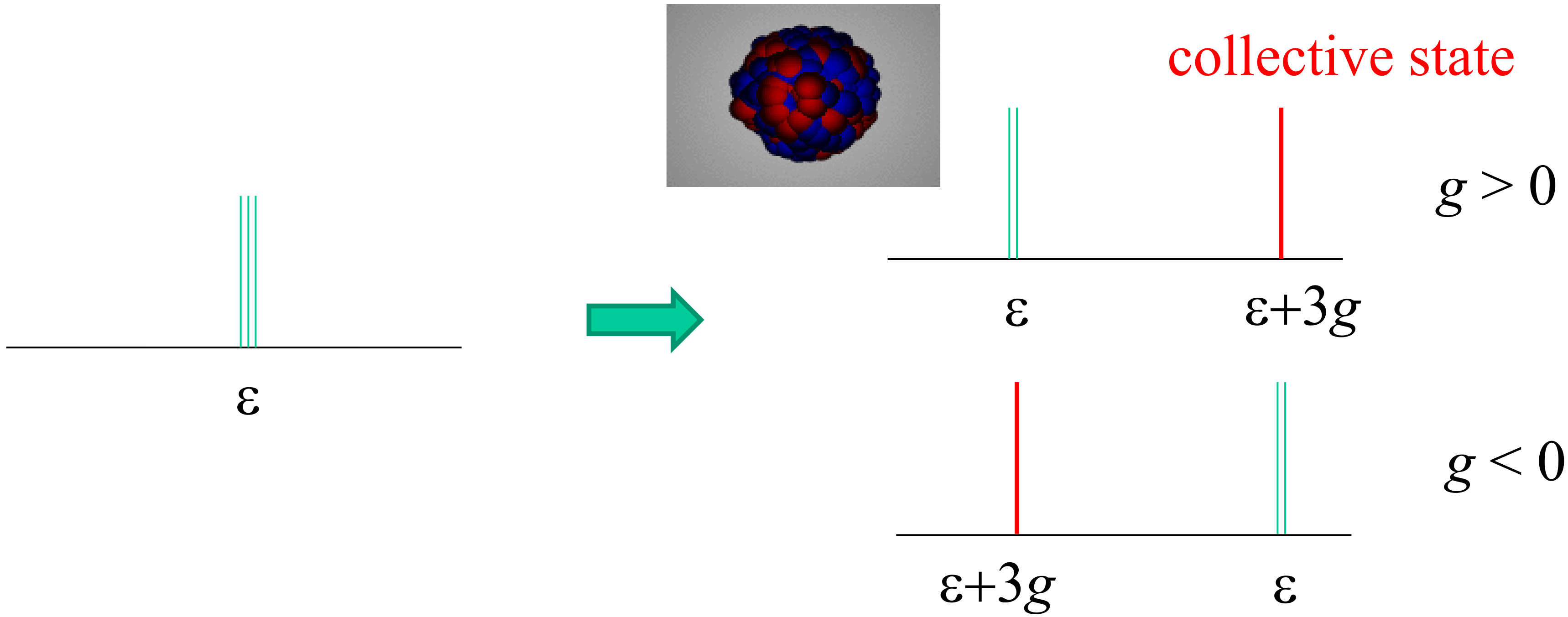
$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

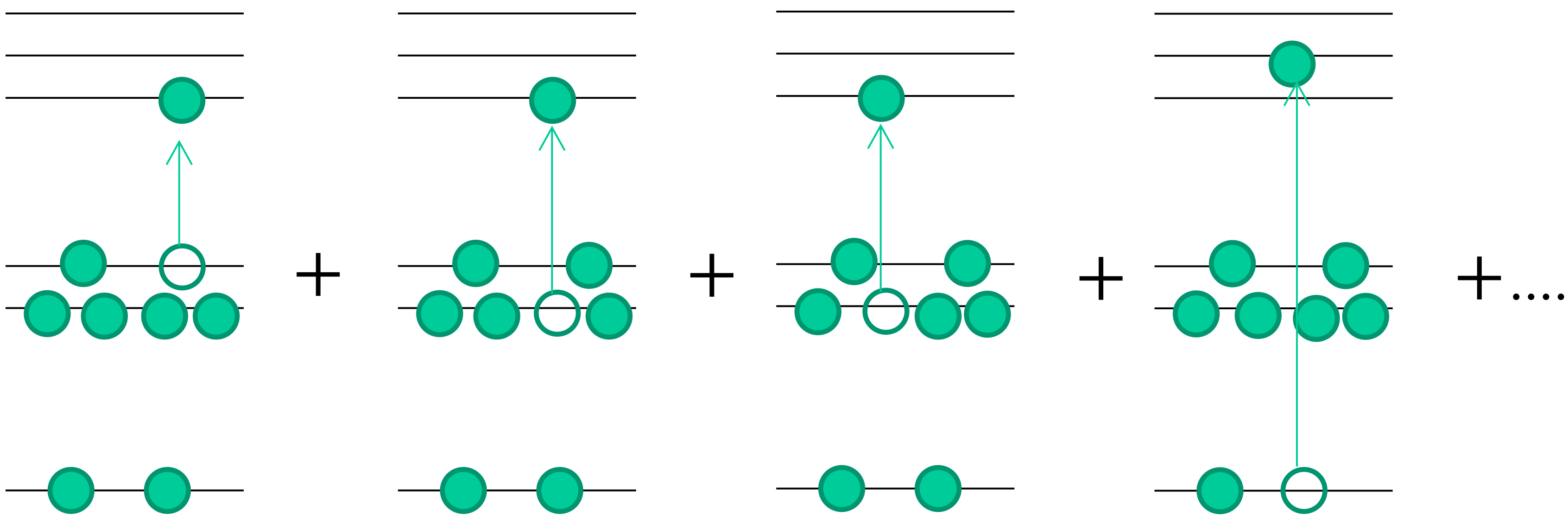
$$\begin{pmatrix} \epsilon+g & g & g \\ g & \epsilon+g & g \\ g & g & \epsilon+g \end{pmatrix} \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \epsilon \cdot \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

TDA on a schematic model

$$H = \epsilon \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + g \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

→ Diagonalization: $\lambda = \epsilon, \epsilon, \underline{\epsilon + 3g}$





複数の粒子・空孔状態をコヒーレントに重ね合わせることによって
多数の核子が励起に関与していることを表現する

$$|\nu\rangle \;=\; \sum_{ph} X_{ph}\, a_p^\dagger a_h |HF\rangle \equiv \sum_{ph} X_{ph} |ph^{-1}\rangle$$

$$\left| \left\langle \nu \left| \sum_{ph} f_{ph} a_p^\dagger a_h \right| 0 \right\rangle \right|^2 = \left(\sum_{ph} f_{ph} X_{ph} \right)^2$$

干渉項がすべて
同符号で寄与

Iso-scalar type modes: $E < \epsilon_{ph} \rightarrow \lambda < 0$ (attractive)

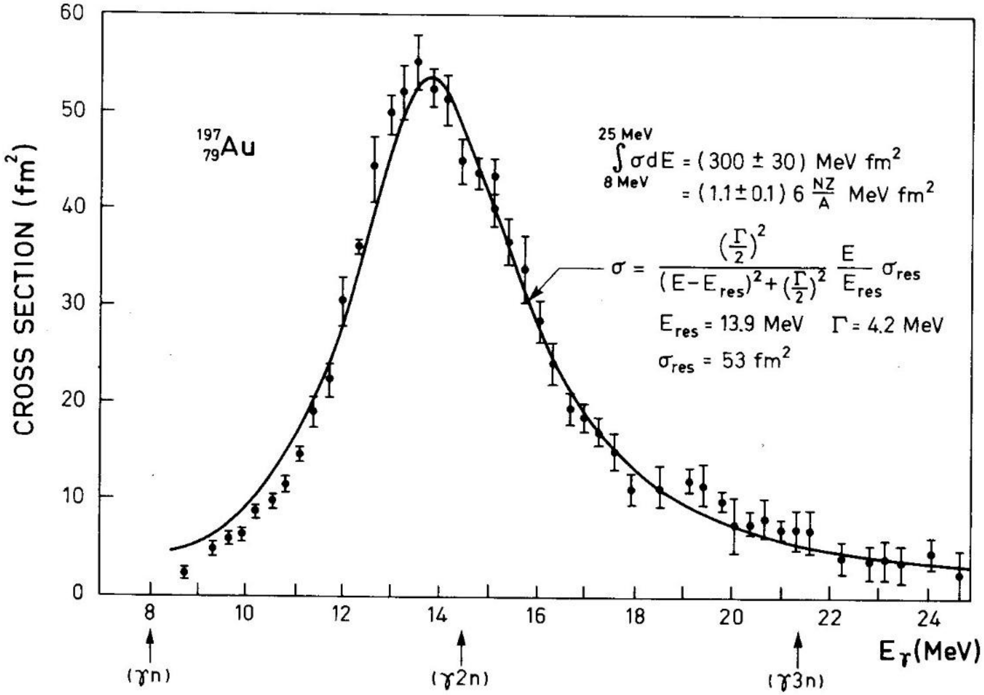
Iso-vector type modes: $E > \epsilon_{ph} \rightarrow \lambda > 0$ (repulsive)

Experimental systematics:

IV GDR: $E \sim 79 A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 41 A^{-1/3}$

IS GQR: $E \sim 65 A^{-1/3}$ (MeV) $\longleftrightarrow \epsilon_{ph} \sim 82 A^{-1/3}$

(note) single particle potential: $\hbar\omega \sim 41 A^{-1/3}$ (MeV)



^{197}Au

$$E_{GDR} = 14 \quad (\text{MeV})$$

$$\epsilon_{ph} \sim 41 \cdot 197^{-1/3}$$

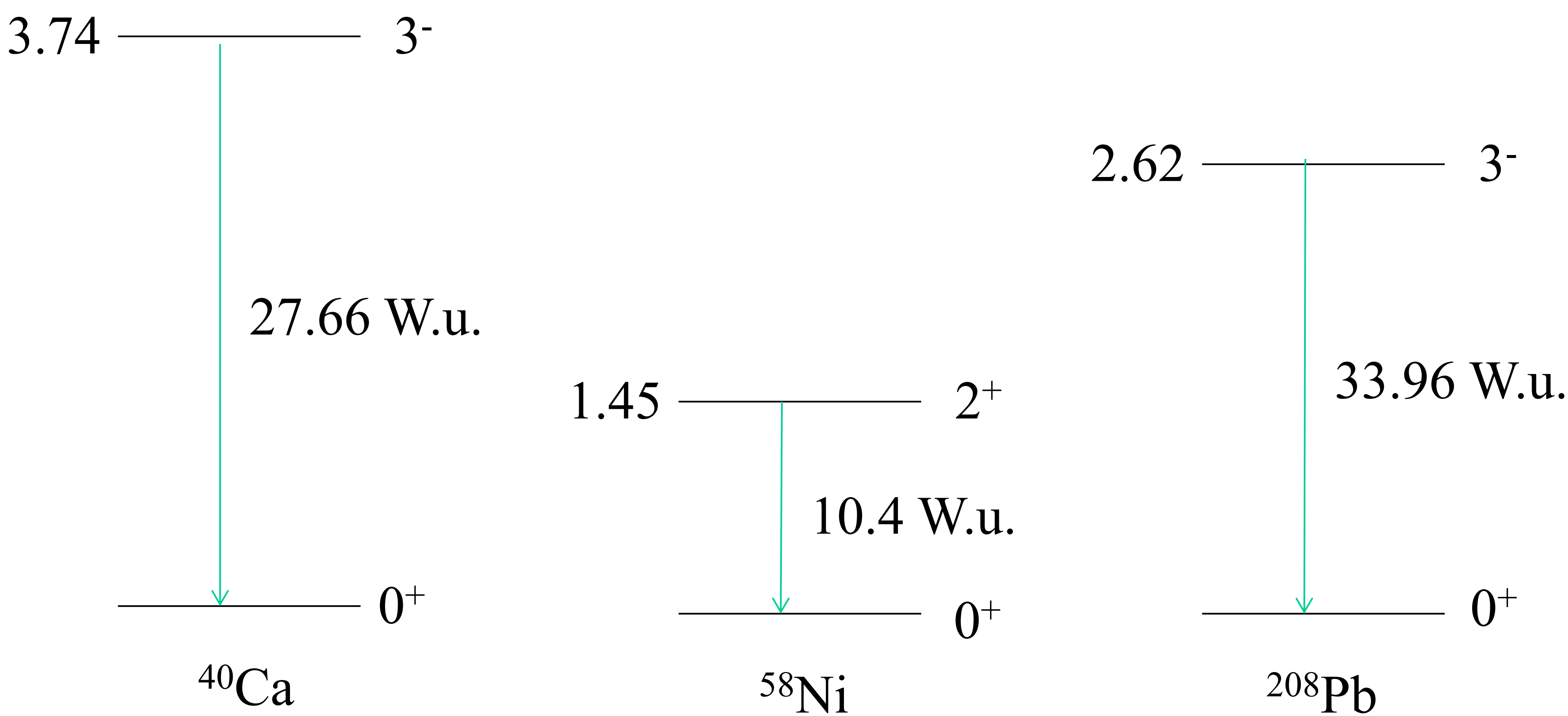
$$\sim 7 \quad (\text{MeV})$$

どれだけの核子が励起に関与しているのか?

Single-particle transition (Weisskopf unit)

$$B(E\lambda : I_i \to I_{gs}) = \frac{(1.2A^{1/3})^{2\lambda}}{4\pi}\left(\frac{3}{\lambda+3}\right)^2 \qquad (e^2\mathrm{fm}^{2\lambda})$$

exp data:



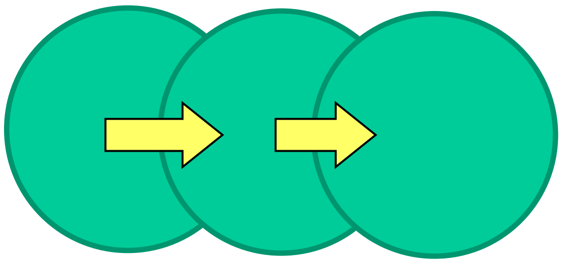
Spurious motion and RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇨ Zero energy mode (Nambu-Goldstone mode)



does not require an extra energy → zero energy mode

A drawback of TDA:

Zero modes are mixed with physical excitations
→ Zero modes appear at finite excitation energies.

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph} X_{ph}\, a_p^\dagger a_h |0\rangle \qquad \text{(TDA)}$$

A better approximation:

the random phase approximation (RPA)

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

規格化:

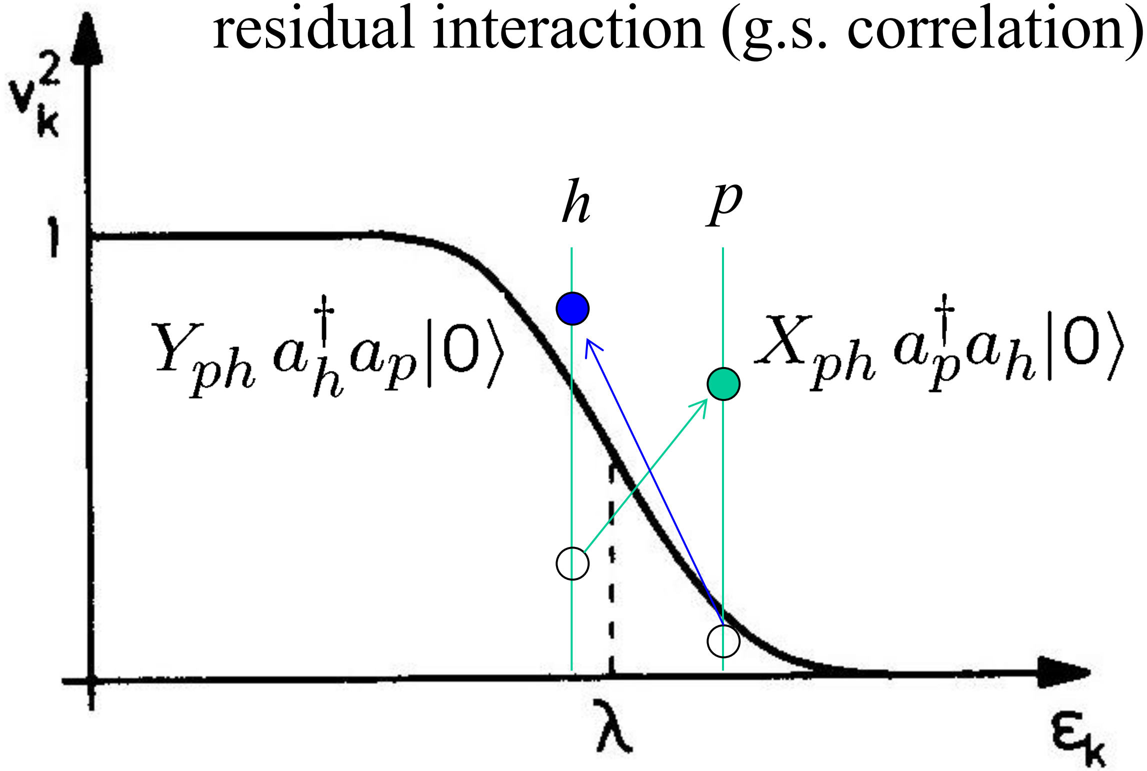
$$\langle\nu|\nu\rangle = 1 \rightarrow \sum_{ph}\left(|X_{ph}|^2 - |Y_{ph}|^2\right) = 1$$

A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

smearing of Fermi surface due to the residual interaction (g.s. correlation)



A better approximation: **the random phase approximation (RPA)**

$$|\nu\rangle = Q_\nu^\dagger|0\rangle = \sum_{ph}\left(X_{ph}\,a_p^\dagger a_h - Y_{ph}\,a_h^\dagger a_p\right)|0\rangle$$

(superposition of 1p1h states)

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

↓

$$\langle HF|[\delta Q, [H, Q_\nu^\dagger]]|HF\rangle = E_\nu\langle HF|[\delta Q, Q_\nu^\dagger]|HF\rangle$$

$\rightarrow$ coupled equations for X and Y

$$\delta Q = a_h^\dagger a_p, \quad a_p^\dagger a_h$$

$$\langle HF|[\delta Q,[H,Q_\nu^\dagger]]|HF\rangle = E_\nu \langle HF|[\delta Q, Q_\nu^\dagger]|HF\rangle$$

$$Q_\nu^\dagger = \sum_{ph} X_{ph}\, a_p^\dagger a_h - Y_{ph}\, a_h^\dagger a_p \qquad \delta Q = a_h^\dagger a_p, \quad a_p^\dagger a_h$$

RPA equation:

$$\begin{aligned} \sum_{p'h'} A_{ph,p'h'} X_{p'h'} + B_{ph,p'h'} Y_{p'h'} &= E_\nu X_{ph} \\ \sum_{p'h'} B^*_{ph,p'h'} X_{p'h'} + A^*_{ph,p'h'} Y_{p'h'} &= -E_\nu Y_{ph} \end{aligned}$$

$$\begin{aligned} A_{ph,p'h'} &= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle \\ B_{ph,p'h'} &= \langle pp'|\bar{v}|hh'\rangle \end{aligned}$$

or

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E_\nu \begin{pmatrix} X \\ Y \end{pmatrix}$$

Spurious motion in RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero mode (Nambu-Goldstone mode)

$$[H, \hat{O}] = 0$$

RPA

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

$\hat{O}$ is a solution of RPA with E=0

$$Q^\dagger = \hat{O} = \sum_{ph} (O_{ph} a_p^\dagger a_h + O_{hp} a_h^\dagger a_p)$$

(note) $Q^\dagger_{\mathrm{TDA}} = \sum_{ph} O_{ph} a_p^\dagger a_h$ ⇒ $[H, Q^\dagger_{\mathrm{TDA}}] \neq 0$

Spurious motion in RPA

Mean-Field Approximation ⟷ Broken symmetiries

- Center of mass localization (single center)
- Rotational motion

Restoration of broken symmetries

⇒ Zero mode (Nambu-Goldstone mode)

RPA

$$[H, Q_\nu^\dagger] \sim E_\nu Q_\nu^\dagger$$

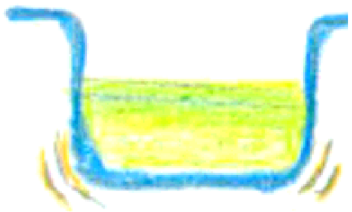
if $[H, \hat{O}] = 0$

Then $\hat{O}$ is a solution of RPA with E=0

The physical solutions are completely separated out from the spurious modes.

他のRPAの定式化

- 線形応答理論

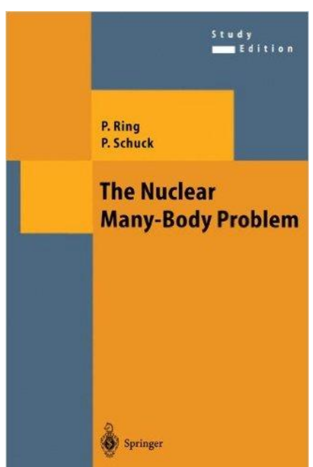


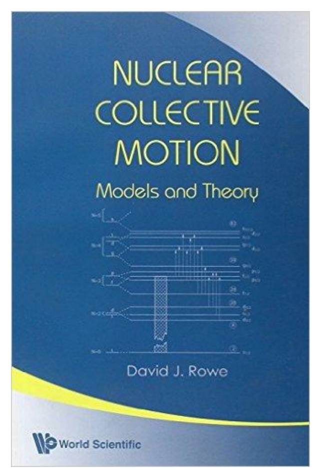
外場で原子核を揺すった時に、
原子核がどのように応答するか摂動論を使って議論する
→固有モードを見つける

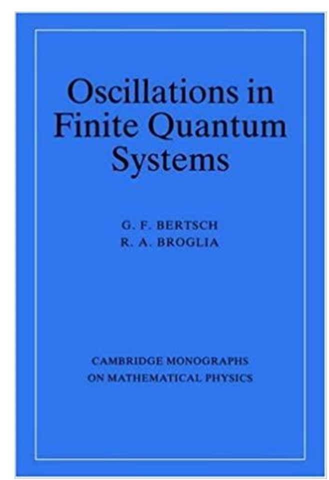
- 時間に依存するハートリー・フォック(TDHF)方程式を線形化

$$i\hbar\dot{\rho}(t) = [h[\rho], \rho] \qquad \longleftarrow \qquad \rho(t) = \rho_0 + \delta\rho(t)$$

詳しくは:







Comparison between Skyrme-(Q)RPA calculation and exp. data

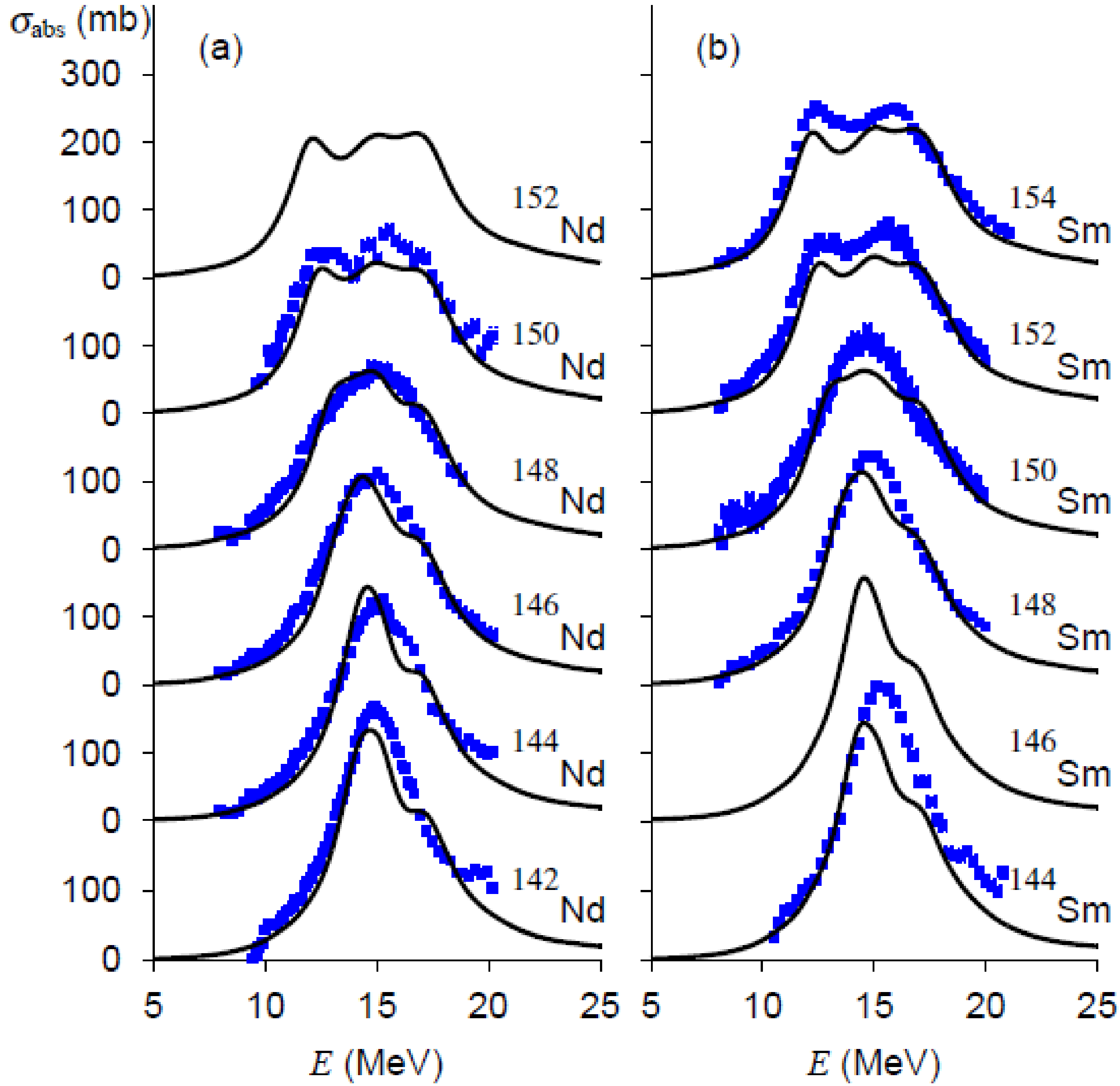
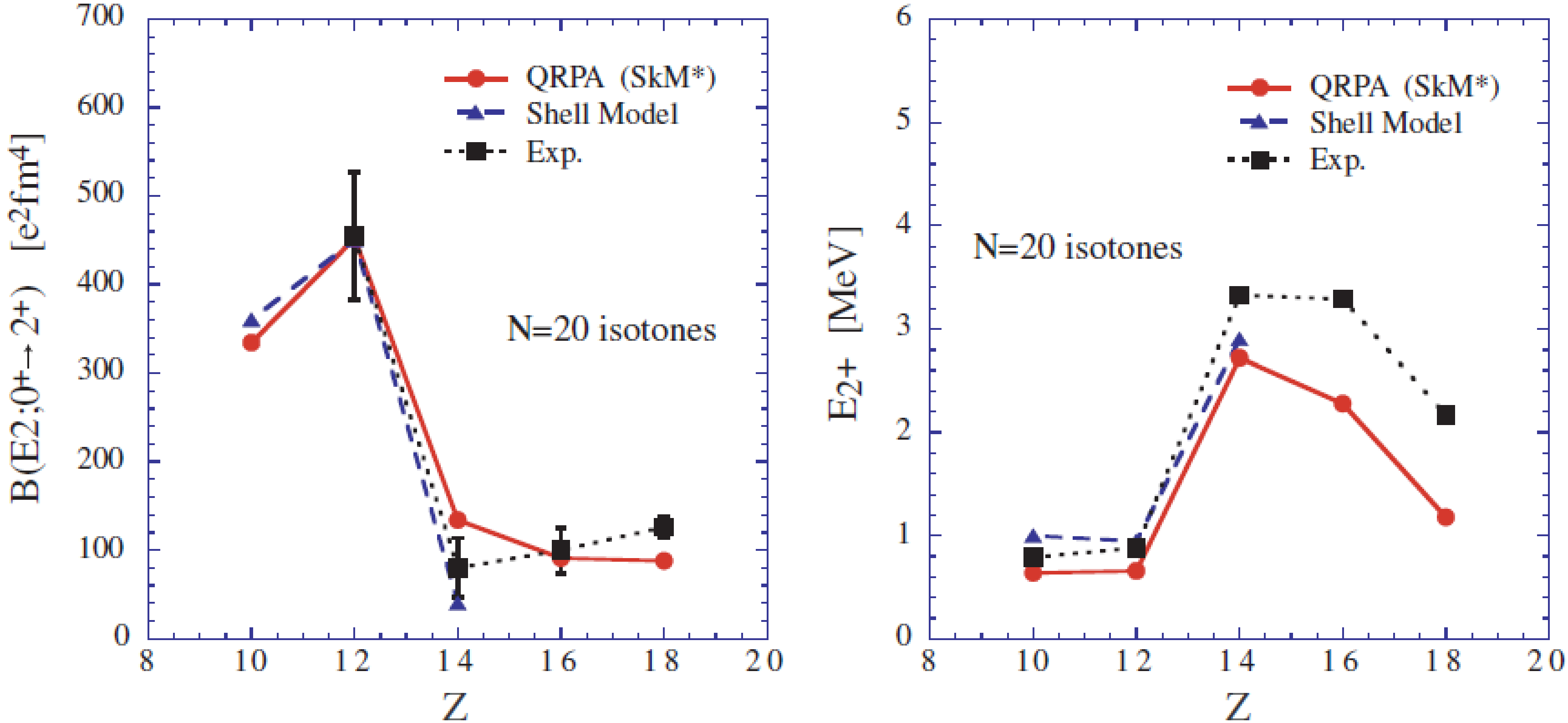


photo-absorption
cross section
(GDR)

K. Yoshida
and T. Nakatsukasa,
PRC83(‘11)021304



M. Yamagami and Nguyen Van Giai, PRC69 ('04) 034301

RPA on a schematic model

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\begin{aligned} A_{ph,p'h'} &= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle \\ B_{ph,p'h'} &= \langle pp'|\bar{v}|hh'\rangle \end{aligned}$$

Separable interaction:

$$\begin{aligned} \langle ph'|\bar{v}|hp'\rangle &= \lambda D_{ph} D^*_{p'h'} \\ \langle pp'|\bar{v}|hh'\rangle &= \lambda D_{ph} D_{p'h'} \end{aligned}$$

Note: a separable interaction

$$H\psi = E\psi; \qquad \psi = \sum_i C_i \phi_i$$

$$\longrightarrow \quad \sum_j H_{ij} C_j = E C_i; \qquad H_{ij} = \langle \phi_i | H | \phi_j \rangle$$

suppose $H_{ij} = \epsilon_i \delta_{i,j} + \lambda f_i^* f_j$ (separable form)

$$(\epsilon_i - E) C_i + \lambda f_i^* \underbrace{\sum_j f_j C_j}_{\equiv T} = 0$$

$$C_i = -\lambda \frac{T f_i^*}{\epsilon_i - E}$$

$$T = -\lambda \sum_j \frac{|f_j|^2}{\epsilon_j - E} T \quad \longrightarrow \quad \boxed{\frac{1}{\lambda} = \sum_i \frac{|f_i|^2}{E - \epsilon_i}}$$

RPA on a schematic model

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} X \\ Y \end{pmatrix}$$

$$\begin{aligned} A_{ph,p'h'} &= (\epsilon_p - \epsilon_h)\delta_{ph,p'h'} + \langle ph'|\bar{v}|hp'\rangle \\ B_{ph,p'h'} &= \langle pp'|\bar{v}|hh'\rangle \end{aligned}$$

Separable interaction:

$$\begin{aligned} \langle ph'|\bar{v}|hp'\rangle &= \lambda D_{ph} D^*_{p'h'} \\ \langle pp'|\bar{v}|hh'\rangle &= \lambda D_{ph} D_{p'h'} \end{aligned}$$

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

Cf. TDA dispersion relation: $$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}}$$

RPA on a schematic model

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$$
$$\langle pp'|\bar{v}|hh'\rangle = \lambda D_{ph} D_{p'h'}$$

Separable interaction:

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

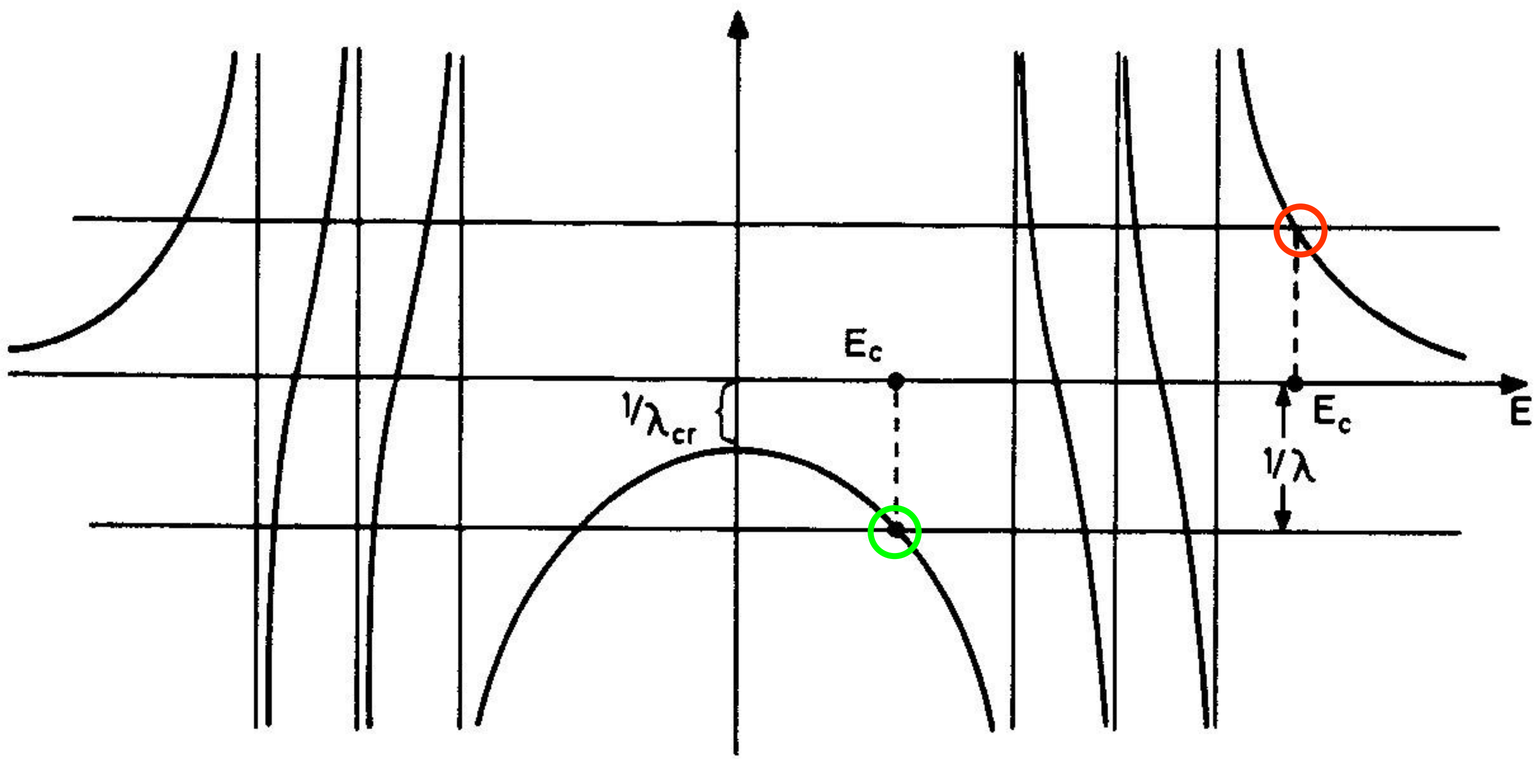


Figure 8.11. Graphical solution of the dispersion relation (8.135).

RPA on a schematic model

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'}$$
$$\langle pp'|\bar{v}|hh'\rangle = \lambda D_{ph} D_{p'h'}$$

Separable interaction:

$$\frac{1}{\lambda} = \sum_{ph} \frac{|D_{ph}|^2}{E - \epsilon_{ph}} - \frac{|D_{ph}|^2}{E + \epsilon_{ph}}$$

(RPA dispersion relation)

$\epsilon_{ph} = \epsilon$ のとき、

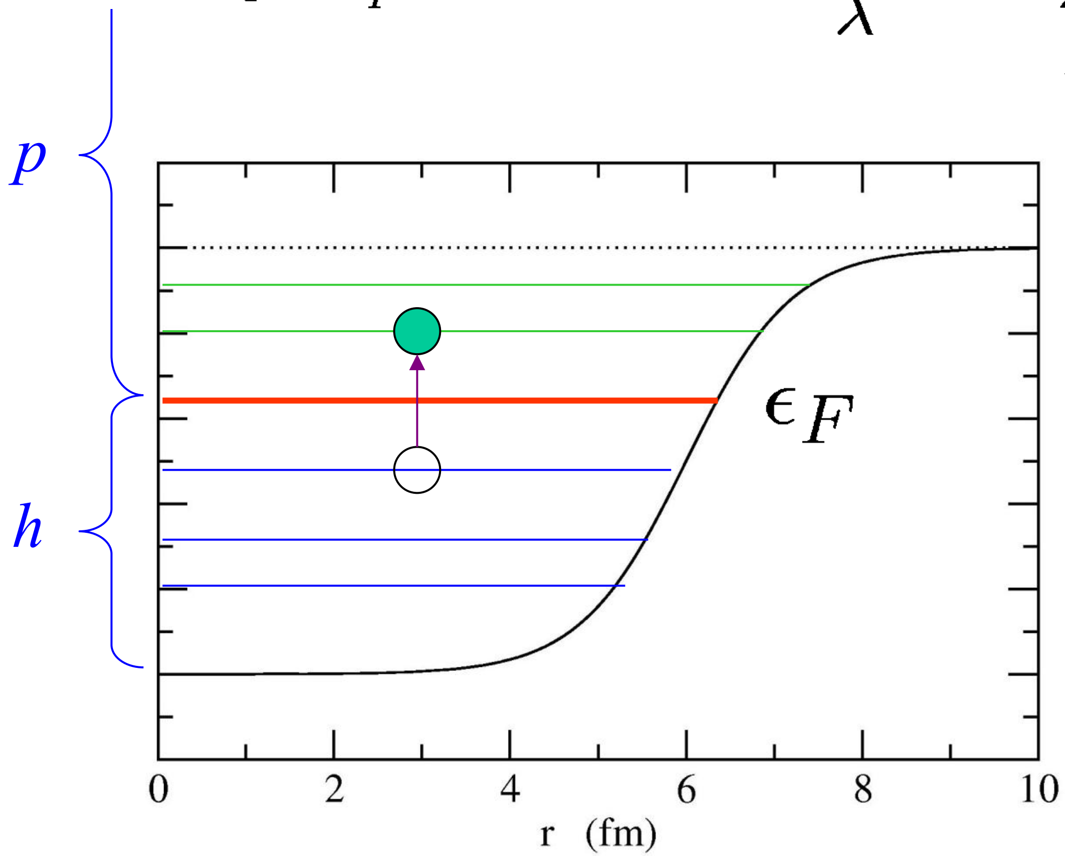
$$\frac{1}{\lambda} = \sum_{ph} |D_{ph}|^2 \left(\frac{1}{E - \epsilon} - \frac{1}{E + \epsilon} \right) = \sum_{ph} |D_{ph}|^2 \frac{2\epsilon}{E^2 - \epsilon^2}$$

$$\longrightarrow \quad E^2 = \epsilon^2 + 2\epsilon\lambda \sum_{ph} |D_{ph}|^2$$

λ が負(引力)だと、どこかで $E^2 < 0$ となる

Continuum Excitations

$$\langle ph'|\bar{v}|hp'\rangle = \lambda D_{ph} D^*_{p'h'} \quad \Longrightarrow \quad \frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E}$$



h: all the occupied (bound) states

p: the bound excited states + continuum states

$$\frac{1}{\lambda} = -\sum_{ph} \frac{|D_{ph}|^2}{\epsilon_p - \epsilon_h - E} = -\sum_{ph} \langle \phi_h | D^\dagger | \phi_p \rangle \frac{1}{\epsilon_p - \epsilon_h - E} \langle \phi_p | D | \phi_h \rangle$$

(note) $\hat{h}\phi_p = \epsilon_p \phi_p$

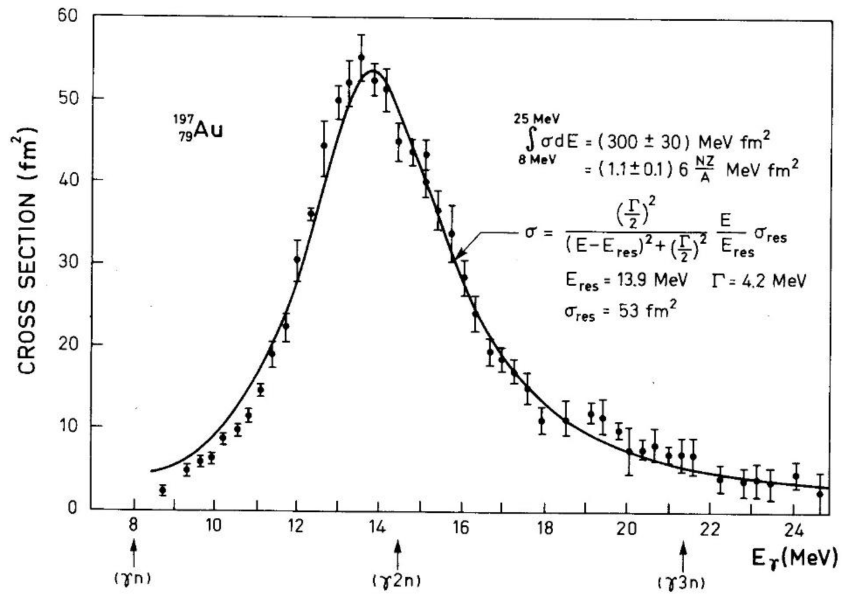
$$\frac{1}{\lambda} = -\sum_{ph} \langle \phi_h | D^\dagger \frac{1}{\hat{h} - \epsilon_h - E} | \phi_p \rangle \langle \phi_p | D | \phi_h \rangle$$

$$1 = \sum_i |\phi_i\rangle\langle\phi_i| = \sum_p |\phi_p\rangle\langle\phi_p| + \sum_h |\phi_h\rangle\langle\phi_h|$$

$$\frac{1}{\lambda} = -\sum_h \langle \phi_h | D^\dagger \frac{1}{\hat{h} - \epsilon_h - E} \left[1 - \sum_{h'} |\phi_{h'}\rangle\langle\phi_{h'}| \right] D | \phi_h \rangle$$

particle 状態の和がなくなった→連続状態もすべて自動的に入る

巨大共鳴の幅



i) 連続状態との結合（粒子放出）

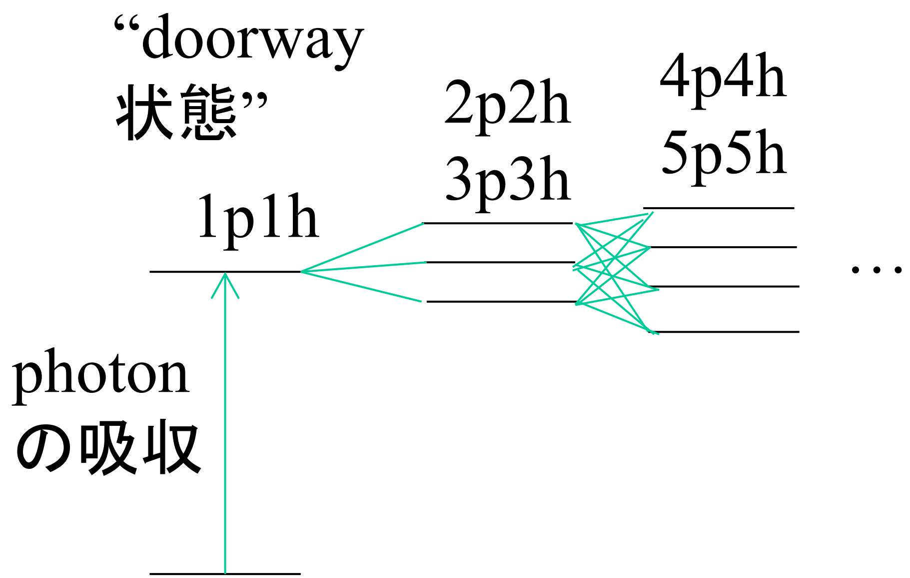
escape width $\Gamma^{\uparrow}$

continuum RPA

ii) より複雑な状態との結合

spreading width $\Gamma^{\downarrow}$

軽い核を除き
幅の主成分



1体演算子

レポート問題5（〆切：1月27日（火）23:55）

RPA型の連立固有値方程式：

$$\begin{cases} \sum_j (A_{ij} X_j + B_{ij} Y_j) = E X_i \\ \sum_j (B^*_{ij} X_j + A^*_{ij} Y_j) = -E Y_i \end{cases}$$

において、行列 A, B がそれぞれ

$$\begin{cases} A_{ij} = \epsilon_i \delta_{i,j} + \lambda D_i D^*_j \\ B_{ij} = \lambda D_i D_j \end{cases}$$

で与えられるとき、RPA dispersion relation

$$\frac{1}{\lambda} = \sum_i \frac{|D_i|^2}{E - \epsilon_i} - \frac{|D_i|^2}{E + \epsilon_i}$$

を導け。ただし、λ, ε_i は実数とする。

レポート問題6（〆切：1月27日（火）23:55）

RPA のA 行列、B行列が

$$A = \begin{pmatrix} \epsilon + g & g \\ g & \epsilon + g \end{pmatrix}, \qquad B = \begin{pmatrix} g & g \\ g & g \end{pmatrix}$$

で与えられているときに RPA方程式を

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ Y_1 \\ Y_2 \end{pmatrix} = E \begin{pmatrix} X_1 \\ X_2 \\ Y_1 \\ Y_2 \end{pmatrix}$$

解き、（正の）固有値2つを求めよ。ただし、ε は正とする。

g が負のとき、その大きさをゼロから大きくしていくと、ここで求めた2つの解のどちらかはどこかで 0 になる。そのときの g を求めよ。
g がさらに小さくなると解は複素数になるが、それは物理的にどういう状況に対応するか考察せよ。

Generator Coordinate Method （生成座標法）

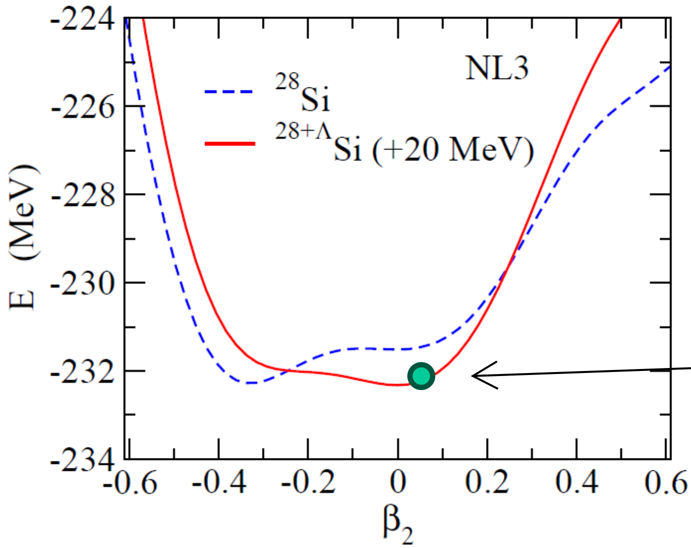
拘束条件付き Hartree-Fock 法

$$\hat{H} \to \hat{H}' = \hat{H} + \lambda(\hat{Q}_{20} - Q_{20})^2; \quad \hat{Q}_{20} \propto \sum_i r_i^2 Y_{20}(\hat{\boldsymbol{r}}_i)$$

→ Hartree-Fock 近似で最適化： $|\Psi_{gs}(Q_{20})\rangle$

$$\langle \Psi_{gs}(Q_{20})|\hat{Q}_{20}|\Psi_{gs}(Q_{20})\rangle \sim Q_{20}$$

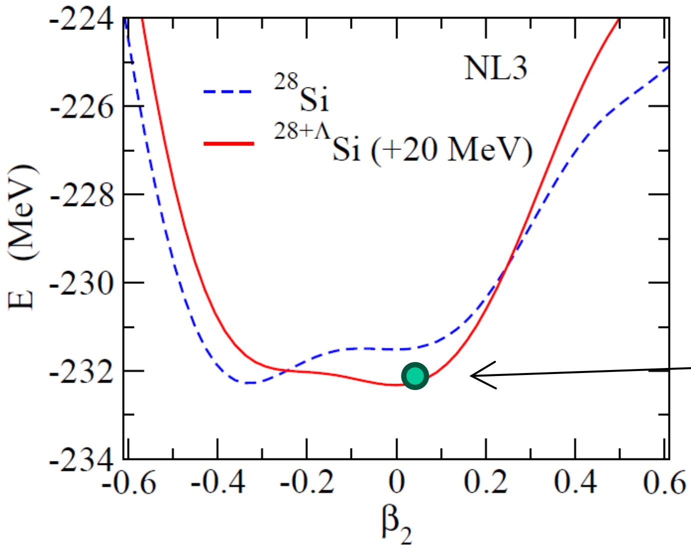
エネルギー曲線： $E(Q_{20}) = \langle \Psi_{gs}(Q_{20})|\hat{H}|\Psi_{gs}(Q_{20})\rangle$



$Q_{20} \propto R^2 \beta_2$

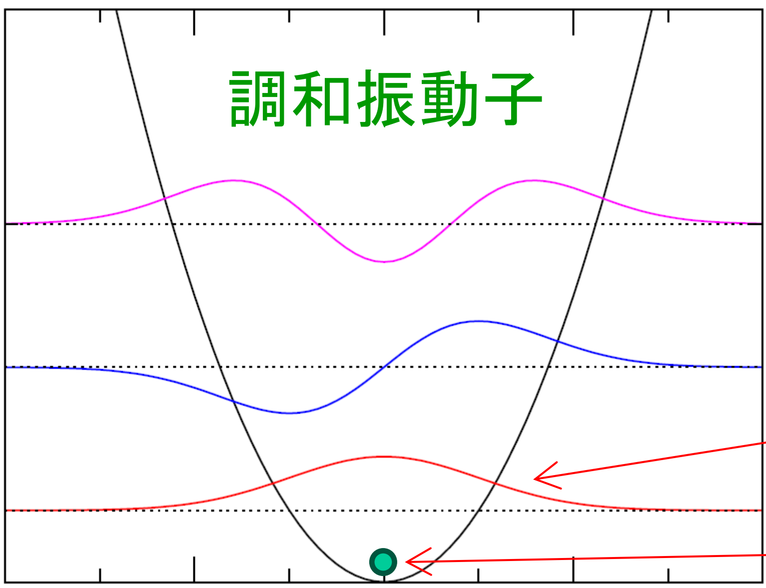
absolute minimum
（拘束をかけないとここが求まる）

Generator Coordinate Method （生成座標法）



$$Q_{20} \propto R^2 \beta_2$$

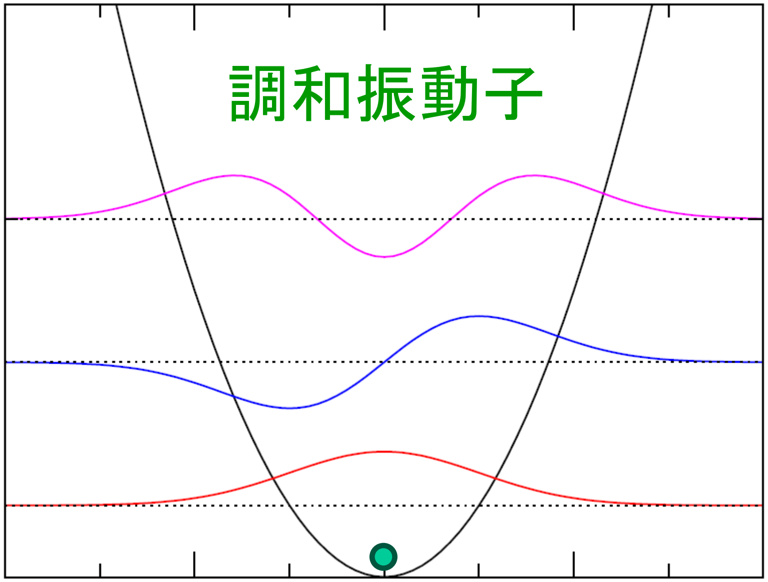
absolute minimum
（拘束をかけないとここが求まる）



量子力学（ゼロ点振動）

古典力学

Generator Coordinate Method （生成座標法）

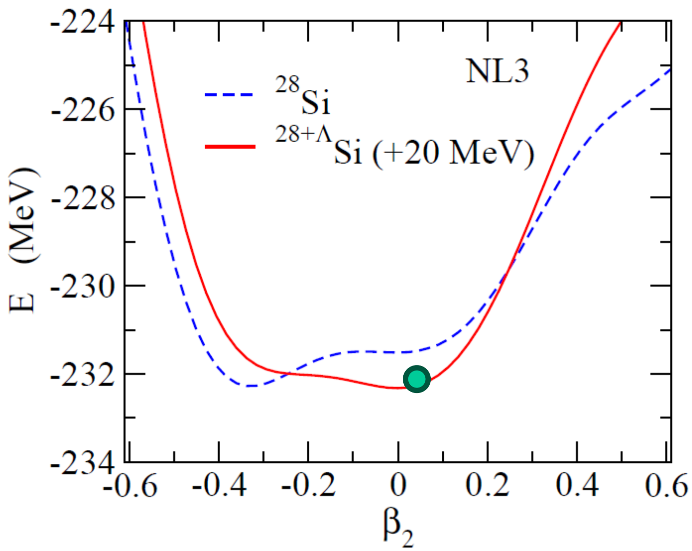


古典力学的基底状態:

$$|\Psi\rangle = |r=0\rangle$$

量子力学的基底状態:

$$|\Psi\rangle = |\phi_0\rangle = \int d\boldsymbol{r}\langle\boldsymbol{r}|\phi_0\rangle|\boldsymbol{r}\rangle$$



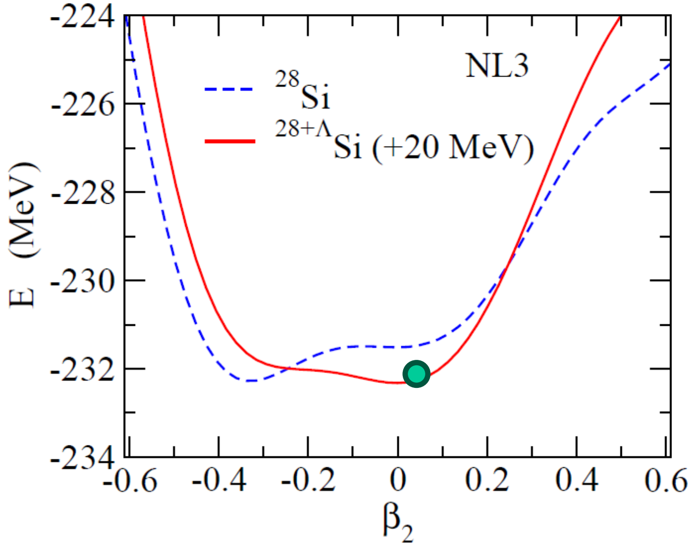
Hartree-Fock状態（“古典”）

$$|\Psi\rangle = |\Psi_{gs}(\beta_0)\rangle$$

量子ゆらぎ（生成座標法）

$$|\Psi\rangle = \int d\beta\, f(\beta)|\Psi_{gs}(\beta)\rangle$$

Generator Coordinate Method （生成座標法）



Hartree-Fock状態（"古典"）

$$|\Psi\rangle = |\Psi_{gs}(\beta_0)\rangle$$

量子ゆらぎ（生成座標法）

$$|\Psi\rangle = \int d\beta\, f(\beta)|\Psi_{gs}(\beta)\rangle$$

変分で決める

$$\frac{\delta}{\delta f^*}(\langle\Psi|\hat{H} - E|\Psi\rangle) = 0$$

$$\to \int d\beta' \langle\Psi_{gs}(\beta)|\hat{H}|\Psi_{gs}(\beta')\rangle\, f(\beta') = E\int d\beta' \langle\Psi_{gs}(\beta)|\Psi_{gs}(\beta')\rangle\, f(\beta')$$

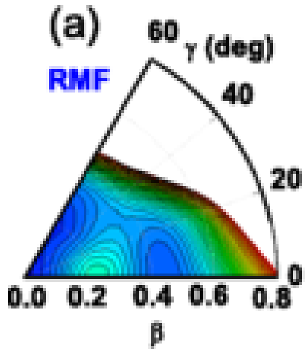
「Hill-Wheeler 方程式」

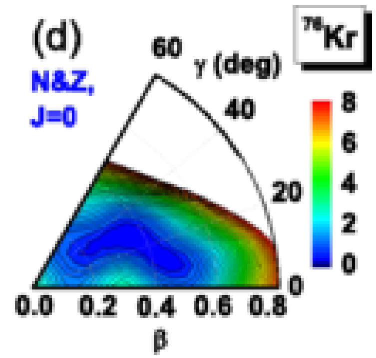
$$N(\beta,\beta') = \langle\Psi_{gs}(\beta)|\Psi_{gs}(\beta')\rangle$$

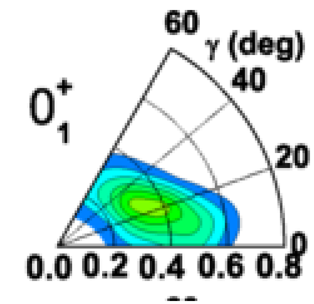
（非直交基底になっている）

Generator Coordinate Method （生成座標法）

^{76}Kr

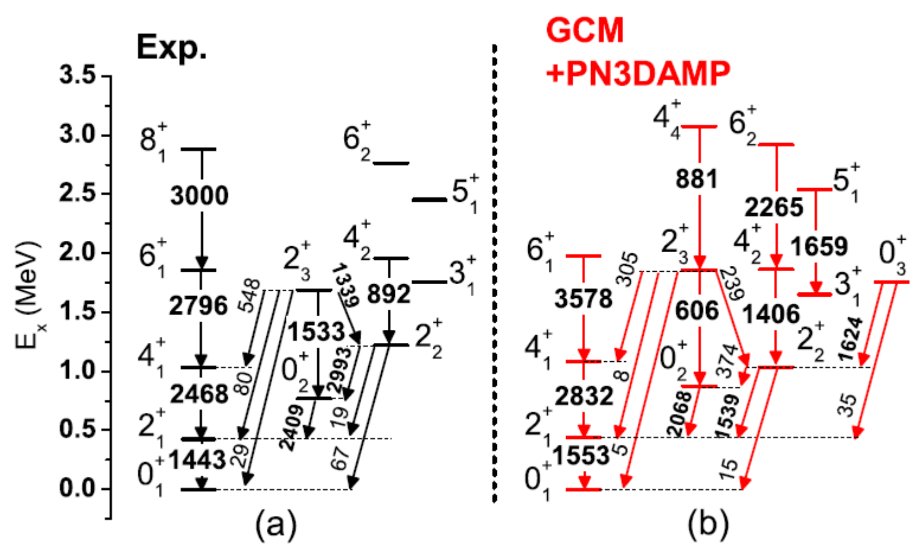






PES（平均場）　　PES（量子数射影）　　波動関数 (0$^+$)

スペクトル



J.M. Yao, K. Hagino, Z.P. Li, J. Meng, and P. Ring, PRC89 (‘14) 054306.