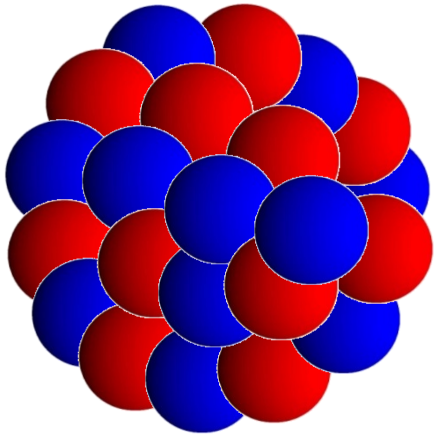
原子核反応論

- 原子核物理：核子多体系としての原子核の振る舞い

  ← 核子間相互作用から理解する

➢ 静的な振る舞い：原子核構造論

✓ 基底状態の性質（質量、大きさ、形など）
✓ 励起状態の性質

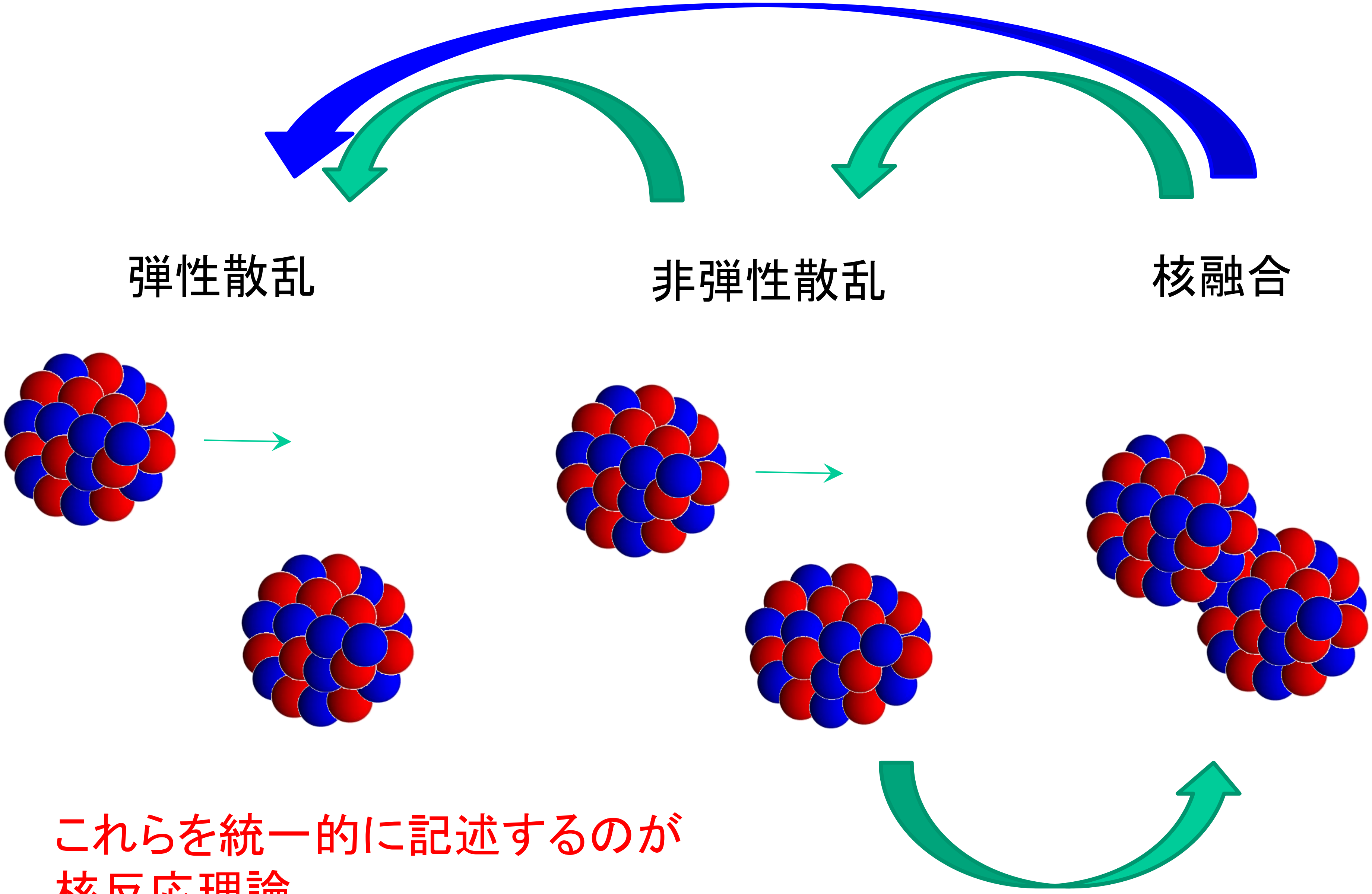


➢ ダイナミックス：原子核反応論

原子核は複合粒子
✓ 豊富な反応様式
✓ 核構造と核反応
の織り成す様々なインタープレイ

- 弾性散乱
- 非弾性散乱
- 核子移行反応
- 核融合反応

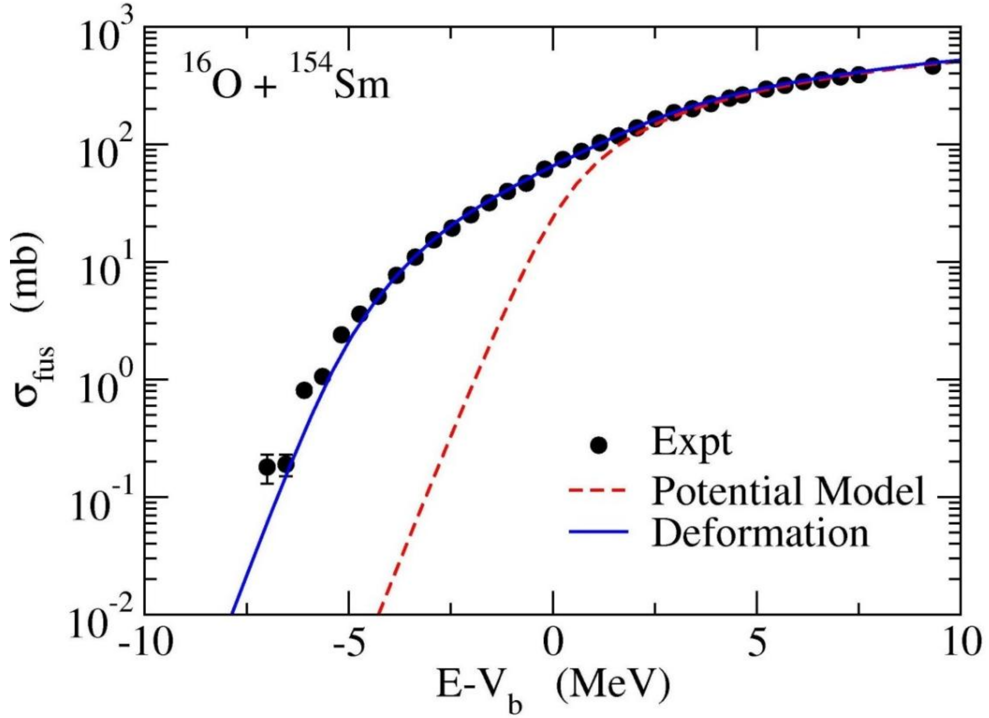
量子多体系のダイナミックス（原子核反応）

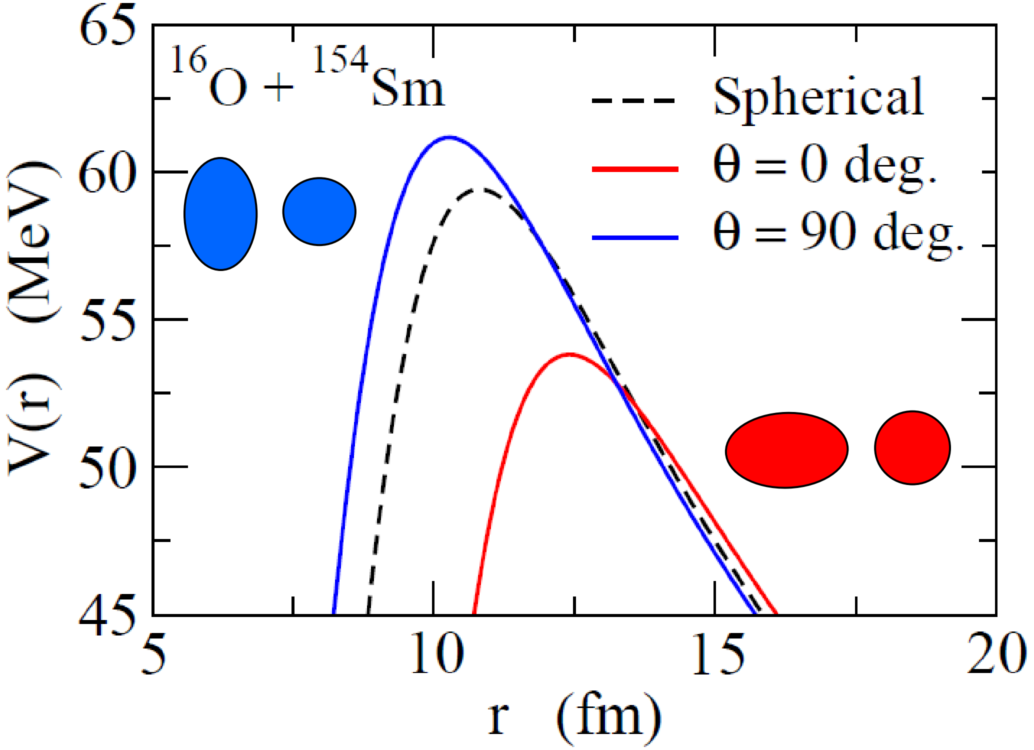


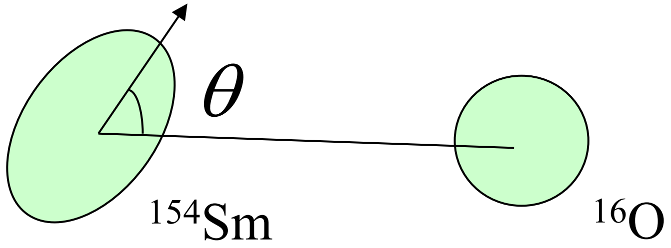
これらを統一的に記述するのが
核反応理論

核構造と核反応のインタープレイのいい例の一つ

重イオン核融合反応断面積の増大: 原子核の変形が大きな効果をもたらす







^{154}Smは変形→向きによってポテンシャルが変わる

$$\sigma_{\rm fus}(E) = \int_0^1 d(\cos\theta)\sigma_{\rm fus}(E;\theta)$$

K.Hagino and N. Takigawa, PTP128 (2012) 1061

Nuclear Reactions

Shape, interaction, and excitation structures of nuclei ⟵ scattering expt.

cf. Experiment by Rutherford (α scatt.)

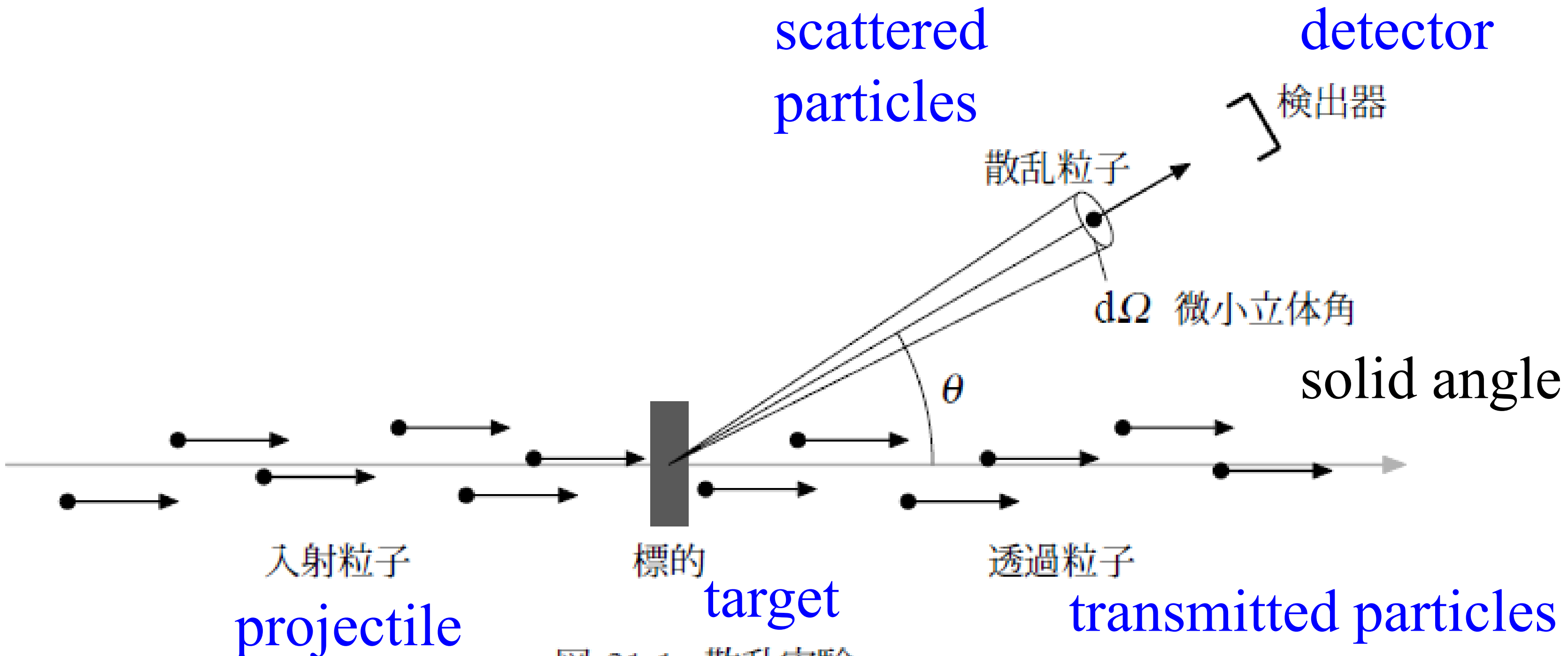


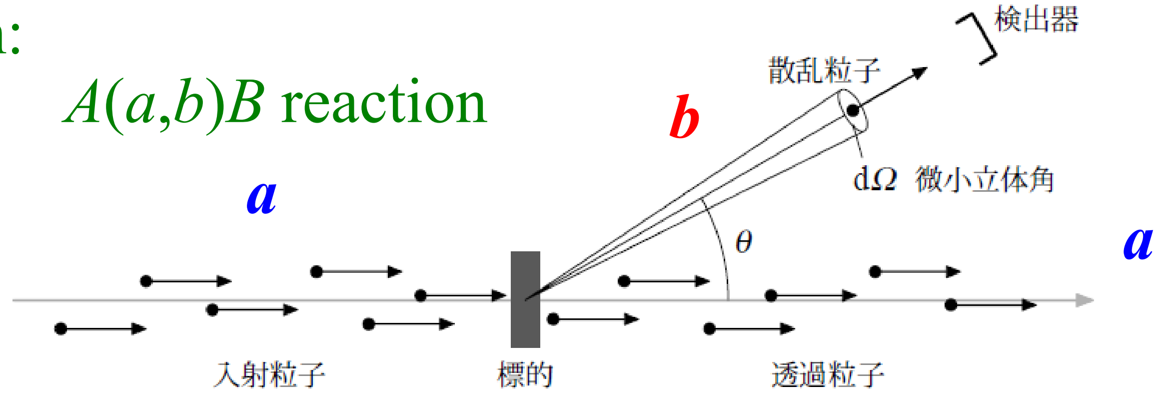
図 21.1: 散乱実験

http://www.th.phys.titech.ac.jp/~muto/lectures/QMII11/QMII11_chap21.pdf

K. Muto (TIT)

notation:

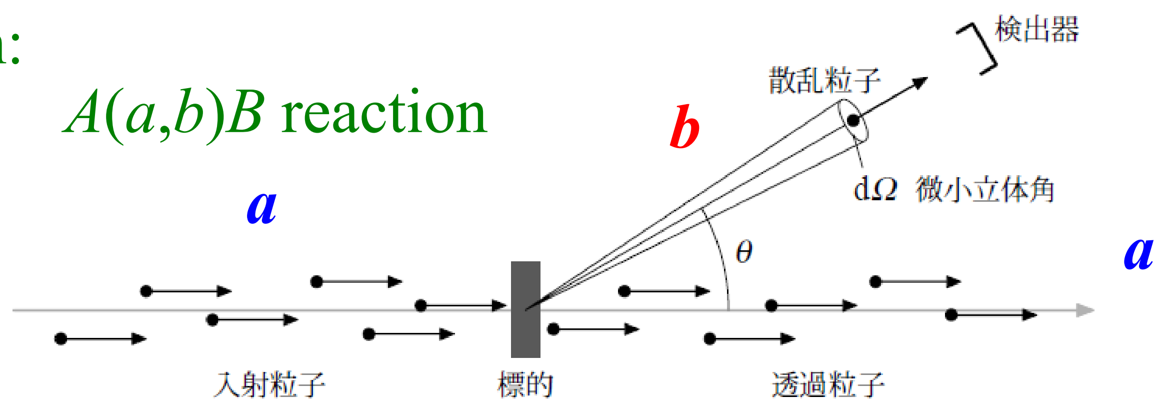
$A(a,b)B$ reaction



$A \rightarrow B$

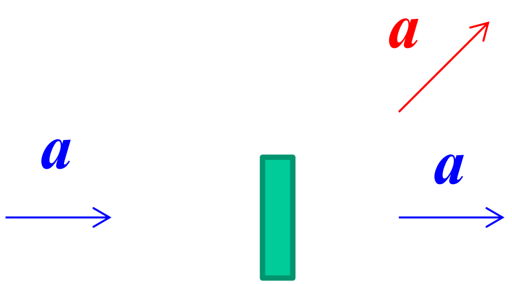
notation:

A(*a*,*b*)*B* reaction



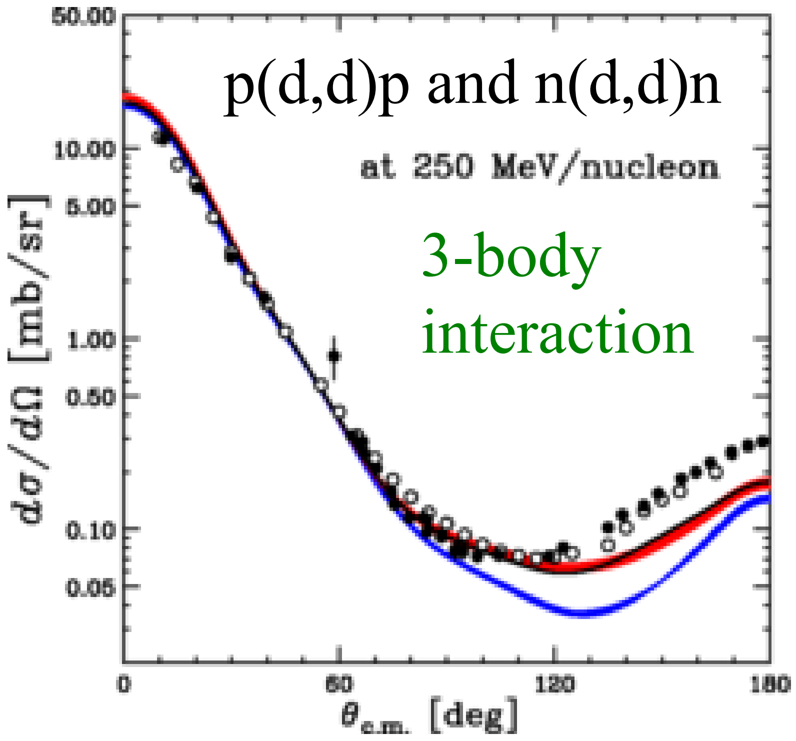
$A \rightarrow B$

✓elastic scattering



$A_{gs} \rightarrow A_{gs}$

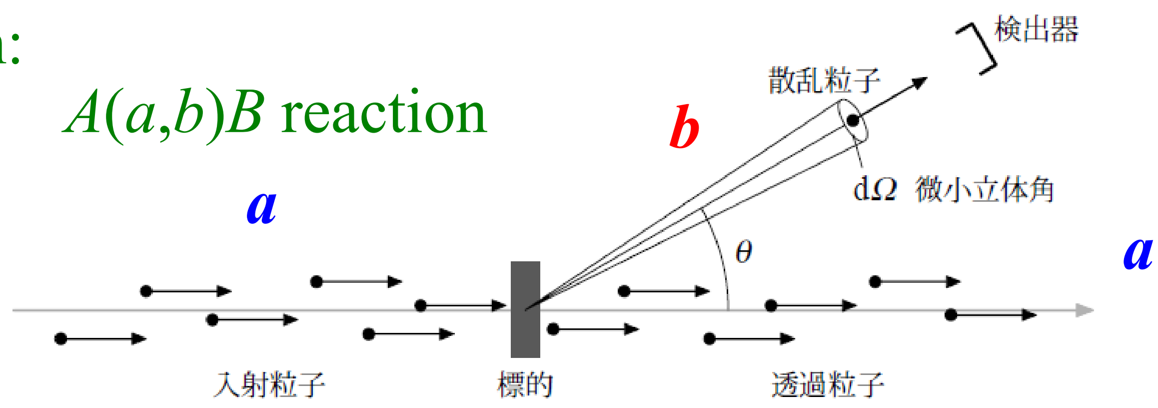
fundamental interaction between *a* and *A*



K. Sekiguchi et al., PRC89('14)064007

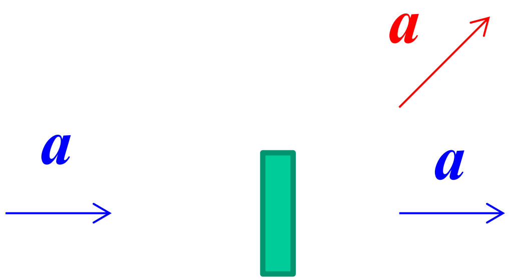
notation:

$A(a,b)B$ reaction



$A \rightarrow B$

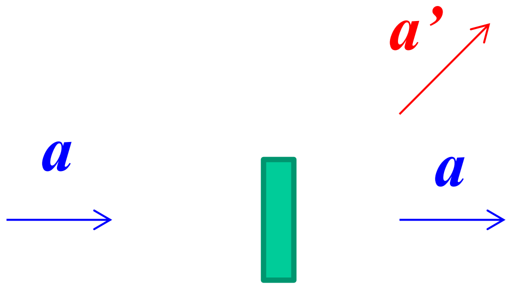
✓elastic scattering



$A_{gs} \rightarrow A_{gs}$

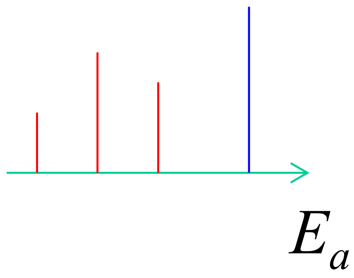
fundamental interaction between a and A

✓inelastic scattering



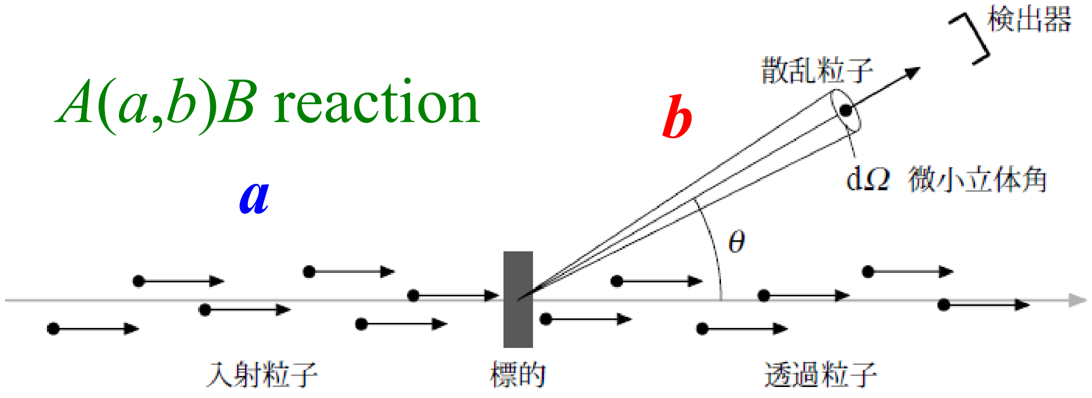
$A_{gs} \rightarrow A^*$

excitation spectrum of a nucleus A



励起状態の
- ✓ エネルギー
- ✓ 角運動量

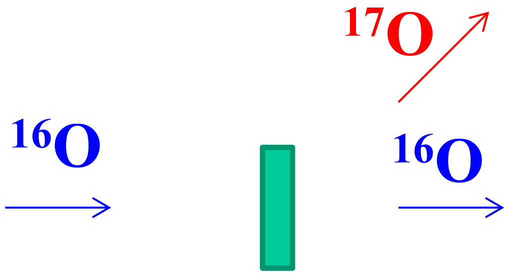
A(*a*,*b*)*B* reaction



$A \rightarrow B$

transfer reactions

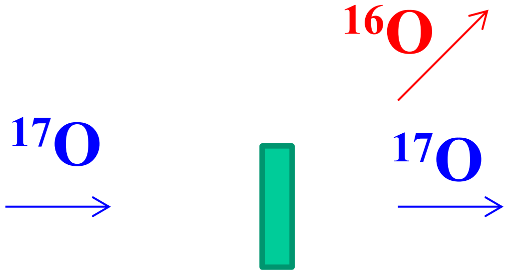
✓transfer reaction
(pick-up reaction)



$^{208}Pb_{gs} \rightarrow {}^{207}Pb$

level schem of ^{207}Pb

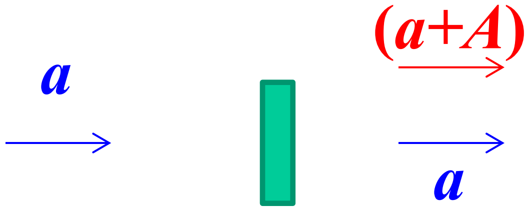
✓transfer reaction
(stripping reaction)



$^{208}Pb_{gs} \rightarrow {}^{209}Pb$

level schem of ^{209}Pb

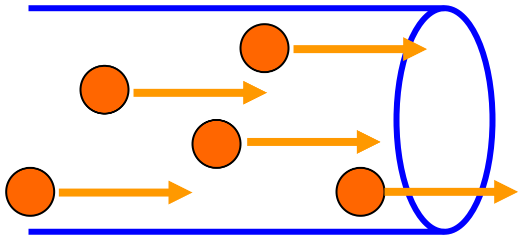
✓fusion reaction



$A_{gs} \rightarrow X$

- interaction between *a* and *A*
- structure of *a* and *A*
- 未知核の生成

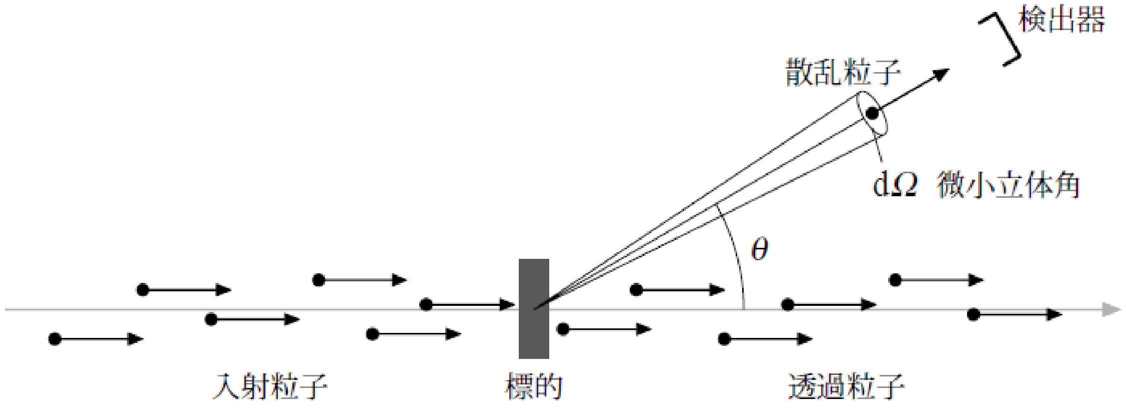
Cross sections (断面積)



incident beam

flux = the number of particles
crossing unit area per unit time

$$j = \rho_P \cdot v$$



event rate (the number of event per unit time
per target nucleus)
: proportional to the incident flux

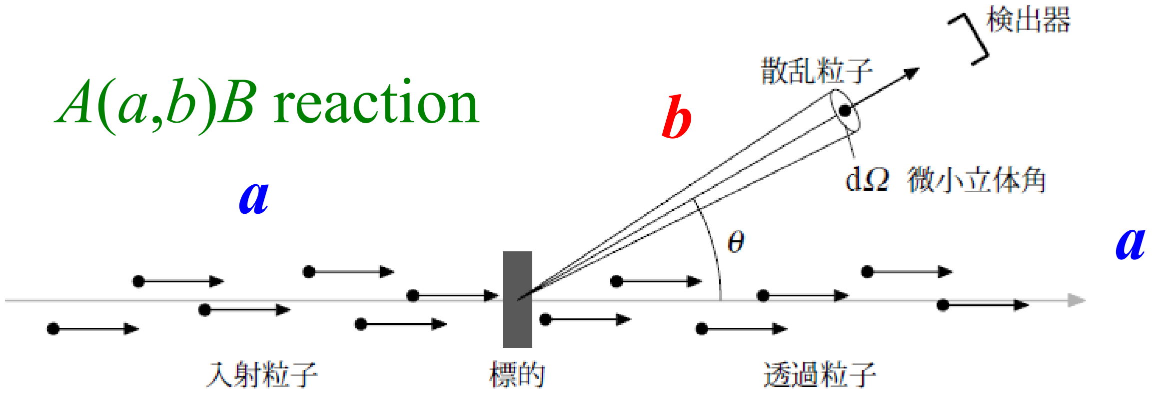
differential cross sections
(angular distribution)

$$dR(\theta, \phi) = N_T \cdot \frac{d\sigma}{d\Omega} \cdot j \cdot d\Omega$$

total cross section: $\sigma = \int d\Omega \frac{d\sigma}{d\Omega}$

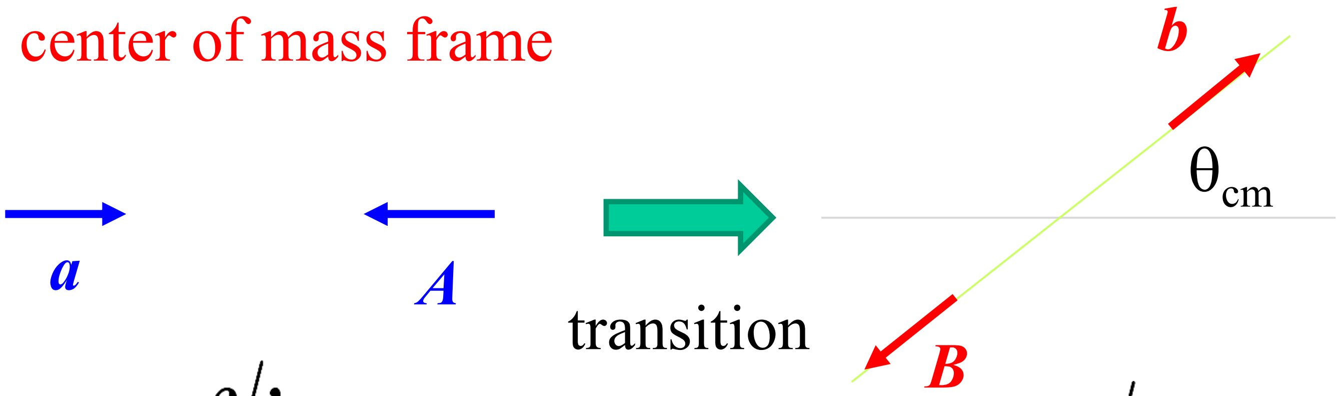
断面積（量子力学）

$A(a,b)B$ reaction



$A \rightarrow B$

center of mass frame



ψ_i ψ_f

$$\frac{d\sigma}{d\Omega} = \frac{R}{j_{\rm in}}$$

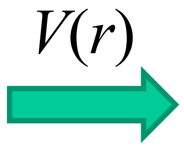
散乱振幅

$$\psi(\boldsymbol{r}) \rightarrow e^{i\boldsymbol{k}\cdot\boldsymbol{r}} + f(\theta)\,\frac{e^{ikr}}{r}$$

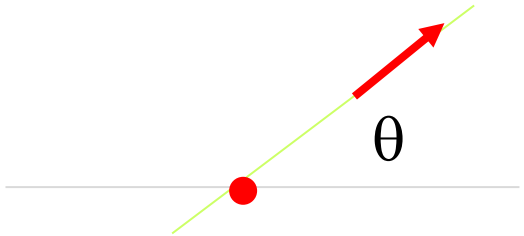
$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

Born approximation

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$



$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$



$$\left(-\frac{\hbar^2}{2\mu}\boldsymbol{\nabla}^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

transition rate for elastic scattering:

$$\begin{aligned} W_{fi} &= \frac{2\pi}{\hbar}\int\frac{d\boldsymbol{p}_f}{(2\pi\hbar)^3}|\langle\psi_f|V|\psi_i\rangle|^2\delta(E_f - E_i) \\ &= \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega\left|\tilde{V}(\boldsymbol{q})\right|^2 \end{aligned}$$

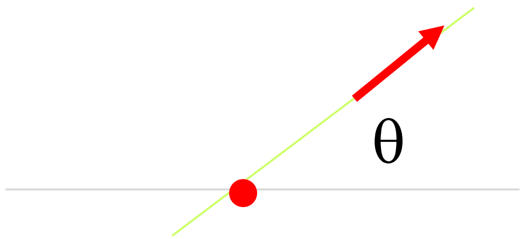
$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r} e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar}V(r) \equiv \int d\boldsymbol{r} e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(r)$$

Born approximation

$$\psi_f(\boldsymbol{r}) = e^{i\boldsymbol{p}_f\cdot\boldsymbol{r}/\hbar}$$

$$\psi_i(\boldsymbol{r}) = e^{i\boldsymbol{p}_i\cdot\boldsymbol{r}/\hbar}$$

$V(r)$



$$W_{fi} = \frac{\mu p_i}{4\pi^2\hbar^4}\int d\Omega \left|\tilde{V}(\boldsymbol{q})\right|^2$$

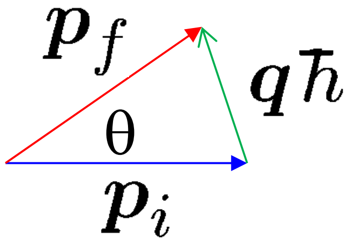
momentum transfer

$$\tilde{V}(\boldsymbol{q}) = \int d\boldsymbol{r}e^{i(\boldsymbol{p}_i-\boldsymbol{p}_f)\cdot\boldsymbol{r}/\hbar}V(r) \equiv \int d\boldsymbol{r}e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}V(r)$$

incident flux: $j_{\text{inc}} = \rho_i v = p_i/\mu$

$$\sigma = \frac{W_{fi}}{j_{\text{inc}}} = \int d\Omega \boxed{\frac{\mu^2}{4\pi^2\hbar^4}\left|\tilde{V}(\boldsymbol{q})\right|^2}$$

$$= \frac{d\sigma}{d\Omega}$$



$$q\hbar = 2p_i \sin\frac{\theta}{2}$$

Electron scattering

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho_{\text{ch}}(\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

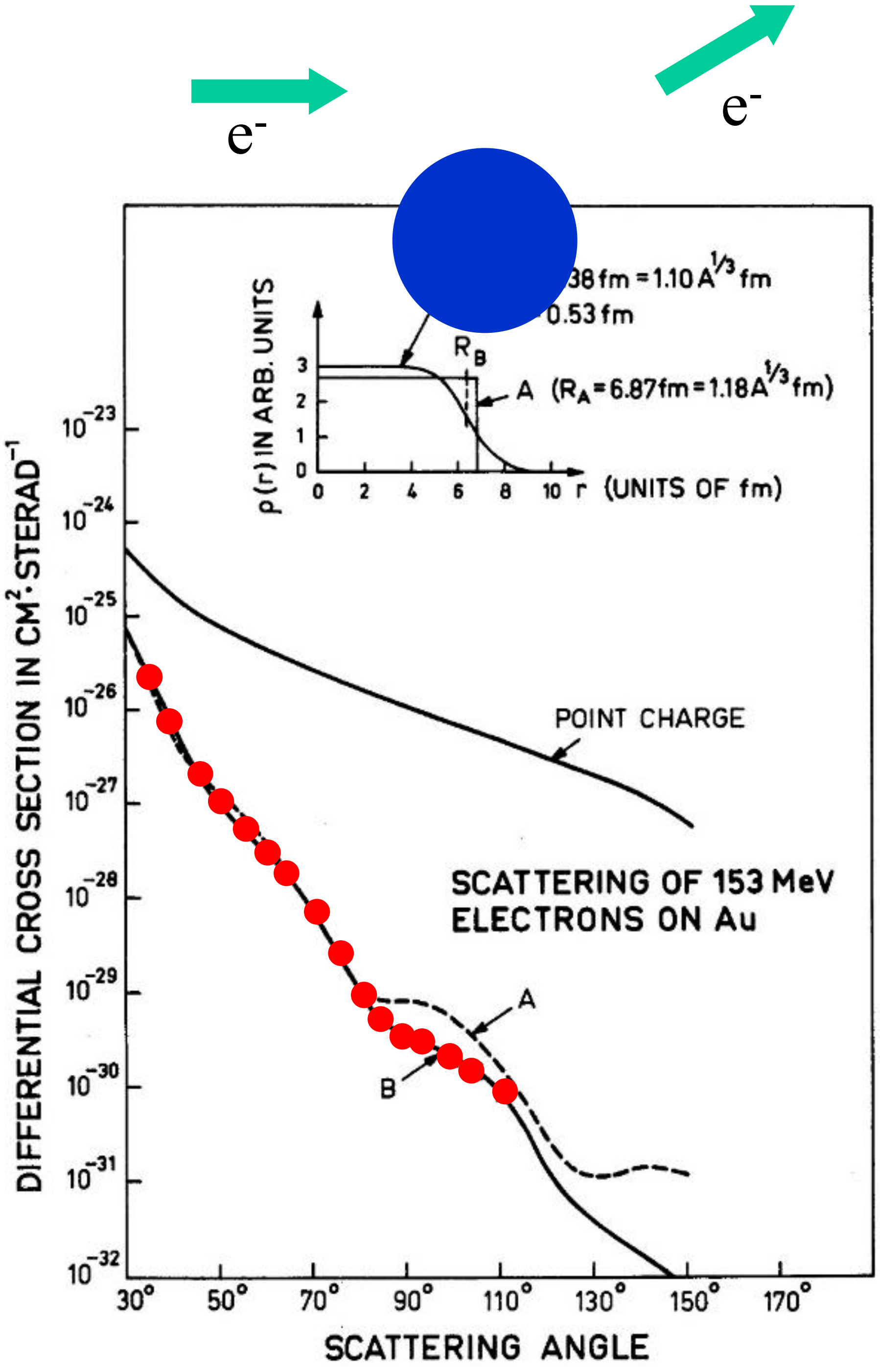
$$\frac{d\sigma}{d\Omega} = \frac{e^4}{(4E\sin^2\theta/2)^2}|F(\boldsymbol{q})|^2 = \left(\frac{d\sigma_{\text{Ruth}}}{d\Omega}\right)|F(\boldsymbol{q})|^2$$

Form factor

$$F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\,\rho_{\text{ch}}(\boldsymbol{r})\,d\boldsymbol{r}$$

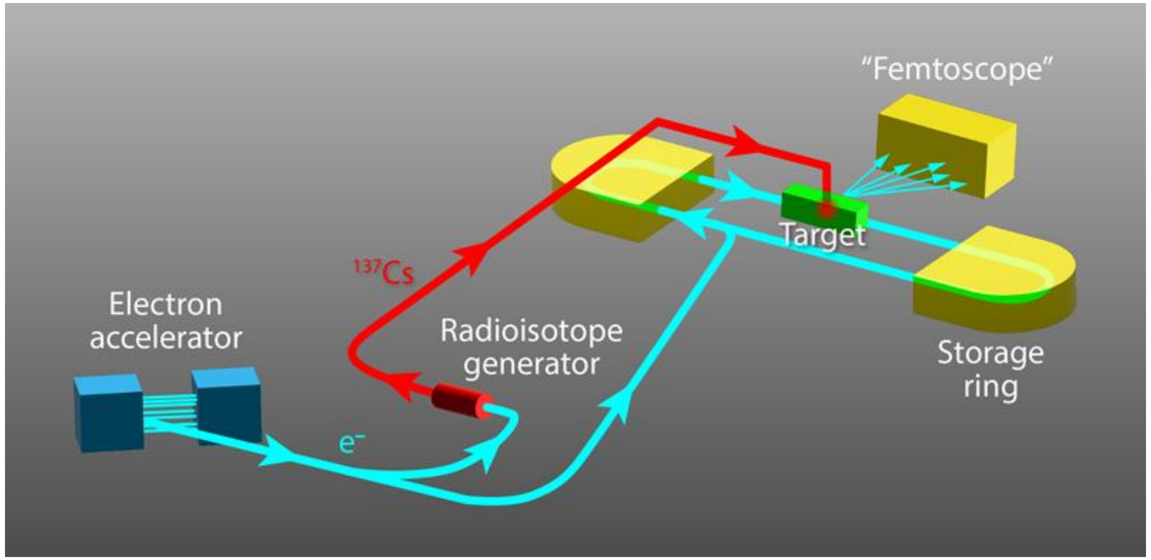
* relativistic correction:

$$\frac{d\sigma_{\text{Ruth}}}{d\Omega} \to \frac{d\sigma_{\text{Mott}}}{d\Omega} = \frac{d\sigma_{\text{Ruth}}}{d\Omega}\cdot\left(1-\frac{v^2}{c^2}\sin^2\frac{\theta}{2}\right) \sim \frac{d\sigma_{\text{Ruth}}}{d\Omega}\cdot\cos^2\frac{\theta}{2} \qquad (v\to c)$$

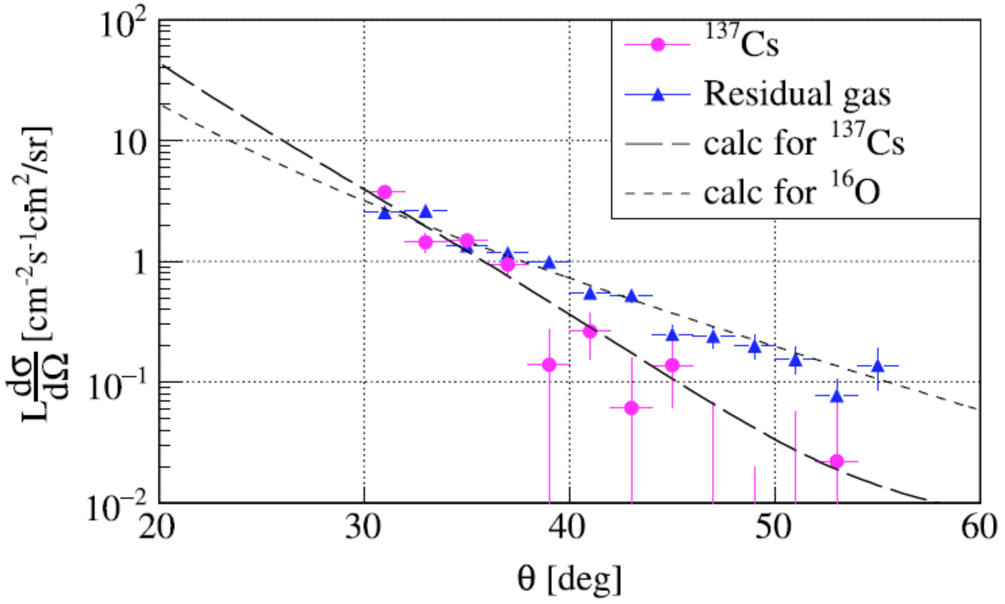


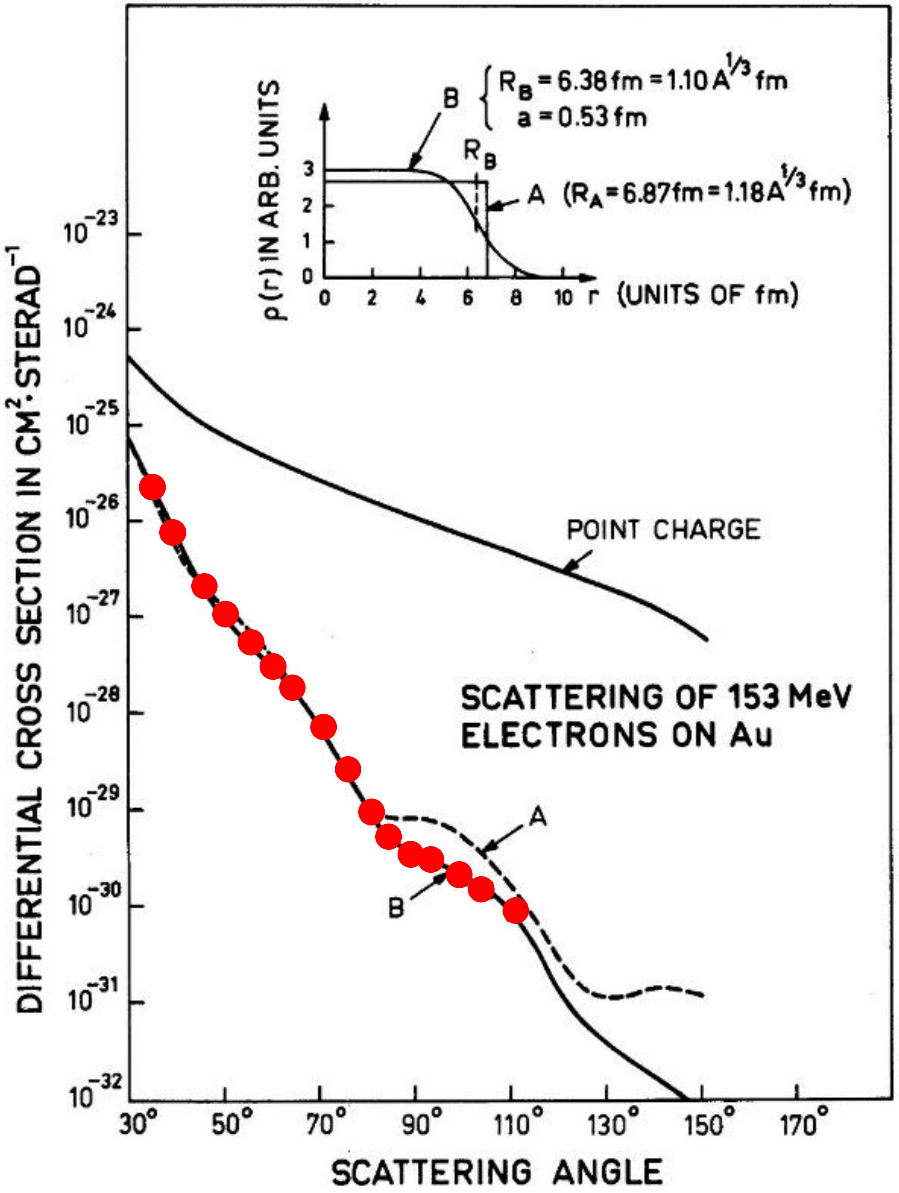
cf. 不安定原子核に対する電子散乱実験 (SCRIT)

安定核の電子散乱



^{137}Cs ($T_{1/2}$=30.08 y)





K. Tsukada et al., PRL131, 092502 (2023)

レポート問題7（〆切：1月27日（土））

電子と原子核の相互作用が

$$V(r) = -e^2 \int d\boldsymbol{r}' \frac{\rho_{\mathrm{ch}}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

で与えられているとする。ここで、ρ_{ch} は原子核の電荷密度で $\int d\boldsymbol{r}\, \rho_{\mathrm{ch}}(\boldsymbol{r}) = Z$

と規格化されているとする。ボルン近似を用いて弾性散乱の断面積を求めると、

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4E \sin^2\theta/2}\right)^2 |F(\boldsymbol{q})|^2 \qquad F(\boldsymbol{q}) = \int e^{-i\boldsymbol{q}\cdot\boldsymbol{r}}\, \rho_{\mathrm{ch}}(\boldsymbol{r})\, d\boldsymbol{r}$$

となる。

qが小さいところで電子散乱の形状因子 $F(q)$ を決めることにより、原子核の荷電半径

$$\langle r^2 \rangle = \frac{1}{Z} \int d\boldsymbol{r}\, r^2 \rho_{\mathrm{ch}}(\boldsymbol{r})$$

を求められることを示せ。

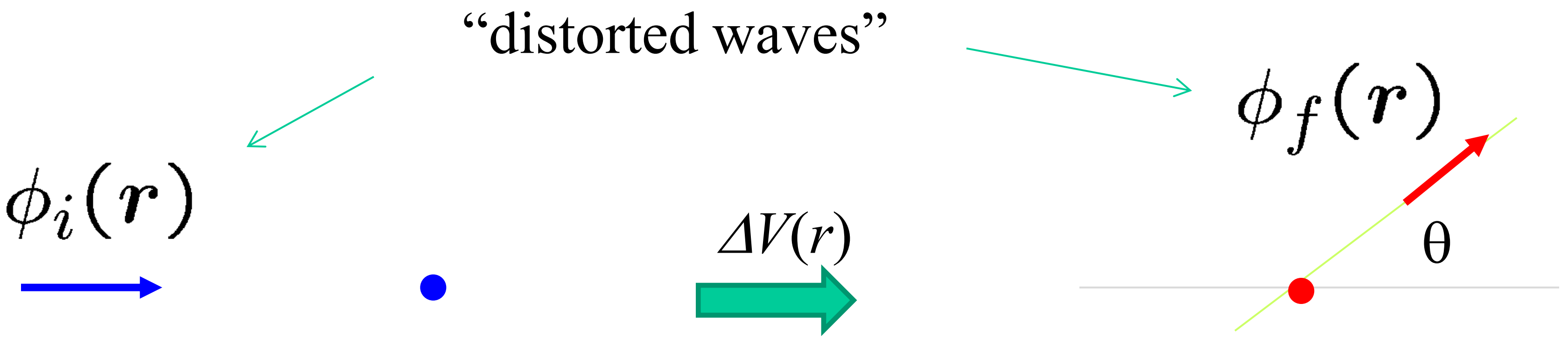
Distorted Wave Born approximation (DWBA)

$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underline{V(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation

$$\Rightarrow \left(-\frac{\hbar^2}{2\mu}\nabla^2 + \underline{V_0(r)} + \underline{V(r) - V_0(r)} - E\right)\psi(\boldsymbol{r}) = 0$$

perturbation



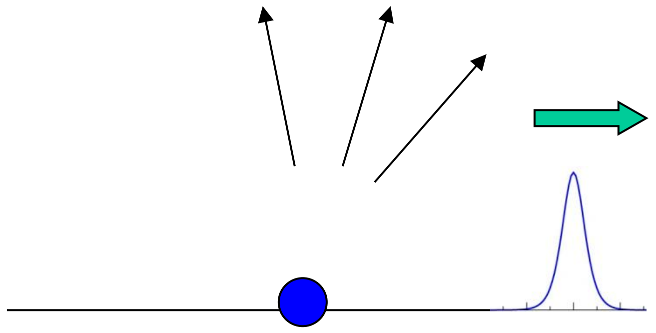
✓inelastic scattering

✓transfer reactions

How to choose $V_0(r)$? : Optical model

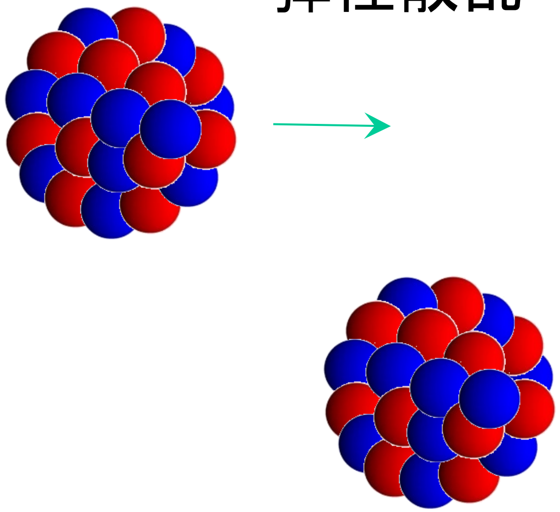
Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)

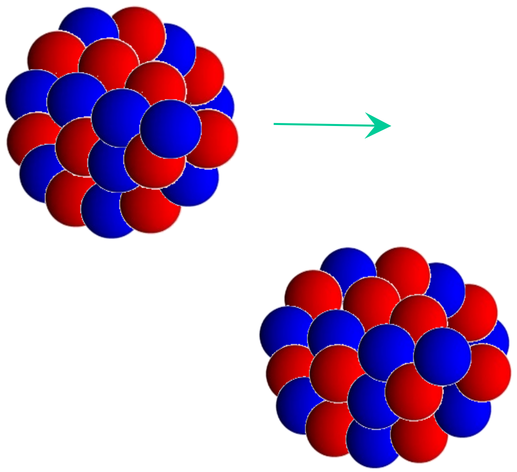


Loss of incident flux (absorption)

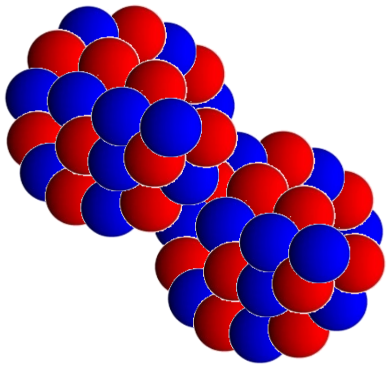
弾性散乱



非弾性散乱



核融合



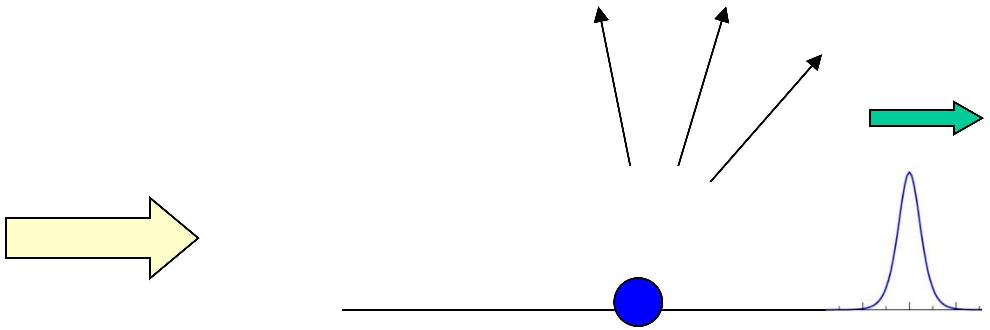
→ 光学ポテンシャル

$$V_{\rm opt}(r) = V(r) - iW(r) \qquad (W > 0)$$

How to choose $V_0(r)$? : Optical model

Reaction processes

- Elastic scatt.
- Inelastic scatt.
- Transfer reaction
- Compound nucleus formation (fusion)



Loss of incident flux (absorption)

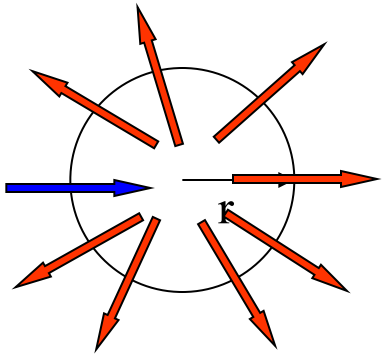
Optical potential

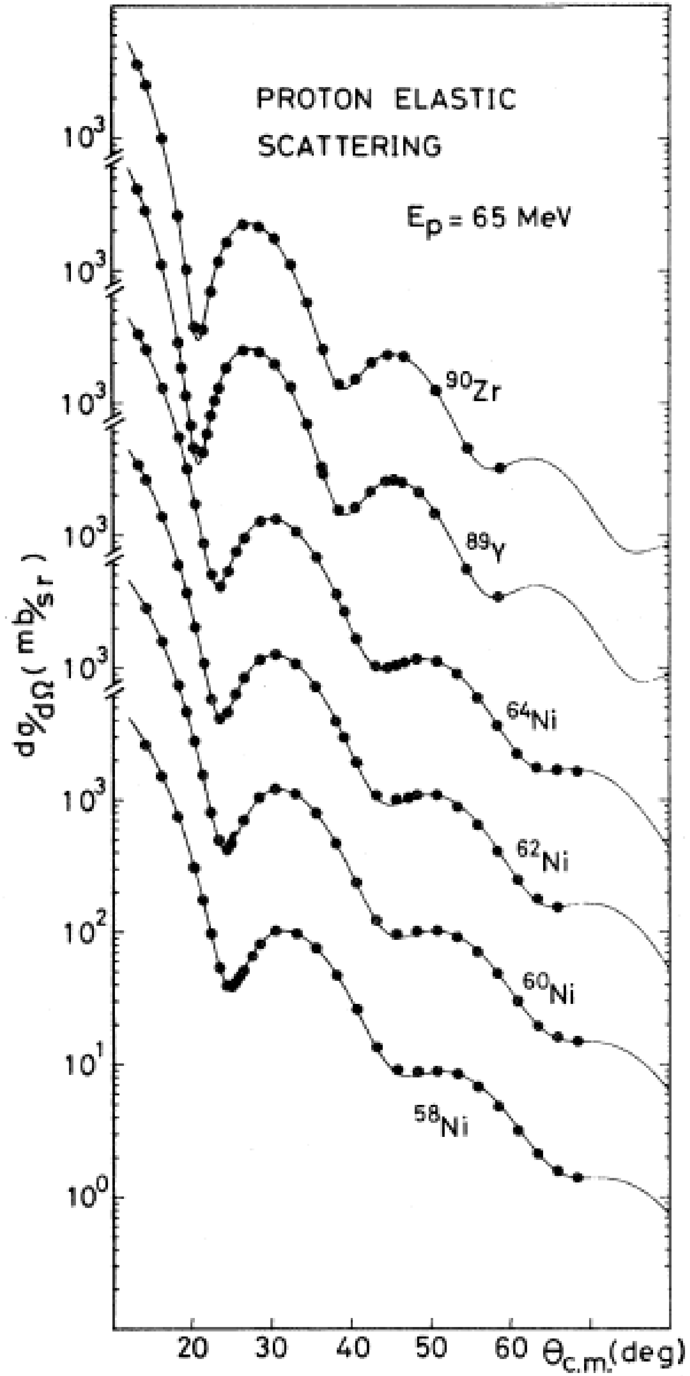
$$V_{\rm opt}(\boldsymbol{r}) = V(\boldsymbol{r}) - iW(\boldsymbol{r}) \qquad (W > 0)$$

$$\Longrightarrow \quad \boldsymbol{\nabla}\cdot\boldsymbol{j} = \cdots = -\frac{2}{\hbar}W|\psi|^2$$

(note) Gauss's theorem

$$\int_S \boldsymbol{j}\cdot\boldsymbol{n}\,dS = \int_V \boldsymbol{\nabla}\cdot\boldsymbol{j}\,dV$$





$$\left(-\frac{\hbar^2}{2\mu}\nabla^2 + \frac{Z_P Z_T e^2}{r} + V_{\text{opt}}(r) - E\right)\psi(r) = 0$$

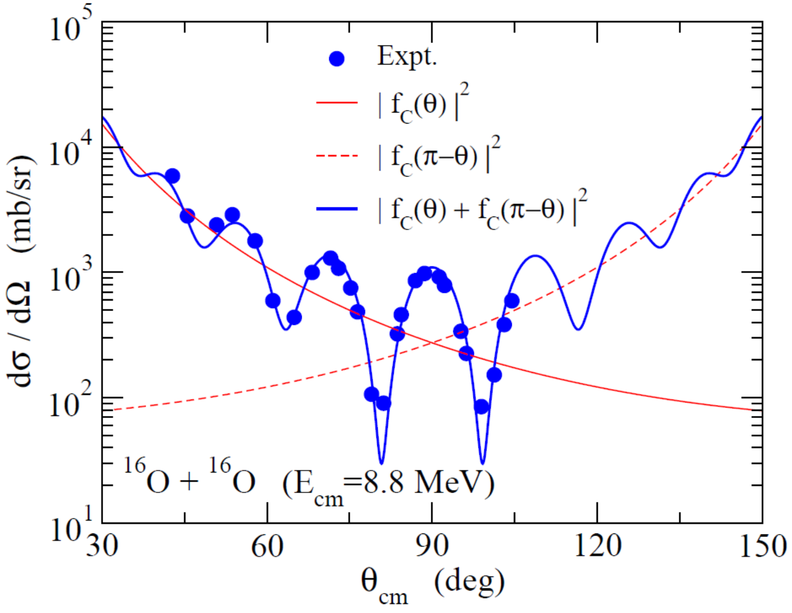
Woods-Saxon + volume &surface imaginary parts

H. Sakaguchi et al., PRC26 (1982) 944

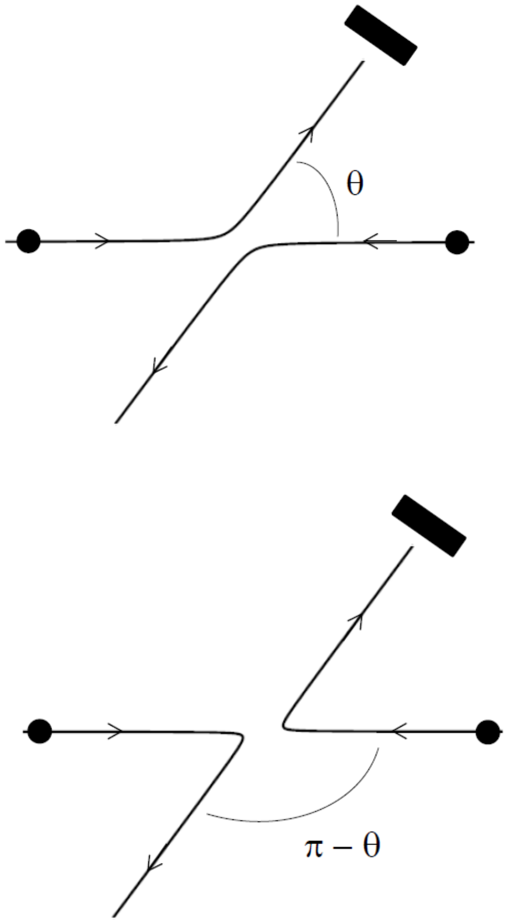
imaging quantum interference phenomena

quantum interference phenomena in heavy-ion reactions

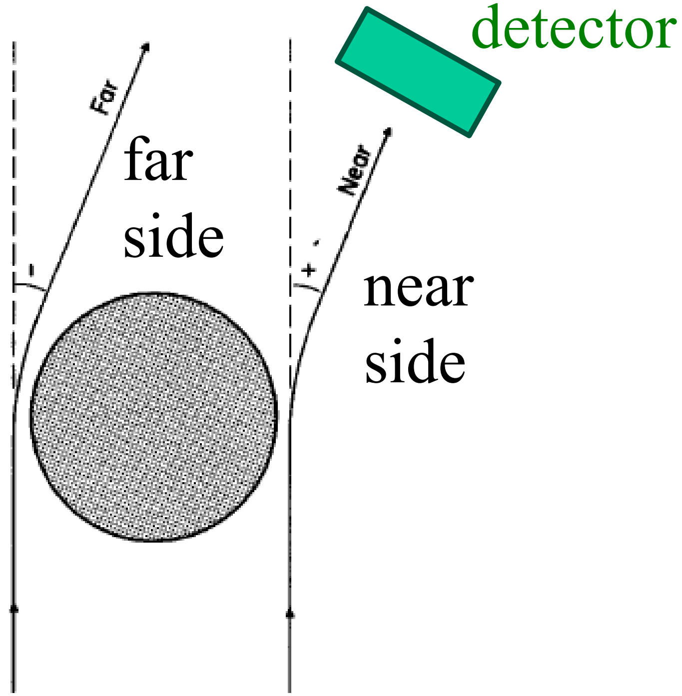
✓ Mott Scattering



expt: D.A. Bromley et al., Phys. Rev. 123 ('61)878



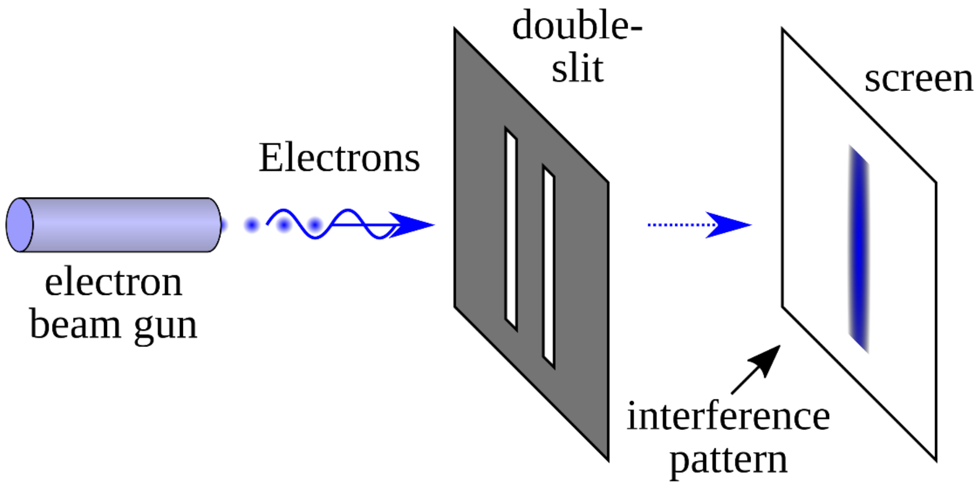
✓ near side-far side interference

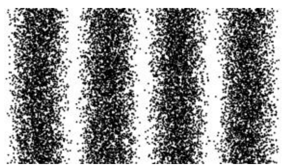


✓ nuclear-Coulomb interference
✓ barrier wave-internal wave interference

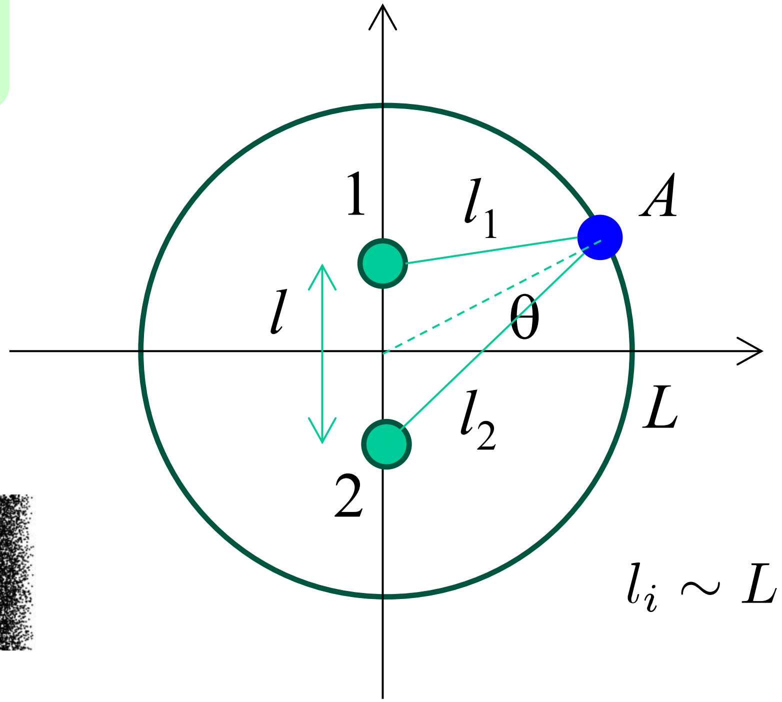
imaging quantum interference phenomena

a double slit problem





Wikipedia



$$l_i \sim L\left(1 \pm \frac{l}{2L}\sin\theta\right)$$

the amplitude at A

$$f(\theta) = f_1(\theta) + f_2(\theta) \to P = |f_1(\theta) + f_2(\theta)|^2$$

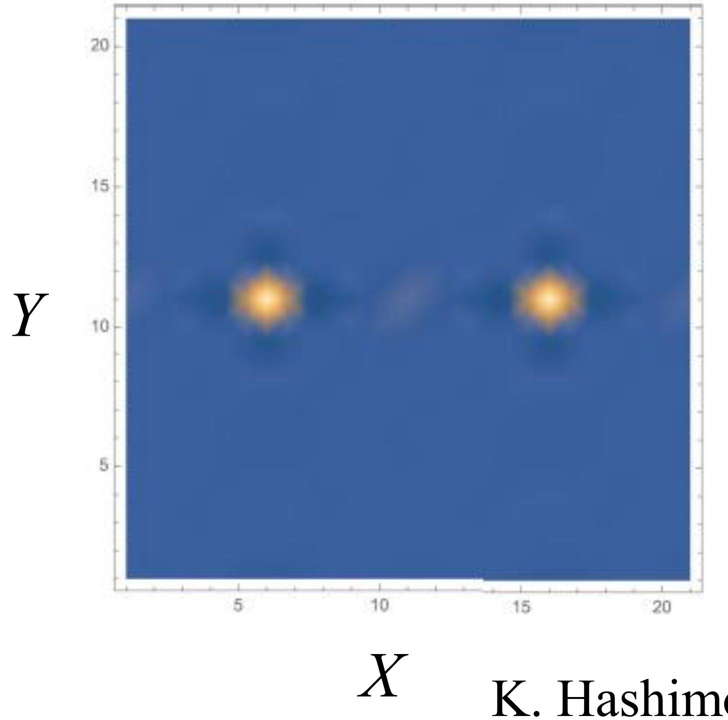
$$f_i(\theta) = A\sin\left(\frac{2\pi}{\lambda} l_i - \omega t\right)$$

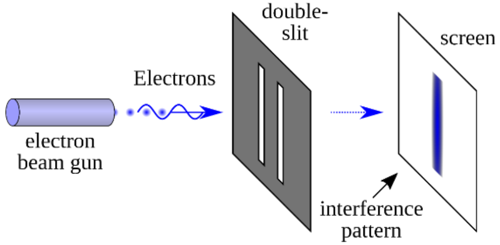
the Fourier transform of $f(\theta)$ in a limited region:

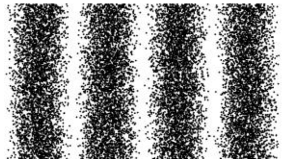
$$\Phi(X,Y) \propto \int_{\theta_2}^{\theta_2} d\theta \int_{\varphi_1}^{\varphi_2} d\varphi\, e^{ik(\theta X + \varphi Y)} f(\theta,\varphi)$$

imaging quantum interference phenomena

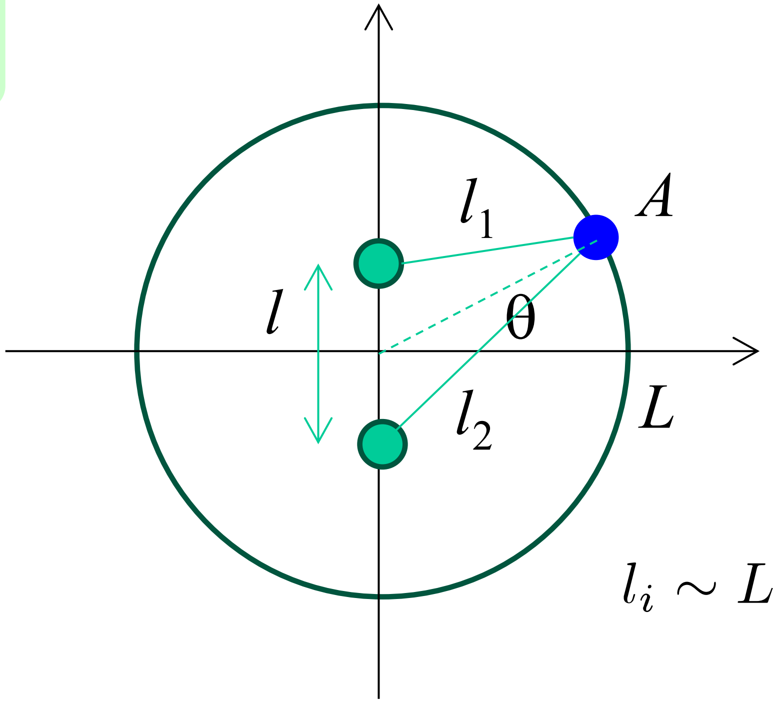
$|\Phi(X,Y)|^2$







K. Hashimoto et al., PTEP2023, 043B04 (2023)



$$l_i \sim L\left(1 \pm \frac{l}{2L}\sin\theta\right)$$

the amplitude at A

$$f(\theta) \;=\; f_1(\theta) + f_2(\theta) \rightarrow P = |f_1(\theta) + f_2(\theta)|^2$$

$$f_i(\theta) \;=\; A\sin\left(\frac{2\pi}{\lambda}l_i - \omega t\right)$$

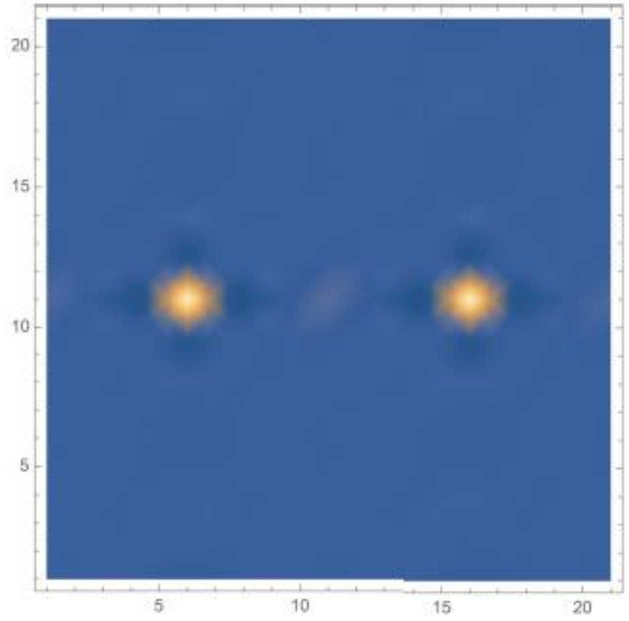
the Fourier transform of $f(\theta)$ in a limited region:

$$\Phi(X,Y) \propto \int_{\theta_2}^{\theta_2} d\theta \int_{\varphi_1}^{\varphi_2} d\varphi\, e^{ik(\theta X + \varphi Y)} f(\theta,\varphi)$$

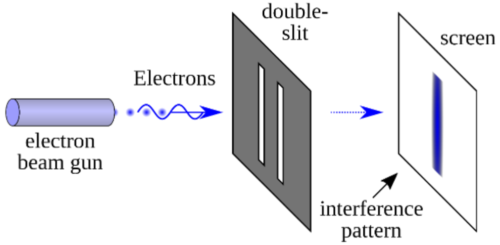
imaging quantum interference phenomena

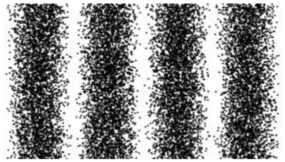
$|\Phi(X,Y)|^2$

Y



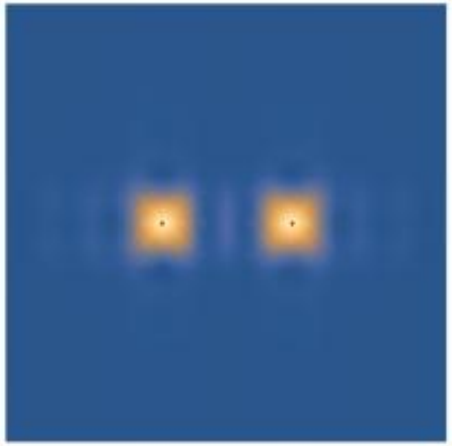
X



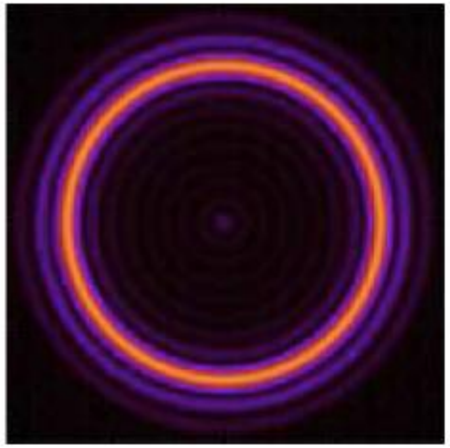


similar

Other applications in particle physics



scattering of string



imaging black holes through AdS/CFT

K. Hashimoto, Y. Matsuo, and T. Yoda, PTEP2023, 043B04 (2023) : “String is a double slit”

K. Hashimoto, S. Kinoshita, and K. Murata, PRL123, 031602 (2019), PRD101, 066018 (2020)

imaging quantum interference phenomena

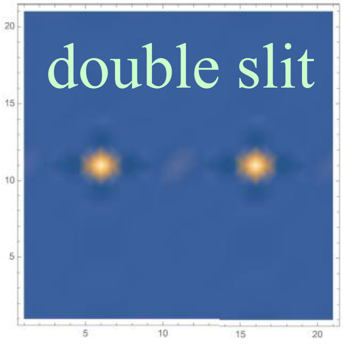
K. Hagino and T. Yoda, PLB848, 138326 (2024).
K. Heo and K.Hagino, PRC111, 034612 (2025).

Application to nuclear reactions:

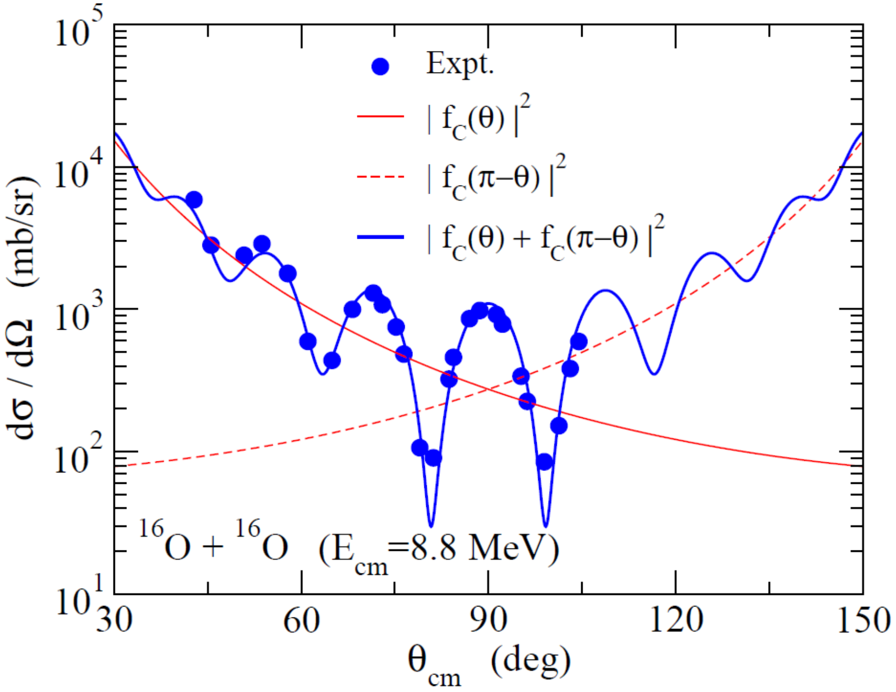
$$V_{\rm opt}(r) = V(r) - iW(r) \longrightarrow f(\theta) \longrightarrow \frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

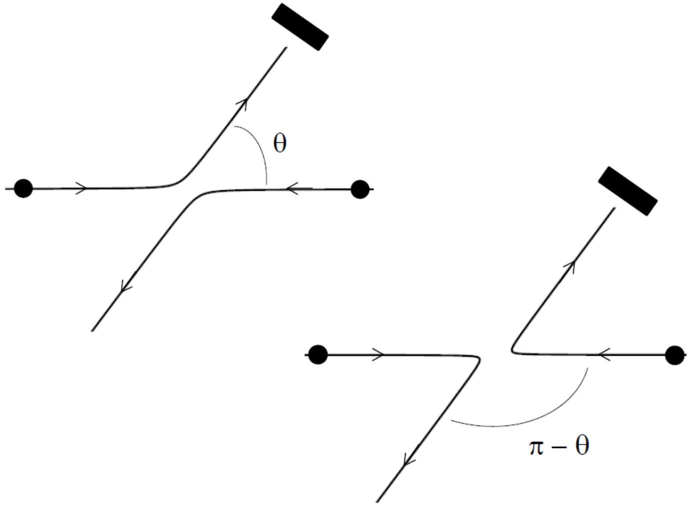
F.T.

$$\Phi(X,Y) \to |\Phi(X,Y)|^2$$



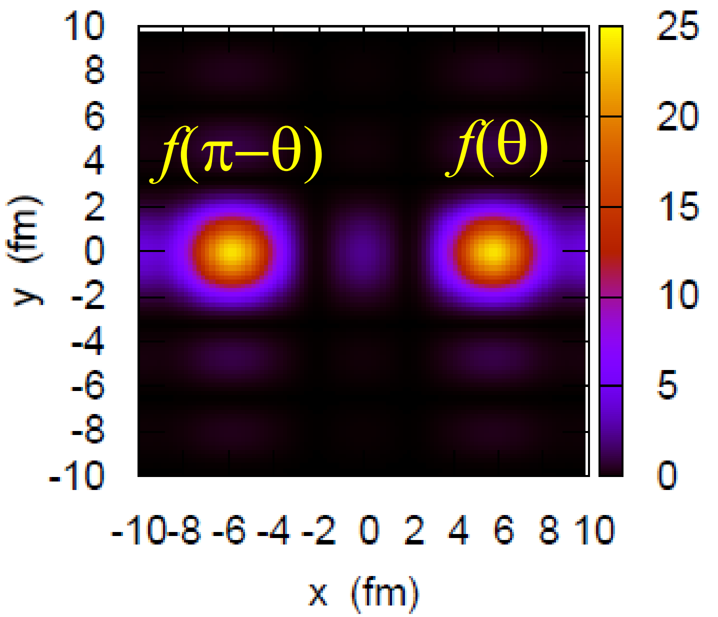
$^{16}O + ^{16}O$ at E = 8.8 MeV



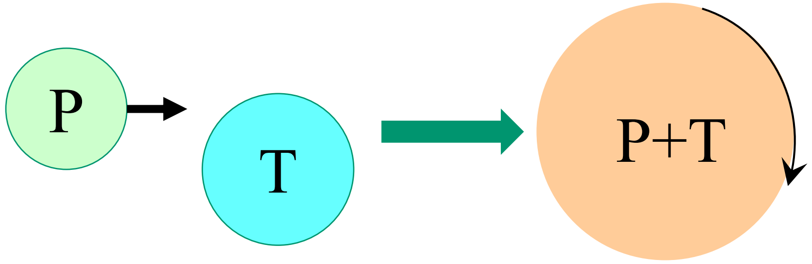


$$\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi - \theta)|^2$$

θ_0=90 deg., $\Delta\theta$=30 deg.



Fusion reactions: compound nucleus formation

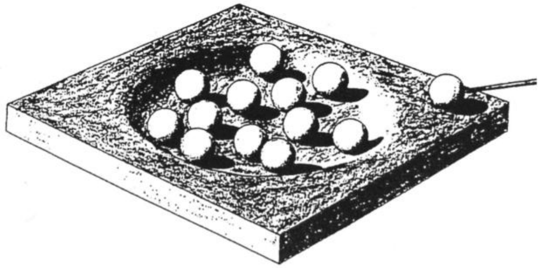


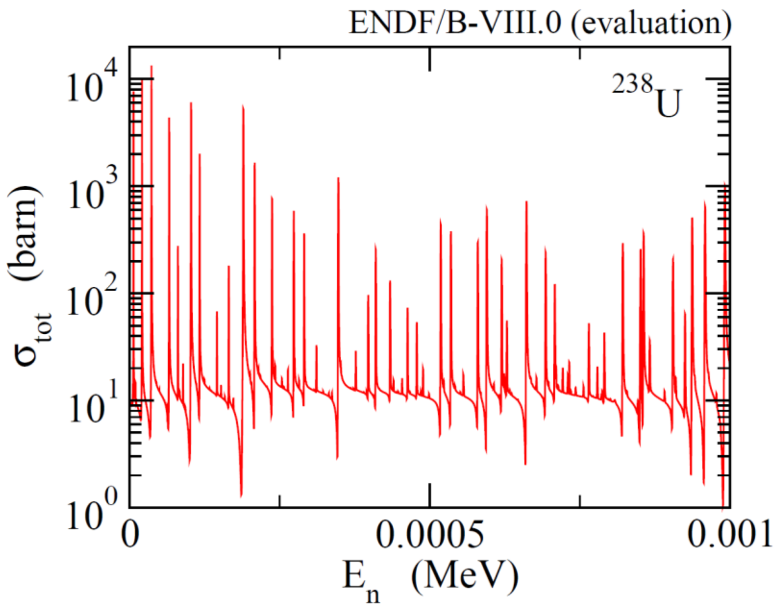
compound nucleus

Niels Bohr (1936)

Wikipedia

neutron capture reactions

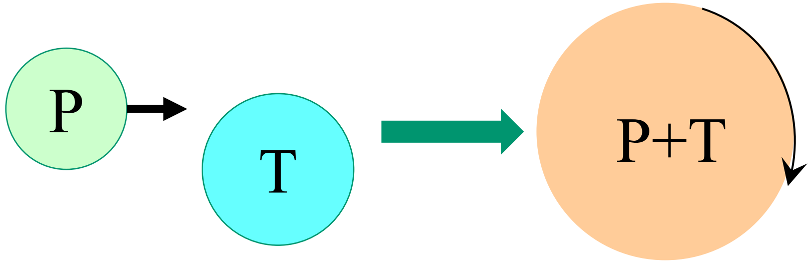




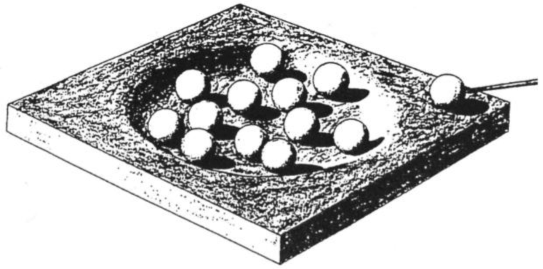
Exp: Enrico Fermi (1935)

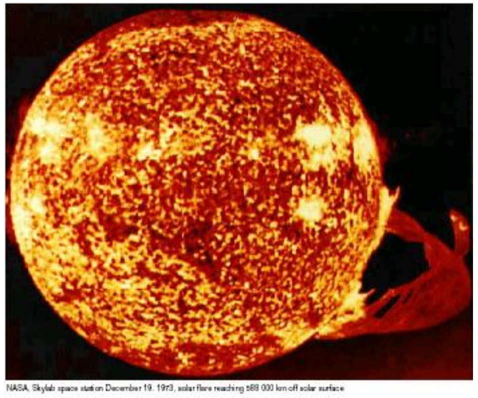
many very narrow resonances (width ~ eV)

Fusion reactions: compound nucleus formation

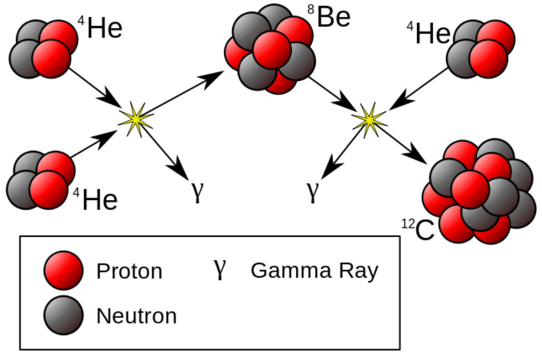


compound
nucleus

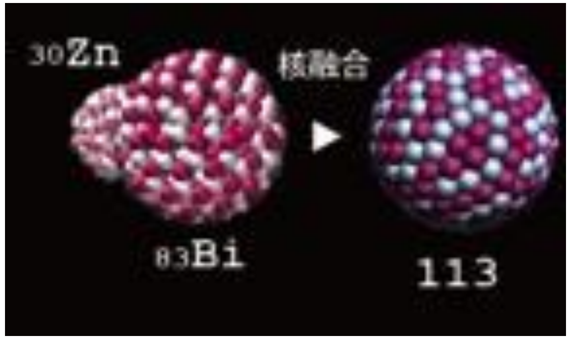




energy production
in stars (Bethe '39)



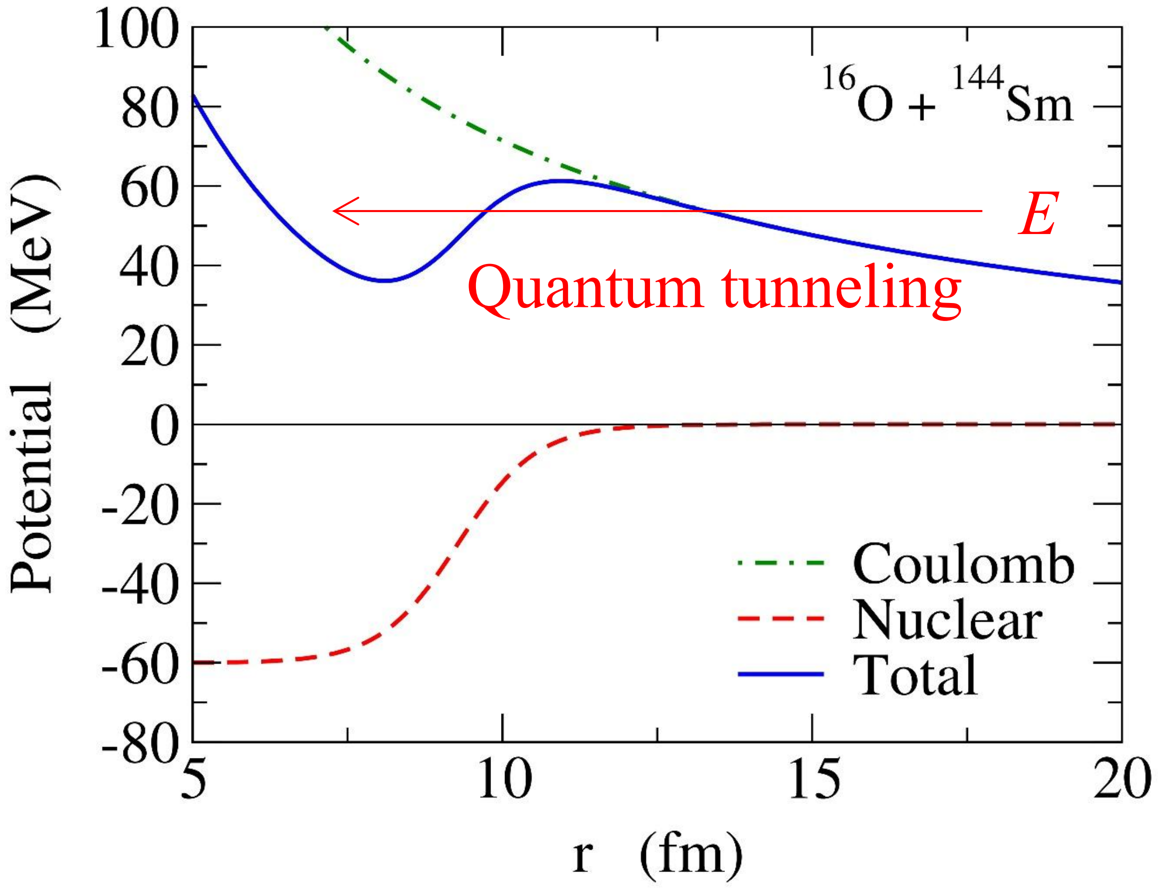
nucleosynthesis



superheavy elements

Fusion and fission: large amplitude motions of quantum many-body systems
← microscopic understanding: an ultimate goal of nuclear physics

Coulomb barrier



1. Coulomb interaction
   long range, repulsion
2. Nuclear interaction
   short range, attraction

Potential barrier (Coulomb barrier)

Fusion: takes place by overcoming the barrier

the barrier height → defines the energy scale of a system

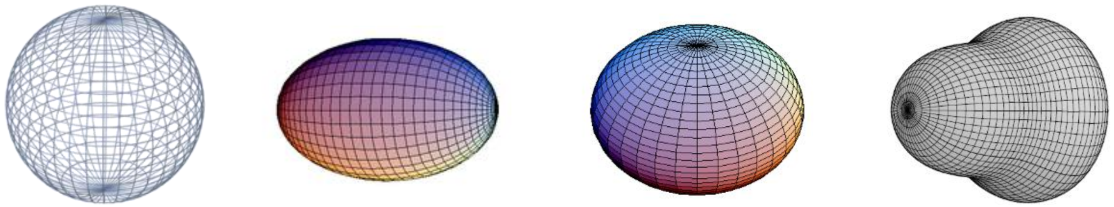
Fusion reactions at energies around the Coulomb barrier

Sub-barrier fusion reactions and quantum tunneling

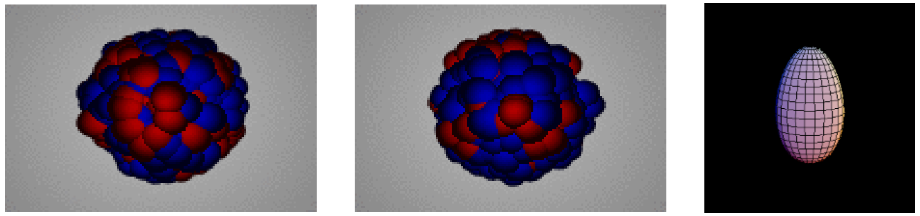
Fusion with quantum tunneling

with many degrees of freedom

- several nuclear shapes



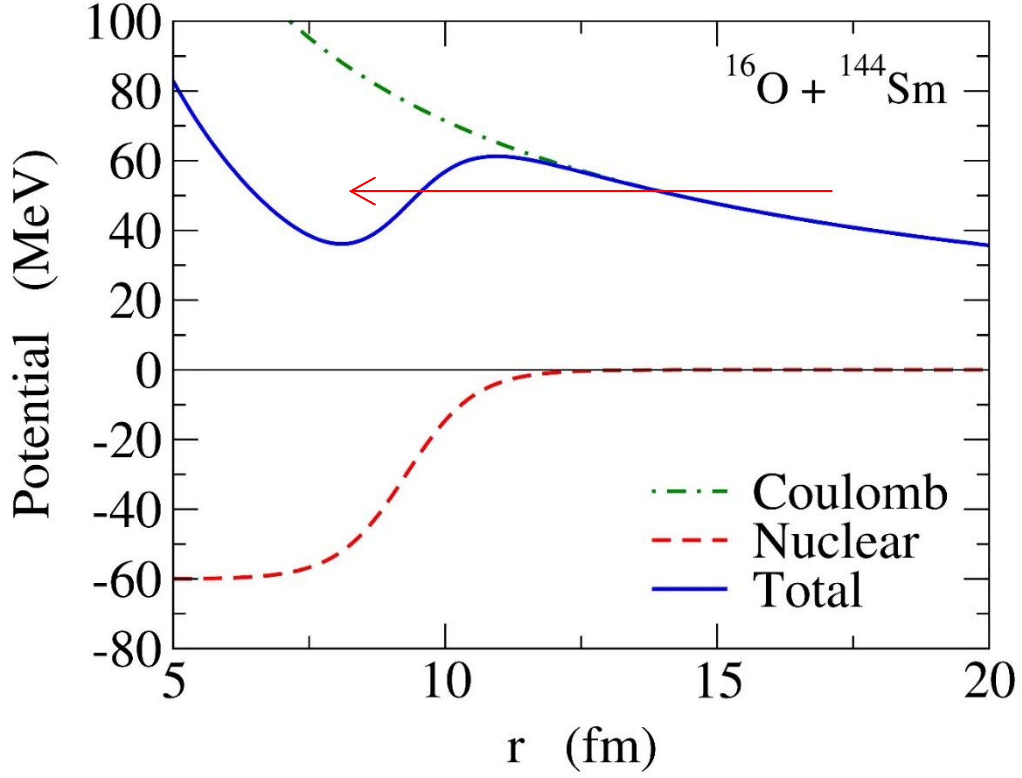
- several surface vibrations



several modes and adiabaticities

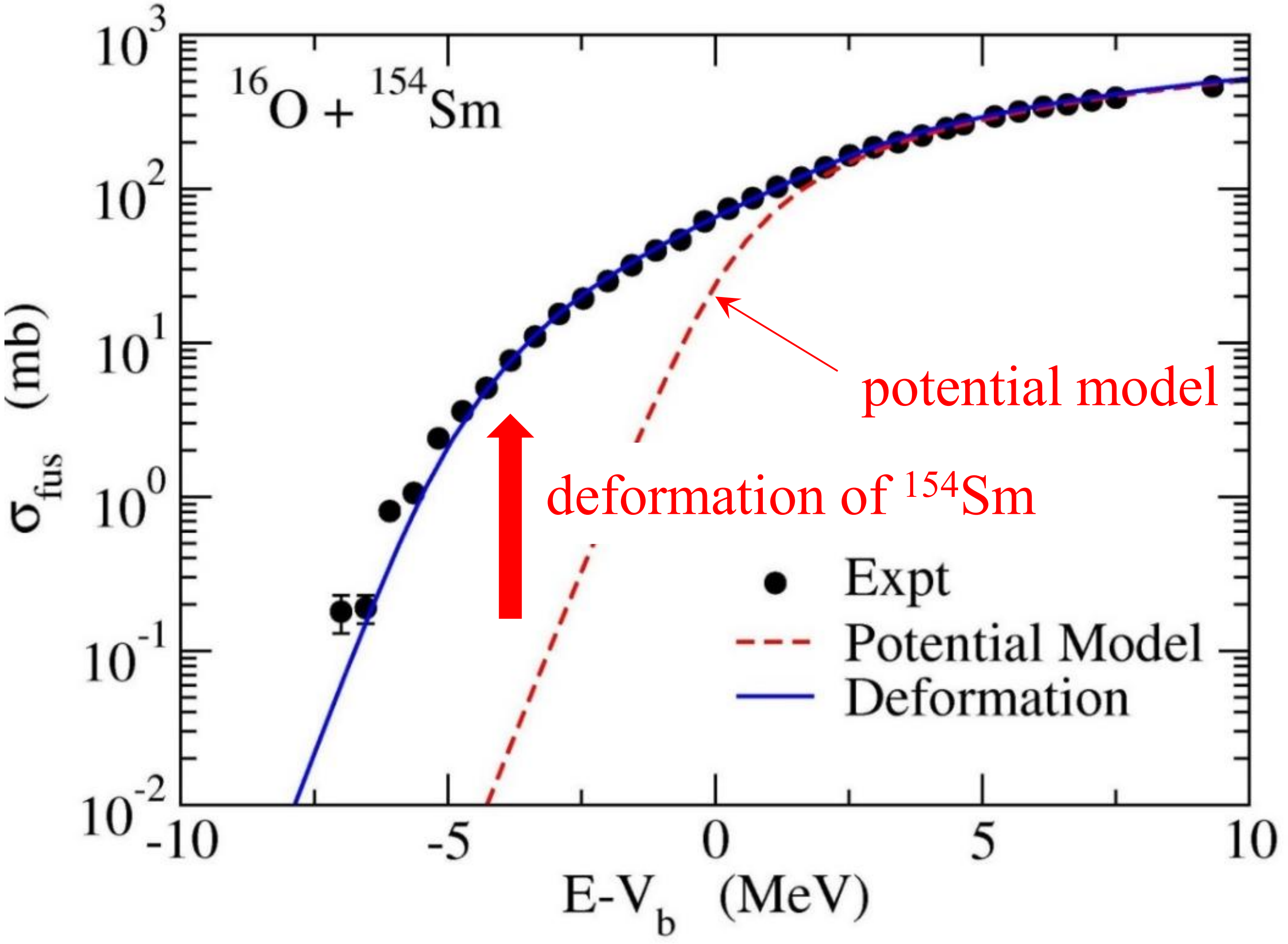
- several types of nucleon transfers

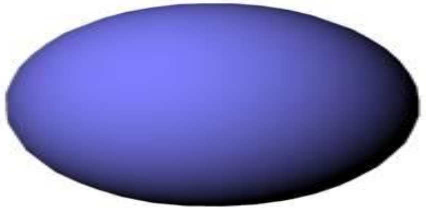
Sub-barrier fusion reactions



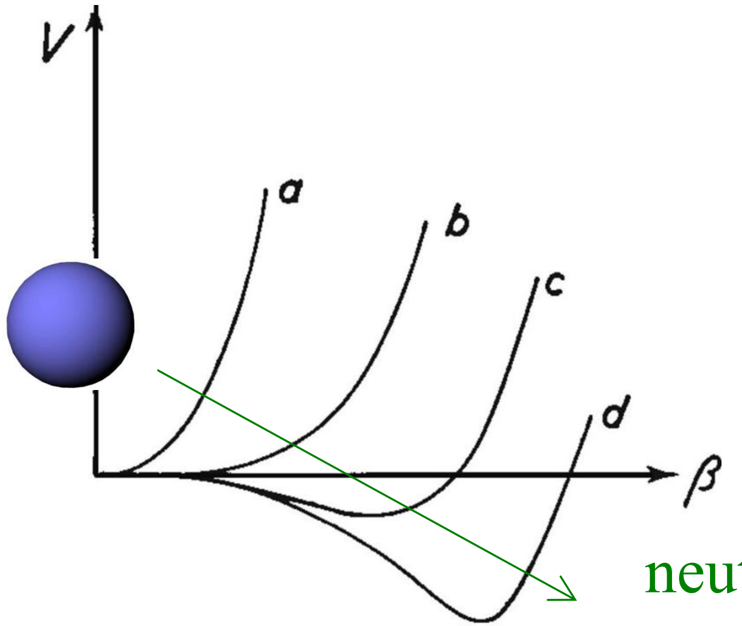
Tunneling probabilities: the exponential *E* dependence
→ nuclear structure effects are amplified

Sub-barrier fusion reactions and quantum tunneling

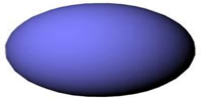




^{154}Sm

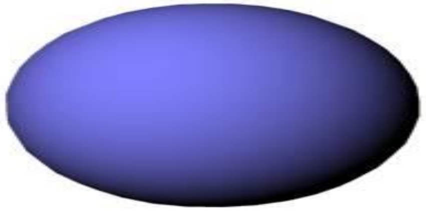


neutron number



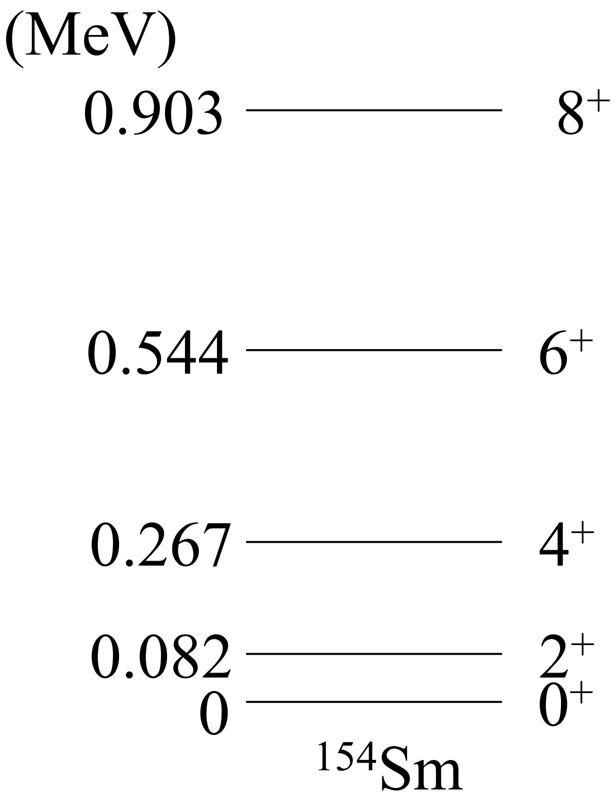
K. Hagino and N. Takigawa, Prog. Theo. Phys.128 (2012)1061.

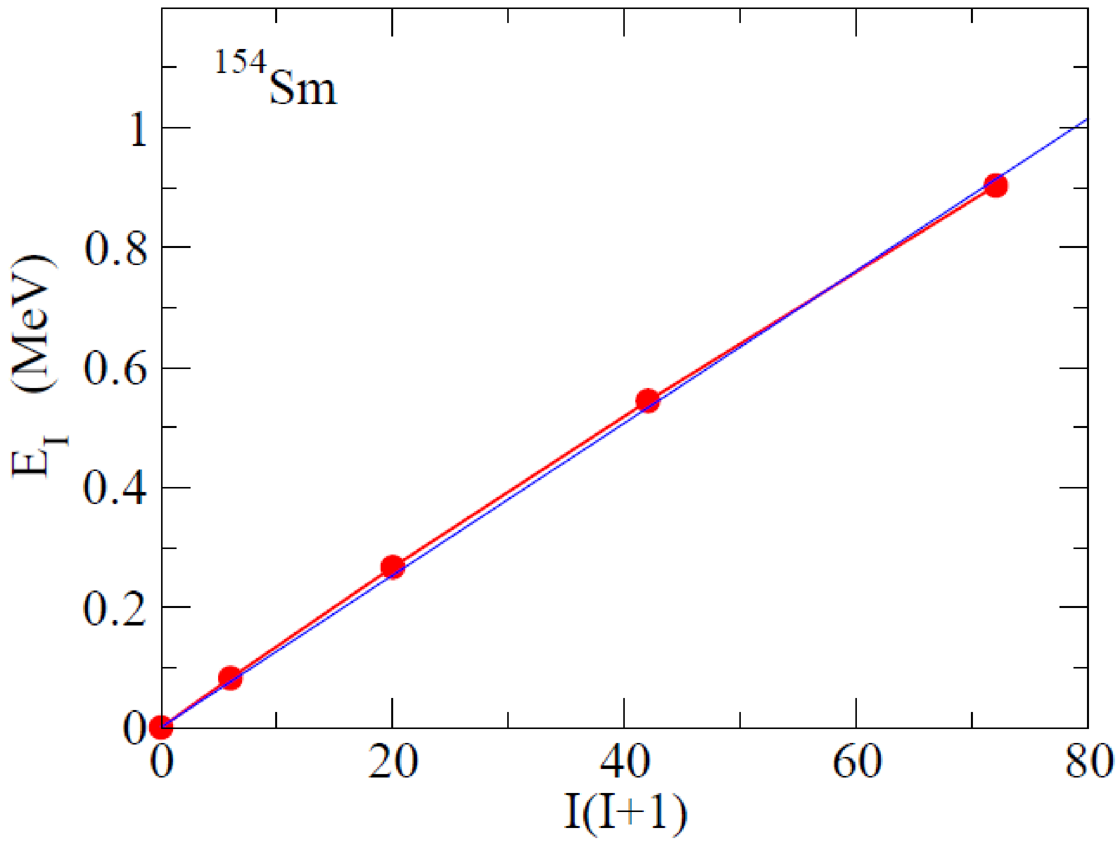
Effects of nuclear deformation on fusion



^{154}Sm

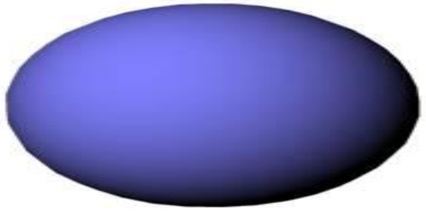
$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$



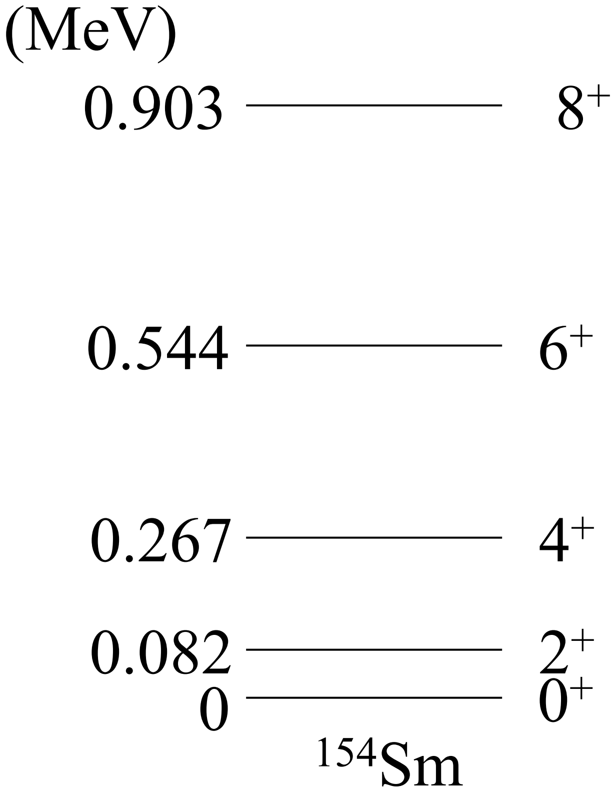


rotational spectrum

Effects of nuclear deformation on fusion



^{154}Sm



rotational spectrum

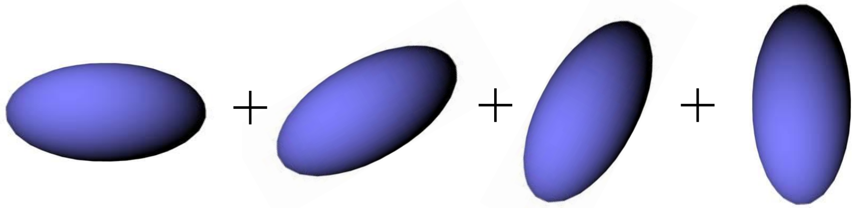
a small rotational energy

$$E_I = \frac{I(I+1)\hbar^2}{2\mathcal{J}}$$

→ a large moment of inertia J

→ rotation: a slow deg. of freedom

$$E_{\rm rot} \sim E_{2^+} = 82 \text{ keV}$$
$$E_{\rm tunnel} \sim \hbar\Omega_{\rm barrier} \sim 3.5 \text{ MeV}$$

$\Psi_{0^+} =$ 

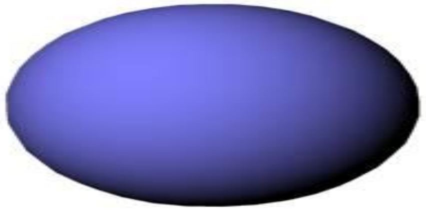
→ a spherical state in the lab. system

fix the orientation angle to calculate the fusion probability

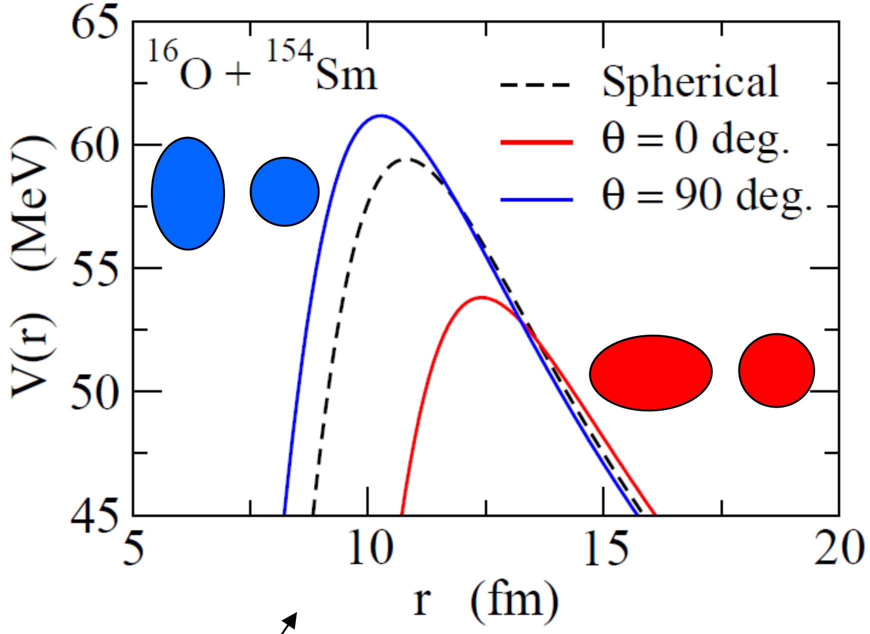
"a snapshot of a rotating nucleus"

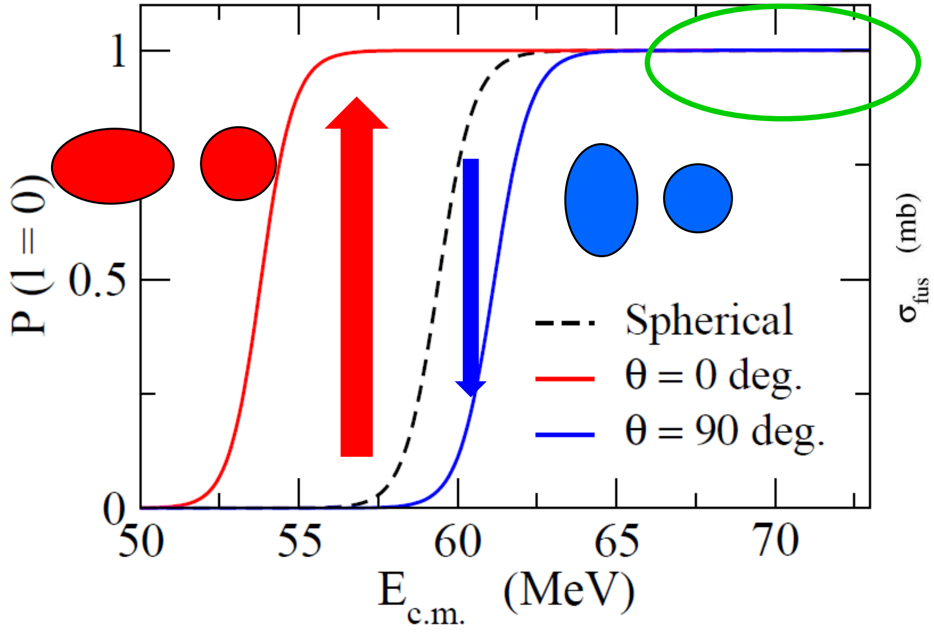
Effects of nuclear deformation on fusion

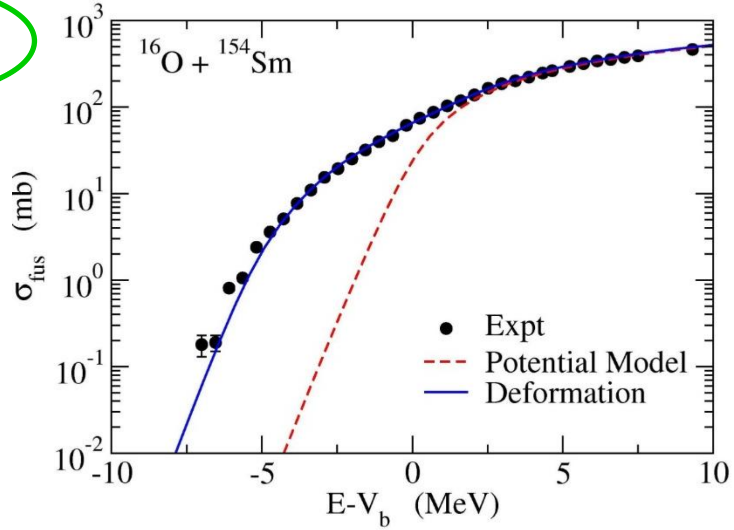
^{154}Sm : a typical deformed nucleus

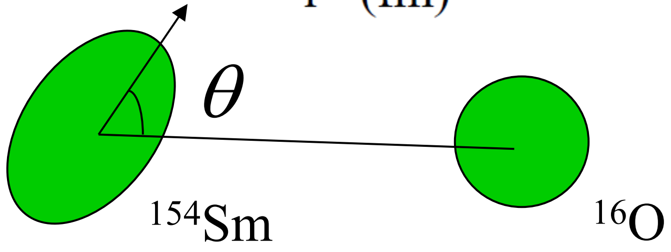


^{154}Sm









$$\sigma_{\text{fus}}(E) = \int_0^1 d(\cos\theta)\sigma_{\text{fus}}(E;\theta)$$

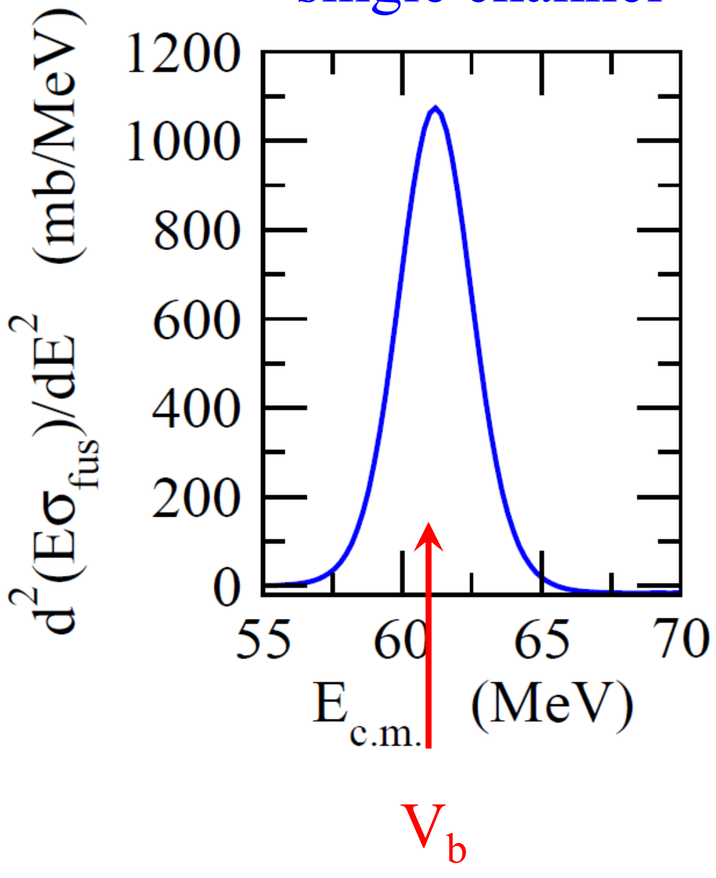
Fusion: strong interplay between nuclear structure and reaction

Fusion barrier distribution

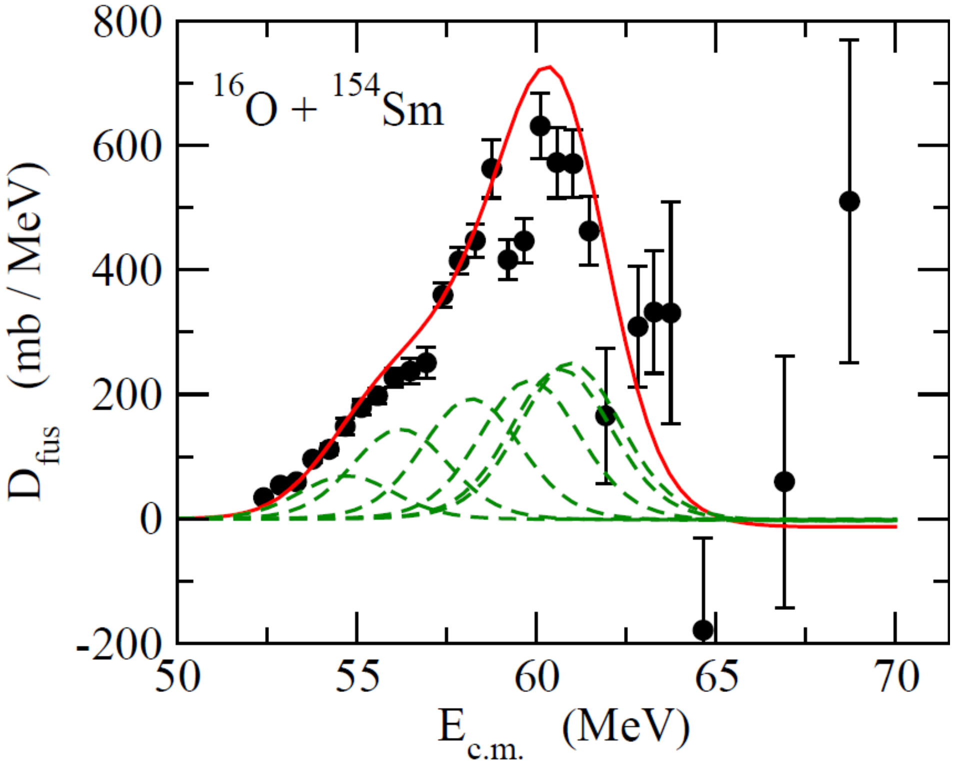
$$D_{\rm fus}(E) = \frac{d^2(E\sigma_{\rm fus})}{dE^2} \propto \frac{dP_{l=0}}{dE}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

single channel



with multi-barriers

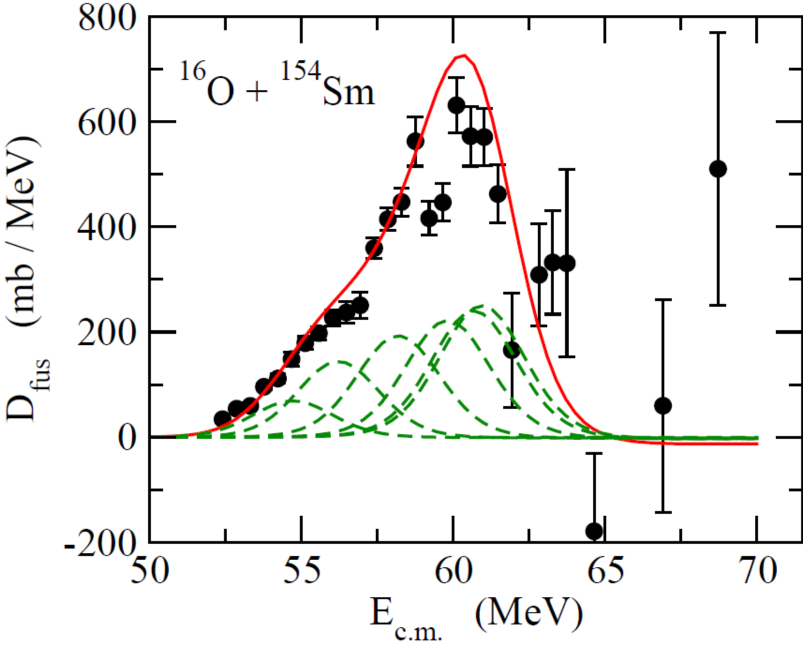


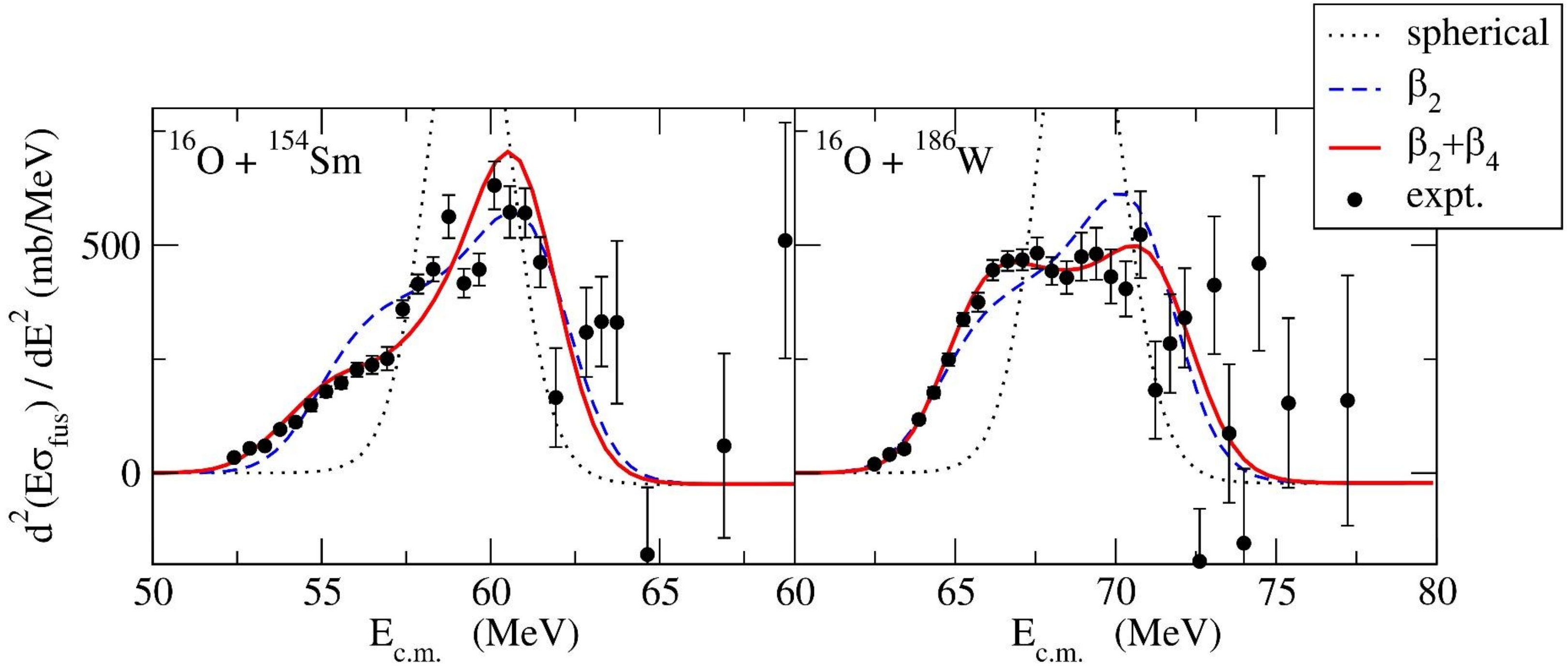
K.H. and N. Takigawa, PTP128 ('12) 1061

Fusion barrier distribution

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2}$$

N. Rowley, G.R. Satchler, and P.H. Stelson, PLB254 ('91) 25

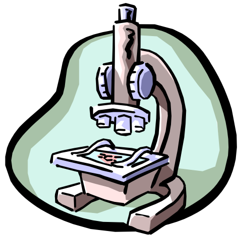


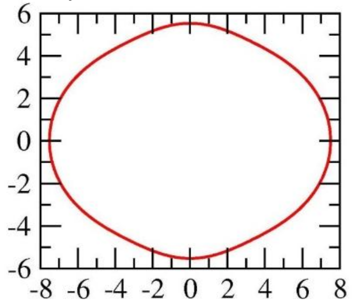


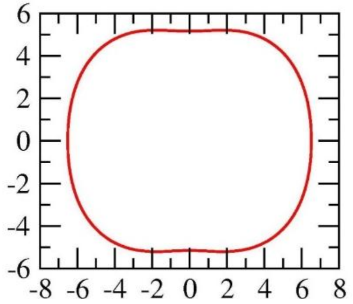
$$R(\theta) = R_0(1 + \beta_2 Y_{20}(\theta) + \beta_4 Y_{40}(\theta) + \cdots)$$

$$\beta_2 = 0.33 \qquad \beta_4 = +0.05$$

$$\beta_2 = 0.29 \qquad \beta_4 = -0.03$$







sensitive to the sign of β_4!

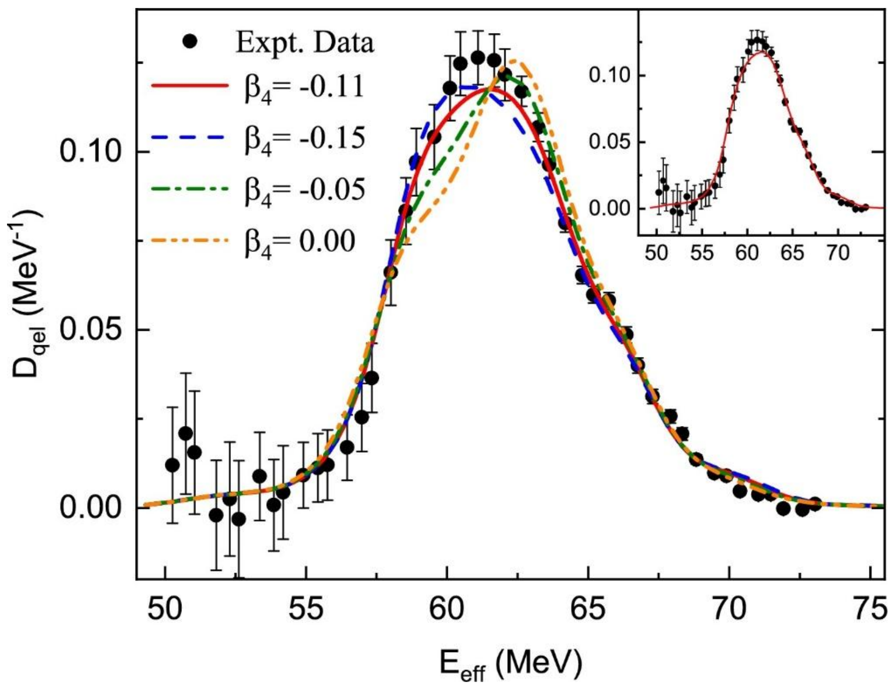
→ Fusion as a quantum tunneling microscope for nuclei

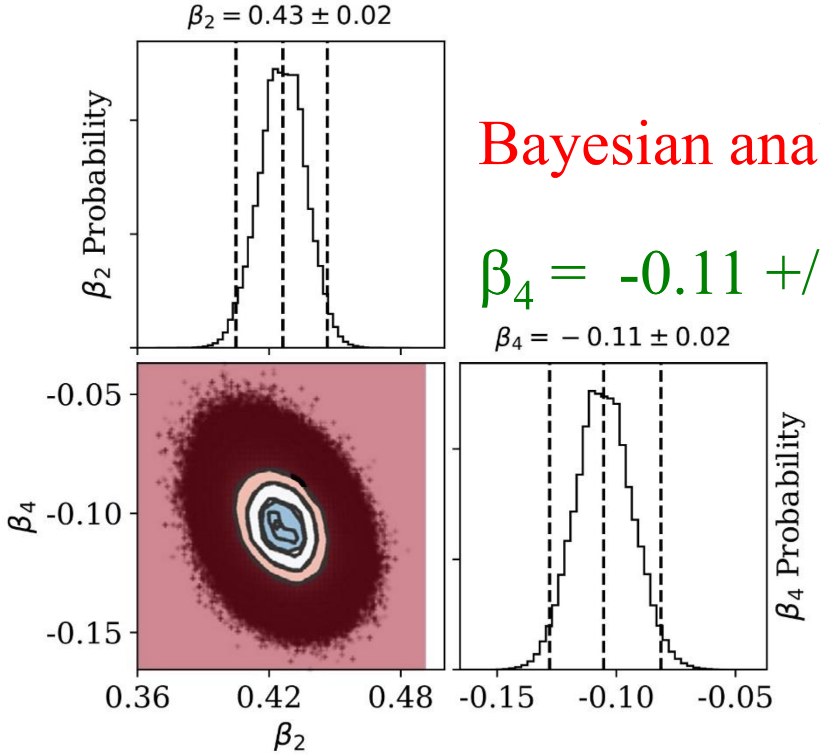
Determination of β_4 of ^{24}Mg with quasi-elastic barrier distributions

Y.K. Gupta, B.K. Nayak, U. Garg, K.H., et al., PLB806, 135473 (2020).

$\beta_2 =$ 0.43 +/- 0.02

^{24}Mg + ^{90}Zr





Bayesian analysis

$\beta_4 =$ -0.11 +/- 0.02

high precision determination of β_4
→ for the first time

cf. (p,p'): β_4 = -0.05 +/- 0.08

R. De Swiniarski et al., PRL23, 317 (1969)