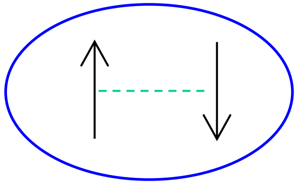
原子核の対相関現象

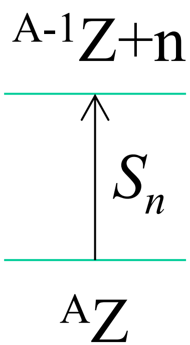
原子核の中で2つの核子がペアを組む

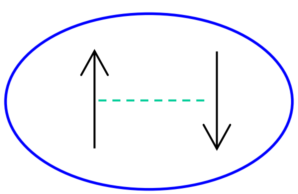


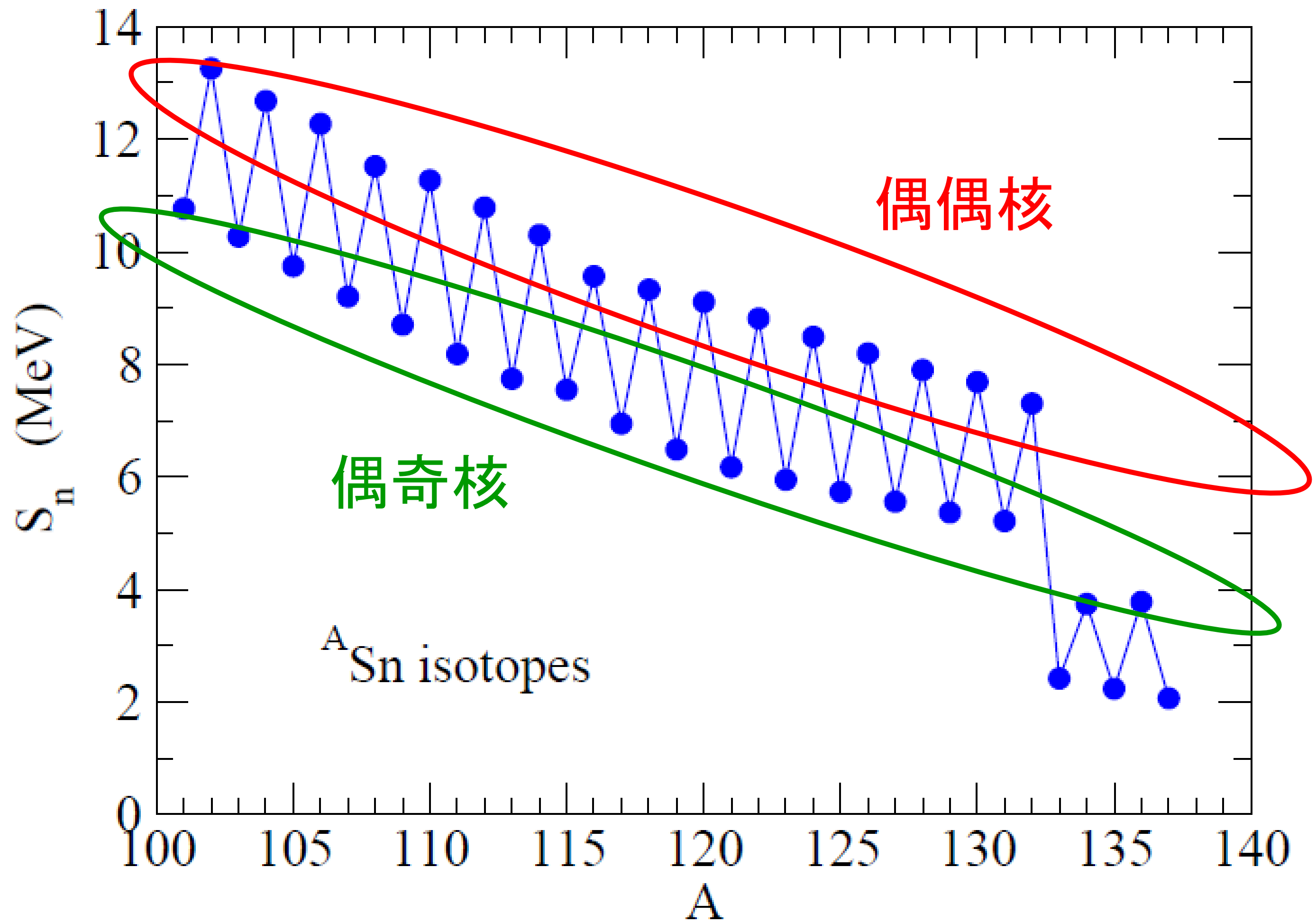
対相関エネルギー

偶数個の中性子から1つ中性子を取る方が奇数個から取るより大きなエネルギーが必要：対相関

even-odd staggering



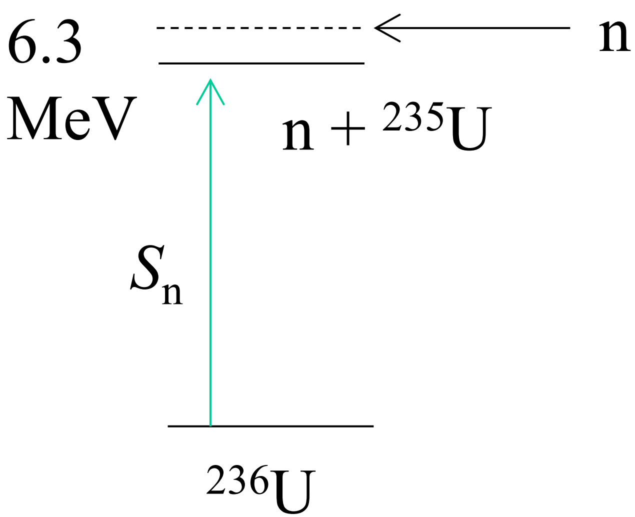


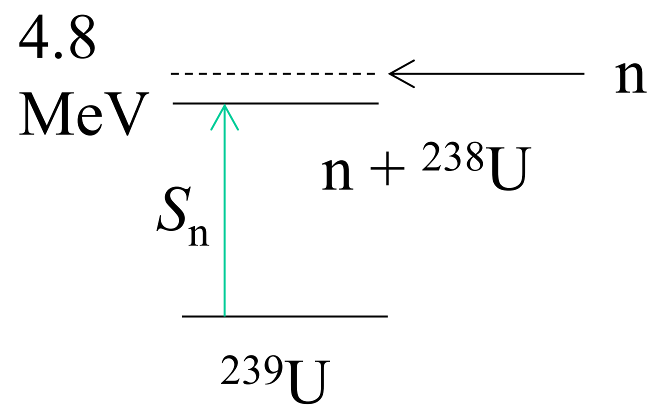


1n separation energy: $S_n\,(A,Z) = B(A,Z) - B(A\text{-}1,Z)$

（参考）中性子誘起核分裂

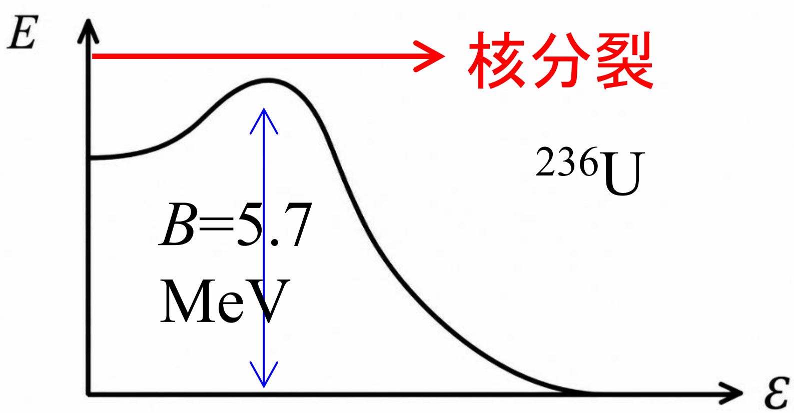
$$\mathrm{n} + {}^{235}\mathrm{U} \longrightarrow {}^{236}\mathrm{U}^* \longrightarrow \mathrm{fission}$$

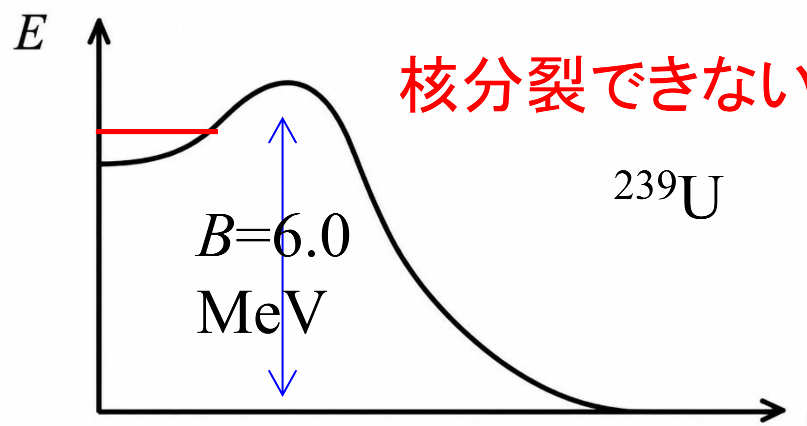




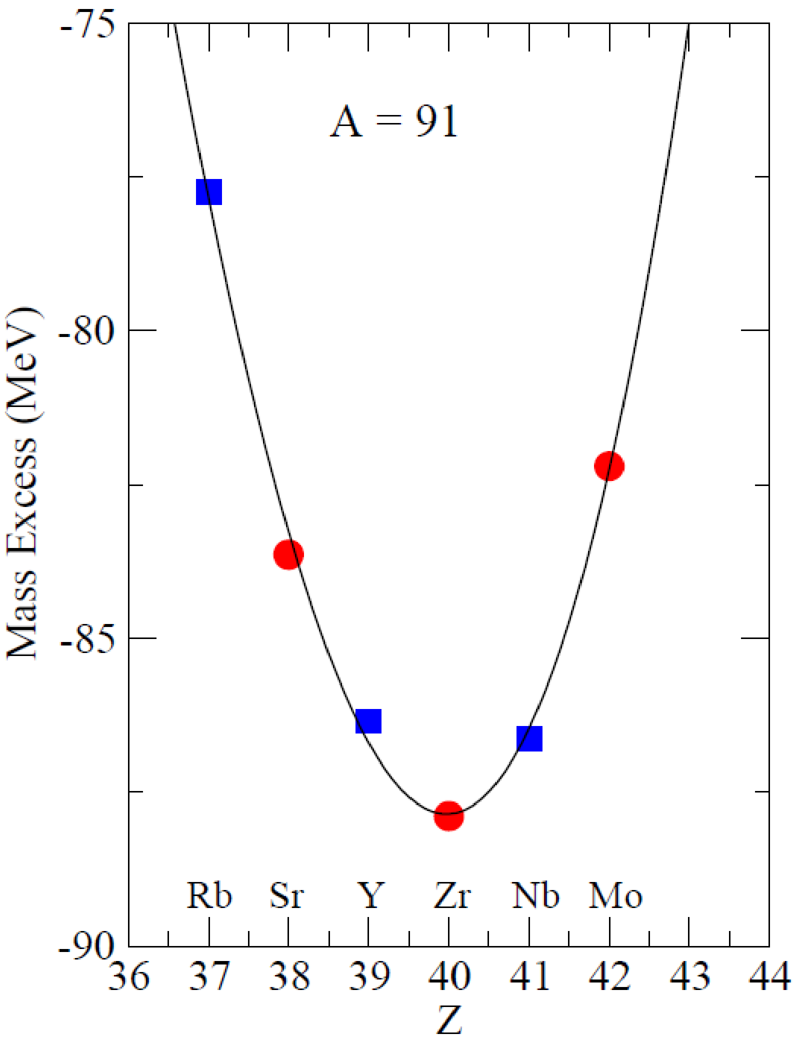
$E^* = S_\mathrm{n} + E_\mathrm{n} \sim 6.3$ MeV
($E_\mathrm{n} \sim 1/40$ eV)

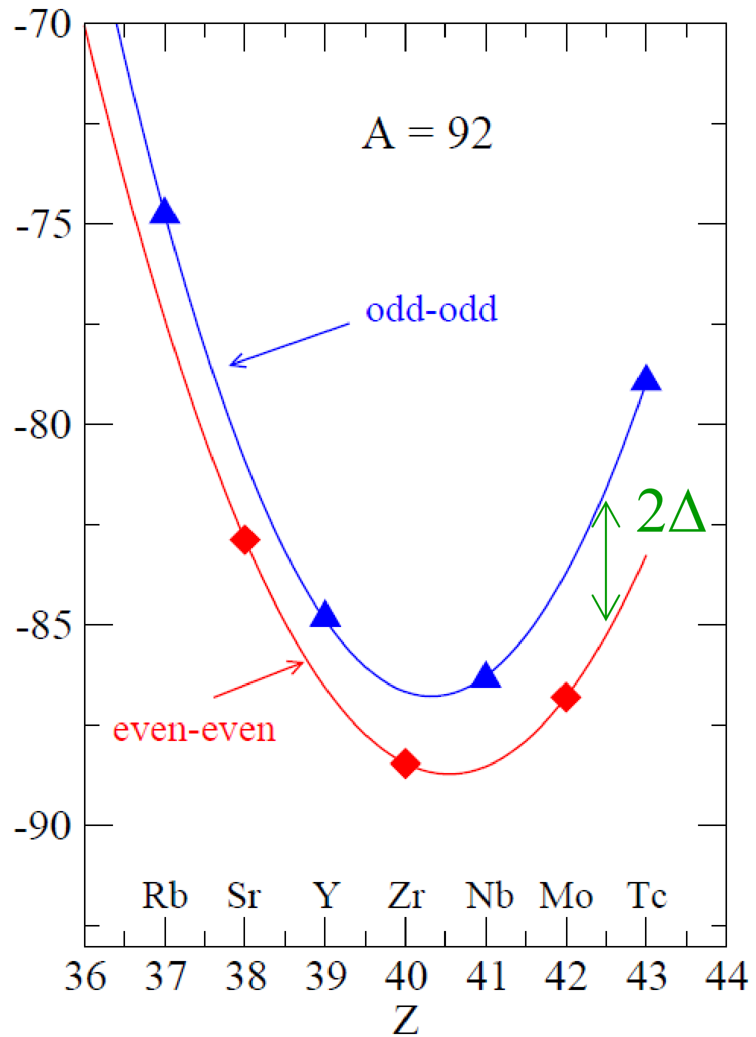
$E^* = S_\mathrm{n} + E_\mathrm{n} \sim 4.8$ MeV
($E_\mathrm{n} \sim 1/40$ eV)





Aが奇数の場合　　Aが偶数の場合



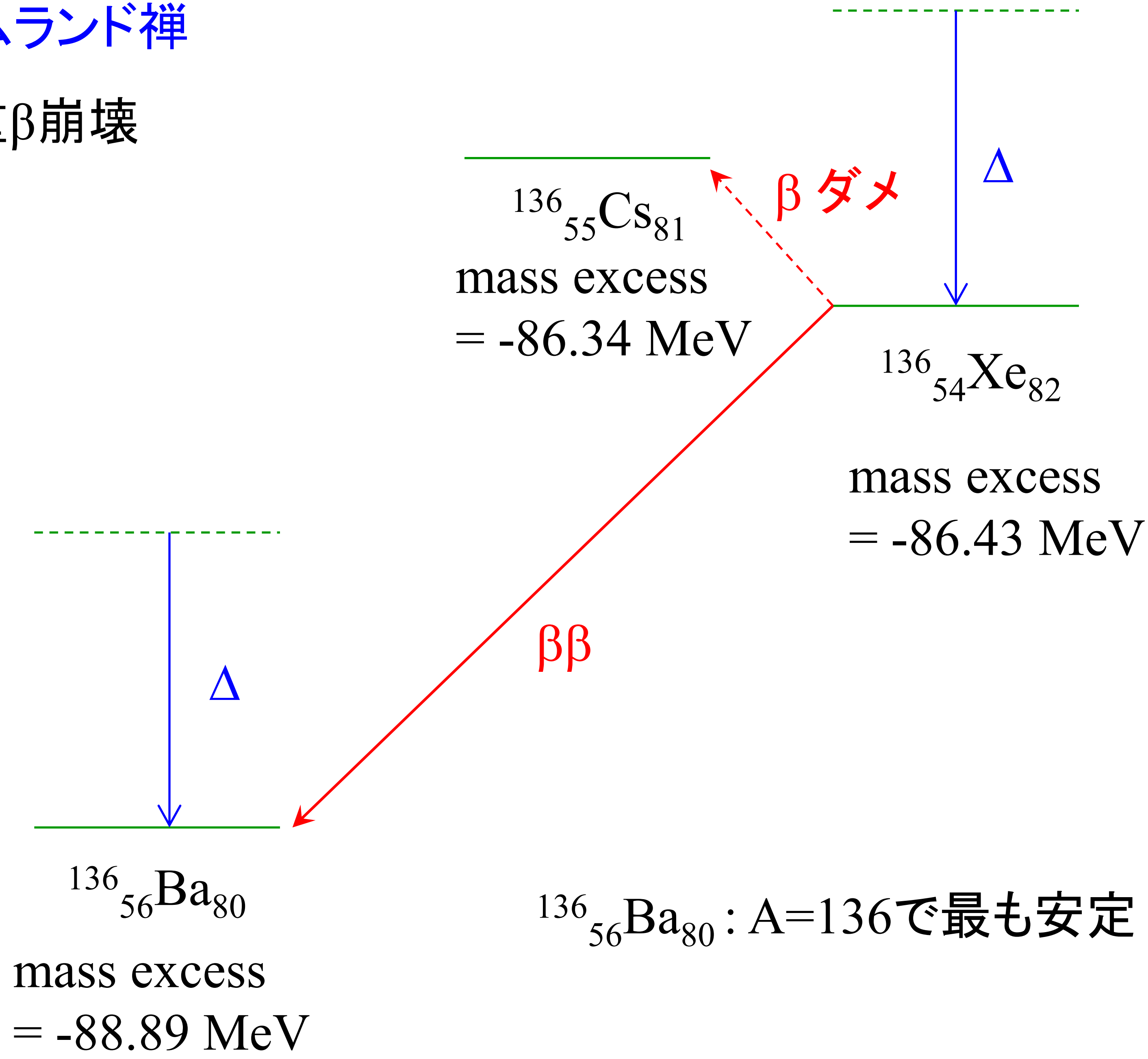


mass excess（縦軸）は $M(A,Z) - Au$ という定義

（u は原子質量単位で ^{12}C の質量が $12u$ という定義）

（参考）カムランド禅

^{136}Xeの2重β崩壊



$^{136}_{56}Ba_{80}$: A=136で最も安定

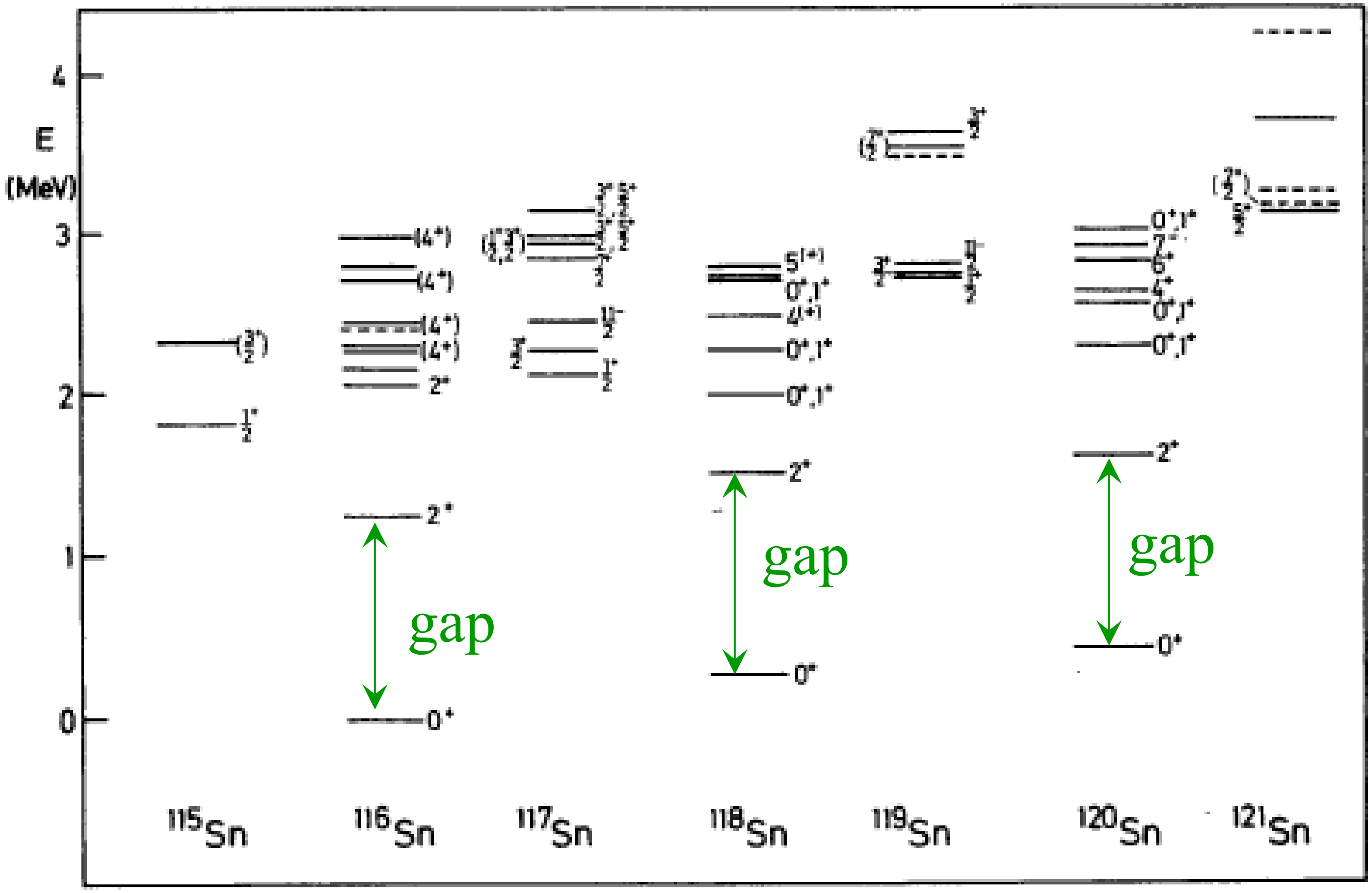
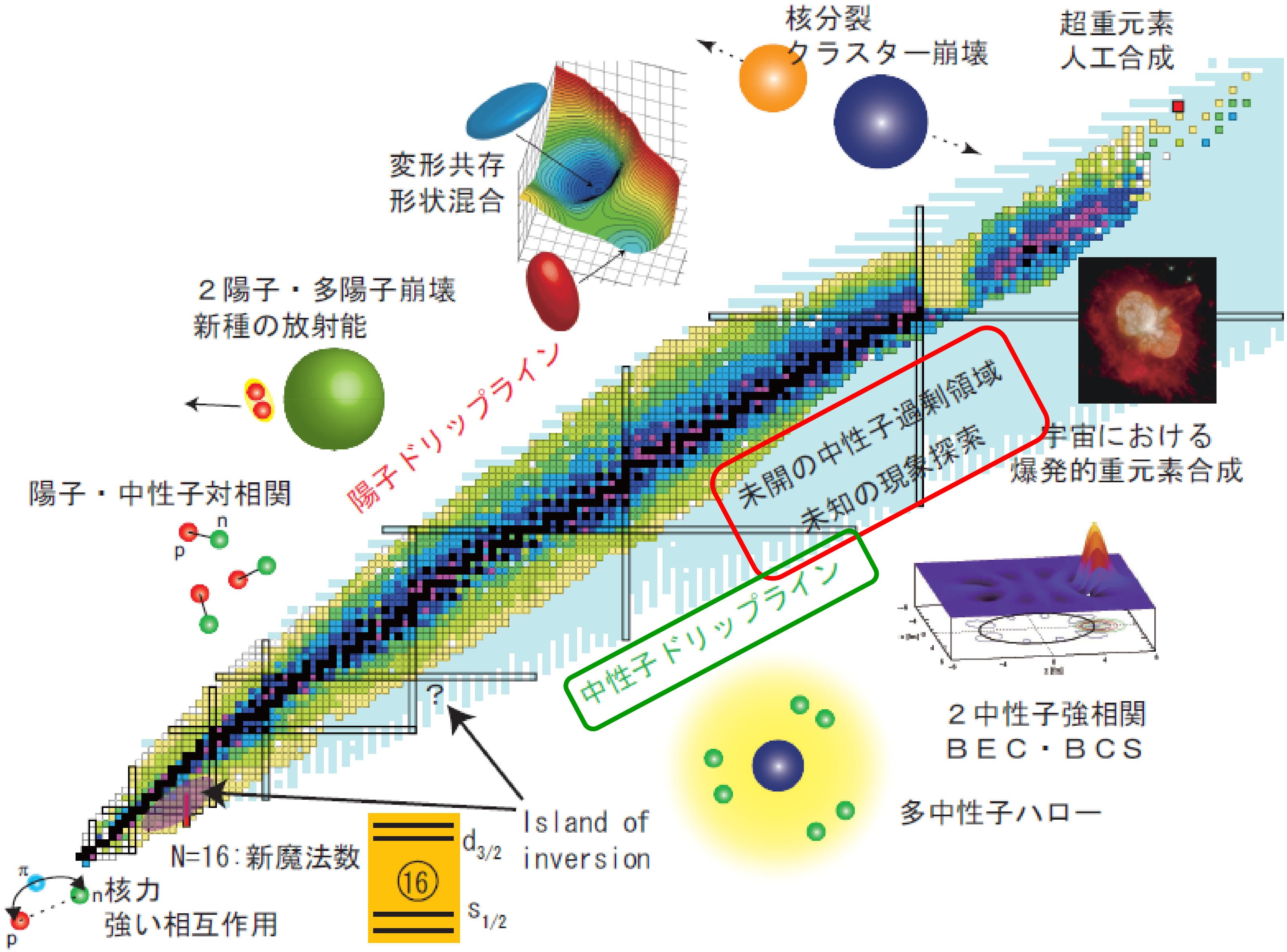


Figure 6.1. Excitation spectra of the $_{50}$Sn isotopes.

Ring-Schuck

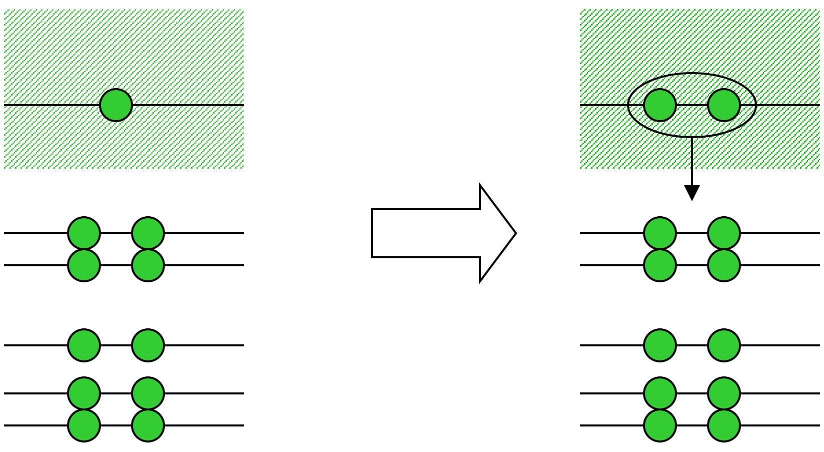
中性子過剰核と対相関



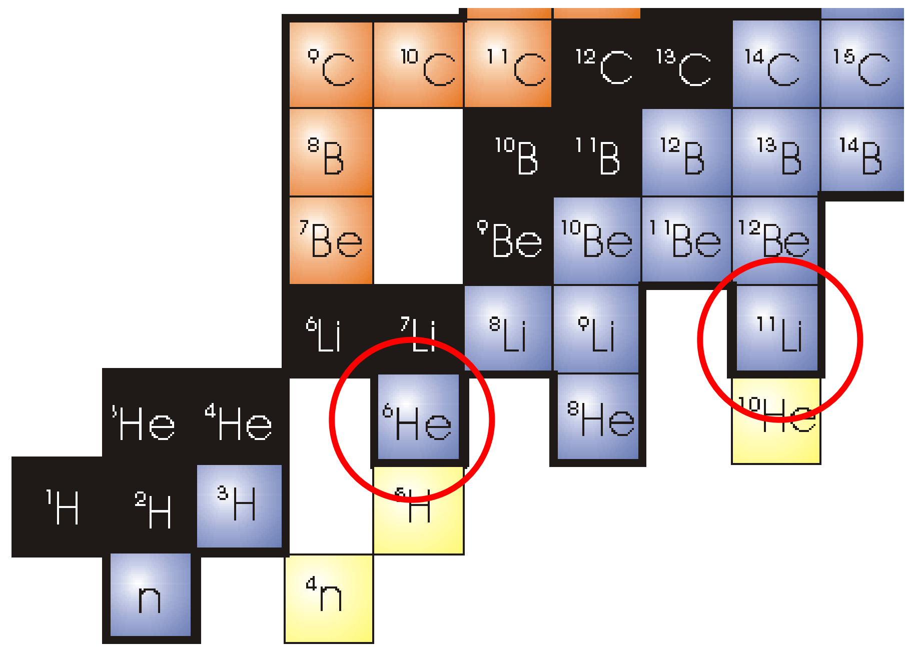
核分裂
クラスター崩壊
超重元素
人工合成
変形共存
形状混合
２陽子・多陽子崩壊
新種の放射能
陽子ドリップライン
陽子・中性子対相関
p
n
未開の中性子過剰領域
未知の現象探索
宇宙における
爆発的重元素合成
中性子ドリップライン
?
２中性子強相関
ＢＥＣ・ＢＣＳ
多中性子ハロー
Island of
inversion
N=16：新魔法数
16
$d_{3/2}$
$s_{1/2}$
π
p
n
核力
強い相互作用用

弱束縛核における対相関：ボロミアン原子核

残留相互作用 → 引力



不安定　　　　安定

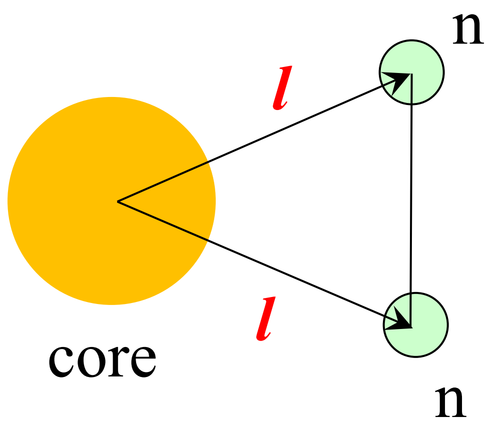


“ボロミアン核”

- ✓ ハロー現象
- ✓ 電気双極子遷移
- ✓ ダイニュートロン相関
- ✓ 核反応

ダイニュートロン相関：2中性子間の空間的相関

$^{18}\mathrm{O} = {}^{16}\mathrm{O} + \mathrm{n} + \mathrm{n}$

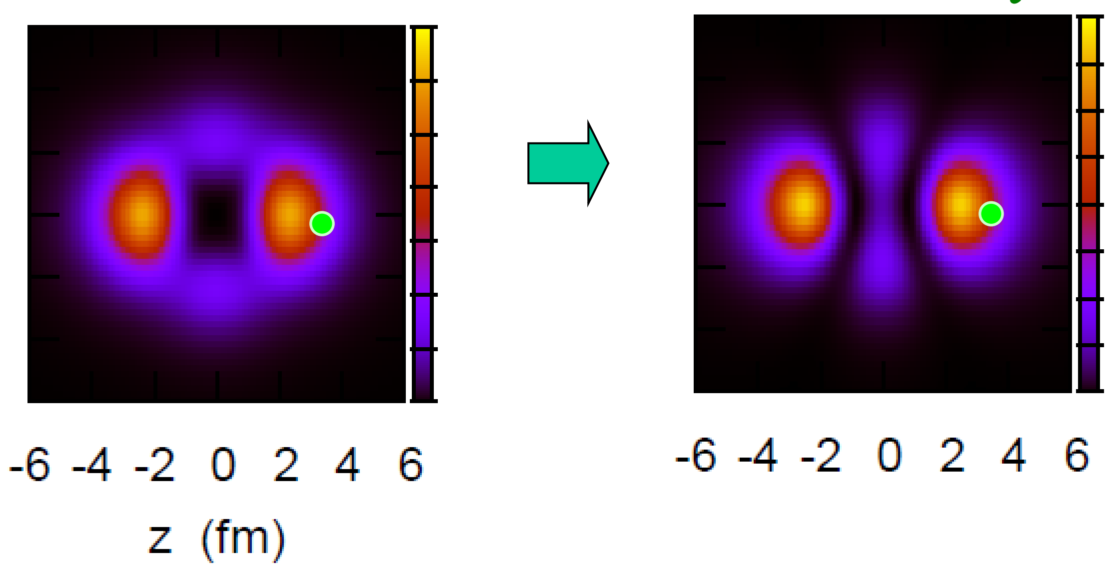


$$\Psi_{\mathrm{g.s.}}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \mathcal{A} \sum_{nn'jl} C_{nn'jl} [\phi_{njl}(\boldsymbol{r}_1)\phi_{n'jl}(\boldsymbol{r}_2)]^{(I=0)}$$

$$\to \rho_2(\boldsymbol{r}) = |\Psi_{\mathrm{g.s.}}(\boldsymbol{r}, \boldsymbol{r}')|^2_{\boldsymbol{r}'=z_0}$$

single-*l*

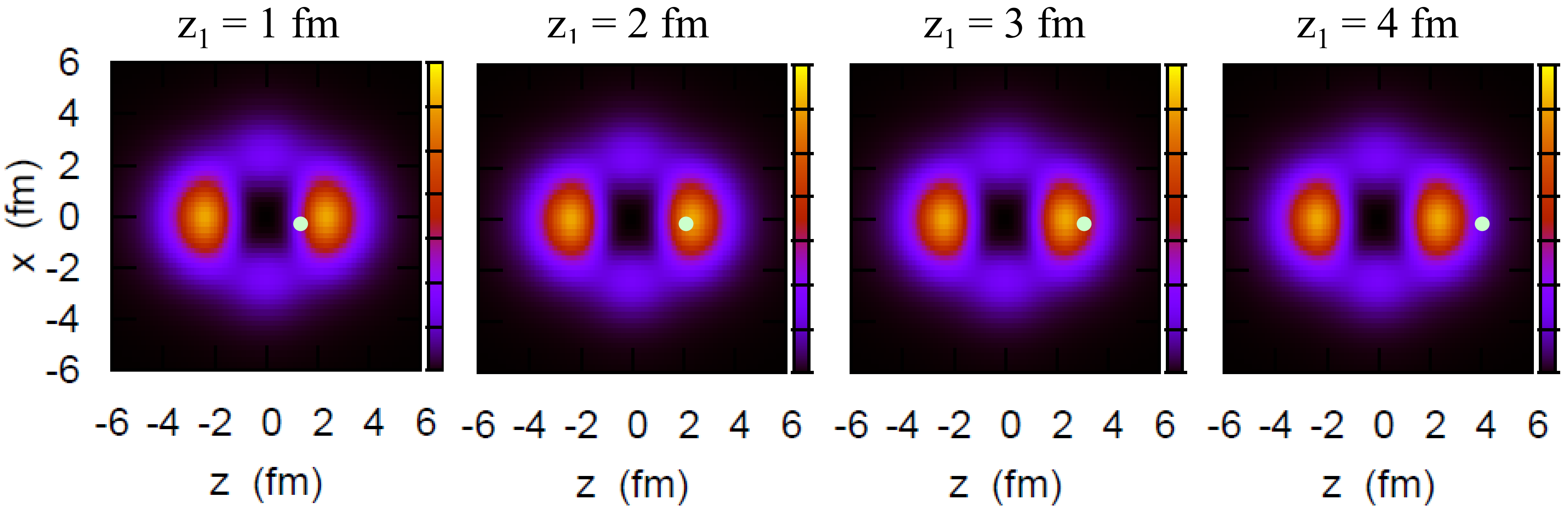
multi-*l*, but even *l* only



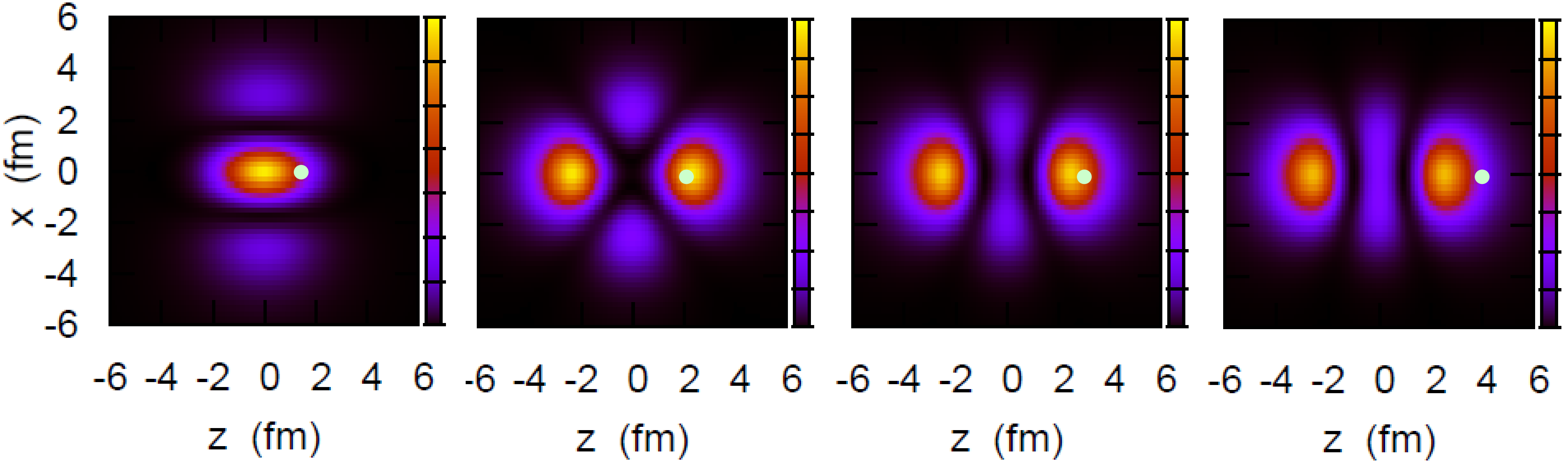
コア核をはさんで
左右対称な分布

K. Hagino, H. Sagawa, P. Schuck, J. of Phys. G 37, 064040 (2010)

相関なし

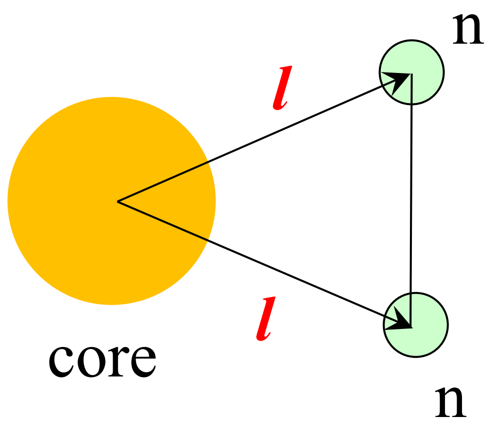


相関あり



ダイニュートロン相関：2中性子間の空間的相関

$^{18}\mathrm{O} = {}^{16}\mathrm{O} + \mathrm{n} + \mathrm{n}$



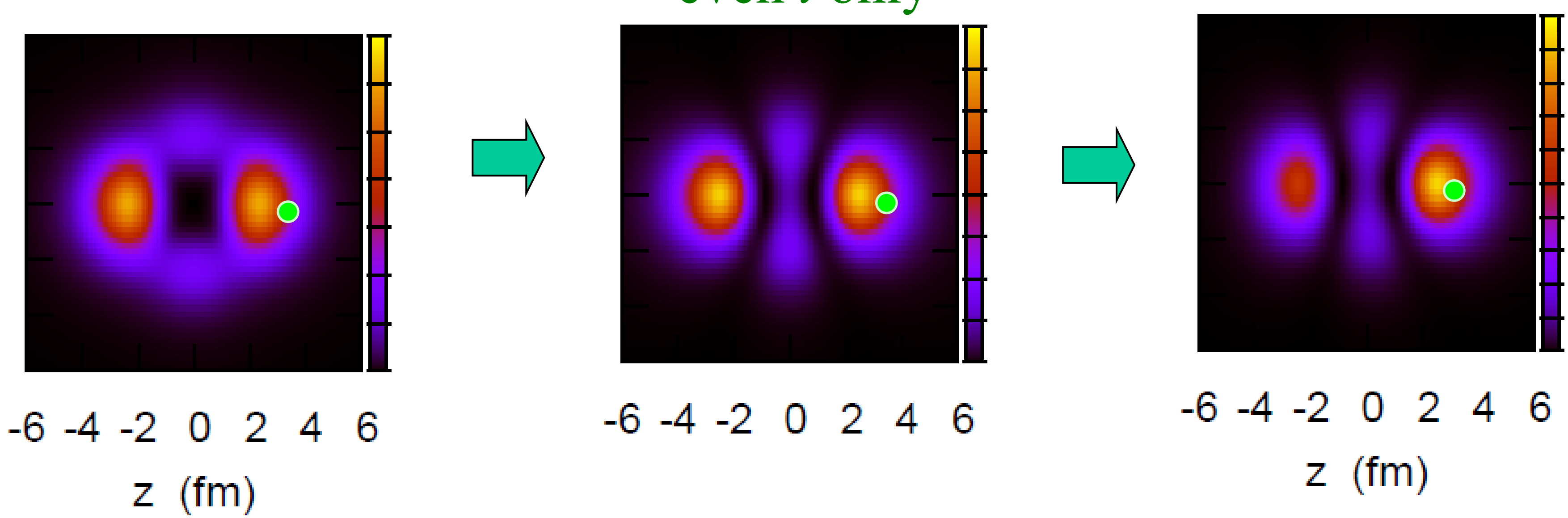
$$\Psi_{\mathrm{g.s.}}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \mathcal{A} \sum_{nn'jl} C_{nn'jl}[\phi_{njl}(\boldsymbol{r}_1)\phi_{n'jl}(\boldsymbol{r}_2)]^{(I=0)}$$

$$\to \rho_2(\boldsymbol{r}) = |\Psi_{\mathrm{g.s.}}(\boldsymbol{r}, \boldsymbol{r}')|^2_{\boldsymbol{r}'=z_0}$$

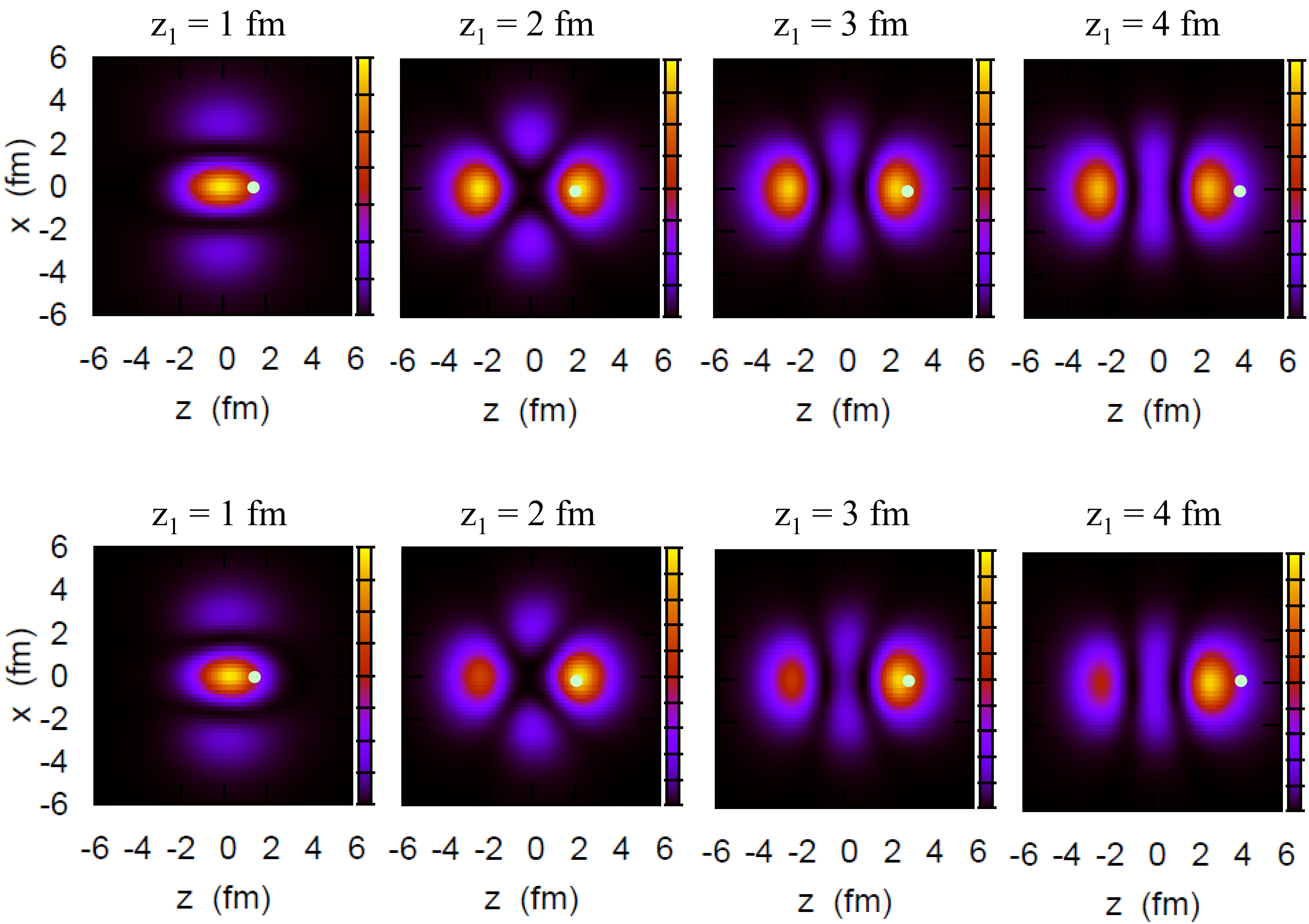
single-*l*

multi-*l*, but even *l* only

multi-*l*, both even and odd *l*

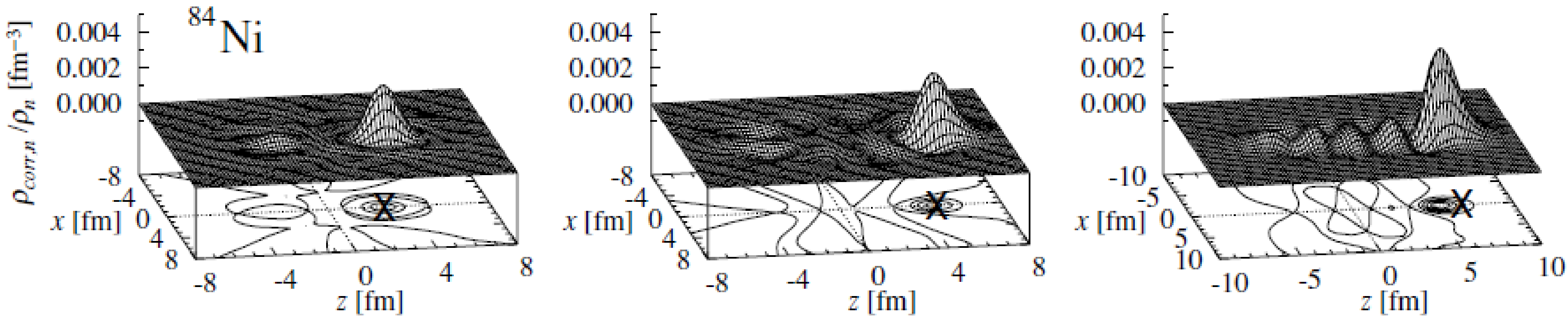


K. Hagino, H. Sagawa, P. Schuck, J. of Phys. G 37, 064040 (2010)

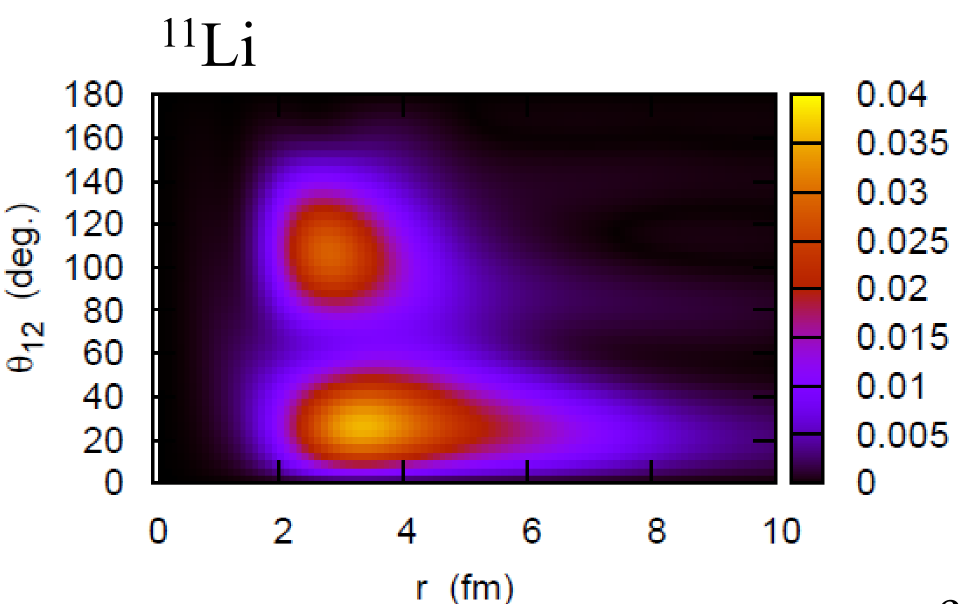


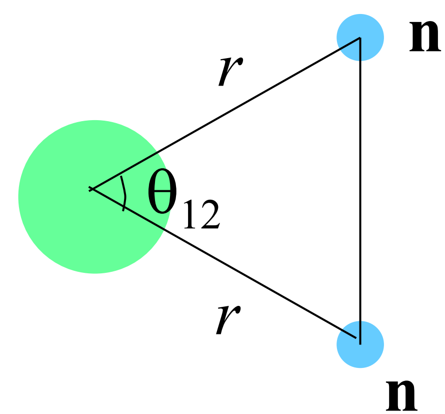
z1 = 1 fm
z1 = 2 fm
z1 = 3 fm
z1 = 4 fm
x (fm)
z (fm)
6
4
2
0
-2
-4
-6

中性子過剰核におけるダイニュートロン相関



M. Matsuo, K. Mizuyama, and Y. Serizawa, PRC71('05)064326
Skyrme HFB



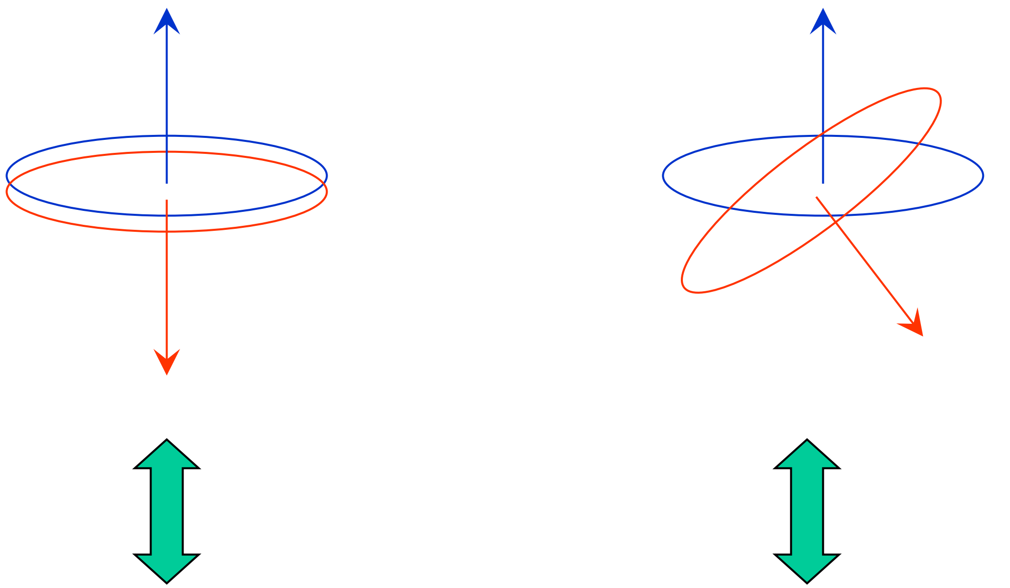


K. Hagino and H. Sagawa,

Phys. Rev. C72 ('05) 044321

cf. Y. Kanada-En'yo, PRC76 ('07) 044323

対相関の本質:

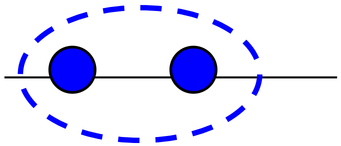


L=0 対　　　　　$L \neq 0$ 対

L=0 対に対して空間的重なりが最大（エネルギー的に得）

“対相関”

対相関（ペアリング）



$$\boxed{|(ll)LM\rangle = \sum_{m,m'} \langle lmlm'|LM\rangle \psi_{lm}(\boldsymbol{r})\psi_{lm'}(\boldsymbol{r}')}$$

$$\Downarrow$$

残留相互作用によるエネルギー変化：

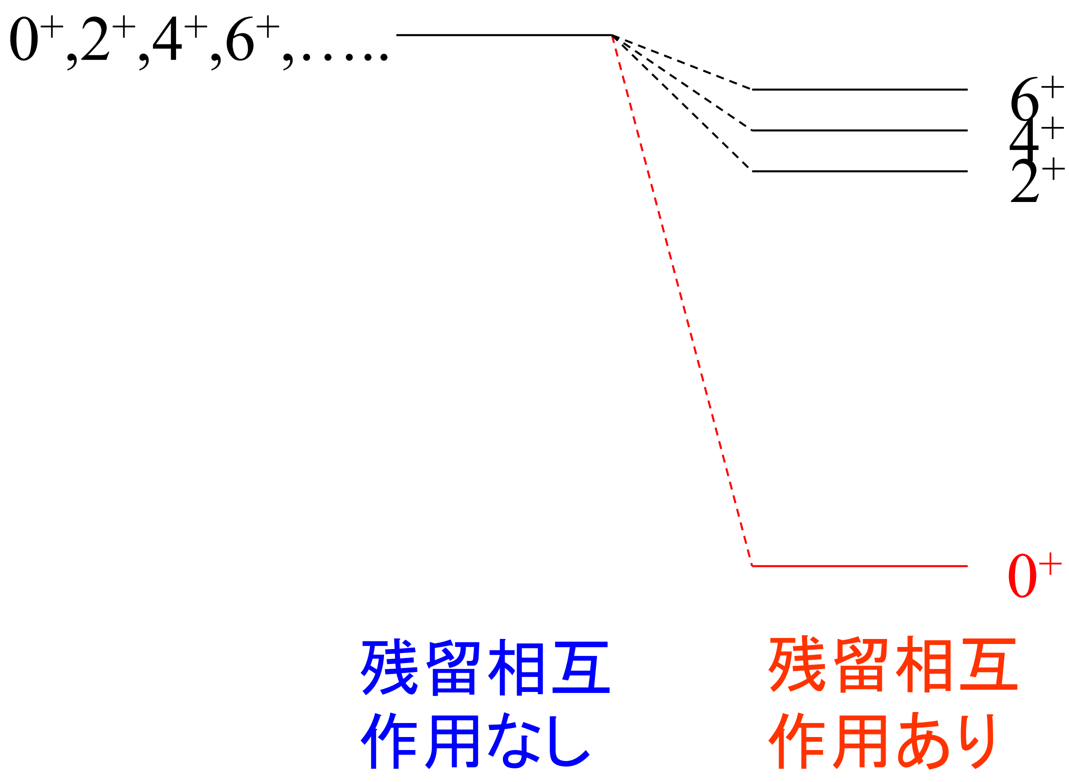
$$v_{\rm res}(\boldsymbol{r},\boldsymbol{r}') \quad \sim -g\,\delta(\boldsymbol{r}-\boldsymbol{r}')$$

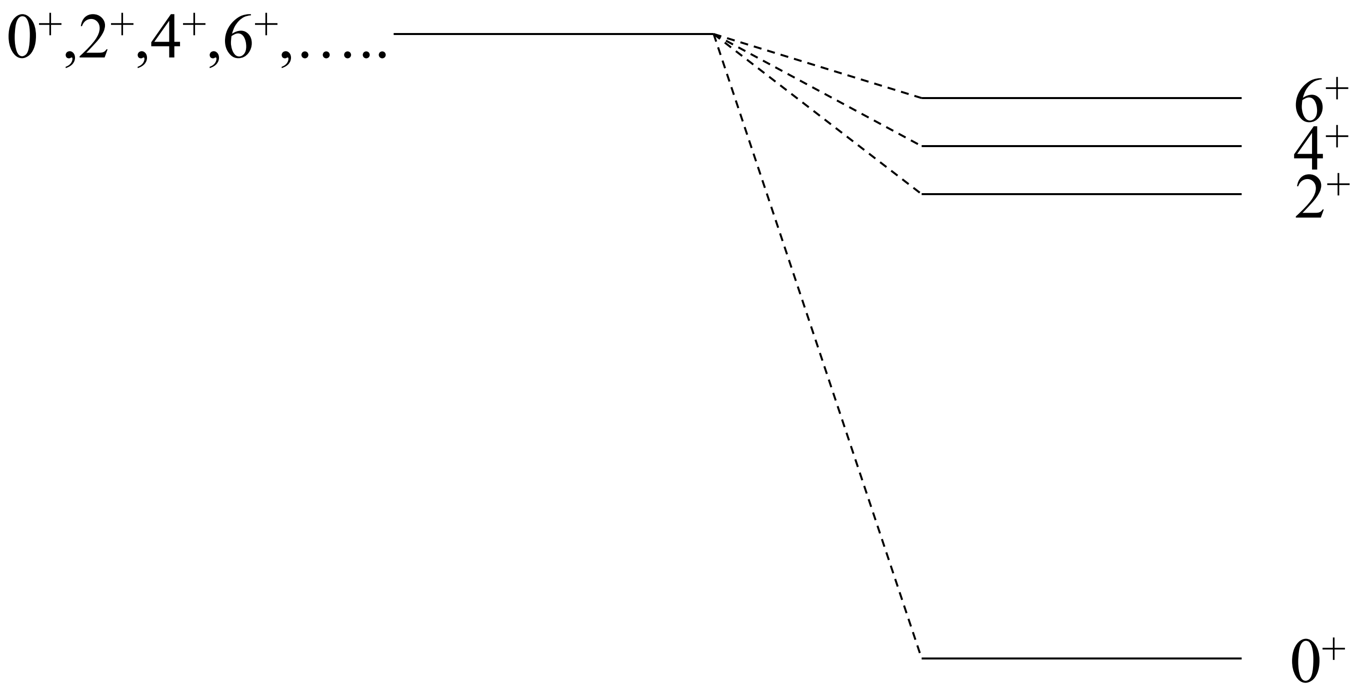
$$\begin{aligned}\Delta E_L &= \langle (ll)LM|v_{\rm res}|(ll)LM\rangle \\ &= -g\,I_r^{(l)}\,\frac{(2l+1)^2}{4\pi}\begin{pmatrix} l & l & L \\ 0 & 0 & 0 \end{pmatrix}^2\end{aligned}$$

$$I_r^{(l)} = \int_0^\infty r^2 dr\,(R_l(r))^4$$

$$\Delta E_L = -g\, I_r^{(l)} \frac{(2l+1)^2}{4\pi} \begin{pmatrix} l & l & L \\ 0 & 0 & 0 \end{pmatrix}^2 \equiv -g\, I_r^{(l)} \frac{A(ll;L)}{4\pi}$$

$A(ll;L)$	$L=0$	$L=2$	$L=4$	$L=6$	$L=8$
$l=2$	5.00	1.43	1.43	---	---
$l=3$	7.00	1.87	1.27	1.63	---
$l=4$	9.00	2.34	1.46	1.26	1.81





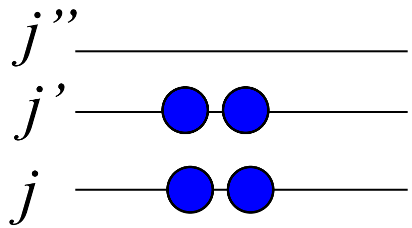
残留相互作用なし

残留相互作用あり

<u>原子核の基底状態のスピン</u>

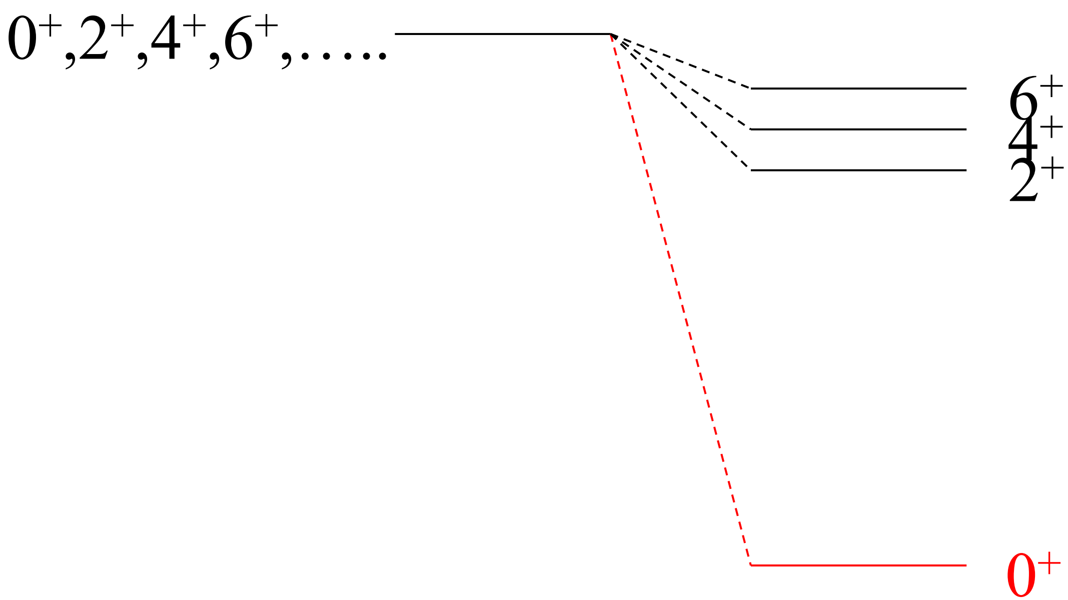
- 偶々核:例外なしに 0^+
- 奇核: 最外殻核子の角運動量と一致

対相関のより高度な理論的記述:BCS近似



複数個のレベルに
複数個のペアがある問題

$$v_{\mathrm{res}}(\boldsymbol{r},\boldsymbol{r}') \quad \sim -g\,\delta(\boldsymbol{r}-\boldsymbol{r}')$$



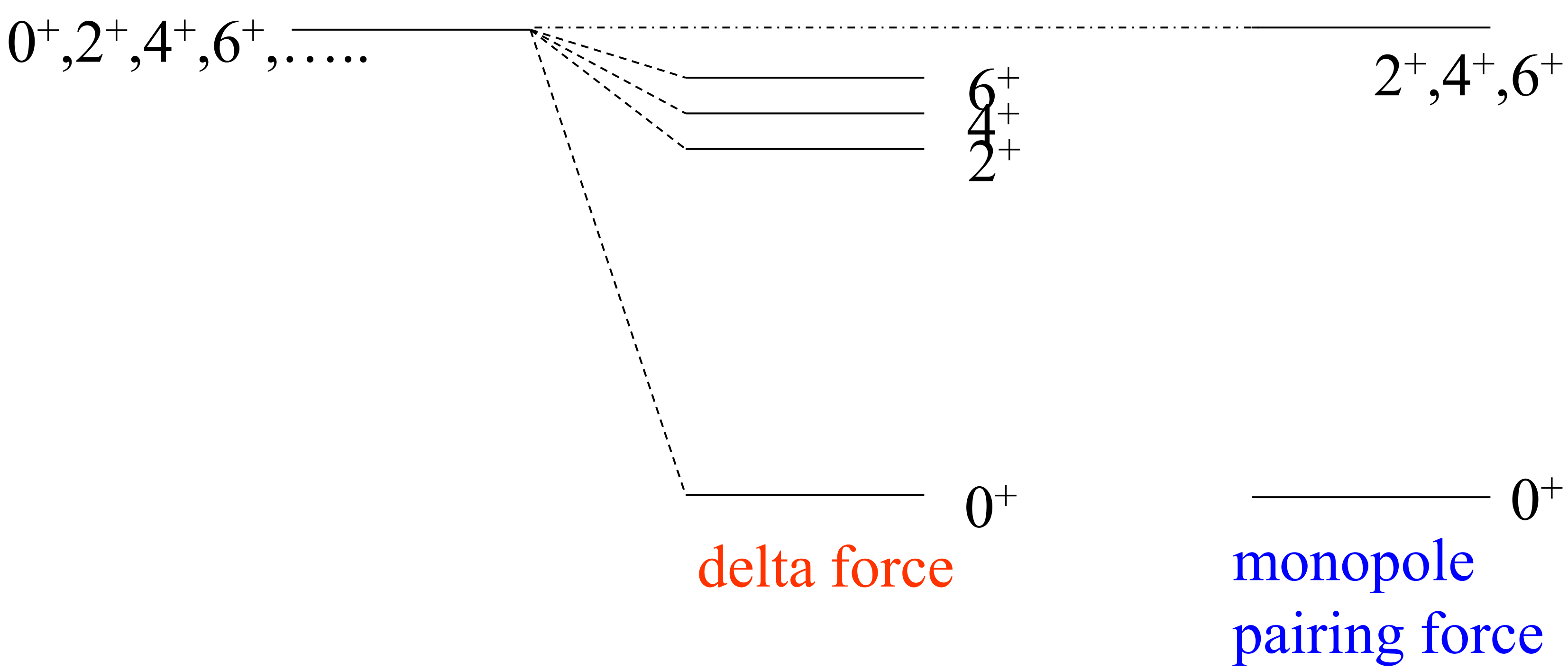
デルタ関数のままでもいいが、説明を簡単にするために
もう少し簡単にした相互作用を導入する。

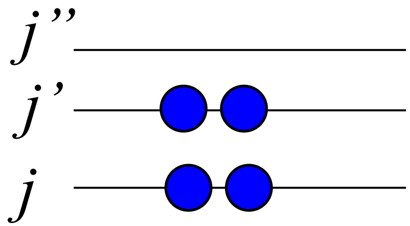
Simplified pairing interaction

$$\boxed{V = -G\,P^\dagger P; \quad P^\dagger = \sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}}}$$

$\bar{\nu}$: ν の時間反転した状態

e.g., $|\nu\rangle = |njlm\rangle, \quad |\bar{\nu}\rangle = |njl-m\rangle$



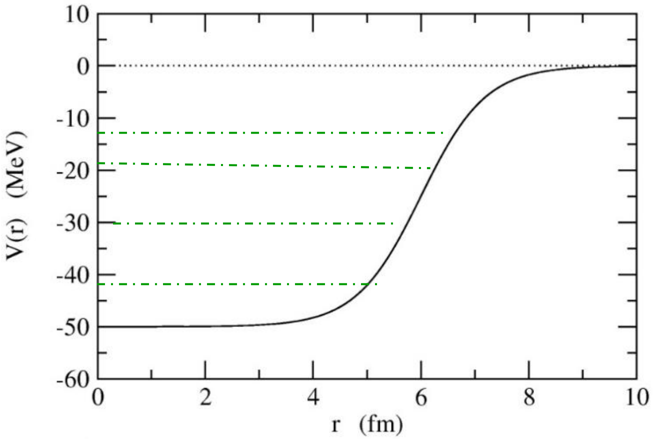


複数個のレベルに
複数個のペアがある問題

BCS理論: もともと、超伝導を説明するために Bardeen, Cooper, Schrieffer によって1957年に定式化された理論
→これを原子核の対相関現象に適用 (Bohr, Mottelson, Pines, 1958)

HF+BCS theory

①平均場近似をして核子の感じるポテンシャルを求める
（平均的な振る舞いをまず決める）



$$H = \sum_k \epsilon_k (a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G \left(\sum_{k>0} a_k^\dagger a_{\bar{k}}^\dagger \right) \left(\sum_{k>0} a_{\bar{k}} a_k \right)$$

②各準位の占有確率を決める。
決め方は、残留相互作用も含めてエネルギーが最小になるようにする。

$$H = \sum_{\nu} \epsilon_\nu (a^\dagger_\nu a_\nu + a^\dagger_{\bar{\nu}} a_{\bar{\nu}}) \underbrace{- G \left(\sum_{\nu>0} a^\dagger_\nu a^\dagger_{\bar{\nu}} \right) \left(\sum_{\nu>0} a_{\bar{\nu}} a_\nu \right)}_{\substack{\text{2体の相互作用} \\ \rightarrow\ \text{1体近似をする}}}$$

cf. HF potential

$$V_H(\boldsymbol{r}) = \int v(\boldsymbol{r}, \boldsymbol{r}') \rho_{\mathrm{HF}}(\boldsymbol{r}') d\boldsymbol{r}$$

Solve the pairing Hamiltonian

$$\boxed{H = \sum_{\nu} \epsilon_\nu (a_\nu^\dagger a_\nu + a_{\bar{\nu}}^\dagger a_{\bar{\nu}}) - G \left(\sum_{\nu>0} a_\nu^\dagger a_{\bar{\nu}}^\dagger \right) \left(\sum_{\nu>0} a_{\bar{\nu}} a_\nu \right)}$$

in the mean-field approximation

- Mean-field approximation:

$$V = -G\, P^\dagger P \to -G \left(\langle P^\dagger \rangle P + P^\dagger \langle P \rangle \right) \equiv -\Delta (P^\dagger + P)$$

$\Longleftrightarrow$ particle number violation

$$\Delta \equiv G\langle P \rangle = G\langle P^\dagger \rangle = G \sum_{\nu>0} \langle a_\nu^\dagger a_{\bar{\nu}}^\dagger \rangle$$

we consider $H' = H - \lambda \hat{N}$ instead of H :

$$
\begin{aligned}
H' &= \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G\hat{P}^\dagger \hat{P} \\
&\to \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta(\hat{P}^\dagger + \hat{P}) \\
&= \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)
\end{aligned}
$$

$$H' = \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G\hat{P}^\dagger \hat{P}$$

$$\rightarrow \sum_{k>0}(\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0}(a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)$$

ハミルトニアン H_k は $|0\rangle,\ |1\rangle = a_k^\dagger a_{\bar{k}}^\dagger |0\rangle$ を基底にして対角化できる

$$H_k \equiv (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta(a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)$$

行列表示をとると: $H_k = \begin{pmatrix} \langle 0|H_k|0\rangle & \langle 0|H_k|1\rangle \\ \langle 1|H_k|0\rangle & \langle 1|H_k|1\rangle \end{pmatrix} = \begin{pmatrix} 0 & -\Delta \\ \Delta & 2(\epsilon_k - \lambda) \end{pmatrix}$

対角化:

$$\begin{pmatrix} 0 & -\Delta \\ \Delta & 2(\epsilon_k - \lambda) \end{pmatrix}\begin{pmatrix} u_k \\ v_k \end{pmatrix} = E\begin{pmatrix} u_k \\ v_k \end{pmatrix}$$

$\rightarrow E = \epsilon_k - \lambda - \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$ （基底状態→小さい方をとる）

$$u_k = \frac{\Delta}{\sqrt{E^2 + \Delta^2}} \rightarrow u_k^2 = \frac{\Delta^2}{E^2 + \Delta^2} = \cdots = \frac{1}{2}\left(1 + \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\right)$$

$$H' = \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - G\hat{P}^\dagger \hat{P}$$
$$\to \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k)$$

もう少しエレガントに解く:

● Transform H' in a form of $H' = \sum_{k>0} E_k(\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$

→ g.s.: $\alpha_k|BCS\rangle = 0$

1st excited state: $|1_k\rangle = \alpha_k^\dagger |BCS\rangle$ at E_k

…. and so on.

Bogoliubov transformation

$$\boxed{\alpha_\nu^\dagger = u_\nu a_\nu^\dagger - v_\nu a_{\bar{\nu}}, \quad \alpha_{\bar{\nu}}^\dagger = u_\nu a_{\bar{\nu}}^\dagger + v_\nu a_\nu}$$

(Quasi-particle operator)

Bogoliubov transformation

$$\boxed{\alpha^\dagger_k = u_k a^\dagger_k - v_k a_{\bar{k}}, \qquad \alpha^\dagger_{\bar{k}} = u_k a^\dagger_{\bar{k}} + v_k a_k}$$

(Quasi-particle operator)

$$\begin{pmatrix} \alpha^\dagger_k \\ \alpha_{\bar{k}} \end{pmatrix} = \begin{pmatrix} u_k & -v_k \\ v_k & u_k \end{pmatrix} \begin{pmatrix} a^\dagger_k \\ a_{\bar{k}} \end{pmatrix}$$

(note) $\{\alpha_\nu, \alpha_{\nu'}\} = 0, \quad \{\alpha_\nu, \alpha^\dagger_{\nu'}\} = \delta_{\nu,\nu'}$ を要請

$$\rightarrow \quad u_\nu^2 + v_\nu^2 = 1$$

$$\begin{pmatrix} a^\dagger_k \\ a_{\bar{k}} \end{pmatrix} = \begin{pmatrix} u_k & -v_k \\ v_k & u_k \end{pmatrix}^{-1} \begin{pmatrix} \alpha^\dagger_k \\ \alpha_{\bar{k}} \end{pmatrix} = \begin{pmatrix} u_k & v_k \\ -v_k & u_k \end{pmatrix} \begin{pmatrix} \alpha^\dagger_k \\ \alpha_{\bar{k}} \end{pmatrix}$$

Bogoliubov transformation

$$\boxed{\alpha_k^\dagger = u_k a_k^\dagger - v_k a_{\bar{k}}, \qquad \alpha_{\bar{k}}^\dagger = u_k a_{\bar{k}}^\dagger + v_k a_k}$$

(Quasi-particle operator)

or $$a_k^\dagger = u_k \alpha_k^\dagger + v_k \alpha_{\bar{k}}, \qquad a_{\bar{k}}^\dagger = u_k \alpha_{\bar{k}}^\dagger - v_k \alpha_k$$

$$\begin{aligned} H' &= \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \\ &\rightarrow \end{aligned}$$

using the quasi-particle operators:

$$
\begin{aligned}
H' &\sim \sum_{k>0} (\epsilon_k - \lambda)(a_k^\dagger a_k + a_{\bar{k}}^\dagger a_{\bar{k}}) - \Delta \sum_{k>0} (a_k^\dagger a_{\bar{k}}^\dagger + a_{\bar{k}} a_k) \\
&= \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\
&\quad + \sum_{k>0} [2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)
\end{aligned}
$$

if $\quad 2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2) = 0$

then $\quad H' = \sum_{k>0} E_k (\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$

with $\quad E_k = (\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k$

$$H' = \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$
$$+ \sum_{k>0} [2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)$$

$$\left\{ \begin{array}{lcl} 0 & = & 2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2) \\ 1 & = & u_k^2 + v_k^2 \end{array} \right.$$

$$\boxed{\begin{array}{lcl} u_k^2 & = & \frac{1}{2}\left(1 + \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\right) \\ v_k^2 & = & \frac{1}{2}\left(1 - \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\right) \end{array}}$$

$$
\begin{aligned}
H' \;=\; & \sum_{k>0} [(\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k](\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}}) \\
& + \sum_{k>0} [2(\epsilon_k - \lambda) u_k v_k - \Delta(u_k^2 - v_k^2)](\alpha_k^\dagger \alpha_{\bar{k}}^\dagger + \alpha_{\bar{k}} \alpha_k)
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
u_k^2 &= \frac{1}{2}\left(1 + \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\right) \\
v_k^2 &= \frac{1}{2}\left(1 - \frac{\epsilon_k - \lambda}{\sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}}\right)
\end{aligned}
}
$$

$$
\begin{aligned}
E_k &= (\epsilon_k - \lambda)(u_k^2 - v_k^2) + 2\Delta u_k v_k \\
&= \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}
\end{aligned}
$$

$$H' = \sum_{k>0} E_k(\alpha_k^\dagger \alpha_k + \alpha_{\bar{k}}^\dagger \alpha_{\bar{k}})$$

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

Ground state wave function:

$$\alpha_k |BCS\rangle = 0$$

$$|BCS\rangle \propto \prod_{k>0} \alpha_k \alpha_{\bar{k}} |0\rangle$$

$$= \prod_{k>0} v_k \left(u_k + v_k\, a_k^\dagger a_{\bar{k}}^\dagger\right) \Big| 0 \Big\rangle$$

$$\boxed{|BCS\rangle = \prod_{k>0} \left(u_k + v_k\, a_k^\dagger a_{\bar{k}}^\dagger\right) \Big| 0 \Big\rangle}$$

Ground state wave function:

$$\alpha_k|BCS\rangle = 0$$

$$\boxed{|BCS\rangle = \prod_{k>0}\left(u_k + v_k\, a_k^\dagger a_{\bar{k}}^\dagger\right)\Big|\,0\Big\rangle}$$

(note) $\left\langle BCS|a_k^\dagger a_k|BCS\right\rangle = |v_k|^2$: occupation probability

(note)

$$E'_{\rm BCS} = \langle BCS|H'|BCS\rangle \sim 2\sum_{k>0}(\epsilon_k - \lambda)v_k^2 - \frac{\Delta^2}{G}$$

Ground state wave function:

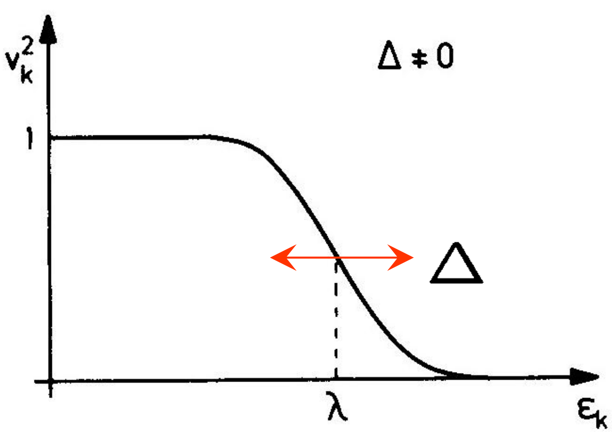
$$\alpha_k|BCS\rangle = 0$$

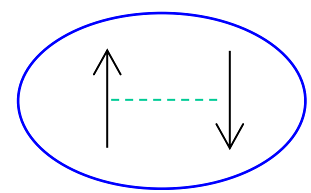
$$\boxed{|BCS\rangle = \prod_{k>0}\left(u_k + v_k\, a_k^\dagger a_{\bar{k}}^\dagger\right)\Big|0\Big\rangle}$$

(note) $$\left(1 + \frac{v_k}{u_k}\, a_k^\dagger a_{\bar{k}}^\dagger\right)\Big|0\Big\rangle = \exp\left(\frac{v_k}{u_k}\, a_k^\dagger a_{\bar{k}}^\dagger\right)\Big|0\Big\rangle$$

$$|\Psi\rangle \propto \exp\left(\sum_{k>0}\frac{v_k}{u_k}\, a_k^\dagger a_{\bar{k}}^\dagger\right)\Big|0\Big\rangle$$

(pair condensed wave function)





クーパー対の凝縮

Gap equation

$$\begin{cases} u_k^2 = \dfrac{1}{2}\left(1 + \dfrac{\epsilon_k - \lambda}{E_k}\right) \\ v_k^2 = \dfrac{1}{2}\left(1 - \dfrac{\epsilon_k - \lambda}{E_k}\right) \end{cases}$$

$$E_k = \sqrt{(\epsilon_k - \lambda)^2 + \Delta^2}$$

$$\boxed{\begin{aligned} \Delta &= G\langle BCS|\hat{P}|BCS\rangle = G\sum_{k>0} u_k v_k \\ &= \frac{G}{2}\sum_{k>0}\frac{\Delta}{E_k} \end{aligned}}$$

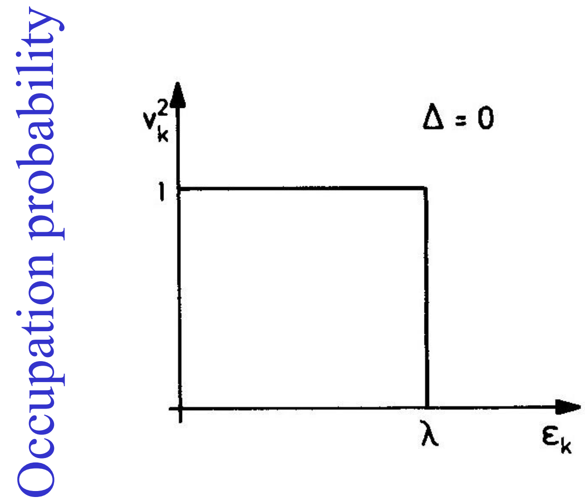
(Gap equation)

$$N = 2\sum_{k>0} v_k^2 \qquad \leftarrow \lambda$$

$$\Delta = \frac{G}{2} \sum_{k>0} \frac{\Delta}{E_k}$$

i) Trivial solution: always exists

$$\Delta = 0$$



$$\Delta = G \sum_{k>0} u_k v_k$$

$$\begin{aligned} v_k^2 &= 1 \quad (\epsilon_k \leq \lambda) \\ &= 0 \quad (\epsilon_k > \lambda) \end{aligned}$$

$$|\Psi\rangle = \prod_{k>0} a_k^\dagger a_{\bar{k}}^\dagger |0\rangle$$

$$\Delta = \frac{G}{2}\sum_{k>0}\frac{\Delta}{E_k}$$

i) Trivial solution: always exists

$$\Delta = 0$$

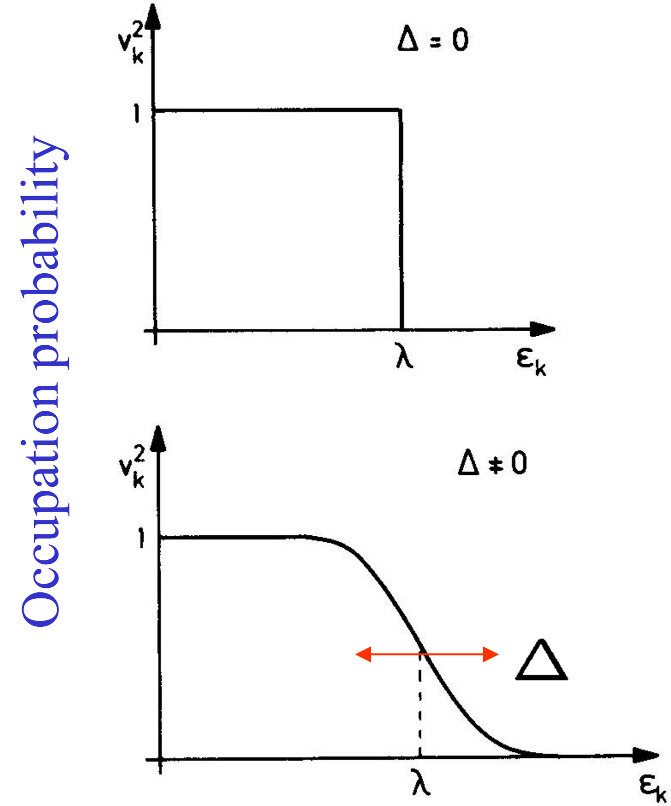
G a/o $N \longrightarrow$ large

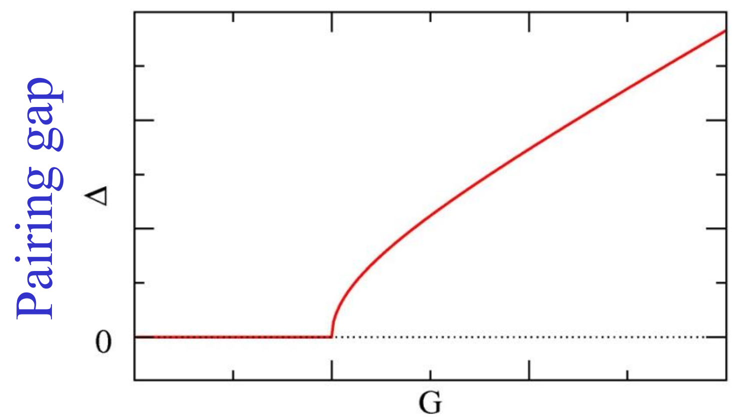
ii) Superfluid solution

$$\Delta \neq 0$$

$$v_k < 1$$

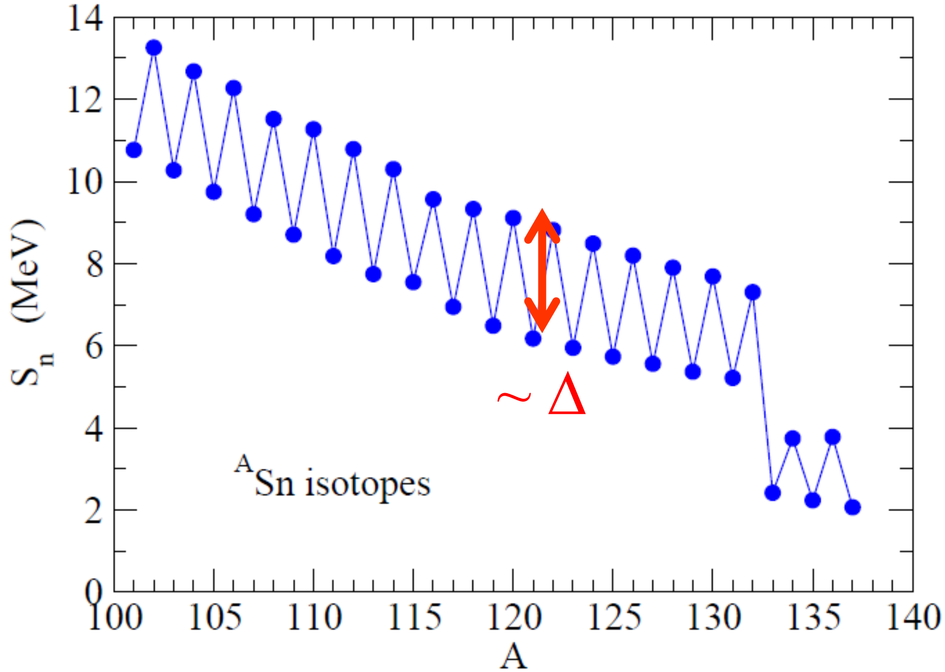
$$|BCS\rangle = \prod_{k>0}\left(u_k + v_k\, a_k^\dagger a_{\bar{k}}^\dagger\right)\big|0\rangle$$

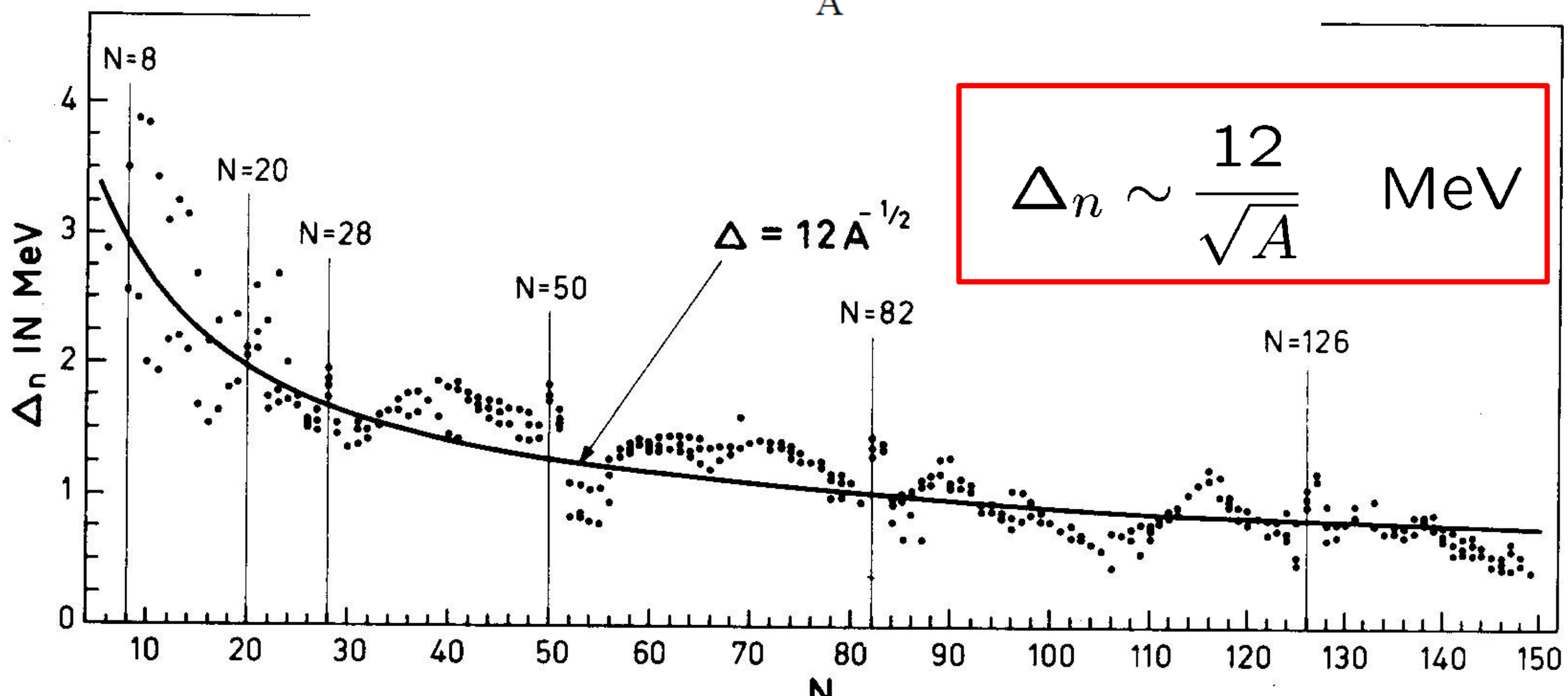




Number fluctuation

Normal-Superfulid phase transition





$$\boxed{\Delta_n \sim \frac{12}{\sqrt{A}} \quad \text{MeV}}$$

粒子数非保存と粒子数射影法

$$|BCS\rangle = \prod_{k>0} \left(u_k + v_k\, a^\dagger_k a^\dagger_{\bar{k}}\right)\Big|\,0\Big\rangle$$

様々な粒子数の状態が混ざっている $|BCS\rangle = \sum_{N_k} C_{N_k}|N_k\rangle$

ただし、平均値だけは正しく設定されている:

$$\langle BCS|\hat{N}|BCS\rangle = 2\sum_{k>0} v_k^2 = N$$

粒子数のゆらぎの度合い:

$$(\Delta N)^2 = \langle BCS|\hat{N}^2|BCS\rangle - N^2 = 4\sum_{k>0} u_k^2 v_k^2$$

粒子数非保存と粒子数射影法

$$|BCS\rangle = \prod_{k>0}\left(u_k + v_k\, a^\dagger_k a^\dagger_{\bar{k}}\right)\Big|0\Big\rangle$$

様々な粒子数の状態が混ざっている $|BCS\rangle = \sum_{N_k} C_{N_k}|N_k\rangle$

粒子数射影: $\hat{P}_N|BCS\rangle = C_N|N\rangle$

$$\hat{P}_N = \frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{i(\hat{N}-N)\phi}$$

(note)

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{i(N'-N)\phi} = \delta_{N,N'}$$

粒子数非保存と粒子数射影法

ハミルトニアンは粒子数を保存:

$$[H,\hat{N}]=0\to U^\dagger(\phi)HU(\phi)=H;\quad U(\phi)=e^{i\phi\hat{N}}$$

U(1) 対称性

BCS状態は U(1) 対称性が自発的に破れた状態

$$|BCS\rangle=\prod_{k>0}\left(u_k+v_k\,a_k^\dagger a_{\bar{k}}^\dagger\right)\Big|0\Big\rangle$$

$$|BCS(\phi)\rangle\equiv U^\dagger(\phi)|BCS\rangle=\prod_{k>0}\left(u_k+v_k\,e^{-2i\phi}\,a_k^\dagger a_{\bar{k}}^\dagger\right)\Big|0\Big\rangle$$

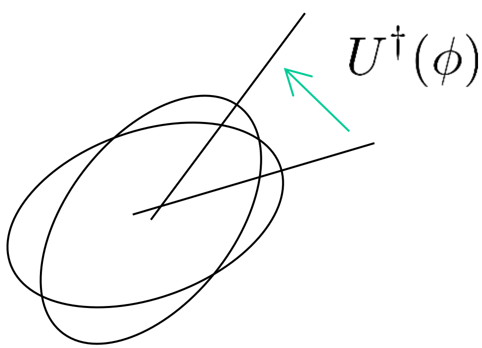
ゲージ空間で「変形」している状態

粒子数非保存と粒子数射影法

$$[H,\hat{N}]=0 \rightarrow U^\dagger(\phi)HU(\phi)=H;\quad U(\phi)=e^{i\phi\hat{N}}$$

BCS状態は U(1) 対称性が自発的に破れた状態

$$|BCS(\phi)\rangle \equiv U^\dagger(\phi)|BCS\rangle=\prod_{k>0}\left(u_k+v_k\,e^{-2i\phi}\,a_k^\dagger a_{\bar{k}}^\dagger\right)\Big|\,0\Big\rangle$$



ちょうど角運動量と角度の関係のようなもの：　$\hat{L}_z=\frac{\hbar}{i}\frac{\partial}{\partial\phi}$

（この場合、角度 φ はゲージ角）　$\longrightarrow$　$\hat{N}=\frac{\hbar}{i}\frac{\partial}{\partial\phi}$

レポート問題8 : 〆切 7/28（火） 23:59 までKULMSから提出

$$\widehat{P}_N = \frac{1}{2\pi}\int_0^{2\pi} d\phi\, e^{i(\widehat{N}-N)\phi}$$

が粒子数Nに対する射影演算子になっていることを示せ。